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Chapter 9

Special Tips and Tricks for the Physics GRE

One of the main reasons we wanted to write a Physics GRE review book is that none of the existing review materials address *both* general test-taking strategies and strategies specific to physics problems. You can find general test-taking tips anywhere: hopefully things like "don't study the night before the exam" and "work through the easier problems first, marking the harder ones to come back to later" are intimately familiar to you already. One important thing to mention is the quarter-point wrong-answer penalty: this exists so that random guessing has an expected value of zero, and thus eliminating even one answer choice gives you a positive expectation if you guess randomly. However, random guessing should be a last resort (especially because, with a sample size of 1 exam, expected value arguments don't really apply), and the tips and tricks detailed below will often make it possible to narrow down the answer choices *completely* without ever actually solving the problem from first principles!

9.1 Derive, don't memorize

If you're just beginning your GRE preparation, and you've started looking through your freshman year textbook, you're probably overwhelmed by the sheer number of formulas. If you're like most physics students, you probably don't even remember learning many of them! But for better or for worse, the Physics GRE is a test of outside knowledge, and you need to know certain formulas to answer many of the questions. And the formula sheet provided at the beginning of the test is worse than useless: numerical values of constants you'll never need, a couple random definitions, and three moments of inertia. Obviously we're going to need an efficient way to remember all the missing formulas.

Richard Feynman (famous 20th century physicist and co-inventor of quantum electrodynamics) has a wonderful piece of advice on this sort of thing: "knowledge triangulation." No one can possibly remember all the formulas, but if you can remember a few key facts, you can reconstruct most of the rest of your knowledge, and "triangulate" unknown facts from known ones. The key to this is remembering the basic steps in the important derivations in all the key areas tested on the physics GRE.

Try this: divide up your formulas into categories based on how involved the derivations are. Class 1 would be the absolute basics, things like $\mathbf{F} = m\mathbf{a}$, expressions for kinetic energy ($\frac{1}{2}mv^2$ for translational, $\frac{1}{2}I\omega^2$ for rotational), the rest energy $E_0 = mc^2$ of a particle, and so on. These are essentially definitions of important physical quantities, rather than actual formulas. Class 2 would be formulas which you could quickly derive in a couple steps from the Class 1 formulas. This might include formulas for recoil velocities in 1-dimensional elastic collisions where one mass is at rest (apply conservation of momentum and energy) and the cyclotron frequency of a charged particle in a magnetic field (use the fact that the magnetic field provides the centripetal force required for uniform circular motion). Class 3 is any formula or equation that you expect will take more than 2 or 3 lines of algebra to derive, such as normal mode frequencies for a pair of coupled springs or 2nd order energy shifts in quantum mechanical perturbation theory.

Now, focus your attention on memorizing the Class 1 formulas, and the steps in the derivations that lead to the Class 2 formulas. Start a formula sheet containing the Class 3 formulas, adding them as you come across them in your studying, and memorize them as you go. Also, include a sketch of the derivations of the Class 2 formulas, but *don't* include the formula itself. Your notes might look like this:

EM boundary conditions at a conductor: apply Maxwell's equations using infinitesimally thin pillboxes and loops

That way, every time you review your formula sheet, you'll force yourself to rederive these formulas. If you find you can't do this after several tries, promote it to a Class 3 formula and write it down.

Of course, this classification is a very individual process, and will depend strongly on which subjects you consider your strengths or weaknesses. But a good target is to have no more than 10 Class 3 formulas for the major subjects (classical mechanics, electricity and magnetism), and no more than 5 Class 3 formulas for each of the smaller subject areas. Anything else is probably overkill, assuming you're familiar enough with the basics to know the Class 1 formulas by heart. And despite what the GRE formula sheet may suggest, moments of inertia are *not* worth memorizing. We would consider the formula $I = mr^2$ for a point mass a Class 1 formula, and everything else Class 2 (just integrate, or use the parallel axis theorem).

