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SAMPLE EXAM 2

Time — 170 minutes

100 questions

Directions: Each of the questions or incomplete statements below is followed by five suggested answers or completions. Select the one that is best in each case and then fill in the corresponding space on the answer sheet.

TABLE OF INFORMATION

Rest mass of the electron	m_e	$=$	9.11×10^{-31} kg
Magnitude of the electron charge	e	$=$	1.60×10^{-19} C
Avogadro's number	N_A	$=$	6.02×10^{23}
Universal gas constant	R	$=$	8.31 J/mol $\cdot$ K
Boltzmann's constant	k	$=$	1.38×10^{-23} J/K
Speed of light	c	$=$	3.00×10^{8} m/s
Planck's constant	h	$=$	$6.63 \times 10^{-34}\mathrm{J \cdot s} = 4.14 \times 10^{-15}$ eV $\cdot$ s
	$\hbar$	$=$	$h/2\pi$
	hc	$=$	1240 eV $\cdot$ nm
Vacuum permittivity	ϵ_0	$=$	8.85×10^{-12} $\mathrm{C^2/(N \cdot m^2)}$
Vacuum permeability	μ_0	$=$	$4\pi \times 10^{-7}$ T $\cdot$ m/A
Universal gravitational constant	G	$=$	6.67×10^{-11} $\mathrm{m^3/(kg \cdot s^2)}$
Acceleration due to gravity	g	$=$	9.80 $\mathrm{m/s^2}$
1 atmosphere pressure	1 atm	$=$	1.0×10^{5} $\mathrm{N/m^2}$ $= 1.0 \times 10^{5}$ Pa
1 angstrom	1 Å	$=$	1×10^{-10} m $= 0.1$ nm

Prefixes for Powers of 10

10^{-15}	femto	f
10^{-12}	pico	p
10^{-9}	nano	n
10^{-6}	micro	μ
10^{-3}	milli	m
10^{-2}	centi	c
10^{3}	kilo	k
10^{6}	mega	M
10^{9}	giga	G
10^{12}	tera	T
10^{15}	peta	P

Moments of inertia about center of mass

Rod	$\frac{1}{12}M\ell^2$
Disc	$\frac{1}{2}MR^2$
Sphere	$\frac{2}{5}MR^2$

1. A ball of mass m is dropped from a tall building, and experiences a velocity-dependent air resistance force $F = bv$. What is its terminal velocity?

   (A) $\dfrac{b}{mg}$

   (B) $\dfrac{mb}{g}$

   (C) $e^{b/m}$

   (D) $\dfrac{mg}{b}$

   (E) $\dfrac{mg}{b}(1 - e^{-b/m})$

2. A charged particle moving in the x-direction enters a region of uniform magnetic field $\mathbf{B} = B_0(\hat{\mathbf{x}} + \hat{\mathbf{y}})$. The path of the particle after it enters the field is a

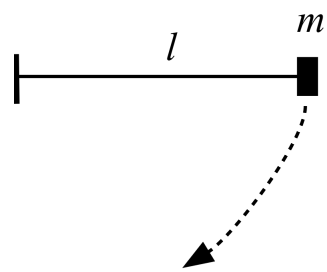
   (A) circle

   (B) cycloid

   (C) helix

   (D) straight line

   (E) logarithmic spiral



3. A massless rope of length l, attached to a fixed pivot at one end and with a mass m at the other end, is held horizontally and then released, as shown in the diagram. When the mass is at its lowest point, the tension in the rope is

   (A) 0

   (B) $gl/2$

   (C) mg

   (D) $2mg$

   (E) $3mg$

4. A particle of charge q and mass m is suspended from a massless string. A constant electric field of known magnitude is turned on, perpendicular to the direction of gravity, and the rope forms some angle α with the vertical. A measurement of α determines which of the following quantities?

   (A) m

   (B) q

   (C) q/m

   (D) qm

   (E) None of the above

5. A hydrogen atom transitions from the $n = 3$ to $n = 2$ states by emitting a photon. What is the wavelength of the photon?

   (A) 347 nm

   (B) 657 nm

   (C) 985 nm

   (D) 2.32 μm

   (E) 1.34 mm

6. For a quantum operator to represent a physical observable, it must be

(A) Hermitian
(B) positive-definite
(C) finite-dimensional
(D) symmetric
(E) none of the above

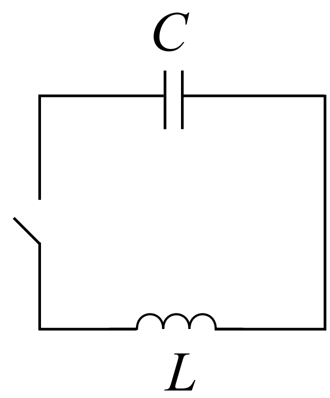
7. If the net force on an object is zero, which of the following MUST be true?

I. Its angular momentum is constant.
II. Its velocity is zero.
III. Its acceleration is zero.

(A) I only
(B) II only
(C) III only
(D) I and II
(E) I and III

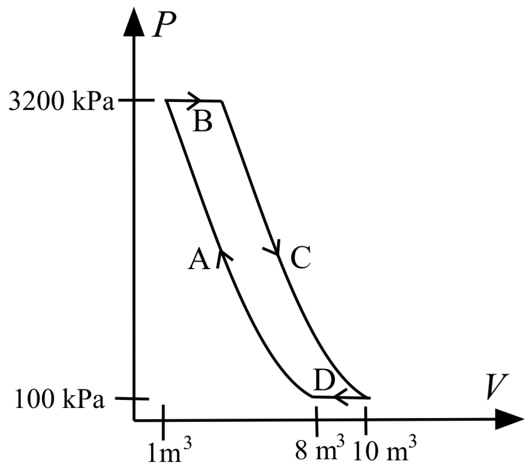
8. Fluorine is not naturally found as free atoms, but rather in compounds as the ion Fl^-. The electron configuration of a neutral fluorine atom is

(A) $1s^2\,2s^1$
(B) $1s^2\,2s^2$
(C) $1s^2\,2s^2\,2p^1$
(D) $1s^2\,2s^2\,2p^5$
(E) $1s^2\,2s^2\,2p^6$



9. In the circuit shown in the diagram, the capacitor is initially charged, and the switch is closed at $t = 0$. Assuming all circuit elements have negligible resistance, the peak magnitude of the current is achieved at

(A) $t = \frac{\pi}{4}\sqrt{\frac{L}{C}}$
(B) $t = \frac{\pi}{\sqrt{LC}}$
(C) $t = 2\pi\sqrt{LC}$
(D) $t = \frac{\pi}{2}\sqrt{LC}$
(E) $t = \frac{\pi}{2\sqrt{LC}}$



Questions 10-11 refer to the $P-V$ diagram of an ideal gas undergoing the Brayton cycle. Steps A and C are isentropic, and steps B and D are isobaric.

10. What is the approximate work done by the gas over one cycle?

(A) -6200 kJ

(B) -3100 kJ

(C) 0

(D) 3100 kJ

(E) 6200 kJ

11. The gas used in the cycle is most likely

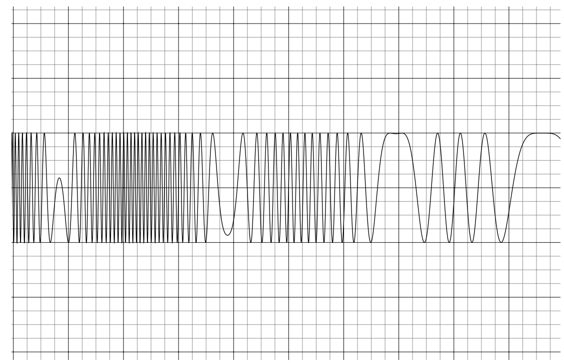
(A) monoatomic

(B) diatomic

(C) triatomic

(D) ionized

(E) heteronuclear



12. The oscilloscope trace shown in the diagram is an example of

(A) frequency modulation

(B) amplitude modulation

(C) pulse-code modulation

(D) single-sideband modulation

(E) clipping

13. Kepler's third law states that the ratio T_1/T_2 of the periods of two planets orbiting the sun is proportional to $(a_1/a_2)^{3/2}$, where a_1 and a_2 are the semi-major axes of the orbits of the two planets. Ignoring relativistic corrections and the gravitational influence of bodies other than the sun, this statement of Kepler's third law is only *approximately* true because

(A) The inverse-square force law fails at long distances

(B) Different planets have different masses

(C) The periods T_1 and T_2 are not constant

(D) Planetary orbits are not precisely elliptical

(E) Gravitational potential energy is not constant along each orbit

14. A spin-1/2 particle has the angular wavefunction

$$\psi(\theta,\phi) = \frac{1}{\sqrt{2}}\left(Y_3^0(\theta,\phi) + Y_2^1(\theta,\phi)\right)$$

where $Y_l^m(\theta,\phi)$ are the normalized spherical harmonics. Which of the following is a possible result of measuring the particle's total spin quantum numbers j and m_j?

