Investor Sentiment and Global Economic Conditions

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Abstract

Investor Sentiment is measured at both global and local levels as the common component of pricing errors investors make when valuing stocks. Investor sentiment and macroeconomic factors are jointly modelled within a hierarchical dynamic factor model allowing for time-varying parameters and stochastic volatility. We extend existing methods to enable estimation of the model with the prescribed hierarchy which permits a cross-country analysis. Our approach allows us to control for macroeconomic conditions that may contaminate investor sentiment indices. We find that global investor sentiment is a key driving force behind domestic sentiment and global economic conditions.

Keywords: Business Cycles ; Hierarchical Dynamic Factor Model; Stock Market Sentiment.
JEL: C32, E32, E51.

1 Introduction

Asset prices are correlated with business cycles. Stock markets tend to rise in good times and are occasionally subject to spectacular reversals, which tend to happen near business cycle turning points and in sync across countries. Such events are difficult to reconcile with standard finance models in which investors force prices to equal the present value of cash flows. Investor sentiment,

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which refers to beliefs about future cash flows and investment risks that are not fully justified by fundamentals (cf. Baker and Wurgler (2007)), can provide a compelling explanation for why the market price of risk varies over time (see Barberis et al. (1998)).

This paper proposes a new approach to measure investor sentiment and goes on to study the global macroeconomic consequences of investor sentiment shocks, which represent unanticipated changes to the difference between stock returns and their fundamental value as prescribed by theory. We ask two related questions. First, does the component of stock price movements, which is unrelated to fundamentals, drive macroeconomic fluctuations? Second, should investor sentiment be regarded as a global variable and therefore exogenous to domestic economies or is it best thought of as being domestically determined? Our empirical study is based on a hierarchical dynamic factor model, which treats investor sentiment and international macroeconomic factors as unobserved variables. Investor sentiment is measured as the common component of mispricing factors which are obtained as the expected return of holding stocks beyond what is implied by fundamentals. The hierarchical structure of our model allows us to explore cross-country spillovers of sentiment and the link between financial and real sectors of each country. In doing so, we build on the Diebold-Yilmaz Connectedness Index (DYCI) proposed by Diebold and Yilmaz (2009). Contrary to the authors, and in the spirit of Korobilis and Yilmaz (2018) we extract a connectedness index by explicitly modelling time-varying parameters instead of using rolling-window estimates. This framework allows us to i) measure investor sentiment at both local and global levels and ii) study the spillovers of investor sentiment across countries and how their importance changed through time.

Our paper relates to a large body of literature attempting to measure investor sentiment in the stock market. The common thread in most studies is the need to find proxies for sentiment and combine several imperfect measures so as to mitigate potential confounding influences that each proxy is subject to. Candidate proxies include surveys, mood proxies, retail investor trades, mutual fund flows, option implied volatility, first day return on Initial Public Offerings (IPOs), volume of IPOs, new equity issues and insider trading (see Baker and Wurgler (2007) for a review). In their influential studies, Baker and Wurgler (2006, 2007) construct an investor sentiment index (BW index, hereafter) that aggregates the information in 6 proxies – the closed end fund discount rate, NYSE share turnover, number of IPOs, average first day returns of IPOs, dividend premium and the equity share in new issues. From an econometric point of view, the index is the first principal component of those proxies. Following the same approach, Huang et al. (2015) improve the BW index by accounting for a potential common approximation error in all proxies by applying a Partial Least Squares (PLS) algorithm to extract the investor sentiment index. In both studies the emphasis is to find the common component of investor sentiment proxies that predicts aggregate stock market returns. In these studies, even though investor sentiment is assumed to be unobservable, an implicit assumption is that it spans the space of the proxies considered.

Our approach differs from the above-cited papers in several ways. First, investor sentiment is measured as the common component of firm-level mispricing innovations, according to a dividend discount model-type asset pricing equation derived by Campbell and Shiller (1988). The underlying assumption is therefore that investor sentiment can be thought of as commonalities in over/under-valuation of stocks by investors. Therefore, our investor sentiment index spans the space of pricing errors directly rather than relying on proxies for over/under-valuation. While the literature on asset pricing offers other alternatives to derive stock price valuations, a large number of alternative models are variants of the dividend discount model.[1] Although factor models are popular in asset pricing, there is currently no consensus on which factors should be included in the pricing kernel (see Cochrane (2011) and Welch and Goyal (2008) for a discussion). Nevertheless, we evaluate the sensitivity of our investor sentiment indices to the inclusion of a wider set of measures of firm fundamentals and show that the results do not change significantly.

Second, our investor sentiment index is determined within a dynamic macro-finance model whereby mispricing innovations and macroeconomic factors are jointly determined. This mitigates the possibility of investor sentiment indices being contaminated with macroeconomic innovations which act as confounding factors. We therefore address the critique that existing sentiment indices

[1] See e.g. Ang and Liu (2001, 2007), Bakshi and Chen (1997, 2005), Ang and Bekaert (2007), Bekaert et al. (2010) among others, for non-linear present value models in discrete and continuous time.

are mainly driven by business-cycle components (see Sibley et al. (2016)). Our approach also enables structural analysis that helps to understand how investor sentiment is formed and its macroeconomic relevance. Third, we explicitly model heteroscedasticity, which is a common concern when working with financial data, and time-variation in the parameters, thereby capturing changes in the relationships amongst the variables in the model through time. Our approach allows measurement of investor sentiment across countries – an exercise that is typically challenging due to lack of available data and is made easier through our approach.[2]

There is a growing consensus that investor sentiment is important in understanding economic fluctuations. This paper presents empirical evidence which lends support to such a hypothesis. The literature studying the propagation of financial market shocks to the real economy is large. Many studies emphasise financial market frictions as the key elements whereby price-to-fundamental fluctuations propagate and amplify shocks to the real economy (e.g. Bernanke and Gertler (1989); Kiyotaki and Moore (1997)). Contrary to these theories, recent work in behavioural finance hypothesise that time variation in investor sentiment, reflecting changes in risk appetite and beliefs, are a key determinant of the expected return of assets in the economy. Gennaioli et al. (2015), Greenwood et al. (2016) and others stress that investors update their beliefs in a way which is not fully rational due to behavioral bias such as overoptimism and extrapolation of past good/bad news, which leads to mispricing. Other explanations for why sentiment may arise include the existence of noise traders (Montone and Zwinkels (2020)) and rational investors which are limited by arbitrage contraints (Shleifer and Vishny (1997)). A common thread in sentiment based theories is the role of an endogenous unwinding of overoptimistic beliefs, which is the key mechanism leading to rapid swings in asset prices. The idea is that buoyant sentiment translates in high valuations when risk is aggressively priced.

Much of the related literature focus on how credit market sentiment drives macroeconomic dynamics (eg. Greenwood and Hanson (2013) and López-Salido et al. (2017)). Nonetheless, the

[2] Baker et al. (2012) extend the BW index to 6 major stock markets by using only 4 of the 6 original proxies of investor sentiment due to lack of data. Our approach allows the construction of an investor sentiment index for any country with reliable stock market data.

same arguments carry through to stock market sentiment, since equity is also key in determining the external finance premium. Although a growing consensus is emerging that investor sentiment plays a role in understanding asset pricing, its relationship to the macroeconomy is still unclear. Despite recent progress in establishing a link between credit market sentiment and real activity by López-Salido et al. (2017), investor sentiment associated to stock prices remained largely unexplored up until more recently. A number of papers argue that the key link between investor sentiment and the macroeconomy occurs through investment decisions (Morck et al., 1990; Polk and Sapienza, 2009). Buoyant investor sentiment leads to an increase in firms' real investment, prompting rising employment (McLean and Zhao, 2014). Moreover, Montone and Zwinkels (2020) show that shocks to US investor sentiment affect employment growth domestically and across-borders due to spillover effects which are also documented by Baker et al. (2012).

From an econometric point of view, we extend existing methods introduced by Koop and Korobilis (2014) to estimate hierarchical factor models. The Kalman filter proposed by the authors is adjusted such that dynamic factor models with time-varying parameters and stochastic volatility can be estimated with constraints placed on the state space. The hierarchy is achieved by imposing some structure on the loadings matrix which defines the way idiosyncratic sentiment variables load onto the latent global and domestic sentiment factors.

Three key results emerge from our empirical exercise. First, both global and domestic investor sentiment are well described by few factors. In particular, the seven factors we use in our empirical exercise, which include one global and six local investor sentiment factors capture 45% of the dynamics of firm-level investor sentiment across countries. Second, global investor sentiment is an important driver of real economic developments. Shocks to global and local investor sentiment together explain between 5% and 12% of the dynamics of the five macroeconomic factors that capture movements in industrial production, inflation, unemployment, interest rates and term-spreads across countries. Third, global investor sentiment is also relevant in understanding the dynamics of domestic investor sentiment factors, explaining roughly between 28% and 51% of the movements in domestic investor sentiment. We find that the global investor sentiment factor is a '*net transmitter*' of spillovers in the system, while domestic sentiment factors are '*net receivers*'. Despite a significant dependence on global investor sentiment shocks, a material fraction of the variation of domestic investor sentiment factors is idiosyncratic. We find that the sign of the net directional connectedness between investor sentiment and macroeconomics varies over time which reflects a complex two-way cause and effect relationship between investor sentiment and macroeconomic conditions.

The remainder of the paper proceeds as follows. Section 2 explains the theoretical underpinnings of our approach to measure investor sentiment. Section 3 presents the econometric approach and Section 4 discusses the data. Results are presented in Section 5 while Section 6 examines the sensitivity of our results to different specifications. Finally, Section 7 concludes.

