Growth Fragility and Systemic Risk under Model Uncertainty

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Abstract

The link between systemic risk and output growth is hard to study for two reasons. First, the relationship is believed to be nonlinear. Second, systemic risk is unobservable and the myriad of measures proposed in the literature impose the additional challenge of having to deal with model uncertainty. This paper examines the relationship between the quantiles of output growth and systemic risk through the lens of a Bayesian quantile regression. We study, in particular, the relevance of 33 systemic risk indicators to explain lower quantiles of output growth that measure growth fragility. Model uncertainty is tackled by using sparse-modelling techniques that perform both model selection and shrinkage. Some systemic risk indicators studied have a relevant predictive content for output growth. However, instability of predictive relations based on such indicators is the norm.

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1 Introduction

The global financial crisis that started in 2007 has brought systemic risk to the forefront of the research agenda of academics and policymakers. On the empirical side, the literature has focused on measuring systemic risk in an effort to provide indicators of financial imbalances in the economy, hoping that these could serve as early warnings of forthcoming recessions. This has lead to the proposal of a myriad of indicators, aimed at capturing different dimensions of systemic risk. Billio et al. (2012) and Bisias et al. (2012a) survey over 30 such indicators, many more have been proposed since. The staggering amount of systemic risk indicators proposed in the literature owes to the fact that the definition of systemic risk is rather diffuse. As a result, different measures capture different dimensions of the concept. Nevertheless, a synthesis and working definition is provided by Peydro et al. (2015), that introduces systemic risk as *"the risk of threats to financial stability that impair the functioning of a large part of the financial system with significant adverse effects on the broader economy."*[1].

A key element that distinguishes a systemic event from otherwise systemically unimportant shocks, that are constantly hitting the financial system without resulting in a crisis, are its macroeconomic relevance. Whether an episode is classified as systemic therefore depends on its impact on welfare and real activity [2]. Overall, systemic risk indicators are expected to signal downside risks to real activity and reflect the likelihood of a recession.

This paper studies the relationship between output growth and systemic risk through the lens of a Bayesian quantile regression model. As opposed to the common approach in the literature, we use sparse-modelling techniques to address model uncertainty concerns, allowing for both model selection and shrinkage. We address two major challenges that complicate the study of this relationship.

First, systemic risk and real activity are thought to relate in a nonlinear way. For instance, Brunnermeier and Sannikov (2014) study the full dynamics of the equilibrium of an economy, characterized as inherently unstable due to nonlinear amplification channels that expose it to *"volatility crisis episodes"*. Because the system's reaction to shocks is nonlinear, even small shocks may be subject to amplification, pulling the economy away from its steady state. Moreover, the stationary distribution is bimodal, which in practice means the economy might remain persistently in a

[1]This definition is broadly consistent with that proposed by the IMF, FSB, ECB and BIS (see International Monetary Fund (2009); Hartmann et al. (2009); Caruana (2010)).

[2]Laeven and Valencia (2013) compile a comprehensive dataset of cross-country banking crisis from 1970 to 2011 and identify 147 systemically important banking crisis that result in an average output loss of 23 per cent.

depressed regime due to endogenous risk. One step further, Christiano et al. (2014) augment a dynamic general equilibrium model with a financial accelerator mechanism and suggest that fluctuations in the severity of information asymmetry in the economy account for a material portion of business cycle fluctuations. Contrary to the extant literature, risk shocks stemming from the cross-sectional idiosyncratic uncertainty in the allocation of capital are not a mere propagation channel but the source of economic fluctuations. Consistent with these ideas, Adrian et al. (2016) model the full distribution of GDP growth that is found to be skewed and exhibit a long lower tail. Furthermore, financial conditions are found to determine downside growth vulnerability.

Second, any attempt to empirically model the link between systemic risk and real economic activity must deal with inherent model uncertainty plaguing such an exercise. On one hand, there are multiple dimensions of systemic risk and it is unclear which dimension is relevant in explaining each quantile of macroeconomic fluctuations. On the other hand, measurement error associated with each systemic risk measure might mean some indicators are too noisy and not informative. In practice, the researcher is forced to choose a single set of regressors within the possible 2^K candidate models, supposing there are K different systemic risk measures to choose from [3]. Adrian et al. (2016) and Giglio et al. (2016) deal with the large number of possible predictors by employing dense modelling techniques. The financial conditions index used by Adrian et al. (2016) is simply a linear combination of a large set of financial variables. Whereas, Giglio et al. (2016) explicitly uses principal component analysis to shrink the space of potential predictors within a quantile regression. The author examines the predictive content of 19 different systemic risk indicators in explaining the lower tail of the predictive distribution of output, one year ahead. They find that systemic risk skews the distribution of real activity, shifting its left tail outwards.

Principal Component Analysis has proven effective in reducing model dimensionality and is widely used in the literature[4]. It is based on the principle that all potential predictors might be relevant and thus shrinks the information content in all variables into a few factors that explain the maximum amount of variation of the pool of regressors. Its main disadvantage in this framework has to do with interpretability. Although shrinking the information content of a large pool of systemic risk indicators into few factors is effective in dealing with model uncertainty and

[3]The 33 systemic risk measures surveyed in this paper result in over 10^9 potential candidate models to choose from when relating systemic risk to real activity.

[4] see for example Bernanke et al. (2005); Koop (2013); Boivin et al. (2018); Bok et al. (2018) and Stock and Watson (2016a) for a review

avoiding the curse of dimensionality, it is unable to inform which systemic risk indicators signal risks to output growth, since that information is lost in the shrinkage process.

Instead, we examine the relevance of 33 of the most popular systemic risk indicators proposed in the literature in light of three sparse-modelling methods that deal with model uncertainty. We specify a "spike-and-slab" prior following the original proposal of Mitchell and Beauchamp (1988) and Korobilis (2017) in a quantile regression framework. To understand the sensitivity of our results to the degree of sparsity imposed, we re-run the model with a Bayesian Lasso prior suggested by Park and Casella (2008) and a Ridge prior type following Giannone et al. (2017).

Our approach has several advantages that allow us to study new questions that were not addressed in the literature. First, by employing quantile regression methods, we explore the nonlinear nature of the relationship between output growth and systemic risk. Our approach is flexible enough to examine how the link between the two variables changes across quantiles. The novelty of this work lies in the variable selection and shrinkage algorithm, that automatically selects the most relevant systemic risk indicators in explaining each quantile of output growth in a data-driven way. Second, we assess the empirical relevance of 33 systemic risk measures in a common setting, avoiding over-fitting and model dimensionality constraints. Third, we perform in-sample and out-of-sample analysis through a rolling window forecasting exercise to understand which systemic risk measures carry relevant information content for real activity at each point in time.

Two main results emerge from our exercise. First, few systemic risk measures possess relevant information content to predict output growth developments. We find that it is particularly hard to predict lower quantiles of GDP growth, which suggests that systemic risk indicators contain very little recession relevant information content. This result is in line and extends the findings of Giglio et al. (2016) and Stock and Watson (2003). Second, from the 33 systemic risk measures assessed, the term spread is the most relevant predictor, chosen by the algorithm as containing relevant information to forecast GDP growth across quantiles for the full sample. However, by performing a rolling window forecasting exercise we find that its forecasting power diminishes post-2000's and other indicators such as the Market Leverage of financial institutions are favoured. This result lends support to the hypothesis that the yield curve is loosing importance as a driver of real economic developments, as suggested by Bordo and Haubrich (2008) amongst others.

The remainder of the paper proceeds as follows. Section 2 presents the data and section 3 reviews the literature on the 33 systemic risk indicators assessed. Section

4 explains the econometric framework, estimation technique, variable selection and shrinkage procedure. Section 5 discusses the main results and findings. Section 6 concludes.

2 Data

An overwhelming quantity of systemic risk measures have been proposed in the literature. We restrict our attention to those which data is available, relying on the surveys of Bisias et al. (2012b) and Giglio et al. (2016). In addition to the indicators considered by the aforementioned authors, we study a number of related indicators that were not considered in their articles but are relevant to our analysis.

