Two stage least squares

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- Recap of ordinary least squares: (OLS)

→ model with treatment A and outcome Y: $Y_i = \beta_0 + A_i \beta_1 + \varepsilon_i$

- usual assumption: error-term ε_i and covariate A_i are independent: $A_i \perp\!\!\!\perp \varepsilon_i$.

↳ however: if there is confounding, A_i and ε_i will be correlated. ⇒ OLS fails.

→ using OLS to determine β_1 would not represent a valid causal effect b/c of the confounding ($A_i \not\!\perp\!\!\!\perp \varepsilon_i$).

Solution: Use of the two stage least squares method:

- 2 stage least squares: a method to estimate causal effects in the instrumental variable (IV) setting.

(1) assume Z is a valid IV (it affects treatment & meets exclusion restriction)

STAGE 1: Regress treatment received, A, on the instrumental variable Z:

$$\boxed{A_i = \alpha_0 + Z_i \alpha_1 + \varepsilon_i}$$ (regressing A on Z)

- where ε_i is assumed independent with $\mu_{\varepsilon_i} = 0$ and $\sigma^2_{\varepsilon_i} = \text{const.}$
- by randomization of Z_i; Z_i and ε_i should be independent

Using OLS, estimate α_0 as $\hat{\alpha}_0$ and α_1 as $\hat{\alpha}_1$; then obtain a predicted value of treatment, $\hat{A}_i$ for each person:

$$\hat{A}_i = \hat{\alpha}_0 + \hat{\alpha}_1 \cdot Z_i \quad \text{(for each subject)}$$

↳ the predicted value of A given Z.

STAGE 2: Regress the outcome Y, on the fitted value, $\hat{A}$, from stage 1:

$$\boxed{Y_i = \beta_0 + \underbrace{\hat{A}_i}_{\text{regressor}} \beta_1 + \varepsilon_i}$$ (regressing Y on $\hat{A}$)

- where ε_i is zero-mean and const. variance
- by exclusion restriction: Z independent of Y given A
  ↳ $\hat{A}$ is a projection of A onto space spanned by Z

→ estimating β_1 using OLS represents the causal effect.

Two Stage Least Squares Estimator: 2SLS Estimator:

- assume that both A and Z are binary:

A: treatment (yes/no) & Z: encouragement (yes/no)

Stage 1: $\hat{A}_i$ is an estimate of $E(A \mid Z)$

↳ $P(A \mid Z_i)$

Stage 2: $Y_i = \beta_0 + \hat{A}_i \beta_1 + \varepsilon_i$

↳ What is the interpretation of β_1?

$$\beta_1 = (\cancel{\beta_0} + \beta_1 \cdot (\hat{A}_i = 1) \cancel{+ \varepsilon_i}) - (\cancel{\beta_0} + \beta_1 \cdot (\hat{A}_i = 0) \cancel{+ \varepsilon_i}) = E(Y \mid \hat{A} = 1) - E(Y \mid \hat{A} = 0) = \beta_1$$

- there are two values of $\hat{A}$ in the 1st stage model:

1. $\hat{\alpha}_0$ (for $Z = 0$)
2. $\hat{\alpha}_0 + \hat{\alpha}_1$ (for $Z = 1$)

↳ if we have some non-compliance, then these two values will not necessarily be exactly $0 = \hat{\alpha}_0$ and $1 = \hat{\alpha}_1$ but s.th. between 0 and 1.

- when we go from $Z = 0$ to $Z = 1$, we observe a change in $\hat{A}$ from $\hat{\alpha}_0$ to $\hat{\alpha}_0 + \hat{\alpha}_1$.

- with a $\hat{\alpha}_1$ unit change in $\hat{A}$, we observe a mean difference change of $\hat{E}(Y \mid Z=1) - \hat{E}(Y \mid Z=0)$ in the mean of Y.

ITT effect

↳ if Z changes from $Z = 0 \to Z = 1$, $\hat{A}$ changes by $\hat{\alpha}_1$ units

⇒ ITT-effect: is a change in $\hat{A}$ by $\hat{\alpha}_1$ units whereas
β_1: is a change in $\hat{A}$ by 1 full unit

$$ITT = \underbrace{\underbrace{\hat{E}(Y \mid Z=1)}_{\substack{\text{for } Z_i = 1 \\ \text{we get:} \\ \hat{\alpha}_0 + \hat{\alpha}_1}} - \underbrace{\hat{E}(Y \mid Z=0)}_{\substack{\text{for } Z_i = 0 \\ \text{we get:} \\ \hat{\alpha}_0}}}_{\substack{\text{Change in Z by one full unit} \\ \text{corresponds to change in } \hat{A} \text{ by} \\ \hat{\alpha}_1 \text{ units.}}}$$ which requires $\hat{A}$ to change by $\hat{\alpha}_1$ units

- if we see a change in the mean of Y for a $\hat{\alpha}_1$ units change in $\hat{A}$, then we should see a $\dfrac{\hat{E}(Y \mid Z=1) - \hat{E}(Y \mid Z=0)}{\hat{\alpha}_1}$ unit change in the mean of Y for a 1 unit change in $\hat{A}$.

→ because we know that $\hat{\alpha}_1 \in [0, 1]$, we have to assume that $\hat{\alpha}_1$ unit change will be smaller than a 1 unit change. Therefore, the observed change in the mean of Y MUST increase. This is achieved by dividing by $\hat{\alpha}_1$ (b/c dividing a number X by a value < 1 increases the value of X).

- the 2SLS estimator is a consistent estimator of the complier average causal effect (CACE):

$$\beta_1 = CACE = \frac{E(Y \mid Z=1) - E(Y \mid Z=0)\}\ \text{ITT-effect}}{E(A \mid Z=1) - E(A \mid Z=0)\}\ \text{causal effect of treatment on compliance} \Rightarrow \hat{\alpha}_1}$$

2SLS more generally:

- 2SLS can also be used with covariates X and non-binary data (e.g. continuous valued treatment A)

Stage 1: regress A on Z and covariates X
↳ obtain fitted value of A: $\hat{A}$.

Stage 2: regress Y on $\hat{A}$ and covariates X
↳ the coefficient of $\hat{A}$, i.e. β_1, will be the complier average causal effect of treatment (CACE).

Sensitivity analysis to check IV assumptions:

(1) Exclusion restriction: if Z does directly affect Y by an amount ρ, would my conclusion change? Vary value of ρ.

(2) Monotonicity: if the proportion of defiers was π, would my conclusions change?

↳ the idea is to check whether conclusions would change if assumptions were to be violated.