2 Measuring Investor Sentiment

The construction of our sentiment index is theoretically based on the dividend discount model (DDM) and its linear approximation introduced in Campbell and Shiller (1988). The realized one-period gross return on equity at time $t \geq 0$ is given by

$$\exp(r_{t+1}) = \frac{P_{t+1} + D_{t+1}}{P_t}, \tag{1}$$

where P_t denotes the per-share price of a firm's stock at time t and D_t is the dividend paid at time t. In order to apply time-series analysis, we follow Campbell and Shiller (1988) and linearize the price-dividend ratio to obtain an approximate linear expression for the relationship between log returns, log dividends and log prices. Using first order Taylor approximation, the aforementioned authors approximate the log gross return

$$r_{t+1} = \log\left(\frac{P_{t+1} + D_{t+1}}{P_t}\right) \approx k + g_{t+1} - \rho dy_{t+1} + dy_t, \tag{2}$$

where $dy_t = \log(D_t/P_t)$ is the dividend yield and $g_{t+1} = \log(D_{t+1}/D_t)$ is the one-period growth rate of dividends. The constant $k = -\log(\rho) - (1-\rho)\log(1/\rho - 1)$ and ρ is the average ratio of the stock price P_t to the sum of $P_t + D_t$.

We rewrite equation (2) as a predictive regression as follows

$$r_{t+1} = k + dy_t + g_{t+1} - \rho dy_{t+1} + \eta_{t+1} \tag{3}$$

with residuals η_{t+1}. For a given firm j, the above expression allows us to compare realized returns r^j_{t+1} with expected returns $\hat{r}^j_{t+1}$, which are obtained as the linear projection of returns onto a vector of fundamentals consisting of the dividend yield dy_{t+1}, its own lag dy_t, and the dividend growth rate g_t. The difference between expected and realized returns is measured by η^j_t, the residuals of regression (3). This term captures the movements in realized returns which are not accounted by fundamentals. In this setting, the component of the stock price of a firm that is not explained by its own fundamentals, is hereby interpreted as a measure of investor sentiment for firm j. The next sections will present the econometric framework we use to study the macroeconomic relevance of our measure of investor sentiment which originates at a firm level and is an aggregate of the panel η^j_t which we obtain when running regression (3) for all firms in our dataset.

3 Econometric Approach

3.1 Hierarchical Dynamic Macro-Finance Factor Model

We describe the joint behavior of the unobserved macroeconomic factors and country specific investor sentiment factors using a hierarchical dynamic factor model. Therefore, we assume that five macroeconomic variables - industrial production, inflation, unemployment rates, interest rates and term-spreads for the k countries considered in our exercise are driven by some common factors. Denoting by y_t the $N_y \times 1$ vector of macroeconomic variables across countries with $N_y = 5k$, we thus assume the following factor model

$$y_t = \Gamma_{t,yy} F_t^y + v_t^y, \tag{4}$$

where F_t^y comprises the five latent macroeconomic factors in the model which summarize the dynamics of the corresponding macroeconomic variables in all countries. $\Gamma_{t,yy}$ is an $N_y \times 5$ matrix of factor loadings which is restricted in such a way that the factors can be identified as depicting industrial production, inflation, unemployment, interest rates, and term spreads. v_t^y is an $N_y \times 1$ vector of idiosyncratic components.

In addition to the latent macroeconomic factors F_t^y, there are k factors which capture country specific investor sentiment and one global factor which captures global investor sentiment. We denote by N_η the total number of firms, across countries considered in the empirical study, and model each firms' expected returns according to (3). Let η_t be the $N_\eta \times 1$ vector of innovations to equation (3) for all firms. We then allow these pricing errors to be described by a factor model that involves both macroeconomic as well as investor sentiment factors. Specifically, we have

$$\eta_t = \Gamma_{t,\eta y} F_t^y + \Gamma_{t,\eta\eta} F_t^\eta + v_t^\eta, \tag{5}$$

where F_t^η is a $k+1$ vector of latent investor sentiment factors which describe comovement of mispricing in k countries and in addition one global factor which is driven by global mispricing across all countries considered. $\Gamma_{t,\eta\eta}$ is an $N_\eta \times (k+1)$ matrix of factor loadings which is constrained so that all elements of η_t load onto the global factor and only some of these elements load onto domestic sentiment factors, according to the firms' country of domicile. Finally, v_t^η is an $N_\eta \times 1$ vector of idiosyncratic components.

The macroeconomic and sentiment factors are then jointly determined by

$$\begin{pmatrix} y_t \\ \eta_t \end{pmatrix} = \begin{bmatrix} \Gamma_{t,yy} & \Gamma_{t,y\eta} \\ \Gamma_{t,\eta y} & \Gamma_{t,\eta\eta} \end{bmatrix} \begin{pmatrix} F_t^y \\ F_t^\eta \end{pmatrix} + \begin{pmatrix} v_t^y \\ v_t^\eta \end{pmatrix}, \quad \begin{pmatrix} v_t^y \\ v_t^\eta \end{pmatrix} \sim N(0, V_t), \tag{6}$$

with $\Gamma_{t,y\eta} = 0$. Here $\Gamma_t = (\Gamma_{t,ij})_{i,j\in\{\eta,y\}}$ are matrices of factor loadings. The idiosyncratic components collected in $v_t = (v_t^y, v_t^\eta)^\top$ are assumed to follow a Gaussian distribution centered in zero with time varying covariance matrix V_t. Moreover, we adopt the common identifying assumption that $V_t = diag(V_{t,yy}, V_{t,\eta\eta})$ is diagonal, which guarantees that v_t is a vector of idiosyncratic shocks and F_t^y contains information common to the underlying variables. We assume macroeconomic variables are standardized prior to their inclusion in the model and η_t are zero mean pricing errors, therefore equation (6) does not include intercepts.

In addition, the joint dynamics of the investor sentiment factors and the macroeconomic factors follow a VAR process of the form

$$\begin{pmatrix} F_t^y \\ F_t^\eta \end{pmatrix} = \begin{bmatrix} \Phi_{t,yy} & \Phi_{t,y\eta} \\ \Phi_{t,\eta y} & \Phi_{t,\eta\eta} \end{bmatrix} \begin{pmatrix} F_{t-1}^y \\ F_{t-1}^\eta \end{pmatrix} + \begin{pmatrix} u_t^y \\ u_t^x \end{pmatrix}, \quad \begin{pmatrix} u_t^y \\ u_t^\eta \end{pmatrix} \sim N\left(0, \begin{bmatrix} \Sigma_{t,yy} & \Sigma_{t,y\eta} \\ \Sigma_{t,\eta y} & \Sigma_{t,\eta\eta} \end{bmatrix}\right), \tag{7}$$

with time-varying covariance matrix $\Sigma_t = (\Sigma_{t,ij})_{i,j\in\{\eta,y\}}$ and VAR coefficients $\Phi_t = (\Phi_{t,ij})_{i,j\in\{\eta,y\}}$.

Three final remarks that complete the model defined by (4)–(7) are in order. First, the model allows for time varying parameters Γ_t and Φ_t in the measurement equation (6) and the state equation (7). Both matrices depend on time t and their dynamics are defined as

$$\gamma_t = \gamma_{t-1} + \varepsilon_t \quad \varepsilon_t \sim N(0, W_t), \tag{8}$$

$$\phi_t = \phi_{t-1} + h_t \quad h_t \sim N(0, Q_t), \tag{9}$$

where γ_t and ϕ_t are stacked versions of matrices Γ_t and Φ_t. Second, all covariance matrices in the model (6)–(9) which include V_t, Σ_t, W_t and Q_t are time dependent. Time dependence of V_t and Σ_t allows us to capture stochastic volatility. Whereas, the extent to which W_t and Q_t vary with time determines the amount of time-variation in the parameters Γ_t and Φ_t, respectively. Lastly, it is important to note why we chose to leave $\Gamma_{t,\eta y}$ unrestricted. By doing so, we mitigate the possibility that our measure of investor sentiment F_t^η is contaminated by macroeconomic innovations. It is clear from equation (5) that $\Gamma_{t,\eta y}$ is crucial in certifying that F_t^η only captures the common movements in sentiment that are not due to the state of the macroeconomy summarized by F_t^y. This is a key advantage of measuring investor sentiment in this framework.

3.2 Factor Hierarchy

In our application we require a factor model which is able to account for potential common dynamics between the macroeconomy, global and local investor sentiment. Therefore, each factor is identified according to which individual series are allowed to load onto it which is prescribed by a hierarchical structure that applies to the loadings matrix. This strategy is common and has been adopted for instance by Breitung and Eickmeier (2016); Ha et al. (2020); Stock and Watson (2016) and Potjagailo and Wolters (2020). Each factor is given an interpretation according to the group of variables which are allowed to load onto it. To be specific, we impose a set of restrictions on Γ_t which are summarized by the restrictions matrix below

$$R = \left(\begin{array}{c|cccc} I_5 \otimes \mathbf{1}_k & & \mathbf{0} & & \\ \hline & \mathbf{1}_{N_\eta^1} & \mathbf{1}_{N_\eta^1} & \cdots & \mathbf{0} \\ \mathbf{1} & \vdots & \vdots & \ddots & \vdots \\ & \mathbf{1}_{N_\eta^k} & \mathbf{0} & \cdots & \mathbf{1}_{N_\eta^k} \end{array}\right) \tag{10}$$

where $\otimes$ denotes the Kronecker product. The matrix R has the same dimensions as Γ_t (i.e., $(N_y + N_\eta) \times (k+5)$) and comprises 4 blocks which correspond to each block in Γ_t. While Γ_t is time-varying and depends on t, the restrictions matrix R is time-invariant. Each column dictates which variables are loaded into which factor. The upper left block dictates the way that each macroeconomic variable loads onto the five macroeconomic factors which are common components of industrial production, inflation, unemployment, interest rates and term-spreads across countries. For instance, the first k entries in column 1 are a vector of ones (i.e., $\mathbf{1}_k$) and all other rows in this block are zero. These restrictions allow industrial production for all countries to load onto the first factor, excluding all other variables. The bottom-right block specifies the way mispricing innovations load onto sentiment factors. Recall that there is one global sentiment factor and k domestic sentiment factors, one for each country. Notice that all mispricing innovations are allowed to load onto the global sentiment factor while only a subset of these variables are allowed to load onto the domestic sentiment factors. The total series allowed to load onto local investor sentiment, correspond to the total number of firms considered for each country $\{N_\eta^1, \ldots, N_\eta^k\}$. Lastly, there are two blocks which correspond to matrices $\Gamma_{t,\eta y}$ and $\Gamma_{t,y\eta}$. The upper-right block is a matrix of zeros since sentiment factors are absent from the measurement of macroeconomic factors (see equation (4)). Whereas, the lower-left block is a matrix of ones because we wish to include macroeconomic factors in the measurement equation of sentiment factors (see equation (5)), we have seen that doing so is key in ensuring investor sentiment indices are uncontaminated by macroeconomic innovations. The importance of R will become apparent in the next section where we discuss estimation.