#	Systemic Risk indicator	Sample	Reference
1	absorption	1947-2011	Kritzman et al. (2011)
2	Delta Absortion	1947-2011	Kritzman et al. (2011)
3	AIM	1947-2011	Amihud (2002)
4	CatFin	1947-2011	Allen et al. (2012)
5	GZ spread	1973-2011	Simon Gilchrist et al. (2012)
6	Baa/Aaa Bond yield	1947-2011	Giglio et al. (2016)
7	TED spread	1984-2011	Stock and Watson (2003)
8	Term Spread	1947-2011	Stock and Watson (2003)
9	Baa/10-yr T-rate spread	1962-2011	Stock and Watson (2003)
10	Mortg-GS10 Spread	1971-2011	Stock and Watson (2016b)
11	Comm. paper-3mT-Bill spread	1959-2011	Stock and Watson (2016b)
12	Excess Bond Premium	1973-2011	Simon Gilchrist et al. (2012)
13	Intl. Spillover	1963-2011	Diebold and Yilmaz (2011)
14	CoVaR	1947-2011	Adrian and Brunnermeier (2016)
15	Delta CoVaR	1947-2011	Adrian and Brunnermeier (2016)
16	Book lvg.	1969-2011	Giglio et al. (2016)
17	Mkt. Lvg.	1969-2011	Giglio et al. (2016)
18	DCI	1947-2011	Billio et al. (2012)
19	MES	1947-2011	Acharya et al. (2017)
20	MES-BE	1947-2011	Brownlees and Engle (2012)
21	Volatility	1947-2011	Giglio et al. (2016)
22	Size conc.	1947-2011	Giglio et al. (2016)
23	Turbulence	1947-2011	Kritzman and Li (2010)
24	PQR	1947-2011	Giglio et al. (2016)
25	Average DD.	2008-2011	Saldías (2013)
26	portfolio DD.	2008-2011	Saldías (2013)
27	MRI CITI Index	1997-2011	Adrian et al. (2010)
28	CAPE	1947-2011	Shiller (2005)
29	VXO	1962-2011	Bloom (2009)
30	Sent. Index	1965-2011	Baker and Wurgler (2006)
31	Credit-to-gdp gap	1962-2011	Aldasoro et al. (2018)
32	Debt Service Ratio	1999-2011	Aldasoro et al. (2018)
33	Loan Supply	1990-2011	Lown and Morgan (2006)

Table 1: Measures of systemic risk considered and respective sample dates.

The table above summarizes all measures that we consider as potential predictors of downturns in economic activity, the sample period for which they are available and respective references. A more detailed explanation of the data, transformations and sources used is provided in the Appendix. In the next section we briefly explain each measure considered.

3 An overview of systemic risk indicators

In what follows, we briefly discuss each systemic risk measure covered by this article, grouping them by category, according to their characteristics. A thorough review is provided by Bisias et al. (2012b) and Giglio et al. (2016) and readers are referred to these articles for further details.

3.0.1 Institution specific risk

Institution specific measures of systemic risk capture entity-level financial stress. In contrast to macroeconomic systemic risk indicators, their level of granularity allows for a microeconomic view of systemic risk. It is the case of the CoVaR and ΔCoVaR of Adrian and Brunnermeier (2016) that expresses the contribution of each financial institution to the overall risk in the system by measuring the loss stemming from the distress of a specific institution. Next, the MES - Marginal Expected Shortfall as proposed by Acharya et al. (2017), measures the conditional expectation of a firm's equity loss, given that the market falls below a pre-determined threshold, at a given horizon. A number of measures proposed in the literature follow the same logic, and are nested in the MES. It is the case of the SRISK of Acharya et al. (2012), the LRMES of Acharya et al. (2010) and others. With regards to financial institution liquidity risk, Amihud (2002) proposes the AIM - the ratio of absolute stock return to dollar volume. Another class of models focus on interdependence of equity returns of financial institutions. These include the Absorption ratio and its sibling, the delta Abortion, of Kritzman et al. (2011) that measure the portion of variance of the financial system explained by the first principal component. The International Spillover Index of Diebold and Yilmaz (2009) captures cross-country comovement in Macroeconomic variables. The Dynamic Causality Index - DCI of Billio et al. (2012) also focuses on entity level equity returns as a source of systemic risk. The authors measure the number of Granger causal relationships between bank equity returns, as a measure of interconnectedness. In addition to the aforementioned off-the-shelf systemic risk measures, Giglio et al. (2016) constructs Book and Market leverage and volatility for the 20 largest US financial institutions. The author also calculates the size concentration of the US financial sector using an Hirschman-Herfindahl index. Saldías (2013) proposes to measure systemic stress by calculating the average distance-to-default of the banking system, that reflects the market implied insolvency risk of the US banking system. The author complements this measure with the portfolio distance-to-default that signals systematic insolvency of the financial system that is regarded as a portfolio of banks. These measures are calculated by using market prices of options on Exchange-Traded-Funds (ETFs) tracking the banking system.

Each entity specific measure of systemic risk discussed is included in our analysis by aggregating it across institutions.

3.0.2 Macroeconomic measures of systemic risk

Macroeconomic measures of systemic risk comprise those that are not specific to any institution in particular. It is the case of the CatFin of Allen et al. (2012) that takes a holistic approach by measuring the Value at Risk of the overall financial system. The Cyclical Adjusted Price-to-Earning ratio of Shiller (2005) is used to gauge the relationship between price and valuation of equity markets across the business cycle. The cycle described by the ratio of price to earnings of stock market constituents on aggregate reaches its peak before financial crisis. Another example of such measures are those related to sentiment. The MRI CITI index is a risk aversion index, constructed by Citi Bank that summarizes the dynamics of key variables that determine risk aversion in the financial markets. These include the implied volatility of options on foreign exchange related assets, Emerging Market sovereign yield spreads, corporate CDS spreads, the implicit volatility of equity options and an interbank market spread. The case for considering risk aversion as a measure of financial imbalances is made by Adrian et al. (2010) that stresses the key role of movements in the price of risk in determining the build-up and unfolding of financial imbalances. The main argument is that, when the price of risk changes, so does the risk bearing capacity of financial intermediaries. Therefore, risk appetite determines credit supply decisions and consequently potential financial imbalances. The VIX is one of the most widely used measures of risk aversion (see Rey and Evgenia (2015) Baskaya et al. (2017)) and It has been related to bank risk-taking, international capital flows, leverage and credit cycles [5]. The sentiment index of Baker and Wurgler (2006) is also considered for reasons analogous to those explained above. The authors document that their sentiment index explains financial market valuation across the business cycle. The last indicator related to sentiment is the Excess Bond Premium (EBP) of Gilchrist and Zakrajšek (2012). This indicator is interpreted by the authors as a measure of credit market sentiment or risk appetite in credit markets. Analytically it is constructed as the residual component of their corporate bond market spread (GZ Spread), net of expected defaults.

[5] We include the VXO which is similar to the VIX except that the underlying index from which it derives its value is narrowed down to a smaller set of firms. This is to avoid the structural break in the VIX series resulting from the change in basis for its calculation.

Credit has been found to play an important role in triggering financial crisis (see Jordà et al. (2010); Schularick and Taylor (2012b); Jordà et al. (2015); Schularick and Taylor (2012a). In fact, recent research by Dell'Ariccia et al. (2012) suggests that since 1970s one out of three credit booms resulted in a financial crisis . Motivated by this evidence, our analysis comprises the credit-to-GDP gap - that measures the macroeconomic significance of the amount of debt contracted and the Debt Service Ratio, that reflects the debt burden of all agents in the economy. Some evidence presented by Aldasoro et al. (2018) suggests that these indicators signal forthcoming banking crisis. We supplemented these measures with a Loan Supply indicator, that is taken from the Senior Loan Officer Opinion Survey, a quarterly survey on US bank lending practices carried out by the Federal Reserve banks. The macroeconomic significance of this series is discussed in Lown and Morgan (2006).

The last category of systemic risk indicators considered are various yield spreads. There is a large strand of early literature documenting the predictive content of the term spread (given by the slope of Treasury yields) in forecasting economic downturns in the medium-term horizon (for instance Estrella and Hardouvelis (1991), Estrella and Mishkin (1998), Stock and Watson (2003) and Piazzesi et al. (2006)). Term spreads are though to aggregate a large amount of recession-relevant information in the economy. On one hand, they are forward looking variables, summarizing expectations of economic agents about the future economic outlook. On the other hand, they reflect the current stance of monetary policy which is a powerful tool steering the business cycle. In addition to the term spread, we include in our analysis the TED spread, given by the difference between the 3-month LIBOR and the Tresury Bill yield for the same maturity.

Beyond term spreads, recent research has examined the importance of other yield spreads as leading indicators of future economic conditions. The information content of corporate bond spreads for the US and European countries' real economic activity has been studied by Gilchrist et al. (2009), Gilchrist and Zakrajšek (2012), Bleaney et al. (2016) and López-Salido et al. (2017). There are two main channels explaining the predictive power of credit spreads emphasized in the literature.

