

*Financial condition indices in an incomplete data environment**

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Abstract

We construct a Financial Conditions Index (FCI) for the United States using a mixed-frequency dataset that features many missing observations. The novel combination of probabilistic principal component techniques and a mixed-frequency Bayesian factor-augmented VAR model resolves the challenges posed by data points being unavailable within a high-frequency data-set. Even with up to 62% of the data missing, the new approach yields a less noisy FCI that tracks the movement of 22 underlying financial variables more accurately both in-sample and out-of-sample.

JEL classification: C11, C32, C52, C53, C66.

Keywords: Financial Conditions Index, Mixed-Frequency, Bayesian Methods

1 Introduction

Assessing financial conditions, particularly risks potentially posed by disruptions to credit markets for the business cycle, has become a priority for central banks since the Global Financial Crisis ([Bordo 2017](#)). While quantifying risks to financial stability remains a challenge for the profession, a key step in this direction has been the development of financial conditions indices; a financial conditions index (FCI) provides a quantitative summary of common movements in an array of financial variables that describe financial conditions in an economy.

Since current financial conditions may reflect relevant information about future risks to the real economy ([Adrian et al. 2019](#) and [Giglio et al. 2016](#)), a timely high-frequency FCI is of considerable appeal to the policy-maker. A significant challenge that confronts the econometrician when she sets about the task of constructing a timely FCI is the incompleteness

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of the dataset of interest. High-frequency financial datasets are often hampered by missing values while macroeconomic time series are usually only reported at low frequencies. This paper offers solutions to problems related to data incompleteness, paving the way to constructing FCIs that are reliable and timely.

This paper estimates FCIs for the United States at a weekly, monthly and quarterly frequencies, utilising several unbalanced panel techniques based on probabilistic principal components to deal with the noise in the data at higher frequencies. Our baseline framework is a mixed-frequency factor-augmented vector autoregressive model with time-varying coefficients and stochastic volatility. Our key contribution is to extend the model of [Koop and Korobilis \(2014\)](#) to accommodate incomplete data, featuring many missing values and mixed-frequency data. This extension is critical so that the model can be estimated at higher frequencies.

To do so, we leverage the probabilistic view of a Principal Component Regression (PPCA) proposed by [Tipping and Bishop \(1999a\)](#) which admits missing values and various levels of regularization that address the problem that conventional principal component techniques (PCA) overfit the data at higher frequencies. We integrate PPCA into the two-step estimation procedure for large approximate dynamic factor models of [Doz et al. \(2011\)](#). The estimated weekly, monthly and quarterly FCIs track the dynamics of 22 key financial variables. At the weekly frequency, even with 62% of the data missing, our approach delivers an FCI which is more meaningful from an economic point of view and which yields superior in-sample fit. We also find evidence that for some frequencies and forecast horizons, the FCI adds value to macroeconomic forecasting. In the pseudo out-of-sample forecasting exercise, PCA tends to be outperformed by the alternative probabilistic signal extraction methods used, although this result is more discrete.

Our factor model can be regarded as a large bridge model similar to that of [Giannone et al. \(2008\)](#) and others, which estimate factors out of a large number of high-frequency time-series to bridge lower-frequency macroeconomic data. Therefore, our approach to deal with mixed-frequency data differs from [Brave and Butters \(2011\)](#) who use Harvey accumulators. Other alternative approaches are discussed thoroughly by [Forni and Marcellino \(2013\)](#) and [Schorfheide and Song \(2015\)](#).

While the methodology adopted in this paper inherits all the advantages of the approaches introduced by [Koop and Korobilis \(2014\)](#), it also offers a more flexible way to measure financial conditions that allows for: i) time-varying weights, which define the way each financial variable load into the FCI; ii) structural instability of the relationship between the FCI combining time-series data available at different frequencies.

Most FCI models avoid the problem of having to deal with incomplete datasets by restricting the sample size and aggregating higher frequency variables to a lower base frequency. Neglecting the mixed-frequency nature of the data leads to less informative indicators which may fail to capture potentially relevant movements in financial variables at high frequencies. An exception is [Brave and Butters \(2011\)](#) that employs the EM algorithm of [Stock and Watson \(2002b\)](#) to estimate a Principal Component in a mixed-frequency setting. This allows the authors to estimate a FCI for the US economy dating back to the early 1970s.

Most FCIs are estimated under the assumption that the underlying parameters and volatilities are time-invariant. However, there are good reasons to believe that, neither the importance of a financial variable in determining broad financial conditions, nor the relation between financial conditions and the macroeconomy, are constant over time. Financial intermediation was subject to important institutional changes in recent decades, characterized by deregulation, financial innovation and globalization. These long-run trends have resulted in a larger size of the financial sector in the economy which, in principle, could increase the relevance of shocks to financial conditions for macroeconomic fluctuations. Indeed, after the Great Financial Crisis of 2007/09, a large literature has established the importance of modelling time-varying parameters, especially when modelling financial time-series (see for instance [Stock and Watson \(2007\)](#) and [Koop and Korobilis \(2012b\)](#)).

The remainder of the paper proceeds as follows. Section 2 reviews the literature on the transmission channels through which financial conditions influence macroeconomic developments, Section 3 explains the data used, Section 4 outlines the model and the estimation methods. Section 5 discusses the results and Section 6 concludes.

2 Measuring financial conditions with incomplete data

The data underpinning the estimation of a high-frequency FCI may be incomplete for four key reasons. First, time series are reported at different frequencies. For instance, when building a weekly index, the time series of GDP growth which is only available at a quarterly frequency, will feature many missing observations. Second, the starting dates of many variables differ. In our empirical exercise, for instance, macroeconomic variables are available for the full sample period from 1971 to 2018. On the contrary, many financial variables start later. Third, when measuring financial conditions in real time, it is often that many variables have missing observations for the most recent periods. Fourth, researchers often encounter missing observations in the middle of the sample, for some time-series, due to data provider issues.

When encountering these problems, many researchers often choose to aggregate the data in a way that the mixed-frequency property of the original dataset is compromised. The missing observations are often replaced by the mean of the corresponding time series. However, such procedures may lead to relevant information loss that can lead to 'aggregation bias'. By aggregating the data and setting the remaining missing observations to the corresponding time-series' unconditional mean, the researcher implicitly neglects the information in higher frequency movements in the variables. Next, we describe the data used to construct our FCI.

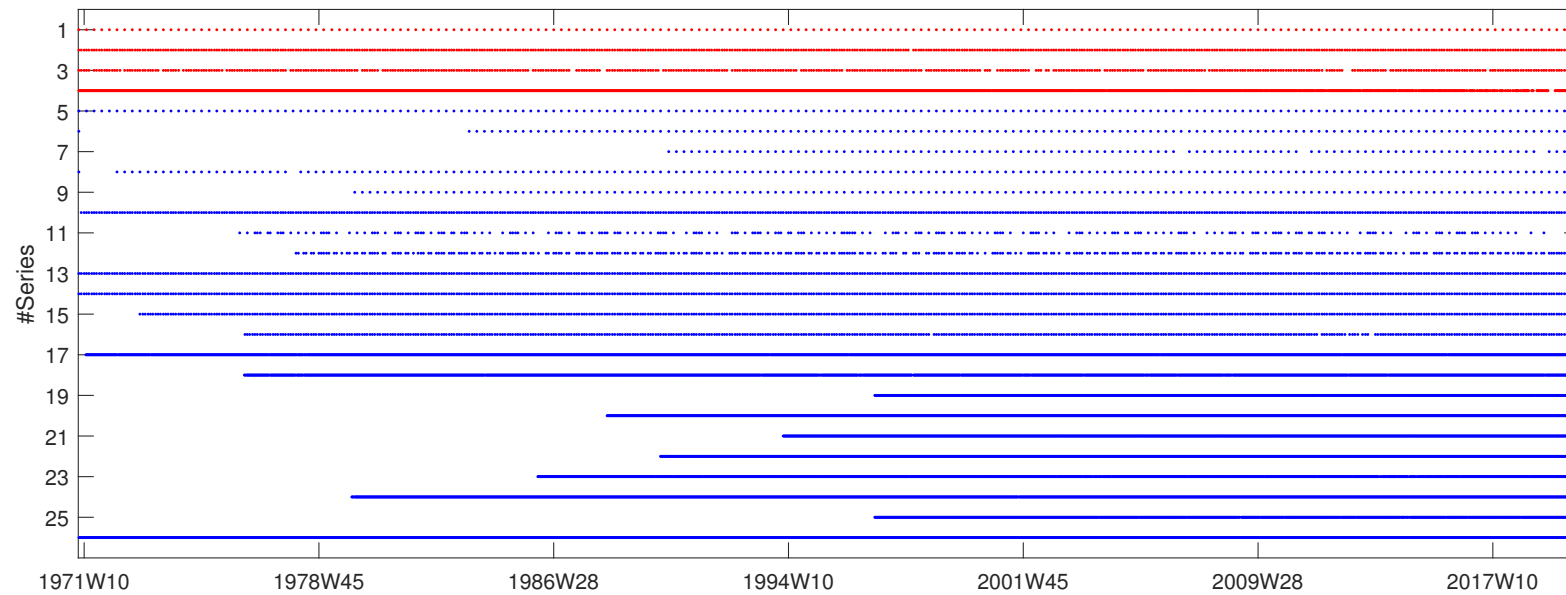
FCIs should capture movements in a wide range of variables that contain relevant information about the current state of financial conditions. With this in mind, we select a broad range of variables that extend the data considered by [Koop and Korobilis \(2014\)](#) (see [Table B.1](#) for further details). We include variables covering the following categories:

1. We consider variables that measure credit volumes and leverage. These include household debt as a percentage of GDP and household's debt service ratio, the total amount

outstanding of asset-backed securities issuance, as a percentage of GDP and total consumer credit owned and securitized as a percentage of GDP. Credit supply, a common determinant of financial distress, is considered by including a survey indicator of credit standards (DRTSCILM).