3.3 Estimation

The hierarchical factor model we use is described by equations (6)–(9). We follow Koop and Korobilis (2014) in setting priors for all parameters in the model, which are explained in Appendix A. Estimation follows a Bayesian Kalman Filter routine similar to the authors. The key novelty of our approach is its adaptation to estimate the factor model with the prescribed hierarchical structure. Estimation can be summarized by the following steps:

i) **Step 1:** Estimate investor sentiment at a firm level by estimating the residuals from regression (3);

ii) **Step 2:** Initialize the factors $\{F_t^y, F_t^\eta\}$ through PCA;

iii) **Step 3:** Estimate the system (6)–(7) with a Kalman filter and smoother based on Koop and Korobilis (2014).

The first step consists in obtaining an estimate of firm-level mispricing innovations η_t. These are residuals of the asset pricing equation discussed in Section 2. Figure B.1 provides an intuition for this procedure. It shows average expected and realized returns for each country. Next, in step 2 the factors are initialized with the PCA according to the hierarchy defined by (10). These estimates are a key input of step 3 which consists in estimating the system (6)–(7). The Kalman filter and smoother algorithm used for estimation at this stage is described in detail in Appendix A. The method functions iteratively by estimating all parameters conditional on the data and the initial estimates of the factors. Once the estimates for the parameters are found, it then updates the factors, conditional on the parameters. A final remark with regards to the way time-variation is modelled is in order. Following Koop and Korobilis (2014), the variance-covariance matrix of the innovations to equations (6)–(9) is modelled as an Exponential Weighted Moving Average process. This implies that time variation occurs smoothly as prescribed by forgetting factors which capture persistence. There are four key parameters $\{\kappa_1, \kappa_2, \kappa_3, \kappa_4\}$ which control time-variation in the model. Therefore, time-variation may be collapsed by setting these parameters to 1 (see Appendix A for more details).

3.4 Diebold-Yilmaz Connectedness for Dynamic Factor Models

In a series of papers, Diebold and Yilmaz (2009, 2012, 2014) develop what is known as the Diebold Yilmaz Connectedness Index (DYCI) methodology to study the link between variables in a vector autoregressive VAR(p) system. The application of such a framework to dynamic factor models is straightforward. Moreover, as shown by Barigozzi and Hallin (2017) dynamic factor models offer the possibility of performing variance decomposition exercises at the level of individual series. However, the emphasis in this paper is in studying the link between the factors in system (7). Let us write equation (7) of the previous subsection in moving average (MA) form. For $F_t = (F_t^y, F_t^\eta)^\top$ we obtain

$$F_t = \sum_{j=0}^{\infty} A_{j,t} u_{t-j}, \tag{11}$$

where at given time t, $A_{j,t} = \Phi_{t-j+1} A_{j-1,t}$, $A_0 = I_N$ for N factor innovations. Equation (11) gives the impulse responses of each factor to factor innovation shocks. The forecast error variance decomposition, which is the basis of the DYCI, can then be written as

$$\theta_{ij,t}(H) = \frac{\sigma_{jj,t}^{-1} \sum_{h=0}^{H-1} (e_i^\top A_{h,t} \Sigma_t e_j)^2}{\sum_{h=0}^{H-1} e_i^\top (A_{h,t}) \Sigma_t (A_{h,t})^\top e_i}, \tag{12}$$

where Σ_t is the time-varying variance-covariance matrix of factor innovations u_t, $\sigma_{jj,t}$ is the standard deviation of the j^{th} factor equation at time t and e_i is a selection vector with one in the i^{th} element and zeros otherwise. Thus, $\theta_{ij,t}(H)$ is interpreted as the contribution of a shock to the j^{th} factor equation to factor i's H-step ahead forecast error variance, at each point in time t.

Equation (12) results in a connectedness table that varies over time. It is symmetric and delivers three important statistics. First, the pairwise directional connectedness defined as

$$C^H_{i\leftarrow j,t} = \frac{\theta_{ij,t}(H)}{\sum_{j=1}^{N} \theta_{ij,t}(H)}, \tag{13}$$

which measure the importance of a shock to factor j in explaining the dynamics of factor i at time t. Second, the total directional connectedness to and from, respectively, given by

$$C^H_{\bullet\leftarrow j,t} = \sum_{\substack{j=1\\ i\neq j}}^{N} C^H_{i\leftarrow j,t}, \quad C^H_{i\leftarrow \bullet} = \sum_{\substack{i=1\\ i\neq j}}^{N} C^H_{i\leftarrow j,t}. \tag{14}$$

The total directional connectedness (from) $C^H_{i\leftarrow\bullet}$, can be interpreted as the portion of variation in i that is explained by all other shocks in the system at time t. Whereas, the total directional connectedness (to) $C^H_{\bullet\leftarrow j}$, measures the significance of factor j in driving all other variables in the system. Finally, the total directional connectedness

$$C^H_t = \sum_{\substack{i,j=1\\ i\neq j}}^{N} C^H_{i\leftarrow j,t}, \tag{15}$$

which simply measures the level of dependence between variables in the system. Overall, the generalization of the DYCI methodology to an environment where parameters change with time is straightforward.

4 Data

4.1 Data Sources and Transformations

The data used throughout the paper is divided in two groups – macroeconomic data and firm-level data. The period covered spans 1991M7 to 2018M12, the frequency is monthly. Although it would be interesting to estimate the model with data prior to 1991M7, firm-level data is not available for some countries such as Germany. We also chose to use data until 2018M12 to minimize the impact of revisions to more recent macroeconomic data which can be significant.

Firm-level variables are available through Thomson Reuters, Eikon/Datastream. They include the following variables (Eikon-mnemonic in parentheses): the Dividend-Yield (DY), expected growth of dividends per share with 1 year horizon (WC08611) and the Price Index (PI) with which yearly log-returns for all firms were constructed. For the robustness exercise we additionally collect firm-level Price-to-Earning (PE), 12M forward Price-to-Earnings (PEFD12), Dividend-Earning Ratio - the Dividend-Payout per share (WC09504) and Price to Book Value (PTBV). These variables were sourced for all firms that constitute the reference stock market for each country (i.e., US 500 firms (S&P 500), Canada 60 firms (S&P/TSX), Japan 125 firms (NIKKEI), UK 102 firms (FTSE), Germany 30 firms (DAX), France 40 firms (CAC)). Because it is necessary to take the logs of Dividend-Yields, we consider log(100+DY) to avoid extreme values generated by many observations close to zero.

Macroeconomic series are sourced from FRED, Federal Reserve Bank of St. Louis, the Monthly Monetary and Financial Statistics (MEI) section of OECD Statistics, the Statistical Data Warehouse (SDW) maintained by the European Central Bank and Eikon (codes in parenthesis). They include industrial production for the US (INDPRO), Canada (CANPROINDMISMEI), Japan (JPNPROINDMISMEI), UK (GBRPROINDMISMEI), Germany (DEUPROINDMISMEI), France (FRAPROINDMISMEI). CPI inflation is calculated for the US (USACPIALLMINMEI), Canada

(CANCPIALLMINMEI), Japan (JPNCPIALLMINMEI), UK (GBRCPIALLMINMEI), Germany (DEUCPIALLMINMEI), France (FRACPIALLMINMEI) based on those time-series. Unemployment rates are sourced from Eikon except for France whose data may be found through the SDW, ECB. Data for industrial production and CPI are transformed so as to consider their growth rates. Interest rate statistics are fetched from OECD Statistics. Long-term interest rates refer to secondary market yields of 10 year government bonds for each country. Whereas, short-term interest rates relate to money market instruments for each country. We calculate the term-spread for each country as the difference between long-term and short-term interest rates. In addition, we consider the first difference of short-term interest rates to ensure stationarity. All variables are standardized prior to their inclusion in the dynamic factor model.

