www.physicsgreprep.com
Second edition, printing 1.1 (updated February 2015)

ISBN-13 978-1479274635

Chapter 7

Laboratory Methods

The Laboratory Methods section of the GRE is an odd duck. Some questions (like graph reading or basic statistics) cover things you learned in middle school, while others (like lasers or radiation detection) deal with things you'll never see until a lab class, or even (if you're a theorist like one of the authors) until your second year of graduate school. The purpose of this chapter is to remedy that problem and briefly review *all* the material you may see on a GRE. Keep in mind that Lab Methods questions only make up 6% of the GRE, which means that it's *not* worth memorizing every type of laser medium in detail only to get one question right on your exam. Use this chapter as a reference to shore up any knowledge you may be missing, but by all means don't spend too much time on it.

7.1 Graph reading

We won't insult your intelligence by telling you how to read a graph. But here are some less common features to watch out for.

7.1.1 Dimensional analysis

Problem solving with dimensional analysis is mostly discussed in Chapter 9. Here we just mention one rather obvious application to graph reading, because it showed up on a recently-released GRE:

- **Read the axis labels.** In particular, note if they carry dimensions. One question on a recent GRE asked for the expression of a slope of a line in terms of some fundamental constants, and the question could be solved entirely by finding the dimensions of the ratio y/x by looking at the respective axes.

7.1.2 Log plots

Linear plots are useful for displaying data obeying linear relations, but for data obeying a power law or an exponential relation, a log plot is most useful. In this kind of graph,

equal intervals on the x or y axes correspond to constant *multiples*, rather than constant differences. Or in terms of logarithms, equal intervals correspond to constant differences in $\log_{10} x$ or $\log_{10} y$.[1] For example, equally spaced ticks on the y axis may represent 1, 10, 100, 1000, etc. Most likely, if you encounter this kind of plot on the GRE, you will see a log-log plot, where *both* axes are divided logarithmically as described above. Occasionally, you may see a log-linear plot, where one axis has a linear scale and the other has a log scale. In either case, examining the scale of the graph will tell you which you're dealing with.

Here are a few facts that come in handy when working with log plots:

- Log-log plots *never* show zero on the axes. This is because if $x = 0$ or $y = 0$, $\log x$ or $\log y$ is $-\infty$. Instead, the graph is simply cut off at some point.
- A straight line on a log-log plot corresponds to a power-law, $y = ax^b$. The slope is b and the constant a can be determined by finding the y value corresponding to $x = 1$.
- A straight line on a log-linear plot, where the y-axis is logarithmic and the x-axis is linear, corresponds to an exponential growth law, $y = C \cdot 10^{bx}$. C is the y-intercept, and the slope is b.

7.2 Statistics

You are undoubtedly familiar with the basic statistical concepts of mean, median, and mode; we will not review these here. We will also not review the general theory of statistics, because the only kinds of statistics questions you'll see on the GRE will be applied to particular scenarios. Specifically, you'll see questions dealing with data where measurement error must be quantified, and counting problems where the Poisson distribution is the appropriate technique.

7.2.1 Error analysis

For a *sample* of data points $\{x_1, x_2, \ldots, x_n\}$ taken from a much larger underlying *population*, one can get an estimate of their spread by computing the *sample variance*,

$$\sigma_S^2 = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \overline{x})^2, \tag{7.1}$$

where $\overline{x}$ is the sample mean, i.e. the average of the sample data points. You may be used to seeing n rather than $n - 1$ in the denominator. The difference between the two is clearly only important for small sample sizes, and is meant to correct for the fact that the *true* variance (rather than the sample variance) must be calculated using the *true* mean, instead of the sample mean. This distinction is unlikely to be important on the GRE, but it is easy to remember which to use. If you are estimating the variance of a full population using all

[1] This is one of the very few times in your scientific life you will use base-10 logs rather than base-e.

members of the population, put n in the denominator. For example, if you want the variance of the height of students in a school, and you have measured the heights of every student, then there is an n in the denominator. If you are using a sample to estimate the variance, then put $n-1$ in the denominator. For example, if you measured the heights of a random sample of 50 out of the 1000 students in the school, then you use $n-1$.

Measurement errors are typically quoted in standard deviations, which are the square root of variance, as in 5 ± 1 kg. There, the mean $\overline{x}$ of the sample is 5 kg, and the standard deviation σ is 1 kg. For a large number of measurements, the distribution of measurement results approaches a Gaussian with mean $\overline{x}$ and standard deviation σ, so the $\pm$ means that the probability of the true value of the measured quantity falling outside the range $\overline{x} \pm \sigma$ is 32%. Turning this around, a measurement error may be computed by some other means, and *used* as if it represents the variance of a Gaussian. There are a couple standard manipulations you'll be expected to do with measurement error:

- **Propagation of error.** Suppose that you measure a number $X \pm \sigma_X$ and a number $Y \pm \sigma_Y$. What is the uncertainty on, say, the ratio r of X and Y? This is the question answered by *propagation of error*. *Uncorrelated* errors σ add in quadrature. That is, if a measurement is quoted with two separate uncorrelated sources of error, say statistical σ_{stat} and systematic σ_{sys}, the *total* error is

$$\sigma_{tot} = \sqrt{\sigma_{stat}^2 + \sigma_{sys}^2} \tag{7.2}$$

  This simple relation can be generalized quite easily for a variable which is a function of some other variables, $z(x_1, x_2, \ldots, x_n)$. Given errors on the x_i, the error on z is essentially given by the chain rule,

$$\sigma_z^2 = \sum_{i=1}^{n} \left(\frac{\partial z}{\partial x_i} \right)^2 \sigma_{x_i}^2. \tag{7.3}$$

