

Growth Fragility and Systemic Risk under Model Uncertainty

Abstract

The link between systemic risk and economic growth is hard to study because the relationship is believed to be nonlinear and systemic risk is unobservable. The myriad of measures proposed in the literature add model uncertainty as an additional difficulty. We use a Bayesian quantile regression to study the relevance of 33 systemic risk indicators to explain lower quantiles of output growth. Model uncertainty is tackled with sparse-modelling techniques that perform both model selection and shrinkage. Less than halve of the indicators considered are selected to explain lower tails of economic growth.

JEL classification: G10, E37, C11, C32, C58.

Keywords: Quantile Regression, Systemic Risk, Growth Fragility, Forecasting.

1 Introduction

The global financial crisis that started in 2007 has brought systemic risk to the forefront of the research agenda of academics and policymakers. On the empirical side, the literature has focused on measuring systemic risk in an effort to provide indicators of financial imbalances in the economy, hoping that these might serve as early warnings of forthcoming recessions. This has led to the proposal of a myriad of indicators, aimed at capturing different dimensions of systemic risk. [Billio et al. \(2012\)](#) and [Bisias et al. \(2012a\)](#) survey over 30 such indicators, many more have been proposed since. The staggering amount of systemic risk indicators proposed in the literature owes to the fact that the definition of systemic risk is rather diffuse. As a result, different measures capture different dimensions of the concept. Nevertheless, a synthesis and working definition is provided by [Peydro et al. \(2015\)](#), that introduces systemic risk as *the risk of threats to financial stability that impair the functioning of a large part of the financial system with significant adverse effects on the broader economy*¹. A key element that distinguishes a systemic event from otherwise systemically unimportant shocks, that are constantly hitting the financial system without resulting in a crisis, is its macroeconomic relevance. Whether an episode is classified as systemic therefore depends on its impact on welfare and real activity². Overall, systemic risk indicators are expected to signal downside risks to real activity and reflect the likelihood of a recession.

This paper studies the relationship between output growth and systemic risk through the lens of a Bayesian quantile regression model. As opposed to the common approach in the literature, sparse-modelling techniques are used to address model uncertainty concerns, allowing for both model selection and shrinkage. We address two major challenges that complicate the study of this relationship. First, systemic risk and real activity are thought to relate in a nonlinear way (see for example [Brunnermeier and Sannikov \(2014\)](#); [Christiano et al. \(2014\)](#) and [Adrian et al. \(2019\)](#)). Second, any attempt to empirically model the link between systemic risk and real economic activity must deal with inherent model uncertainty plaguing such an exercise. On one hand, there are multiple dimensions of systemic risk and it is unclear which dimension is relevant in explaining each quantile of macroeconomic fluctuations. On the other hand, measurement error associated with each systemic risk indicator might mean some indicators are too noisy and uninformative. In practice, the researcher is forced to choose a single set of regressors within the possible 2^K candidate models, supposing there are K different systemic risk measures from which to choose. [Adrian et al. \(2019\)](#) and [Giglio et al. \(2016\)](#) deal with the large number of possible predictors by employing dense modelling techniques. The financial conditions index used by [Adrian et al. \(2019\)](#) can be viewed a linear combination of a large set of financial variables. They find that systemic risk skews the distribution of real activity, shifting its left tail outwards. Whereas, [Giglio et al. \(2016\)](#) explicitly uses principal component analysis to shrink the space of potential

¹This definition is broadly consistent with that proposed by the IMF, FSB, ECB and BIS (see [International Monetary Fund \(2009\)](#); [Hartmann et al. \(2009\)](#); [Caruana \(2010\)](#)).

²[Laeven and Valencia \(2013\)](#) compile a comprehensive dataset of cross-country banking crisis from 1970 to 2011 and identify 147 systemically important banking crisis that result in an average output loss of 23 per cent.

predictors within a quantile regression. The authors examine the predictive content of 19 different systemic risk indicators in explaining the lower tail of the predictive distribution of output, one year ahead.

Principal Component Analysis has proven effective in reducing model dimensionality and is widely used in the literature³. It is based on the principle that all potential predictors might be relevant and thus shrinks the information content in all variables into a few factors that explain the maximum amount of variation in the pool of regressors. The main disadvantage of this framework has to do with interpretability. Although shrinking the information content of a large pool of systemic risk indicators into a few factors is effective in dealing with model uncertainty and avoiding the curse of dimensionality, it is unable to inform which systemic risk indicators signal risks to output growth, since that information is lost in the shrinkage process. Instead, we examine the relevance of 33 of the most popular systemic risk indicators proposed in the literature in light of two sparse-modelling methods that deal with model uncertainty. We specify a *Stochastic Search Variable Selection* (SSVS) prior following the original proposal of [Mitchell and Beauchamp \(1988\)](#) and [Korobilis \(2017\)](#) in a quantile regression framework. To understand the sensitivity of our results to the degree of shrinkage imposed, we re-run the model with a Bayesian Lasso prior suggested by [Park and Casella \(2008\)](#) and a Ridge prior type following [Giannone et al. \(2017\)](#).

Our approach has several advantages that allow us to study new questions that have not been addressed in the literature so far. First, by employing quantile regression methods, we explore the nonlinear nature of the relationship between output growth and systemic risk. Quantile regressions yield complex predictive distributions that need not be symmetric or unimodal. Our approach is flexible enough to examine how the link between the variables changes across quantiles. Second, we assess the empirical relevance of 33 systemic risk measures in a common setting, avoiding over-fitting and model dimensionality constraints. Third, we perform in-sample and out-of-sample analysis through a recursive forecasting exercise to understand if systemic risk measures carry relevant information content for predicting real activity at each point in time.

From an econometric point of view, the novelty of this work lies in the variable selection and shrinkage algorithms, that select the most relevant systemic risk indicators in explaining each quantile of output growth in a data-driven way. We show how the SSVS, Lasso and Ridge Quantile Regressions can be obtained in a common setting with very few changes to the model specification. In addition, we build on [Kozumi and Kobayashi \(2011\)](#) and [Korobilis \(2017\)](#) and extend the Bayesian Lasso to allow for data-dependent shrinkage in a quantile regression environment.

The exercise offers insights on the relevance of financial variables in macroeconomic prediction. A number of papers have documented the unpredictability of GDP growth during the Great Moderation (see eg. [D’Agostino et al. \(2006\)](#); [Rossi and Sekhposyan \(2010\)](#)) and the fragile and unreliable predictive content of financial indicators ([Stock and Watson \(2003\)](#)). [Giglio et al. \(2016\)](#) finds that few systemic risk measures possess significant predictive content for downside quantiles

³See for example [Bernanke et al. \(2005\)](#); [Koop \(2013\)](#); [Boivin et al. \(2018\)](#); [Bok et al. \(2018\)](#) and [Stock and Watson \(2016a\)](#) for a review.

of macroeconomic shocks. Two main results emerge from our exercise and contribute to this strand of the literature. First, less than half of the indicators considered are selected to explain lower tails of economic growth. Systemic Risk indicators are expected to signal macroeconomic risks and predict growth fragility. However, the number of indicators selected as relevant predictors of lower tails of GDP are no more than those selected as relevant covariates of middle and upper quantiles. From the 33 systemic risk measures assessed, the term spread, size concentration of the financial sector, the Cyclical Adjusted Price-to-earnings ratio (CAPE) and the Debt Service Ratio are the most relevant predictors, selected by the model in a data-driven way as containing relevant information to forecast GDP growth across quantiles, forecast horizons and shrinkage methods for the full sample. Second, in the out-of-sample forecasting exercise we carry out, we do not find evidence that systemic risk indicators improve economic growth forecasting.