4.2 Treatment of Missing Values

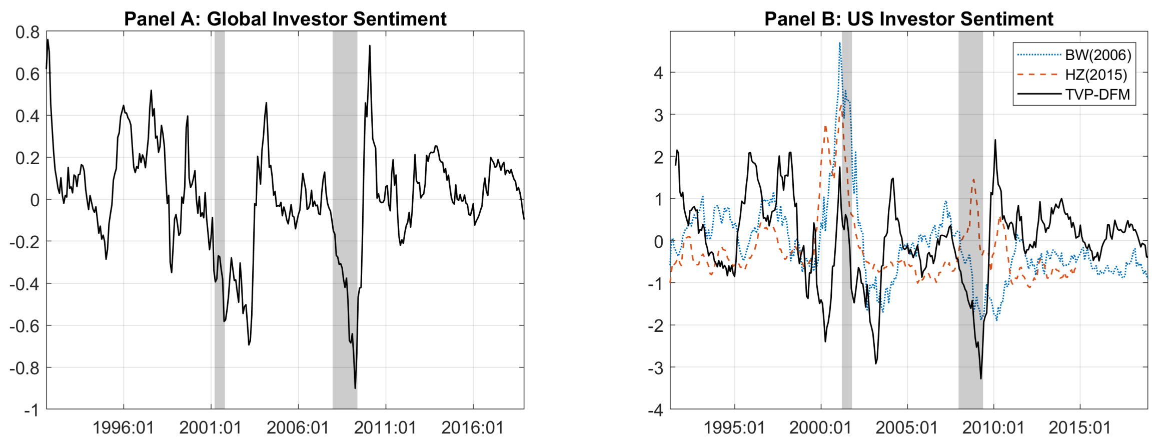
Missing values occur in the first step estimation procedure outlined in Section 3.3. We collect firm-level data covering the period between 1991M7-2018M12. The data for many firms is incomplete and therefore, running the first stage regression involves imputation of missing values for firm-level data. The specific algorithm used consists in fitting an ARMA model that minimizes the Akaike information criterion over the remaining sample. Each NaN value is replaced by the fitted values from the ARMA model. We drop any firm for which at least one variable is missing for more than 25% of our sample. After doing so we are left with a balanced panel of 586 firms.

5 Discussion of the Results

We begin by discussing the investor sentiment indices that our model delivers and how they compare with some of the most popular measures in the literature. Panel A in Figure 1 presents the global investor sentiment index, which summarizes commonalities in mispricing in all countries. Panel B compares our investor sentiment index with two popular alternatives which have only been developed for the US. Although the alternative measures correlate with ours, they also disagree in some periods, noticeably at the beginning of the sample.

These alternative measures consist in a first principal component of the cross section of several proxies which include the close-end fund discount rate, NYSE turnover, number of initial public offerings (IPOs), average first-day returns of IPOs, dividend premium, and the equity share in new issues. Our approach imposes more structure by extracting investor sentiment factors jointly at a global and local level while purging the effect of macroeconomic innovations. Throughout the sample we consider, from 1991M7–2018M12, it is possible to observe that investor sentiment tends to correlate with business cycles. It plummets before and during economic recessions and rises in times of recovery.

Figure 1: Investor Sentiment Indices

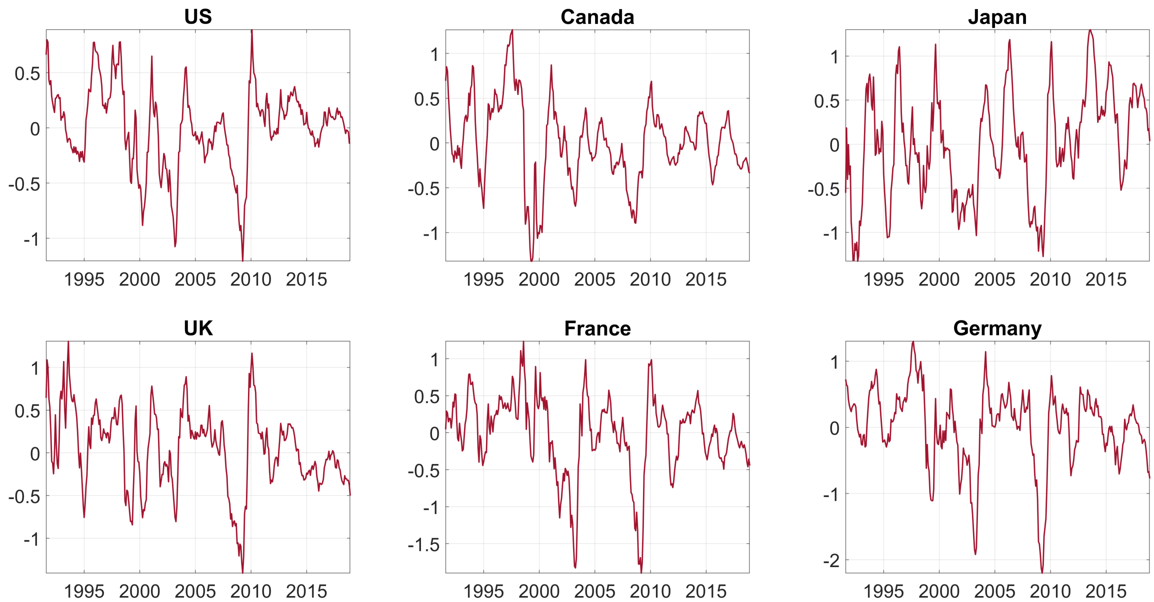


Notes: Global investor sentiment index (Panel A) and US investor sentiment index (Panel B) calculated with a time-varying parameter hierarchical factor model versus those found for the US economy by Baker and Wurgler (2006) and Huang et al. (2015), standardized for comparison purposes.

Although each country's own investor sentiment index exhibits its own idiosyncrasy, they share in common a pattern of overvaluation followed by a period of undervaluation around recessions. Figure 2 shows the domestic investor sentiment indices extracted within our hierarchical dynamic factor model with six countries: US, UK, Canada, Japan, France and Germany.

An outstanding question relates to the appropriateness of our factor model to describe mispricing innovations for all countries in our dataset. We examine the properties of the factors estimated using firm-level data which include all firms across the countries considered. After the factors are estimated we regress the i^{th} series in the dataset on the set of 7 factors which feature in our dataset following McCracken and Ng (2016) and Stock and Watson (2016). For $k = 1, \ldots, 7$ this yields $R_i^2(k)$ for series i. The incremental explanatory power of factor k is given by $mR_i^2(k) = R_i^2(k) - R_i^2(k-1)$, $k = 2, \ldots, 7$ with $mR_i^2(1) = R_i^2(1)$. The average importance of factor k is given by $R^2(k) = \frac{1}{N_k} \sum_{i=1}^{N_k} R_i^2(k)$. Table B.1 summarizes the above-mentioned statistics.

Figure 2: Domestic Sentiment Indices



Notes: The six domestic investor sentiment factors as extracted with the hierarchical dynamic factor model with time varying parameters.

Overall, the seven global and local investor sentiment factors used explain 0.45 of the total variation in firm-level investor sentiment. We see that the global investor sentiment factor explains 0.22 of the variation in the data and is dominated by firms in the US, although the UK, France and Canada also feature in the group of 10 series that load most heavily on this factor. It explains between 0.57 and 0.68 of the dynamics of firm-level investor sentiment of the top 10 firms that load onto this factor. Next, the marginal contribution of investor sentiment factors for each country seem to help explaining the data further and show marginal R-squares between 0.06 and 0.24. They capture features of the data which are not yet described by the global factor and are therefore specific to each country. All domestic investment sentiment factors accurately describe the underlying series, explaining as much as 0.53 of the firm-level investor sentiment in Japan.

The macroeconomic sector of the model consists of five macroeconomic factors presented in Figure B.2. There are three factors that capture real activity - industrial production, inflation and unemployment. Two additional factors capture changes in monetary conditions - an interest rate factor that describes the behaviour of short-term interest rates; and a term-spread factor that measures the slope of the yield curve. These factor are expected to capture in part monetary policy shocks. The coloured dashed lines show the five corresponding time-series in each of the six countries included in the model. The extracted latent macroeconomic factors capture the global behavior of macroeconomic variables considered.

5.1 Cross-Country Linkages

Table 1 summarizes system-wide interconnection. These statistics are obtained from our hierarchical factor model with time-invariant parameters. It shows the set of pairwise directional connectedness which give the portion of variation of macroeconomic factors which is due to global and domestic investor sentiment shocks and vice versa.

Table 1: Macro - Investor Sentiment Static Connectedness Table.

		Macro. Factors					Inv. Sent. Factors							
		Ind Prod	Inflation	Unempl.	Interest rate	Term-Spread	Global Sent	US Sent	CAD Sent	JP Sent	UK Sent	DE Sent	FR Sent	**From others**
Macro. Factors	Ind. Prod.	50.28	11.54	11.69	6.32	3.01	6.20	1.93	1.06	0.22	1.70	1.05	0.25	**44.98**
	Inflation	10.77	66.55	1.16	1.39	1.66	1.42	0.39	1.14	0.86	1.58	0.39	1.23	**22.00**
	Unempl.	33.53	11.29	26.64	9.37	1.62	4.14	0.87	0.90	2.35	1.52	1.03	0.25	**66.86**
	Interest rate	5.50	1.97	1.31	81.70	3.03	0.37	1.21	0.68	0.99	0.55	0.69	0.32	**16.59**
	Term-Spread	7.23	3.73	2.47	45.42	27.24	1.13	0.77	0.50	1.04	0.91	0.57	0.67	**64.45**
Inv. Sent. Factors	Global Sent	8.61	5.57	1.44	3.21	1.75	63.93	0.83	4.57	0.66	1.25	0.68	0.77	**29.33**
	US Sent	8.53	7.32	1.72	2.18	2.19	51.20	9.24	4.31	0.84	1.86	0.42	0.90	**81.47**
	CAD Sent	5.97	6.32	1.63	5.48	1.89	30.92	1.68	35.77	0.61	1.50	0.33	1.00	**57.32**
	JP Sent	4.80	5.67	1.88	4.05	4.03	28.96	9.75	1.36	28.28	2.25	1.65	0.37	**64.78**
	UK Sent	6.00	2.43	2.80	3.03	2.65	43.83	2.20	6.18	2.22	18.99	0.99	1.34	**73.67**
	DE Sent	6.06	3.39	2.01	6.45	1.71	50.88	2.39	9.76	2.52	1.66	5.74	0.20	**87.04**
	FR Sent	6.43	5.95	3.96	5.37	1.55	48.72	3.10	4.40	1.71	2.30	1.63	5.64	**85.11**
	To others	**103.43**	**65.18**	**32.06**	**92.27**	**25.09**	**267.77**	**25.12**	**34.87**	**14.02**	**17.08**	**9.41**	**7.30**	**693.60**
	NET	**58.46**	**43.19**	**-34.80**	**75.68**	**-39.35**	**238.43**	**-56.35**	**-22.45**	**-50.76**	**-56.59**	**-77.64**	**-77.81**	

Notes: The sample is 1991M1 through 2018M12 and the predictive horizon is 24 months. Pairwise-connectedness figures displayed result from the estimation of a constant-parameter hierarchical factor model with the full sample of data. Each row/column in the table represents a factor in the model. Factors are divided in macroeconomic factors and sentiment factors according to the hierarchy explained in the methodology section. The last column (FROM) gives total directional connectedness (from); i.e., row sums (from all others to i). The penultimate row (TO) gives total directional connectedness (to); i.e., column sums (to all others from j). Whereas the last row (NET) gives the difference in total directional connectedness (to–from). The bottom-right element (in bold) is total connectedness.