First, a part of credit spreads is due to default risk. Investors demand a premium for the likelihood of default of bond issuers. Hence, changes in the default premium are reflected in corporate yield spreads thus signalling future macroeconomic conditions. Second, it has been found that a part of corporate yield spreads that is unrelated to default risk may help predict economic activity. This component is often referred to as credit market sentiment, translating investor beliefs and the stance of credit supply. Why might credit market sentiment be a leading indicator of real activity ? Two main arguments justify this relationship. The first one is related to investor sentiment. The main idea is that investors price-in noisy news about the financial condition of borrowers when only a part of it reflects economic fundamentals. The noise component generates volatility in credit spreads as investors update their beliefs with regards to credit risk as new information comes to public. Second, credit spreads mirror changes in credit supply. Changes in the willingness to lend by major financial institutions result in tighter lending standards and less favourable financial conditions that exert a negative shock on economic activity consistent with the financial accelerator mechanism pointed out by Kiyotaki and Moore (1997) and Bernanke et al. (1999).

The adage that credit spreads signal potential downturns in economic activity motivates the inclusion of a number of additional yield spreads in our analysis. We include the spread between Baa and Aaa Bond yields that summarize the price of risk at the end of Moody's rating spectrum of investment-grade corporate bonds. This series is popular and is used by López-Salido et al. (2017) in deriving their measure of credit market sentiment. Next, we include the Gilchrist and Zakrajšek (2012) spread (GZ spread), that is built using secondary market prices of senior unsecured bonds issued by large US non-financial firms. We supplement these measures by including the spread between the 30-year conventional mortgage rate, disclosed by Freddie Mac and the 10 year Treasury Bond yield. The spread given by the 3-Month AA Financial Commercial Paper Rate and the 3-Month Treasury Bill secondary market rate is also considered.

4 Bayesian quantile regression

We wish to explain the τth quantile of output growth h-steps ahead, denoted by y_{t+h}, by regressing this quantity on a set of explanatory variables that include systemic risk indicators and also relevant own lags of real activity, organized in a vector x_t of dimensions $T \times K$, where T is the time dimension and K the number of regressors.

Analytically, the τth quantile of y_{t+h} is given by its inverse probability distribution function denoted

$$\mathbb{Q}_\tau(y_{t+h}) = inf\{y : P(y_{t+h} \leq y) \geq \tau\}. \tag{1}$$

The quantile function can be expressed as the solution of the minimization problem

$$\mathbb{Q}_\tau(y_{t+h}) = min_q \mathbb{E}(\rho_\tau(y_{t+h} - q)), \tag{2}$$

where $\rho_\tau(u) = u(\tau - I(u < 0))$ is refered to in the literature as the quantile loss function. In the seminal paper by Koenker and Bassett (1978) the conditional quantiles of y_{t+h} are expressed as functions of the set of observables in a form similar to the following equation

$$\mathbb{Q}_\tau(y_{t+h}|I_t) = x_t'\beta_\tau + \varepsilon_t. \tag{3}$$

The main advantage of quantile regression is that the coefficients β_τ are allowed to vary across quantiles τ, capturing non-linear dynamics between real activity and systemic risk as prescribed by theory. It also gives a richer picture about the uncertainty surrounding point forecasts and how such uncertainty depends on measures of systemic stress. We are particularly interested in lower quantiles of real activity that depict economic downturns.

Estimation is straightforward in a classical framework and proceeds by solving the optimization routine specified in 2. However, it is advantageous to formalize the model in a bayesian setting to address our concerns over model uncertainty and to deal with the large number of predictors that have been found to result in in-sample overfitting due to proliferation of parameters in different frameworks (see Stock and Watson (2006) and Koop and Korobilis (2011)).

Following closely the approach of Korobilis (2017) we rewrite the error distribution as

$$\varepsilon_t = \theta z_t + \phi\sqrt{z_t}u_t \tag{4}$$

where $z_t \sim Exp(1)$ and u_t is a standard normal distribution. $\theta = (1-2\tau)/\tau(1-\tau)$ and $\phi = 2/\tau(1-\tau)$, for a given quantile $\tau \in [0,1]$. By plugging expression 4 into equation 1 we get a new quantile regression that can be estimated with bayesian methods

$$\mathbb{Q}_\tau(y_{t+h}|I_t) = x_t'\beta_\tau + \theta z_t + \phi\sqrt{z_t}u_t. \tag{5}$$

This reparametrization is licit because solving the optimization problem expressed in 2 is shown to be equivalent to maximizing a likelihood function under the asymmetric Laplace error distribution (see Yu and Moyeed (2010)).

4.1 Model Selection and Shrinkage

The first step in Bayesian inference is the specification of prior distributions for all relevant parameters. Priors allow the researcher to include additional relevant information in the analysis. In this exercise, we use priors as a device to address our concerns over both model uncertainty and in ensuring a parsimonious representation of the relationship between systemic risk and real activity.

Model uncertainty arises from the large number of systemic risk indicators that aim at approximating different aspects of systemic risk. Understanding which particular indicator merit inclusion in the regression to explain growth fragility is unclear a priori but important for two main reasons. First, it is essential to understand when and from which part of the financial system are risks originating. Secondly, from a statistical viewpoint, it is necessary to assess the predictive content of each systemic risk measure to understand its suitability as an early warning indicator of a recession.

In parallel, describing the relation between systemic risk and real activity in a parsimonious way is motivated by statistical reasons - to avoid overfitting and unnecessary parameter proliferation (see Stock and Watson (2006) and Koop and Korobilis (2011)); but it is also justified on economic grounds - Giglio et al. (2016) suggests that a small subset of systemic risk indicators are relevant in describing the interaction between systemic risk and output.

We specify the following priors

$$\beta_\tau | \gamma_\tau, \delta_\tau \sim N(0, \gamma_\tau \delta_\tau^2) \tag{6}$$

$$\delta_\tau^{-2} \sim Gamma(a_1, a_2) \tag{7}$$

This multi-level prior specification where β_τ is conditionally normal, allows for automatic shrinkage and model selection through the parameter γ_τ, that pushes β_τ to zero for any coefficient delivering poor fit. This setting offers several possibilities to impose sparsity in the model. In what follows we review three such alternatives that are nested in this framework.

4.1.1 Spike-and-Slab

Similar to the spike-and-slab prior originally proposed by Mitchell and Beauchamp (1988), in our setting each coefficient $\beta_{i,\tau}$ takes non-zero values with probability π_0. We refer to this hyperparameter as the probability of inclusion. Our approach follows that of George and McCulloch (1993), Korobilis (2013) in the sense that our prior for the coefficients is conditionally Gaussian albeit in the context of quantile regression closer to the work of Korobilis (2017). Formally, this prior forms a hierarchical structure that extends 6 and 7 by adding the following hyperpriors to that setting

$$\gamma_\tau|\pi_0 \sim Bernoulli(\pi_0) \tag{8}$$

$$\pi_0 \sim Beta(b_0, b_1), \tag{9}$$

Because γ_τ is a binomial variable, only the systemic risk indicators with the highest predictive power will be included in the equation. Moreover, the probability of inclusion of each indicator π_0 is also random, thus controlling the degree of shrinkage automatically. Hence, if $\gamma_{i,\tau} = 0$, the parameter $\beta_{i,\tau}$ is shrank to zero. Whereas, if $\gamma_{i,\tau} = 1$, $\beta_{i,\tau}$ will follow a normal distribution centered in zero. On the other hand, since γ_τ is estimated from the data, within a standard Gibbs Sampler, examining the posterior of γ_τ will inform on which variables are most relevant in explaining each quantile of real activity.

4.1.2 Bayesian Lasso

An alternative method capable of selecting relevant variables in a linear regression framework has been proposed by Tibshirani (1996) and is widely known as least absolute shrinkage and selection operator (Lasso). The Lasso is part of a wider class of penalized regression models that work by adding a penalty term to the objective function from which the coefficient estimates derive and has been shown to be effective in quantile regression (see Wu and Liu (2009); Li and Zhu (2008)). It works by solving the following optimization problem,

$$\beta^{LASSO} = \underset{\beta}{\operatorname{argmin}} \, \rho_\tau \varepsilon' \varepsilon + \lambda \left\| \beta \right\|_1, \tag{10}$$

where $\|\beta\|_1 = \sum_{j=1}^{p} \beta_j$ and λ controls the amount of regularization, that ensures shrinkage towards zero and prevents overfitting [6]. The lasso owes its name to the form of penalty imposed. It works by adding an L1-norm regularizer on the prediction weights standing for the absolute value of magnitude of the coefficients.

