Problems week 3

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```python
from sympy import *
import numpy as np
import matplotlib.pyplot as plt
init_printing()
from scipy.linalg import lu
x = symbols('x')
```

1 Multiple choice

1.1 A 1.1.12 d

Answer: 2 down, $z = x^{n+1}$ og $v = \sum_{i=0}^{n} a_i x^i$

1.2 B

Ombyt række 1 og 3.

1.3 C

Vi har

$$LUx = b \to (LU)^{-1}LUx = (LU)^{-1}b \to x = U^{-1}L^{-1}b$$

1.4 D

Vi har

$$\|x\|_2 = \sqrt{1^2 + 2^2 + 3^2 + 0^2} = \sqrt{14} \approx 3.74$$

2 Matrixfaktoriseringer

```python
A = np.array([[2, 1, 1], [1, 2, -1], [1, -1, 3]])
P, L, U = lu(A)
display(L, U)
```

```
array([[ 1. ,  0. ,  0. ],
       [ 0.5,  1. ,  0. ],
       [ 0.5, -1. ,  1. ]])

array([[ 2. ,  1. ,  1. ],
       [ 0. ,  1.5, -1.5],
```

```
       [ 0. ,   0. ,   1. ]])
```

```
Linv = np.linalg.inv(L.T)
display(U @ Linv)
```

```
array([[2. , 0. , 0. ],
       [0. , 1.5, 0. ],
       [0. , 0. , 1. ]])
```

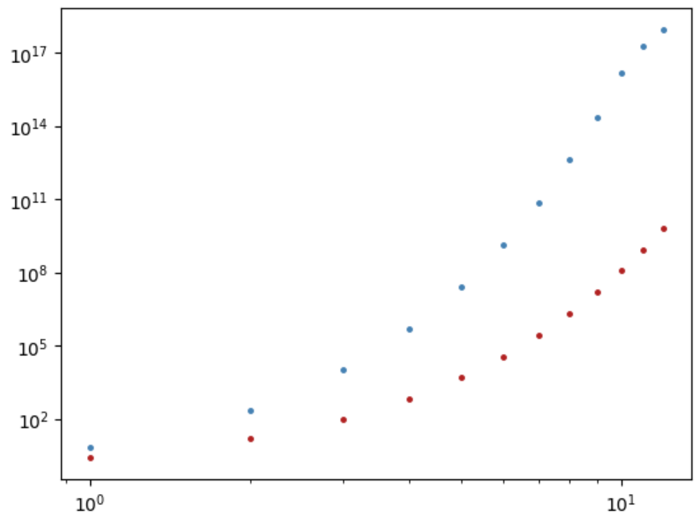
3 Vandermonde matricer og normalligninger

```
def M(n):
    arr, conds, mtm = [], [], []
    for i in n:
        nval = np.linspace(0, 1, i+1)
        A = np.vander(nval)
        cond = np.linalg.cond(A)
        d_n = np.linalg.cond(A.T @ A)
        mtm.append(d_n)
        arr.append(A)
        conds.append(cond)
    return arr, conds, mtm

n = range(1, 13)
M_n, conds, d_n = M(n)

plt.loglog(n, conds, '.', markersize=5, color="firebrick")
plt.loglog(n, d_n, '.', markersize=5, color="steelblue")
```

```
[<matplotlib.lines.Line2D at 0x112ea3750>]
```



```python
#display([x**2 for x in conds], d_n)
display(np.isclose([x**2 for x in conds], d_n))
```

```
array([ True,  True,  True,  True,  True,  True,  True,  True, False,
       False, False, False])
```

Ja, vi ser at $c_n^2 \approx d_n$ for $n = 1, \ldots, 10$.

Rule of thumb: If $\kappa(A) = 10^k$, then one can expect to lose at least k digits of precision in solving the system $Ax = b$.

```python
def significant_digits(c):
    arr = []
    for cond in c:
        k = symbols('k')
        eq1 = Eq(10**k, cond)
        arr.append(round(15 - solve(eq1, k)[0]))
    return arr

display("We can expect to use this amount of significant digits:")
significant_digits(conds)
```

```
'We can expect to use this amount of significant digits:'
```

```
[15, 14, 13, 12, 11, 10, 10, 9, 8, 7, 6, 5]
```

For the normalligningssystemet, we have:

```python
display("We can expect to use this amount of significant digits:")
significant_digits(d_n)
```

```
'We can expect to use this amount of significant digits:'
```

```
[14, 13, 11, 9, 8, 6, 4, 2, 1, -1, -2, -3]
```

4 Teori - konditionstal for normalligningerne

Det gælder, at B's egenværdier er positive.

Da B er symmetrisk, har vi at

$$B^T = B \rightarrow B^T B = B \cdot B = B^2$$

Så har vi at

$$Bv = \lambda v \rightarrow B^2 v = \lambda^2 v \rightarrow (B^T B)v = \lambda^2 v$$

Lad λ være matricen B største egenværdi. Dermed har vi, at

$$\|M^T M\|_2 = \sqrt{\lambda^2} = \lambda = \|M\|_2^2$$

Hvilket var det, vi gerne ville vise. $\square$