  Because the trend of the GRE seems to be to eschew calculus entirely, you probably won't need this formula, but we include it here for completeness. A few specific instances of this formula for combining multiple sources of uncertainty tend to arise frequently and are easy to remember. If $A \pm \sigma_A$ and $B \pm \sigma_B$ are two measurements with *uncorrelated* errors and a is a constant factor with no uncertainty (e.g. π), then we have the following combinations and associated uncertainties,

$$\begin{aligned}
f = aA \qquad & \sigma_f = a\sigma_A \\
f = A + B \quad & \sigma_f = \sqrt{\sigma_A^2 + \sigma_B^2} \\
f = AB \qquad & \sigma_f = f\sqrt{\left(\frac{\sigma_A}{A}\right)^2 + \left(\frac{\sigma_B}{B}\right)^2} \\
f = A/B \qquad & \sigma_f = f\sqrt{\left(\frac{\sigma_A}{A}\right)^2 + \left(\frac{\sigma_B}{B}\right)^2}.
\end{aligned}$$

- **Weighted averages.** Suppose the same quantity is measured in two different ways, yielding two different values x and y with two different errors σ_x and σ_y. These measurements can be combined to give a single value X using a weighted average, where the weights are the errors themselves:

$$X = \frac{x/\sigma_x^2 + y/\sigma_y^2}{1/\sigma_x^2 + 1/\sigma_y^2} \tag{7.4}$$

$$\sigma_{tot}^2 = \frac{1}{1/\sigma_x^2 + 1/\sigma_y^2} \tag{7.5}$$

  In other words, the data points with smaller errors are weighted more strongly in the weighted average, and the total variance is the harmonic mean of the variances. These equations generalize simply to more than two measurements.

- **Uncertainty.** A statement like "this measurement has an uncertainty of 10%" means that the sample mean $\overline{x}$ and the error σ satisfy $\sigma/\overline{x} = 0.1$.

Finally, we will mention one silly piece of nomenclature which was probably drilled into your head in high school chemistry, but which you've long since forgotten:

- *Precise* measurements have small variance.

- *Accurate* measurements are close to the true value.

You can come up with examples where any combination of these two are true or false: precise but not accurate, accurate (on average) but not precise, and so on.

7.2.2 Poisson processes

The *Poisson distribution* describes the probability of counting 1, 2 or 3, etc. events in a fixed time, when the events occur randomly at a known constant rate. Some classic examples described by the Poisson distribution are the clicks in a Geiger counter measuring radioactive decays or photons arriving in a telescope. Since the Poisson distribution describes the probability of "counting" a certain number of events, it is often called *counting statistics*. Mathematically,

$$P(n) = \frac{\lambda^n e^{-\lambda}}{n!}. \tag{7.6}$$

Here, λ is the expected (or average) number of counts in a given time interval, and $P(n)$ is the probability of observing exactly n counts in that same time interval. You should probably memorize this formula, but it's really not that hard to remember where all the n's and λ's go by noting that, like any probability distribution, we must have $\sum_{n=0}^{\infty} P(n) = 1$. Indeed, summing over n gives the Taylor series for e^{λ}, which cancels with $e^{-\lambda}$ to give 1 as required. Even if you don't memorize equation (7.6), definitely memorize these important facts:

- **The error of a Poisson measurement is $\sigma \approx \sqrt{N}$ for large N.** When we observe a Poisson process, we usually are interested in measuring the rate. We might, for example, measure $N = 100$ decays of a radioactive source in 1 second. But then what is the error of the measurement? The Poisson distribution has the useful property that if N events are expected, then the standard deviation of the distribution is $\sigma = \sqrt{N}$ (this approximation is typically safe for $N > 20$). This means that the error on a measurement of N events is just $\sqrt{N}$. In our example above, we would say that the measured rate was 100 ± 10 Hz.

- $\boldsymbol{P(0) = e^{-\lambda}}$. This is a measure of how rare the process is: if λ is small, you are relatively likely to observe no events.

- **The time between Poisson events follows an exponential distribution.** More specifically, if one event from a Poisson with mean λ occurs at time $t = 0$, the time t of the next event's arrival is distributed as a function of time according to $P(t) = \lambda e^{-\lambda t}$. Here t is measured in whatever units of time are used to define λ.

7.3 Electronics

The electronics portion of Lab Methods takes over where the circuits portion of Electricity and Magnetism left off. Now, instead of just applying a constant voltage to a simple circuit, we are interested in the response of basic circuit elements to a time-varying voltage, the behavior of more advanced circuit elements, and the basics of digital logic.

7.3.1 AC behavior of basic circuit elements

You're already familiar with the three basic circuit elements (resistors, capacitors, and inductors) from E+M. In the context of electronics, these devices are usually described in terms of their AC behavior; in other words, their response to an alternating current. Roughly, capacitors and inductors can behave as if they carry resistance when hit with an alternating current of various frequencies, and it's convenient to treat all three circuit elements on the same footing. This is done using the concept of *impedance* Z, a complex number which obeys Ohm's law, $V = IZ$. For an alternating current $V = e^{i\omega t}$, Z contains information about *both* the magnitude and the phase of the resulting current, allowing considerable calculational simplifications. It's probably useful to remember the following:

$$\text{Capacitor} : Z = \frac{1}{i\omega C} \tag{7.7}$$

$$\text{Inductor} : Z = i\omega L \tag{7.8}$$

$$\text{Resistor} : Z = R \tag{7.9}$$

As always, ω refers to the angular frequency of the supply voltage V.