(A) $j = 3,\ m_j = 0$
(B) $j = 2,\ m_j = 1/2$
(C) $j = 7/2,\ m_j = -1/2$
(D) $j = 7/2,\ m_j = 3/2$
(E) $j = 9/2,\ m_j = -1/2$

15. The normalized energy eigenfunctions of the infinite square well of size L are $\psi_n(x) = \sqrt{\frac{2}{L}}\sin(n\pi x/L)$. The expectation value of energy of the state

$$\Psi = \frac{1}{\sqrt{2}}\psi_2 + \frac{1}{\sqrt{3}}\psi_3 + \frac{1}{\sqrt{6}}\psi_4$$

for a particle of mass m is

(A) $\frac{4\pi^2\hbar^2}{3mL^2}$
(B) $\frac{8\pi^2\hbar^2}{3mL^2}$
(C) $\frac{17\pi^2\hbar^2}{4mL^2}$
(D) $\frac{14\pi^2\hbar^2}{3mL^2}$
(E) $\frac{23\pi^2\hbar^2}{6mL^2}$

16. An ideal gas is maintained at a temperature of 250 K through contact with a thermal reservoir and is free to expand against a piston. If 5000 J of heat is slowly added to the gas, what is the change in entropy of the gas?

(A) 10 J/K
(B) $10\ln 2$ J/K
(C) 20 J/K
(D) 40 J/K
(E) 500 J/K

17. The photoelectric effect provides direct experimental evidence for which of the following properties of light?

I. It has two linearly independent polarization states
II. It carries kinetic energy proportional to its frequency.
III. It travels at a constant speed c in vacuum.

(A) I only
(B) II only
(C) I and II
(D) II and III
(E) I, II, and III

18. The Hamiltonian $H = eE_0z$, describing an atomic electron of charge $-e$ interacting with a uniform electric field in the z-direction, is responsible for

(A) the Zeeman effect
(B) the Lamb shift
(C) hyperfine splitting
(D) the Stark effect
(E) stimulated emission

19. A block of mass m_1 moving with velocity v collides elastically with an block of mass m_2 at rest. If m_1 continues moving in the same direction it did prior to the collision, one can conclude

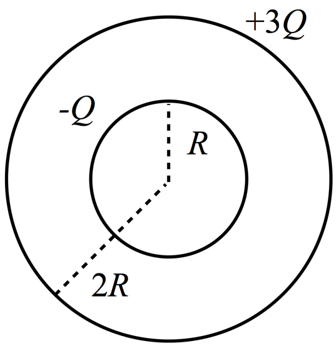
    (A) $m_1 > m_2$
    (B) $m_1 = m_2$
    (C) $m_1 < m_2$
    (D) Momentum was not conserved in this collision
    (E) None of the above

20. A 20 cm tall slice of a spherical mirror is placed on an ATM such that the image of a 2-meter tall person 1 meter away from the ATM will just fill the surface of the mirror. What must the radius of curvature R and convexity of the mirror be?

    (A) $R = 22\,\text{cm}$, convex
    (B) $R = 40\,\text{cm}$, concave
    (C) $R = 80\,\text{cm}$, concave
    (D) $R = 4.5\,\text{m}$, convex
    (E) $R = 18\,\text{m}$, convex

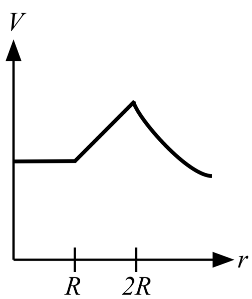
21. The quantized vibrations of a crystal lattice are called

    (A) photons
    (B) anyons
    (C) phonons
    (D) vibrons
    (E) rotons

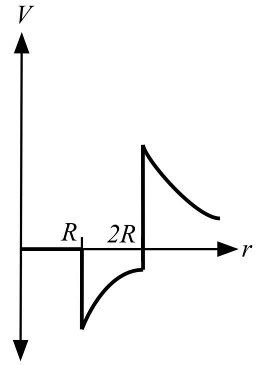


22. Shown in the diagram are two concentric thin spherical shells of radii R and $2R$, the outer one carrying charge $+3Q$ and the inner one carrying charge $-Q$. Setting the electric potential equal to zero at infinity, which of the following graphs best represents the electric potential as a function of r, the distance from the center of the shells?

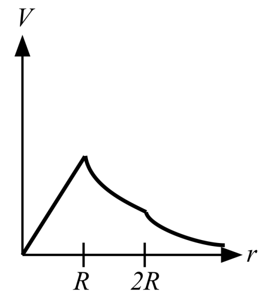
(A)



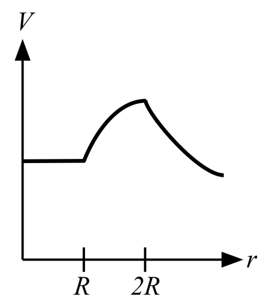
(B)



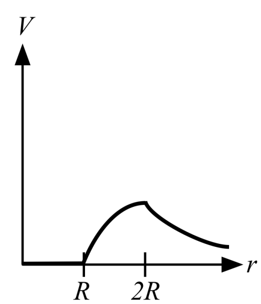
(C)



(D)



(E)



23. The force-carrying particle responsible for binding the quarks in the proton is called the

(A) photon
(B) gluon
(C) W boson
(D) Z boson
(E) Higgs boson

24. A charge $-e$ at the origin is subject to a uniform electric field $\mathbf{E} = -E\hat{\mathbf{x}}$. After traveling to $(3, 4)$, what is the change in potential energy of the charge?

(A) $-3Ee$
(B) $3Ee$
(C) $-5Ee$
(D) $5Ee$
(E) $-7Ee$

25. Which of the following are true statements about Gauss's law for magnetism, $\nabla \cdot \mathbf{B} = 0$?

I. It implies that magnetic monopoles do not exist in nature.
II. It is incompatible with the continuity equation for $\mathbf{J}$.
III. It allows the definition of a vector potential.

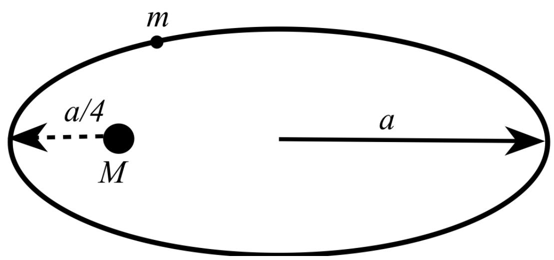
(A) I only
(B) II only
(C) III only
(D) I and II
(E) I and III

26. A relativistic particle of mass m has momentum $p = mc$. What is the particle's energy?

(A) mc^2
(B) $\sqrt{2}mc^2$
(C) $2mc^2$
(D) $4mc^2$
(E) None of the above

27. The hot, dense gas of electrons and positive ions known as a plasma is capable of supporting charge density waves known as plasma oscillations. Let n_e be the number density of electrons, e the charge of the electrons, and m^* an effective mass of the electrons in the plasma. The frequency of plasma oscillations is

(A) $\omega = \dfrac{\epsilon_0}{n_e e^2 m^*}$
(B) $\omega = \dfrac{m^* e^2}{n_e \epsilon_0}$
(C) $\omega = \dfrac{e^2}{n_e \epsilon_0}$
(D) $\omega = \sqrt{\dfrac{n_e e^2 m^*}{\epsilon_0}}$
(E) $\omega = \sqrt{\dfrac{n_e e^2}{m^* \epsilon_0}}$



28. A planet of mass m orbits a star of mass M in an elliptical orbit with semi-major axis a, as shown in the diagram. The distance of closest approach to the planet is $a/4$. Assuming $m \ll M$, the ratio of the planet's speed at perigee (when the planet is closest to the star) to the planet's speed at apogee (when the planet is furthest away from the star) is

(A) 1/4

(B) 1/3

(C) 4

(D) 7

(E) 16

29. Two identical sailboats race across a lake, starting from rest. Boat 1 reaches the finish line first with velocity v_1, and boat 2 arrives later with velocity $v_2 > v_1$. Let F_{t1} and F_{t2} be the average force per unit *time* on boats 1 and 2, respectively, and let F_{d1} and F_{d2} be the average force per unit *distance* on boats 1 and 2. Which of the following MUST be true?