For instance, shocks to global and local sentiment are together responsible for roughly 12% of the dynamics of industrial production, 7% of inflation, 11% of unemployment and about 5% of interest rates and term-spreads. We also observe that the global component of investor sentiment is the indicator with most macroeconomic significance - accountable for about 13% of the movement of all five macroeconomic factors in our study. Furthermore, global investor sentiment is also responsible for a significant portion of the variation in domestic sentiment. For instance, as much as 51% of the dynamics of US investor sentiment is driven by global sentiment shocks. Whereas, it only explains 28% of the movements in Japanese investor sentiment. Overall, these results highlight a significant level of commonalities in investor sentiment across borders. They also suggest that, investor sentiment is partly determined internationally. However, we observe that for all countries, a very significant portion of the movements in country-specific investor sentiment are idiosyncratic. This is the case of Japan but also of Canada where only 30% of the movement in domestic investor sentiment is due to global sentiment shocks. Reversely, shocks to macroeconomic factors explain a large portion of the variation in global and domestic investor sentiment factors. It can be observed from Table 1, column 3 to 7, that all macroeconomic factors explain a significant portion of the variation in both global and local sentiment factors. This result highlights a bilateral cause and effect relationship between investor sentiment and the macroeconomy.

The final column of Table 1 'from' gives the sum of the pairwise directional connectedness for a given factor with respect to all other shocks. It measures the portion of the forecast error variance of a shock to each variable due to shocks to all the other variables in the system. Whereas, the penultimate row 'to' measures the significance of each variable in determining all others. The final row 'net' is simply the difference between total directional connectedness 'from' and 'to'. Positive net total directional connectedness suggests a given factor is a net 'transmitter' and therefore an influential source of variation in the system. We observe that the net directional connectedness of all domestic investor sentiment factors is negative, whereas the global sentiment factor has positive net directional connectedness. This suggests global sentiment is a key driving force in the system, shaping both domestic investor sentiment and global macroeconomic conditions.

Is investor sentiment a global phenomenon or should it be regarded as a domestically determined variable? Our results suggest that domestic investor sentiment has a significant idiosyncratic component. However, it is influenced to a large extend by foreign investor sentiment shocks. Notwithstanding, the results show a lot of heterogeneity in the extent to which domestic sentiment is determined abroad. The country less exposed to foreign investor sentiment shocks is Japan, where approximately 29% of the variation of investor sentiment is determined by global investor sentiment. While for the US, Germany, UK and France this figure amounts to nearly 50%, which reveals a strong foreign exposure.

Taken together, the evidence seems to suggest that (i) a material portion of the variation of domestic investor sentiment is due to global investor sentiment shocks but the idiosyncratic component of its dynamics is material; (ii) The cause and effect relationship between investor sentiment and macroeconomics is complex. Overall, this relationship seems to be two sided with causality running in both directions.

5.2 Dynamic Cross-Country Connectedness

The main focus of our exercise is to understand the nature of the linkage between real economic activity and investor sentiment. We have discussed the evidence of a bilateral link between the macroeconomy and investor sentiment and found global investor sentiment to be the predominant shock driving the macroeconomy. This result stems from the estimation of our model assuming fixed parameters on a full sample from 1991M7 through 2018M12. However, there are good reasons to suspect that the parameters that describe this relationship have been subject to changes. Clearly, the rise of linkages amongst global financial markets, capital mobility and size of the financial sector in the economy that took place in the last 30 years ought to contribute to a continuous evolution of how important investor sentiment is in steering the macroeconomy. To examine this point deeper, we study dynamic links between global investor sentiment and the macroeconomy by re-estimating the model with time-varying parameters. We present directional connectedness statistics in Figure

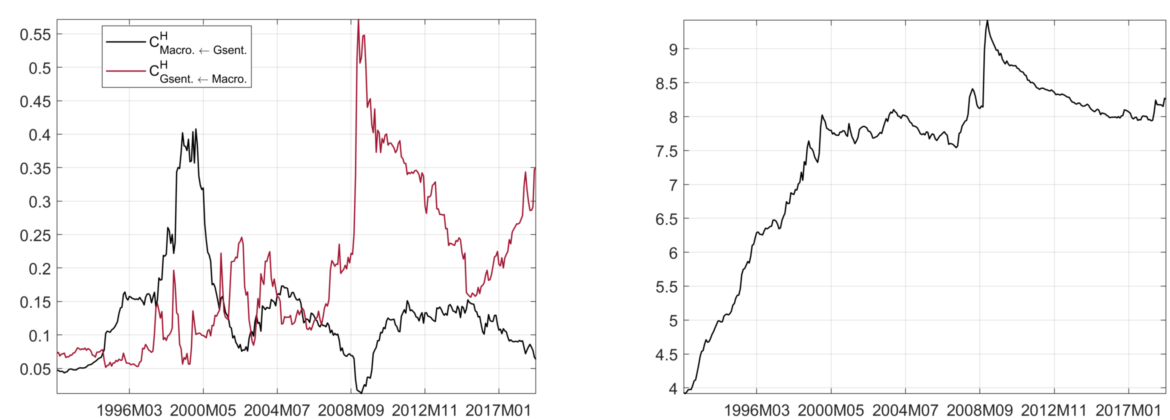
3 below. It shows the average portion of variation from global investor sentiment to real activity - the first three macroeconomic factors - and vice versa.

Our results confirm the complex nature of the relationship between real economic developments and investor sentiment. While it is difficult to establish an overall lead-lag relation, it is possible to observe that from 1993 to 2003, global investor sentiment explained on average more than 15% of the movements in real activity. Whereas, shocks to real activity explain a smaller fraction of the movement in investor sentiment. This pattern seems to revert in 2007-08. During the Great Recession, the macroeconomy seems to be the key driving force, explaining more than 30% of the dynamics of global investor sentiment.

Another interesting result from our analysis is that, during the past 30 years, total connectedness in the system has increased drastically (cf. right panel Figure 3). Moreover, total connectedness reaches local maxima during episodes of financial distress, namely the 2007/08 financial crisis.

Overall the findings discussed highlight a bilateral link between investor sentiment and macroeconomics. However, the direction of the cause and effect relationship between real activity and investor sentiment is state dependent and seems to vary over time.

Figure 3: Connectedness Macro-Investor Sentiment



Notes: Left panel: Directional connectedness (from) measures for global sentiment (Gsent.) and the real activity factors in the macroeconomic sector of the model which include three factors (industrial production, inflation and unemployment), obtained with a time-varying parameter hierarchical factor model. Right panel: Total directional connectedness measures obtained with a time-varying parameter hierarchical factor model.

