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Chapter 8

Specialized Topics

The Specialized Topics questions on the Physics GRE are probably the most unique aspect of the test. It's hard to think of any other test (other than TV game shows) in which a full 10% is random assorted knowledge. This may seem daunting, but with smart preparation, these questions actually offer a huge advantage.

The special topics questions are almost entirely pure knowledge recall, otherwise known as fact regurgitation. This is the kind of knee-jerk memorization you probably hated in high-school chemistry or biology. When confronted by a special topics question, you'll either know it or you won't. If you know it, that's one question down in under 10 seconds, which gives you a *huge* bonus on time for the more difficult calculational questions. If you don't know it, you probably won't be able to figure out the answer just by reasoning through it, and you may waste 5 or more minutes second-guessing yourself when stuck between two equally appealing answer choices. The optimum strategy, then, is to amass a basic knowledge of as many areas of cutting-edge physics as possible, *just enough* to make the associations between "buzzwords" and concepts that will allow you to recall the required knowledge.

Luckily, this kind of studying is *dead easy.* Every couple days, take a break from your normal Physics GRE practice and just *read.* Pick up a basic textbook in an advanced subject you're unfamiliar with (for example, if you're aiming towards high-energy, choose an introductory solid-state physics or electrical engineering textbook), and don't bother working any problems; just read the book as if it were a novel. You might learn something new and interesting, but that's not really the point: by reading this way, you'll be forming connections and associations in your memory that you might not even be aware of. It's likely you won't be able to remember exactly what you read, but if prompted by a keyword that shows up on the GRE, your memory will spring into action with that feeling of "I've seen this somewhere before." That's really all you need for these kinds of questions.

To give you a head start, we've collected here some of the material which is most likely going to be tested in these kinds of questions. The tone of this chapter will be much more informal than the rest of the book, and some concepts are purposefully not explained in gory detail: as mentioned above, you won't need to know these kinds of details, so consider this leisure reading. When you're sick of doing problems, revisit this chapter and read a few paragraphs.

8.1 Nuclear and particle physics

8.1.1 The Standard Model: particles and interactions

The modern description of the particles and forces found in nature is a relativistic quantum field theory called the Standard Model. Relativistic quantum field theory is a framework for reconciling quantum mechanics with special relativity. This framework has the surprising result that each charged[1] particle has an *antiparticle*, with identical mass but opposite charges – the first particle such discovered was the anti-electron, more commonly known as the positron. The spin-1 *bosons* of the Standard Model mediate the fundamental forces: photons mediate the electromagnetic force, $W^{\pm}$ and Z bosons mediate the weak nuclear force, and gluons mediate the strong nuclear force. (There are also hypothetical spin-2 gravitons, which mediate the gravitational force, but despite our best efforts these have not been experimentally observed.) The photon and gluons are massless, whereas the W and Z bosons are extremely heavy, about 90 times the mass of the proton. For group-theoretical reasons, there are eight gluons. The photon and the Z are their own antiparticles, whereas the W^+ is the antiparticle of the W^-.

The spin-1/2 *fermions* of the Standard Model, collectively known as *matter*, can be organized in three *generations*. In each generation, there are two *quarks*, one electron-type particle, and one *neutrino*. The first generation contains the up quark (u), down quark (d), electron (e), and electron neutrino (ν_e); the second contains the charm quark (c), strange quark (s), muon (μ), and mu neutrino (ν_μ); and the third contains the top quark (t), bottom quark (b), tau (τ), and tau neutrino (ν_τ). Each generation is successively heavier than the next, with the muon more than 200 times the mass of the electron, and the tau about 1000 times the mass of the muon. Because of the large mass hierarchy between generations, third- and second-generation particles will tend to decay to first-generation particles. Indeed, *all* the stable matter in the universe consists of first-generation particles (plus all flavors of neutrinos floating around – more on this later).

The quarks interact via the strong nuclear force, also known as *color*. The mathematical description of the strong force involves assigning one of three "colors" (red, green, blue) to every quark, such that each generation really contains six quarks (up-red, up-blue, up-green, etc.). Quarks also carry electric charge: up-type quarks have charge $+2/3$, and down-type have $-1/3$, in units of the magnitude of the electron charge. In fact, they interact via the weak nuclear force as well: emitting or absorbing a W-boson can change the *flavor* of a quark, from an up-type to a down-type or vice-versa, and can also change its generation. The electron-type particles and neutrinos, collectively known as *leptons*, interact via the electromagnetic and weak forces, but *not* the strong force. The electron, muon, and tau all have charge -1, but the neutrinos are electrically neutral. Antiparticles of the quarks and leptons are usually either given the prefix "anti-" or labeled by their opposite charge (μ^+, pronounced "mu-plus," for the anti-muon), with the exception of the positron, men-

[1]Here "charged" is meant in a general sense and does not refer only to electric charge: for example, the neutron is electrically neutral but carries both weak charge and baryon number, and hence has an antiparticle, the antineutron.

tioned above. Similarly, the quarks and neutrinos also have antiparticles, usually denoted by putting a bar over each particle's symbol; for example: $\bar{u}$ for up antiquark, or $\bar{\nu}_e$ for electron antineutrino. Like the leptons, the antiquarks are distinguished from the quarks by having the opposite charge. Since the u has a charge of $+2/3$, for example, the $\bar{u}$ has a charge of $-2/3$.