2. We include one variable that captures exchange rate movements of several important currencies towards the dollar (JPMNEER) and a commodity index which tracks the movements of the price of a number of important commodities (CRY Index).
3. A number of indicators which generally describe the market price of risk are considered. These include, for instance, the TED spread, the spread on 2 year and 10 year Treasury Bills. We include a number of risk indicators targeting in particular credit markets. These include a Loan Performance index, commercial paper spread, high yield spreads, mortgage spreads, the spread between the bank prime loan rate and the Libor, the finance rate on commercial bank loans. We also include the loan-to-deposit ratio of commercial banks, which measures liquidity risk.
4. Variables measure equity valuations in the economy include the S&P500 price index and the Wilshire 5000 Full Cap Price index.
5. We include variables related to beliefs, expectations and uncertainty. We consider the VIX, which is commonly related to risk-aversion and uncertainty, a survey index of expected changes in financial conditions and a yield curve weighted index of the implied volatility of short-term treasury options.
6. Finally, we also use macroeconomic data to construct the FCI but do not include them in the index. These include real GDP growth, inflation measured as the percent variation in the Consumer Price Index and the unemployment rate. The interest rate used in the observable sector of the model is the effective federal funds rate. The macroeconomic sector of the model allows us to make sure our FCI is uncontaminated by macroeconomic innovations. Figure 1 gives a visual description of the data used to construct the FCI.

Figure 2.1: Incomplete data used to build the FCI



Notes: The frequency of the data is weekly. The white squares indicate missing values; red/blue squares indicate the availability of macroeconomic/financial variables considered. Macroeconomic variables include 4 time-series, while the financial variables include the 22 financial variables used. The *#Series* refers to the numbering of the time-series according to Table B.1.

The first rows in red describe the observations in our dataset that relate to macroeconomic variables. It can be observed that, with the exception of interest rates, all macroeconomic variables in our model are only available at quarterly and monthly frequency, in contrast to several of the financial variables that are available at the weekly frequency. These are examples of the first reason why some data is missing in large datasets that are used to estimate FCIs - some variables are observed at a lower frequency than others. The rows in blue describe the pattern of sparsity in financial variables. Here observations are missing for three main reasons. First, most financial variables are missing for many periods at the beginning of the sample. Second, there are some missing values in the middle of the sample due to data limitations of the data provider. Lastly, it is possible to observe few variables which are not available for the very recent periods. This is often caused by lags in data availability. In total, the FCI is based on 22 financial variables and 4 macroeconomic series which are explained in detail in Table B.1. All variables are transformed so that they are stationary and standardized prior to the estimation of the model.

3 A mixed-frequency financial conditions index with structural instabilities

We estimate the FCI using a mixed-frequency time-varying parameter FAVAR (MF-TVP-FAVAR). This approach offers several advantages over its alternatives. First, the factor-augmented structure of the model nests a standard VAR and a Dynamic Factor Model (DFM). Therefore, it is easy to assess the marginal contribution of the additional information content of the FCI, to understand the behavior of key observable real economic variables, while retaining the statistical power of DFMs which have been found to be effective in summarizing the information from a large number of time series (see [Stock and Watson 2016](#) for a review). Second, the model accounts for time-varying parameters. It is essential to account for time-variation of the relationships between the FCI, its constituents and the macroeconomy. This concern is important because FCIs span a long period of time and cover several macroeconomic eras. For instance, it is well documented that the dynamics of macroeconomic variables have been more volatile in the 70s, early 80s and after the Great Financial Crisis as compared to the period in between (1987-2007), dubbed the Great Moderation (see [Stock and Watson \(2002a\)](#)). Finally, the estimation respects the mixed-frequency nature of the data, capturing high-frequency movements in financial conditions which are important to monitor financial stability in real time.

3.1 The MF-TVP-FAVAR

Consider a Mixed-Frequency Time-Varying Parameter Factor-Augmented VAR (MF-TVP-FAVAR), written as follows

$$X_t = \lambda_t^F F_t + \lambda_t^Y Y_t + u_t, \quad (1)$$

$$\begin{bmatrix} Y_t \\ F_t \end{bmatrix} = \beta_{t,1} \begin{bmatrix} Y_{t-1} \\ F_{t-1} \end{bmatrix} + \dots + \beta_{t,p} \begin{bmatrix} Y_{t-p} \\ F_{t-p} \end{bmatrix} + \varepsilon_t, \quad (2)$$

where X_t is an $(N \times T)$ matrix of N mixed-frequency financial variables that load onto the FCI denoted in this setting by F_t . λ_t^F are time-dependent loading parameters that define which financial variables x_{it} load onto F_t at each point in time t . λ_t^Y denotes the coefficients of a set of observable mixed-frequency macroeconomic variables organized in Y_t , in the measurement equation (1) which are also time-dependent. This term is key in ensuring that the variation in the financial variables that are attributable to changes in real economic conditions are purged from F_t . The state equation (2) describes the relationship between macroeconomic variables and the FCI. The set of parameters that determine this relationship $\{\beta_{t,1}, \dots, \beta_{t,p}\}$ are also allowed to vary over time. Lastly the innovations to the measurement equation $u_t \sim N(0, V_t)$ and to the state equation $\varepsilon_t \sim N(0, Q_t)$ are Gaussian and their volatilities are allowed to vary with time.

The mixed-frequency nature of the model allows us to obtain a FCI at a weekly, monthly as well as quarterly frequency. Our strategy involves writing the model for the highest available frequency, in which lower frequency variables feature in the model and are bridged with factors calculated at higher frequencies. This methodology combines principal components with the Kalman filter, as proposed by [Giannone et al. \(2008\)](#) and [Doz et al. \(2011\)](#) and is known as "bridging with factors". It should be noted that it is not possible to construct the FCI at higher frequencies by using the standard estimation procedure of [Koop and Korobilis \(2014\)](#) since some macroeconomic variables are only reported at a quarterly frequency.

The model is completed by the last 2 state equations below.

$$\beta_t = \beta_{t-1} + \eta_t, \quad \eta_t \sim N(0, R_t), \quad (3)$$

$$\lambda_t = \lambda_{t-1} + v_t, \quad v_t \sim N(0, W_t), \quad (4)$$

which allow the parameters to smoothly change over time. A final remark regarding all variance-covariance matrices $\{V_t, Q_t, R_t, W_t\}$ is in order. While time-variation in V_t and Q_t describes the dynamics of stochastic volatility, R_t and W_t determine the amount of time-variation in the parameters β_t and λ_t . For V_t and Q_t , Exponential Weighted Moving Average (EWMA) estimators are used which involves the use of recursive simulation-free variance discounting methods and depend on decay factors κ_1 and κ_2 ¹. EWMA estimators accurately approximate integrated GARCH processes whereas R_t and W_t are estimated through forgetting factor methods described in [Koop and Korobilis \(2012a, 2013\)](#) and are driven by κ_3 and κ_4 ².

¹See Appendix A.1, equations 27-28.

²See Appendix A.1, equations 17-18.

Variance discounting and forgetting factors are commonly used since they offer a computationally feasible way of modelling stochastic volatility and time-variation in the parameters by recursively updating all variance-covariance matrices which are assumed to evolve smoothly over time.

The model defined by equations (1)-(4) differs from that proposed by [Koop and Korobilis \(2014\)](#) in two important dimensions. Firstly, the time series that are included in Y_t and X_t need not have the same frequency. [Koop and Korobilis \(2014\)](#) estimate their model using data that are easily available at a quarterly frequency, eliminating the need to use mixed-frequency techniques. In contrast, our model is estimated at higher frequencies but our dataset (Y_t) includes variables such as inflation or GDP growth which are only available at a quarterly frequency. This feature of our model is very useful for policymakers who often need indicators of financial conditions at higher frequencies. Secondly, we treat the missing values in X_t as latent variables. While [Koop and Korobilis \(2014\)](#) set any missing values in X_t to zero (which is the unconditional mean of all the standardized financial time-series), we estimate any missing values in X_t with probabilistic principal component methods. We will discuss the latter methodology in the next section.

We estimate our model at a weekly, monthly and quarterly frequencies. Monitoring financial conditions at higher frequencies is appealing but the noise in financial data at higher frequencies may jeopardize the relevance of higher frequency FCIs. We will later see the value of PPCA in dealing with this problem.

3.2 Estimation

The model defined by equations (1)-(4) can be cast in state-space form and is estimated through a Bayesian Kalman filtering and smoothing routine which is based on [Koop and Korobilis \(2014\)](#) (see Appendix A.1 for a detailed explanation of the algorithm). The algorithm extends the one proposed by the authors to a mixed-frequency data environment and involves the following steps:

- **Step 1:** Estimate F_t using PCA methods which accommodate missing values.
- **Step 2:** Conditional on the initial values of the factors, \tilde{F}_t , estimate the parameters in the MF-TVP-FAVAR by applying Kalman filtering and smoothing.
- **Step 3:** Conditional on the parameters estimated in Step 2, estimate \hat{F}_t , which is used as our FCI, by applying Kalman filtering and smoothing.

