

# A Bond Graphs approach to Physical Modelling of Musical Instruments

**Andrew McGregor<sup>(2)</sup> Eduardo Miranda<sup>(3,4)</sup> Peter  
Gawthrop<sup>(1)</sup>**

(1) Centre for Control Systems, University of Glasgow, UK,  
peterg@eng.gla.ac.uk

(2) Gonville and Caius College, University of Cambridge, UK,  
aism2@cam.ac.uk

(3) Sony Computer Science Laboratory - Paris, France,  
miranda@csl.sony.fr

(4) Lamesm, Universidade Federal de Santa Maria, Brazil,  
lamesm@geocities.com

## Abstract

In this paper we introduce the use of bond graphs for the implementation of classic physical models of musical instruments. Bond graphs are a diagram-based technique for representing physical systems, such as an electrical circuit or a mechanical engine. They provide a precise and unambiguous modelling tool which allows for the specification of hierarchical physical structures. The paper begins with an introduction to the bond graphs technique, followed by an example implementation of the clarinet. The paper concludes with final remarks and future developments.

## 1 INTRODUCTION TO BOND GRAPHS

The classic approach to physical modelling of musical instruments attempts to emulate the behaviour of vibrating media using a network of interconnected mechanical units, called mass and spring. On a computer, this network is implemented as a set of differential equations, whose solution describes the waves produced by the model in operation. Sound samples result from the computation of these equations.

Bond graphs are a diagram-based method, first introduced in the late 1950s, for representing physical systems (Paynter 1961; Gawthrop and Smith 1996). It consists of a standard set of elements connected

together in a structure representative of the physical system being modelled. The connections between the elements are referred to as bonds, hence the name bond graphs. On a basic level, bond graphs modelling is an extension and a refinement of the premise that almost every physical system, be it acoustic, mechanic, hydraulic, thermodynamic or magnetic, has an electrical counterpart. For example, the electric circuit in Figure 1 is equivalent to the mechanic mass-spring-damper (MSD) system portrayed in Figure 2. In this analogy, force in the MSD is the equivalent of voltage (or potential difference)  $p$  over the capacitor  $C$ , and the velocity is the equivalent of the current  $i$  flowing in the electric circuit. These equivalencies follow from the fact that there is a common factor in all types of physical systems, namely energy. Energy always tends to dissipate and thus will flow to anywhere there is presently less energy. For example, a warm room will not cool if all the adjacent rooms are equally warm, whereas it will cool if adjacent rooms are cooler. When there is a flow of energy, there will always be a rate of this flow and a difference between the energy levels of the place the energy moves from and the place the energy moves to. In bonds graph parlance, this difference is referred to as the effort. In this case, the greater the effort, the greater the flow. In an electrical circuit, for example, the rate of flow is referred to as the current and the effort as the voltage (or potential difference); in a mechanical system this rate would be referred to as the velocity and the effort as the force.

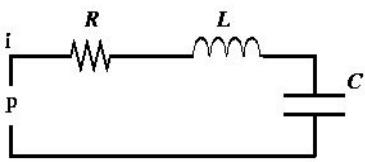


Figure 1: An electrical model.  
spring-damper.

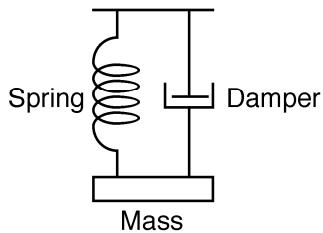


Figure 2: A simple mass-

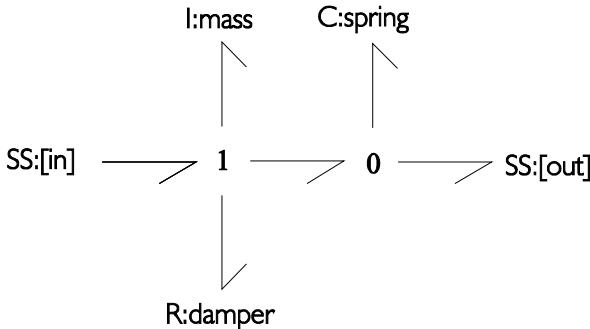


Figure 3: The corresponding bond graph of the MSD shown in Figure 2.