The strong, weak, and electromagnetic forces vary widely in their strengths and experimental signatures. As the name suggests, the strong force is the strongest, causing strongly-interacting particles to decay with lifetimes on the order of 10^{-23} seconds. Next is the electromagnetic force: particles which decay electromagnetically have lifetimes of about $10^{-18} - 10^{-16}$ seconds (note that longer lifetimes mean *weaker* forces), and the telltale signature is the emission of a photon, though this is not required. Finally, particles decaying by the weak force have the longest lifetimes, about $10^{-10} - 10^{-8}$ seconds, and the telltale signature is the emission of a neutrino, though once again this is not always present. The weak decay you're probably most familiar with is beta decay (discussed further below) which provided the first evidence for the neutrino because the varying energy spectrum of the emitted electron suggested a 3-body rather than 2-body decay. Questions about which force is responsible for which process are relatively common among special topics questions on the GRE. As we've emphasized, it's impossible to tell which force is responsible just by looking at the decay products, or even the lifetime: there are particles which conspire to have extraordinarily long lifetimes despite decaying strongly, and particles which can decay into the same final state by two different forces. But the combination of these two factors is a useful guide: if you see a particle with a lifetime of 10^{-17} seconds which decays to two photons, you can be pretty sure electromagnetism was responsible.

8.1.2 Nuclear physics: bound states

All ordinary matter in the universe is protons, neutrons, and electrons. We've already addressed electrons above: these are elementary particles. However, protons and neutrons (collectively known as *nucleons*) are *composite* – in the framework of quantum field theory, they are teeming seas of quarks and gluons constantly popping in and out of existence. At low energies, where nuclear physics is applicable, we can simplify this description considerably using the *quark model*. Here, the proton is considered a bound state of two up quarks and a down quark, written uud, for a total charge of $2(2/3)-1/3 = +1$, and the neutron is a bound state of two down quarks and an up quark (udd), for a total charge of $2(-1/3) + 2/3 = 0$. Due to a property of the strong force called *confinement*, free quarks cannot be seen in nature, and instead they collect themselves into bound states like protons and neutrons. All of these bound states are color-neutral, also referred to as *color singlets*.

If we collide strongly-interacting particles together at higher and higher energies, we can form all kinds of different bound states, heavier than the proton and neutron. Some may be thought of as excited states of nucleons: for example, the first excited state of the proton is known as the Δ^+, which has the same quark content as the proton but is so much heavier that it can be considered a distinct particle. In general, bound states of quarks fall into two categories: *mesons*, made of a quark and an antiquark, and *baryons*, made of three quarks.

(Anti-baryons have three antiquarks.) Note that because of the rules for adding angular momentum, mesons may have either spin-1 or spin-0, whereas baryons may have spin-3/2 or spin-1/2. In the 1960's, it was observed that mesons and baryons made out of only the three lightest quarks (up, down, and strange) arranged themselves into interesting patterns which Gell-Mann called *the Eightfold Way.* Historically, mesons and baryons were the vehicles by which new generations of quarks were discovered: see Griffiths for more information. Color was originally introduced as a description of the strong force in order to explain the quark content of some of these baryons: a baryon made of three identical quarks, for example sss, would violate the Pauli exclusion principle since it's supposed to be a fermion but its wavefunction is symmetric. Putting the quarks in an antisymmetric color state, the color singlet mentioned above, fixes this problem.

The lightest baryons are the proton and the neutron. The fact that the neutron is slightly heavier than the proton means that the neutron can decay via[2] $n \to p + e^- + \bar{\nu}_e$. Indeed, free neutrons *do* decay, with a lifetime on the order of 15 minutes, but inside a nucleus the constant strong interactions with protons keep them from decaying immediately. In certain nuclei, though, the neutron can decay via quantum tunneling, in a process better known as *beta decay.* As the nuclei get bigger and bigger, the electromagnetic repulsion between protons starts to cancel the attractive effects of the strong force, and whole chunks of the nucleus can break off: this leads to *alpha decay*, emission of bound states of two protons and two neutrons (in other words, helium-4 nuclei). Speaking of nuclear sizes, a good fact to remember is that typical nuclear diameters are femtometers, or 10^{-15} m. The final type of radiation, *gamma* radiation, is the emission of photons from an excited state of a nucleus, which doesn't change the proton/neutron composition of the nucleus.