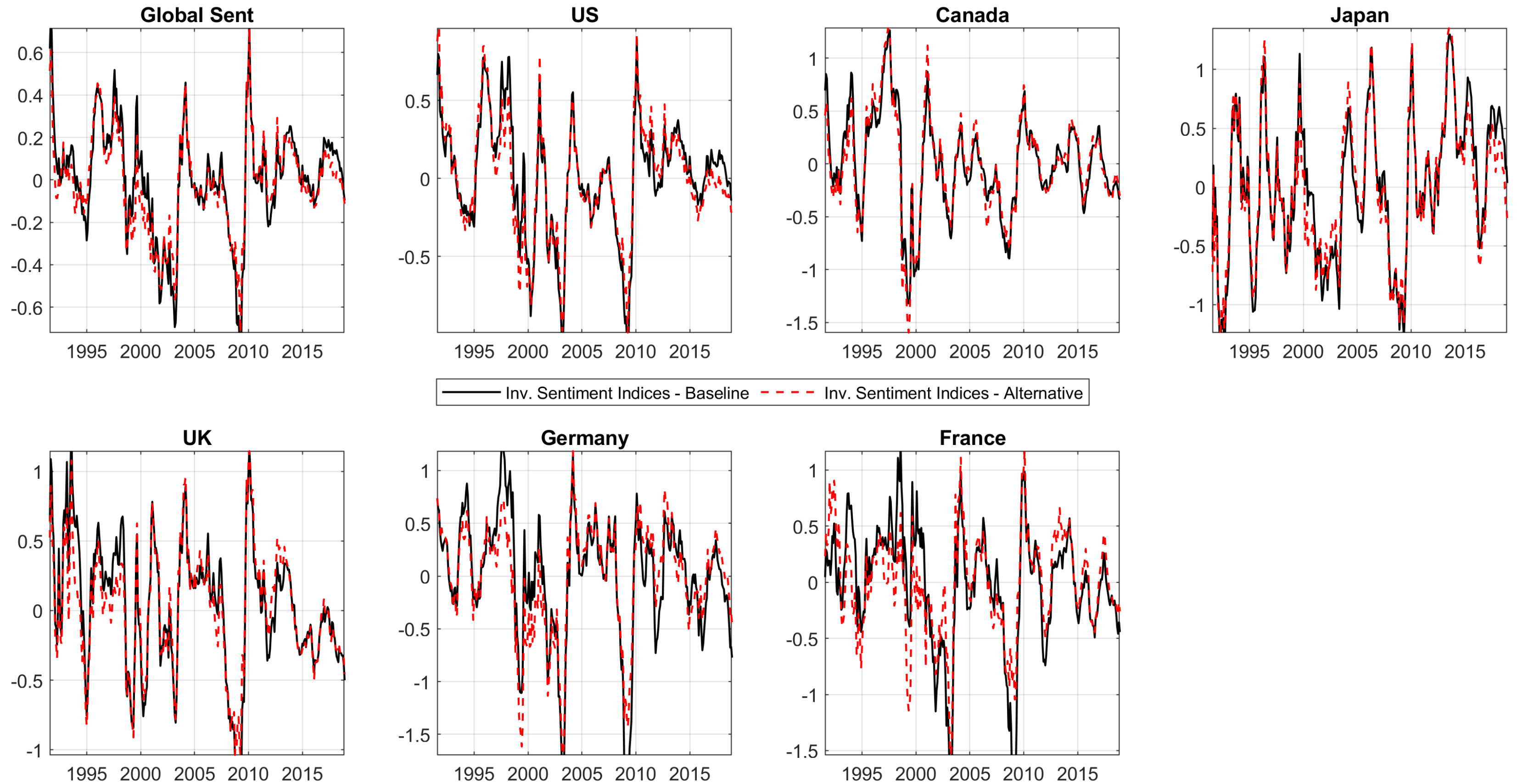
6 Robustness

Our investor sentiment indices are common components of the innovations to the Campbell-Shiller log-linearized description of mean returns (equation (3)). A plausible concern is that this baseline specification may miss potentially relevant predictors that capture firm fundamentals and are thus orthogonal to investor sentiment. In this section, we augment our baseline specification (equation (3)) so as to include other potentially relevant predictors of mean returns based on Welch and Goyal (2008).

The additional predictors considered are selected taking into consideration data availability and relevance of the predictors discussed in Welch and Goyal (2008). Overall, the baseline Campbell-Shiller regression is augmented by considering five additional regressors. These consist of two measures of firm earnings - a price-to-earnings ratio which is based on current earnings reported by each firm; plus a forward price-to-earnings ratio which is based on analysts' consensus surrounding expected earnings, one year ahead. The inclusion of realized and expected earnings may help predict mean returns, in particular for growth stocks. Furthermore, by considering expected earnings we mitigate the possibility that our investor sentiment indices are contaminated by news regarding future earnings which are also part of a firm's fundamental value. We also consider the price-to-book value, which reflects balance-sheet changes, dividend-earnings (payout) ratio and a measure of stock variance, calculated according to Welch and Goyal (2008).

Figure 4 presents our set of investor sentiment indices calculated by augmenting the Campbell-Shiller regression with the set of predictors explained above. It can be seen that, although some differences are apparent, the main signal in all indices does not suffer material distortions. However, some remarkable differences can be noticed for France and Germany. An important and reassuring aspect to note is that, the shrinkage performed by our factor model neglects any noise or idiosyncratic variation in innovations to the mean return process that is too feeble to represent a common thread across a sufficiently large number of firms.

Figure 4: Investor Sentiment Indices based on Alternative Specification



Notes: (black line) Global and country-specific Investor Sentiment Indices estimated according to our baseline specification (i.e. equation 3) . (red line) Investor Sentiment Indices re-estimated by augmenting the baseline specification with a set of predictor that include a wider spectrum of measures of firm fundamentals.

7 Conclusion

This paper studies the link between investor sentiment and the macroeconomy across countries. Investor sentiment is measured as the common factor of movements in equity prices across countries which are not accounted by fundamentals based on a dividend discount type asset pricing model, following Campbell and Shiller (1988). We show that our index shares similar dynamics of other measures of investor sentiment available in the literature and is robust to the inclusion of several other measures of firm fundamentals. In our setting, investor sentiment is measured within the framework of a macro-finance hierarchical factor model which allows us to parse out confounding factors such as macroeconomic conditions, a common concern in the literature. We construct usable indices of global and country-specific sentiment for six countries. Because our index relies on data which is publicly available, it can be easily extended for a large cross-section of countries. Most of the literature focuses on the US economy partly because it is difficult to find proxies of investor sentiment for many countries. Our paper may help guide future work on the global relevance of investor sentiment.

We allow for smooth structural changes between the variables in the model by accounting for time-varying parameters and stochastic volatility - features of the data which have long been a concern when modelling financial time series. The vector-autoregressive nature of the model offers the possibility of leveraging on standard tools for structural analysis. We build on the DYCI methodology of Diebold and Yilmaz (2009) and study the connectedness of investor sentiment and macroeconomic factors. The time-varying parameter feature of our model allows us to built dynamic connectedness indices without having to consecutively estimate the model on a rolling window.

We find that global investor sentiment is key in driving domestic investor sentiment and steering global macroeconomic conditions. However, evidence suggests the idiosyncratic component of the dynamics of domestic investor sentiment indices is still very important. We find evidence of a two-way cause and effect relationship between investor sentiment and macroeconomic conditions. Through the lens of our time-varying parameter model estimation we uncover the state dependent nature of this relationship (i.e., the sign of the net directional connectedness between investor sentiment and macroeconomics varies over time). Our contribution from an econometric viewpoint consists in extending prior studies by allowing the estimation of hierarchical factor models with time-varying parameters.

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A Econometric Methods

A.1 Bayesian Kalman Filter with Forgetting Factors

The model defined in (6) - (9) configures a Time-Varying Parameter Dynamic Factor Model and can be written compactly, in state space form as follows

$$\begin{aligned} X_t &= F_t\Gamma_t + v_t, \quad v_t \sim N(0, V_t), \\ F_t &= F_{t-1}\Phi_t + u_t, \quad u_t \sim N(0, \Sigma_t), \\ \gamma_t &= \gamma_{t-1} + \varepsilon_t \quad \varepsilon_t \sim N(0, W_t), \\ \phi_t &= \phi_{t-1} + h_t \quad h_t \sim N(0, Q_t), \end{aligned}$$

where $X_t = [y_t, \eta_t]'$ include all macroeconomic data and mispricing factors and $F_t = [\tilde{F_t^y}, \tilde{F_t^\eta}]'$ denotes the estimated factors.[3] Furthermore, the loadings Γ_t are subject to the restrictions

$$\gamma_{i,t} = 0 \quad \text{if} \quad r_i = 0 \tag{A.1}$$

[3]The estimated factors are key inputs to the algorithm and result from a priori estimation of the measurement equation with a PCA.

where $r = vec(R)$ is the vectorized version of the matrix R defined in (10), which defines the hierarchy of the factors. In addition, let $\theta_t = \{\Gamma_t, \Phi_t\}$ denote the parameter set and $D_t = \{X_t, F_t\}$ the data for $t = \{1, \ldots, T\}$. Assuming that we know the posterior of θ at time $t-1$, Bayesian filtering/smoothing is based on the equations below

$$p(\theta_t, \theta_{t-1}|D_{t-1}) = p(\theta_t|\theta_{t-1}, D_{t-1})p(\theta_{t-1}|D_{t-1}), \tag{A.2}$$

$$p(\theta_t|D_{t-1}) = \int_P p(\theta_t|\theta_{t-1}, D_{t-1})p(\theta_{t-1}|D_{t-1})d\theta_{t-1}, \tag{A.3}$$

where P is the support of θ_{t-1}. The prediction step is given by the Chapman-Kolmogorov equation (A.2). At each iteration t, the prior $p(\theta_t|D_{t-1})$ is updated according to equation (A.3) and the likelihood $p(D_t|\theta_t)$ is augmented by an additional observation of D_t. Hence the posterior distribution is updated according to Bayes' rule

$$p(\theta_t|D_t) = \frac{1}{H_t} p(D_t|\theta_t, D_{t-1})p(\theta_t|D_{t-1}). \tag{A.4}$$

Here $H_t = \int p(D_t|\theta_t)p(\theta_t|D_{t-1}))$ is the normalizing constant. Equation (A.4) is referred to as the updating step. The algorithm outlined below extends the one derived in Koop and Korobilis (2014) to hierarchical factor models, where Γ_t reflects the desired hierarchy which is configured through R, a matrix of zeros and ones, which constrains the state space. It consists in 2 steps, iterating through prediction (A.3) and updating (A.4) after the system is initialized. These two main steps are repeated for $t = \{1, \ldots, T\}$.

A.2 Implementation of the Kalman Filter Approach

A.2.1 Priors and Initialization

The choice of the priors follows the diffuse choices of Koop and Korobilis (2014):

$$F_t \sim N(0, \Omega^F_{t|t}), \quad \gamma_t \sim N(0, \Omega^\gamma_{t|t}), \quad \phi_t \sim N(0, \Omega^\phi_{t|t})$$

The variances of these priors can be seen as hyperparameters and are initialized to $\Omega^F_{0|0} = 4 * I_r$ and $\Omega^{\gamma}_{0|0} = I_N$. The parameters ϕ_t are given Minessota priors that penalizes more distant lags. Therefore, $\Omega^{\phi}_{t|t}$, is a diagonal covariance matrix and the coefficient for lag p is initialized to $0.1/p^2$ for $p = 1, ..., 4$. In addition, the variances of the innovations are initialized to: $V_0 = 0.1 \times I_N$, $\Sigma_0 = 0.1 \times I_r$, $W_0 = 10^{-5} \times I_{N \times r}$ and $Q_0 = 10^{-5} \times I_m$, where m is the number of VAR parameters and r is the total number of factors. These state variables are allowed to smoothly change over time, following an Exponentially Weighted Moving Average (EWMA).