The remainder of the paper proceeds as follows. Section 2 presents the data and section 3 explains the econometric framework, estimation technique, variable selection and shrinkage procedure. Section 4 discusses the main results and findings. Section 5 concludes.

2 Data

An overwhelming quantity of systemic risk measures have been proposed in the literature. We restrict our attention to those for which data is available, relying on the surveys of [Bisias et al. \(2012b\)](#) and [Giglio et al. \(2016\)](#). In addition to the indicators considered by the aforementioned authors, we study a number of related indicators that were not considered in their articles but are relevant to our analysis.

Table 1: Measures of systemic risk considered and respective sample dates.

#	Systemic Risk indicator	Sample	Reference
1	absorption	1947-2011	Kritzman et al. (2011)
2	Delta Absortion	1947-2011	Kritzman et al. (2011)
3	AIM	1947-2011	Amihud (2002)
4	CatFin	1947-2011	Allen et al. (2012)
5	GZ spread	1973-2011	Simon Gilchrist et al. (2012)
6	Baa/Aaa Bond yield	1947-2011	Giglio et al. (2016)
7	TED spread	1984-2011	Stock and Watson (2003)
8	Term Spread	1947-2011	Stock and Watson (2003)
9	Baa/10-yr T-rate spread	1962-2011	Stock and Watson (2003)
10	Mortg-GS10 Spread	1971-2011	Stock and Watson (2016b)
11	Comm. paper-3mT-Bill spread	1959-2011	Stock and Watson (2016b)
12	Excess Bond Premium	1973-2011	Simon Gilchrist et al. (2012)
13	Intl. Spillover	1963-2011	Diebold and Yilmaz (2011)
14	CoVaR	1947-2011	Adrian and Brunnermeier (2016)
15	Delta CoVaR	1947-2011	Adrian and Brunnermeier (2016)
16	Book lvg.	1969-2011	Giglio et al. (2016)
17	Mkt. Lvg.	1969-2011	Giglio et al. (2016)
18	DCI	1947-2011	Billio et al. (2012)
19	MES	1947-2011	Acharya et al. (2017)
20	MES-BE	1947-2011	Brownlees and Engle (2012)
21	Volatility	1947-2011	Giglio et al. (2016)
22	Size conc.	1947-2011	Giglio et al. (2016)
23	Turbulence	1947-2011	Kritzman and Li (2010)
24	PQR	1947-2011	Giglio et al. (2016)
25	Average DD.	2008-2011	Saldias (2013)
26	portfolio DD.	2008-2011	Saldias (2013)
27	MRI CITI Index	1997-2011	Adrian et al. (2010)
28	CAPE	1947-2011	Shiller (2005)
29	VXO	1962-2011	Bloom (2009)
30	Sent. Index	1965-2011	Baker and Wurgler (2006)
31	Credit-to-gdp gap	1962-2011	Aldasoro et al. (2018)
32	Debt Service Ratio	1999-2011	Aldasoro et al. (2018)
33	Loan Supply	1990-2011	Lown and Morgan (2006)

The table above summarizes all measures that we consider as potential predictors of downturns in economic activity, the sample period for which they are available and respective references. A more detailed explanation of the data, transformations and sources used is provided in the Appendix. In what follows we explain the econometric approach we adopt to investigate the link between the 33 measures described and different quantiles of economic growth.

3 Bayesian Quantile Regression

We wish to explain the τ th quantile of output growth h-steps ahead, denoted by y_{t+h} , by regressing this time series on a set of explanatory variables that include systemic risk indicators and also relevant own lags of real activity, organized in a matrix x_t of dimensions $T \times K$, where T is the time dimension and K the number of regressors. Analytically, the τ th quantile of y_{t+h} is given by its inverse probability

distribution function denoted

$$\mathbb{Q}_\tau(y_{t+h}) = \inf\{y : P(y_{t+h} \leq y) \geq \tau\}. \quad (1)$$

The quantile function can be expressed as the solution of the minimization problem

$$\mathbb{Q}_\tau(y_{t+h}) = \min_q \mathbb{E}(\rho_\tau(y_{t+h} - q)), \quad (2)$$

where $\rho_\tau(u) = u(\tau - I(u < 0))$ is referred to in the literature as the quantile loss function. In the seminal paper by [Koenker and Bassett \(1978\)](#) the conditional quantiles of y_{t+h} are expressed as functions of the set of observables in a form similar to the following equation

$$\mathbb{Q}_\tau(y_{t+h}|I_t) = x'_t \beta_\tau + \varepsilon_t. \quad (3)$$

The main advantage of quantile regression is that the coefficients β_τ are allowed to vary across quantiles τ , capturing non-linear dynamics between real activity and systemic risk as prescribed by theory. It also gives a richer picture about the uncertainty surrounding point forecasts and how such uncertainty depends on measures of systemic stress. We are particularly interested in lower quantiles of real activity that depict economic downturns.

Estimation is trivial in a classical framework and proceeds by solving the optimization routine specified in (2). However, it is advantageous to formalize the model in a bayesian setting to address our concerns over model uncertainty and to deal with the large number of predictors that have been found, in different contexts, to result in in-sample overfitting due to the proliferation of parameters (see [Stock and Watson \(2006\)](#) and [Koop and Korobilis \(2011\)](#)). The application of Bayesian methods to equation (3) is however, not straightforward *a priori* because the innovations ε_t are not normally distributed. Indeed, [Yu and Moyeed \(2001\)](#) show that the solution to (2) is equivalent to the maximization of a likelihood function under the asymmetric Laplace error distribution. Because the asymmetric Laplace distribution can be represented as a scale mixture of normals (see [Kotz et al. \(2001\)](#)), the quantile regression (3) may be respecified such that Gibbs Sampling methods can be applied. Following [Kozumi and Kobayashi \(2011\)](#) we rewrite the error distribution as

$$\varepsilon_t = \theta z_t + \phi \sqrt{z_t} u_t, \quad (4)$$

where $z_t \sim \text{Exp}(1)$ and u_t is a standard normal distribution. $\theta = (1 - 2\tau)/\tau(1 - \tau)$ and $\phi = 2/\tau(1 - \tau)$, for a given quantile $\tau \in [0, 1]$. By plugging expression 4 into equation 3 we obtain a new quantile regression that can be estimated with Bayesian methods

$$\mathbb{Q}_\tau(y_{t+h}|I_t) = x'_t \beta_\tau + \theta z_t + \phi \sqrt{z_t} u_t. \quad (5)$$

In the next section we show that the choice of specific priors allow for shrinkage and model selection in regression (5).

3.1 Model Selection and Shrinkage

The first step in Bayesian inference is the specification of prior distributions for all relevant parameters. Priors allow the researcher to include additional relevant information in the analysis. In this exercise, we use priors as a device to address our concerns over both model uncertainty and in ensuring a parsimonious representation of the relationship between systemic risk and real activity.

Model uncertainty arises from the large number of systemic risk indicators that aim at approximating different aspects of systemic risk. Understanding which particular indicator merits inclusion in the regression to explain growth fragility is unclear *a priori* but important for two main reasons. First, it is essential to understand from which part of the financial system risks are originating. Secondly, from a statistical viewpoint, it is necessary to assess the predictive content of each systemic risk measure to understand its suitability as an early warning indicator of a recession.