You can go even further and develop mnemonics for memorizing Class 3 formulas by treating them as Class 2 formulas, and doing a quick-and-dirty "derivation." Here are a couple examples. The formula for the Bohr radius of the hydrogen atom, $a_0 = \dfrac{4\pi\epsilon_0\hbar^2}{m_e e^2}$, is both completely ubiquitous in quantum mechanics, and a huge mess. But instead of memorizing the expression, you can cheat slightly and derive it using mostly classical mechanics and a little quantum mechanics. Apply the uncertainty principle in the form $\Delta r \Delta p \sim \hbar$ to the Bohr model of the hydrogen atom, where we assume the electron executes uniform circular motion in the Coulomb field of the proton. Putting $\Delta r = r$ and $\Delta p = p$, and turning the $\sim$ sign into an $=$ sign, we obtain precisely the Bohr radius. (Try it yourself!) Strictly speaking, of course, this derivation is completely bogus: the p appearing in the uncertainty relation should really be the *radial* momentum, the right-hand side should be $\hbar/2$, and setting $\Delta r = r$ is dubious at best. However, if you just treat this derivation as a mnemonic, you have a 2-line derivation of a Class 3 formula, which takes it off your list of formulas to memorize. A simpler example, but one which may be a little too advanced for the Physics GRE, is the Schwarzschild radius of a black hole. Treat light like a "particle" of mass m and kinetic energy $\frac{1}{2}mv^2$, and find the starting radius R for which the escape velocity from a body of mass M is the speed of light $v = c$. You'll find the mass m cancels out, and that light can only escape to infinity for $R > 2GM/c^2$, the Schwarzschild radius. Again, the right answer for the wrong reasons, but it's quick and it works.

Keep an eye out for mnemonics like this, and you should be able to keep your formula sheet to a manageable size. That way you can devote more of your study time to reviewing and doing practice problems, rather than cramming your brain full of formulas.

9.2 Dimensional analysis

Physical quantities have units. This may not seem like a profound statement, but it is an extraordinarily powerful tool for getting order-of-magnitude answers to physical questions, without ever doing involved computations. On the GRE, it offers an interesting alternative problem-solving method thanks to the multiple-choice format. The *very first* thing you should do when you see a tough-looking question is to scan the answer choices to see if they all have the same units. If *not*, there's a decent chance that only one of the answer choices has the correct units, and by identifying the units you want for the problem in question, you can get to the correct answer by dimensional analysis alone. Based on the tests released by ETS, it appears that pure dimensional analysis problems were very common on older tests, fell out of favor around 2000, but are possibly making a comeback. Here's an example similar to answer choices which appeared on a 2008 test:

(A) h/f

(B) hf

(C) h/λ

(D) λf

(E) $h\lambda$

Without even knowing the question, only one of these choices can possibly be correct, because they all have different units. A question this easy is relatively rare, though: you might expect to see 5 or so per test, at most.

A somewhat more common example is:

(A) $R\sqrt{l/g}$

(B) $R\sqrt{g/l}$

(C) $R\sqrt{2l/g}$

(D) Rg/l

(E) $R^2l/2g$

Assuming R and l stand for lengths (g always has its usual meaning of gravitational acceleration), a quick scan shows that A and C have the same units, while all the others are different. So once we know which units we're looking for, at best we've solved the problem, and at worst we're down to two choices, A and C.

Because dimensional analysis applies to nearly every problem on the GRE, it's an excellent fallback tool in case you forget exactly how to approach a problem, or draw a complete blank. It pays to get *very* comfortable with computing units for quantities, so here's an example to practice with.

Example: Which of the following gives the uncertainty Δx^2 for the ground state of the harmonic oscillator?

(A) $\dfrac{\hbar}{2m\omega}$

(B) $\dfrac{\hbar^2}{m\omega}$

(C) $\dfrac{\hbar\omega}{m}$

(D) $\dfrac{\omega}{2\hbar m}$

(E) $\dfrac{\hbar\omega}{m^2}$

We're looking for a quantity with units of (length)^2. First, let's do the dimensional analysis the straightforward way, listing the dimensions of all the variables as powers of mass M, length L, and time T, the three fundamental units in the SI system:

- $\hbar$: ML^2T^{-1}
- m: M
- ω: T^{-1}

The most general combination we can form is $\hbar^a m^b \omega^c$, and we want this to have units of L^2, so we get a system of linear equations in a, b, and c that we can solve:

$$\begin{aligned} a + b &= 0 \\ 2a &= 2 \\ -a - c &= 0. \end{aligned}$$

It's straightforward to see that $a = 1$, $b = -1$, and $c = -1$; in other words, $\hbar/m\omega$, choice A. We're off by a factor of two, but who cares: only choice A has the correct units. In point of fact, writing down the linear equations was probably a waste of time, since we could have just as easily stared at the list of units for $\hbar$, m, and ω and determined that the quantity we were looking for was $\hbar/m\omega$ right away.