A bond graphs model is composed of one or a combination of the following basic elements:

- Energy Source: this can either be a source of flow or a source of effort; for example, a pressure imposed on the mouth piece of a clarinet or gravity acting on a falling apple.
- Energy Store: this can either store flow (e.g. a spring or a capacitor) or effort (e.g. mass). The accumulation of either flow or effort variables gives the system its state. The consequence of this is that the system's reaction to a certain input is dependent on the system's previous behaviour.
- Energy Dissipater: this is an element that simply dumps energy out of the system. For example, resistors and friction; both dissipate energy in the form of heat.
- Energy Transfer: these are elements that are responsible for transferring energy within the system. An energy transfer element can be either a junction, a transformer or a gyrator.

There are two types of junctions: the effort junction and the flow junction. In an effort junction (or '0' junction), the effort on the bond coming into the junction is equal to the effort in each of the bonds coming out of the junction (Figure 4). In a flow junction (or '1' junction) the flow on the bond coming into the junction is equal to the flow in each of the bonds coming out of the junction (Figure 5).

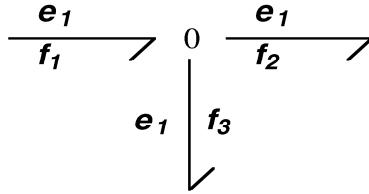


Figure 4: A common effort junctions where  $e_1f_1 = e_1f_2 + e_1f_3$ .

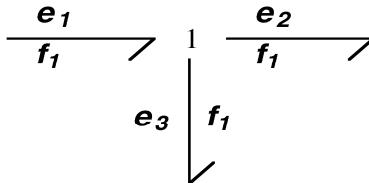


Figure 5: A common flow junction where  $e_1f_1 = e_2f_1 + e_3f_1$ .

Although Figures 4 and 5 portray only examples using 3-port junctions, the reader should bear in mind that a junction can, in principle, have any number of ports. Transformers and gyrators, however, have only two ports (Figure 6).



Figure 6: The transformer element.

The transformer (TR) specifies an output effort  $e_2$  using the relationship  $e_2 = ke_1$ , where  $k$  is the transformer ratio. The value of  $f_2$  is then forced to be  $f_1/k$  so that  $e_1f_1 = e_2f_2$ . The gyrator (GY) is similar to the transformer, except that it specifies the output flow  $f_2$  using the relationship  $f_2 = ge_1$ , where  $g$  is referred to as the mutual inductance.

Bond graphs are highly modular, lending themselves to the modelling of complex systems that can be decomposed into simpler subsystems at various levels. Within this context, the model of a musical instrument such as the clarinet can be specified by decomposing it into functional parts (e.g. mouth piece, pipe, etc.), each of which is then modelled separately. The bond graphs that encapsulate each part of the

instrument is then considered as a new bond graphs component that can be called by any other high- level graph (Figure 8). The bond graphs technique is appealing because it allows for the specification of models that visually resemble the structure of the mechanisms of the instruments they represent. Here, the system automatically produces the respective differential equations that govern the states of the model (i.e. the values for each energy store) and the output of the model.

## 2 AN EXAMPLE STUDY: A BOND GRAPHS MODEL OF THE CLARINET

### 2.1 Understanding the Mechanism of the Clarinet

When a musician blows the clarinet, the pressure in the mouth ( $P_o$ ) becomes greater than the pressure in the mouthpiece ( $P_m$ ), resulting in a flow of air through the aperture of the reed x (Figure 7). The aperture of the reed tends to decrease because the reed starts to close as soon as the pressure begins. While this is happening, a pressure wave is released into the pipe of the instrument. As the pressure wave travels down the pipe, the reed vibrates due to the stiffness of the reed, the effect of the flow of air  $U_f$  and the inertia. The effect of inertia dictates that the movement of the reed will overshoot a position of equilibrium, that is, the position where the two forces would balance. The frequency of this initial vibration depends upon the stiffness of the reed and a function of  $U_f$  and when the pressure waves reach the end of the pipe, they attempt to adjust themselves to the atmospheric pressure. The air on which the pressure waves are carried have mass and therefore an inertia when this equalisation takes place. Similar to the reed itself, this inertia causes the equalisation to overshoot and the result is that sections that were previously high pressure are now low pressure and vice- versa; that is, the wave has been inverted. This inverted wave propagates itself from the end of the pipe of the instrument in all directions, including back up the pipe. The resulting superimposition of the original pressure waves travelling down the pipe and the inverted reflected waves travelling back to the pipe originates nodes (areas of alternation between high and low pressure) and antinodes (areas that stay at average pressure, typically atmospheric). A node cannot exist at the end of the clarinet bore or adjacent to a tone hole. Using this knowledge, it is possible to have a control over the fundamental frequency (i.e. pitch) of the sound produced.