Looking only at the magnitudes of these quantities, we see that at high frequencies, capacitors have small impedances – in other words, they tend to cause only small voltage

drops, and behave like short circuits. Inductors, on the other hand, have the opposite behavior: at high frequencies, they behave like open circuits, where no current can flow. This makes sense because at high frequencies, the capacitor is barely being charged, and easily goes through many tiny charge-discharge cycles without saturating its maximum charge for a given voltage. Inductors are hindered by their self-inductance, which tends to resist large changes in current, so at very high frequencies they don't let any current pass at all.

One important example of a high-frequency voltage is when a constant (DC) voltage source V is suddenly switched on at $t = 0$. The above arguments then tell you that the voltage across an uncharged capacitor at $t = 0$ is zero, but increases to V as $t \to \infty$; on the other hand, the voltage across an inductor is V at $t = 0$, but tends to 0 as $t \to \infty$. Similarly, at $t = 0$ there is a large current going through the capacitor, but no current going through the inductor. This is helpful in recognizing the correct graph of V, Q, or I for an RL or RC circuit, a very standard GRE question.

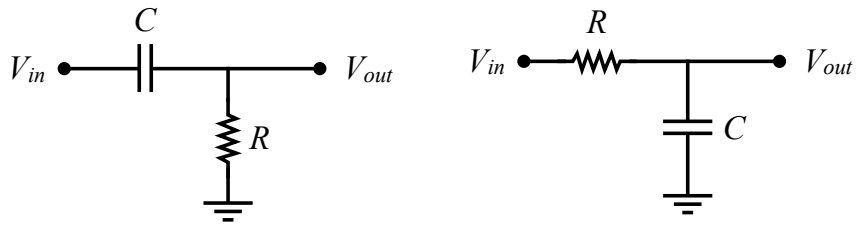
Essentially, impedance is just a clever way for remembering all these arguments and encoding them in a simple mathematical formula so you don't have to reproduce the argument every time. It also tells you how things behave when you add them in series or in parallel: using $V = IZ$, we get

$$\text{Series} : Z_{tot} = Z_1 + Z_2 + \cdots Z_n \tag{7.10}$$

$$\text{Parallel} : Z_{tot}^{-1} = Z_1^{-1} + Z_2^{-1} + \cdots Z_n^{-1} \tag{7.11}$$

These formulas contain all the usual formulas for resistors, capacitors, and inductors in series, as well as all the information about phase lag in RLC circuits, in one convenient package.

For the GRE, the most common application of these impedance formulas will be to high-pass and low-pass filters. These related circuits look like this:



(a) High-pass filter (b) Low-pass filter

In both cases, the circuit has distinctive behavior because the capacitor acts like a short circuit at high frequencies. If the resistor is connected to ground as in (a), at high frequencies the impedance of the resistor will dominate, and the voltage drop across C will be negligible – in other words, high frequencies will pass but low frequencies are attenuated. On the other hand, if the capacitor is connected to ground, the reverse is true: at high frequencies the capacitor shorts and all the current flows to ground, so V_{out} is near zero. You may find it a helpful mnemonic to remember that in a low-pass filter, the capacitor is "low," that is, connected to ground. One could build RL high-pass and low-pass filters, with the roles of

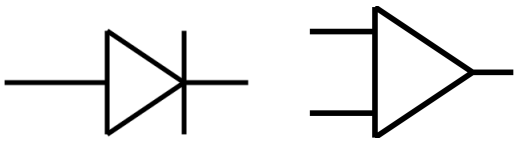
the two circuit elements reversed because of the opposite impedance behavior of capacitors and inductors.

Finally, we should mention how the resonant behavior of LC circuits can be derived using impedance. For a circuit with just one inductor and one capacitor, we have

$$Z_{LC} = \frac{1}{i\omega C} + i\omega L = \frac{i(1-\omega^2 LC)}{\omega C}.$$

The numerator vanishes when $\omega = 1/\sqrt{LC}$, the resonant frequency of an LC circuit. Including a resistor means that the impedance is never perfectly zero, but frequencies near $1/\sqrt{LC}$ still have small impedance, so this circuit acts as a *bandpass filter*. Indeed, adding a resistor will end up giving a real part to Z, such that Z can never exactly vanish no matter what the frequency: however, resonance is still defined as *the frequency where the imaginary part of the impedance vanishes*.[2]

7.3.2 More advanced circuit elements



(a) Diode (b) Op-amp

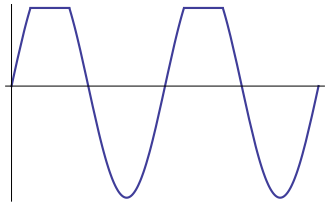
The most interesting thing we can make from just the three basic elements is an RLC circuit, which we already examined in Chapter 2. If we add just a few other key circuit elements, whose circuit diagram icons are shown above, we get all sorts of interesting behavior.

- **Diode.** This device uses properties of semiconductors to ensure that *current can only flow in one direction*. In a circuit diagram, the arrow points in the direction current is allowed to flow. However, no current can flow at all until a minimum *bias voltage* is applied across the diode – typically this is about 0.7 V for a silicon diode. Apart from that bias voltage, the voltage drop across a diode is approximately independent of the current. Uses of diodes include turning an alternating current into a direct current (this is known as a rectifier circuit) and to reroute current away from sensitive electrical components (if the voltage surges, the diode starts conducting, resulting in an almost short circuit if the voltage is high enough).