For an alternate method, we could have avoided the ugly units of $\hbar$ by remembering that $\hbar\omega$ has nice units of energy. One form of energy is kinetic energy, $\frac{1}{2}mv^2$, so to get units of L^2 we need to divide energy by one power of M and multiply by two powers of T. This gives

$$\hbar\omega \times \frac{1}{m} \times \frac{1}{\omega^2} = \frac{\hbar}{m\omega},$$

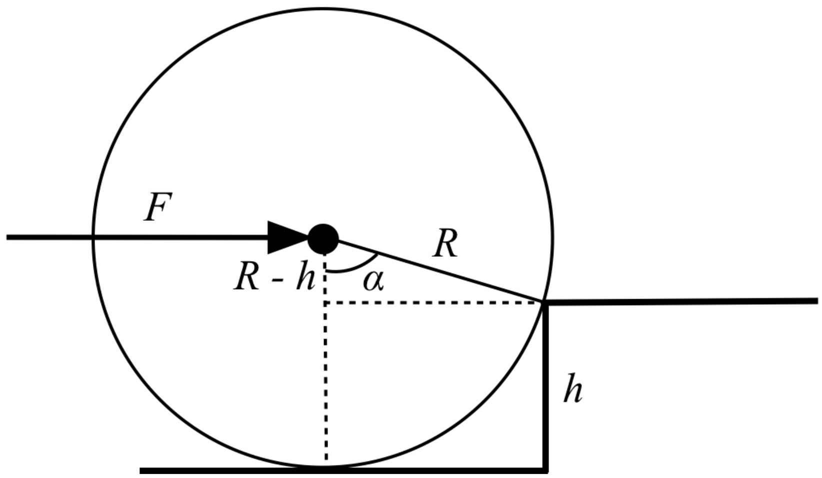
as before. Note how much faster this was than actually computing the uncertainty for the harmonic oscillator, either by using operator methods or the position-space wavefunction! Since this kind of dimensional analysis comes up so often, we *strongly* recommend coming up with your own method for solving these dimensional equations. Some combination of memorizing the MLT units for common constants, remembering useful combinations of constants with nice units like q^2/ϵ_0, and mnemonic methods would be an excellent start.

9.3 Limiting cases

A careful analysis of limiting cases is one of the most efficient ways to check your work on physics problems. This is especially true for the GRE, where you'll often be able to hone in on the correct answer choice by considering limiting cases, even when dimensional analysis fails.

What exactly constitutes a "limiting case," of course, depends on the problem. Some of the more common ones include letting a quantity like a mass, velocity, or energy go to zero or infinity, and seeing if the result makes sense in this limit. Here's a simple example: say you have a block of mass m on an inclined plane at an angle θ from the horizontal, and you can't remember whether the component of the gravitational force along the ramp is $mg\cos\theta$ or $mg\sin\theta$. Instead of fussing around with similar triangles, just consider what happens when θ is either 0 or $\pi/2$. In the first case, the ramp is horizontal, so the block doesn't slide; in other words, gravity does't act at all along the direction of the ramp. In the second case, the ramp is vertical, so the entire force of gravity mg acts in downwards and the block just falls straight down. Either of these tell you immediately that the force we're looking for is $mg\sin\theta$.

Let's do a slightly more involved example.



Consider the classic problem of a wheel of mass M and radius R up against a ledge of height h. What horizontal force F do you have to apply at the axle to roll the wheel up over the ledge? (Try this problem yourself before reading the rest of the discussion.) This problem is solved most simply by considering the torques about the contact point with the ledge; the ledge exerts some complicated force on the wheel, but we can ignore this entirely because it exerts no torque about the contact point. The wheel will roll up if the torque due to the horizontal force exceeds the torque due to gravity:

$$\begin{aligned}
\tau_g &= RMg\sin\alpha = Mg\sqrt{R^2-(R-h)^2} \\
\tau_F &= RF\sin(\pi/2-\alpha) = RF\cos\alpha = F(R-h) \\
\tau_F > \tau_g &\implies F > Mg\frac{\sqrt{2Rh-h^2}}{R-h}
\end{aligned}$$

Now, let's say we made a mistake calculating $\sin\alpha$, and wrote $\sin\alpha = \dfrac{\sqrt{R^2-h^2}}{R}$. This gives $F > Mg\sqrt{\dfrac{R+h}{R-h}}$. This sort of looks right: it has the right dimensions, and it goes to infinity as $h \to R$, which makes sense (you're never going to be able to push the wheel over the ledge using just a sideways force if the ledge is as high as the radius). However, taking the limiting case of $h \to 0$, we find $F > Mg$. This certainly doesn't make sense: if the ledge disappears, then any force, however small, will allow the wheel to keep rolling. So we know we've made a mistake somewhere.