The mixed-frequency nature of the model may give rise to a sparse dataset; 'sparsity' implies that at some points across the time dimension, there are very few observations. This is particularly relevant for early periods in the sample for which a small subset of all variables considered are observed. One of the great advantages of estimation through Kalman filtering techniques is that it is straightforward to deal with missing values. The algorithm of [Koop and Korobilis \(2014\)](#) (Steps 2 and 3) is modified in accordance to [Koopman and Com-mandeur \(2008\)](#) by simply setting the Kalman Gain term to zero in each updating equation,

whenever an observation is missing. This guarantees that in the absence of any new information regarding a variable (*i.e.* when it is missing), our best guess for the current state of the respective parameter remains unchanged. In Step 1, prior to the MF-TVP-FAVAR stage, we estimate the factor F_t and reconstruct the data matrix of financial variables. The challenge is to reconstruct the unbalanced panel of data X_t which contains missing observations. The reconstructed matrix is then used as an input to Step 2. In Step 3, a Kalman filter/smoothing is passed through the pre-estimated factor yielding our final estimate of the FCI. Steps 2 and 3 are similar to the method proposed by Doz et al. (2011).

3.2.1 Mixed-frequency adjustments and the treatment of missing values

In order to take full advantage of the data available, we need to deal with the mixed-frequency and incomplete nature of the data. There are many alternatives in the literature to deal with mixed-frequency data.³ We follow the strategy proposed by Giannone et al. (2008) and Doz et al. (2011) that relies on a two-step estimation previously explained that combines principal components with the Kalman filter and smoother. First, the parameters are estimated conditional on the estimated factors, where the latter are obtained through principal components calculated on a balanced version of the dataset. Then, the Kalman smoother is used to update the estimate of the signal variable on the basis of the full unbalanced dataset. As noted by Foroni and Marcellino (2013), the framework can be interpreted as a large bridge model in which a large number of variables bridge high-frequency time-series with forecasts of the quarterly variables. Missing values in the measurement equation (1) and state equation (2) are treated by modifying a Kalman filter, and we provide details in section A.1. However, the existence of missing values in X_t need to be taken into account in Step 1 to pre-estimate F_t . The missing values in X_t can be dealt with using a variety of unbalanced panel methods, which we discuss next.

3.2.2 Principal Component Analysis in the presence of missing values

One of the most popular approaches to PCA is based on singular value decomposition (SVD) of the covariance matrix of the data,

$$C = N^{-1} X^T X = U D U^T, \quad (5)$$

where D contains the eigenvalues and U the eigenvectors of the covariance matrix C , normalized to have unit-length. This approach is only valid when the panel is balanced, *i.e.* when no observations are missing. In the presence of missing values, Least-Square techniques need to be modified. We explore the following four alternative solutions to finding principal components in incomplete datasets.

- i) *Standard PCA with missing values set to zero (PCA)*
- ii) *Least Squares Expectation-Maximization PCA (EM-PCA)*

³Foroni and Marcellino (2013) and Schorfheide and Song (2015) provide a comprehensive review of the different techniques.

iii) *Probabilistic PCA (PPCA)*

iv) *Variational Bayesian PCA (VBPCA)*

These methods vary in several important ways.⁴ The standard PCA removes missing values by setting them to zero. Because the variables in X are standardized, this is equivalent to setting each missing value to the respective variable's unconditional mean. Next, the Expectation-Maximization PCA (EM-PCA) as proposed by [Stock and Watson \(2002b\)](#), results in an imputed matrix X where the missing values are reconstructed such that the Least-Squares Likelihood is maximized. In each iteration any missing points are replaced with their conditional mean. The PCA and EM-PCA belong to the family of Least-Squares methods since they work by minimizing the squared residuals of the PC regression. In contrast, the probabilistic PCA and the Variational Bayes PCA approach the PC regression from a probabilistic point of view.

A probabilistic formulation of PCA offers a number of benefits, including a well-founded treatment of missing values, extendability and regularization. The PPCA is based on the work of [Tipping and Bishop \(1999b\)](#). Estimation in an incomplete data setting is discussed in [Ilin and Raiko \(2010\)](#) which use an EM algorithm treating the latent factors as hidden variables. Although the PPCA performs some regularization (*i.e.* the Gaussian priors set on the factors penalize large values in F), this might be insufficient if the data is very sparse. An extension that allows for more strict penalization is proposed by [Oba et al. \(2003\)](#). This approach which is called Variational Bayesian PCA (VBPCA) involves imposing priors over the remaining parameters in the model that penalize more complex explanations of the data.

To summarize, estimation of the proposed FCI is based on the MF-TVP-FAVAR which is estimated in three steps. In the first step, a large number of financial data series X are compressed into a common factor by using one of the four methods suggested above. Secondly, taking this factor as given, a Kalman filter and smoother is applied to estimate time-varying parameters and stochastic volatilities. Third, conditional on the parameters obtained, the Kalman filter and smoother is applied to the factors resulting in the FCI.

4 Results

4.1 Measuring financial conditions with incomplete data

The main purpose of our exercise is to construct an index that tracks the development of financial conditions even while fully internalising the incomplete nature of the underlying data. Figure 4.1 presents the Financial Conditions Index (FCI) estimated at a weekly, monthly and quarterly frequency based on the Variational Bayes approach to PCA, which is the model that yields the best in-sample fit across frequencies.⁵ Figure B.2 presents in-sample Mean Squared Errors obtained when estimating a Principal Component (PC) Regression with each of the four alternative methods we employ. The key result is that probabilistic solutions to

⁴We discuss the main idea and intuition for each method. More details are provided in Appendix A

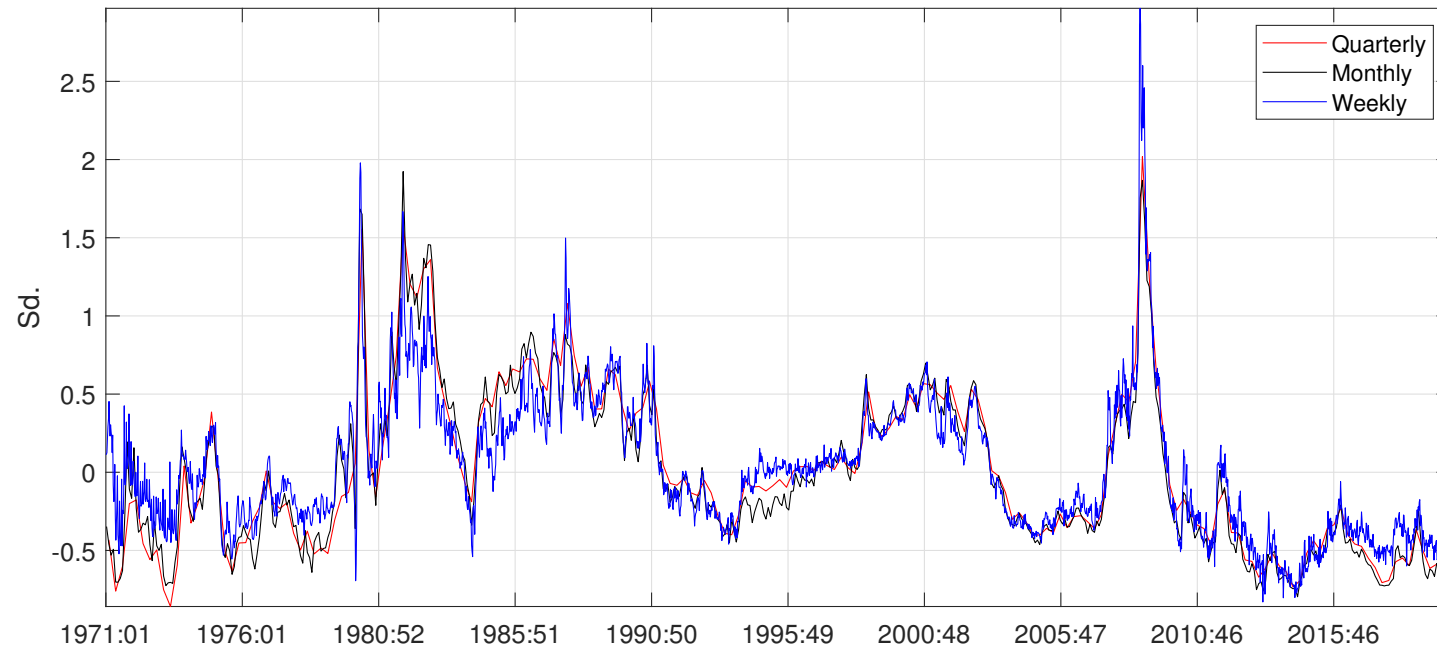
⁵Figure B.1 presents the FCI calculated with alternative PCA methods, across the different frequencies considered.

PC estimation yield better in-sample fit. This finding is particularly evident when moving from quarterly to weekly frequency. At a weekly frequency, the probabilistic PCA noticeably outperforms PCA and EM-PCA. Moreover, Figure B.1 highlights that at higher frequencies, Least Squares solutions to PC estimation (*i.e.* PCA and EM-PCA) deliver very noisy indices compromising interpretability. On the other hand, the probabilistic measures, PPCA and VBPCA deliver interpretable indices.

