

# Analysing data after matching

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- assumption:
  - (1) we have matched the data
  - (2) we have checked for covariate balance btw. groups

⇒ goal: outcome analysis with matched and balanced data

Analysis of outcomes in data after successful matching:

- test for treatment effect (e.g. hypothesis testing)
- estimate the treatment effect and its confidence interval

⇒ always we need to take the matching into account

Randomization tests: (a.k.a.: permutation tests or exact tests)

- idea:

- (1) compute test statistic from observed data  $T^{\text{obs}}$
- (2) assume null-hypothesis ( $H_0$ ) of no treatment effect is true
- (3) randomly permute treatment assignment within pairs and re-compute test-statistic  $T^{\text{perm}}$

- (4) repeat step (3) many times and see how unusual the observed test statistic from step (1) is (relative to the test statistics computed on the permuted data)

⇒ if  $H_0$  is true, then  $T^{\text{obs}}$  should be consistent with the test statistics  $T^{\text{perm}}$ , that we computed in step (3)

Example:

- 13 matched pairs w/ (binary outcome)
- we Test statistic  $T_{..}$  # of events in the treated group (arun i.e. # of outcomes in the treated group)

Observed data		Outcomes		ex. permutation 1:		* pair changed
Matched pair	Treated	Control		T	C	
$T^{\text{obs}} = 2 \text{ old}$	1	0	0	*	-	" "
+ 1 new = 5	2	1	0	*	-	" "
	3	1	0	*	0	" 1*
	4	0	0	*	-	" "
	5	1	1	*	-	" "
	6	0	0	*	-	" "
	7	0	1	*	1	" 0*
	8	0	1	*	-	" "
	9	0	0	*	-	" "
	10	1	0	*	0	" 1*
	11	0	0	*	-	" "
	12	1	0	*	-	" "
	13	1	0	*	-	" "

- \* concordant pairs b/c same outcome in both treatment of the pair
- \* discordant pairs b/c different outcomes

↳ permute discordant pairs b/c this changes the outcome of the treatment (e.g. matched pair #2 goes from (1,0) → (0,1))

↳ permute pairs based on Bernoulli random variable (meaning)

that NOT ALL discordant pairs should be permuted !!!)

McNemar test: (for testing null hypothesis using a randomization test w/ matched (paired) binomial data)

- hypothesis test used for paired binomial ( $n=1$ ) / bernoulli data control

treat	Outcome	Outcome	
		1	0
1	1	5	
0	2	5	

Concordant pairs

discordant pairs

↓

Contain the information whether or not there is a treatment effect (i.e. comparing these 5 vs. 2 pairs)

if there is no treatment effect, we would expect these numbers to be close / similar on average (in the long run of running the permutation simulation above)

Randomization tests: (aka permutation tests!)

- the basic idea also works with continuous data (e.g. systolic blood pressure)
- any other test statistics (e.g. difference in sample means  $\bar{X}$ )

⇒ randomly permute treatment labels and re-compute test statistic  $T^{\text{perm}}$ , and compare it to  $T^{\text{obs}}$ !

SBP

→ for paired continuous data we can use the paired t-test instead

Other outcome models:

- Conditional logistic regression: for matched binary outcome data

- stratified Cox model: for time-to-event (survival) outcome data

· Baseline hazard stratified on matched sets

using the "match-ID" variable for performing

the actual stratification

- Generalized estimating equations (GEE):

· cluster data on "match-ID" variable

· useful for binary outcome data

· can estimate:  
(i) causal risk ratio

(ii) causal odds ratio

(iii) causal risk difference

when log-link fct. used

depends on the link fct.

when identity-link fct. used