(A) $F_{t1} > F_{t2}$

(B) $F_{t1} < F_{t2}$

(C) $F_{d1} > F_{d2}$

(D) $F_{d1} < F_{d2}$

(E) None of the above

30. The first two normalized position-space energy eigenfunctions of the harmonic oscillator Hamiltonian $H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2$ are

$$\psi_0(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} e^{-m\omega x^2/2\hbar}$$
$$\psi_1(x) = \sqrt{2}\left(\frac{m\omega}{\pi\hbar}\right)^{1/4} xe^{-m\omega x^2/2\hbar}$$

A delta-function perturbation $V(x) = \epsilon\delta(x)$ is added to the harmonic oscillator Hamiltonian. What are the new energies E_0 and E_1 to first order in perturbation theory?

(A) $E_0 = \hbar\omega/2$, $E_1 = 3\hbar\omega/2$

(B) $E_0 = \hbar\omega/2 + \epsilon\left(\frac{m\omega}{\pi\hbar}\right)^{1/4}$, $E_1 = 3\hbar\omega/2$

(C) $E_0 = \hbar\omega/2 + \epsilon\sqrt{\frac{m\omega}{\pi\hbar}}$, $E_1 = 3\hbar\omega/2$

(D) $E_0 = \hbar\omega/2 + \epsilon\sqrt{\frac{m\omega}{\pi\hbar}}$, $E_1 = 3\hbar\omega/2 + 2\epsilon\sqrt{\frac{m\omega}{\pi\hbar}}$

(E) $E_0 = \hbar\omega + \epsilon\sqrt{\frac{m\omega}{\pi\hbar}}$, $E_1 = 2\hbar\omega$

31. At sufficiently high temperature T, which of the following contributes to the total energy of a diatomic molecule?

I. Translational kinetic energy
II. Rotational kinetic energy
III. Vibrational potential energy

(A) I only

(B) II only

(C) I and II

(D) I and III

(E) I, II and III

32. During the adiabatic expansion phase of a Carnot cycle, one mole of gas expands to twice its original size. The change in entropy of the gas during this process is

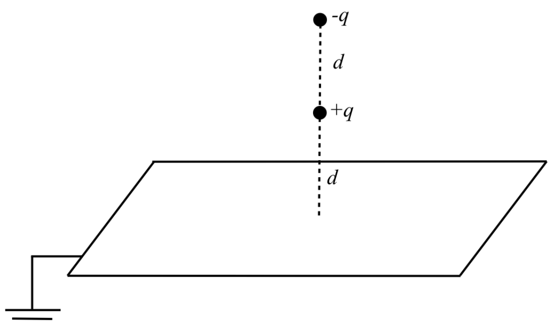
(A) $R \ln 2$

(B) $-R \ln 2$

(C) $2R$

(D) $-2R$

(E) 0

33. 

A particle of charge $+q$ is placed at the point $(0, 0, d)$, between an infinite grounded conducting plate at $z = 0$ and a stationary charge $-q$ at $(0, 0, 2d)$, as shown in the diagram. What is the force on the charge $+q$?

(A) $\dfrac{q^2}{8\pi\epsilon_0 d^2}\hat{\mathbf{z}}$

(B) $\dfrac{7q^2}{24\pi\epsilon_0 d^2}\hat{\mathbf{z}}$

(C) $\dfrac{11q^2}{72\pi\epsilon_0 d^2}\hat{\mathbf{z}}$

(D) $\dfrac{25q^2}{72\pi\epsilon_0 d^2}\hat{\mathbf{z}}$

(E) $\dfrac{31q^2}{144\pi\epsilon_0 d^2}\hat{\mathbf{z}}$

Questions 34-35 refer to the following scenario. A new star is discovered with an optical telescope, from which it is deduced that it emits most of its power in the orange region of the visible spectrum, at wavelength approximately 600 nm.

34. Assuming the star behaves as a black-body and neglecting possible redshift, what is its approximate temperature?

(A) 200 K

(B) 5000 K

(C) 6×10^4 K

(D) 3×10^6 K

(E) 2×10^9 K

35. It is later discovered that the star is receding at a peculiar velocity of $0.2c$. The ratio between the true temperature T_{true} and the measured temperature T_{meas} is

(A) 1/5

(B) 2/3

(C) 3/2

(D) $\sqrt{2/3}$

(E) $\sqrt{3/2}$

36. A one-dimensional system has Lagrangian

$$L(q, \dot{q}, t) = A\dot{q}^2 + \sin(q/L - \omega t)$$

for constants A, L, and ω. What is the Euler-Lagrange equation of motion?

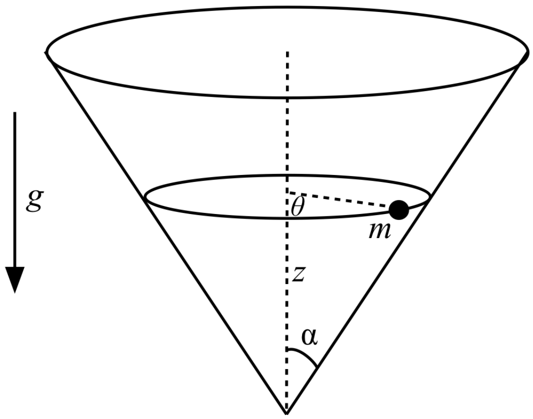
(A) $\dot{q} = \dfrac{\omega}{2AL}\sin(q/L - \omega t)$

(B) $\dot{q} = -\dfrac{\omega}{A}\cos(q/L - \omega t)$

(C) $\dot{q} = -\dfrac{\omega}{2A}\cos(q/L - \omega t)$

(D) $\ddot{q} = \dfrac{1}{2AL}\cos(q/L - \omega t)$

(E) $\ddot{q} = -\dfrac{1}{AL}\cos(q/L - \omega t)$



37. A particle of mass m is constrained to move on a cone of opening angle α, oriented as shown in the diagram. What is a possible Hamiltonian for this system, in terms of the coordinates z and θ and their conjugate momenta?

(A) $H = \frac{p_z^2}{2m}\tan^2\alpha + \frac{p_\theta^2}{2mz^2}\tan^2\alpha - mgz$

(B) $H = \frac{mp_z^2}{2} + \frac{mp_\theta^2 z^2}{2}\tan^2\alpha - mgz$

(C) $H = \frac{p_z^2}{m}\cos^2\alpha + \frac{p_\theta^2}{mz^2}\cot^2\alpha + mgz$

(D) $H = \frac{p_z^2}{2m}\cos^2\alpha + \frac{p_\theta^2}{2mz^2}\cot^2\alpha + mgz$

(E) $H = \frac{1}{2}mz^2\tan^2\alpha\,\dot{\theta}^2 + \frac{1}{2}m\tan^2\alpha\,\dot{z}^2 + mgz$

38. The specific heat at constant volume, C_V, of a solid is observed at low temperatures T to follow the Debye law $C_V = AT^3$ with A a constant. What is the internal energy of the solid $U(T)$ as a function of temperature, assuming $U(0) = 0$, in the regime of validity of the Debye law?

(A) $3AT^2$

(B) AT^3

(C) $\frac{1}{3}AT^3$

(D) $\frac{1}{4}AT^4$

(E) AT^4

39. Let $|n\rangle$ denote a set of real, orthonormal energy eigenfunctions of a Hamiltonian $\hat{H}$ in one dimension, with energies E_n. Let $\hat{p}$ denote the momentum operator. Which of the following must be true?