The indices presented in Figure 4.1 summarize the movement in the underlying 22 financial variables relevant for real economic developments in the US (see Table B.1 in the Appendix for details). By construction, the FCI fluctuates around zero. Times of tight financial conditions such as the period marked by the Global Financial Crisis in 2009, drive the index upwards. In contrast, periods of accommodative financial conditions lead to negative values of the FCI.

The figure reveals the size of the 'bias' that results from neglecting information at higher frequencies. For instance, the FCI built at quarterly frequency tends to underestimate the size of financial distress in 2009, during the Great Recession.

Figure 4.1: Financial Conditions Indices at different data frequencies

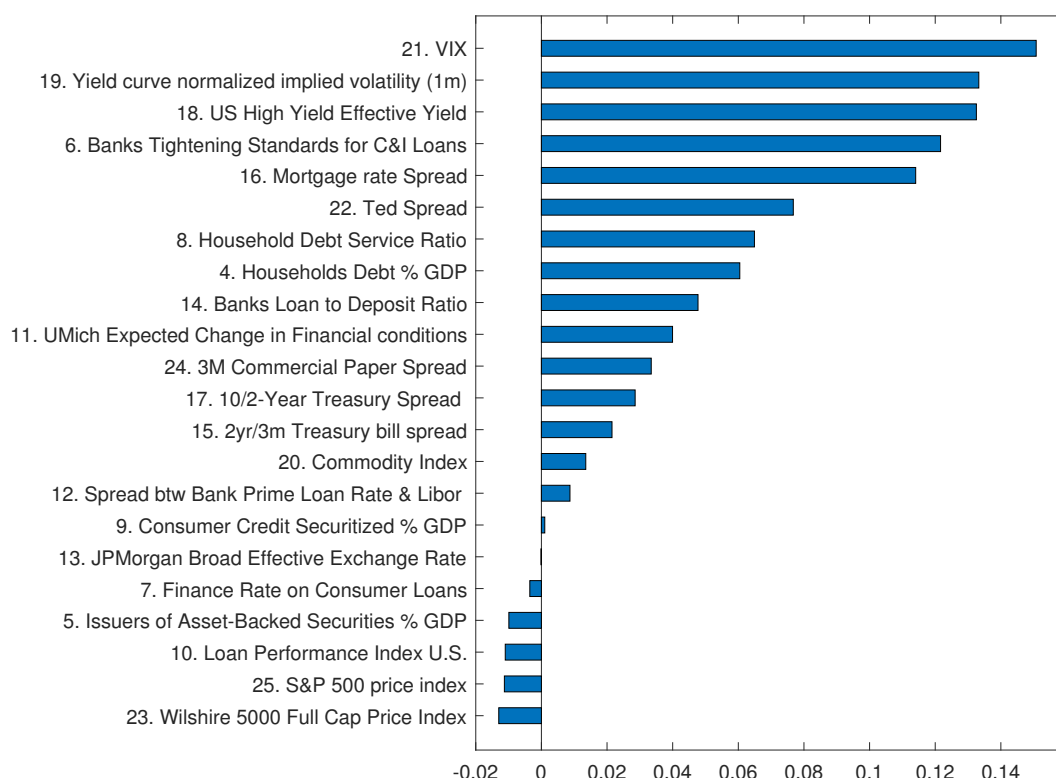


Notes: The Financial Conditions Index (FCI) in this figure is estimated for the period 1971-2018 with the Variational Bayes variant of our algorithm for a weekly, monthly and quarterly frequencies. Positive values of the FCI indicate financial conditions are tighter than average, while negative values indicate financial conditions that are looser than average. Deviations from the mean are measured in Standard Deviations (Sd.).

4.2 Individual contributions to the dynamics of the FCI

We now examine the driving forces and macroeconomic relevance of financial conditions as measured by the best-fitting FCI, the VBPCA. Figure 4.2 shows the average importance of the underlying financial variables which contribute the most in driving the FCI, throughout the period from 1990-2018.

Figure 4.2: Average weights of individual financial variables loading into the Financial Conditions Index



Notes: The weights represent the average time-varying loadings of each financial indicator into the FCI for the full sample period 1971-2018.

The indicators with the highest positive contribution for the movement in the FCI include the VIX which measures the markets' expectation of near term volatility and the volatility implied in Treasury options. Credit supply, as measured by the change in credit standards by commercial banks, also features in the most influential variables driving the FCI. Other relevant indicators include two important measures of leverage and liquidity - such as the Household Debt Service Ratio and the Bank Loans to Deposit Ratio; Most of the remaining influential variables driving financial conditions relate to credit spreads. These include Mortgage Spreads, the TED spread and Spreads on US government debt.

In addition, the financial variables which contribute negatively to the developments of financial conditions are mainly quantity variables (in dollars) which relate to equity valuations and mortgage lending. This result is intuitive, since in times of tight financial conditions, the

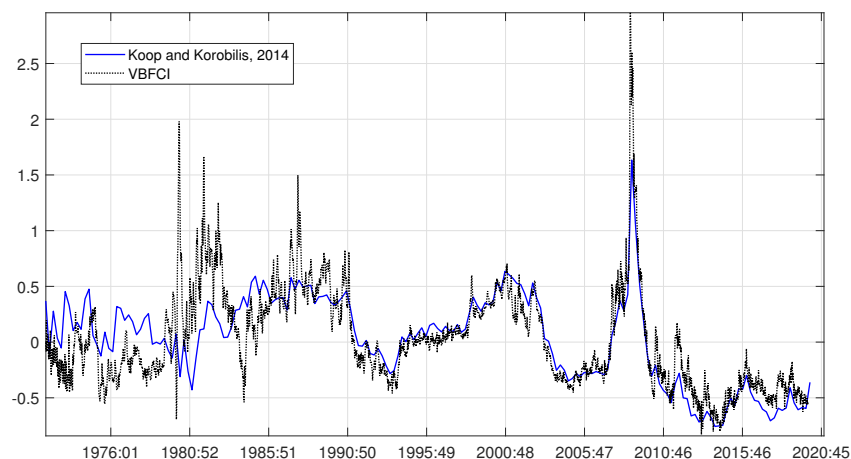
performance of outstanding loans tends to deteriorate and stock markets tend lose value. Some interest rates and spreads also have a countercyclical contribution to the index. These include commercial the finance rate on consumer loans. This may reflect the fact that the yield curve tends to invert in recessions, when financial conditions also tend to tighten.

4.3 Comparison with alternative econometric approaches

The key novelty of the econometric approach used throughout this paper is its treatment of missing data and regularization of noise. The model of [Koop and Korobilis \(2014\)](#) only allow the construction of the FCI at a quarterly frequency. [Brave and Butters \(2011\)](#) high-frequency FCI is a Dynamic Factor Model with no observables and no time-varying dynamics. Our mixed-frequency model allows us to construct the FCI at a weekly and monthly frequencies and retain all modelling flexibility offered by [Koop and Korobilis \(2014\)](#).

Figure 4.3 below allows for a comparison between our approach and [Koop and Korobilis \(2014\)](#) estimated on the same dataset at a quarterly frequency. The FCI is naturally more informative than the analogous model estimated at a quarterly frequency. This is because it captures more movements in the underlying data that occur at higher frequencies, but also because it estimates missing data points in financial time series. Overall, the FCI seems to provide more timely signals of financial distress. This is particularly noticeable during the Great Recession.

Figure 4.3: Comparison between the FCI and alternative methods.



Notes: The FCI is estimated with the methods described in section 2, with the Variational Bayes variant of the PCA algorithm presented, at a weekly frequency for the sample period between 1971-2018. The FCI by [Koop and Korobilis \(2014\)](#) is estimated on the same dataset at a quarterly frequency.

4.4 The macroeconomic relevance of financial conditions

We now focus on the macroeconomic relevance of financial conditions by studying the out-of-sample performance of our FCI in forecasting GDP growth, inflation, unemployment and interest rates. We calculate the FCI at a weekly, monthly and quarterly frequencies and com-

pare Mean Square Forecast Error (MSFE) statistics calculated for a given frequency f and forecast horizon h as follows

$$MSFE_h^f = \frac{\sum_{j \in O_j} (Y_j - \hat{Y}_j)^2}{T - h},$$

where O_j denotes the set of indices j for which Y_j is observed at frequency f and T is the total number of time-series observations. To be clear, \hat{Y}_j is our forecast, h -steps ahead, for all macroeconomic time-series at a given frequency f . When working with higher-frequency data - say monthly, we only observe GDP once every three months. In this case, we calculate MSFE statistics based on the difference $(Y_j - \hat{Y}_j)^2$ series for $j = \{1, 3, 6, 9, \dots\}$.

Our goal is to compare the forecasting performance of FCIs constructed with different signal extraction methods (i.e. PCA, PPCA, EM-PCA and VBPCA). Although it is tempting to explore how the forecasting performance varies across the data-frequencies considered, this analysis would be spurious in this setting because weekly, monthly and quarterly models are estimated with a different number of parameters. We estimate the weekly, monthly and quarterly models with 12, 6 and 4 lags, respectively. The choice of the number of lags to include in the model is a compromise between computational cost, the risks of over-parameterization and neglecting persistence. Moreover, the FCIs estimated for different frequencies also differ, making this comparison even harder. For these reasons, a comparison of forecast performance across frequencies is out of the scope of our current work and is left for future research.

