



# Differential Spatial Modulation

MIMO Project Report

Varshil Gandhi (160108016)

Aniket Agrawal (160108005)

Mohit Chandak (160108026)

Department of EEE

Indian Institute of Technology Guwahati

Supervisor: Prof. Rakhes Singh Kshetrimayum

# Differential Spatial Modulation

4th November, 2019

## Abstract

In a recently developing modulation technique, Spatial modulation (SM), only a single antenna is activated during MIMO transmission. The major limitation associated with the SM is that channel state information (CSI) is required at the receiver end. Estimation of CSI makes the decoding process fairly complex and difficult to implement. In a recent trial to overcome the difficulties posed by SM, a differential SM scheme(DSM) is formulated that completely bypasses any CSI at the transmitter or receiver, while preserving the single active transmit antenna property. This scheme can be applied to any constant energy constellation such as phase-shift keying (PSK) and to systems with arbitrary numbers of transmit and receive antennas.

## 1 Introduction

Although, MIMO systems support high data rate transfers attributing to its large numbers of antennas, this significantly introduces hunger for high energy. Further, multi-stream data requires complex receiver decoding techniques. Some of these issues were addressed by the recent single-RF chain based transmission scheme known as spatial modulation. In SM technique, the information bits are partitioned into two parts, of which one modulates the symbol and the other activates one out of all transmit antennas. This allows SM to eliminate interchannel interference and to avoid synchronization among the transmit antennas. The requirement of CSI in SM scheme is satisfied by transmission of pilot signals. Although SM is more robust than previous methods like V-Blast, the estimation of CSI in highly mobile scenarios, where mobility is comparable to data transmission rates, becomes complex and costly.

A common solution to the mentioned limitations is differential modulation. A raft of techniques have been proposed in response to these challenges including the differential detection scheme proposed for the Alamouti scheme to avoid CSI. Borrowing the concept of SM, a new technique STSK emerged and showing that a differential scheme is possible on STSK, differential space-time-shift keying (DSTSK) emerged. However, the present documentation on DSTSK fails to account for two major limitations, and further, the design details are not clearly mentioned.

The paper referred in this report presents a new differential scheme tailored to SM. In addition to completely avoiding CSI in both receiver and transmitter, DSM scheme also exploits the time dimension along with the space to facilitate the differential (de)modulation. Unlike DSTSK, the transmission matrices can be systematically designed based on the

order of transmit antennas being activated and additionally, DSM allows multiple symbols to be transmitted per block. This releases the constraints posed on DSTSK and allows a improved spectral efficiency.

## 2 Spectral Efficiency

The set  $G$  contains  $N_t!2^{N_t b}$  blocks, where  $N_t$  represents the number of antennas and  $b$  signifies  $2^b$ -PSK constellation. Based on the Stirling's formula, the theoretical spectral efficiency of **DSM** can be given as:

$$R_{DSM,theory} = \frac{1}{N_t} \log_2(N_t!) + b \quad (1)$$

$$R_{DSM,theory} \approx \log_2(N_t) - \left( \log_2(e) - \frac{1}{N_t} \log_2(\sqrt{2\pi N_t}) \right) + b \quad (2)$$

where the middle term indicates the loss of spectral efficiency, which is upper bounded by  $\log_2(e)$ . While, spectral efficiency of a conventional **SM** is given as:

$$R_{SM,theory} = \log_2(N_t) + b \quad (3)$$

Usually  $N_t!$  is not an integral power of 2, which is usually tackled by fractional bit encoding(FBE). However, FBE leads to longer decoding delay and worse bit error rate (BER) performance.

So, in practice we assume that  $N_t$  is an integer power of 2. Hence, the spectral efficiency of SM and DSM is given as:

$$\bar{R}_{SM} = \lfloor \log_2(N_t) \rfloor + b \quad (4)$$

$$\bar{R}_{DSM} = \frac{1}{N_t} \lfloor \log_2(N_t!) \rfloor + b \quad (5)$$

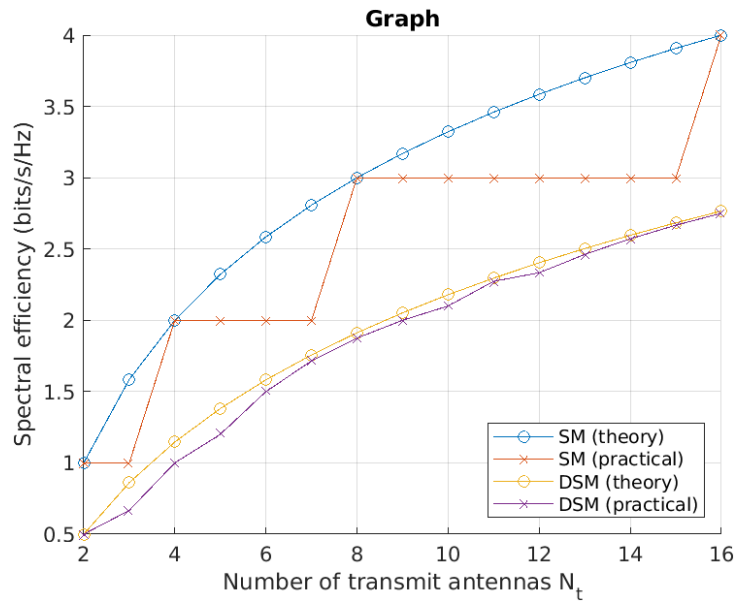


Figure 1: Spectral Comparison of SM and DSM

### 3 System Model Specifications

The system here is assumed to be consisting of  $N_t$  transmitting antennas and  $N_r$  receiving antennas. In the SM system, the constellation vector  $s_t$  of dimension  $N_t$  consists of only a single non-zero symbol as only one antenna operates at a particular time instant. Therefore, the transmit-to-receive wireless link is time hopping which makes differential operation difficult in SM.

So, the system design is based on a  $N_t \times N_t$  matrix made by stacking constellation vectors transmitted from time  $[k, k + N_t - 1]$  where  $k$  is some integer. The matrix designed is known as the **space-time block** indicated by **S**. According to this design, each antenna is activated only once during each block, thus making differential operations possible as long as the wireless channel remains unchanged over the two consecutive blocks.

Let us consider a set **G** containing all possible combinations of matrices **S** for a system with  $N_t$  transmitting antennas and  $2^b$ -PSK constellation. It is very clear that the number of such matrices will be  $N_t! \times 2^{N_t b}$ . But,  $N_t!$  is not an integer power of 2 for  $N_t > 3$ . So, in practical implementations of the DSM system, a subset of **G** represented as  $G_M$  is considered with  $2^{\lfloor \log_2 N_t! \rfloor} \times 2^{N_t b}$  transmitting blocks. This allows the system to encode each transmitting block with  $\lfloor \log_2 N_t! \rfloor + N_t b$  bits.

#### 3.1 Algorithmic Details

The transmission process starts with input data-bits and the initial transmission block,  $S_0$ . As the current system assumes a  $2^b$ -PSK constellation for encoding bits into symbols, 1 always remain in the symbol set for all values of  $b$ . So, without any loss of generality,  $S_0$  is defined as an identity matrix.

As mentioned earlier, DSM encodes the bits into two parts. The first  $\lfloor \log_2 N_t! \rfloor$  are encoded in the spatial dimension using specific mapping algorithms (section 3.3) and remaining  $N_t \times b$  bits are encoded into  $N_t$  constellation symbols in the signal domain. Specifically, the first batch of bits determines the order of transmit antennas being activated in a space-time block by an index-mapping procedure. And the second batch of bits denote which symbol must the transmitting antenna send. Using both these methods, we get a unique transmission block  $X_t$  when multiplied with  $S_{t-1}$  gives us the transmission block  $S_t$  to be transmitted. The above process goes on till all the data bits are transmitted. In case the last bits are less than the required bits to encode, zero bits are padded with the message data.

The receiver end receives the transmitted block after fading and noise in the channel affecting the transmitted block. The receiver starts with  $Y_0$  equal to Identity matrix. For or discussions, we assume  $N_r = N_t$ . The differential detection can be explained as below:

$$Y_{t-1} = H_{t-1}S_{t-1} + N_{t-1} \quad (6)$$

$$Y_t = H_t S_t + N_t \quad (7)$$

For DSM to work equivalently good as the theoretical spectral efficiency methods, the channel is assumed to be quasi-static. The fading coefficients remain constant over two adjacent DSM transmission blocks. With the given assumption, we could solve eq (6) and (7) to get

$$Y_t = Y_{t-1}X_t - N_{t-1}X_t + N_t \quad (8)$$

The optimal maximum likelihood detector can be derived as

$$X_{predicted} = \underset{X \in G_M}{\operatorname{argmax}} \operatorname{trace}[\operatorname{Re}(Y_t^H Y_{t-1} X)] \quad (9)$$

This explains the iterative procedure used by the receiver in Subsection 3.2.