In addition to processes where nuclei can break apart (*fission*), sufficiently small nuclei can also join together through *fusion.* This requires enormous temperatures and pressures in order to overcome the electromagnetic repulsion between the protons, but can also release enormous amounts of energy; indeed, both the sun and the hydrogen bomb are powered by fusion reactions. In the sun, successive protons are fused onto larger and larger nuclei, with some being converted to neutrons along the way, all the way up to ${}^4_2\text{He}$. In the standard picture of the genesis of heavy elements in the early universe, light nuclei continued to fuse as a result of supernovas, all the way up to iron (atomic number 26), which is the most stable nucleus. Heavier nuclei become progressively more and more unstable, up to lead (atomic number 82), beyond which all heavier nuclei will eventually decay.

8.1.3 Symmetries and conservation laws

The general rule of particle physics is that anything that *can* happen, *will* happen, unless it is forbidden by a symmetry or a conservation law. For example, the electron is the lightest negatively charged particle (excluding quarks, which as we've discussed can't exist free in nature), so it is forbidden from decaying by conservation of charge. But the proton is heavier

[2] In particle physics, reactions are sometimes written without the + sign for multiple particles in the initial or final state, so neutron decay may also be written $n \to pe^-\bar{\nu}_e$.

than the positron – why doesn't it decay? To explain this, and other similar observations, a new law called *conservation of baryon number* was introduced. Baryons get baryon number $+1$, anti-baryons get -1, and everything else is assigned zero. Similarly, the fact that an extra neutrino is *always* produced in beta decay suggests *conservation of lepton number*, which is really three separate conservation laws (conservation of electron number, muon number, and tau number), where in each generation the lepton and its associated neutrino are given lepton number $+1$ and corresponding antiparticles get -1.

We can apply these rules to find the possible decay modes of the muon μ^-. By conservation of mu number, we must have a mu neutrino among the decay products. By conservation of charge, we must have a lighter negatively charged particle: the only stable option is the electron. But by conservation of electron number, we must have an accompanying neutral particle with electron number -1, namely the anti-electron-neutrino. This fixes one decay mode (in fact, the dominant one) to be

$$\mu^- \to e^- + \nu_\mu + \bar{\nu}_e.$$

Other decays are possible, but they must all contain extra pairs of particles with net charge and lepton number zero: for example, a pair of photons, or an electron-positron pair. In fact, a general rule of thumb says that the dominant decay mode will be the one with the fewest number of particles in the final state, because each final-state particle comes with a suppression factor of $1/(2\pi)^4$ in the quantum-mechanical amplitude.

There are also various *discrete* symmetries which are important in nature. The symmetry operation P, or *parity*, reverses the orientation of space, which is another way of saying P takes a configuration of particles to its mirror image. The symmetry operation C, *charge conjugation*, exchanges particles and antiparticles. Finally, T, or *time-reversal*, does what it sounds like. A very important theorem in quantum field theory states that all Lorentz-invariant local quantum field theories must be symmetric under the combined action of all these operations, known as *CPT*. However, it is an important and striking fact that the Standard Model does *not* respect each of these symmetries individually. The weak interaction is said to be *maximally parity-violating*: the classic experiment which proved this looked at beta-decay of cobalt-60, and found that the decay products were preferentially produced in one direction relative to the spin of the nucleus. In fact, the weak interaction doesn't conserve CP either: the main evidence for this comes from the neutral kaon system (for the record, kaons are the lightest mesons which contain strange quarks). By the CPT theorem, this means that T by itself must also be violated, and there are various experiments involving the precession of muon spins in a magnetic field which demonstrate this.

8.1.4 Recent developments

For the Standard Model to be mathematically consistent, it must contain at least one additional particle: the *Higgs boson*, whose discovery was at long last announced on July 4, 2012. This particle is responsible for giving mass to all elementary particles, via a mechanism known as *spontaneous symmetry breaking*. For more details on both, see Section 8.4 below –

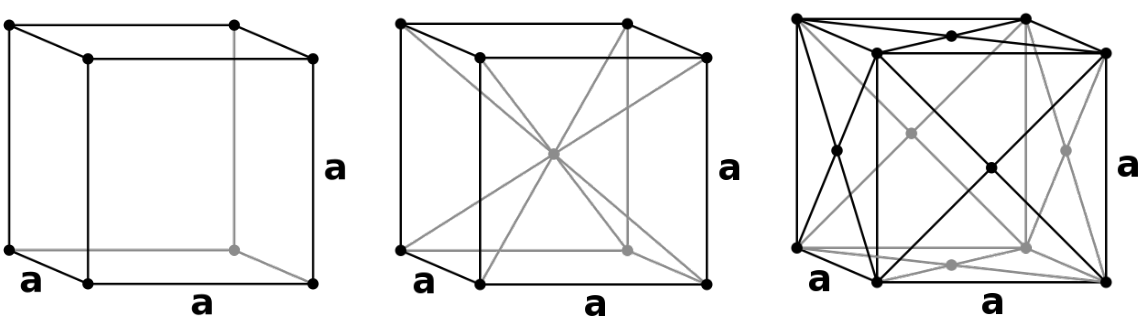
both the particle and the mechanism are important enough to each deserve their own Nobel prize! For the measured value of the Higgs boson mass, 126 GeV, to be consistent with the principles of quantum field theory, many physicists believe that there must exist a further symmetry of nature known as *supersymmetry*. If this is the case, each elementary particle has a *superpartner* with exactly the same charges, but with spin differing by 1/2. As of this writing, the Large Hadron Collider is hot on the trail of these hypothetical particles, but none have yet been discovered.