Describing the relation between systemic risk and real activity in a parsimonious way is motivated by statistical reasons - to avoid overfitting and unnecessary parameter proliferation (see [Stock and Watson \(2006\)](#) and [Koop and Korobilis \(2011\)](#)); but it is also justified on economic grounds - [Giglio et al. \(2016\)](#) suggests that a small subset of systemic risk indicators are relevant in describing the interaction between systemic risk and output. We specify the following priors

$$\beta_\tau | \gamma_\tau, \delta_\tau \sim N(0, \gamma_\tau \delta_\tau^2), \quad (6)$$

$$\delta_\tau^{-2} \sim \text{Gamma}(a_1, a_2). \quad (7)$$

This multi-level prior specification where β_τ is conditionally normal, allows for automatic shrinkage and model selection through the parameter γ_τ , that shrink β_τ to zero for any coefficient delivering poor fit. This setting offers several possibilities to impose sparsity in the model. In what follows we review three such alternatives that are nested in this framework.

3.1.1 Stochastic Search Variable Selection

Similar to the Stochastic Search Variable Selection (SSVS) prior originally proposed by [Mitchell and Beauchamp \(1988\)](#), in our setting each coefficient in β_τ takes non-zero values with probability π_0 . We refer to these hyperparameters as the probabilities of inclusion. Our approach follows that of [George and McCulloch \(1993\)](#) and [Korobilis \(2013\)](#) in the sense that our prior for the coefficients is conditionally Gaussian albeit in the context of quantile regression closer to the work of [Korobilis \(2017\)](#). Formally, this prior forms a hierarchical structure that extends (6) and (7) by adding the following hyperpriors to that setting

$$\gamma_\tau | \pi_0 \sim \text{Bernoulli}(\pi_0), \quad (8)$$

$$\pi_0 \sim \text{Beta}(b_0, b_1). \quad (9)$$

Because γ_τ is a binomial variable vector, only the systemic risk indicators with the highest predictive power will be included in the regression. Moreover, the probability of inclusion of each indicator defined by π_0 is also random, thus controlling the degree of shrinkage automatically. Hence, if $\gamma_{i,\tau} = 0$, the parameter $\beta_{i,\tau}$ is shrank to zero.

Whereas, if $\gamma_{i,\tau} = 1$, the prior for $\beta_{i,\tau}$ will follow a normal distribution centered in zero. On the other hand, since γ_τ is estimated from the data, within a standard Gibbs Sampler, examining the posterior of γ_τ will inform which variables are most relevant in explaining each quantile of real activity.

3.1.2 Bayesian Lasso

An alternative method capable of selecting relevant variables in a linear regression framework has been proposed by Tibshirani (1996) and is widely known as Least Absolute Shrinkage and Selection Operator (Lasso). The Lasso is part of a wider class of penalized regression models that work by adding a penalty term to the objective function from which the coefficient estimates derive, and has been shown to be effective in quantile regression (see Wu and Liu (2009); Li and Zhu (2008))⁴. The Lasso owes its name to the form of penalty imposed. It works by adding an L1-norm regularizer on the prediction weights standing for the absolute value of magnitude of the coefficients.

The Lasso regression estimates can be given a Bayesian interpretation as it has been shown that, for specific choices of priors, the mean or mode of the posterior distribution of the parameters are equivalent to penalized regression results. Park and Casella (2008) show that the Lasso estimate can be interpreted as a Bayesian posterior mode estimate when the parameters have independent Laplace priors. Moreover, the authors show that the Laplace distribution can be written as a scale mixture of normals with an exponential mixing density. Hence, the Bayesian Lasso can be obtained by specifying a hierarchical structure that extends (6) and (7) by adding the following hyperpriors to that setting

$$\gamma_\tau | \lambda \sim \mathcal{E}(2/\lambda^2), \quad (10)$$

$$\lambda^2 \sim \text{Gamma}(c_0, c_1). \quad (11)$$

Where $\mathcal{E}(\cdot)$ denotes an exponential distribution with mean $2/\lambda^2$. The Normal-Inverse Gamma prior layer in (6)-(7) remains valid, except that in (6), the prior variance should be changed to $\Omega = \text{diag}(\gamma_\tau)$ ⁵.

Contrary to its frequentist sibling, the Bayesian Lasso in our framework allows for the automatic choice of the degree of shrinkage λ . In our application the degree of shrinkage is driven by data since λ is random and has its own posterior density. The main idea underlying these prior choices is similar. We wish to shrink nuisance parameters and sparsify the model such that a clearer pattern of the most important systemic risk indicators might emerge.

⁴It works by solving the following optimization problem, $\beta^L = \text{argmin}_\beta \rho_\tau \varepsilon' \varepsilon + \lambda \|\beta\|_1$, where $\|\beta\|_1 = \sum_{j=1}^p \beta_j$ and λ controls the amount of regularization, that ensures shrinkage towards zero and prevents overfitting (see Kapetanios et al. (2018) for a review of penalised regression techniques in a linear regression setting and Li et al. (2010) that discusses variable selection and shrinkage in a Bayesian quantile regression setting.)

⁵see Kozumi and Kobayashi (2011) section 3.2.

3.1.3 Bayesian Ridge

The Ridge estimator is another special case of the penalized regression where an L2-norm regularizer is added to the objective function of the estimation problem⁶. The Ridge regression was first introduced by [Hoerl and Kennard \(1970\)](#) and the main idea resembles the previous methods discussed. Similar to the Lasso, the Ridge regression also has a Bayesian analogue, that can be obtained simply by specifying Normal-Inverse Gamma priors for the regression parameters such as in (6) and (7). Hence, if no other hyperpriors are added to the primary specification defined by (6) and (7), or if we set $\pi_0 = 1$ in the SSVS setting, the Ridge regression emerges by default (see [Kapetanios et al. \(2018\)](#) and [Giannone et al. \(2017\)](#)).

3.2 Estimation

The parameters in (5) can be estimated with a Gibbs Sampler since the respective likelihood function is conditionally Gaussian written as

$$f(\mathbf{y}|\beta_\tau, \mathbf{z}) \propto \prod_{i=1}^T z_t^{-1/2} \times \exp\left\{-\frac{1}{2} \sum_{i=1}^T \frac{(y_t - x_t' \beta_\tau - \theta z_t)^2}{\phi^2 z_t}\right\}, \quad (12)$$

with $\mathbf{y} = (y_1, \dots, y_T)'$ and $\mathbf{z} = (z_1, \dots, z_T)'$. A full characterization of a Gibbs Sampler pass-through in our exercise is provided in Appendix B. The algorithm builds on the procedure suggested by [Korobilis \(2017\)](#) using different priors in order to implement the three variable selection techniques discussed. In Appendix D we assess the convergence of the Gibbs Sampler employed.

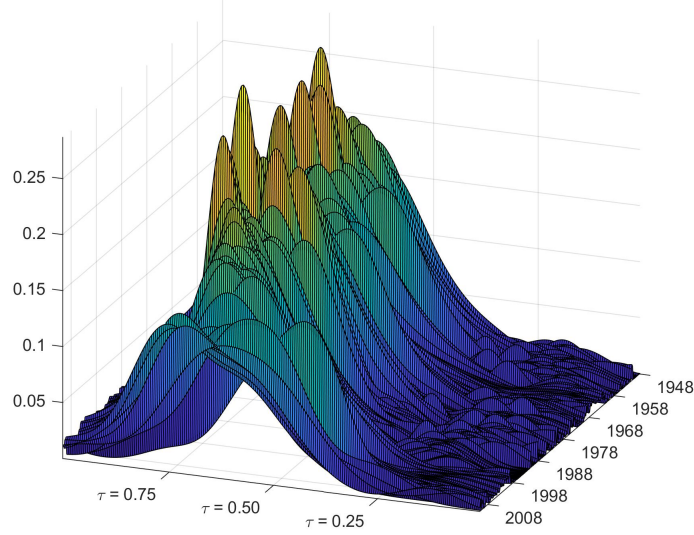
4 Discussion of the Results

In this section we discuss the results that emerge from the estimation of the quantile regression which yields a predictive distribution of GDP growth, conditional on the set of systemic risk indicators considered. We focus on two main questions. First, which systemic risk indicators explain in-sample the variation of different quantiles of real activity and in particular lower quantiles, that are associated with recessions. Second, we assess the out-of-sample performance of the quantile regression estimated with the SSVS prior, which is our baseline specification.