Our baseline specification is obtained by estimating our model (described in section 3.1) without any latent factor included; a VAR with mixed-frequency data. The first line of Table 4.4 for each macroeconomic variable reports MSFE for the baseline case, without the inclusion of any FCI. The lines below report the ratio of the MSFEs obtained from the baseline case and the exact same model augmented with the FCI, computed via one of the four methods discussion thus far (i.e. PCA, EM-PCA, PPCA and VBPCA). A ratio lower than 1 signals that the FCI measured with one of the aforementioned signal extraction methods outperforms the benchmark model.

The main message of the out-of-sample exercise is that the value of the econometric methods proposed throughout this paper becomes more obvious when working at higher frequencies with noisy data. Importantly, the value of using FCIs to forecast macroeconomic variables is higher in the very short run, at weekly and monthly frequencies. At a weekly frequency, the FCI computed via VBPCA and PPCA improves forecasting most significantly for unemployment and interest rates. What is particularly striking is the relatively superior performance of the VBPCA and PPCA methods over the EM-PCA used by Brave and Butters and in many other economic applications. This result emphasises the importance of regularization in econometric models, in particular when the data is observed at high-frequency and is noisy.

On the other hand, there is little evidence that the alternative signal extraction methods outperform PCA at a quarterly frequency. This finding is consistent with the previously re-

ported result that these methods add value when the data is very noisy (which tends to be the case at higher frequencies).

Quarters ahead	1q	4q	8q	12q	20q	Months ahead	1m	4m	6m	12m	24m	Weeks ahead	1w	4w	8w	9w	12w
GDP growth						GDP growth						GDP growth					
MF-VAR (no FCI)	0.68	0.78	0.73	0.57	0.54	MF-VAR (no FCI)	0.77	1.09	1.03	1.15	0.89	MF-VAR (no FCI)	0.67	0.68	0.72	0.72	0.73
EM-PCA	1	1.02	1.11	1.08	1.03	EM-PCA	1.31	1.56	1.46	1.16	1.13	EM-PCA	0.97	0.95**	0.96*	0.99	1.01
PCA	0.98	1	0.99	1	1.01	PCA	1.14	1.24	1.17	1.08	1.15	PCA	0.97	0.99	0.94	0.96	0.98
PPCA	0.98	1.02	1	0.99	1.02	PPCA	1.14	0.94	0.88*	0.74***	0.70***	PPCA	0.99	1	1	1.01	1
VBPCA	0.95	1.01	1	0.99	1.01	VBPCA	1.2	1.10	1	0.95***	0.99	VBPCA	0.96	1.01	1.05	1.08	1.04
CPI inflation						CPI inflation						CPI inflation					
MF-VAR (no FCI)	0.59	0.61	0.71	0.68	0.66	MF-VAR (no FCI)	0.56	0.71	0.69	0.69	0.69	MF-VAR (no FCI)	0.55	0.56	0.61	0.63	0.73
EM-PCA	0.95**	0.96*	0.90	0.84	0.90	EM-PCA	0.94***	0.98	1.04	1.17	1.04	EM-PCA	0.99	1.05	1	0.97	0.86*
PCA	1.04	1.07	1.12	1.03	0.95	PCA	0.91*	1.03	1.05	1.11	1.23	PCA	1.03	1.12	1.24	1.19	1.23
PPCA	1.06	1.12	1.21	1.13	1.01	PPCA	0.93***	0.94***	0.96**	0.96***	0.95***	PPCA	0.96	0.99	1	0.98	0.92
VBPCA	1	1.06	1.12	1.06	1	VBPCA	0.92**	0.95**	0.98	1.02	0.96*	VBPCA	0.94	1	1.04	1	0.86*
Unemployment						Unemployment						Unemployment					
MF-VAR (no FCI)	0.80	0.97	0.93	0.87	0.87	MF-VAR (no FCI)	0.81	0.88	0.95	1.19	1.13	MF-VAR (no FCI)	0.85	0.86	0.89	0.89	0.94
EM-PCA	1	1	1.02	1.04	1.01	EM-PCA	0.93***	0.96*	0.98	0.95*	0.97	EM-PCA	0.97**	0.98	1.01	1.02	1
PCA	0.92	1	1	1.01	1	PCA	1.03	1.06	1.08	1.13	1.07	PCA	0.93**	0.95	0.94	0.94	0.92
PPCA	0.94	1.01	0.99	1.01	1.01	PPCA	0.97**	0.96**	0.95*	0.89*	0.84**	PPCA	0.92***	0.93*	1.02	1.03	1
VBPCA	0.98	1	1.01	1.02	1	VBPCA	0.96**	0.95***	0.92**	0.90*	0.96	VBPCA	0.94**	0.94	1.03	1.05	1.01
Interest rates						Interest rates						Interest rates					
MF-VAR (no FCI)	0.04	0.16	0.35	0.48	0.61	MF-VAR (no FCI)	0.02	0.21	0.32	0.71	0.89	MF-VAR (no FCI)	0.08	0.42	1.20	1.38	1.78
EM-PCA	0.95	1.04	1.03	0.95	0.89	EM-PCA	1.09	0.90	0.90	0.95	0.86	EM-PCA	0.73***	0.53***	0.57***	0.61***	0.77***
PCA	0.97*	1.16	1.24	1.22	1.07	PCA	0.79***	0.87**	0.96	1.36	1.62	PCA	0.83***	0.81***	0.94*	0.98	1.10
PPCA	1	1.12	1.17	1.15	1.04	PPCA	1.28	1.03	0.96	0.86**	0.70***	PPCA	0.67***	0.58***	0.67***	0.70***	0.85***
VBPCA	0.90*	1.09	1.15	1.13	1.04	VBPCA	0.97	0.81***	0.78***	0.89***	1.04	VBPCA	0.58***	0.55***	0.65***	0.68***	0.84***

Table 4.1: Out-of-Sample MSFE for different signal extraction methods, data frequencies (i.e. quarterly, monthly and weekly) and horizons for predictions with a Mixed-Frequency VAR (MF-VAR) for GDP growth, inflation, unemployment and interest rates on a training sample 1971-1981 and testing 1981-2020. The methods are (i) standard Principal Component Analysis with missing values set to 0 (PCA) (ii) Least Squares Expectation-Maximization (EM-PCA) (iii) Probabilistic Principal Component Analysis (PPCA) and (iv) Variational Bayesian Probabilistic Principal Component Analysis (VBPCA). The MSFE statistics for the MF-VAR (first line in each table) are obtained by running the model without the inclusion of any FCI. The lines below show the ratio between the MSFE of the model in concern augmented with an FCI and the baseline MF-VAR model without any FCI. Values below 1 in each table for the PCA, EM-PCA, PPCA and VBPCA imply that the relevant model outperforms the baseline model for a given horizon, frequency and macro variable. ***,** and * refer to p-values lower than 0.01, 0.05 and 0.1, respectively, associated to the Diebold-Mariano Statistic with automatic lag-selection.

5 Conclusion

We constructed a Financial Conditions Index with data reported at different frequencies, starting dates and many missing observations. In doing so, we address a challenging methodological aspect of estimating such indices; the incompleteness of the underlying dataset. Our novel estimation approach accounts for both missing values in financial time series and the fact that macroeconomic time series are only available at lower frequencies.

Traditional macroeconometric techniques cope poorly with jointly addressing the mixed-frequency nature of the data, the proliferation of missing values and the need to build indexes at high frequencies. The factor-augmented VAR we estimated through a combination of probabilistic PCA methods and Kalman filtering and smoothing routines offer a solution to cope with those features of the data. The Financial Condition Index obtained shows a better fit and is also meaningful from an economic perspective. In-sample fit as well as out-of-sample forecasting results suggest that the alternative signal-extraction methods that we employ are important particularly when the data is noisy, which usually tends to be the case at higher frequencies.

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A Econometric Methods

A.1 Mixed-Frequency Bayesian Kalman Filter with Forgetting Factors

The model defined in (1)-(4) configures a MF-TVP-FAVAR and can be written compactly, in state space form as follows

$$X_t = Z_t \Lambda_t + u_t, \quad u_t \sim N(0, V_t), \quad (6)$$

$$Z_t = Z_{t-1} \beta_t + \varepsilon_t, \quad \varepsilon_t \sim N(0, Q_t), \quad (7)$$

$$\beta_t = \beta_{t-1} + \eta_t \quad \eta_t \sim N(0, R_t), \quad (8)$$

$$\Lambda_t = \Lambda_{t-1} + v_t \quad v_t \sim N(0, W_t). \quad (9)$$

Where $\Lambda_t = [\lambda_t^y, \lambda_t^f]'$, $Z_t = \begin{bmatrix} Y_t \\ F_t \end{bmatrix}$. Note that Z_t depends on the latent factor F_t that is taken as data⁶. Let $\theta_t = \{\Lambda_t, \beta_t\}$ denote the parameter set and $D_t = \{X_t, Z_t\}$ the data for $t = \{1, \dots, T\}$. Assuming that we know the posterior of θ at time $t-1$, Bayesian filtering/smoothing is based on the equations below

$$p(\theta_t, \theta_{t-1} | D_{t-1}) = p(\theta_t | \theta_{t-1}, D_{t-1}) p(\theta_{t-1} | D_{t-1}), \quad (10)$$

$$p(\theta_t | D_{t-1}) = \int_{\Omega} p(\theta_t | \theta_{t-1}, D_{t-1}) p(\theta_{t-1} | D_{t-1}) d\theta_{t-1}, \quad (11)$$

where Ω is the support of θ_{t-1} . The prediction step is given by the Chapman-Kolmogorov equation A6.