- **Op-amp.** Short for operational amplifier, this device has two inputs and one output. The output voltage is proportional to the difference between the two input voltages, usually by factors as large as 10,000. However, the op-amp has a maximum possible output voltage, so if the difference between input voltages is too large, the output will

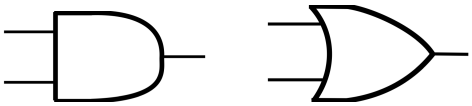
[2]Depending on whether the circuit elements are in series or parallel, the resonant frequency may actually be a maximum of $|Z|$, rather than a minimum, but the definition of the resonant frequency is still the same.

saturate and distort the signal; that is, there will no longer be a linear relationship between input and output voltage. This is known as *clipping*, and has a very distinctive waveform (as well as a distinctive sound if the signal is audio), shown below. Note how the top of the sine wave has been "flattened."



7.3.3 Logic gates

The basis of modern electronics is digital circuitry, where circuit element output voltages take discrete values rather than continuous ones. A "high" output voltage is interpreted as the digit 1, and a "low" voltage is interpreted as 0, so Boolean logic can be implemented in electronic circuits. The two main logic gates are AND and OR, and their symbols are shown below.



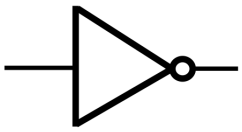
(a) AND gate (b) OR gate

For two inputs A and B, AND outputs $A \cdot B$ and OR outputs $A + B$. Here we are using Boolean logic notation (which also shows up on the GRE): note that this is *not* the same as binary arithmetic. Instead, it's easiest to decipher with 0 standing for "false" and 1 standing for "true." So AND returns true only if inputs A *and* B are true, otherwise it returns false. Similarly, OR returns true if inputs A *or* B are true, so only returns false if both A and B are false. These results can be summarized in "truth tables," for example the OR gate has

A	B	A OR B
0	0	0
0	1	1
1	0	1
1	1	1

but it's easier to just remember what these gates do by their names.

Both gates can be modified by inverting either of the inputs or the output, which is symbolized with a circular "bubble" on the circuit diagram. Alternatively, the circuit element which inverts an input is called a NOT gate, and looks like the symbol for an op-amp but with only one input and a bubble on the output:



In Boolean logic, inversion is represented with a bar: $\overline{A}$. Inverting the outputs to AND and OR gates give so-called NAND and NOR: it's a possibly useful fact that *all* basic logic gates can be constructed exclusively from combinations of either NAND and NOR gates.

One final piece of trivia on which you may be tested is De Morgan's laws, which are stated in Boolean algebra as follows:

$$\overline{A \cdot B} = \overline{A} + \overline{B} \tag{7.12}$$

$$\overline{A + B} = \overline{A} \cdot \overline{B} \tag{7.13}$$

In other words, a NAND or NOR gate (LHS) is just an OR or AND gate with inverted inputs (RHS), respectively.

7.4 Radiation detection and instrumentation

This section is based largely on Knoll, mentioned in the Resources list at the very beginning of this book. As always, we'll be brief, but feel free to check out that reference if you want more details.

A useful general concept when dealing with subatomic particle interactions is the *cross section*. Imagine that you're shooting a stream of bullets at a bowling ball of radius R. The surface area of the ball is $4\pi R^2$, but the surface that the bullets "see" is the area of the shadow cast by the ball, πR^2. This effective area where collisions can take place is called the cross section, and usually given the symbol σ. Subatomic particles are point particles, so this analogy breaks down in that regime, but we can still associate an effective cross section with a collision event by taking into account the quantum-mechanical probability for the collision to occur. So whenever you see "cross section" in a problem on the GRE, think of "effective collision probability." Occasionally you might be asked to compute a cross section, given other numbers like luminosity (number of particles per unit time); this will *always* be pure dimensional analysis.

7.4.1 Interaction of charged particles with matter

Charged particle come in many different types, but for purposes of the GRE, you really only need to know how electrons and nuclei interact as they pass through bulk matter. Common nuclei could include protons, alpha particles (helium-4 nuclei), or heavier nuclei which are the byproduct of fission reactions. For kinetic energies below the approximate binding energy per nucleon of most elements (a few MeV), both electrons and nuclei overwhelmingly interact with matter by the electromagnetic force. If the interaction occurs with atomic electrons,

the electrons can either be excited to higher energy levels (*excitation*) or stripped from the atom (*ionization*), both of which cause the incident particles to lose energy. The different masses of electrons and nuclei, however, produce a variety of differences in the interactions.

- **Range**. Nuclei are stopped faster than electrons: average path length for an alpha particle is 10^{-5} m, while for a high-energy electron it is 10^{-3} m. Another way of stating this is that the energy loss per length dE/dx is much higher for nuclei than for electrons (for kinetic energies $\lesssim 1$ GeV).

- **Path shape**. Nuclei tend to travel in straight lines, because they interact primarily with atomic electrons which are much lighter (think of a bowling ball continuously colliding with a sea of ping-pong balls). Electrons tend to bounce around and scatter through wide angles much more easily.

- **Collision target**. Nuclei interact almost exclusively with atomic electrons; interactions of heavy particles with nuclei are so rare that they can be ignored for most practical purposes, although historically they did play a role in Rutherford's gold foil experiment, which established the existence of the nucleus from scattering by incident alpha particles. Electrons can either interact with atomic electrons or atomic nuclei – the latter is still rare, but can lead to measurable effects in detectors.

