Causal effects

17.04.21 11:11

Average Causal Effect (ATE):

pop. of interest

World 1: (w1) Everyone receives A=0 → $\text{mean}(Y)$

World 2: (w2) Everyone receives A=1 → $\text{mean}(Y)$

$ATE = \text{mean}^{w1}(Y) - \text{mean}^{w2}(Y)$, here, the difference is the average causal effect (i.e. over the whole population)

$\mathbb{E}(Y^1 - Y^0)$: if Y is binary, this would be a risk difference $(P(Y=1|\text{group } 1) - P(Y=1|\text{group } 2))$.

Conditioning on versus setting treatment:

based on conditioning outcomes i.e. after data was collected/observed

- in general: $\mathbb{E}(Y^1 - Y^0) \neq \mathbb{E}(Y \mid A=1) - \mathbb{E}(Y \mid A=0)$

means MANIPULATING

based on idea of potential outcomes: i.e. setting treatments

$\mathbb{E}(Y^1 - Y^0)$: average causal effect

$\mathbb{E}(Y \mid A=1) - \mathbb{E}(Y \mid A=0) \neq$ average causal effect b/c:

↳ $\mathbb{E}(Y \mid A=a)$ is restricted to subpopulation of people who actually had $A=a$.

↳ THIS ISN'T INTERVENTIONING!!!

REAL WORLD IS DIFFERENT:

pop. of interest

people got A=0 → $\text{mean}(Y)$

people got A=1 → $\text{mean}(Y)$

average diff. in outcome between 2 totally different subpopulations defined by treatment group

⇒ this doesn't result in an isolated treatment effect because the minuend ($\mathbb{E}(Y \mid A=1)$) is from a different population than the subtrahend ($\mathbb{E}(Y \mid A=0)$)!

→ causal effect estimates should thus always be based on outcomes from the EXACT SAME POPULATION!!!

Example:

$\mathbb{E}(Y \mid A=1)$: mean of Y among people with A=1

$\mathbb{E}(Y^1)$: mean of Y if the whole population was treated with A=1

$\mathbb{E}(Y \mid A=1) - \mathbb{E}(Y \mid A=0)$ is not a causal effect, because it compares two different populations of people!!!

$\mathbb{E}(Y^1 - Y^0)$ ... is a causal effect b/c it compares the same people under different treatments A.

Other causal effects:

- $\mathbb{E}(Y^1 / Y^0)$: causal relative risk
- (*) $\mathbb{E}(Y^1 - Y^0 \mid A=1)$: causal effect of treatment on the treated

restrict to subpopulation

- $\mathbb{E}(Y^1 - Y^0 \mid V=v)$: average causal effect in the subpopulation with covariate $V=v$.
  ↳ aka heterogeneity of treatment effects
  ↳ ex.: V=age, V=race, etc. → defines a subpopulation, where V=v is true.

(*) example: CE of treatment on the treated:

treated population (20%) (A=1)

World 1: everyone with A=0 → $\text{mean}(Y)$

World 2: everyone with A=1 → $\text{mean}(Y)$

causal effect of treatment on the treated