I. $\langle m|n\rangle = m + n$

II. $\langle n|\hat{p}|n\rangle = 0$

III. $\langle m|\hat{H}|n\rangle = \delta_{mn}E_n$

(A) I only

(B) II only

(C) III only

(D) I and II

(E) II and III

40. The Meissner effect refers to the tendency of superconductors to

(A) develop a surface charge density

(B) expel magnetic fields

(C) acquire a finite resistance at a critical temperature T_c

(D) spontaneously develop an internal electric field

(E) have persistent currents

41. The hydrogen isotope tritium, ^{3}H, contains one proton and two neutrons and has a half-life of approximately 12 years. The binding energy of tritium is closest to

(A) 8.5 eV

(B) 8.5 keV

(C) 8.5 MeV

(D) 8.5 GeV

(E) 8.5 TeV

42. A positive muon stopped by matter can attract an electron to form an exotic bound state known as muonium, where the muon (which has the same charge as the proton) acts as the nucleus. Let m_μ be the mass of the muon, m_e the mass of the electron, and m_p the mass of the proton. What is the Bohr radius of muonium, in terms of the Bohr radius of ordinary hydrogen a_0 and the masses of the particles?

(A) $a_0 \dfrac{m_p}{m_\mu}$

(B) $a_0 \dfrac{m_\mu}{m_p}$

(C) $a_0 \left(\dfrac{m_\mu}{m_p}\right)^2$

(D) $a_0 \dfrac{m_p(m_e + m_\mu)}{m_\mu(m_e + m_p)}$

(E) $a_0 \dfrac{m_p(m_e + m_p)}{m_\mu(m_e + m_\mu)}$

43. Let $\hat{x}$ and $\hat{p}$ be the quantum-mechanical position and momentum operators, respectively. The commutator $[\hat{x}, \hat{p}^2]$ is equivalent to which of the following?

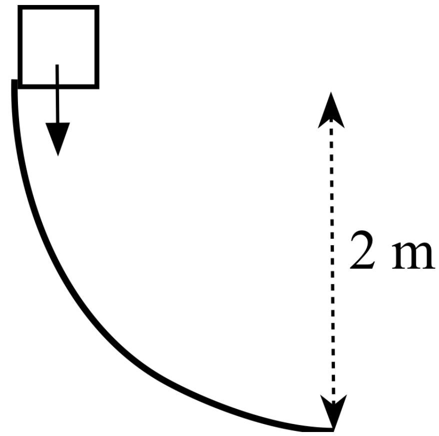
(A) 0

(B) $i\hbar$

(C) $-i\hbar\hat{x}$

(D) $2i\hbar\hat{p}$

(E) $2i\hbar$



44. A block of mass 3.14 kg slides down a ramp in the shape of a quarter-circle of radius 2 m, as shown in the diagram. If the block reaches the bottom of the ramp with velocity 4 m/s, then ignoring air resistance, the work done by friction during the slide down the ramp is most nearly

(A) 10.8 J

(B) 11.6 J

(C) 22.7 J

(D) 36.4 J

(E) 75.2 J

45. Consider an infinite charge-carrying wire, of charge per unit length λ. Setting the zero of electric potential at distance a from the wire, what is the electric potential as a function of the distance r from the wire?

(A) $\dfrac{\lambda}{2\pi\epsilon_0} \ln(a/r)$

(B) $\dfrac{\lambda}{4\pi\epsilon_0} \ln(a/r)$

(C) $\dfrac{\lambda}{2\pi\epsilon_0} \ln(r/a)$

(D) $\dfrac{\lambda}{4\pi\epsilon_0 r}$

(E) $\dfrac{\lambda(r-a)}{4\pi\epsilon_0 r}$

46. A ideal beam-splitter is an optical device which lets part of an incident beam of light pass through and reflects the remainder, with no absorption taking place in the beam-splitter. Let the incident beam have complex amplitude E, the reflected beam have amplitude E_r, and the transmitted beam have amplitude E_t. Which of the following MUST be true?

(A) $E_r = E/\sqrt{2}$

(B) $E_r = E/2$

(C) $E = E_r + E_t$

(D) $|E|^2 = |E_r|^2 + |E_t|^2$

(E) $|E|^4 = |E_r|^4 + |E_t|^4$

47. The Hamiltonian operator for a free particle of mass m moving in three dimensions is

(A) $-\dfrac{\hbar^2\nabla^2}{m}$

(B) $-\dfrac{\hbar^2\nabla^2}{2m}$

(C) $-i\hbar\nabla$

(D) $\dfrac{\hbar^2\nabla^2}{2m}$

(E) 0

48. In proton therapy, medium-energy protons are directed at a cancer patient's tumor in order to irradiate it. Which of the following pieces of information would be MOST useful in determining the correct energy and angle with which to fire the protons?

(A) The charge-to-mass ratio of the proton

(B) A plot of the proton's energy loss per unit distance traveled in human tissue

(C) The distance traveled in human tissue as a function of energy

(D) The mean lifetime of the proton

(E) The binding energy of the proton

49. Student A, of mass 100 kg, stands 2 meters from the center of a circular platform which is free to rotate on frictionless bearings. Student B, not standing on the platform, tosses student A a baseball of mass 0.09 kg, which reaches student A with a velocity of 20 m/s directed perpendicular to the line joining student A and the center of the platform. If the platform has moment of inertia 50 kg·m^2, what is its angular velocity immediately after student A catches the baseball?

(A) 0.008 rad/s

(B) 0.009 rad/s

(C) 0.0144 rad/s

(D) 0.018 rad/s

(E) 0.072 rad/s

50. In Compton scattering, the change in wavelength of the scattered light is given in terms of the electron mass m_e and the scattering angle θ by which of the following?

(A) $\dfrac{h}{m_e c}(1-\cos\theta)$

(B) $\dfrac{m_e}{hc}(1+\cos\theta)$

(C) $\dfrac{h}{m_e c}(1+\sin\theta)$

(D) $\dfrac{m_e}{hc}\sin^2\theta$

(E) $\dfrac{1}{hcm_e}(1-\sin\theta)$

51. The binding energy of the electron in the Li^{++} ion is approximately

(A) 1.51 eV

(B) 13.6 eV

(C) 40.8 eV

(D) 122.4 eV

(E) 1102 eV

52. Which of the following is NOT a true statement about the third law of thermodynamics?

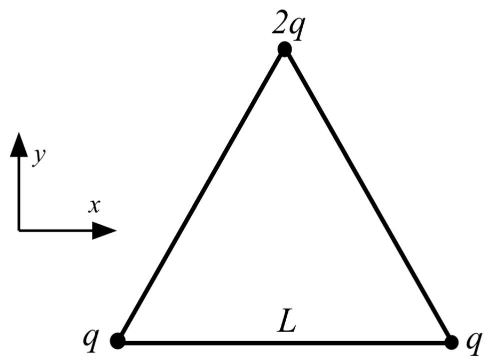
(A) It implies that the entropy of a perfect crystal of a pure substance must approach zero at absolute zero

(B) It implies that the entropy of an isolated system can sometimes decrease

(C) It is a consequence of the fact that the ground state degeneracy of a system determines its entropy

(D) It implies that absolute zero can never be reached in experiments

(E) It permits the entropy of a system to be nonzero at absolute zero



53. Questions 53-54 refer to the diagram above, with charges q, q, and $2q$ are placed at the corners of an equilateral triangle of side length L, and the $\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$ axes oriented as shown. What is the electric field at the center of the triangle?