The lasso regression estimates can be given a Bayesian interpretation as it has been shown that, for specific choices of priors, the mean or mode of the posterior distribution of the parameters are equivalent to penalized regression results. Park and Casella (2008) show that the Lasso estimate can be interpreted as a Bayesian posterior mode estimate when the parameters have independent Laplace priors. Moreover, the authors show that the Laplace distribution can be written as a scale

[6] see Kapetanios et al. (2018) for a review of penalised regression techniques in a linear regression setting and Li et al. (2010) that discusses variable selection and shrinkage in a Bayesian quantile regression setting.

mixture of normals with an exponential mixing density. Hence, the Bayesian Lasso can be obtained by specifying a hierarchical structure that extends 6 and 7 by adding the following hyperpriors to that setting

$$\gamma_\tau|\lambda \sim \prod_{j=1}^{p} \frac{\lambda^2}{2} e^{-\lambda^2\gamma_j^2/2} \tag{11}$$

$$\lambda^2 \sim Gamma(c_0, c_1), \tag{12}$$

Contrary to its frequencist sibling, the Bayesian Lasso allows for the automatic choice of the degree of shrinkage λ. The main idea underlying these prior choices is similar. We wish to shrink nuisance parameters and sparsify the model such that a clearer pattern of the most important systemic risk indicators might emerge.

4.1.3 Bayesian Ridge

The Ridge estimator is another special case of the penalized regression where an L2-norm regularizer is added to the objective function of the estimation problem as follows.

$$\beta^{RIDGE} = \underset{\beta}{\operatorname{argmin}}\, \rho_\tau \varepsilon' \varepsilon + \lambda \left\|\beta\right\|_2, \tag{13}$$

with $\|\beta\|_1 = \sum_{j=1}^{p} \beta_j^2$.

Ridge regression was first introduced by Hoerl and Kennard (1970) and the main idea resembles the previous methods discussed. Similar to the Lasso, the Ridge regression also has a Bayesian analogue, that can be obtained by simply specifying Normal-Inverse Gamma priors for the regression parameters such as 6 and 7. Hence, if no other hyperparameters are added to the primary specification defined by 6 and 7, or if we set $\pi_0 = 1$ in the Spike-and-Slab setting, the Ridge regression emerges by default (see Kapetanios et al. (2018) and Giannone et al. (2017)).

4.2 Estimation

The parameters in 5 can be estimated with a Gibbs sampler since the respective likelihood function is conditionally Gaussian written as

$$f(\mathbf{y}|\beta_\tau, \mathbf{z}) \propto \prod_{i=1}^{T} z_t^{-1/2} \times exp\Big\{ -\frac{1}{2} \sum_{i=1}^{T} \frac{(y_t - x_t'\beta_\tau - \theta z_t)^2}{\phi^2 z_t} \Big\} \tag{14}$$

With $\mathbf{y} = (y_1, ..., y_T)'$ and $\mathbf{z} = (z_1, ..., z_T)'$. A full characterization of a Gibbs Sampler pass-through in our exercise is provided in Appendix B. The algorithm builds on the procedure suggested by Korobilis (2017) using different priors in order to implement the three variable selection techniques discussed.

5 Discussion of the results

In this section we discuss results that emerge from the estimation of the predictive distribution of GDP growth, allowed by the quantile regression and described in the previous sections. We focus on two main questions. First, which systemic risk indicators explain in-sample the variation of the lower tail of real activity, that is associated to recessions. Second, what is the out-of-sample performance of the various systemic risk measures proposed in the literature.

To motivate our discussion we compare the quantiles of the distribution of GDP growth against the distribution of each and every systemic risk measure considered. Results of the respective Quantile-Quantile plots are reported in Figures 3, 4 and 5. A Q-Q plot that is reasonably linear suggests that two datasets have similar distributions and a linear relationship between the variables. However, the vast majority of measures assessed exhibit a nonlinear relationship with GDP growth since the points in most graphs fall along a line in the middle of the graph but curve off in the tails. It is clear that i) nonlinearity characterizes the relationship between GDP growth and systemic risk; ii) distributions diverge mostly in the left tail, which represent economic downturns and recessions.

An interesting exercise that quantile regressions allow for is to examine the full predictive distribution of GDP. Unlike normal linear models, quantile regressions yield complex predictive distributions that need not be symmetric. We are particularly interested in the shape of its lower tails, since they reflect the likelihood of economic downturns. Figure 1 shows the predictive distribution of GDP, 4 steps ahead, for 2007Q4, four quarters ahead. The last quarter of 2007 marks the peak of the US business cycle and the start of a recession that ended the 73 month economic expansion that started in November 2001, bringing the Great Moderation to an end.

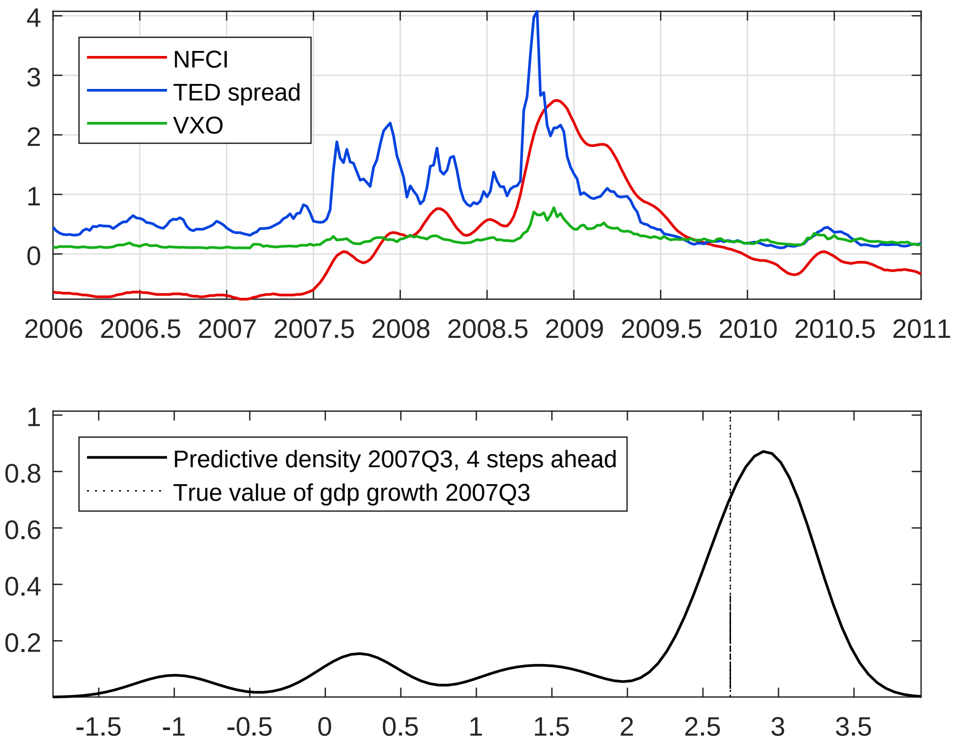

Figure 1: Time series for three measures of financial distress - the National Financial Conditions Index (NFCI), TED spread and the VXO (see data Appendix for a full description) and predictive density of GDP, 4 quarters ahead in 2007Q3.

The first panel of figure above shows three important measures of financial distress in the run-up to the 2008 recession, that are often regarded as harbinger of a crisis. In the second panel, the predictive distribution of quarterly GDP growth (annualized) in 2007Q3, four steps ahead, is plotted together with the actual value of GDP growth for that quarter. It can be observed that the distribution of GDP forecasts exhibit a long lower tail, with significant probability placed on negative outcomes. This reflects the uncertainty of the economic climate, even before the actual recession started. Overall, results suggest the shape of the distribution of predictive GDP growth is not specific to this particular quarter. On the contrary, it can be seen from from Figure 6 that the distribution of GDP growth forecasts for all quarters since the 1980's is skewed to the left, where the median tends to be lower than the mean, with a fat lower tail.

We now focus on the relevance of systemic risk measures in explaining the full spectrum of the GDP growth distribution. The Spike-and-Slab prior is our benchmark choice for implementing variable selection. It enforces sparcity by guaranteeing that not all systemic risk measures are selected and only the most relevant variables are picked by the algorithm.

		predictors at horizon h = 1			predictors at horizon h = 4			predictors at horizon h = 8		
#	Systemic risk indicator	$p \leq 0.25$	$0.4 \leq p \leq 0.6$	$p \geq 0.75$	$p \leq 0.25$	$0.4 \leq p \leq 0.6$	$p \geq 0.75$	$p \leq 0.25$	$0.4 \leq p \leq 0.6$	$p \geq 0.75$
1	absorption									
2	Delta Absortion									
3	AIM									
4	CatFin							•		
5	GZ spread									
6	Baa/Aaa Bond yield									
7	TED spread									
8	Term Spread				•	•		•	•	•
9	Baa/10-yr T-rate spread									
10	Mortg-GS10 Spread									
11	Comm. paper-3mT-Bill spread	•	•	•	•	•			•	
12	Excess Bond Premium									
13	Intl. Spillover									
14	CoVaR									
15	Delta CoVaR									
16	Book lvg.				•	•			•	
17	Mkt. Lvg.									
18	DCI				•	•	•			
19	MES									
20	MES-BE									
21	Volatility									
22	Size conc.		•	•	•	•	•	•	•	•
23	Turbulence									
24	PQR									
25	Average DD.									
26	portfolio DD.									
27	MRI CITI Index									
28	CAPE		•	•						
29	VXO									
30	Sent. Index									
31	Credit-to-gdp gap									
32	Debt Service Ratio					•		•	•	
33	Loan Supply				•	•		•	•	

Table 2: Systemic Risk indicators selected by the Spike-and-Slab prior as relevant in explaining different quantile intervals of the predictive distribution of GDP growth at different horizons.