On firmer experimental footing is the discovery that *neutrinos have mass*. This was deduced by observing *neutrino oscillations*, a phenomenon by which one flavor of neutrino (say an electron neutrino) is emitted from a source, but later detected as another flavor (say a tau neutrino). Unfortunately, this kind of measurement only permits one to determine mass differences, not absolute masses, but it is known that all neutrinos are extremely light, with masses less than 1/1000 the mass of the electron.

8.2 Condensed matter physics

8.2.1 Crystal structure

An ideal crystal is constructed by the infinite repetition of identical structural units in space. In the simplest crystals the structural unit is a single atom, as in copper, silver, gold, iron, aluminum, and the alkali metals. But for other materials the smallest structural unit may contain many atoms or molecules. For simplicity, we'll call this unit an "atom" in everything that follows, just to keep from using too many words. The examples below illustrate some common crystal lattices.



(a) Simple Cubic (b) Body-centered cubic (BCC) (c) Face-centered cubic (FCC)

The key here is *repetition*: the crystal structure is infinite, so we have to define a repeating pattern, known as a *unit cell*. The cubes drawn above are known as *conventional unit cells*, and they make the cubic symmetry apparent by containing a cube with atoms at every vertex, but some add extra points to it: the body-centered cubic contains an atom at the center, and the face-centered cubic contains atoms at the centers of each of the faces. To construct the rest of the crystal, you can tesselate space in all three dimensions using these unit cells.

Despite making the symmetry manifest, it may be the case that the conventional unit cell is not the smallest repeating pattern, where smallest means "containing the least number of atoms." This smallest pattern is known as the *primitive unit cell*. This is the case for the cubics: the primitive unit cell for the BCC is an octahedron with half the volume of the conventional cell, and the primitive unit cell for the FCC is a parallelepiped with one quarter the volume of the conventional cell. Similarly, the volumes of the conventional unit cells are all equal, but the interatomic distances are all different: for a cube of side a, the simple cubic has distance a, the BCC has distance $a\sqrt{3}/2$, and the FCC has distance $a\sqrt{2}/2$. A favorite GRE question type gives you the volume of a primitive unit cell and asks for the interatomic distance or the volume of the conventional unit cell. By the way, these lattice structures are examples of *Bravais lattices*: there are fourteen of them, but the only ones which show up on the GRE with any frequency are the three cubic types given above.

There are two other ideas associated with crystal lattices that may show up on the exam. The *reciprocal*, or *dual*, lattice is the Fourier transform of the original lattice. Just like the Fourier transform of a collection of position vectors $\mathbf{x}$ (position space) is a collection of momentum vectors $\mathbf{p}$ (momentum space), the reciprocal lattice can also be considered a "space" in its own right. The vectors which make up the dual lattice, called the *reciprocal lattice vectors*, are the normal vectors to the planes formed by the original lattice. This is rather complicated to visualize, but the following facts are true:

- The simple cubic is its own dual lattice. For a lattice of side length a, the dual lattice has side length $2\pi/a$ (the 2π comes from the Fourier transform, and the $1/a$ since the lattice vectors are supposed to have units of wavenumber).

- The body-centered cubic and face-centered cubic lattices are dual to one another.

- The dual to a hexagonal lattice is another hexagonal lattice, but rotated through a $30°$ angle.

The primitive unit cell of the reciprocal lattice is so important that it is given its own name: the *(first) Brillouin zone*.

8.2.2 Electron theory of metals

In solid-state physics, the free electron model is a simple model for the behavior of electrons in a crystal structure of a metallic solid: the atomic nuclei are pinned in place, and all the atomic electrons are *delocalized*, not belonging to any nucleus and free to roam around the metal. Roughly speaking, this explains why metals conduct electricity. If electrons behaved according to the laws of classical mechanics, at zero temperature they would just sit still with zero energy. But electrons are quantum particles, more specifically *fermions*, and according to the Pauli exclusion principle, no two electrons can occupy the same quantum state. So one electron can have zero momentum with spin up, the next can have zero momentum with spin down, but any additional electrons are going to have nonzero momentum.