An interesting exercise that quantile regressions allow for is to examine the full predictive distribution of GDP growth. Unlike normal linear models, quantile regressions yield complex predictive distributions that need not be symmetric. We are particularly interested in the shape of its lower tails, since they reflect the likelihood of economic downturns. Figure 1 shows the predictive distribution of GDP growth, one-year ahead, conditional on systemic risk, which we call growth surface. It results from the estimation of our baseline quantile regression specification (5) for a grid of quantiles $\tau = \{0.05, \dots, 0.95\}$ for the full sample period.

⁶The Ridge estimator is found by solving $\beta^R = \operatorname{argmin}_\beta \rho_\tau \varepsilon' \varepsilon + \lambda \|\beta\|_2$ with $\|\beta\|_1 = \sum_{j=1}^p \beta_j^2$.

Figure 1: Growth Surface. Predictive density of GDP growth, conditional on Systemic Risk.



Notes: The growth surface results from the estimation of the Quantile Regression with the baseline SSVS prior for each quantile $\tau = \{0.05, \dots, 0.95\}$ and interpolated with a gaussian kernel.

The figure suggests the predictive distribution of GDP growth is left-skewed, a finding that is not new and consistent with the results discussed by [Adrian et al. \(2019\)](#). Only a closer look at the coefficient estimates can inform about which Systemic Risk indicators contain relevant information in predicting GDP growth. We examine the indicators picked as relevant predictors of the full spectrum of the GDP growth distribution by the different shrinkage algorithms. We start by discussing estimation results for the quantile regression with the SSVS prior which is our benchmark choice for implementing variable selection. It enforces sparsity in the model because the posterior distribution of the coefficients inherits the priors' shape, with a considerable probability mass in zero.

Table 2: Systemic Risk indicators selected as relevant in explaining different quantiles of the predictive distribution of GDP growth at different horizons.

#	Systemic risk indicator	predictors at horizon $h = 1$			predictors at horizon $h = 4$			predictors at horizon $h = 8$		
		$\tau = 0.2$	$\tau = 0.5$	$\tau = 0.8$	$\tau = 0.2$	$\tau = 0.5$	$\tau = 0.8$	$\tau = 0.2$	$\tau = 0.5$	$\tau = 0.8$
1	absorption									
2	Delta Absortion									
3	AIM									
4	CatFin									
5	GZ spread									
6	Baa/Aaa Bond yield									
7	TED spread									
8	Term Spread			•	•	•	•	•	•	•
9	Baa/10-yr T-rate spread									
10	Mortg-GS10 Spread			•			•			
11	Comm. paper-3mT-Bill spread	•		•			•			
12	Excess Bond Premium									
13	Intl. Spillover									
14	CoVaR									
15	Delta CoVaR			•						
16	Book lvg.	•								
17	Mkt. Lvg.	•								
18	DCI									
19	MES									
20	MES-BE									
21	Volatility									
22	Size conc.		•	•	•	•	•		•	•
23	Turbulence									
24	PQR									
25	Average DD.									
26	portfolio DD.									
27	MRI CITI Index									
28	CAPE	•	•	•	•	•	•			
29	VXO			•			•			
30	Sent. Index									
31	Credit-to-gdp gap									
32	Debt Service Ratio	•		•	•	•	•	•	•	•
33	Loan Supply									

Notes: The dots in the table refer to the indicators for which the associated coefficient $\beta_{i\tau}$ estimate (ie, its posterior median) is different from zero and therefore selected for inclusion in the quantile regressions for $\tau = \{0.20, 0.5, 0.80\}$, and horizons $h = \{1, 4, 8\}$ in quarters. The regression includes a constant, two own lags of GDP growth and the Chicago Fed National Activity Index (CFNAI) which are not subject to variable selection and therefore always feature in the regressions and are not reported.

Table 2 shows the systemic risk indicators selected, from the set of all measures considered, for a given quantile τ and prediction horizon h . The reported τ depict a lower $\tau = 0.20$, middle $\tau = 0.5$ and upper $\tau = 0.80$ quantile of the predictive growth distribution. A systemic risk indicator deemed relevant in explaining economic downturns is expected to be selected for the lower quantiles. Moreover, we consider three projection horizons h - one, four and eight quarters ahead. Thus, we can distinguish between indicators that signal future economic developments in the very short term and with some time in advance (up to two years). This is important for policy reasons as addressing risks with measures such as macroprudential policies might take some time.

Several messages can be taken from Table 2. First, few systemic risk indicators are selected to explain the predictive distribution of GDP growth. This may reflect either an overall poor in-sample fit or potential multicollinearity stemming from overlapping information that many of these measures contain. Second, systemic

risk measures are expected to signal risks to growth and thus explain the lower tail of the predictive GDP growth distribution. However, results suggest that less than halve of the indicators considered are selected for inclusion in the lower quantile regression ($\tau = 0.20$). Table 5 in Appendix C provides a more granular view of this result. Even though some other indicators are selected for lower quantiles (ie, the Mortgage spread, commercial paper spread and the Turbulence index are selected for the regression with $\tau = 0.10$), this is also the case for upper quantiles ($\tau = 0.8$). Therefore, most systemic risk measures do not appear to contain specific information that help predict growth fragility since the number of indicators that feature in the lower quantile regression is not significantly different than the number of measures selected to explain middle and upper quantiles.

Some indicators are picked up by the algorithm for most quantiles considered. The term spread, debt service ratio, the measure of size concentration in the financial sector and the cyclical adjusted price to earning ratio (CAPE) stand out as being consistently relevant. Several other measures such as the loan supply, the VIX, the Moody’s Baa/Treasury spread, the mortgage and commercial paper spreads are selected for some horizons and quantiles considered (see Table 5 Appendix C). Nevertheless, most systemic risk measures are rarely selected. The question of whether this reflects a high degree of collinearity with other indicators which deliver superior fit or irrelevance in predicting real economic developments can not be answered in this framework.

It is also possible to distinguish between systemic risk indicators that tend to be relevant in the short term, up to one year ahead, and those that signal risks to growth in the medium run, for $h=8$ quarters. The term spread and debt service ratio and the concentration index are relevant across quantiles in short to medium run ($h=4$ and $h=8$) while the predictive power of the CAPE and other measures of credit spreads seems to be concentrated in short-term horizons of up to one year ahead.

4.1 Robustness

To understand how sensitive the results presented so far are to the specific shrinkage technique used, we employ the additional methods discussed - the Bayesian Lasso and the Ridge. Table 3 summarizes the main findings. The coefficients signalled as relevant are those that significantly differ from zero - at least 50 % of the posterior density around the median does not include zero. This calculation is done for comparison purposes, since contrary to the SSVS, both the Ridge and the Lasso deliver continuous posterior distributions. Figure 4 in Appendix C gives a clear picture of this idea. It shows how the shrinkage performed by the Ridge and the Lasso differs from the SSVS for some of the most relevant systemic risk indicators. Both the alternative methods result in a continuous posterior distribution.