Checking limiting cases is an extremely powerful strategy if you're running out of time at the end of the test. Remember, eliminating even *one* answer choice gives you a positive expected value for that question. If you can quickly identify the relevant limiting cases, and check them against the answer choices, you can often eliminate up to three wrong answers in under a minute.

9.4 Numbers and estimation

Broadly speaking, there are two kinds of physicists: theorists and experimentalists. If you're a theorist, you're probably more comfortable with formulas than numbers, and you might not remember the last time you had to calculate an explicit temperature, energy, or pressure. But a large part of the Physics GRE requires you to think like an experimentalist, estimating rough orders of magnitudes for various physical quantities. Here we'll talk about some strategies for doing so.

First of all, there are some numbers you should just know *cold*. These are the numbers that show up so often in real physics problems that if you haven't already memorized them, you will have after less than a few months of graduate research in the relevant field. Perversely, many of these are *not* the numbers that show up on the Table of Information on the first page of the GRE. Here's the most important example: the binding energy of hydrogen is 13.6 eV. You could memorize the formula for the Bohr energies, $E_n = -\frac{1}{n^2}\frac{m_e e^4}{2(4\pi\epsilon_0)^2\hbar^2}$, plug in all the constants given in the table, and find E_1 after a ton of arithmetic…or you can memorize this one number.

Actually, this number tells you quite a lot: if you remember the mass of the electron is about 0.5 MeV/c^2 (another number to memorize – see below), this means you can treat the hydrogen atom non-relativistically, because the electron's binding energy is much less than its mass. If you know that X-rays have energies of the order of keV, you know that hydrogen atom transition energies are safely below this range, in the ultraviolet. And you know that atoms close to hydrogen in the periodic table will have roughly similar ionization energies: more specifically, the binding energy of each electron increases as the square of the nuclear charge Z, so the ground state energy of helium is about $(13.6)(2^2)(2) \approx 110$ eV, and the binding energy of lithium is about $(13.6)(3^2)(3) \approx 370$ eV. To be clear, these numbers are just approximations – you've probably treated the helium atom using the variational principle in your quantum mechanics class, and you've seen the ground state energy is somewhat less than 110 eV. But these rough estimates are plenty for the GRE – in fact, estimating the binding energy of lithium is a practice question on the Sample Question set released by ETS.

Other important numbers show up as commonly-used combinations of fundamental constants. If you're like us, you probably had to memorize the value for h in high school chemistry – but when's the last time you actually had to use the value for h *by itself* in a calculation? If you're calculating anything in quantum mechanics, you use $\hbar$, and if you're doing anything relativistic, you use $\hbar c$. These combinations are worth memorizing, because they're the ones that you'll actually need. Similarly, Boltzmann's constant k is almost *never* used by itself, but always in combination with temperature. But if you remember that room temperature is about 300 K, and 1K $\approx \frac{1}{40}$eV, you can get the value if you need it. When dealing with combinations of constants, equally important is remembering the units: $\hbar c \approx 200\,\text{MeV}\cdot\text{fm}$ has units of energy $\times$ distance, which tells you that the characteristic distance associated to an object with energy 0.5 MeV is $(200/0.5)\times 10^{-15}$ m, or about 4×10^{-13} m: this is a rough estimate of the Compton wavelength of the electron. (Actually, this is off by a factor of 2π, but who cares? It's good enough as an order of magnitude.)