As the inverted reflected waves reach the mouthpiece of the clarinet, they cause the reed to open (when a high pressure section of the wave reaches the reed) or close (when a low pressure section of the wave reaches the reed). Since the resulting pressure wave emitted when the reed has been 'forcibly' opened or closed is larger than it otherwise would be, the period of the reed becomes an exact divisor of the time it takes for a pressure wave to travel the length of the tube and back again. In this case, the pipe of the clarinet will tend to resonate only those partials that set up an antinode at the open end of the instrument; these normally are odd partials. It is this interaction between the pipe and the reed that form the basis of the sound production of the clarinet (Rossing 1990; Campbell and Greated 1987).

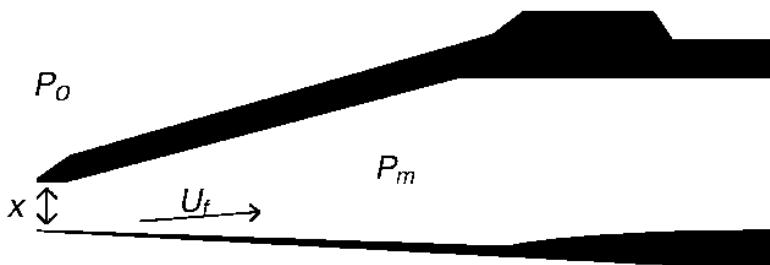


Figure 7: The mouthpiece of the clarinet.

## 2.2 The Clarinet Model

At the highest level, the clarinet model consists of a source-sensor SS component referred to as the player, that exerts an effort (i.e. pressure) on the reed component which is connected to the tube (or pipe) component. The tube imposes a pressure on a second SS component referred to as the listener (Figure 8). As the SS component is predefined by default (Gawthrop 1995), the system already knows how it works, but the reed and the tube components need to be created.

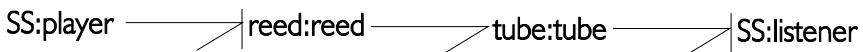


Figure 8: The implementation of the clarinet model at its highest level.

### 2.2.1 The reed Component

The reed component broadly follows the physical description given by de Bruin and van Walstijn (1995). However, rather than writing the model as a set of equations, it is graphically described by the bond graph of Figure 11. The reed component consists of two sub-models: one for the actual reed and one for the air interacting with the reed. As discussed below, these are coupled together to the input and output of the reed component.

The input and output of the reed component can be thought of as the pressure applied to the mouthpiece from the player and the pressure in the interior of the mouthpiece. These correspond to the two ports of the reed component of the clarinet model and are specified as SS components. As the flow of air will be the same at both SS components, they are linked by a common flow function, that is, a '1' junction, in bond graphs parlance. As explained earlier, the effort carried by the bond entering the junction will equal the sum of the efforts carried by the bonds exiting the junction. This means that it is the difference in pressure between the mouthpiece exterior and interior that drives the movement of the reed.

The reed is modelled as a MSD system (Figure 3): the mass, spring and dumper components are labelled  $m_r$ ,  $c_r$  and  $d_r$ , respectively, and they share the common velocity of the '1' junction with that of the air immediately adjacent to the reed associated with the bond attached to the TF component labelled  $S_r$ . This TF component transforms the volumetric flow of air  $U_f$  associated with the reed motion to the corresponding velocity  $x$  ( $x$  is a time derivative).