The columns in Table 3 for the SSVS show the main result previously discussed, that features in Table 2. While the next columns show the indicators found to be relevant when re-running the model with the Lasso and Ridge priors. The indicators deemed relevant are quite consistently signalled by the alternative methods employed. Some differences can however be observed. Compared to the SSVS prior,

the Bayesian Lasso selects less systemic risk indicators as relevant covariates to explain the three quantiles of GDP growth considered. In total, only 6 of such indicators are chosen by the algorithm out of the 33 eligible covariates for the three quantiles considered. On the contrary, the Ridge prior selects 8 predictors. This compares with 7 variables selected by the SSVS.

Overall, the main result that only a select few systemic risk indicators contain additional relevant information in predicting future economic developments holds across both sparse and dense methods. The most relevant predictors selected which include the term spread, the debt service ratio, the CAPE and size concentration of the financial sector are consistently relevant in all alternatives.

Table 3: Systemic Risk indicators selected as relevant in explaining different quantiles of the predictive distribution of GDP growth, one year ahead.

#	Systemic risk indicator	SSVS			Lasso			Ridge		
		$\tau = 0.20$	$\tau = 0.50$	$\tau = 0.80$	$\tau = 0.20$	$\tau = 0.50$	$\tau = 0.80$	$\tau = 0.20$	$\tau = 0.50$	$\tau = 0.80$
1	absorption									
2	Delta Absortion									
3	AIM									
4	CatFin									
5	GZ spread									
6	Baa/Aaa Bond yield									
7	TED spread									
8	Term Spread	•	•	•	•	•	•	•	•	•
9	Baa/10-yr T-rate spread									
10	Mortg-GS10 Spread			•	•			•	•	
11	Comm. paper-3mT-Bill spread			•				•	•	•
12	Excess Bond Premium									
13	Intl. Spillover									
14	CoVaR									
15	Delta CoVaR									
16	Book lvg.									
17	Mkt. Lvg.									
18	DCI									
19	MES									
20	MES-BE									
21	Volatility									
22	Size conc.	•	•	•		•	•	•	•	•
23	Turbulence				•					
24	PQR									
25	Average DD.									
26	portfolio DD.									
27	MRI CITI Index									
28	CAPE	•	•	•	•	•	•	•	•	•
29	VXO			•					•	•
30	Sent. Index									
31	Credit-to-gdp gap									
32	Debt Service Ratio	•	•	•	•	•	•	•	•	•
33	Loan Supply							•	•	•

Notes: The dots in the table refer to the indicators selected by the quantile regressions with a SSVS, Lasso and Ridge priors. For the Lasso and Ridge the signalled indicators are those for which at least 50 % of the posterior density around the median does not include zero. The quantile regressions results are for $\tau = \{0.20, 0.5, 0.80\}$ and for the horizons $h = \{1, 4, 8\}$ in quarters.

4.2 Forecasting Output Growth using Systemic Risk indicators

In this section, we evaluate the out-of-sample forecasting performance of the quantile regression with the SSVS prior, which is our baseline specification following [Adrian et al. \(2019\)](#). We re-run the model recursively on a training sample of data between 1947Q1 and 1976Q1. At each out-of-sample point, we predict GDP growth for a

given quantile τ , at a given horizon h , using the full set of systemic risk indicators deemed relevant by the SSVS prior. We then compare our prediction with the observed value of GDP growth and the in-sample estimates. This procedure is then iterated, expanding the estimation sample one period at a time, until the end of the sample (2011Q4). Panels A and B in Figure 3, Appendix C summarize the main outcomes of this exercise. These two figures compare the in-sample fit of the median ($\tau = 0.5$) and upper/lower quantiles ($\tau = 0.10$ and $\tau = 0.90$) with the out-of-sample prediction of the median obtained by re-estimating the model recursively at each out-of-sample point in time.

In-sample estimates do not vary significantly from out-of-sample prediction for the median, although some differences can be observed at the beginning of the out-of-sample period between 1947Q1 until 1990Q1 for the prediction one year ahead. It is also possible to observe that the realized GDP growth is within the 10th-90th percentile range, for most observations. There are however some exceptions, most noticeably at the beginning of the sample.

Next, we evaluate the out-of-sample accuracy and calibration of the density forecasts by analysing the predictive score and the probability integral transform (PIT) statistics which measure the predictive density and the cumulative distribution evaluated at the realized value of GDP growth at a given horizon. Results of this exercise are presented in Panels C-F of Figure 3. Panels C and D plot predictive scores for the model conditional on the full set of systemic risk indicators and real activity, vis-a-vis the model which only includes real activity measures (ie, GDP growth, its own lags and the CFNAI), estimated four and one quarters ahead. Higher values of predictive scores suggest more accurate prediction since they show that the realized value is close to the outcomes to which the model attributes greater likelihood. We observe that it is difficult to distinguish between predictive scores of the model estimated with the full set of systemic risk indicators from the baseline model which only includes real activity measures as regressors. This result points toward the hypothesis that systemic risk indicators do not seem to significantly add value when for out-of-sample forecasting of real activity.

PIT statistics tell a similar story. Panels E and F of Figure 3 plot the PIT statistics which measure the cumulative density below the realized value of GDP growth for each observation in the testing sample for the quantile regressions estimated for the median (ie, $\tau = 0.50$). In a perfectly calibrated model PIT statistics for the quantile regression for the median would coincide with $\tau = 0.50$. For most of the testing sample period PIT statistics are above 0.5 which highlights the skewed nature of the predictive distribution of GDP growth. Again, it is difficult to distinguish between the predictive distribution of GDP conditional on systemic risk indicators and only on real activity.

5 Conclusion

In this paper we study the link between real economic growth and systemic risk in light of a Bayesian quantile regression model that captures the nonlinear nature of this relationship. The quantile regression employed yields a complex predictive growth distribution. We empirically assess the relevance of 33 systemic risk indica-

tors to explain the full spectrum of the distribution of GDP growth and analyse the practical utility of these indicators in improving economic growth forecasting. To deal with a large number of possible predictors, we employ sparse-modelling techniques to perform variable selection and shrinkage. These include the use of priors which implement Stochastic Search and Variable Selection (SSVS) and the Bayesian Lasso algorithms which are then compared to a Ridge which belongs to the class of dense methods. Describing the relation between systemic risk and real activity in a parsimonious way is motivated by statistical reasons - to avoid overfitting and unnecessary parameter proliferation; but it is also justified on economic grounds since earlier literature suggests that only a subset of systemic risk indicators are relevant in describing the interaction between systemic risk and output. Understanding which particular indicator merits inclusion in the regression to explain the lower tails of the growth distribution, which we call growth fragility, is unclear a priori but important for two main reasons. First, it is essential to understand when and from which part of the financial system risks are originating. Secondly, from a statistical viewpoint, it is necessary to assess the predictive content of each systemic risk measure to understand its suitability as an early warning indicator of a recession. The SSVS prior is used to derive the main results and the alternative methods are used to analyse the sensitivity of the findings to the specific shrinkage technique employed.

In-sample results suggest that only a subset of the systemic risk measures in study contain relevant information that explain future developments of GDP growth. Less than half of the systemic risk indicators considered are selected as relevant predictors of lower quantiles of GDP growth, suggesting that most systemic risk indicators contain limited recession-relevant information. The term spread, CAPE, the debt service ratio and the size concentration in the financial sector stand out as relevant predictors of the lower tail of economic activity. This result is robust to different shrinkage methods. In the out-of-sample forecasting exercise it is difficult to distinguish the performance of the model which includes measures of real activity and systemic risk as compared to the baseline case which does not include systemic risk indicators.