Next, at each iteration t , the prior $p(\theta_t | D_{t-1})$ gets updated according to equation A6 and the measurement likelihood $p(D_t | \theta_t)$ is augmented by an additional observation of D_t . Hence the posterior distribution is updated according to the Bayes rule

$$p(\theta_t | D_t) = \frac{1}{H_t} p(D_t | \theta_t, D_{t-1}) p(\theta_t | D_{t-1}). \quad (12)$$

Where $H_t = \int p(D_t | \theta_t) p(\theta_t | D_{t-1})$ is the normalizing constant. Equation A7 is referred to as the updating step. To summarize, the algorithm extends the one derived in [Koop and Korobilis \(2014\)](#) to a mixed-frequency environment where D_t is allowed to contain missing values. It consists in 2 stFCI, iterating through prediction (A6) and updating (A7) after the system is initialized. These two main stFCI are repeated for $t = \{1, \dots, T\}$.

1. Kalman Filter

1.1 Initialization (Priors) All quantities are initialized according to their priors which are chosen following the diffuse choices of [Koop and Korobilis \(2014\)](#): $f_0 \sim N(0, 10)$, $\Lambda_0 \sim N(0, I)$, $\beta_0 \sim N(0, I)$. The variances of the innovations in (A1)-(A4) can be seen as hyperparameters and are set to $\hat{V}_0 = 0.1 * I$, $\hat{Q}_0 = 0.1 * I$, $\hat{R}_0 = 10^{-5} * I$ and $\hat{W}_0 = 10^{-5} * I$. However,

⁶This quantity is the first principal component calculated in Step 1 by employing one of the four algorithms discussed in the body of the paper and detailed in section A2 to reconstruct the incomplete data matrix X_t

in this setting the hyperparameter are allowed to smoothly change over time following a Exponentially Weighted Moving Average (EWMA).

1.2 Prediction

$$\Lambda_t \sim N(\Lambda_{t|t-1}, \Sigma_{t|t-1}^\Lambda), \quad (13)$$

$$\beta_t \sim N(\beta_{t|t-1}, \Sigma_{t|t-1}^\beta). \quad (14)$$

Where $\Lambda_{t|t-1} = \Lambda_{t-1|t-1}$, $\beta_{t|t-1} = \beta_{t-1|t-1}$ and

$$\Sigma_{t|t-1}^\beta = \Sigma_{t-1|t-1}^\beta + \hat{R}_{t|t-1}, \quad (15)$$

$$\Sigma_{t|t-1}^\Lambda = \Sigma_{t-1|t-1}^\Lambda + \hat{W}_{t|t-1}. \quad (16)$$

The state covariances in the equations above are estimated by

$$\hat{R}_{t|t-1} = \frac{1}{\kappa_3} \hat{R}_{t-1|t-1}, \quad (17)$$

$$\hat{W}_{t|t-1} = \frac{1}{\kappa_4} \hat{W}_{t-1|t-1}. \quad (18)$$

where κ_3 and κ_4 are forgetting factors that define the law of motion of the parameters. We set these quantities to $\kappa_3 = \kappa_4 = 0.99$.⁷ The prediction step allows us to compute measurement equation prediction errors that are necessary inputs for the updating step and computed as

$$\hat{u}_t = x_t - \hat{x}_{t|t-1}, \quad (19)$$

$$\hat{\varepsilon}_t = z_t - \hat{z}_{t|t-1}. \quad (20)$$

Where $\hat{x}_{t|t-1} = z_t \Lambda_{t|t-1}$ and $\hat{z}_{t|t-1} = z_{t-1} \beta_{t|t-1}$. With missing data, we simple set to zero the error corresponding to variables missing at a given point in time t .

1.3 Update

Update each Λ_{it} for $i = 1, \dots, n$ and β_t

$$\Lambda_{it} \sim N(\Lambda_{it|t}, \Sigma_{ii,t|t}^\Lambda), \quad (21)$$

$$\beta_t \sim N(\beta_{t|t}, \Sigma_{t|t}^\beta). \quad (22)$$

The terms in (A16) are calculated as

$$\Lambda_{it|t} = \Lambda_{it|t-1} + \Sigma_{ii,t|t-1}^\Lambda z'_t (\hat{V}_t + z_t \Sigma_{ii,t|t-1}^\Lambda F'_t)^{-1} \hat{\varepsilon}_t, \quad (23)$$

$$\Sigma_{ii,t|t}^\Lambda = \Sigma_{ii,t|t-1}^\Lambda - \Sigma_{ii,t|t-1}^\Lambda z'_t (\hat{V}_t + z_t \Sigma_{ii,t|t-1}^\Lambda F'_t)^{-1} z_t \Sigma_{ii,t|t-1}^\Lambda. \quad (24)$$

Where the term $\Sigma_{ii,t|t-1}^\Lambda z'_t (\hat{V}_t + z_t \Sigma_{ii,t|t-1}^\Lambda F'_t)^{-1}$ is the Kalman Gain for each time period t and is set to zero for the variables missing at each step.

⁷In practice these two equations are approximations of $\hat{R}_{t|t-1} = \hat{R}_{t-1|t-1} + \hat{\eta}_{t-1} \hat{\eta}'_{t-1}$ and $\hat{W}_{t|t-1} = \hat{W}_{t-1|t-1} + \hat{v}_{t-1} \hat{v}'_{t-1}$ in the standard Kalman Filter (see [Koop and Korobilis \(2013\)](#) and [Raftery et al. \(2010\)](#))

The terms in (A17) are calculated as

$$\beta_{t|t} = \beta_{t|t-1} + \Sigma_{t|t-1}^\beta z_t' (\hat{Q}_t + z_{t-1} \Sigma_{t|t-1}^\beta z_{t-1}')^{-1} (z_t - z_{t-1} \hat{\beta}_{t-1}) \quad (25)$$

$$\Sigma_{t|t}^\beta = \Sigma_{t|t-1}^\beta - \Sigma_{t|t-1}^\beta z_t' (\hat{Q}_t + z_t \Sigma_{t|t-1}^\beta z_t')^{-1} z_t \Sigma_{t|t-1}^\beta \quad (26)$$

Where the term $\Sigma_{t|t-1}^\beta z_t' (\hat{Q}_t + z_{t-1} \Sigma_{t|t-1}^\beta z_{t-1}')^{-1}$ is the Kalman Gain for each time period t and is set to zero for the variables missing at each step.

The only outstanding terms that need defining are the measurement equation error covariance matrices that can be obtained using EWMA as follows

$$\hat{V}_t = \kappa_1 \hat{V}_{t-1} + (1 - \kappa_1) \hat{u}_t \hat{u}_t', \quad (27)$$

$$\hat{Q}_t = \kappa_2 \hat{Q}_{t-1} + (1 - \kappa_2) \hat{\varepsilon}_t \hat{\varepsilon}_t'. \quad (28)$$

where κ_1 and κ_2 are forgetting factors that define the law of motion of the idiosyncratic volatilities in the measurement equation and the volatilities of the observable variables and the factors in the state equation. We set these quantities to $\kappa_1 = \kappa_2 = 0.99$.

2. Kalman Smoother

The kalman filter algorithm described in (1.1)-(1.3) above works by forward recursion and outputs estimates of $E(\theta|D^t)$ for all parameters $\theta = [\hat{V}_t, \hat{Q}_t, \hat{R}_t, \hat{W}_t, \hat{\beta}_t, \hat{\Lambda}_t]$ in the model using the data available D^t for $t = 1, \dots, t$. However, we are ultimately interested in an estimate of $E(\theta|D^T)$ which yields the parameter states conditional on the entire sample $t = 1, \dots, T$. Therefore, the kalman smoother is applied to the output of the kalman filter working by backward recursion. Given that the Kalman filter takes into account missing data, no alteration is necessary to the Kalman smoother.

3. Kalman Smoother/filter for Factors

The full three step procedure described in section 3.2 is complete by applying the Kalman filter and smoother algorithm to the factors, which are initialized with a PCA estimate. The algorithm and mixed frequency extensions are analogous to that previously described.

A.2 PCA algorithms for Incomplete Data Environments

A.2.1 Least-Squares PCA in the presense of missing data

Several simple ways to deal with missing values in a classical LS PCA framework consist in setting missing values to zero or using an Expectation Maximization PCA (EM PCA). The later technique is used by [Stock and Watson \(2002b\)](#) and consists of an iterative procedure that alternates between imputing missing values in X (E-step) and applying standard PCA to the pseudo-balanced panel of data (M-step) until convergence is reached. To summarize the algorithm proceeds as follows:

- **E-step:** Reconstruct X_t by filling in its missing values:

$$X_t^* = \begin{cases} X_t & \text{for observed values} \\ \hat{\Lambda}^k \hat{F}_t^k & \text{for missing values.} \end{cases} \quad (29)$$

- **M-step:** Perform standard PCA by SVD on the infilled matrix X_t^* and obtain new values for $\{\hat{\Lambda}^k, \hat{F}_t^k\}$.