The details aren't important, but what happens is that at zero temperature, the electrons in the metal fill up a sphere in momentum space, the *Fermi sphere.* The electrons which are free to roam around the metal sit on the surface of this sphere (the *Fermi surface*), with wave vectors $|\mathbf{k}| = k_F$, the *Fermi wave vector.* (Everything in this business is named after Fermi.) Recalling the usual relations from quantum mechanics, this means the Fermi momentum is $p_F = \hbar k_F$, and the Fermi energy is $E_F = \hbar^2 k_F^2/2m$. The remaining ingredient is a formula for k_F in terms of the number density n of electrons:

$$k_F = (3\pi^2 n)^{1/3}. \tag{8.1}$$

Let's pause for a quick sanity check: n has units $1/(\text{length})^3$, so $n^{1/3}$ has units of 1/length, appropriate for a wave vector. The Fermi energy is then

$$E_F = \frac{\hbar^2}{2m}(3\pi^2 n)^{2/3} \tag{8.2}$$

The *density of states* is the number of possible free electron states at a given energy E, given by

$$\rho(E) = \frac{V\sqrt{2}}{\pi^2\hbar^3} m^{3/2}\sqrt{E}, \tag{8.3}$$

where we have collected factors to emphasize the scaling which the GRE will care about, $\rho(E) \propto m^{3/2}\sqrt{E}$. Note that integrating the density of states should give the total number of electrons,

$$N = \int_0^{E_F} \rho(E)\, dE. \tag{8.4}$$

Combining the expressions for the Fermi energy and the density of states gives the density of states at the Fermi surface,

$$\rho(E_F) = \frac{3}{2}\frac{N}{E_F}. \tag{8.5}$$

Since most of the electrons are buried in the Fermi sphere, at low temperatures, the number of conduction electrons N_C in the metal is *not* N, but instead is approximately

$$N_C \approx \rho(E_F)(k_B T) \sim \frac{N k_B T}{E_F}, \tag{8.6}$$

which represents the number of electrons close enough to the Fermi surface to be able to escape due to thermal fluctuations. As always, the prefactors of π and 3 and such are not worth remembering (indeed, we dropped the prefactor 3/2 in the heuristic discussion above), but the power-law dependences ($k_F \propto n^{1/3}$, $E_F \propto n^{2/3}$) are very standard GRE questions. As we noted above, a quick way to remember this is to get k_F from dimensional analysis, then E_F from the dispersion relation for a massive particle.

8.2.3 Semiconductors

A semiconductor has electrical conductivity intermediate in magnitude between that of a conductor and an insulator. Semiconductors differ from metals in their characteristic property of decreasing electrical resistivity with increasing temperature.

Current conduction in a semiconductor occurs via mobile or "free" electrons and holes (the not-so-creative name for the absence of an electron), collectively known as charge carriers. Electrons, of course, have negative charge, so the absence of an electron (a hole) has *positive* charge; in semiconductor physics, one thinks of the hole as an actual positively-charged particle in its own right. Adding a small amount of impurity atoms to the semiconductor, known as *doping*, greatly increases the number of charge carriers within it. There are two ways to do this:

- **p-type doping.** When a doped semiconductor contains excess holes it is called *p-type* (the "p" stands for "positive"). An example of p-type doping is on silicon: an atom from group 13 of the periodic table, such as boron or aluminum, is substituted into the crystal lattice for silicon.

- **n-type doping.** When a doped semiconductor contains excess free electrons it is known as *n-type* ("n" for "negative"). The most common example for n-type doping is atomic substitution in group 14 solids (silicon, germanium, or tin), which contain four valence electrons, by group 15 elements (phosphorus, arsenic, antimony) which contain five loosely bound valence electrons.

8.2.4 Superconductors

Superconductivity is a phenomenon where certain materials, when cooled below a characteristic critical temperature, have exactly *zero* electrical resistance. Superconductivity is a quantum mechanical phenomenon, and cannot be understood within the confines of classical electromagnetism. There are two key aspects of superconductivity which are GRE favorites:

- **Meissner effect.** The Meissner effect is an expulsion of a magnetic field from a superconductor during its transition to the superconducting state. In a weak applied field, a superconductor "expels" nearly all magnetic flux, meaning that the magnetic field inside the superconductor is nearly zero (more specifically, it falls off exponentially fast as a function of distance from the surface). It does this by setting up electric currents near its surface. The magnetic field of these surface currents cancels the applied magnetic field within the bulk of the superconductor. As the field expulsion, or cancellation, does not change with time, the currents producing this effect (called *persistent currents*) do not decay with time. Therefore the conductivity can be thought of as infinite: a superconductor.

- **Cooper pairs.** Cooper pairs are part of the *BCS theory*, which was used to explain superconductivity. A Cooper pair is a specific state of two electrons, weakly bound to each other, such that the pair has total energy lower than the Fermi energy, which would be the energy of the electrons at zero temperature. Thus it is energetically favorable for the electrons to pair up. There are many more details, but just remember that the Cooper pair is "responsible" for superconductivity, and quantum-mechanically this pair of electrons behaves collectively like a *boson*, rather than a fermion.