Table 2 shows the systemic risk indicators selected, from the set of all measures considered, for a given quantile interval and a prediction horizon. The quantiles p are divided in three intervals - lower $p \leq 0.25$, middle $0.4 \leq p \leq 0.6$ and upper $p \geq 0.75$. Hence, a systemic risk indicator deemed relevant in explaining economic downturns is expected to be selected for the lower interval. In addition, we consider three projection horizons - one, four and eight quarters ahead. Thus, we can distinguish between indicators that signal future economic developments in the very short term and with some time in advance (up to two years). This is important for policy reasons as addressing risks with measures such as macroprudential policies might take some time.

There are several messages suggested by Table 2. Less than halve of the 33 systemic risk indicators considered are selected to explain the dynamics of GDP growth, across quantiles. This reflects an overall poor in-sample fit of many measures and potential multicollinearity stemming from overlapping information that many of these measures give. In particular, systemic risk measures are expected to signal risks to growth and thus explain the lower tail of the predictive GDP growth distribution. However, results suggest that even less indicators are selected for inclusion in the lower quantile regression ($p \leq 0.25$). This highlights that predicting economic downturns such as recessions adds even more difficulty to the already challenging task of predicting middle quantiles of real economic growth.

Strikingly, only the spread between commercial paper and short term treasury bills is selected as a predictor for lower quantiles of GDP one quarter ahead. Notwithstanding, the algorithm picks significantly more predictors when explaining GDP growth one and two years ahead. It can be seen that Book leverage and the concentration index of financial institutions, Credit Supply, the term spread, the DCI, the CatFin and the Debt Service Ratio contain information content relevant in signaling future economic developments 4 quarters ahead.

5.1 Robustness

To understand the extent to which the results presented so far change with the specific technique chosen to impose sparsity, we use the additional methods discussed - Bayesian Lasso and Ridge - and report the finding in Table 3. The first column shows the main result previously discussed and featured in Table 2, while columns 2 and 3 show the variables selected by the Lasso and Ridge priors as predictors of the distribution of GDP growth one year ahead.

Compared to the Spike-and-Slab prior, The Bayesian Lasso selects a similar number of systemic risk indicators as relevant covariates to explain lower quantiles of GDP growth. In total only 6 of such indicators are chosen by the algorithm out of the 33 eligible covariates. On the contrary the Ridge prior selects 8 predictors. This compares with 6 variables selected by the Spike-and-Slab. However, the main result that only a select few systemic risk indicators contain relevant information in predicting future economic developments is common across methods.

The variables selected also differ across methods. However, some predictors such as the term spread, the size concentration in the financial sector are chosen by all methods as relevant explanatory variables of economic downturns. This is consistent with the well established causal relationship between the term spread and real economic activity (see Stock and Watson (2003) for a review of the literature on this subject). A salient difference refers to the importance attributed to stock market variables such as the VXO and CAPE. Contrary to what is implied by the Spike-and-Slab algorithm, both the Bayesian Lasso and the Ridge suggest that these indicators meaningfully signal economic downturns. Moreover, the Ridge prior selects both the Spread between the Mortgage rate and the 10 year Treasury Bond and The Commercial Paper / Treasury Bill as relevant predictors, across quantiles of output growth.

#	Systemic risk indicator	Spike-and-Slab			Lasso			Ridge		
		$p \leq 0.25$	$0.4 \leq p \leq 0.6$	$p \geq 0.75$	$p \leq 0.25$	$0.4 \leq p \leq 0.6$	$p \geq 0.75$	$p \leq 0.25$	$0.4 \leq p \leq 0.6$	$p \geq 0.75$
1	absorption									
2	Delta Absortion									
3	AIM									
4	CatFin									
5	GZ spread									
6	Baa/Aaa Bond yield									
7	TED spread									
8	Term Spread	•	•		•	•	•	•	•	•
9	Baa/10-yr T-rate spread				•	•	•			
10	Mortg-GS10 Spread							•	•	
11	Comm. paper-3mT-Bill spread	•	•					•	•	•
12	Excess Bond Premium									
13	Intl. Spillover									
14	CoVaR									
15	Delta CoVaR									
16	Book lvg.	•	•							
17	Mkt. Lvg.									
18	DCI	•	•	•						
19	MES									
20	MES-BE									
21	Volatility									
22	Size conc.	•	•	•	•	•	•	•	•	•
23	Turbulence				•	•	•			
24	PQR									
25	Average DD.									
26	portfolio DD.									
27	MRI CITI Index									
28	CAPE				•	•	•	•	•	•
29	VXO				•	•	•	•	•	
30	Sent. Index									
31	Credit-to-gdp gap									
32	Debt Service Ratio		•					•	•	•
33	Loan Supply	•	•					•	•	•

Table 3: Systemic Risk indicators selected by the Spike-and-Slab, Bayesian Lasso and Ridge priors as relevant in explaining different quantile intervals of the predictive distribution of GDP growth, one year ahead.

5.2 Time-varying relevance of Systemic Risk indicators

One important limitation of the in-sample analysis discussed so far is that it does not account for possible structural breaks of the parameters. In other words, full sample results are unable to capture potential changes in the relevance of individual variables across the sample considered. This is important from an economic point of view because, each systemic risk indicator summarizes the information of a particular sector of the financial system, forcefully leaving out other pieces of information that might be more important in driving economic outcomes in some periods and less relevant in others. Each recession is characterized by its own idiosyncrasies. Therefore, it is natural that some systemic risk indicators are only relevant in some subset of the sample considered.

To examine this issue, we re-estimate the model over a rolling window of 60 quarters. The rolling sample length is chosen to include roughly 3 business cycles that last on average roughly 5 years [7]. In practice, we re-estimate the model iteratively every quarter and perform variable selection.

Figure 2 summarizes the results for the iterative estimation exercise, one year ahead. The graph shows which of the 33 systemic risk measures listed and numbered in Table 1 are automatically selected by the spike-and-slab algorithm at each time period, throughout the full sample. For a given quarter, the selected indicators are highlighted with a coloured ring. The colour of the ring is defined by the probability of inclusion of each variable, which is given by the posterior median of γ (see methodology section) should the variable be included. The probability of inclusion measures the likelihood that a given variable is selected by the algorithm within each Gibbs Sampler pass through. The closer it is to 1, the stronger the predictive content of each indicator.

Each panel in Figure 2 describes the predictors included in the quantile regression for a given quantile interval. For instance, the upper panel shows the set of variables relevant in explaining economic downturns - the lower 25th quantile of GDP growth. Some main results are worth highlighting. First, consistent with the finding previously discussed, less systemic risk measures are picked as predictors of lower quantiles of GDP growth, comparing to middle quantile, suggesting they are less informative about economic downturns. Second, it is possible to observe that the term spread is consistently selected by the algorithm from the beginning of the sample until the 2000's and is not selected thereafter. This finding corroborates the argument of many studies suggesting that the ability of the term spread to forecast output growth has diminished in recent years (see Paya et al. (2004); Haubrich (2004); D'Agostino et al. (2006); Bordo and Haubrich (2008)). Third, From the 2000's, the Market leverage of financial institutions emerges as the most relevant predictor of future economic developments in general and economic downturns in particular, within the systemic risk measures considered. Between 1995-2000, the CAPE, CoVaR and delta CoVaR are selected as predictors of economic downturns consistently across quantiles. Some pockets of predictability can be observed across the sample. However, no other indicators are selected consistently for the one year

[7] According to the NBER's Business Cycle Dating Committee, between 1945 and 2009 there were 11 cycles.

ahead forecasting horizon.