(A) $\frac{q}{4\pi\epsilon_0 L^2}(\hat{\mathbf{x}} + \sqrt{3}\hat{\mathbf{y}})$

(B) $-\frac{3q}{4\pi\epsilon_0 L^2}\hat{\mathbf{y}}$

(C) $\frac{4q}{3\pi\epsilon_0 L^2}\hat{\mathbf{y}}$

(D) $-\frac{q}{\pi\epsilon_0 L\sqrt{3}}\hat{\mathbf{y}}$

(E) 0

54. What is the electric potential at the center of the triangle?

(A) $\frac{q\sqrt{3}}{\pi\epsilon_0 L}$

(B) $\frac{q\sqrt{3}}{4\pi\epsilon_0 L}$

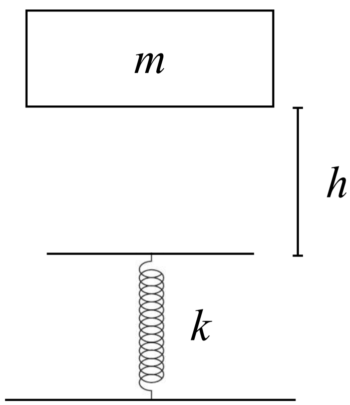
(C) $\frac{4q\sqrt{3}}{3\pi\epsilon_0 L}$

(D) $\frac{4q}{3\pi\epsilon_0 L^2}$

(E) 0

55. A car engine operates between a cold reservoir at temperature 27°C and a hot reservoir at 127°C. What is the minimum amount of heat the engine must absorb from the hot reservoir in a period of 1 minute to obtain a power output of 100 kW?

(A) 1500 kJ
(B) 6000 kJ
(C) 8000 kJ
(D) 18,000 kJ
(E) 24,000 kJ



56. A brick of mass m falls onto a massless spring with spring constant k from a height h above it. What is the maximum distance the spring will be compressed from its equilibrium length?

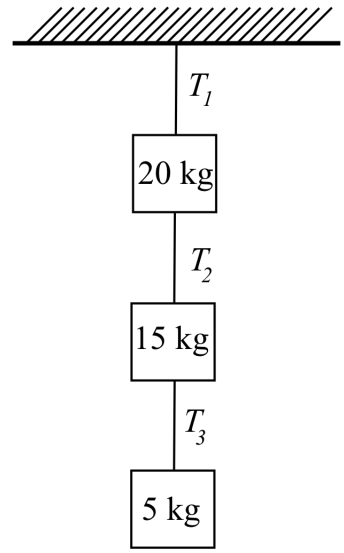
(A) $\dfrac{mg}{k}$

(B) $\dfrac{mk}{2gh}$

(C) $\dfrac{k}{mg}\left(1+\sqrt{\dfrac{mg}{kh}}\right)$

(D) $\dfrac{mg}{k}\left(1+\sqrt{\dfrac{2kh}{mg}}\right)$

(E) $\dfrac{mg}{k}\left(1+\sqrt{1+\dfrac{2kh}{mg}}\right)$



57. Three weights are suspended from a ceiling using massless ropes, as shown in the diagram. The tensions in the ropes are T_1, T_2, and T_3. What is T_1/T_3?

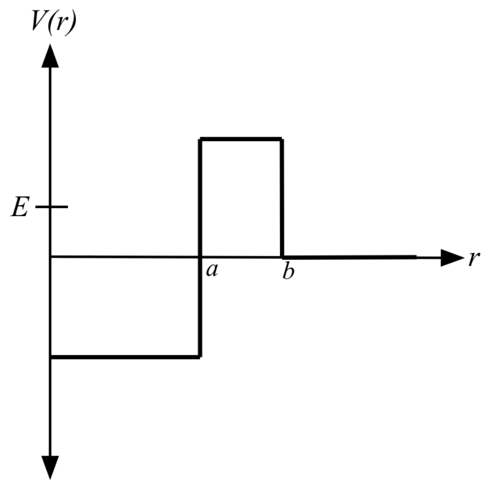
(A) 0.25
(B) 1
(C) 3
(D) 4
(E) 8

58. A particle's normalized spin wavefunction has the form

$$\psi(\theta, \phi) = \sqrt{\frac{3}{2\pi}} \sin\theta \cos 2\phi \sin\phi$$

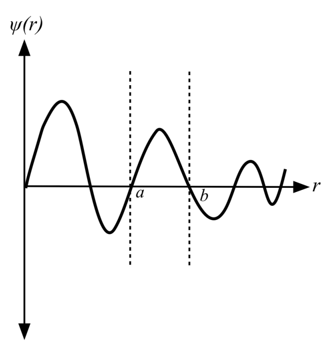
What is the expectation value of the particle's z-component of orbital angular momentum L_z?

(A) 0
(B) $-3\hbar/2\pi$
(C) $3\hbar/2\pi$
(D) $-3\hbar/\pi$
(E) $3\hbar/\pi$

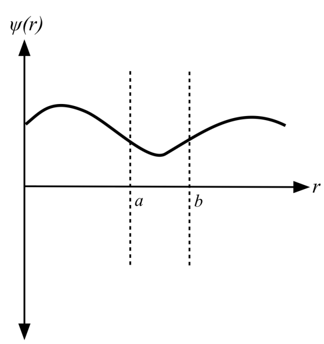


59. The strong nuclear force binding an alpha particle to a nucleus can be modeled the potential shown in the diagram. Which of the following plots best illustrates the radial wavefunction of an alpha particle with energy E which tunnels out of the nucleus in alpha decay?

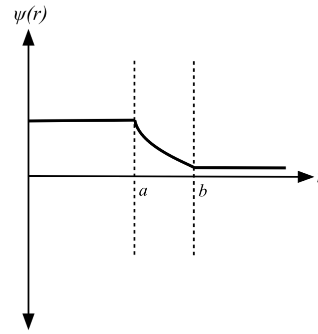
(A)



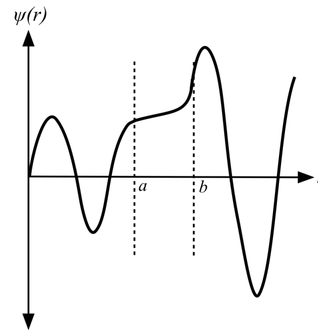
(B)



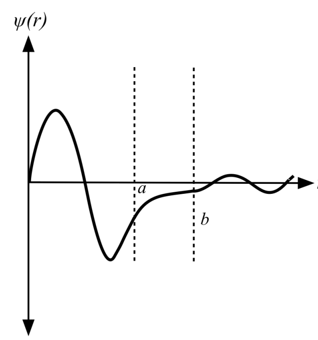
(C)

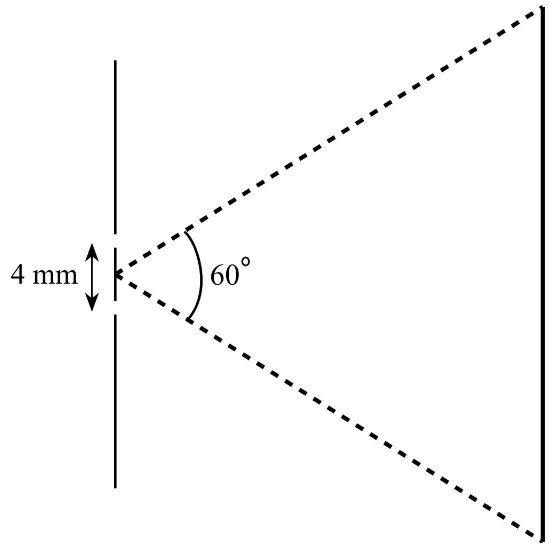


(D)



(E)





60. A coherent beam of monochromatic light of wavelength 500 nm is directed towards two very thin slits separated by a distance 4 mm. Far behind the slits is a screen covering an angular region of 60° as shown in the diagram. Approximately how many bright interference bands are visible on the screen?

(A) 0
(B) 4000
(C) 6800
(D) 8000
(E) 13600

61. A stationary telescope monitoring a rocket ship observes the ship emitting flashes of light at 1 second intervals. If the ship begins moving toward the telescope at speed $0.6c$, with what period does the telescope observe the light flashes?

(A) 0.5 s
(B) 0.8 s
(C) 1 s
(D) 1.2 s
(E) 2 s

62. A particle moving at speed $0.8c$ enters a tube of length 30 m and hits a target at the end of the tube. How far away is the target when the particle enters the tube, in the reference frame of the particle?