A.2.2 Prediction

The Kalman filter prescribes that

$$\gamma_t | D_{1:t-1} \sim N(\gamma_{t|t-1}, \Omega^{\gamma}_{t|t-1}), \quad \phi_t | D_{1:t-1} \sim N(\phi_{t|t-1}, \Omega^{\phi}_{t|t-1}),$$

where $\gamma_{t|t-1} = \gamma_{t-1|t-1}$, $\phi_{t|t-1} = \phi_{t-1|t-1}$ and

$$\Omega^{\phi}_{t|t-1} = \Omega^{\phi}_{t-1|t-1} + \hat{Q}_{t|t-1}, \quad \Omega^{\gamma}_{t|t-1} = \Omega^{\gamma}_{t-1|t-1} + \hat{W}_{t|t-1}.$$

The state covariances in the equations above are estimated by

$$\hat{Q}_{t|t-1} = \frac{1}{\kappa_3} \hat{Q}_{t-1|t-1}, \quad \hat{W}_{t|t-1} = \frac{1}{\kappa_4} \hat{W}_{t-1|t-1}.$$

where κ_3 and κ_4 are forgetting factors that define the law of motion of the parameters and are set to 0.99 following the authors.[4] The prediction step allows us to compute measurement and state equation prediction errors, that are necessary inputs for the updating step, as

$$\hat{\upsilon}_{it} = X_{it} - F_t \gamma_{i,t|t-1}, \quad \hat{u}_t = F_t - F_{t-1} \Phi_{t|t-1}.$$

[4] In practice these two equations are approximations of $\hat{Q}_{t|t-1} = \hat{Q}_{t-1|t-1} + \hat{\eta}_{t-1}\hat{\eta}'_{t-1}$ and $\hat{W}_{t|t-1} = \hat{W}_{t-1|t-1} + \hat{\upsilon}_{t-1}\hat{\upsilon}'_{t-1}$ in the standard Kalman filter (see Koop and Korobilis (2013) and Raftery et al. (2010)).

A.2.3 Updating

- Update each γ_{it} for $i = 1, \ldots, n$

$$\gamma_{it}|D_{1:t} \sim N(\gamma_{it|t}, \Omega^{\gamma}_{ii,t|t}),$$

where

$$\gamma_{it|t} = \gamma_{it|t-1} + \Omega^{\gamma}_{ii,t|t-1} F'_t (\hat{V}_{i,t} + F_t \Omega^{\gamma}_{ii,t|t-1} F'_t)^{-1} \hat{v}_t,$$
$$\Omega^{\gamma}_{ii,t|t} = \Omega^{\gamma}_{ii,t|t-1} - \Omega^{\gamma}_{ii,t|t-1} F'_t (\hat{V}_{i,t} + F_t \Omega^{\gamma}_{ii,t|t-1} F'_t)^{-1} F_t \Omega^{\gamma}_{ii,t|t-1},$$

where the term $\Sigma^{\gamma}_{ii,t|t-1} F'_t (\hat{V}_{it} + z_t \Sigma^{\gamma}_{ii,t|t-1} F'_t)^{-1}$ is the Kalman gain for each time period t and is set to zero for the loadings which are constrained, according to (A.1).

- Update each ϕ_t as

$$\phi_t|D_{1:t} \sim N(\phi_{t|t}, \Omega^{\phi}_{t|t}),$$

where

$$\begin{aligned} \phi_{t|t} &= \phi_{t|t-1} + \Omega^{\phi}_{t|t-1} F'_{t-1} (\hat{\Sigma}_t + F_{t-1} \Omega^{\phi}_{t|t-1} F'_{t-1})^{-1} \hat{u}_t \\ \Omega^{\phi}_{t|t} &= \Sigma^{\phi}_{t|t-1} - \Omega^{\phi}_{t|t-1} F'_{t-1} (\hat{\Sigma}_t + F_{t-1} \Omega^{\phi}_{t|t-1} F'_{t-1})^{-1} F_{t-1} \Omega^{\phi}_{t|t-1}, \end{aligned}$$

where the term $\Omega^{\phi}_{t|t-1} F'_{t-1} (\hat{\Sigma}_t + F_{t-1} \Omega^{\phi}_{t|t-1} F'_{t-1})^{-1}$ is the Kalman gain for each period t.

- Update V_t and Σ_t given information at time t using EWMA specifications as follows

$$\hat{V}_{i,t|t} = \kappa_1 \hat{V}_{i,t-1} + (1 - \kappa_1) \hat{v}_{t|t} \hat{v}'_{t|t},$$
$$\hat{\Sigma}_{t|t} = \kappa_2 \hat{Q}_{t-1} + (1 - \kappa_2) \hat{u}_{t|t} \hat{u}'_{t|t}.$$

where κ_1 and κ_2 are forgetting factors that define the law of motion of the idiosyncratic volatilities in the measurement equation and the volatilities of the observable variables and the factors in the state equation. They are both set to 0.96 following the authors original proposal.

A.3 Smoother for the parameters

Obtain smoothed estimates of Γ_t, Φ_t, V_t and Σ_t given the information at time t + 1 for $t = T-1, \ldots, 1$.

- Update each $\gamma_{i,t}$ using the fixed interval smoother

$$\gamma_{i,t}|D_{1:T} \sim N(\gamma_{it|T}, \Omega^{\gamma}_{ii,t|T}),$$

  where

$$\begin{aligned}\gamma_{i,t|T} &= \gamma_{i,t|t} + C^{\gamma}_t(\gamma_{i,t+1|T} - \gamma_{i,t+1|t}),\\ \Omega^{\gamma}_{ii,t|T} &= \Omega^{\gamma}_{ii,t|t} + C^{\gamma}_t(\Omega^{\gamma}_{ii,t|T} - \Omega^{\gamma}_{ii,t|t})C'^{\gamma}_t\end{aligned}$$

  and $C^{\gamma}_t = \Omega^{\gamma}_{ii,t|t}(\Omega^{\gamma}_{ii,t+1|t})^{-1}$.

- Update ϕ_t using the fixed interval smoother

$$\phi_t|D_{1:T} \sim N(\phi_{t|T}, \Omega^{\phi}_{t|T}).$$

  where

$$\begin{aligned}\phi_{t|T} &= \phi_{t|t} + C^{\phi}_t(\phi_{t+1|T} - \phi_{t+1|t}),\\ \Omega^{\phi}_{t|T} &= \Omega^{\phi}_{t|t} + C^{\phi}_t(\Omega^{\phi}_{t|T} - \Omega^{\phi}_{t|t})C'^{\phi}_t\end{aligned}$$

  and $C^{\phi}_t = \Omega^{\phi}_{t|t}(\Omega^{\phi}_{t+1|t})^{-1}$.

- Update V_t and Σ_t given information at time t + 1 using the following equations

$$V^{-1}_{t|t+1} = \kappa_1 V^{-1}_{t|t} + (1-\kappa_1)V^{-1}_{t+1|t+1},$$

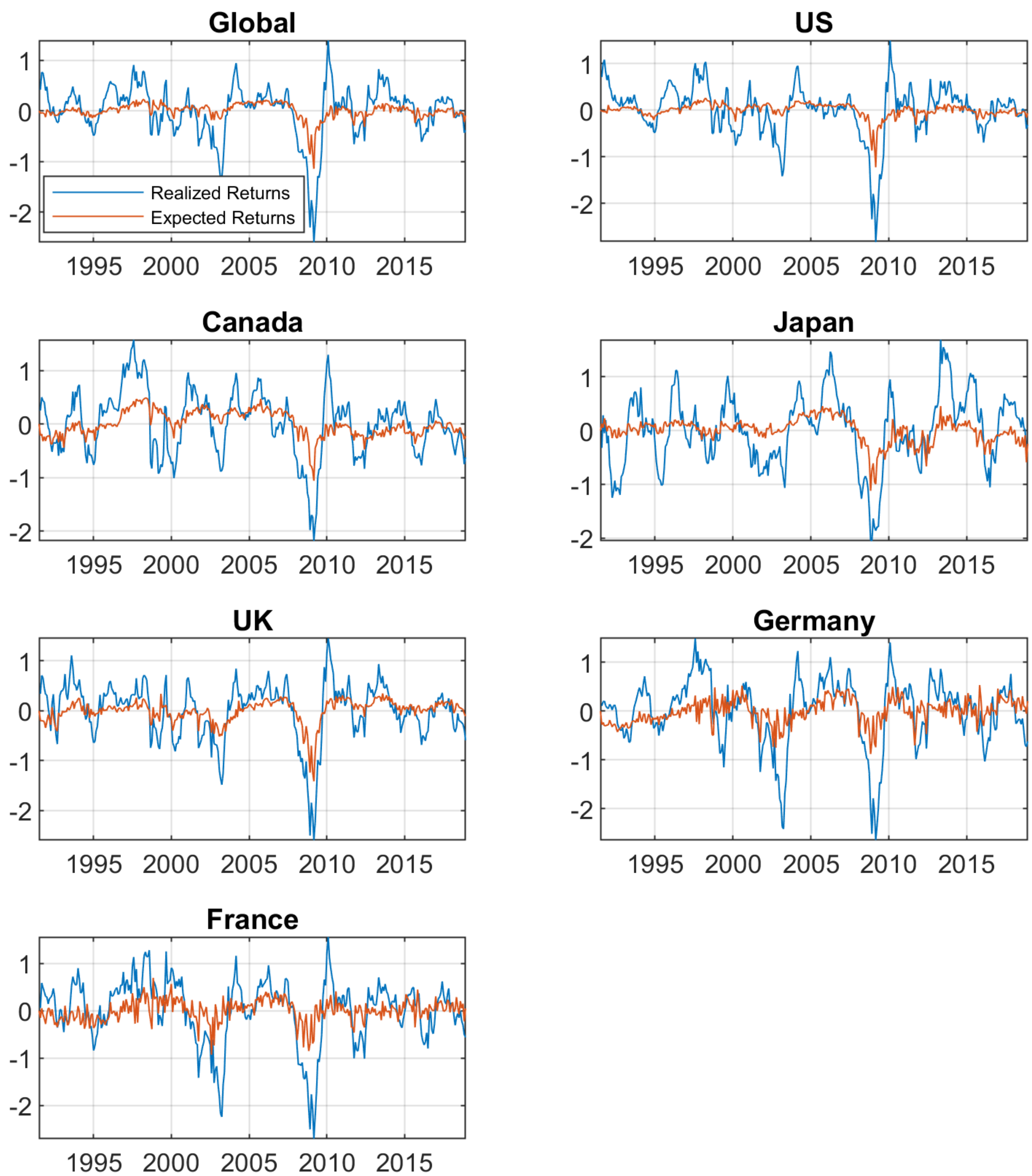
$$\Omega^{-1}_{t|t+1} = \kappa_2 \Omega^{-1}_{t|t} + (1-\kappa_2)\Omega^{-1}_{t+1|t+1}.$$