8.3 Astrophysics

The most likely astrophysics topic you'll be tested on is *cosmological redshift*, simply because there are some easy formulas associated with it. Astrophysical evidence clearly indicates that our universe is expanding, but that statement needs to be made more precise for it to have any physics content. What we mean is that the spacetime metric ds^2, which defines what we mean by distance, is *not* constant like we assume in special relativity, but actually changes with time, and does so in such a way that the distances between spacelike-separated points grows with time. In equations

$$ds^2 = dt^2 - a(t)^2(dx^2 + dy^2 + dz^2),$$

where $a(t)$ is called the *scale factor*. The fact that space is expanding is just the statement that $a(t)$ is an increasing function of t. There's not much we can do with this metric without the tools of general relativity (which you *certainly* don't have to know for the GRE!), but we can write down a few true facts without derivation:

- **Photons are redshifted by the scale factor.** If λ_0 is the wavelength of a photon today, and λ_T was its wavelength at time T, then

$$\frac{\lambda_0}{\lambda_T} = \frac{a(\text{today})}{a(T)}. \tag{8.7}$$

  Since $a(t)$ is increasing, this means that the wavelength of a photon is *increased* by a factor of the ratio of distances *now* to distances *then*: this is the origin of the term redshift.

- **Black-body temperatures are inversely proportional to the ratio of scale factors.** This just follows from (8.7) and the Wien displacement law, $\lambda_{\text{max}} \propto 1/T$; as the universe expands by a factor of 2, black-bodies cool by a factor of 2. The typical application of this result is to the cosmic microwave background temperature, which to an excellent approximation follows a perfect black-body spectrum. For more about the cosmic microwave background, see the discussion of the 2006 Nobel Prize in Section 8.4.

- **Hubble's law: recession velocity is proportional to distance.** Due to the expansion of space, distant objects appear to be receding from us. If a galaxy appears to be moving away from us with velocity v, then Hubble's law states

$$v = H_0 D \tag{8.8}$$

where D is the measured distance between the distant galaxy and us, and H_0 is the *Hubble parameter*. H_0 is sometimes called the Hubble "constant," but this is misleading, because H_0 actually changes with time. The details of this are not important for the GRE: just remember that if a galaxy is twice as far away, it will appear to be moving twice as fast.

- **Redshift can be used as a measure of time.** This is just a convention, but a very useful one. Define the *redshift* z by

$$z(T) = \frac{\lambda_0}{\lambda_T} - 1. \tag{8.9}$$

In other words, z measures how much a photon has been redshifted since time T. The -1 is to ensure that $z(\text{today}) = 0$. Thus, positive redshifts correspond to times in the past, and negative redshifts to times in the future. It's certainly a funny way to measure time, but astronomers often refer to events which happened "at redshift 3," which just means the time T such that $\frac{\lambda_0}{\lambda_T} - 1 = 3$; note that this means the photon wavelength has actually been redshifted by a factor of *four*, not three! This is a tricky convention which may well show up on the test.

Finally, a couple qualitative statements about the expanding universe.

- **Cosmological redshift is *not* a Doppler shift.** It's true that the distances between galaxies are increasing as time goes on, but that does *not* mean they are moving apart from each other. This is a common misconception, and the picture to keep in mind is an inflating balloon: if you glue coins to the surface, as the balloon inflates each coin will move away from each other coin, but the coins are not actually changing their position on the surface. Thus there is no motion of the source or the receiver to cause a Doppler shift; cosmological redshift is due to *the expansion of space itself*. That being said, there *can* be a component of redshift which comes from the motion of galaxies relative to each other (known as *peculiar velocities*). On recent GRE's the wording of the relevant questions did not distinguish between Doppler and cosmological redshift, but the meaning should be clear from context. If you're given or asked for a *speed*, the question is asking about the relativistic Doppler shift; if you're asked about *distances*, the question refers to cosmological redshift.

- **Gravitationally bound systems are exempt.** The metric with the expanding factor $a(t)$ only applies to the universe on large scales, and is *not* the metric for every point in space. Indeed, general relativity tells us that the presence of matter changes the spacetime metric, and in particular systems which contain enough matter to become gravitationally bound (like our solar system) do *not* undergo expansion. In the cartoon picture above, the coins that we glued to the surface of the balloon are not themselves expanding. Thus distances do not seem to expand within our solar system, although we observe distant galaxies receding from each other.

We'll end this section on a topic central to both authors' research: *dark matter*. Since 1933, astronomers have noticed a mismatch between the amount of visible mass in galaxies, and the mass inferred from applying Newtonian gravity and the virial theorem to these galaxies. Specifically, galaxies were rotating too fast for their dynamics to be accounted for solely by the light-emitting matter visible inside. Further astrophysical observations involving the expansion of the universe and the cosmic microwave background radiation, as well as direct observations of colliding galaxy clusters such as the Bullet Cluster, strengthened the case for some kind of gravitating but non-light-emitting matter, dubbed dark matter, whose mass abundance in the universe is more than 5 times that of ordinary matter. Because it doesn't emit light (photons), dark matter can't be charged (if it is, it must have extraordinarily small electric charge, much smaller than the electron charge), but it may interact with the weak nuclear force. This is the model for the most popular candidate for dark matter, the WIMP or "weakly interacting massive particle," though there are a huge number of other models. Dark matter has not yet been directly detected on Earth, nor has it been unambiguously produced at particle accelerators, but many experiments are underway to try to determine its mass and interactions.

