

Marginal structural models (MSM)

13.05.21

14:19

PRIMARY DIFFERENCE: MSMs model potential outcomes Y^a as opposed to observed outcomes Y .

Motivation:

- IPTW estimation is used for simple causal effects s.a. average causal effect (ACE)
- ↳ BUT we can also use IPTW methods to estimate causal effect params. from models

Marginal: model that is not conditional on the confounders (thus allowing use to infer population average estimates)

Structural: model for potential outcomes, NOT observed outcomes

1) Linear MSM: (continuous outcome) params. we want to estimate

$$E[Y^a] = \psi_0 + \psi_1 a \quad a = 0, 1$$

then: $E[Y^0] = \psi_0$ and $E[Y^1] = \psi_0 + \psi_1$

thus: ψ_1 is the average causal effect $E[Y^1] - E[Y^0] = (\psi_0 + \psi_1) - (\psi_0) = \psi_1$

MSM: is a model for the mean of the potential outcomes Y^a .
(i.e. $E[Y^0]$)

2) Logistic MSM: (binary outcome) (equivalent to $P(Y^a=1)$ b/c Y^a is binary.)

$$\text{logit}(E[Y^a]) = \psi_0 + \psi_1 a \quad a = 0, 1$$

- so $\exp(\psi_1)$ is the causal odds ratio

→ the mean of a binary outcome is a PROBABILITY of the outcome being equal to 1.

$$\text{entire pop. odds ratio} = \frac{\frac{P(Y^1=1)}{1 - P(Y^1=1)}}{\frac{P(Y^0=1)}{1 - P(Y^0=1)}} \left\{ \begin{array}{l} \text{odds that } Y^1=1 \\ \text{odds that } Y^0=1 \end{array} \right.$$

MSM with Effect Modification (a.k.a. heterogeneity of treatment effect):

- suppose V is a variable that modifies the effect of treatment A
→ linear MSM w/ effect modifier: (e.g. $V \equiv \text{diabetes}$, where $V(\text{diabetes}=\text{Yes})=1$
 $V(\text{diabetes}=\text{No})=0$)

instead of cond. on all confounders X , only cond. on V .

$$E[Y^a|V] = \psi_0 + \psi_1 a + \psi_3 V + \psi_4 a \cdot V \quad (a=0,1)$$

main effect for interaction term $a \cdot V$
main effect for modifier V
main effect for treatment a

$$\text{then } E[Y^1|V] - E[Y^0|V] = \psi_1 + \psi_4 V$$

→ Idea of effect modifier: At different values of V , there might be different treatment effects.

- V ... effect modifier

General MSM:

$$g\{E[Y^a|V]\} = h(a, V; \psi)$$

$g\{\cdot\}$... link function

$h(\cdot)$... is a fct. specifying parametric form of a and V
(typically additive, linear)