The algorithm alternates between the E-M stFCI until convergence is reached in which case new estimates for the parameters in iteration $k - 1$ do not improve the Least-squares minimization problem solved in iteration k .

A.2.2 Variation view of the Expectation Maximization (EM) algorithm for incomplete data environments

Consider the standard PC regression

$$X_t = \Lambda F_t + \xi_t, \quad \xi_t \sim N(0, v_x I). \quad (30)$$

Where $\theta = \{\Lambda, F_t, v_y\}$ are model parameters and a subset X_{mis} of the data matrix is missing and treated as hidden variables. The variational view of the EM algorithm (see [Neal and Hinton \(1999\)](#) and [Attias \(2000\)](#)) consists in minimizing the objective function

$$V(\theta, p(X_{mis})) = \int p(X_{mis}) \log \frac{p(X_{mis})}{p(X|\theta)} dX_{mis} = \quad (31)$$

$$\int p(X_{mis}) \log \frac{p(X_{mis})}{p(p(X_{mis}|\theta))} dX_{mis} - \log p(X_{obs}|\theta), \quad (32)$$

wrt the model parameters θ and the density over the missing data $p(X_{mis})$. X_{obs} denotes the observed data such that $X = X_{mis} \oplus X_{obs}$.

E-step. Equation A26 is the Kullback-Leibler divergence between the pdfs over observable and unobservable data. The minimization of this expression wrt $p(X_{mis})$, given θ is shown to yield

$$p(X_{mis}|\theta) = \prod_{ij \in O} N(\hat{x}(\theta)_{ij}, v_x). \quad (33)$$

where O is the set of indices for which observation x_{ij} is observed, $\hat{x}(\theta)_{ij}$ result from the reconstruction of the incomplete data matrix X from expression A24, for a given θ . This procedure is referred to as the E-step of the algorithm.

M-step. Next, the proceedings from the E-step are substituted back into expression (A26). The terms in the resulting expression which depend on θ are given by

$$- \int p(X_{mis}) \log p(X|\theta) dX_{mis}. \quad (34)$$

It can be shown that minimization of (A28) wrt. θ is equivalent to minimizing the LS objective function in case of no missing data ([Neal and Hinton, 1999](#)). Thus, the M-step of the

algorithm consists in performing SVD decomposition to the imputed data matrix X . The algorithm alternatives between the E-M stFCI until convergence is reached (ie, when the reconstruction error stabilizes).

A.2.3 Probabilistic PCA (PPCA)

A probabilistic PCA specification has been found to provide a good foundation to handle missing data [Ilin and Raiko \(2010\)](#). The probabilistic PCA set forth by [Tipping and Bishop \(1999b\)](#) can be written as follows

$$X_t = m + \Lambda F_t + \xi_t, \quad (35)$$

where both the principal component and the noise term are assumed normally distributed as follows

$$p(F_t) \sim N(0, I_K), \quad (36)$$

$$p(\xi_t) \sim N(0, \tau^{-1} I_N), \quad (37)$$

where $\theta = \{m, \Lambda, \tau\}$ are model parameters, I_N and I_T denote identity matrices and τ is the scalar inverse variance of ξ . It can be shown that the Maximum Likelihood (ML) estimation of the PPCA is identical to PCA in the case of non-missing data. The great advantage of the PPCA is that, in case of incomplete data, it allows for regularization that arises naturally from the choice of Gaussian priors. The model can then be estimation with a standard EM algorithm. The necessary extensions to handle missing data are discussed in [Ilin and Raiko \(2010\)](#). Below I summarize their procedure.

E-step. Estimate the conditional distribution of the hidden variables F given the data X and model parameters θ ,

$$p(F|X, \theta) = \prod_{j=1}^K N(\bar{F}_j, \Sigma_{F_j}), \quad (38)$$

based on the following updating rules

$$\Sigma_{F_j} = \tau^{-1}(\tau^{-1} I + \sum_{i \in O_j} \lambda_i \lambda_i^T)^{-1}, \quad (39)$$

$$\bar{F}_j = \tau \Sigma_{F_j} \sum_{i \in O_j} \lambda_i (x_{ij} - m_i), j = 1, \dots, K, \quad (40)$$

$$m_i = \frac{1}{|O_i|} \sum_{j \in O_i} (x_{ij} - \lambda_i^T \bar{F}_j). \quad (41)$$

M-step. re-estimate the model parameters as

$$\lambda_i = \left(\sum_{j \in O_i} \bar{F}_j \bar{F}_j^T + \Sigma_{F_j} \right)^{-1} \sum_{j \in O_i} \bar{F}_j (x_{ij} - m_i), i = 1, \dots, N, \quad (42)$$

$$\tau = \left[\frac{1}{N} \sum_{ij \in O} [x_{ij} - \lambda_i^T \bar{x}_j - m_i]^2 + \lambda_i^T \Sigma_{F_j} \lambda_i \right]^{-1}. \quad (43)$$

where O_i, O_j and O denote the set of indices i, j for which x_{ij} is observed.

A.2.4 Variational Bayesian PCA (VBPCA)

Some studies suggest that the standard PPCA is still vulnerable to over-fitting (see for example [Ilin and Raiko \(2010\)](#)). One possible reason that might lead to overfitted solution might be the nontrivial choice of the number of principal components to include in (A30). Including a large number of common components F_t might cause the model to over-learn the data.

One possible solution to this problem consists in penalizing large values in the matrices Λ and F_t . The probabilistic pca model is flexible enough to allow for an automatic, data-driven selection of relevant common components by shrinking to zero the solutions λ_j that are small relative to the noise variance. This can be achieved through a variational bayesian PCA algorithm as explained below. We follow [Oba et al. \(2003\)](#) in imposing additional regularization to penalize parameter values that yield more complex explanations of the data. Hence, in addition to (A31)-(A32) one can further impose

$$p(m|\tau) \sim N(0, (\gamma_{m0}\tau)^{-1}I_T), \quad (44)$$

$$p(\lambda_j|\tau, \alpha_j) \sim N(0, (\alpha_j\tau)^{-1}I_T), \quad (45)$$

$$p(\tau) \sim G(\tau|\bar{\tau}_0, \gamma_{\tau_0}), \quad (46)$$

where $\psi = \{\gamma_{m0}, \gamma_{\tau_0}, \bar{\tau}_0, \alpha_j\}$ are hyperparameters and λ_j are the parameters in column j of the loadings matrix Λ that define the importance of each principal component F_j , $j = \{1, 2, \dots, K\}$. The prior $p(\Lambda|\alpha, \tau)$, which has a hierarchical structure, is called an automatic relevance determination (ARD) prior. This structure plays a key role in guaranteeing parsimony of the model. Its variance $(\alpha_j\tau)^{-1}$ is determined by a hyperparameter α_j that becomes large when the euclidean distance $\|\lambda_j\|$ is small relative to the noise variance τ^{-1} .

Estimation of the model now requires a variational EM algorithm as proposed by [Attias \(2000\)](#) to cope with the unknown analytical form of the posterior of the parameters $p(\Lambda, F_t, m|X, \psi)$, which invalidates the E-step of the standard EM algorithm. To overcome this difficulty, the author proposes an approximation to this quantity by a simpler $q(\Lambda, F_t, m)$. Using a variational approach, the E-step is modified such that the objective function approximates $p(\theta|X)$ with a simpler pdf $p(\theta)$, written as follows

$$V(p(\theta), \psi) = \int p(\theta) \log \frac{p(\theta)}{p(X, \theta|\psi)} d\theta = \quad (47)$$

$$\int p(\theta) \log \frac{p(\theta)}{p(X, \theta|\psi)} d\theta - \log p(X|\psi), \quad (48)$$

E-step. Equation (A33) is the Kullback-Leibler divergence between the true posterior and its approximation. In this step the approximation $p(\theta)$ is updated. This corresponds to minimizing this distance wrt $p(\theta)$.

M-step. Next, the approximation $p(\theta)$ is used as if it was the actual posterior $p(\theta|X, \psi)$ in order to increase $p(X|\psi)$. This consists in deriving the expression (A33) wrt ψ .

The algorithm alternatives between the E-M stFCI until convergence is reached (i.e. when the reconstruction error stabilizes).