Appendix A Data

Series	Description	t-code	Source
GDP	Real Gross Domestic Product.	5	St. Louis Fed
CFNAI	Chicago Fed National Activity Index, aggregates the most important real activity variables.	1	St. Louis Fed
absorption	Captures the fraction of the total variance of a set of assets explained or "absorbed" by a the first eigenvectors/principal components.	1	Giglio et al. (2016)
Delta Absortion	First difference of absorption.	1	Giglio et al. (2016)
AIM	The ratio of absolute stock return to dollar volume.	2	Giglio et al. (2016)
CatFin	Value at Risk measure of a cross-section of financial firms.	1	Giglio et al. (2016)
GZ spread	Corporate Bond credit spread calculated using secondary market prices of senior unsecured bonds issued by a large representative sample of US non-financial firms.	5	S. Gilchrist Website
Baa/Aaa Bond yield	Spread btw Moody's Seasoned Baa Corporate Bond Yield and 10-Year Treasury rate	5	St. Louis Fed
TED spread	The difference between 3M Treasury bill and 3M LIBOR based on US dollars.	1	Stock and Watson (2016b)
Term Spread	The difference between 10yr and 3M Treasury bill.	1	Stock and Watson (2016b)
Baa/10-yr T-rate spread	The spread between Moody's Seasoned Baa Corporate Bond and the 10Yr Treasury bill.	1	St. Louis Fed
Mortg-GS10 Spread	The spread btw 30-Year Conventional Mortgage Rate and 10Yr Treasury bill.	1	Stock and Watson (2016b)
CP3FM-TB3MS	The spread btw 3-Month AA Financial Commercial Paper Rate and 3M Treasury bill.	1	Stock and Watson (2016b)
Excess Bond Premium	The residual component of the GZ spread that reflects investor attitudes toward corporate credit risk.	1	S. Gilchrist Website
Intl. Spillover	Cross-country comovement in a set of macroeconomic variables.	5	Giglio et al. (2016)
CoVaR	Value at Risk (VaR) of the financial system conditional on institutions being under distress.	1	Giglio et al. (2016)
Delta CoVaR	Derives from the CoVaR.	1	Giglio et al. (2016)
Book lvg.	Book leverage for the 20 biggest institutions in the US.	1	Giglio et al. (2016)
Mkt. Lvg.	Market leverage for the 20 biggest institutions in the US.	1	Giglio et al. (2016)
DCI	Number of Granger causal relationships between bank equity returns, as a measure of interconnectedness.	1	Giglio et al. (2016)
MES	A firm's expected equity loss when market falls below a certain threshold over a given horizon.	1	Giglio et al. (2016)
MES-BE	Derives from the MES.	1	Giglio et al. (2016)
Volatility	Average volatility of the equity returns of the 20 biggest financial firms in the US.	1	Giglio et al. (2016)
Size conc.	The market equity Herfindal index of the 100 biggest financial firms in the US.	1	Giglio et al. (2016)
Turbulence	Covariance relative to a longer-term covariance estimate of top US financial firms.	1	Giglio et al. (2016)
PQR	Principal Components of a set of systemic risk measures.	1	Giglio et al. (2016)
Average DD.	Average Distance to Default measures the market's perception of the average risk of insolvency among major US banks.	1	Cleveland Fed
Portfolio DD.	Portfolio Distance to Default measures the market's perception of the systematic insolvency risk of the banking system as a whole.	1	Cleveland Fed
MRI CITI Index	CitiBank Risk Aversion Indicator, aggregates a set of indicators capturing risk aversion in the financial markets.	1	Bloomberg
CAPE	Cyclical Adjusted Price-to-Earnings ratio, calculated as the ratio of the current price of the S&P500 by its inflation adjusted historical earnings record over the past 10 years.	5	R. Shiller's Website
VXO	Implied volatility of near-the-money options of the S&P500.	1	Bloomberg
Sent. Index	Stock Market Investor Sentiment Index.	1	J. Wurgler Website
Credit-to-gdp gap	Difference between the credit-to-GDP ratio and its long-term trend given in percentage points.	1	BIS, Data Warehouse
Debt Service Ratio	The ratio of interest payments plus amortisations to income for the private non-financial sector.	1	BIS, Data Warehouse
Loan Supply	Net Percentage of Domestic Banks Tightening Standards for Commercial and Industrial Loans to Large and Middle-Market Firms as reported by Senior Loan Officers.	1	St. Louis Fed

Notes: The transformation codes (t-code column) are 1:levels; 2:1st dif; 5:1st dif of the logarithm.

Table 4: Data Description and Sources.

Appendix B Algorithm for Posterior Inference

The model is based on the priors for the regression parameters and their variance as defined in the body of the paper and outlined below

$$\begin{aligned}\beta|\gamma, \delta &\sim N(0, \gamma\delta^2) \\ \delta^{-2} &\sim \text{Gamma}(a_0, a_1)\end{aligned}$$

These priors are common to all three methods presented to perform model selection and shrinkage. The key parameter to enforce sparsity is γ and we define a hierarchical structure where the hyperprior distribution dictates which method from the three (SSVS, Bayesian Lasso or Ridge) is used. In what follows we present the conditional posteriors necessary to set up the Gibbs Sampler used for full posterior inference. To ease the notation, we drop the subscript τ from all parameters. However, it is important to note that the procedure described applies to each quantile and inference is based on estimation for a predefined quantile grid.

B.1 SSVS

A Stochastic Search Variable Selection Algorithm in a quantile regression setting is proposed by [Korobilis \(2017\)](#) based on the Gibbs Sampler of [Kozumi and Kobayashi \(2011\)](#). We follow the authors, define a grid and set the quantiles considered $\tau = \{5, \dots, 95\}$ with increments of 5. In practice, each quantile defined gives rise to an additional regression to estimate. Thus parameters vary across quantiles τ . The following points describe a full pass-throw of the Gibbs Sampler, for each quantile τ .

- 1 The conditional posterior of β is given by

$$\begin{aligned}p(\beta|y, z, \delta^{-2}, \gamma, \pi_0) &\sim N(\bar{\beta}, \bar{V}), \\ \text{with } \bar{\beta} &= \bar{V} \left[\sum_{t=1}^T \tilde{x}_t(y_t - \theta z_t)/\phi^2 z_t \right], \bar{V} = \left[\sum_{t=1}^T \frac{\tilde{x}_t' \tilde{x}_t}{\tau^2 z_t} + \text{diag}(\gamma\delta^{-2}) \right].\end{aligned}$$

- 2 The conditional posterior of δ^{-2} is given by

$$\begin{aligned}p(\delta^{-2}|y, z, \beta, \gamma, \pi_0) &\sim \text{Gamma}(\bar{a}_0, \bar{a}_1), \\ \text{with } \bar{a}_0 &= a_0 + 1/2 \quad \text{and} \quad \bar{a}_1 = \beta^2/2 + a_1.\end{aligned}$$

Where a_0 and a_1 are hyperparameters that are set to 0.1.

- 3 The conditional posterior of z_t is given by

$$\begin{aligned}p(z_t|y, \beta, \gamma, \delta^{-2}, \pi_0) &\sim \mathcal{GIG}\left(\frac{1}{2}, \bar{\kappa}_0, \bar{\kappa}_1\right), \\ \text{with } \bar{\kappa}_0 &= \sum_{t=1}^T (y_t - x_t \beta_\tau)/\phi \quad \text{and} \quad \bar{\kappa}_1 = \sqrt{2 + \theta^2}/\phi.\end{aligned}$$

- 4 The conditional posterior of each element of γ is given by

$$p(\gamma|y, z, \beta, \delta^{-2}, \pi_0) \sim \text{Bernoulli}(\pi_0).$$

5 The conditional posterior of π_0 is given by

$$p(\pi_0|y, z, \beta, \gamma, \delta^{-2}) \sim \text{Beta}(\bar{b}_0, \bar{b}_1),$$

with $\bar{b}_0 = 1 + b_0$ and $\bar{b}_1 = n - 1 + b_1$.