A.4 Kalman Filter/Smoother for factors

With estimates for all parameters in hand, the Kalman filter and smoother algorithm can be applied to the factors F_t, which had been initialized with a PCA estimate. The algorithm is analogous to the one described above.

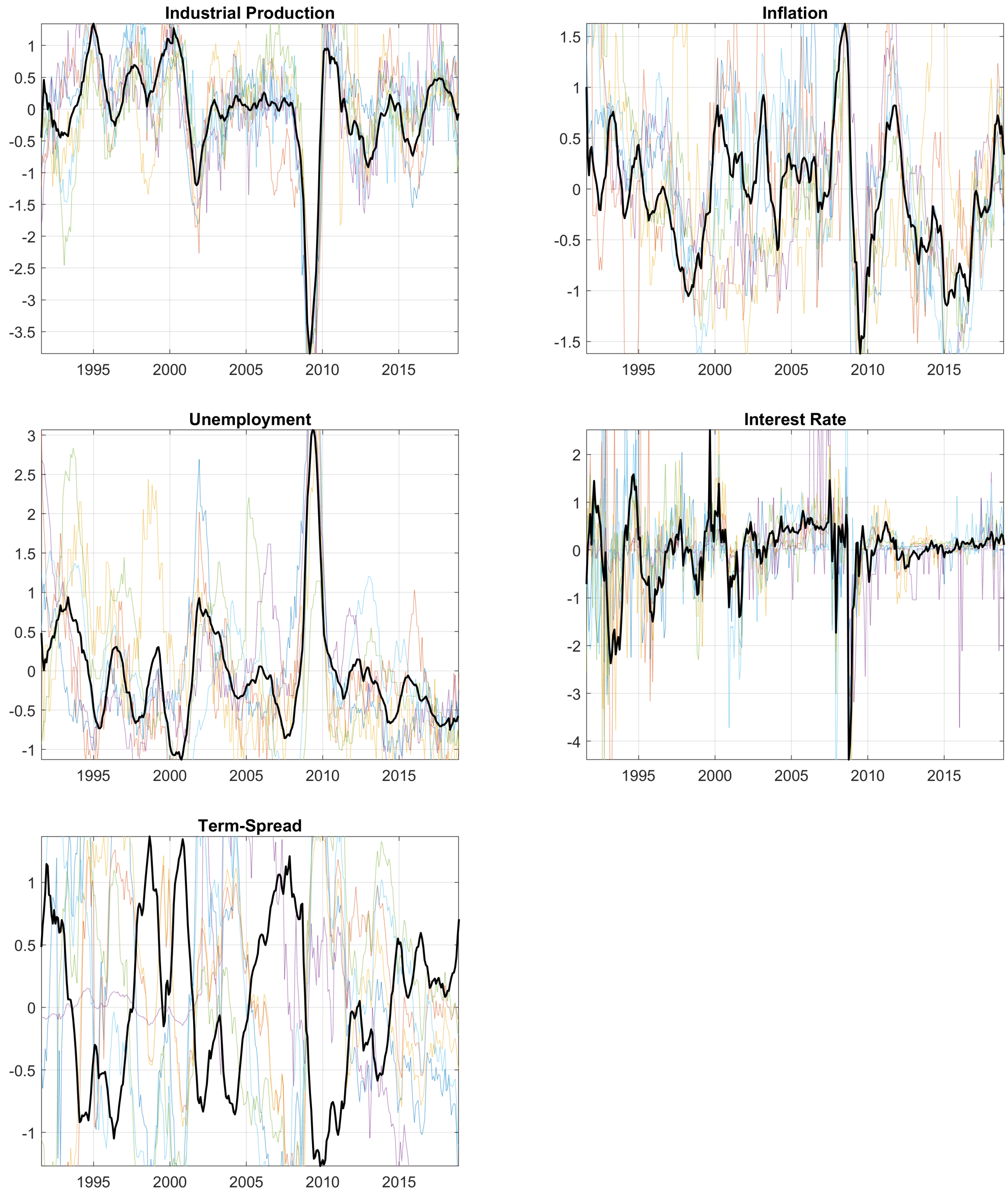
B Supplementary Material

Figure B.1: Realized and Expected Returns Across Countries



Notes: Average realized and expected returns across firms (per country) and for all firms regardless of domicile (Global) for the sample considered 1991M7-2018M12. Expected returns are constructed by estimating the model of Campbell and Shiller (1988) (equation (3)).

Figure B.2: Macroeconomic Factors



Notes: The five macroeconomic factors extracted from the hierarchical dynamic factor model with time varying parameters. Coloured lines plot the time series for industrial production, inflation, unemployment, the first difference of short-term interest rates and the term spread for each of the 6 countries included in the model: US, UK, Canada, Japan, France and Germany.

Table B.1: Investor Sentiment Factors: Total Variance Explained, 0.45.

$mR^2(GlobalSent)$	0.22	$mR^2(USSent.)$	0.10	$mR^2(CanSent)$	0.10
US - AMERICAN EXPRESS	0.68	US - SOUTHERN	0.43	CAD - ROYAL BANK OF CANADA	0.30
CAD - TORONTO-DOMINION BANK	0.65	US - CONSOLIDATED EDISON	0.42	CAD - CANADIAN IMP.BK.COM.	0.24
US - BANK OF NEW YORK MELLON	0.64	US - LOCKHEED MARTIN	0.42	CAD - NATIONAL BANK OF CANADA	0.23
US - HONEYWELL INTL.	0.63	US - MCKESSON	0.41	CAD - BANK OF MONTREAL	0.23
UK - PRUDENTIAL	0.62	US - DTE ENERGY	0.40	CAD - FORTIS	0.23
US - T ROWE PRICE GROUP	0.61	US - STERIS	0.39	CAD - CANADIAN IMP.BK.COM.	0.22
US - DOVER	0.60	US - ORACLE	0.36	CAD - BK.OF NOVA SCOTIA	0.18
US - JP MORGAN CHASE & CO.	0.60	US - ALTRIA GROUP	0.36	CAD - WESTON GEORGE	0.16
US - BANK OF NEW YORK MELLON	0.58	US - HERSHEY	0.36	CAD - TORONTO-DOMINION BANK	0.13
FR - AXA	0.57	US - ORACLE	0.35	CAD - TC ENERGY	0.13
$mR^2(JPSent)$	0.24	$mR^2(UKSent)$	0.11	$mR^2(DESent)$	0.15
JP - MITSUBISHI HEAVY INDS.	0.53	UK - CRODA INTERNATIONAL	0.39	DE - BAYER	0.29
JP - TOKYO GAS	0.50	UK - FERGUSON	0.34	DE - BMW	0.27
JP - KUBOTA	0.47	UK - SMITH (DS)	0.30	DE - BAYER	0.27
JP - KIKKOMAN	0.46	UK - HALMA	0.29	DE - E ON N	0.25
JP - TOYOTA INDS.	0.45	UK - SPIRAX-SARCO ENGR.	0.29	DE - VOLKSWAGEN PREF.	0.23
JP - HANKYU HANSHIN HDG.	0.44	UK - BARRATT DEVELOPMENTS	0.25	DE - SIEMENS	0.22
JP - KIKKOMAN	0.43	UK - LAND SECURITIES GROUP	0.23	DE - MUENCHENER RUCK.	0.19
JP - TOYOTA TSUSHO	0.42	UK - STANDARD CHARTERED	0.23	DE - SAP	0.15
JP - SEKISUI HOUSE	0.42	UK - BRITISH AMERICAN TOBACCO	0.22	DE - BASF	0.14
JP - KUBOTA	0.41	UK - WHITBREAD	0.22	DE - CONTINENTAL	0.13
$mR^2(FRSent)$	0.06				
FR - BOUYGUES	0.22				
FR - PUBLICIS GROUPE	0.18				
FR - CAPGEMINI	0.17				
FR - BNP PARIBAS	0.12				
FR - ATOS	0.10				
FR - SOCIETE GENERALE	0.09				
FR - AXA	0.09				
FR - CARREFOUR	0.08				
FR - RENAULT	0.08				
FR - KERING	0.08				

Notes: This table lists the 10 series that load most heavily on global and local investor sentiment factors along with $mR^2(k)$, which measures the average importance of factor k to the dynamics of the underlying variables. For example, the global investor sentiment factor explains 0.68 of the variation of the investor sentiment surrounding the firm American Express. This factor has an mR^2 of 0.22. This is the fraction of the variation in 586 series explained by the global investment sentiment. The 7 factors of global and local investor sentiment explain 0.45 of the total variation in all series.