- **Energy loss**. Nuclei lose energy exclusively due to collisions, rather than by emitting radiation. Since mass of an incident nucleus is very different than the electrons with which it interacts, nuclei lose only a small amount of energy in each collision. In other words, they are *continuously* losing energy as they interact. On the other hand, when incident electrons undergo collisions with atomic electrons, the target has the same mass as the incident particle, and so elementary kinematics implies that the electron can lose a large fraction of its energy from a single collision. Furthermore, unlike heavy particles, electrons can lose energy through *bremsstrahlung* (literally "braking radiation"), where in the presence of an electric field the electron emits a high-energy photon (usually in the X-ray region) which carries off a large fraction of its energy. This process is rare compared to collisional losses, but occurs more often in materials with high atomic number because the electromagnetic interaction which provokes bremsstrahlung is proportional to nuclear charge. The rates of energy loss for both nuclei and electrons are in general strongly dependent on the initial energy, but at relativistic speeds, *all* particles lose a roughly similar amount of energy per unit distance travelled (approximately 1 keV/cm in air). This value corresponds to the minimum of the energy-loss curve for both heavy and light particles, so a relativistic particle is referred to as a *minimum ionizing particle* because it deposits the minimum possible amount of energy per unit distance in the medium.

7.4.2 Photon interactions

Photons are uncharged, so they don't interact in quite the same way that charged particles do. However, because they mediate the electromagnetic force, they have strong interactions

with the other charged particles, and produce charged particles as a byproduct of an underlying interaction, which then propagate through the detector as described above. Problems involving a qualitative understanding of these interactions appear frequently on the GRE. There are three important types of underlying interaction:

- **Photoabsorption** (or **photoelectric absorption**). The photon is completely absorbed by an atom, and an electron is emitted in its place, with energy $E_\gamma - E_b$, where E_γ is the incident photon energy and E_b is the electron binding energy. This is the dominant process for low-energy photons, up to a few keV. The binding energy of the atom is often called the *work function* of the material, and denoted by ϕ instead of E_b. The maximum energy that the emitted electrons can have when light is shined on a material is then obviously
$$E_{max} = E_\gamma - \phi. \tag{7.14}$$
Problems involving the work functions of various materials occur often enough that this jargon and notation is worth remembering.

- **Compton scattering**. The photon scatters elastically off an atomic electron, and the scattered electron is ejected from the atom. The wider the photon scattering angle, the more energy it loses to the electron. This is the dominant process for medium-energy photons (tens of keV to a few MeV), and sometimes for low-energy photons as well if the absorber has small atomic number. This is probably a good time to bring up the *Compton wavelength* of a particle of mass m,
$$\lambda = \frac{h}{mc}. \tag{7.15}$$
Unlike the de Broglie wavelength (see Section 5.3.2), the Compton wavelength doesn't depend on the momentum of the particle, but only on its mass. It shows up in the formula for the wavelength shift of light due to Compton scattering,
$$\Delta\lambda = \frac{h}{mc}(1 - \cos\theta). \tag{7.16}$$
This formula is rather difficult to derive (although it's a good exercise in relativistic kinematics), so should be memorized. You may also be asked for the energy shift of the scattered photon, for which you should use the Einstein relation $E = h\nu = hc/\lambda$.

- **Pair production**. If $E_\gamma > 2m_e c^2$, the electric field near a nucleus can induce the photon to produce an electron-positron pair. This is the dominant process for high-energy photons (tens of MeV and above).

Note that photoabsorption is an interaction with the *entire* atom, Compton scattering is an interaction with atomic *electrons*, and pair production is an interaction with the atomic *nucleus*. The probabilities for all three processes are proportional to powers of Z, the atomic number of the absorber, since Z is also the number of atomic electrons available for Compton

scattering. More specifically, the probability of pair production is roughly proportional to Z^2, for Compton scattering it is proportional to Z, and for photoabsorption it is roughly proportional to Z^4. The purpose of using high-Z materials such as tungsten is to increase the likelihood these kinds of interactions will occur, and the strong dependence of the photoabsorption probability on Z explains its dominance at low energies.

7.4.3 General properties of particle detectors

By definition, particle detectors are designed to see incoming particles. Once you know a particle is there, the next obvious thing to do is measure its energy; devices which do this are often called *calorimeters*. To measure energy, the detector takes advantage of the natural process of energy loss in the material, and uses the stuff that absorbs the energy (atomic electrons, photoelectrons, produced electron/positron pairs, and so on) to produce a signal. Since for charged particles, the number of interactions is usually proportional to the incident particle's energy, simply collecting the produced electrons, counting their charge, and turning that into an electrical current may be enough. Other times, it may be necessary to amplify the signal somehow.

The most common case where signal amplification is needed is for photon detection, since only one electron is produced per photon, so we will go through the operation of a generic photon detector in a little more detail because it covers lots of subcomponents which might show up on the GRE. To increase the photon interaction cross-section, we want a high-Z material – a common choice is NaI/Tl, sodium iodide doped with thallium, with the iodine providing the high atomic number of $Z = 53$. An incoming photon produces a single electron by one of the three methods mentioned. It happens that NaI/Tl is also a *scintillator*, which means that a passing charged particle will produce visible light, with the intensity of light produced roughly proportional to the electron energy. These visible photons are then directed to a *photomultiplier tube*, which uses a cascade of photoelectric effects to produce a macroscopic current of electrons. These electrons are finally read out by some kind of analyzer, which converts the current to a digital voltage which becomes the raw data. In an ideal world, the photon energy would be directly proportional to the output voltage, and the detector could be calibrated by irradiating it with a photon source of known energy.