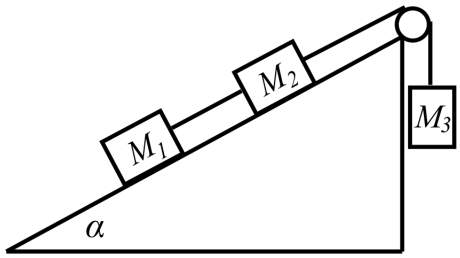
(A) 18 m
(B) 24 m
(C) 30 m
(D) 50 m
(E) 60 m

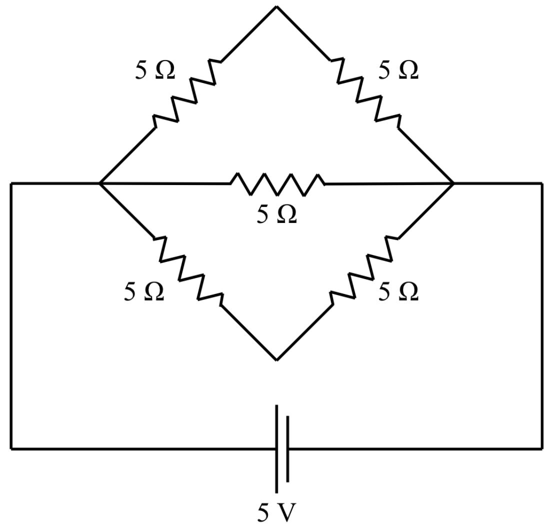
63. The volume of the first Brillouin zone of a simple cubic lattice of lattice spacing a is

(A) a^3
(B) $1/a^3$
(C) $(a/2\pi)^3$
(D) $(2\pi/a)^3$
(E) $a^3/2\pi$

64. Early observations of beta decay of the neutron showed that the emitted electron had a broad energy spectrum, rather than a fixed energy. This was taken as evidence for the existence of the

(A) neutrino
(B) positron
(C) muon
(D) strange quark
(E) pion





65. Blocks of masses M_1, M_2, and M_3 are arranged on a frictionless inclined plane at angle α as shown in the diagram. The pulley at the top of the plane is frictionless and massless, and the system is in static equilibrium. What is α in terms of M_1, M_2, and M_3?

(A) $\tan^{-1}\left(\frac{M_3}{M_1}\right)$

(B) $\sin^{-1}\left(\frac{M_3}{M_1+M_2}\right)$

(C) $\sin^{-1}\left(\frac{M_1+M_2}{M_3}\right)$

(D) $\cos^{-1}\left(\frac{M_1+M_2}{M_3}\right)$

(E) $\cos^{-1}\left(\frac{M_3}{M_1+M_2}\right)$

66. A 5V battery supplies the EMF for the circuit shown in the diagram, where all resistors are 5Ω. What current flows through the circuit? (You may assume the wires are resistanceless and the battery has negligible internal resistance.)

(A) 0.2 A

(B) 0.5 A

(C) 1 A

(D) 2 A

(E) 5 A

67. Questions 67-68 refer to the Pauli matrices,

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$
$$\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

What is the determinant of the matrix

$$M = \sigma_x^2 + \sigma_y^2 + \sigma_z^2?$$

(A) 0
(B) 1
(C) 3
(D) 4
(E) 9

68. The state of a spin-1/2 particle is described by the spinor

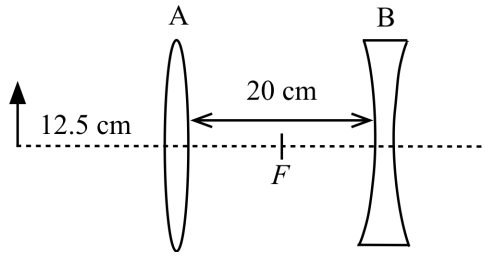
$$\eta = \mathcal{N}\begin{pmatrix} i \\ 2-i \end{pmatrix}$$

where $\mathcal{N}$ is a normalization constant. What is the expectation value of S_x, the spin projection onto the x-axis, in the state η?

(A) $-\hbar$
(B) $-\hbar/3$
(C) $-\hbar/6$
(D) 0
(E) $\hbar/2$

69. Which of the following could represent the displacement of a standing wave?

(A) $\cos(kx - \omega t)$
(B) $\sin(kx - \omega t)$
(C) $(x - vt)^2$
(D) $\sin kx \cos \omega t$
(E) $\omega t \sin^2 kx$



70. A converging lens A and a diverging lens B, both with focal length 10 cm, are arranged so that the midpoint between the lenses F coincides with both lenses' foci. An object is placed 12.5 cm to the left of A. Which of the following gives the correct position and orientation of the the image?

(A) 5 cm to the right of A, inverted
(B) 5 cm to the right of A, upright
(C) 7.5 cm to the right of A, inverted
(D) 30 cm to the right of B, inverted
(E) 30 cm to the right of B, upright

71. A parallel-plate capacitor with capacitance C is at rest in frame $\mathcal{S}$, with the plates of the capacitor parallel to the xy-plane. In frame $\overline{\mathcal{S}}$, moving in the positive $\hat{\mathbf{x}}$ direction at speed v, what is the new capacitance in terms of C?

(A) C
(B) $C\sqrt{1 - v^2/c^2}$
(C) $\dfrac{C}{\sqrt{1 - v^2/c^2}}$
(D) $C(1 - v^2/c^2)$
(E) $\dfrac{C}{1 - v^2/c^2}$

72. In underground particle detection experiments, the main source of naturally occurring ionizing radiation at energies above 200 MeV is

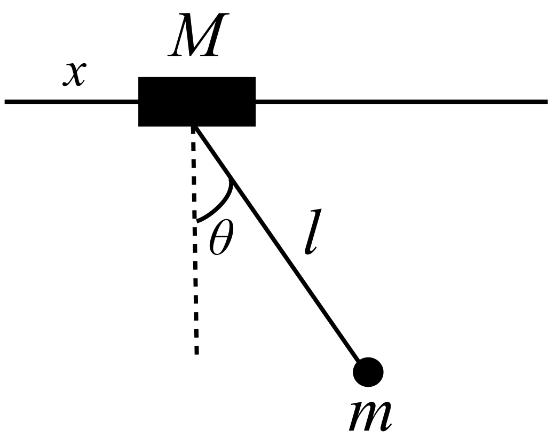
(A) thermal radiation

(B) cosmic ray muons

(C) solar neutrinos

(D) seismic noise

(E) solar flares



73. A point mass m is attached with a massless rod of length l to a pivot of mass M, which is free to slide along a frictionless bar. Letting x be the position of the pivot and θ the angle of the rod, what is a possible Lagrangian for this system?

(A) $L = \frac{1}{2}M\dot{x}^2 + \frac{1}{2}ml^2\dot{\theta}^2 - mgl\cos\theta$

(B) $L = \frac{1}{2}(M+m)\dot{x}^2 + \frac{1}{2}ml^2\dot{\theta}^2 + mgl\cos\theta$

(C) $L = \frac{1}{2}(M+m)\dot{x}^2 + \frac{1}{2}ml^2\dot{\theta}^2 + ml\dot{x}\dot{\theta}\cos\theta + mgl\cos\theta$

(D) $L = \frac{1}{2}(M+m)\dot{x}^2 + \frac{1}{2}ml^2\dot{\theta}^2 + 2ml\dot{x}\dot{\theta}\sin\theta - mgl\cos\theta$

(E) $L = \frac{1}{2}M\dot{x}^2 + \frac{1}{2}ml^2\dot{\theta}^2\sin^2\theta + ml\dot{x}\dot{\theta}\cos^2\theta - mgl\cos\theta$

74. A pipe has cross-sectional area A_1 at one point, but subsequently narrows to a cross-sectional area A_2. If the pressure of an incompressible fluid of density ρ flowing toward the narrow end is p in the first region, and its velocity is v, what is the pressure in the second narrow region?

(A) p

(B) $\frac{1}{2}\frac{A_2^2}{A_1^2}\rho v^2$

(C) $p + \frac{1}{2}\left(\frac{A_2^2}{A_1^2} - 1\right)\rho v^2$

(D) $p + \frac{1}{2}\left(1 - \frac{A_1^2}{A_2^2}\right)\rho v^2$

(E) $p + \frac{1}{2}\left(\frac{A_2^2}{A_1^2} + 1\right)\rho v^2$

75. A particle in three dimensions has radial wavefunction

$$\psi(r) = \begin{cases} 0, & 0 \le r \le a \\ A/r^2, & r > a \end{cases}$$

where A is a normalization constant. What is the probability the particle will be found between $r = a$ and $r = 2a$?