Based on our experience reviewing past GRE's, here is a list of the top 5 numbers to memorize (in order of importance):

- 13.6 eV - energy of the ground state of hydrogen
- 511 keV - mass of the electron in units of c^2
- 1.22 - Coefficient appearing in the Rayleigh criterion, $D \sin\theta = 1.22\lambda$
- 2.9×10^{-3} m·K - Wien displacement law constant
- 2.7 K - temperature of the cosmic microwave background

You can almost certainly get by with just these numbers. Not included in this list are other numbers you can derive in one or two short steps from numbers given in the Table of Information, like $\hbar c$ as discussed above.[1]

9.5 Answer types, a.k.a. what to remember in a formula

The Physics GRE is tricky. Compared to other tests of similar subject matter, like the AP Physics test or the Physics section of the MCAT, the testmakers throw in answer choices which are deliberately designed to mislead you. Being aware of the common patterns of answer choices can help you avoid these traps, and can often suggest the most efficient approach to a problem. In order of increasing difficulty, here are some patterns you should be aware of.

- **Answer choices with different dimensions.** This was covered in Section 9.2 above, and these problems are some of the easiest because of the possibility of eliminating many answer choices without actually doing any calculations.
- **Order of magnitude.** This was touched on in Section 9.4, and similar to dimensional analysis questions, one can get pretty far just by knowing rough orders of magnitude for common physical situations.

[1] Be careful! The most recent GRE included hc on the formula sheet, but often $\hbar c$ is the more useful quantity.

Example: The average intermolecular spacing of air molecules in a room at standard temperature and pressure is closest to

(A) 10^{-12} cm
(B) 10^{-9} cm
(C) 10^{-6} cm
(D) 10^{-3} cm
(E) 1 cm

While you could try to calculate this quantity exactly, using the fact that one mole of gas occupies 22.4 L at STP and so on, it's best just to recognize that A is the scale of nuclear diameters, B is the scale of atomic diameters, and E is macroscopic which just seems incorrect. So by common sense, we've narrowed it down to C and D.

- **"Which power of two?"** This pattern is best illustrated by a couple of examples:

  1. (A) 2
     (B) 4
     (C) 8
     (D) 16
     (E) 32
  2. (A) 0
     (B) $a/3$
     (C) $a/\sqrt{3}$
     (D) a
     (E) $3a$

  While the first set is numeric and the second set is symbolic, they're both testing the same thing: do you know the correct power law for a given variable in a certain formula? Often these answer choices will all have the same dimensions, so dimensional analysis won't help you. But the fact that the choices almost always involve nice numbers suggests that memorizing the various constants which accompany formulas is mostly useless: all that matters is the dependence on the various parameters in the problem. As we've emphasized many times, this is especially apparent in the formula for the Bohr energies, where the dependence on reduced mass, nuclear charge, and principal quantum number are all important. On a similar note, if a formula has a simple power-law dependence, such as the Rayleigh formula for small-particle scattering, it's worth simply committing it to memory without asking too many questions about where it came from. This may not be great physics, but neither is the GRE!

- **Same units, different limiting cases**. This pattern might come from a problem with an angle which can range from 0 to 90°, two unequal masses m and M, or two springs with different spring constants k_1 and k_2. But in any case, while dimensional analysis isn't helpful, taking limiting cases as discussed in Section 9.3 can often help narrow down the answer choices. This pattern lies right on the border between trying to do the problem from the beginning, and forgoing any calculations and just using limiting cases instead. Use your best judgment based on which method you think will be the fastest based on your own strengths and weaknesses.

- **Same units, different numerical factors**. This pattern, which looks like

  (A) $\cos(l/d)$
  (B) $\cos(2l/d)$
  (C) $\cos(l/2d)$
  (D) $\cos(l^2/d^2)$
  (E) $\cos(l^2/2d^2)$

  is tricky, because dimensional analysis is useless, and limiting cases are almost useless. Worse, many of the answer choices only differ by dividing instead of multiplying, increasing the possibility that you land on a trap answer choice by an arithmetic mistake. This pattern is a clue to *slow down*, work through the problem carefully, and try not to refer to the answer choices at any point during your calculation.

- **Random numbers.** Sometimes, you'll have to work out a problem numerically, and all the answer choices will be numbers with no obvious relation to one another. This arises most often in basic kinematics and mechanics problems, where luckily the physics is not an issue – the strategy is just to work slowly and make sure you don't make an arithmetic mistake. Equally as important, many of the wrong answer choices are likely correct answers to an *intermediate* step in the calculation, so just as mentioned above, try *not* to refer to the answer choices until you're absolutely through calculating. This reduces the chance you'll get distracted by a trap answer.