7.4.4 Radioactive decays

Here is one piece of trivia which has shown up on recent exams. A substance which undergoes radioactive decay will have an exponentially decaying number density,

$$N = N_0 e^{-t/\tau} \tag{7.17}$$

where τ is the mean lifetime. If the substance can decay in several ways (through multiple "decay channels"), then the total lifetime is related to the individual lifetimes τ_1, τ_2, etc. by

$$\frac{1}{\tau} = \frac{1}{\tau_1} + \frac{1}{\tau_2} + \cdots \tag{7.18}$$

7.5 Lasers and interferometers

For some reason, questions about names and properties of lasers have become increasingly common on the GRE, despite the fact that the underlying physics of stimulated emission belongs to time-dependent quantum-mechanical perturbation theory and is outside the scope of the test.

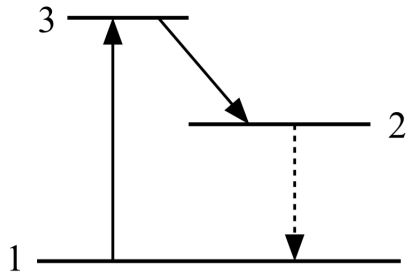
7.5.1 Generic laser operation

Here's a non-technical outline of how a generic laser works. Start with a quantum-mechanical system (the *medium*) with at least two energy levels, a ground state and an excited state. The medium could consist of free atoms, organic molecules, or any number of more exotic substances, several of which will be discussed below. Using some external power source (an *optical pump*), excite more than half of the medium to the excited state: this can be done with an electrical spark, for example. With a majority of the medium in the excited state, we say that *population inversion* has been achieved. Now, the excited states will tend to decay down to the ground state by *spontaneous emission*, emitting a photon in the process. If this photon is absorbed by a particle in the ground state, it will be excited, and there will be no net change in the system. However, time-dependent perturbation theory shows that the photon can also be absorbed by another excited state, which will be "stimulated" to emit *two* photons and drop to the ground state. This process of photons from decaying excited states being absorbed by other excited states, called *stimulated emission*, starts a chain reaction, the product of which is an exponentially large number of photons, all with exactly the same frequency and phase: this is laser light.

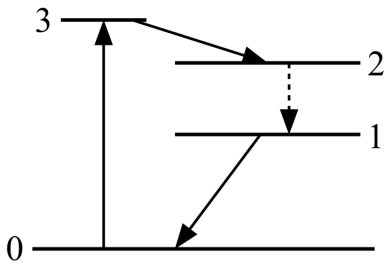
In a real laser, this idealized description must be modified slightly. A careful stat-mech analysis shows that if the system has *only* two levels, it is impossible to achieve population inversion: once the populations of the ground and excited states become equal, the processes of absorption and stimulated emission exactly compensate each other, and there is no chain reaction. Furthermore, the excited state usually decays pretty fast, so we need a *metastable state* between the excited state and the ground state. Referring to the diagrams below, level 3 is the excited state reached by pumping, level 2 is the metastable state, and level 1 some other state between level 2 and the ground state. The decay $3 \to 2$ is fast, but the slower decay $2 \to 1$ (dashed arrows) produces the laser light. If level 1 is (or is very close to) the ground state, the system is said to be a *three-level* laser, but if level 1 is significantly above the ground state, we call it a *four-level* laser.

7.5.2 Types of lasers

Lasers are generally distinguished by their medium and how the transfer between energy levels is achieved. The main examples in the first three categories (indicated in parentheses) do tend to show up as GRE trivia, so you should at least have a passing familiarity with each type.



(a) 3-level laser



(b) 4-level laser

- **Solid-state lasers (Nd:YAG).** The laser medium is a crystal or glass, and the transitions are between atomic energy levels. In a Nd:YAG laser, the crystal is $Y_2Al_5O_{12}$ (yttrium aluminum garnet, or YAG), with some of the Y ions replaced by Nd. The Nd atomic levels are split by the electric field of the YAG crystal, giving a four-level system.

- **Collisional gas lasers (He-Ne).** The laser medium is a gas or mixture of gases, and the transitions are due to collisions between the atoms: an excited electron from one gas transfers its energy to excite an electron in another gas molecule. In a He-Ne laser, there are a huge number of possible laser levels, but a specific wavelength can be selected by placing the laser in a resonant cavity, just as one excites certain EM modes using a conducting cavity in ordinary electrodynamics.

- **Molecular gas lasers (CO_2).** The laser medium is again a gas, and the transitions are *vibrational* energy levels. CO_2 is a standard example because it's cheap, widely available, and its triatomic structure gives it a rather rich vibrational spectrum.

