Causal effect identification and estimation

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- for IV method analysis
- focus on complier average causal effect (CACE)
  ↳ goal: to estimate $E(Y^{a=1} - Y^{a=0} \mid \text{compliers}) =: CACE$

+ assumptions:
  1.) Z is a valid instrument (i.e. Z affects treatment A)
  2.) exclusion restriction (i.e. Z doesn't directly affect Y)
  3.) monotonicity assumption (i.e. there are no defiers)

Let's start w/ intention-to-treat (ITT) effect:

$$\underbrace{E(Y^{z=1} - Y^{z=0})}_{\text{potential outcomes}} = \underbrace{E(Y \mid z=1) - E(Y \mid z=0)}_{\substack{\text{observed data:} \\ \text{(intention-to-treat)} \\ \text{ITT effect}}}$$

} this equality holds b/c of the randomization of Z.

$Y^{z=1}$: value of Y if everyone had been assigned to treatment (Z=1)

$$E(Y \mid z=1) = E(Y \mid z=1, \text{always takers}) \cdot P(\text{always takers} \mid z=1) + E(Y \mid z=1, \text{never takers}) \cdot P(\text{never takers} \mid z=1) + E(Y \mid z=1, \text{compliers}) \cdot P(\text{compliers} \mid z=1). \qquad (1)$$

expected value of Y among people assigned treatment (Z=1) } is a weighted average of expected value of Y given z=1 in the 3 subpopulations.

NOTE on always-takers and never-takers:

$E(Y \mid z=1, \text{always-takers}) = E(Y \mid \text{always-takers})$
$E(Y \mid z=1, \text{never-takers}) = E(Y \mid \text{never-takers})$
} b/c Z has no effect on these subjects.

↳ the encouragement (Z=1) has no impact.

NOTE: $P(\text{always-takers} \mid z=1) = P(\text{always-takers})$
↳ the subpopulations are completely independent from the result/value of the random variable Z.

We can simplify (1) as:

$$E(Y \mid z=1) = E(Y \mid \text{never-takers})\, P(\text{never-takers}) + E(Y \mid \text{always-takers})\, P(\text{always-takers}) + E(Y \mid z=1, \text{compliers})\, P(\text{compliers})$$

- Same for z = 0:

$$E(Y \mid z=0) = E(Y \mid \text{never-takers})\, P(\text{never-takers}) + E(Y \mid \text{always-takers})\, P(\text{always-takers}) + E(Y \mid z=0, \text{compliers})\, P(\text{compliers})$$

equal, cancel out in difference

$$\text{ITT effect} = E(Y \mid z=1) - E(Y \mid z=0) = E(Y \mid z=1, \text{compliers}) \cdot P(\text{compliers}) - E(Y \mid z=0, \text{compliers}) \cdot P(\text{compliers})$$

$$\Rightarrow \frac{E(Y \mid z=1) - E(Y \mid z=0)}{P(\text{compliers})} = E(Y \mid z=1, \text{compliers}) - E(Y \mid z=0, \text{compliers})$$

⇓

z is randomized & we have subpopulation of compliers. So, we are actually randomizing the treatment A.

$$E(Y \mid z=1, \text{compliers}) - E(Y \mid z=0, \text{compliers}) = E(Y^{a=1} \mid \text{compliers}) - E(Y^{a=0} \mid \text{compliers}) = CACE$$

potential outcomes

$$CACE = \frac{E(Y \mid z=1) - E(Y \mid z=0)}{P(\text{compliers})}$$

NOTE that $P(\text{compliers}) = E(A \mid z=1) - E(A \mid z=0)$ b/c the expected value $E(\cdot)$ of a binary variable is just the probability of the binary variable being 1.

* $E(A \mid z=1) = E(A=1 \mid z=1) = P(A=1 \mid z=1)$
  ↳ Proportion of people who are always-takers or compliers

* $E(A \mid z=0) = E(A=1 \mid z=0) = P(A=1 \mid z=0)$
  ↳ Proportion of people who are always-takers

⇒ Proportion of people who are compliers: $E(A=1 \mid \text{compliers}) = P(\text{compliers})$

$$P(\text{compliers}) = E(A \mid z=1) - E(A \mid z=0)$$

$E(A \mid z=1)$: always takers & compliers
$E(A \mid z=0)$: always takers only

CACE:

$$CACE = \frac{\text{ITT-effect}}{\text{proportion of compliers}} = \frac{E(Y \mid z=1) - E(Y \mid z=0)}{P(\text{compliers})} =$$

$$= \underbrace{\frac{E(Y \mid z=1) - E(Y \mid z=0)}{E(A \mid z=1) - E(A \mid z=0)}}_{}$$

numerator: } causal effect of treatment assignment Z on the outcome Y (ITT)
denominator: } causal effect of treatment assignment Z on treatment received

A, Z, Y: all observed variables;
⇒ we can identify the CACE.

Notes:

(1) w/ perfect compliance, we would get CACE = ITT b/c $P(\text{compliers}) = 1$
  ↳ the denominator $\left(E(A \mid z=1) - E(A \mid z=0) \right)$ is the probability $P(\text{compliers})$ and thus always btw. 0 and 1.
  ⇒ $CACE \geq ITT$ $\forall$ values of $P(\text{compliers})$
  ↳ b/c $CACE \geq ITT$, it is an underestimation of CACE

- $CACE \geq ITT$ makes sense b/c there will be subjects among the ITT population that are assigned to treatment (Z=1) but they don't take the treatment → this diminishes the true causal effect of treatment.

Summary of this lecture:

(1) IV methods require 2 assumptions:
  a) exclusion restriction
  b) Z being a "valid" instrument (i.e. affect treatment A)

(2) making the monotonicity assumption, we can identify CACE
  ↳ estimator of CACE is the standard ITT effect estimate divided by the proportion of compliers (i.e. P(compliers))
  ↳ in general, the ITT effect is a lower-bound (or underestimate) of the local average treatment effect among compliers (CACE).