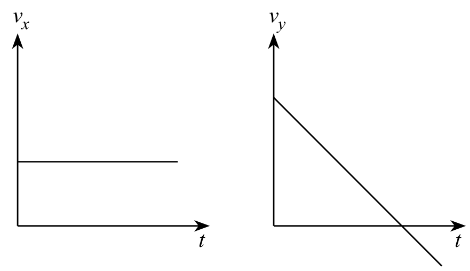
(A) $1/3$

(B) $1/2$

(C) $1/\sqrt{3}$

(D) $1/\sqrt{2}$

(E) 1



76. The diagram above shows plots of components of velocity v_x and v_y versus time t, with identical scales on both plots. Gravity acts in the $-\hat{\mathbf{y}}$-direction. Ignoring air resistance, these plots could represent which of the following scenarios?

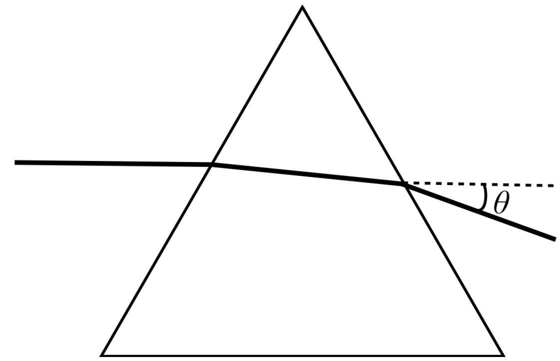
    (A) A ball dropped from the top of a high building

    (B) A rock thrown from a high building at an angle of 45° below the horizontal

    (C) A brick thrown from ground level at an angle of 60° above the horizontal

    (D) A golf ball on an elevated tee struck at an angle of 30° above the horizontal

    (E) A mass attached to a vertical spring which is compressed and then released



77. Monochromatic light in vacuum is incident on an equilateral triangular prism of index of refraction $n = \sqrt{19/12}$. The incident ray is parallel to the base of the prism. What is $\sin\theta$, where θ is the angle between the outgoing ray and the incident ray?

    (A) $\dfrac{12}{19}$

    (B) $\dfrac{3}{4}$

    (C) $\sqrt{\dfrac{19}{48}}$

    (D) $\dfrac{1}{2}(4-\sqrt{3})$

    (E) $\dfrac{1}{8}(3\sqrt{3}-\sqrt{7})$

78. A body of mass m and charge q at the origin is subjected to an electric field $\mathbf{E}(t) = E_0 \sin(\omega t)\hat{\mathbf{x}}$. What is the average power radiated by the charge?

    (A) $\dfrac{q^2 E_0}{4\pi\epsilon_0 c^3 m}$

    (B) $\dfrac{q^2 E_0}{6\pi\epsilon_0 c^3 m}$

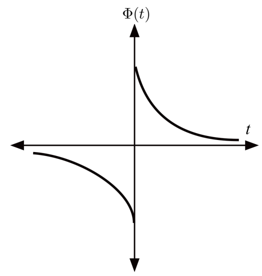
    (C) $\dfrac{q^4 E_0^2}{4\pi\epsilon_0 c^3 m^2}$

    (D) $\dfrac{q^4 E_0^2}{6\pi\epsilon_0 c^3 m^2}$

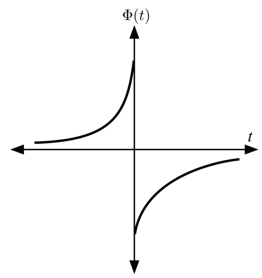
    (E) $\dfrac{q^4 E_0^2}{12\pi\epsilon_0 c^3 m^2}$

79. A magnetic monopole is a hypothetical point particle with magnetic field $\mathbf{B} = \frac{q_m}{r^2}\hat{\mathbf{r}}$. Suppose a magnetic monopole with $q_m > 0$ traveling at constant (nonrelativistic) velocity in the $+\hat{\mathbf{z}}$ direction passed through the center of a loop of wire lying in the xy-plane, at $t = 0$. Which of the following plots best illustrates $\Phi(t)$, the magnetic flux through the loop as a function of t? Assume that the normal to the loop is parallel to the velocity vector of the monopole.

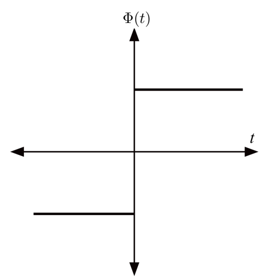
(A)



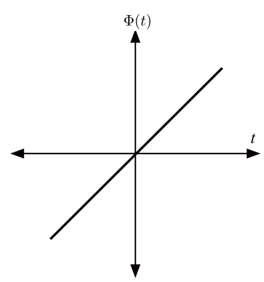
(B)



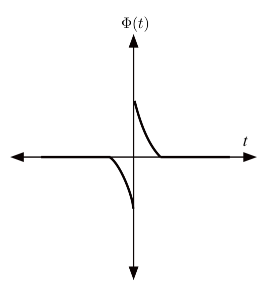
(C)



(D)



(E)



80. For an ideal gas in a container of fixed volume, the most probable speed of the gas molecules as a function of pressure P is proportional to

(A) $P^{-1/2}$
(B) $P^{1/2}$
(C) P
(D) $P^{5/3}$
(E) P^2

81. An ambulance with its siren blaring at frequency f drives in a straight line past an observer, with a distance of closest approach b. In the limit $b \to 0$, what does the observer hear?

(A) A constant frequency f
(B) A frequency starting higher than f which gradually slides to a frequency lower than f
(C) A frequency starting at f which gradually slides to a frequency lower than f
(D) A constant frequency higher than f which jumps to a constant frequency lower than f
(E) None of the above

82. A photon of kinetic energy 25 MeV collides head-on with an electron of energy 50 MeV in the laboratory frame. What is the velocity of the photon in the center of momentum frame of the electron and photon?

(A) $0.33c$
(B) $0.5c$
(C) $0.66c$
(D) $0.95c$
(E) c

83. In the quark model, mesons such as the pion are composed of

(A) two quarks
(B) two antiquarks
(C) a quark and an antiquark
(D) three quarks
(E) two baryons

84. Which of the following is the MAIN factor which prevents neutron stars from gravitationally collapsing?

(A) Pauli exclusion principle
(B) uncertainty principle
(C) tidal forces
(D) exchange forces
(E) strong nuclear force

85. A capacitor C is in an RC circuit with an initial charge Q_0. When the circuit is closed, the energy dissipated in the resistor is used to heat a material of specific heat c_p and mass m, with an efficiency ϵ. Assuming that the material is thermally isolated from everything except the resistor and that the heat capacity of the resistor is negligible compared with the material, what is the change in temperature of the material a long time after the capacitor is discharged?

(A) $\dfrac{Q_0^2\epsilon}{2mc_pC}$
(B) $\dfrac{Q_0^2\epsilon}{mc_pC}$
(C) $\dfrac{2Q_0^2\epsilon}{mc_pC}$
(D) $\dfrac{Q_0^2}{2\epsilon mc_pC}$
(E) $\dfrac{Q_0^2}{\epsilon mc_pC}$

86. Two infinite wires a distance d apart carry equal current I in opposite directions. The force per unit length of one wire acting on the other

(A) has magnitude $\frac{\mu_0 I^2}{2\pi d}$ and is attractive

(B) has magnitude $\frac{\mu_0 I^2}{2\pi d}$ and is repulsive

(C) has magnitude $\frac{\mu_0 I^2}{4\pi d^2}$ and is attractive

(D) has magnitude $\frac{\mu_0 I^2}{4\pi d^2}$ and is repulsive

(E) is zero

87. The partition function Z for the canonical ensemble in statistical mechanics is

$$Z = \sum_n e^{-E_n/kT}$$

where n ranges over all possible microstates of the system, and E_n is the energy of the nth microstate. Which of the following is the partition function for a system of two spin-1/2 particles subject to the Hamiltonian $H = -A\mathbf{S}_1 \cdot \mathbf{S}_2$?