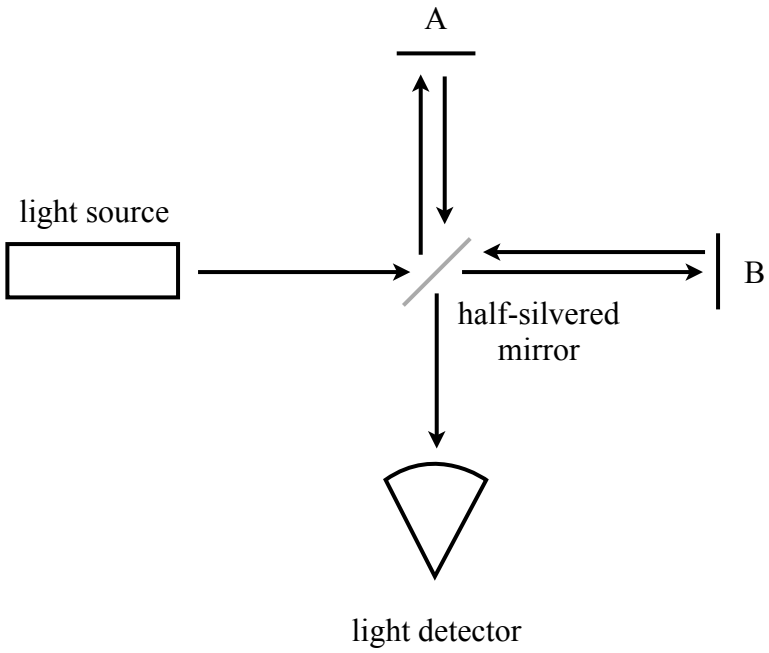
- **Dye lasers.** The laser medium is a liquid, usually an organic dye dissolved in water or alcohol. The transitions are related to the electron-transfer properties along chains of carbon atoms which give dyes their characteristic color. Interestingly, the laser does not tend to operate at the wavelength corresponding to the ordinary visible color of the dye, but because the electron transport chain is extremely efficient, laser operation is still possible at other frequencies.

- **Semiconductor or diode lasers.** The laser medium is a semiconductor (discussed in more detail in Section 8.2). Here, the pumping process excites the conduction band, and the transitions are electron-hole annihilation between electrons in the bottom of the conduction band and holes at the top of the valence band. This gives rise to photons (known as recombination radiation) which form the basis of the laser light.

- **Free-electron lasers.** As the name suggests, the laser medium is simply a collection of electrons, not bound to any atom or molecule. When forced to accelerate back and forth in an external electric field, the electrons will emit bremsstrahlung (already

mentioned in Section 7.4) at a frequency depending on their oscillation frequency. There are no discrete energy levels here, so it's a bit of a stretch to call this a laser, although a semiclassical analysis shows that there is amplification.

7.5.3 Interferometers



An interferometer is a device which takes advantage of the wave properties of light to measure distances and velocities very sensitively. Undoubtedly, the most famous interferometer is the Michelson-Morley model, shown above, used to disprove the idea of the ether in pre-special relativity days. This is the type which will show up on the GRE if you're asked about interferometers, so we'll confine our attention to this model. Monochromatic light is shined on a half-silvered mirror, which reflects half the light to the mirror marked A, and lets the other half through to a second mirror marked B. The light from both mirrors then bounces back to the half-silvered mirror, which splits the incoming light again. The portion which travels to the detector contains contributions from both paths, which interfere with each other when they reach the detector. If the optical path lengths along the two arms are different, the detector will record a pattern of interference fringes, as discussed in much more detail in Chapter 3. One then *counts* the number of fringes visible on the screen: if this number changes, it means that the optical path length difference between the two arms has changed, either by one of the mirrors moving, a change in the index of refraction along one of the arms, or both. By using the double-slit equation $d \sin\theta = m\lambda$, the number of fringes crossing a certain position on the detector (fixed θ) can be used to measure d given λ, or vice versa.

7.6 Review problems

1. In radiation detection, the term "minimum ionizing particle" could refer to

   (A) A photon with energy 10 keV

   (B) A neutron with kinetic energy 1 MeV

   (C) An alpha particle with kinetic energy 5 MeV

   (D) A proton with kinetic energy 10 MeV

   (E) An electron with kinetic energy 50 MeV

2. Event A is drawn from a Gaussian probability distribution with standard deviation σ_A, and event B is drawn from a Gaussian with standard deviation σ_B. If A and B are independent events, the probability distribution for the sum of A and B is a Gaussian with standard deviation:

   (A) $\sigma_A + \sigma_B$

   (B) $\sqrt{\sigma_A \sigma_B}$

   (C) $\sqrt{\sigma_A^2 + \sigma_B^2}$

   (D) $\dfrac{1}{1/\sigma_A + 1/\sigma_B}$

   (E) None of these

3. Which of the following probability distributions best describes the probability of obtaining heads 3 times when a fair coin is flipped 10 times?

   (A) Binomial distribution

   (B) Gaussian distribution

   (C) Student's t distribution

   (D) Log-normal distribution

   (E) χ^2 distribution

4. The number N of radioactive atoms of a particular isotope remaining in a sample as a function of time t is found to obey $N(t) = N_0 e^{-\lambda t}$. What is the half-life of the sample in terms of λ?