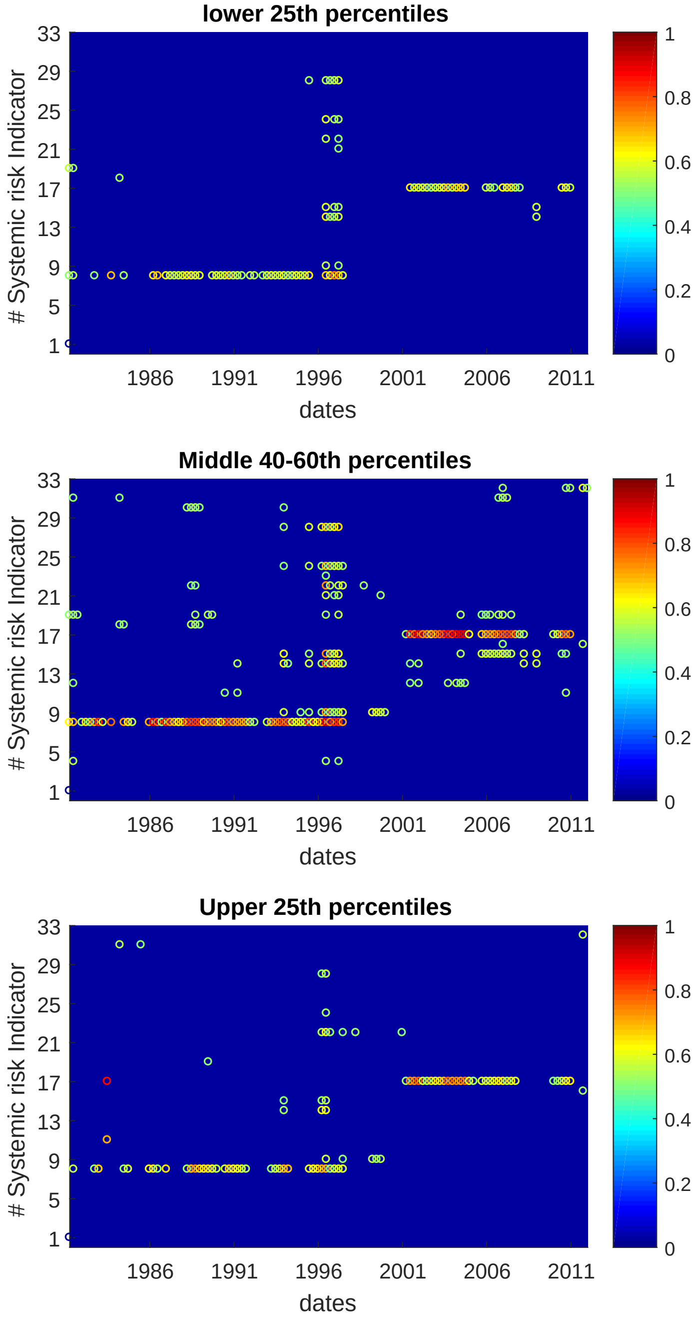

Figure 2: Systemic risk indicators automatically selected as predictors of three quantile intervals of GDP growth, 4 quarters ahead and respective probability of inclusion. Colour of the rings depict the probability of inclusion of each indicator, for each period.

Figures 7 and 8 provide analogous information for different prediction horizons. Figure 7 shows the variables selected for prediction 1 quarter ahead. It suggests an even poorer performance of systemic risk indicators in predicting economic downturns that correspond to lower quantiles of GDP growth. Market leverage remains the leading indicator. However, the DCI is selected between 1983 and 1990, while the Commercial paper spread and the Excess Bond Premium feature in regressions from 2004 onwards.

A larger set of indicators are selected as predictors of middle quantiles of GDP growth. Market leverage, DCI, the Commercial paper spread and Excess Bond Premium are selected for the periods that were also relevant in explaining lower quantiles. Furthermore, other measures emerge as relevant in predicting middle quantiles of the GDP growth distribution. It is the case of the BAA bond corporate spread and MES-BE selected between 1985-1990, the size concentration index and CAPE between 1995 and early 2000's. Lastly, the delta absorption and the GZ spread are picked up by the algorithm intermittently before 1990.

Figure 8 highlights the difficulty of finding informative signals of systemic risk in the longer term, two years in advance. The number of predictors chosen by the algorithm are less when compared to shorter horizons. Literally no systemic risk indicator is found to contain information content of economic downturns post-2007. For middle quantiles however, the term spread remains informative for the period until 2000. Curiously, the credit-to-GDP gap has some predictive content across quantiles. However, this is only observed prior to 1990.

5.3 Forecasting Output Growth using Systemic Risk indicators

The pseudo out-of-sample statistics presented in Table 4 are based on forecasts of GDP growth computed by augmenting an AR(2) model with each systemic risk indicator considered. The idea of this exercise is to understand if the inclusion of a given indicator improves the performance of a random walk model. The Mean Square Forecast Error (MSFE) of this forecast is then compared to the baseline AR(2), which is the benchmark in this framework. Model estimation and variable selection is performed with a 15 year rolling window of data (ie, as the forecasting exercise proceeds through time, only data from the previous 15 years is used for estimation). The sample relative MSFE is computed relative to the AR benchmark from 1978-2009.

#	Systemic risk indicator	1978-85			1985-2000			2000-09		
		horizon			horizon			horizon		
		h = 1	h = 4	h = 8	h = 1	h = 4	h = 8	h = 1	h = 4	h = 8
		MSFE								
	baseline AR(2)	19.29	9.10	5.21	4.11	1.98	1.474	6.93	5.55	3.65
	Quantile regression	14.99	9.50	8.15	5.12	3.55	2.986	8.06	8.06	3.77
		MSFE relative to the AR(2)								
1	absorption	1.04	1.12	1.01	1.03	1.17	1.49	1.03	1.04	1.05
2	Delta Absortion	0.92	0.95	1.00	1.26	1.13	1.11	1.02	1.01	0.99
3	AIM	1.27	1.40	1.53	1.00	1.00	0.99	0.99	0.99	1.00
4	CatFin	1.13	1.10	1.24	1.17	1.23	1.09	1.01	1.11	1.08
5	GZ spread	0.90	0.83	0.95	1.14	1.25	1.06	0.86	0.88	0.95
6	Baa/Aaa Bond yield	0.98	0.87	0.93	1.15	1.26	1.35	0.93	0.90	0.94
7	TED spread				1.05	1.05	1.34	0.79	1.10	0.92
8	Term Spread	1.11	0.77	0.57	1.05	1.36	1.42	1.02	0.99	0.92
9	Baa/10-yr T-rate spread	1.09	1.38	0.97	1.05	1.18	1.40	1.07	1.29	1.07
10	Mortg-GS10 Spread	1.11	1.36	1.64	1.12	1.12	1.21	0.87	0.88	0.88
11	Comm. paper-3mT-Bill spread	0.79	0.71	0.88	1.27	1.34	1.07	0.74	0.90	0.87
12	Excess Bond Premium	1.02	1.15	0.85	0.98	0.77	1.30	0.98	1.20	1.44
13	Intl. Spillover	1.04	1.02	0.97	1.02	1.07	0.99	0.96	1.00	1.02
14	CoVaR	1.04	1.05	0.95	1.15	1.28	0.98	1.02	1.09	1.07
15	Delta CoVaR	1.00	1.09	0.96	1.13	1.09	1.00	1.03	1.08	1.00
16	Book lvg.	1.04	1.08	1.54	1.04	1.17	1.42	1.01	0.99	0.92
17	Mkt. Lvg.	0.93	1.02	1.08	1.14	1.34	1.33	1.05	1.39	1.17
18	DCI	0.84	0.96	1.17	1.65	2.14	1.46	1.08	1.45	1.05
19	MES	1.06	1.07	0.97	1.05	1.31	1.69	1.05	1.11	1.06
20	MES-BE	1.06	1.05	0.99	1.22	1.24	1.28	1.11	1.43	1.42
21	Volatility	1.15	1.15	1.20	1.08	1.17	1.21	1.02	1.11	1.09
22	Size conc.	1.01	1.06	1.02	1.08	1.12	1.89	1.05	1.14	1.17
23	Turbulence	1.06	1.01	0.94	1.13	0.97	0.88	0.95	1.18	0.95
24	PQR	1.00	0.82	0.83	1.08	1.29	1.11	0.95	1.06	1.00
25	Average DD.									
26	portfolio DD.									
27	MRI CITI Index	1.02	1.01	1.00	1.28	1.11	1.06	0.98	1.10	1.14
28	CAPE	0.84	0.78	0.97	1.03	1.12	1.00	0.89	0.91	1.00
29	VXO	1.11	1.14	1.11	1.49	1.24	1.20	1.00	1.02	1.05
30	Sent. Index	1.04	1.15	1.23	1.01	1.27	1.61	1.07	1.17	1.10
31	Credit-to-gdp gap	1.14	1.55	3.73	1.04	1.17	1.84	1.04	0.87	0.67
32	Debt Service Ratio							1.02	0.95	1.02
33	Loan Supply							1.02	1.09	1.23

Table 4: Summary of pseudo out-of-sample forecast accuracy for three periods and three forecast horizons. Entries for each systemic risk indicator refer to the MSFE for the combined iterative forecasts constructed by augmenting an AR(2), relative to the AR(2) bechmark.

