Causal assumptions

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(1) SUTVA (↳ stable unit treatment value assumption)
(2) Ignorability
(2) Consistency
(4) Positivity

- required for identifiability of causal effects
- untestable assumptions

- these assumptions refer to the observed data: $\{Y, A, X\}$

outcome, treatment, pre-treatment covariates X

1) SUTVA: (involves 2 assumptions)

1. No interference: Units do not interfere w/ each other (i.e. the treatment assignment of one unit doesn't affect the treatment effectiveness / outcome / treatment assignment of another unit)

$\Downarrow$

another word: no spillover or contagion btw. units

2. One version of treatment: There is only one varible we can hypothetically intervene on. The treatment is very well defined.

$\Rightarrow$ SUTVA "decouples" units in the observational dataset

2) Consistency:

- Potential outcome under treatment $A=a$, Y^a, is equal to the observed outcome Y, if the actual treatment received is $A=a$

↳ in other words: $Y = Y^a$ if $A=a$, for all a ($\forall a$ in A)

3) Ignorability: (sometimes: "no unmeasured confounders" assumption)

- given pre-treatment covariates X, treatment assignment is independent of potential outcomes

↳ $\underbrace{(Y^0, Y^1) \perp\!\!\!\perp A \mid X}$ "treatment assignment ignorable given confounders X" means: within levels of X, treatment assignment is independent from the potential outcomes.

among people/units with covariates X, this allows us to think of treatment A as being randomly assigned among that subpopulation

↳ example: $Y^0, Y^1 \not\!\perp\!\!\!\perp A$ (because $X \to A \to Y$ & $X \to Y$, $X \to A$, $A \to Y$)

- X ... a single variable (age) with values {young, old}
- fact 1: old people more likely to get treatment $A=1$
- fact 2: old people also more likely to have the outcome (hip fracture) Y, regardless treatment A

$\Rightarrow$ THEN: Y^0 and Y^1 aren't independent from A (marginally)

$\Rightarrow$ **However, within levels of X, treatment might be randomly assigned!**

4) Positivity assumption:

- every unit had some chance of receiving any treatment a ($\forall a, \forall x$)

↳ $\underbrace{P(A=a \mid X=x) > 0 \quad (\forall a, \forall x)}$

- means that treatment assignment is not deterministic (e.g. as a fct. of X)

↳ if for some values/levels of X, treatment was deterministic, then we would have no observed values of Y for one of the treatment groups for those values/levels of X

$\Rightarrow$ **in short: we need variability in treatment assignment for successful treatment effect identifiability!**

Applying these assumptions to causal effect estimation:

(1) $\mathbb{E}[Y \mid A=a, X=x] = \mathbb{E}[Y^a \mid A=a, X=x]$ by consistency assumption in (2). $Y = Y^a$ if $A=a$

$= \mathbb{E}[Y^a \mid X=x]$ by ignorability assumption in (3) $Y^0, Y^1 \perp\!\!\!\perp A \mid X$

$\Rightarrow \mathbb{E}[Y \mid A=a, X=x] = \mathbb{E}[Y^a \mid X=x]$

$\Downarrow$ marginalize over distribution of X

$\mathbb{E}[Y^a]$

Quiz answers:

(1) "No unmeasured confounders" $\equiv$ Ignorability

(2) "No interference between units" $\equiv$ SUTVA