MINIMISE THE COST OF A NEW COFFEE BLEND

SECTION 1. THE PROBLEM

An independent coffee shop is considering creating a unique holiday blend using 4 types of coffee beans: Ethiopian Yirgacheffe (Y), Colombian Supremo (S), and Kenya Peaberry (P), and Guatemalan Huehuetenango (H).

The coffee shop wants to achieve a harmonious blend of floral, fruity, nutty, and chocolatey notes to create a complex and well-rounded flavor profile, offering a delightful taste experience with a balanced acidity and strength level. More information on coffee bean acidity, strength and flavour notes is presented in Appendix A.

1.1 Objective

The roaster aims to minimise the costs of creating the new blend while ensuring the desired flavour of the blend.

1.2 Decision variables

They need to decide on the proportion of each coffee bean (4 types that are mentioned above) to include in the blend following their objective.

1.3 Parameters

The prices and characteristics of each coffee bean is obtained from Coffee-Direct.co.uk, outlined in **Table 1** below.

As only certain amount of a bag is sold, to simplify calculations and measurements during the blending process, prices are calculated per 100g. The acidity and strength levels are evaluated by the provider on a scale from 1-10. The beans' aroma is taken from descriptions provided for each coffee bean type.

Table 1. Details of 4 types of coffee beans

Type	Price (£/908g)	Price (£/100g)	Acidity	Strength	Aroma	Source
Yirgacheffe (Y)	27.99	3.08	5	6	Floral, fruity	Ethopian Yirgacheffe Coffee Beans
Supremo (S)	25.99	2.86	2	2	Nutty	Colombia Supremo Coffee Beans
Peaberry (P)	28.99	3.19	9	8	Earthy, fruity	Peaberry Coffee Beans
Guatemalan (G)	30.99	3.41	6	3	Chocolatey, citrusy	Guatemala Huehuetenango Coffee Beans

It is difficult to get real value for acidity and strength level because they are influenced by various factors. Roast level and bean origin, for example, can influence perceived acidity[1]. The strength of coffee is not solely about caffeine content but also influenced by factors like flavour profiles[2]. Therefore, the scores given by the supplier is utilised in this problem instead. Given the score, there are 3 levels which are **Low (1-3), Medium (4-7) and High (8-10).**

1.4 Constraints

There are 5 constraints need satisfying in this problem, specifically

- The total proportion of all coffee beans in the blend must be equal to 1
- The acidity level of the blend is high (for bright and vibrant taste experience)
- The strength level of the blend is medium (balance between boldness and subtlety, suitable for most coffee drinkers)
- The combined aroma of floral notes (Y), earthy and fruity (G) contributes at least 40% (weight of 1) of the blend's aroma, with floral aroma more preferred than earthy and fruity.
- The nutty aroma (S) should be at maximum of 20%, ensuring a balanced blend with the desired aroma profile.

[1] *Stone Street Coffee*. (n.d.) 'Is Your Coffee Too Acidic?'. Available at: https://stonestreetcoffee.com/blogs/brooklyn-coffee-academy/is-your-coffee-too-acidic

[2] *Hoxton Coffee Co*. (n.d.) 'Coffee Strength - Differences Between Flavour & Caffeine Content'. Available at: https://hoxtoncoffee.com/blogs/coffee/coffee-strength

SECTION 2. MODEL

2.1 Mathematical model

2.1.1 Decision variables

x_1: the proportion of Yirgacheffe in the blend

x_2: the proportion of Supremo in the blend

x_3: the proportion of Peaberry in the blend

x_4: the proportion of Huehuetenango in the blend

$0 \leq x_1, x_2, x_3, x_4 \leq 1$

2.1.2 Objective

Minimise the cost (100g of the blend)

$$C = 3.08\, x_1 + 2.86\, x_2 + 3.19\, x_3 + 3.41\, x_4$$

where the coefficients are the prices per 100g of each type outlined in **Table 1**.

2.1.3 Constraints

(1) Total proportion

$$x_1 + x_2 + x_3 + x_4 = 1$$

(2) High level of Acidity

$$8 \leq \sum_{i=1}^{4} x_i \times a_i \leq 10$$

where a_i is the acidity score of coffee bean x_i

(3) Medium level of Strength

$$4 \leq \sum_{i=1}^{4} x_i \times s_i \leq 7$$

where s_i is the strength score of coffee bean x_i

(4) At least 40% of floral (Y), earthy (P) aroma, floral is more preferred

$$x_1 + 0.5x_3 \geq 0.4$$

(5) Maximum of 20% nutty (S) aroma

$$x_2 \leq 0.2$$

2.2 Accuracy of the model

The model assumes different assumptions including proportionality, additivity, and divisibility. Specifically, the model assumes that the contribution of each coffee bean to the overall characteristics (acidity, strength, aroma) is directly proportional to its proportion in the blend. Also, the overall flavor profile is an additive combination of the flavors of individual beans.

Lastly, the model assumes that the proportions of individual coffee beans can take any real value between 0 and 1.

Regarding the accuracy of the model, there are certain limitations.

First, the model assumes simple linear relationships between proportions and final characteristics, which may not accurately reflect the true complexities of coffee blending. Factors like roasting profiles and interactions between different beans can significantly influence the final taste, especially in terms of aroma.

Second, the model relies on pre-defined thresholds measured in score for acidity, strength, and aroma preferences. These thresholds can be subjective and vary depending on individual preferences. Also, if these scores are not reliable or representative of the actual sensory experience, the model's solution may not accurately reflect the desired flavor profile.

Third, since the aroma plays a crucial role, the requirement of maximum 20% nutty and or at least 40% of floral (Y) and earthy (P) aroma, which coefficient for floral is higher, is very subjective, depending on the roaster. While these constraints might affect the optimal solution, they are included to reflect the real-world scenarios and the creativity and complexity of designing wanted blend flavors.