8.4 Recent Nobel prizes

The Nobel prizes in physics provide an excellent source for random trivia about current developments in physics. Anecdotally (though we don't have enough data to back up this claim), the GRE likes to throw in questions dealing with recent Nobel prizes, so here is a quick summary.

- **2013: The Higgs mechanism and the Higgs boson.** The role of the Higgs boson has already been mentioned in Section 8.1.4 above, but we can elaborate a little further. A puzzling fact about the forces of the Standard Model is that the W and Z bosons are massive, but all the other force-carriers (photons, gluons, and gravitons) are massless. However, putting in a mass "by hand" for these particles results in various inconsistencies, among them the nonsensical prediction that the probability for certain scattering processes is greater than 1! The Higgs mechanism resolves this puzzle by postulating that the W and Z bosons start out massless like the other force carriers, but inherit their mass from the *vacuum expectation value* of the Higgs field, which is constant but nonzero everywhere in space. The same mechanism ends up giving mass to all the other Standard Model matter particles, which also must start out massless for the symmetry properties of the Standard Model to work out correctly. Higgs's insight was that there is an additional particle, the Higgs boson, left over after this mechanism has worked its magic. The Higgs boson couples to matter with strength proportional to the particle's mass; this means it couples most strongly to the heaviest Standard Model particle, the top quark, providing the basis for some of the search strategies which discovered the Higgs boson at the Large Hadron Collider.

- **2012: Measuring and manipulating individual quantum systems.** Quantum systems are hard to study because a measurement of the system typically changes the state of the system. Worse, even if you *try* not to measure a system, the environment which surrounds your system does the measuring for you – this effect is known as *decoherence*, and tends to destroy quantum correlations if the system is at nonzero temperature. The 2012 Nobel Prize was given for two experimental setups which delay this decoherence for as long as possible, and permit certain nondestructive measurements which do not change the state of the system. The former goal is accomplished by cooling systems to very low temperatures and trapping single atoms or photons, and the latter goal uses the clever trick of encoding a measurement of one quantity (for example, the number of photons in a cavity) into the phase of another quantum system (an atom traversing the cavity). A measurement of the phase of the atom will of course destroy the atom's quantum state, but won't touch the photon, allowing it to stay in the cavity and be measured again.

- **2011: Accelerated expansion of the universe.** We've already talked about how the universe is expanding, with a scale factor $a(t)$ that grows with time, $\dot{a}(t) > 0$. The 2011 Nobel prize was given for the discovery that $\ddot{a}(t) > 0$; in other words, *the expansion of the universe is accelerating.* The main evidence for this striking fact came from galaxy surveys which correlated distance with recession velocity, using the more sophisticated time-dependent version of Hubble's law and the relationship between the Hubble parameter and the scale factor. The discovery of accelerated expansion has many important consequences, including the fact that the main component of the energy density of the universe today is not matter, nor dark matter, but whatever substance is responsible for the acceleration, called *dark energy* for lack of a better word. Most evidence suggests that dark energy is actually a *cosmological constant*, a homogeneous energy density throughout all of space with the surprising properties that it exerts negative pressure and does not dilute as the universe expands. But the jury is still out on the nature of dark energy, which is considered one of the most important unsolved problems in fundamental physics today.

- **2010: Isolation of graphene.** Two condensed matter physicists isolated graphene by ripping flakes of it off a lump of graphite using sticky tape. Carbon occurs in many different forms in nature (charcoal, diamond, fullerenes, and so forth), known as *allotropes.* Graphite is one of these, and in fact is made up of a huge number of stacked two-dimensional layers known as graphene. In graphene, carbon atoms are arranged in a hexagonal lattice, which results in highly unusual behavior of the covalently bonded electrons: they behave like *massless particles*, with a linear dispersion relation $\omega \propto k$, rather than massive ones which have quadratic dispersion $\omega \propto k^2$. This makes graphene an excellent conductor, and the fact that it is one atom thick gives rise to many possible engineering applications.

- **2009: Optical fibers and charge-coupled devices.** As you probably know, optical fibers exploit total internal reflection to transmit light over large distances with very little attenuation. The prize was given for a method of fabricating impurity-free glass fibers: this is probably too engineering-heavy to find a place on the GRE.

- **2008: Spontaneously broken symmetry.** One half of this prize was awarded for "the discovery of the mechanism of spontaneous broken symmetry in subatomic physics." To explain how this works in detail would be far beyond what's needed for the GRE, but a few buzzwords might be useful. A system with a certain underlying symmetry can have a ground state which does not respect that symmetry. The standard example is trying to balance a pencil on its tip: even there is no preferred direction for the pencil to fall, it will fall eventually, and pick a direction in doing so. A similar situation can happen in particle physics, and any time the quantum-mechanical ground state does not respect a symmetry which was originally present in the theory, a massless scalar particle appears called a *Nambu-Goldstone boson*. Nambu's original application was to the BCS theory of superconductivity, where gauge invariance is spontaneously broken, giving rise to a massless phonon. But the most well-known application is to the spontaneous breaking of the $\mathrm{SU}(2) \times \mathrm{U}(1)$ gauge symmetry of elementary particle physics, for which the Nambu-Goldstone boson is (the massless partner of) the Higgs boson. The second half of the prize was awarded for an interesting technical result which implies that CP violation requires at least three families of quarks.