B Additional Tables and Figures

#	mnemonic	description	frequency	Sample Start	t-code	Source
1	GDPC1	Real Gross Domestic Product SA. Annual Rate	Q	1971-01-01	5	St. Louis FRED
2	CPIAUCSL	Consumer Price Index for All Urban Consumers	M	1971-01-01	5	St. Louis FRED
3	UNRATE	Unemployment Rate, Percent, SA.	M	1971-01-01	5	St. Louis FRED
4	DFE	Effective Federal Funds Rate	D	1971-01-01	1	St. Louis FRED
5	CMDEBT	Households and Nonprofit Organizations Debt % GDP	Q	1971-01-01	5	St. Louis FRED
6	ABSTICMAHDFS	Issuers of Asset-Backed Securities % GDP	Q	1983-07-01	5	St. Louis FRED
7	DRTSCILM	Net Percentage of Domestic Banks Tightening Standards for Commercial and Industrial Loans	Q	1990-04-01	1	St. Louis FRED
8	TERMCBAUTO48NS	Finance Rate on Consumer Installment Loans at Commercial Banks, New Autos 48 Month Loan, Percent	Q	1972-01-01	5	St. Louis FRED
9	TDSP	Household Debt Service Payments as a Percent of Disposable Personal Income	Q	1980-01-01	1	St. Louis FRED
10	TOTALSL	Total Consumer Credit Owned and Securitized % GDP	M	1971-01-01	1	St. Louis FRED
11	LOANHPI	Loan Performance Index U.S.	M	1976-03-01	5	Bloomberg
12	CONSEXF	UMich Expected Change in Financial conditions	M	1978-02-01	1	Uni Michigan
13	BPLR	Bank Prime Loan Rate / Libor spread	M	1971-01-01	1	St. Louis FRED
14	JPMNEER	JPMorgan Broad Nominal Effective Exchange Rate (2010=100)	M	1971-01-01	5	Bloomberg
15	LDR	All Commercial Banks Loan to Deposit Ratio	M	1973-01-01	1	Haver Analytics
16	2/3TBS	2yr/3m Treasury bill spread	M	1976-06-01	1	St. Louis FRED
17	MORTGAGE30US	Mortgage rate / 10yr Treasury Bill spread	W	1971-04-02	1	St. Louis FRED
18	T10Y2Y	10-Year Minus 2-Year Treasury Constant Maturity yield, Percent	D	1976-06-01	1	St. Louis FRED
19	BAMLH0A0HYM2EY	ICE BofAML US High Yield Master II Effective Yield, Percent	D	1996-12-31	1	Bloomberg
20	MOVE Index	Yield curve weighted index of the normalized implied volatility on 1-month Treasury options.	D	1988-04-04	1	Bloomberg
21	CRY Index	Thomson Reuters/CoreCommodity CRB Commodity Index	D	1994-01-03	1	Bloomberg
22	VXOVIX	Cboe S&P 100/500 Volatility Index	D	1990-01-02	1	St. Louis FRED
23	BASPTDSP	Ted Spread	D	2001-01-02	1	St. Louis FRED
24	WILL5000PRFC	Wilshire 5000 Full Cap Price Index	D	1971-01-01	5	St. Louis FRED
25	CPFF	3-Month Commercial Paper Minus Federal Funds Rate, Percent, Daily, Not Seasonally Adjusted	D	1997-01-02	1	St. Louis FRED
26	SP500	S&P 500 price index	D	1971-01-01	5	St. Louis FRED

Notes: Mnemonic refers to the statistical reference with which the time series can be fetched from the source. Frequency is either Q: quarterly, M: monthly, W: weekly or D: Daily. Sample Start date refers to the first observation for a specific time-series in our sample period 1971-2018. t-code refers to transformation applied to each variable. 1:levels; 5: log-differences.

Table B.1: Description of the data used to construct the Financial Conditions Index.

Figure B.1: FCI calculated with different signal extraction methods within the FAVAR. The four methods include the PCA - simple Principle Components, EM-PCA which stands for Expectation Maximization PCA, PPCA - Probabilistic PCA and VBPCA - Variational Bayes PCA.

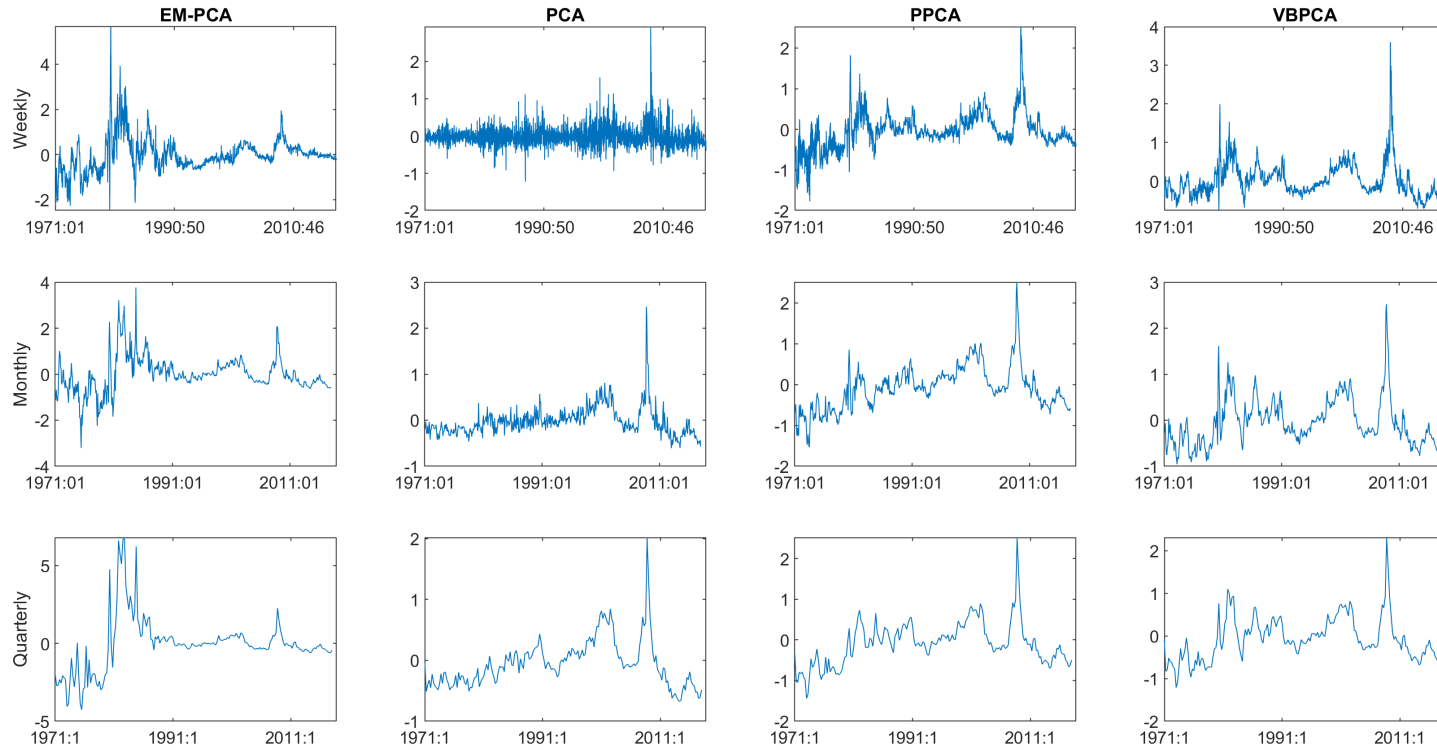
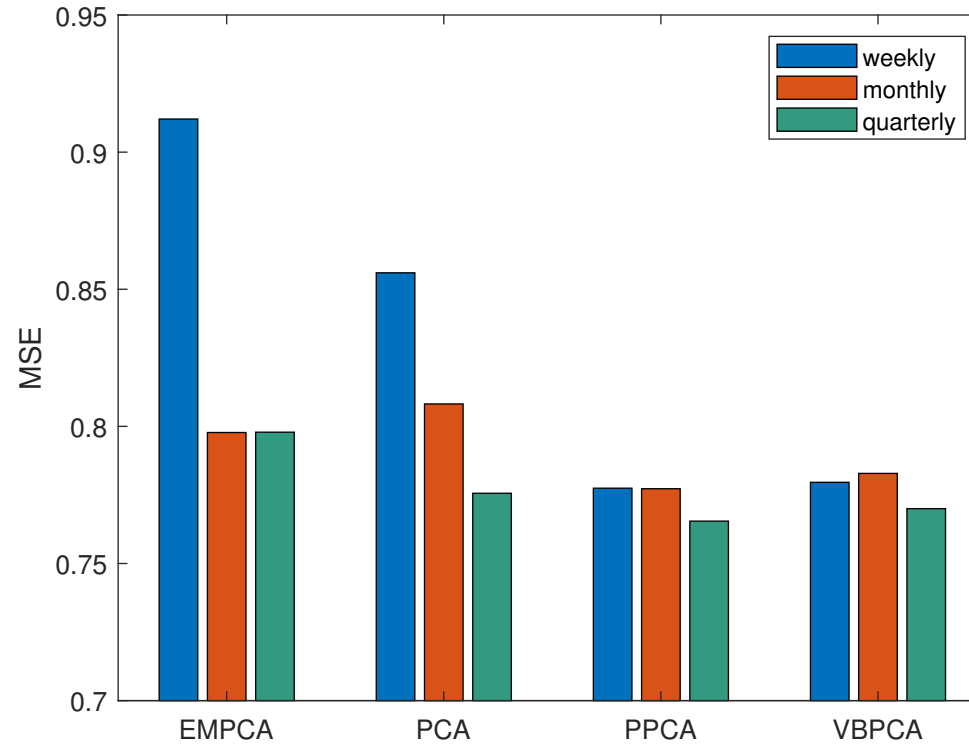


Figure B.2: Reconstruction Mean Squared Error across four alternative PCA methods



Notes: Mean-squared error statistics are calculated in-sample $MSE = \frac{1}{N} \sum_{ij \in O} (x_{ij} - \hat{x}_{ij})^2$, where O includes all indices for which x_{ij} is observed and \hat{x}_{ij} results from the projection of X_t on the factors F_t in equation 2.1, permitting the reconstruction of the incomplete data set of financial variables that loan onto the FCI. for the observable section of the standardized unbalanced panel of financial indicators across the different signal extraction methods.