### 3.2 Algorithm

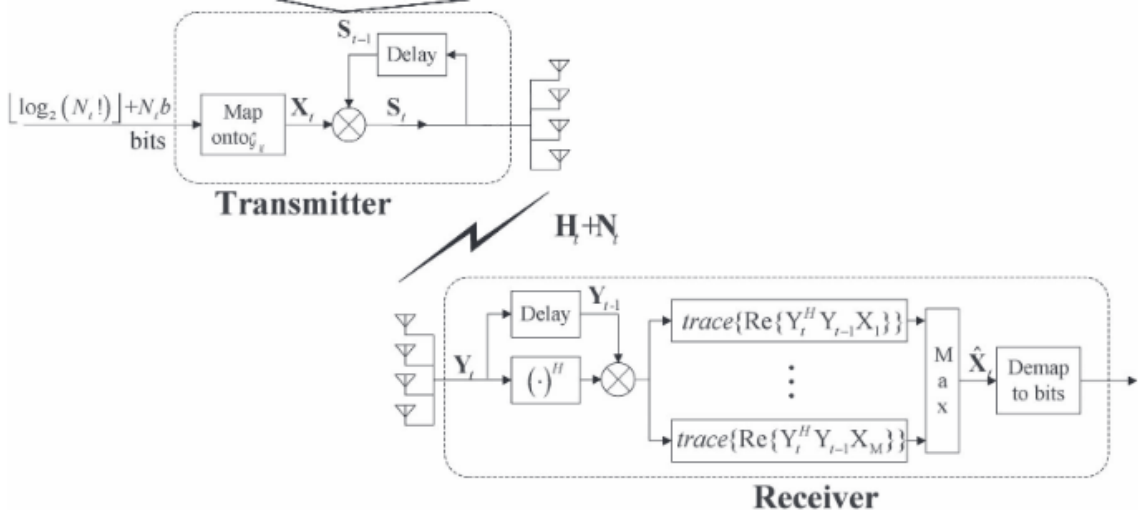


Figure 2: System Overview

The differential process starts with sending  $S_0 = I_{N_t}$  as the first transmission block sent as a default. The algorithms mentioned below assumes the process during time  $t$ ; when the block  $S_{t-1}$  is transmitted.

---

#### Algorithm 1 Transmitter algorithm

---

```

Initialize  $N_t, b$  for  $2^b$ -PSK constellation.
Initialize the transmission with  $S_0 = I_{N_t}$ 
Initialize sequence of bits as inputs
for time = 0: till transmission is complete do
     $S_{t-1}$  = The previous transmitted block
    Bits = Accumulate  $\lfloor \log_2 N_t! \rfloor + N_t b$  bits from the incoming sequence
     $X_t$  = mappingAlgo(Bits,  $N_t, b$ )
     $S_t = S_{t-1} X_t$ 
end for

```

---



---

#### Algorithm 2 Receiver algorithm

---

```

Assuming the algorithm has saved  $Y_{t-1}$ 
for time = 0: till receiving is complete do
     $X_{predicted} = \underset{X \in G_M}{\operatorname{argmax}} \operatorname{trace}[\operatorname{Re}(Y_t^H Y_{t-1} X)]$ 
end for

```

---

---

**Algorithm 3** Mapping algorithm used in transmission

---

Given a number  $m = \text{first } \lfloor \log_2 N_t! \rfloor$  bits in decimal

$b^{(m)} = \text{LehnerCode}(m, N_t)$

Define  $\theta = (1, 2, 3 \dots N_t)$  and container  $a^{(m)}$

**for**  $l = 1 : N_t$  **do**

$a_l^{(m)} = \theta_{b_l^{(m)}}$

$\theta_{b_l^{(m)}}$  is removed from  $\theta$

**end for**

---

### 3.3 Mapping Algorithm

There are two index-based mapping methods that could be used for relating the first  $\lfloor \log_2 N_t! \rfloor$  bits to the transmission block set  $G_M$ .