9.6 Fermi problems

How many piano tuners are there in New York City? This is a classic estimation problem attributed to Enrico Fermi, and it's interesting because it requires more than just a single "educated guess": a good solution will invoke several order-of-magnitude approximations and combine them together using appropriate formulas. For instance, you might start by estimating the population of New York City, then estimating the size of the average family to get an approximate number of families, then multiply by a factor representing the proportion of families that have a piano, and so on. Whole Fermi problems as defined above are unlikely to show up on the GRE, simply because they often require than 2-3 minutes to carry out all the approximations and arithmetic. However, they are an *excellent* way to review your knowledge of important formulas by applying them to a real-world problem. Furthermore, an intermediate calculation in a Fermi problem (maybe involving only one approximation and one formula) is a classic GRE question – see Section 9.5 for an example using intermolecular spacing.

OK, that's enough talking: let's get to an example.

What is the number flux of solar visible-spectrum photons arriving at Earth's atmosphere?

In layman's terms, how bright does the sun appear? (You might want to test your intuition by making an order-of-magnitude guess at that number before continuing.) This is an excellent Fermi problem because it brings in so many different formulas and concepts from several different areas of physics, plus a good helping of geometry and back-of-the-envelope estimating.[2] Here's how we could approach this problem:

1. Assume the sun is a black-body whose spectrum peaks in the visible range, find its temperature using Wien's law, then find the total power from the Stefan-Boltzmann law. Using the shape of the black-body spectrum, convert power to photon flux.

2. Use the average Earth-sun distance to find the photon flux on a sphere at the radius of Earth's orbit. Then find the portion of this sphere subtended by the sun-facing surface of the Earth, since only photons in this solid angle will hit the Earth.

Of course, this probably isn't the only way to approach this problem, but it's pretty straightforward and affords good practice with this style of information. Along the way we'll compare our estimated values with the true values and see how close we come at the end. At each step we'll round to one significant digit, though if you want to be more accurate you can feel free to keep all the numbers and multiply everything at the end.

Step 1: We can justify the approximation about the sun's spectrum just by looking outside: the sun appears bright and mostly white, which means that our eyes receive a large number of photons from all over the visible spectrum. Since the Planck distribution, which is proportional to $\frac{\omega^3}{e^{\hbar\omega/kT}-1}$, falls off rather sharply on either side of the maximum, if the maximum were outside the visible spectrum, the sun would appear either very red or very blue. So we can safely assume the spectral maximum to be at the center of the visible spectrum, approximately 500 nm. (The visible spectrum is about 300-800 nm, a range you should be familiar with.) Using Wien's displacement law $\lambda_{max} \approx 3 \times 10^{-3}\text{m} \cdot \text{K}/T$, we find the surface temperature of the sun is about 6000 K. The actual temperature is 5778 K, with nearly all the difference coming from using a more accurate value for Wien's constant – not bad so far!

Now, the Stefan-Boltzmann law is $P/A = \sigma T^4$, with $\sigma \approx 6 \times 10^{-8}\,\text{W}\,\text{m}^{-2}\,\text{K}^{-4}$ (you'd have to look that one up, since it's not given on the GRE formula sheet) the Stefan-Boltzmann

[2] For an entertaining essay full of such examples, check out Victor Weisskopf's "Modern Physics from an Elementary Point of View," available from the CERN library at `http://cds.cern.ch/record/274976/files/CERN-70-08.pdf?version=1`.

constant. This actually gives the power per unit surface area, as you can see from the units. We're left with

$$\frac{P}{A} \approx 8 \times 10^7 \text{ W m}^{-2}.$$

As we've noted, the black-body distribution is peaked sharply at the maximum, so we can crudely assume that *all* photons have wavelength 500 nm, with energy

$$E_\gamma = hc/\lambda \approx 4 \times 10^{-19} \text{ J}.$$

Since power has units of energy per time, we can use this photon energy to convert from energy to photon number, giving a photon rate per sun surface area of

$$\frac{dN}{dt\, dA_{sun}} = 8 \times 10^7 \text{ J s}^{-1}\text{m}^{-2} \times \frac{1 \text{ photon}}{4 \times 10^{-19} \text{ J}} \approx 2 \times 10^{26} \text{ m}^{-2} \text{ photon/s}.$$