The performance of the various individual systemic risk indicators relative to the autoregressive benchmark is summarized in Table 4 for one, four and eight quarters ahead forecasts of GDP growth, at a quarterly sample frequency. The full sample considered is spit into three sub-samples (1978-85, 1985-2000 and 2000-09) following Stock and Watson (2003).

The first two rows report the Mean Squared Forecast Error of the pseudo out-of-sample benchmark autoregressive forecasts in the three sub-samples considered and quantile regression forecasts for the 50th percentile, corresponding to the median. From the third row onwards relative MSFE are reported. These should be interpreted as the percentage improvement/deterioration of the forecast of GDP growth, relative to the random walk benchmark. A systemic risk indicator is considered to have relevant information content in predicting output growth if the relative MSFE is lower than 1. On the contrary, if a given measures has relative MSFE values greater or equal to 1, we conclude that it does not improve forecasting performance as it fails to outperform the random walk benchmark.

Inspection of Table 4 reveals that some systemic risk indicators forecast relatively well in some subsamples. For example, the GDP growth forecast 8 quarters ahead based on the term spread has a relative MSFE of 0.57 in the first subsample, suggesting a 43 percent improvement in this period relative to the random walk benchmark. Corporate bond spreads such as the GZ spread and the Baa/Aaa yield spread and also the Commercial paper/t-bill spread are found to improve benchmark forecasts between 1978-85. For the most recent subsample between 2000-2009 the CAPE, Commercial paper, mortgage and GZ spreads are found to improve forecasts.

Overall however, very few systemic risk indicators improve forecasting performance of GDP growth. Moreover, improvements in forecasting ability are infrequent and isolated.

6 Conclusion

In this paper we study the link between output growth and systemic risk in light of a Bayesian quantile regression model that captures the nonlinear nature of this relationship. We empirically assess the relevance of 33 systemic risk indicators to explain the full spectrum of the distribution of GDP growth and analyse the practical utility of these indicators in improving economic growth forecasting. To deal with a large number of possible predictors, we employ sparse-modelling techniques to perform variable selection and shrinkage. Describing the relation between systemic risk and real activity in a parsimonious way is motivated by statistical reasons - to avoid overfitting and unnecessary parameter proliferation; but it is also justified on economic grounds since earlier literature suggests that a small subset of systemic risk indicators are relevant in describing the interaction between systemic risk and output. Understanding which particular indicator merits inclusion in the regression to explain growth fragility is unclear a priori but important for two main reasons. First, it is essential to understand when and from which part of the financial system are risks originating. Secondly, from a statistical viewpoint, it is necessary to assess the predictive content of each systemic risk measure to understand its suitability as an early warning indicator of a recession. In this regards, we specify a "spike-and-slab" prior that is used to derive the main results and analyse the sensitivity of the results found by using alternative methods of imposing sparsity in the model. We find that results are robust to the use of a Bayesian Lasso and Ridge prior. In this framework, different systemic risk measures are automatically chosen to predict different quantiles of the GDP growth distribution, across time.

In-sample analysis suggests that very few systemic risk measures contain relevant information that explain future developments of GDP growth. Moreover, even fewer systemic risk indicators are selected as relevant predictors of lower quantiles of GDP growth, suggesting that it is particularly hard to foresee economic downturns and that most systemic risk indicators contain limited recession-relevant information. In another domain, we analyse the extent to which results change when we re-estimate the model using a rolling window of data covering the length of approximately three average business cycles. We find that different systemic risk indicators forecast real activity at different periods of time. The term spread is the most relevant indicator in predicting future develops of GDP before the year 2000. From that year onwards, it is market leverage of financial institutions that emerges as the main predictor within the set we consider.

Out-of-sample analysis corroborates the results previously found. Systemic risk indicators improve economic growth forecasting occasionally but these pockets of predictability are rare and short-lasting.

References

Acharya, V., Engle, R., and Richardson, M. (2012). Capital Shortfall: A New Approach to Ranking and Regulating Systemic Risks. *American Economic Review*, 102(3):59–64.

Acharya, V. V., Pedersen, L. H., Philippon, T., and Richardson, M. (2010). Measuring Systemic Risk. (312417).

Acharya, V. V., Pedersen, L. H., Philippon, T., and Richardson, M. (2017). Measuring Systemic Risk. *Review of Financial Studies*, 30(1):2–47.

Adrian, T., Boyarchenko, N., and Giannone, D. (2016). Vulnerable Growth. (November).

Adrian, T. and Brunnermeier, M. K. (2016). CoVaR. *American Economic Review*, 106(7):1705–1741.

Adrian, T., Moench, E., and Shin, H. S. (2010). Macro risk premium and intermediary balance sheet quantities. *IMF Economic Review*, 58(1):179–207.

Aldasoro, I., Borio, C., and Drehmann, M. (2018). Early Warning Indicators of Banking Crises: Expanding the Family. *BIS Quarterly Review*, (March):29–45.

Allen, L., Bali, T. G., and Tang, Y. (2012). Does Systemic Risk in the Financial Sector Predict Future Economic Downturns? *Review of Financial Studies*, 25(10):3000–3036.

Amihud, Y. (2002). Illiquidity and stock returns: Cross-section and time-series effects. *Journal of Financial Markets*, 5(1):31–56.

Baker, M. and Wurgler, J. (2006). Investor Sentiment and the Cross-Section of Stock Returns. *The Journal of Finance*, 61(4):1645–1680.

Baskaya, Y. S., di Giovanni, J., Kalemli-Ozcan, S., and Ulu, M. F. (2017). International Spillovers and Local Credit Cycles. Technical report, National Bureau of Economic Research, Cambridge, MA.

Bernanke, B. S., Boivin, J., and Eliasz, P. (2005). Measuring the effects of monetary policy: A factor-augmented vector autoregressive (FAVAR) approach. *Quarterly Journal of Economics*, 120(1):387–422.

Bernanke, B. S., Gertler, M., and Gilchrist, S. (1999). Chapter 21 The financial accelerator in a quantitative business cycle framework. pages 1341–1393.

Billio, M., Getmansky, M., Lo, A. W., and Pelizzon, L. (2012). Econometric measures of connectedness and systemic risk in the finance and insurance sectors. *Journal of Financial Economics*, 104(3):535–559.

Bisias, D., Flood, M., Lo, A. W., and Valavanis, S. (2012a). A Survey of Systemic Risk Analytics. *Annual Review of Financial Economics*, 4(1):255–296.

Bisias, D., Flood, M., Lo, A. W., and Valavanis, S. (2012b). A survey of systemic risk analytics. *Annual Review of Financial Economics*, 4(1):255–296.

Bleaney, M., Mizen, P., and Veleanu, V. (2016). Bond Spreads and Economic Activity in Eight European Economies. *Economic Journal*, 126(598):2257–2291.

Bloom, N. (2009). The Impact of Uncertainty Shocks. *Econometrica*, 77(3):623–685.

Boivin, J., Giannoni, M. P., and Stevanović, D. (2018). Dynamic Effects of Credit Shocks in a Data-Rich Environment. *Journal of Business & Economic Statistics*, pages 0–0.

Bok, B., Caratelli, D., Giannone, D., Sbordone, A. M., and Tambalotti, A. (2018). Macroeconomic Nowcasting and Forecasting with Big Data. *Annual Review of Economics*, 10(1).

Bordo, M. D. and Haubrich, J. G. (2008). The yield curve as a predictor of growth: Long-run evidence, 1875-1997. *Review of Economics and Statistics*, 90(1):182–185.

Brownlees, C. T. and Engle, R. (2012). Volatility , Correlations and Tails For Systemic Risk Measurement. *Unpublished manuscript*, (July):16–18.

Brunnermeier, M. K. and Sannikov, Y. (2014). A Macroeconomic Model with a Financial Sector. *American Economic Review*, 104(2):379–421.

Caruana, J. (2010). Systemic risk: how to deal with it? *Bank for International Settlements, Other Publications*, (February):1–21.

Christiano, L. J., Motto, R., and Rostagno, M. (2014). Risk Shocks. *American Economic Review*, 104(1):27–65.

D'Agostino, A., Giannone, D., and Surico, P. (2006). (Un)predictability and Macroeconomic Stability. *ECB Working Paper Series*, 2006(605):43.

Dell'Ariccia, G., Igan, D., Laeven, L., Tong, H., Bakker, B., and Vandenbussche, J. (2012). Policies for Macrofinancial Stability: How to Deal with Credit Booms. *IMF Staff Discussion Note*, pages 1–46.