- Table Lookup method
- Permutation Method

In the first method, a lookup table is created to provide the corresponding permutations for the incoming bits. The selection of permutations may affect the overall performance. For ease of implementation of permutation index mapping, the algorithm discards the lexicographically larger permutations in this development. The main problem is that with increasing transmitting antennas, the table size increases exponentially.

EXAMPLE OF LOOKUP TABLE FOR  $N_t = 3$

Bits	Permutation of Indices	Blocks
00	(1 2 3)	$\begin{bmatrix} s_{11} & 0 & 0 \\ 0 & s_{22} & 0 \\ 0 & 0 & s_{33} \end{bmatrix}$
01	(1 3 2)	$\begin{bmatrix} s_{11} & 0 & 0 \\ 0 & 0 & s_{23} \\ 0 & s_{32} & 0 \end{bmatrix}$
10	(2 1 3)	$\begin{bmatrix} 0 & s_{12} & 0 \\ s_{21} & 0 & 0 \\ 0 & 0 & s_{33} \end{bmatrix}$
11	(2 3 1)	$\begin{bmatrix} 0 & 0 & s_{13} \\ s_{21} & 0 & 0 \\ 0 & s_{32} & 0 \end{bmatrix}$

Figure 3: Lookup table

The second method is based on a one-to-one mapping between integers and permutation of  $N_t$  elements in lexicographic order, known as Lehner Code. The detailed algorithm is mentioned in Subsection 3.2. The algorithm first converts a number  $m$  to its factorial representation  $b^{(m)}$  and then maps the factorial representation to a permutation  $a^{(m)}$ , which is an indexed representation of a matrix with one hot encoding. For example, if the matrix we seek is  $I_{N_t \times N_t}$ , then  $a^{(m)} = [1, 2, 3]$ .

While the second method is better than the first assuming the reduced time and space complexity, all the elements of the set  $G_M$  are required on the receiver side. Thus, the advantage of Lehner Code approach lies only on the transmission side and not the receiving end.

## 4 Experiments

In our experiments, the mapping algorithm encodes bits in  $N_t * N_t$  square matrices, where  $N_t$  denotes the number of transmitting antennas, and each value of the matrix is determined by the encoding constellation given by  $2^b$ . The dimensions of quasi-static Rayleigh flat fading channel used in the experiment is  $N_r * N_t$ , where  $N_r$  denotes the number of receiving antennas.

We have conducted various Bit Error Rate(BER) simulations under various system configurations to examine DSM performance.

### 1. DSM Versus SM:

In this graph, we have compared the BER performance between DSM and SM using  $N_t = 3$  and different values of  $N_r$  ( $N_r = 2, 3$ ), transmit antennas. In our experiments we have used  $N_t$  greater than  $N_r$ . We have used quaternary PSK (QPSK) for DSM and BPSK for SM.