SECTION 3. SOLVE

The solution of the model is outlined in the Excel file.

SECTION 4. TEST

4.1. Optimal solution

The optimal proportion for this problem is **11.8% Supremo, 82.3% Peaberry and 5.9% Guatemalan, no usage of Yirgacheffe**, with the **total cost** of making 100g blend is **£3.17**.
While the price of Peaberry is the second highest, it is used the most due to its richness in acidity and strength and also preferred for the aroma, which is reasonable. On the other hand, while there is a preference for floral aroma from Yirgacheffe, but this coffee bean is not used, which partly did not actually satisfy the coffee shop's requirements.

4.2 Sensitivity Analysis

A screenshot of the sensitivity report from excel is shown in the **Figure 1** below.

Figure 1. Sensitivity Report

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
B5	Proportion Yirgacheffe	0	0.291526302	3.082599119	1E+30	0.291526302
C5	Proportion Supremo	0.117647059	0	2.862334802	0.353996224	1E+30
D5	Proportion Peaberry	0.823529412	0	3.192731278	0.381226703	1E+30
E5	Proportion Guatemalan	0.058823529	0	3.412995595	1E+30	0.361862807

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
B8	Sum_proportion	1	2.810507904	1	0.026190476	0.095238095
B13	Blend_strength	7	-0.149002332	7	0.142857143	0.1
B19	Blend_flavor	0.411764706	0	0.4	0.011764706	1E+30
B13	Blend_strength	7	0	4	3	1E+30
B10	Blend_acidity	8	0.174915781	8	0.4	0.11
B10	Blend_acidity	8	0	10	1E+30	2

Reduced Cost

Regarding Variable cells (Decision variables), reduced costs represent the amount by which the objective coefficient of a variable would need to improve to (decrease for minimisation problems) for that variable to enter the optimal solution. For this model, only proportion of Yirgacheffe has the value while reduced costs of others equal to 0, meaning that if the price of Yirgacheffe declines by £0.29 (9.4%), to £2.79, while others remain, there could be improvements in the objective value. This also explains why Yirgacheffe is not used based on the optimal solution.

To purchase Yirgacheffe at a lower price, the coffee shop has different options. They could subscribe to Coffee-Direct.co.uk for a scheduled delivery which will immediately has 15% discount, covering the required 9.4% decrease. However, if it is still during trial-and-error stage and no need for weekly or monthly supply, they could consider buy in a larger quantity (cheaper price) or find other suppliers that sell at a lower price while still ensuring the coffee bean quality.

Shadow Price

For the shadow prices, which represent the change in the optimal objective function value per unit increase in the right-hand side (RHS) of a constraint. The **shadow price for the Blend_strength** (medium, range from 4-7) constraint indicates that for each unit increase in the upper limit of the strength range for the blend, the optimal objective function value would decrease by approximately £0.149. The blend's strength level is already higher than the

minimum required (4), implying that reducing the strength level may not necessarily impact the optimal solution significantly. However, the coffee shop should be cautious not to compromise the desired flavor profile for increased strength since strong coffee might not suit the majority.

The **shadow price for the Blend_acidity** (high, range from 8-10) constraint indicates that for each unit increase in the acidity level of the blend, the optimal objective function value would increase by approximately £0.175. This suggests that there is potential value in increasing the acidity level of the blend up to the upper limit (10). This could enhance the desired flavor profile and potentially lead to an improvement in the overall blend quality.

Ranges of Certainty

Ranges of certainty represent the allowable range of values, i.e. Allowable Increase and Allowable Decrease, for decision variables or constraint coefficients over which the optimal solution remains unchanged.

With regards to **proportion of Guatemalan**, while its allowable increase is an infinite value, it has an allowable decrease of 0.3619, approximately 36.19%, which is relatively large. This indicates that there is flexibility in reducing the proportion of Guatemalan in the blend without significantly affecting the optimality of the solution. However, it is still essential to consider the potential impact on the flavor profile and overall blend quality when making such adjustments.

The range for **Blend_flavor constraint** has a relatively small allowable increase (0.0118), while allowable decrease is infinite, indicating limited flexibility in increasing the contribution of flavor components of Yirgacheffe and Peaberry, but only Peaberry in this situation based on the optimal solution (Yirgacheffe is not used). However, decreasing the flavor contribution has no practical limit, implying significant flexibility in reducing Peaberry flavor without affecting the optimality of the solution. Since the coffee shop wants a certain strength of earthy and fruity taste from Peaberry, this option of decreasing Peaberry flavor contribution might not be considered.

Non-exact Bounds

Non-exact bounds refer to constraints that are binding but not exact, meaning there is some flexibility in satisfying these constraints without violating optimality.

For example, Proportion Yirgacheffe has reduced Cost: 0.291, increasing the proportion of Yirgacheffe will worsen the objective function (increase cost). However, the allowable increase is set to 1E+30 (infinite value), which indicates that even though increasing Yirgacheffe might worsen the objective function, it is currently not restricted by any upper bound in the model.

APPENDIX

Appendix A: Notes for coffee bean acidity, strength, and aroma

Acidity: Acidity in coffee refers to a pleasant, tangy, or sharp taste sensation, related to the brightness and liveliness of the coffee's flavor.

Strength: Strength in coffee typically refers to its intensity or boldness, and is influenced by factors such as roast level, brewing method, and coffee-to-water ratio.

Aroma: Aroma encompasses the scent of freshly ground coffee beans as well as the aroma released during brewing. Coffee aromas can range from floral and fruity to nutty, chocolatey, or spicy, depending on factors like bean variety, processing method, and roast level. Aroma plays a crucial role in influencing the perceived taste and enjoyment of coffee.