(A) $Z = 4e^{A\hbar^2/4kT}$

(B) $Z = 2e^{A\hbar^2/4kT} + 2e^{-A\hbar^2/4kT}$

(C) $Z = e^{A\hbar^2/4kT} + e^{-3A\hbar^2/4kT}$

(D) $Z = e^{3A\hbar^2/4kT} + e^{-3A\hbar^2/4kT}$

(E) $Z = 3e^{A\hbar^2/4kT} + e^{-3A\hbar^2/4kT}$

88. When ultraviolet light of wavelength 350 nm is shined on a container of gas whose molecules have diameter 1 nm, the intensity of the scattered light is I. If the experiment were repeated with the same incident intensity of red light (wavelength 700 nm), the intensity of the scattered light would be

(A) $I/64$

(B) $I/16$

(C) I

(D) $2I$

(E) $16I$

89. The principal decay mode of the $3s$ state of hydrogen is to

(A) $1s$

(B) $2s$

(C) $2p$

(D) $3p$

(E) nothing; the $3s$ state is stable

90. A new particle's mass is measured in a high-energy physics experiment, and the value reported as 5.43 GeV $\pm$ 0.08 GeV $\pm$ 0.06 GeV, where the first error is systematic and the second is statistical. The total error on the measurement is

(A) 0.0048 GeV

(B) 0.01 GeV

(C) 0.02 GeV

(D) 0.10 GeV

(E) 0.14 GeV

91. A Geiger counter monitoring a radioactive sample records 64 counts in a 1-minute window. The fractional uncertainty on the counting rate is

(A) 1/64

(B) 1/8

(C) 1/4

(D) 1/2

(E) Cannot be determined from the information given

92. What is the electric field in the region $z > 0$ produced by a volume charge density $\rho(z) = \rho_0 e^{z/a}$, for $z \leq 0$? (You may assume $a > 0$.)

(A) $\dfrac{\rho_0 a}{2\epsilon_0}\hat{\mathbf{z}}$

(B) $-\dfrac{\rho_0 a}{2\epsilon_0}\hat{\mathbf{z}}$

(C) $\dfrac{\rho_0 a}{\epsilon_0}\hat{\mathbf{z}}$

(D) $\dfrac{\rho_0 a}{\epsilon_0}e^{-z/a}\hat{\mathbf{z}}$

(E) $-\dfrac{\rho_0 a}{\epsilon_0}e^{z/a}\hat{\mathbf{z}}$

93. A bullet of mass 5 g is fired at a block of wood of mass 1 kg, which is hanging from a massless rigid rod of length 0.4 m. The block is thick enough to stop the bullet entirely inside. Which of the following is closest to the minimum velocity of the bullet such that the block makes a complete vertical revolution?

(A) 200 m/s
(B) 400 m/s
(C) 800 m/s
(D) 1000 m/s
(E) 1600 m/s

94. A radio telescope is trained on a binary star system whose angular separation on the sky is 0.061 radians. What is the minimum diameter of the telescope in order to resolve both stars in the binary by observing radio frequency radiation at 200 MHz? (Ignore any atmospheric effects.)

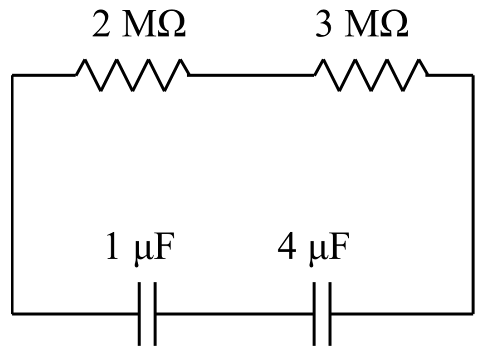
(A) 0.11 m
(B) 1.5 m
(C) 13.3 m
(D) 20 m
(E) 30 m

95. A positron collides with an electron at rest, and another electron-positron pair is produced:

$$e^+ + e^- \to e^+ + e^- + e^+ + e^-$$

What is the minimum energy of the initial positron so that this process can occur, in terms of the speed of light c and the mass of the electron m_e?

(A) $2m_ec^2$
(B) $3m_ec^2$
(C) $4m_ec^2$
(D) $7m_ec^2$
(E) $8m_ec^2$



96. What is the time constant of the circuit shown in the diagram?

(A) 1 s
(B) 4 s
(C) 6 s
(D) 10 s
(E) 25 s

97. The self-inductance of an ideal solenoid is L. If the number of coils per unit length is tripled while all other parameters remain the same, the new self-inductance is

(A) $L/9$
(B) $L/3$
(C) $L\sqrt{3}$
(D) $3L$
(E) $9L$

98. Free-electron lasers produce coherent light through which of the following mechanisms?

(A) Spontaneous emission
(B) Synchrotron radiation
(C) Population inversion
(D) Optical pumping
(E) Electric dipole transitions

99. What is the magnetic field due to an infinite surface current $\mathbf{K} = K_0\hat{\mathbf{y}}$ flowing along the xy-plane?

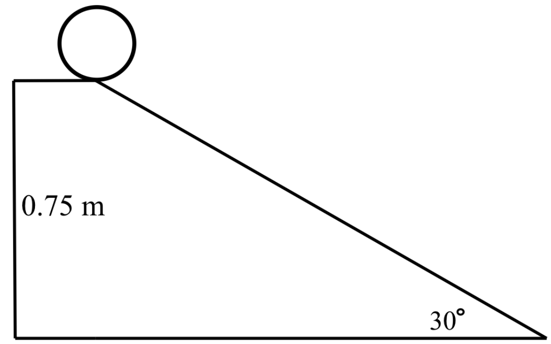
(A) $-\frac{\mu_0 K}{2}\hat{\mathbf{z}}$ for $z < 0$, $\frac{\mu_0 K}{2}\hat{\mathbf{z}}$ for $z > 0$

(B) $\frac{\mu_0 K}{2}\hat{\mathbf{z}}$ for $z < 0$, $-\frac{\mu_0 K}{2}\hat{\mathbf{z}}$ for $z > 0$

(C) $-\frac{\mu_0 K}{2}\hat{\mathbf{x}}$ for $z < 0$, $\frac{\mu_0 K}{2}\hat{\mathbf{x}}$ for $z > 0$

(D) $\frac{\mu_0 K}{2}\hat{\mathbf{x}}$ for $z < 0$, $-\frac{\mu_0 K}{2}\hat{\mathbf{x}}$ for $z > 0$

(E) $-\frac{\mu_0 K}{2}\hat{\mathbf{y}}$ for $z < 0$, $\frac{\mu_0 K}{2}\hat{\mathbf{y}}$ for $z > 0$



100. A sphere of radius 20 cm and mass 45 g is placed atop a ramp of height 0.75 m and inclination angle 30°. If the ramp were frictionless, the sphere would slide down the ramp in a time t. With friction, the sphere would roll without slipping down the ramp, and reach the bottom in a time t'. What is t'/t?

(A) $\sqrt{2/5}$

(B) $\sqrt{7/10}$

(C) 1

(D) $\sqrt{7/5}$

(E) $\sqrt{3}$

Answers to Sample Exam 2

1. D
2. C
3. E
4. C
5. B
6. A
7. C
8. D
9. D
10. E
11. A
12. A
13. B
14. C
15. E
16. C
17. B
18. D
19. A
20. A
21. C
22. D
23. B
24. A
25. E
26. B
27. E
28. D
29. D
30. C
31. E
32. E
33. E
34. B
35. E
36. D
37. D
38. D
39. E
40. B
41. C
42. D
43. D
44. D
45. A
46. D
47. B
48. B
49. A
50. A
51. D
52. B
53. B
54. A
55. E
56. E
57. E

58. A
59. E
60. D
61. A
62. A
63. D
64. A
65. B
66. D
67. E
68. C
69. D
70. B
71. B
72. B
73. C
74. D
75. B
76. C
77. E
78. E
79. B
80. B
81. D
82. E
83. C
84. A
85. A
86. B
87. E
88. B
89. C
90. D
91. B
92. A
93. C
94. E
95. D
96. B
97. E
98. B
99. C
100. D