   (A) $\lambda \ln 2$

   (B) $\dfrac{\lambda}{\ln 2}$

   (C) $\dfrac{\ln 2}{\lambda}$

(D) $\dfrac{1}{\lambda \ln 2}$

(E) $\lambda^{\ln 2}$

Input 1	Input 2	Result
0	0	1
0	1	1
1	0	1
1	1	0

5. The above "truth table" represents which of the following logic gates?

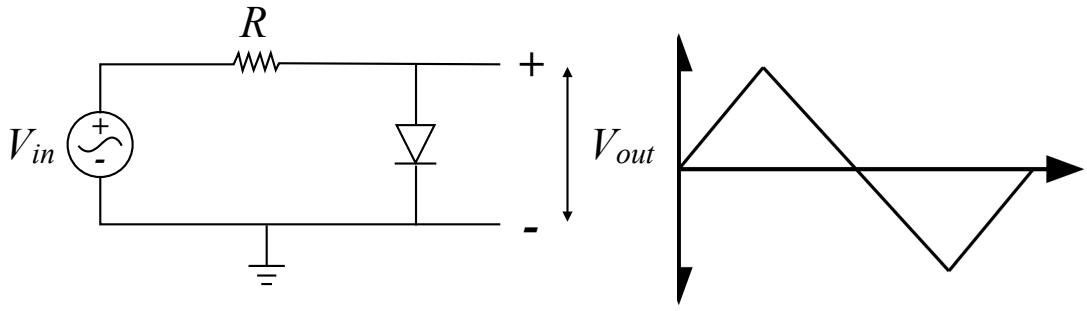
   (A) OR

   (B) AND

   (C) NOR

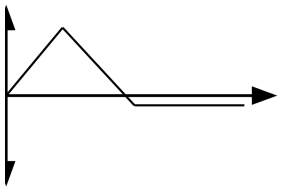
   (D) NAND

   (E) NOT

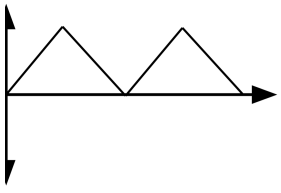


6. The circuit diagram on the left above is driven by an alternating-current generator, whose input voltage V_{in} is shown as a function of time in the plot on the right. Which of the following best represents the shape of the output voltage V_{out}?

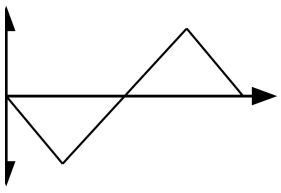
(A)



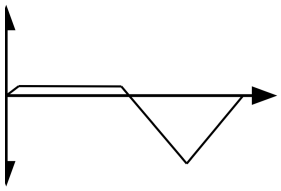
(B)



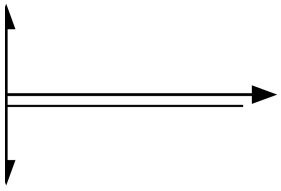
(C)



(D)



(E)



7. A student holding a Geiger counter near a radioactive sample hears five clicks in a ten-second time window. Based on this measurement, what is the probability of hearing exactly one click in a subsequent ten-second time window?

   (A) e^{-5}

   (B) $5e^{-5}$

   (C) $5e^{-2}$

   (D) $\dfrac{5e^{-2}}{2}$

   (E) $2^4 e^{-5}$

8. A narrow bandpass filter is centered at 1 MHz. What combination of inductor and capacitor can be used to create such a filter?

   (A) 25 nF and 10 nH

   (B) 250 nF and 100 nH

   (C) 1 μF and 1 μH

   (D) 20 μF and 10 μH

   (E) 200 μF and 10 μH

7.7 Solutions

1. E - Minimum ionizing particles must be relativistic, and only choice E has energy much greater than its mass. Photons are never minimum ionizing particles because they are neutral and their interactions are qualitatively different from that of charged particles.

2. C - The sum of Gaussian random variables is also a Gaussian random variable. For the same reason that experimental uncertainties add in quadrature, the standard deviation of the probability distribution function for the sum of two Gaussian variables is the quadrature sum of the two distributions.

3. A - The binomial distribution describes any situation where there are a fixed number of trials with binary outcomes (though not necessarily equal odds), and it gives the probability for obtaining $n = 1, 2, 3, ...$ successes. The scenario of flipping a coin is completely analogous to this. The Gaussian distribution is a good approximation to the binomial distribution in the limit of large statistics when the probability of success is near 50%. The other three distributions are asymptotic distributions of test statistics commonly used in hypothesis testing, and they have nothing to do with flipping a coin.

4. C - You might know this off the top of your head, but we can do this systematically very fast. We want $N(t)$ to drop by half, compared to (say) its value at $t = 0$, so we solve:
$$\frac{N_0}{2} = N_0 e^{-\lambda t_{1/2}}.$$
Taking logs gives $-\ln 2 = -\lambda t_{1/2}$, so $t_{1/2} = \ln 2/\lambda$, choice C.

5. D - It's simplest to recognize that if we switch all the 1's and 0's in the "Result" column, we end up with an AND gate, so the given table must represent a NAND gate.

6. D - When V_{in} is positive, the diode is forward biased, which means that the diode is effectively a wire and $V_{out} = 0$. (Actually, because of the built-in 0.7 V bias of silicon diodes, V_{in} must be greater than 0.7 V for forward biasing to occur.) When V_{in} is negative, the diode is reverse biased and effectively an open circuit, so $V_{out} = V_{in}$. Thus up to the 0.7 V diode bias effect, V_{out} keeps only the negative portions of V_{in}.

7. B - This is a straightforward application of the Poisson distribution. The average number of events in a ten-second window is $\lambda = 5$, and the number of desired events is $n = 1$, so plugging into (7.6) gives us B.

8. B - A circuit with an inductor and capacitor acts as a bandpass filter since the inductor filters high frequencies and the capacitor filters low frequency. As an example, the resonant frequency of a circuit containing just an inductor and capacitor in series is given by $\omega = 1/\sqrt{LC}$, or
$$f = \frac{1}{2\pi\sqrt{LC}}.$$
In order to have $f = 1$ MHz, we require
$$LC \sim \frac{1}{3.6 \times 10^{13}} \sim 3 \times 10^{-14} \text{ s}^2,$$
which is closest to B. We have used here the approximation of $\pi = 3$: a convenient trick for estimating order of magnitudes.