Propensity scores

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- propensity score = prob. of receiving treatment, rather than control, given covariates X
- A=1 ... treated & A=0 ... control
- denote propensity score of subject i as π_i:

$$\pi_i = P(A=1 \mid X_i)$$

- π_i is a function of X_i (i.e. prop. score = $f(X)$)

Example:

- if older people more likely receive treatment, then
  - $P(A=1 \mid age=60) > P(A=1 \mid age=30)$
  - $\pi_i > \pi_j$ if $age_i > age_j$

⇒ prob. of receiving treatment is 30% if subject i has prop. score of 0.3.

Balancing scores:

- occurs when 2 subjects have the same propensity score, but they possibly have different covariate values X.

→ both subjects are as likely to be treated

- if we restrict our data to a subpopulation of subjects w/ same value of prop. score, then there should be balance in the 2 treatment groups

⇒ **the propensity score is thus a balancing score**

↳ if we were to condition on the propensity score, then we will have balance btw. both groups

→ Example:

(1) consider only subjects with the same prop. score value (e.g. 0.45), then:

(2) if we stratify that subpopulation of subjects by the factual / actual treatment received, then:

(3) we should see the same dist. of covariates X for the treatment groups.

Formally Balancing Scores would mean:

dist. of the covariates X

$$P(X=x \mid \pi(X)=p, A=1) = P(X=x \mid \pi(X)=p, A=0)$$

↳ implication:

when matching on the propensity score, we should achieve balance btw. the groups A=0 & A=1.

Estimated propensity score:

- in a RT, the prop. score is known (e.g. $P(A=1) = P(A=0) = 0.5$)

↳ in a observational study, prop. score will be unknown

- however, prop. score just involves observed data: A and X, thus we can estimate it!

→ we need to estimate $P(A=1 \mid X)$

↳ the outcome here is A, a binary variable.

→ use e.g. logistic regression (LR) to estimate prop. scores

1. Fit LR model with outcome A, covariates X
2. Use fitted model to get the predicted probability (fitted values) for each subject

⇒ these values are the "estimated" propensity scores.