Where b_0 and b_1 are hyperparameters that are set to 5 and 10, respectively.

B.2 Bayesian Lasso

The Bayesian Lasso has been proposed by [Park and Casella \(2008\)](#). Its implementation for quantile regression requires very few alterations to the Gibbs sampler previously characterized. The descriptions below entails the blocks of the SSVS algorithm that should be changed in order to achieve this specification.

1* The conditional posterior of β is given by

$$p(\beta|y, z, \delta^{-2}, \gamma, \pi_0) \sim N(\bar{\beta}, \bar{\Omega}),$$

with $\bar{\beta} = \bar{\Omega} \left[\sum_{t=1}^T \tilde{x}_t(y_t - \theta z_t) / \phi^2 z_t \right], \bar{\Omega} = \left[\sum_{t=1}^T \frac{\tilde{x}_t' \tilde{x}_t}{\tau^2 z_t} + \Omega^{-1} \right]$.

where $\Omega = \text{diag}(\gamma)$ is the prior variance for β .

4* The conditional posterior of each element j of γ is given by

$$p(\gamma_j|y, z, \beta, \delta^{-2}, \lambda) \sim \mathcal{GIG}\left(\frac{1}{2}, \mu, \lambda\right),$$

with $\mu = |\beta_j|$.

5* The conditional posterior of λ is given by

$$p(\lambda_j|y, z, \beta, \gamma, \delta^{-2}) \sim \text{Gamma}(\bar{c}_0, \bar{c}_1),$$

with $\bar{c}_0 = r$ and $\bar{c}_1 = \gamma_j/2 + \Delta$.

Where Δ and r are hyperparameters that are set to 1 and 1.78 respectively.

B.3 Ridge Regression

A Bayesian interpretation of the Ridge regression is licit and has been discussed in the literature (see [Kapetanios et al. \(2018\)](#); [Giannone et al. \(2017\)](#)). The Ridge regression can be viewed as a particular case of the SSVS model. Indeed, by simply setting $\pi_0 = 1$ the model estimated with a SSVS prior collapses into a Normal Gamma prior model (layers 4 and 5 are eliminated). This alteration is effortless and results in a model with Gaussian priors for the coefficients which has been shown to be equivalent to Ridge Regression.

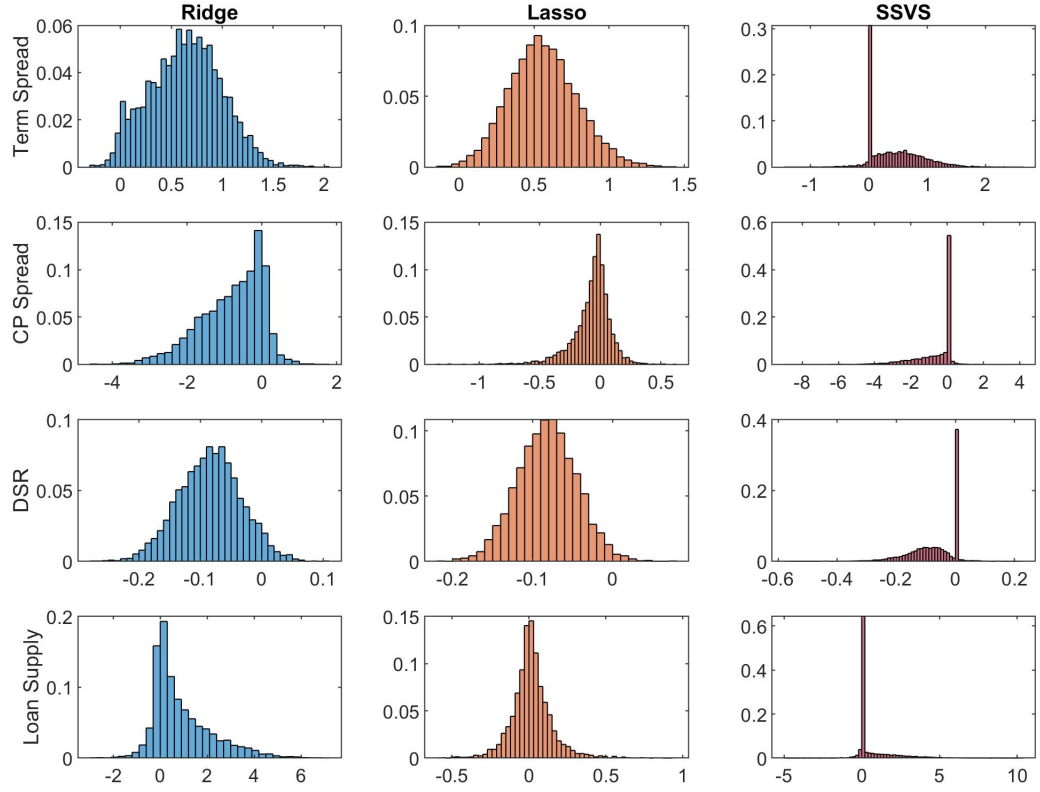
Appendix C Additional Tables & Figures

Table 5: Systemic Risk indicators selected as relevant in explaining different quantiles of the predictive distribution of GDP growth one year ahead.

#	Systemic risk indicator	$\tau = 10$	$\tau = 20$	$\tau = 30$	$\tau = 40$	$\tau = 50$	$\tau = 60$	$\tau = 70$	$\tau = 80$	$\tau = 90$
1	absorption									
2	Delta Absortion									
3	AIM									
4	CatFin									
5	GZ spread									
6	Baa/Aaa Bond yield									
7	TED spread									
8	Term Spread	•	•	•	•	•	•	•	•	•
9	Baa/10-yr T-rate spread									•
10	Mortg-GS10 Spread	•							•	
11	Comm. paper-3mT-Bill spread	•							•	
12	Excess Bond Premium									
13	Intl. Spillover									
14	CoVaR									
15	Delta CoVaR									
16	Book lvg.									
17	Mkt. Lvg.									
18	DCI									
19	MES									
20	MES-BE									
21	Volatility									
22	Size conc.	•	•	•	•	•	•	•	•	•
23	Turbulence	•								•
24	PQR									
25	Average DD.									
26	portfolio DD.									
27	MRI CITI Index									
28	CAPE	•	•	•	•	•	•	•	•	•
29	VXO								•	•
30	Sent. Index									
31	Credit-to-gdp gap									
32	Debt Service Ratio		•	•	•	•	•	•	•	•
33	Loan Supply									•