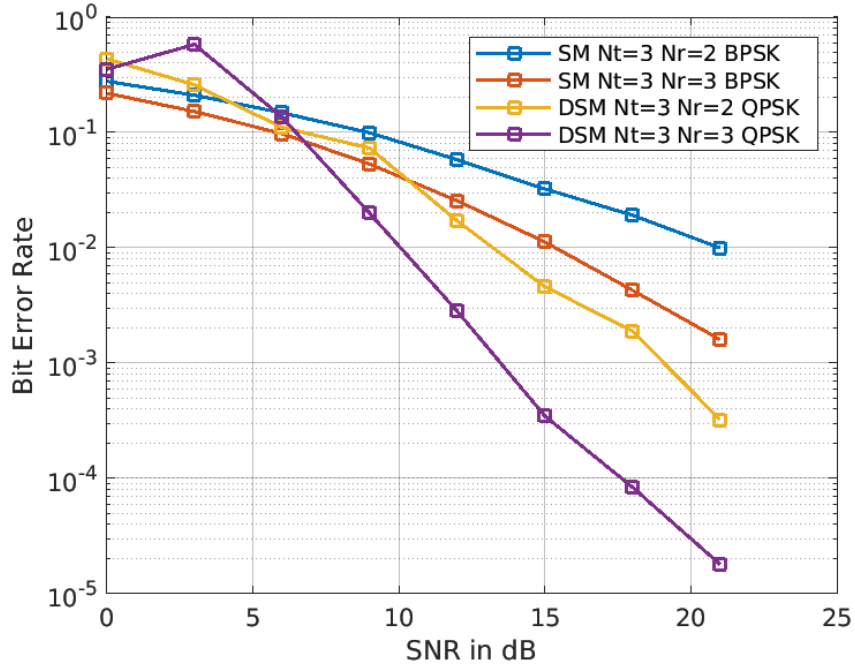


Figure 4: BER performance of differential detection DSM versus coherent detection SM

2. **DSM With Multiple Transmit Antennas:** In this graph, we have compared the DSM BER performance with multiple transmit antennas. For distinct number of transmit antenna, we have experimented for different number or receiver antennas. For all the experiments, the DSM always uses QPSK and SM always uses BPSK.

## 5 Conclusion and Future Work

In this report, we analyzed the concept of Differential Spatial modulation [Bian et al. \(2015\)](#) and its clear advantages wrt spectral efficiency. Using both time and space dimension without the need for CSI at both the transmitter and the receiver makes the method highly beneficial. In comparison with coherent SM [Mesleh et al. \(2008\)](#), DSM exhibits

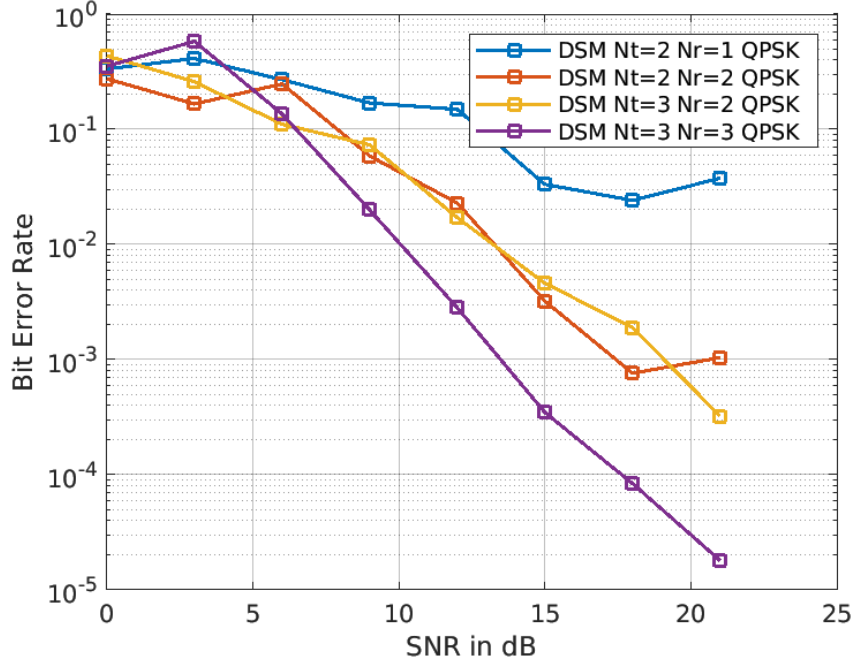


Figure 5: BER performance of differential detection DSM

a spectral efficiency loss that is upper bounded by  $\log_2(e)$  and a performance loss of no more than 3 dB.

The major problems the method faces is the exponential order complexity on the receiver side. The receiver side algorithm iterates on every possible transmission block to search for the block with the maximum trace value. The number of elements in such a set can be calculated as  $2^{\log_2(Nt!) + N_t * b}$  which increases exponentially with increase in  $N_t$  and  $b$ . Future research must focus upon reduced-complexity detection process at the receiver side.

## References

- Bian, Y., Cheng, X., Wen, M., Yang, L., Poor, H. V. and Jiao, B. (2015). Differential spatial modulation, *IEEE Transactions on Vehicular Technology* **64**(7): 3262–3268.
- Mesleh, R. Y., Haas, H., Sinanovic, S., Ahn, C. W. and Yun, S. (2008). Spatial modulation, *IEEE Transactions on Vehicular Technology* **57**(4): 2228–2241.