This '1' junction and the TF component ensure the correct kinematics relationship between the MSD components and the additional volume flow due to reed motion. Moreover, and this is a key attribute of the bond graphs technique, the fact that the '1' junction and the TF are energy-conserving automatically gives the correct relationship between the corresponding force and pressure. This is a great help in obtaining correct models. Similarly, the '0' junction at the other side of the TF

component ensures the correct pressure and flow relationship between the reed and the mouthpiece.

The air sub- model has two components: the inertia of the air above the reed and the (non- linear) air flow resistance labelled  $I_r$  and  $R_r$ , respectively. The former is the straightforward bond graphs I component (mass) considered earlier, but the other component is more complex, as the flow resistance is modulated by the reed position. Thus  $R_r$  is not the simple R component (dumper) but rather an Effort- Modulated Resistor (EMR) component (Figure 9) which is modulated by the displacement of  $x$  of the reed; please refer to book by Gawthrop and Smith (1990) for more information about bond graphs components. The fact that displacement appears as an effort in this context is an artefact of the way we have chosen to model the system. The constitutive relationship embedded in this EMR component follows that of de Bruin and van Walstijn (1995).

At this point, it is important to include a significant feature of the model: the reed *displacement*  $x > x_0$  ( $x_0$  is a constant) as its motion is constrained by the mouthpiece. This is achieved by using a Switched C (CSW) component in place of the standard bond graphs C component (spring). The CSW component (Gawthrop 1995) is an extension of the standard bond graphs C component that allows for discontinuous (or 'switching') behaviour (Figure 10). In this context, it is equivalent to a constrained spring which is not allowed to move beyond a certain point.

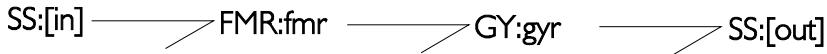


Figure 9: The EMR component.

## C:MTT\_SWITCH



Figure 10: The CSW component.

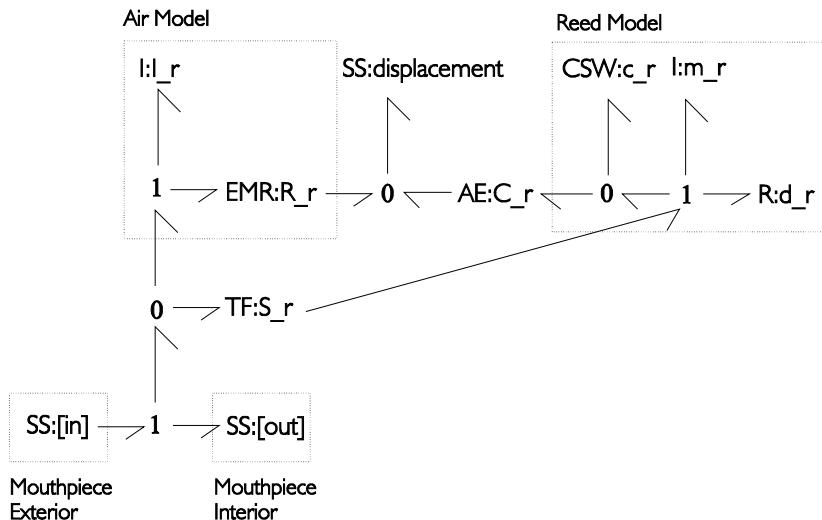


Figure 11: The reed component.

In this model, it is assumed that the reed is driven only by the air pressure, rather than by the air velocity. This appears in the bond graphs model via the effort amplifier (AE) component which prevents direct air velocity/reed interaction. A more complex model could indeed include such as interaction but that is beyond the scope of this example.

### 2.2.2 The tube Component

The tube component is a much simpler model than the reed. As a cross section of air in a pipe has an elasticity and mass, and it loses energy in the form of friction when it moves, the model simply consists of a series of MSDs and TFs (Figure 12).

This can be implemented as a lumped approximation to the partial differential equations describing the motion of the air column. For the sake of simplicity, this example does not include the effect of the air holes, dynamically opening and closing; this could be done simply by adding the appropriate bond graphs components to the model.

### 3 CONCLUSION

In this paper we demonstrated how bond graphs can be successfully employed to model musical instruments. They provide a precise and unambiguous modelling tool which allows for the specification of hierarchical physical structures. Complex models can be created by conceptually decomposing the instrument into simpler parts and then specifying the corresponding bond graphs for each part.