Step 2: We're now going to make some quick-and-dirty approximations. We'll need the fact that the Earth-sun distance is 8 light-minutes (a good fact to memorize, if not for the GRE then for general physics knowledge), which works out to

$$d_{Earth-sun} = 8 \text{ min} \times (3 \times 10^8 \text{ m/s}) \approx 1 \times 10^{11} \text{ m}.$$

We also need the radius of the Earth, which we can estimate as follows. The coast-to-coast distance of the United States is about 3000 miles or about 5000 km, and one can imagine fitting 10 US-sized countries around the equator (certainly more than 1, and certainly less than 100), so the circumference of the Earth is about $C_{Earth} = 5 \times 10^7$ m, giving a radius of

$$r_{Earth} = C_{Earth}/2\pi \approx 1 \times 10^7 \text{ m}.$$

Now, the radius of the sun is somewhere between these two distances, so a reasonable guess would be $r_{sun} = 1 \times 10^9$ m, or halfway between on a logarithmic scale. These guesses are surprisingly close to the true values:

$$\begin{aligned} d_{Earth-sun} &= 1.496 \times 10^{11} \text{ m}, \\ r_{Earth} &= 6.37 \times 10^6 \text{ m}, \\ r_{sun} &= 6.96 \times 10^8 \text{ m}. \end{aligned}$$

Now we can multiply by $4\pi r_{sun}^2$ to get the total photon flux at the surface of the sun:

$$\frac{dN}{dt} \approx (2 \times 10^{26} \text{ m}^{-2} \text{ photon/s})(4\pi \times 10^{18} \text{ m}^2) \approx 2 \times 10^{45} \text{ photon/s}.$$

Assuming these photons are produced isotropically, they will spread out over a sphere, and the above number will also be the total flux at a sphere of any fixed radius. This is still not quite what we want – the Earth only covers a tiny portion of the sphere whose radius is the Earth-sun distance. The surface area of the sphere is $4\pi d_{Earth-sun}^2$, and since the Earth is very small, it cuts out an area approximately equal to its cross-sectional area, πr_{Earth}^2. Multiplying by the ratio of these areas, we have finally

$$\frac{dN_{Earth}}{dt} = 2 \times 10^{45}\,\text{photon/s} \times \frac{\pi r_{Earth}^2}{4\pi d_{Earth-sun}^2} \approx \boxed{5 \times 10^{36}\,\text{photon/s}}.$$

Now, due to all the rounding and approximations, we shouldn't be too surprised to be off by an order of magnitude or two. But this number is fairly important for biology, as it controls things like the growth rate of plants due to photosynthesis, so we can look up the actual value:[3]

$$\frac{dN_{total}}{dt} \approx 2 \times 10^{36} \text{ photon/s}$$
$$\frac{dN_{visible}}{dt} \approx 2.5 \times 10^{35} \text{ photon/s} \quad (400 - 700 \text{ nm}).$$

Not bad! The first thing to note is that the difference between the *total* photon flux and the *visible spectrum* photon flux is only an order of magnitude, which tells us that our approximation of the entire spectrum consisting of visible photons was perfectly fine for a rough estimate. Overall, we were off by about an order of magnitude, but this is not too surprising: we used inaccurate values for various distances, and since they show up squared in many formulas, these errors magnify. Furthermore, the quoted numbers are the flux at sea level, not at the atmosphere, and so ignore the effects of dipole radiation from molecules in the atmosphere that could in principle change the number of visible photons arriving on the ground. But considering how rough we were, being within a factor of 20 is astonishingly good!

The exact number is not so important, but what counts is the excellent physical reasoning we used:

- Using the Wien displacement law to estimate the temperature of the sun.
- Identifying the smallest and largest distance scales in the problem and using them to estimate the radius of the sun.
- Using the shape of the black-body distribution to convert energy into photon number.

And to make progress, we only had to know two numbers that didn't appear on the GRE formula sheet: the Stefan-Boltzmann constant, and the Earth-sun distance. If we had known a different set of numbers, we could still have deduced the rest by the "knowledge triangulation" mentioned in Section 9.1: for example, knowing the mass of the sun instead of the Earth-sun distance, we could have found the distance using the period of Earth's orbit and Newton's law of gravity.

[3] Taken from `www.bionumbers.org`, converting flux per area into total flux by multiplying by $4\pi r_{Earth}^2$. It's unclear whether the total photon flux is normalized to correspond to the total Earth area, or only the cross-section presented to the sun, but this factor of 4 doesn't matter for an estimate like this.