- **2007: Giant magnetoresistance.** Probably too specialized to find a place on the GRE.

- **2006: CMB anisotropy.** This prize was awarded for discovering that the low-temperature bath of photons pervading the universe, known as the *cosmic microwave background*, has an almost perfect blackbody spectrum, and that the deviations from this spectrum may be hints of structure formation in the early universe. Shortly after the Big Bang, the universe was so hot and dense that photons had an extremely small mean free path. Hence we can't use light to determine anything about the very early universe, because the photons didn't travel in straight lines. After about 380,000 years, though, the universe expanded and cooled enough that the photons hit a *surface of last scattering* after which they were free to stream through the universe unimpeded. This coincided with the *epoch of recombination*, when electrons and protons combined to form hydrogen atoms. At this time, the temperature of the universe was about 3000 K, but since then the universe has expanded so drastically that these photons are now highly redshifted, as we discussed earlier. Their temperature today is 2.7 K, corresponding to a blackbody peak at a wavelength of 1.9 mm – this is in the microwave region, hence the name. At a level of about one part in 100,000, the spectrum of this radiation bath deviates from the blackbody spectrum, corresponding to density perturbations in the early universe; many of these have been correlated with the positions of galaxies and galaxy clusters today.

8.5 Review problems

1. Which of the following is a possible decay mode of the π^+? (Note: $\overline{\nu}$ denotes an antineutrino.)

   (A) $e^+\overline{\nu}_e$

   (B) $e^+\nu_e$

   (C) $\mu^+\overline{\nu}_\mu$

   (D) $\mu^-\nu_e$

   (E) $\mu^-\overline{\nu}_\mu$

2. A typical hadron has a lifetime of about 10^{-23} seconds. A new particle is discovered with lifetime of 10^{-10} seconds, and its principal decay mode includes a neutrino. This new particle most likely decays due to the

   (A) weak force

   (B) strong force

   (C) gravitational force

   (D) electromagnetic force

   (E) Higgs force

3. In the quark model, the Δ^{++}, a spin-3/2 resonance of charge +2, consists of which of the following combinations of quarks?

   (A) uuu

   (B) uud

   (C) udd

   (D) ddd

   (E) ud

4. The extreme uniformity of the temperature of the cosmic microwave background radiation is evidence for which of the following theories of the early universe?

   (A) Quintessence

   (B) Big Bang

   (C) Inflation

   (D) Modified Newtonian Dynamics

   (E) Scalar-tensor theory

5. The Meissner effect refers to the expulsion of magnetic fields from a superconductor. The exponential decrease in field strength inside the conductor can be modeled by giving the photon an effective

   (A) positive electric charge

   (B) negative electric charge

   (C) mass

   (D) spin

   (E) color charge

6. The electrical conductivity of a relatively pure semiconductor increases with increasing temperature primarily because:

   (A) The scattering of the charge carriers decreases

   (B) The density of the charge carriers increases

   (C) The density of the material decreases due to volume expansion

   (D) The electric field penetrates further into the material

   (E) The lattice vibrations increase in amplitude

7. Nuclei are held together by the strong nuclear force. At approximately what distance does the Coulomb repulsion between two protons overtake the attractive strong force?

   (A) 2×10^{-6} m

   (B) 2×10^{-9} m

   (C) 2×10^{-12} m

   (D) 2×10^{-15} m

   (E) 2×10^{-18} m

8.6 Solutions

1. B - The only decay which does not violate conservation of lepton number or charge is B, with one anti-electron and one electron neutrino, and a final charge of +1.

2. A - The presence of a neutrino is a dead ringer for the weak force. The longer lifetime is also important, since the weak force is (by its very name) weaker than the strong force, but electromagnetic decays are also slow.

3. A - Recalling that the up quark has charge +2/3, the only combination with charge +2 is uuu. This is also consistent with spin-3/2, which is one possible state from addition of angular momentum applied to three spin-1/2 quarks.

4. C - Inflation was a period of exponential expansion of the universe, responsible for "smoothing out" all inhomogeneities in the early universe. This manifests itself in the fact that the cosmic microwave background has almost the same temperature at diametrically opposite points on the sky, despite the fact that light had no time to travel between them at the surface of last scattering.

5. C - An exponentially decreasing force law is characteristic of a massive particle. This is the same physics responsible for the exponentially decreasing strong force outside the nucleus due to exchange of massive pions.

6. B - The conductivity for semiconductor increases as the number of free electrons increases. The density for free electrons in the material is increased by thermal excitation.

7. D - You should be familiar with the fact that the characteristic range of the nuclear force is 1 fm, or 10^{-15} m.