The bond graphs models are capable of producing realistic emulation of the acoustic counterparts. Due to the highly modular and hierarchical nature of the bond graphs technique, we plan to incorporate a bond graphs-based synthesis engine to ARTIST, an Artificial Intelligence-based sound design system previously presented in these symposia (Miranda 1994; 1996) and elsewhere (Miranda 1995; 1997; 1998a). However, we still need to establish an evaluation framework to compare this technique with other musical physical modelling methods available (e.g., Smith 1992).

As far as our research aims are concerned, the main advantage of the bond graphs method is that it has been designed as a general tool for physical modelling, not necessarily limited to musical instruments (e.g., researchers at Glasgow University are currently using bond graphs to model aircraft's aerodynamics). This is an important factor to consider, as we intend to explore the potential of ARTIST's cognitive engines for other domains; e.g., autonomous robotics (Steels and Vogt 1997).

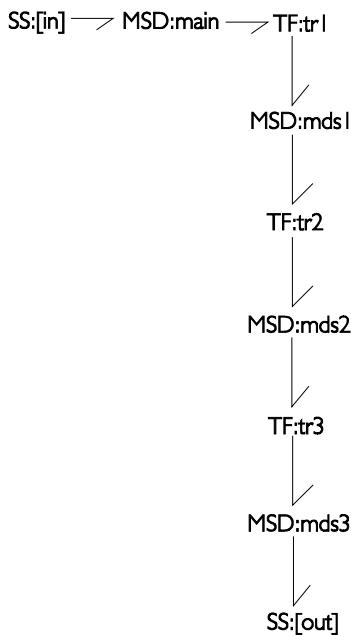


Figure 12: The tube component.

## REFERENCES

- Campbell, M. and Greated, C. 1987. *The Musician's Guide to Acoustics*, London: J.M. Dent & Sons Ltd.
- de Bruin, G. and van Walstijn, M. 1995. "Physical Models of Wind Instruments: A Generalised Excitation Coupled with a Modular Tube Simulation Platform". *Journal of New Music Research*, 24 (1995):148- 163.
- Gawthrop, P. J. 1995. "Physical Model- Based Control: A Bond Graph Approach ". *Journal of the Franklin Institute*, 332B(3):285- 305.
- Gawthrop, P. J. and Smith, L. P. S. 1996. *Metamodeling: Bond Graphs and Dynamic Systems*. Hemel Hempstead: Prentice Hall.

- Miranda, E. R., 1994. "ARTIST:an AI-based Tool for the Design of Intelligent Assistants for Sound Synthesis". *Proceedings of I SBC&M*, pp. 203- 216, Caxambú (Brazil).
- Miranda, E. R. 1995. "An Artificial Intelligence Approach to Sound Design ". *Computer Music Journal*, 19(2):59- 75.
- Miranda, E. R., 1996, "Machine Learning of Sound Attributes: Computer- Assistance in Concept Formation and Musical Invention", *Proceedings of III SBC&M*, pp. 8- 11, Recife (Brazil).
- Miranda, E. R. 1997. "Machine Learning and Sound Design: A Case Study". *Leonardo Music Journal*, 7:49- 55.
- Miranda, E. R. 1998a. "Striking the right note with ARTIST: an AI-based synthesiser". *Recherches et applications en informatique musicale*. Chemillier, M. and Pachet, F. (editors). Paris: Editions Hermes, pp. 227- 239.
- Miranda, E. R. 1998b. *Computer Sound Synthesis for the Electronic Musician*. Oxford (UK): Focal Press.
- Paynter, H. M. 1961. *Analysis and Design of Engineering Systems*. Cambridge (MA): MIT Press.
- Rossing, T. 1990. *The Science of Sound* (2<sup>nd</sup> ed.), Reading (MA): Addison Wesley.
- Smith, J. 1992. "Physical Modeling using Digital Waveguides ". *Computer Music Journal*,16(4):74- 91.
- Steels, L. and Vogt, 1997, P. "Grounding adaptive language games in robotic agents". *Proceedings of 4th European Conference on Artificial Life*, I. Harvey and P. Husband (eds.). Cambridge (MA): MIT Press.