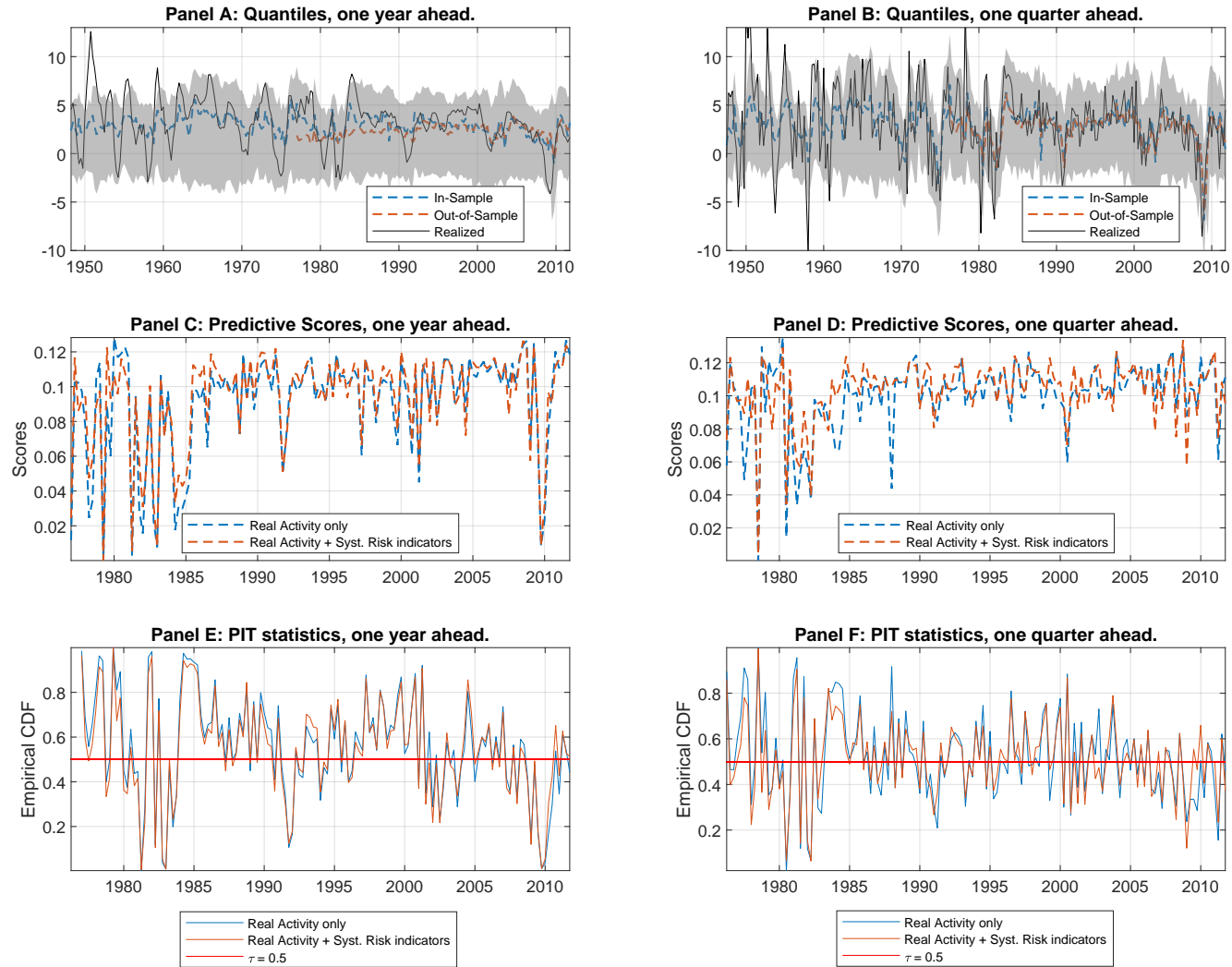
Notes: The dots in the table refer to the indicators for which the probability of inclusion (ie, the posterior mean of γ_p) is greater than 0.5 and therefore selected for the quantile regressions for $\tau = \{0.25, 0.5, 0.75\}$, for the horizons $h = \{1, 4, 8\}$ in quarters. The regression includes a constant, two own lags of GDP growth and the Chicago Fed National Activity Index (CFNAI) which are not subject to variable selection and therefore always feature in the regressions and are not reported.

Figure 2: Posterior Distribution of Some Systemic Risk Indicators.



Notes: Each histogram plots the posterior distribution of the coefficients associated to the four Systemic Risk indicators, estimated from the Quantile Regressions with the SSVS, Ridge and Lasso priors for $\tau = 0.2$ and a forecast horizon of one year ahead. The four systemic risk indicators include the Term Spread, Commercial Paper Spread (CP Spread), the Debt Service Ratio (DSR) and the Loan Supply.

Figure 3: Out-of-Sample Accuracy.



Notes: Panels A and B compare full sample quantile regression in-sample fit for the median $\tau = 0.5$ (blue dashed line), with the out-of-sample fit for the median. Training sample runs from 1947Q1 to 1976Q1 and testing sample from 1976Q2 to 2011Q4. Grey area depicts the distance between the 10th and 90th fitted quantiles. Predictive Scores are estimated with a Quantile Regression with Real Activity indicators only which include GDP and two own lags, the CFNAI and a constant. The regression for Real Activity plus Systemic Risk indicators is estimated with all indicators and an SSVS prior. PIT statistics are constructed for the predictive distribution of a Quantile Regression for the Median $\tau = 0.5$.

Appendix D Convergence Diagnostics

This subsection examines the convergence of the Markov Chain Monte Carlo in the baseline specification. Univariate regressions are less sensitive to starting points of the chain or size of the burn-in period. However, to make sure convergence is satisfactory, we compute inefficiency factors of β_τ . The table below shows these statistics for different quantile regressions employed, using the SSVS prior. These statistics are found to be lower than the inefficiency factors reported in [Kozumi and Kobayashi \(2011\)](#) and therefore acceptable.

Table 6: Inefficiency factors of the parameters of the Quantile Regression estimated with the SSVS prior.

#	Systemic risk indicator	$\beta_{j,\tau=0.20}$	$\beta_{j,\tau=0.50}$	$\beta_{j,\tau=0.80}$
1	absorption	1.18	1.20	0.59
2	Delta Absortion	1.04	1.26	1.17
3	AIM	1.26	1.03	0.82
4	CatFin	0.69	0.74	1.04
5	GZ spread	0.72	0.91	1.08
6	Baa/Aaa Bond yield	0.85	0.83	0.92
7	TED spread	0.77	0.90	0.74
8	Term Spread	0.57	0.55	0.74
9	Baa/10-yr T-rate spread	0.69	0.87	0.79
10	Mortg-GS10 Spread	0.75	0.99	0.91
11	Comm. paper-3mT-Bill spread	0.73	1.74	0.65
12	Excess Bond Premium	0.85	0.98	1.19
13	Intl. Spillover	0.49	0.99	1.05
14	CoVaR	0.55	1.17	1.34
15	Delta CoVaR	0.54	1.20	1.33
16	Book lvg.	0.76	1.04	1.14
17	Mkt. Lvg.	0.78	1.03	1.30
18	DCI	1.22	0.86	0.90
19	MES	0.55	1.14	0.91
20	MES-BE	1.14	0.87	0.83
21	Volatility	0.85	0.85	0.91
22	Size conc.	0.63	0.33	0.63
23	Turbulence	0.66	1.02	0.62
24	PQR	0.58	0.98	0.88
25	Average DD.	0.72	0.62	0.74
26	portfolio DD.	0.70	0.65	0.73
27	MRI CITI Index	1.03	0.66	1.20
28	CAPE	0.52	1.09	0.64
29	VXO	0.83	1.31	0.65
30	Sent. Index	0.91	1.00	0.69
31	Credit-to-gdp gap	0.95	0.74	0.99
32	Debt Service Ratio	0.82	1.29	1.16
33	Loan Supply	0.66	1.40	0.71

Notes: The inefficiency factors are calculated for a MCMC run of 5000 draws and 2000 burn-ins.

The inefficiency factors are a function of the infinite sum of the autocorrelation of the chain, which is estimated using a 4 % tapered window. It is commonplace to consider inefficiency factors equal or lower than 20 satisfactory.

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