Diebold, F. and Yilmaz, K. (2009). Measuring Financial Asset Return and Volatility Spillovers, with Application to Global Equity Markets. *Economic Journal*, 119(January):158–171.

Diebold, F. X. and Yilmaz, K. (2011). On the Network Topology of Variance Decompositions: Measuring the Connectedness of Financial Firms. *SSRN Electronic Journal*.

Estrella, A. and Hardouvelis, G. A. (1991). The Term Structure as a Predictor of Real Economic Activity. *The Journal of Finance*, 46(2):555–576.

Estrella, A. and Mishkin, F. S. (1998). Predicting U.S. Recessions: Financial Variables as Leading Indicators. *Review of Economics and Statistics*, 80(1):45–61.

George, E. I. and McCulloch, R. E. (1993). Variable Selection via Gibbs Sampling. *Journal of the American Statistical Association*, 88(423):881–889.

Giannone, D., Lenza, M., and Primiceri, G. E. (2017). Economic Predictions with Big Data: The Illusion of Sparsity. *CEPR Discussion Paper 12256*, (March 2017):1–21.

Giglio, S., Kelly, B., and Pruitt, S. (2016). Systemic risk and the macroeconomy: An empirical evaluation. *Journal of Financial Economics*, 119(3):457–471.

Gilchrist, S., Boivin, J., Faust, J., Giannone, D., Lucca, D., Mccracken, M., and Gilchrist, S. (2009). Credit Market Shocks and Economic Fluctuations :.

Gilchrist, S. and Zakrajšek, E. (2012). Credit Spreads and Business Cycle Fluctuations. *American Economic Review*, 102(4):1692–1720.

Hartmann, P., Bandt, O. D., Molyneux, P., and Wilson, J. (2009). The concept of systemic risk. *Financial Stability Review*, (December 2006):134–142.

Haubrich, J. G. (2004). and the Credibility of the Monetary Regime :.

Hoerl, A. E. and Kennard, R. W. (1970). Ridge Regression: Biased Estimation for Nonorthogonal Problems. *Technometrics*, 42(1):80–86.

International Monetary Fund (2009). *Global Financial Stability Report - Responding to the financial crisis and measuring systemic risk.*

Jordà, Ò., Schularick, M., and Taylor, A. M. (2010). Financial Crisis, Credit Booms and External Imbalances: 140 Years of Lessons. *NBER*, 53(9):1689–1699.

Jordà, Ò., Schularick, M., and Taylor, A. M. (2015). Leveraged bubbles. *Journal of Monetary Economics*, 76:S1–S20.

Kapetanios, G., Marcellino, M., and Petrova, K. (2018). Analysis of the most recent modelling techniques for big data with particular attention to Bayesian ones. *Statistical Working papers Eurostat.*

Kiyotaki, N. and Moore, J. (1997). Credit Cycles. *Journal of Political Economy*, 105(2):211–248.

Koenker, R. and Bassett, G. (1978). Regression Quantiles. *Econometrica*, 46(1):33.

Koop, G. and Korobilis, D. (2011). UK macroeconomic forecasting with many predictors: Which models forecast best and when do they do so? *Economic Modelling*, 28(5):2307–2318.

Koop, G. M. (2013). Forecasting with Medium and Large Bayesian VARS. *Journal of Applied Econometrics*, 28(2):177–203.

Korobilis, D. (2013). VAR Forecasting using Bayesian Variable Selection. *Journal of Applied Econometrics*, 47(4):36–37.

Korobilis, D. (2017). Quantile regression forecasts of inflation under model uncertainty. *International Journal of Forecasting*, 33(1):11–20.

Kritzman, M. and Li, Y. (2010). Skulls, Financial Turbulence, and Risk Management. *Financial Analysts Journal*, 66(5).

Kritzman, M., Li, Y., Page, S., and Rigobon, R. (2011). Principal Components as a Measure of Systemic Risk. *The Journal of Portfolio Management*, 37(4):112–126.

Laeven, L. and Valencia, F. (2013). Systemic Banking Crises Database. *IMF Economic Review*, 61(2):225–270.

Li, Q., Xiy, R., and Lin, N. (2010). Bayesian regularized quantile regression. *Bayesian Analysis*, 5(3):533–556.

Li, Y. and Zhu, J. (2008). L1-norm quantile regression. *Journal of Computational and Graphical Statistics*, 17(1):163–185.

López-Salido, D., Stein, J. C., and Zakrajšek, E. (2017). Credit-market sentiment and the business cycle. *Quarterly Journal of Economics*, 132(3):1373–1426.

Lown, C. and Morgan, D. P. (2006). The Credit Cycle and the Business Cycle: New Findings Using the Loan Officer Opinion Survey. *Journal of Money, Credit and Banking*, 38(6):1575–1597.

Mitchell, T. J. and Beauchamp, J. J. (1988). Bayesian Variable Selection in Linear Regression. *Journal of the American Statistical Association*, 83(404):1023–1032.

Park, T. and Casella, G. (2008). The Bayesian Lasso. *Journal of the American Statistical Association*, 103(482):681–686.

Paya, I., Venetis, I. A., and Peel, D. A. (2004). Asymmetry in the link between the yield spread and industrial production: Threshold effects and forecasting. *Journal of Forecasting*, 23(5):373–384.

Peydro, J. L., Laeven, L., and Freixas, X. (2015). *Systemic risk, Crises and macroprudential regulation*, volume 53. MIT press.

Piazzesi, M., Ang, A., and Wei, M. (2006). What does the yield curve tell us about GDP growth? *Journal of Econometrics*, 131(1-2):359–403.

Rey, H. and Evgenia, P. (2015). Financial Flows and the International Monetary System. *NBER Working Paper Paper 21172*.

Saldías, M. (2013). Systemic risk analysis using forward-looking distance-to-default series. *Journal of Financial Stability*, 9(4):498–517.

Schularick, M. and Taylor, A. M. (2012a). Credit Booms Gone Bust : Monetary Policy , Leverage Cycles and Financial Crises , 1870 2008. *American Economic Review*, (1):1–5.

Schularick, M. and Taylor, A. M. (2012b). Credit Booms Gone Bust: Monetary Policy, Leverage Cycles, and Financial Crises, 18702008. *American Economic Review*, 102(2):1029–1061.

Shiller, R. J. (2005). *Irrational Exuberance*. Princeton University Press.

Simon Gilchrist, B., Zakrajsek, E., Acharya, V., Adrian, T., Chen, H., Krishnamurthy, A., Li, H., Swanson, E., and Wei, M. (2012). Credit Spreads and Business Cycle Fluctuations. *American Economic Review*, 102(4):1692–1720.

Stock, J. H. and Watson, M. W. (2003). Forecasting Output and Inflation: The Role of Asset Prices. *Journal of Economic Literature*, 41(3):788–829.

Stock, J. H. and Watson, M. W. (2006). Chapter 10 Forecasting with Many Predictors. pages 515–554.

Stock, J. H. and Watson, M. W. (2016a). *Dynamic Factor Models, Factor-Augmented Vector Autoregressions, and Structural Vector Autoregressions in Macroeconomics*, volume 2. Elsevier B.V., 1 edition.

Stock, J. H. and Watson, M. W. (2016b). Factor Models and Structural Vector Autoregressions in Macroeconomics. *Handbook of Macroeconomics*, pages 1–111.

Tibshirani, R. (1996). Regression Shrinkage and Selection via the Lasso. *Journal of the Royal Statistical Society*, 58(1):267–288.

Wu, Y. and Liu, Y. (2009). Variable selection in quantile regression. *Statistica Sinica*, 19:801–817.

Yu, K. and Moyeed, R. A. (2010). Bayesian quantile regression methods. *Journal of Applied Econometrics*, 54(January):1–34.

Appendix A: Data Description

Series	Description	t-code	Source
GDPR	Real Gross Domestic Product.	5	St. Louis Fed
CFNAI	Chicago Fed National Activity Index, aggregates the most important real activity variables.	1	St. Louis Fed
absorption	Captures the fraction of the total variance of a set of assets explained or "absorbed" by a the first eigenvectors/principal components.	1	Giglio et al. (2016)
Delta Absortion	First difference of absortion.	1	Giglio et al. (2016)
AIM	The ratio of absolute stock return to dollar volume.	2	Giglio et al. (2016)
CatFin	Value at Risk measure of a cross-section of financial firms.	1	Giglio et al. (2016)
GZ spread	Corporate Bond credit spread calculated using secondary market prices of senior unsecured bonds issued by a large representative sample of US non-financial firms.	5	S. Gilchrist Website
Baa/Aaa Bond yield	Spread btw Moody's Seasoned Baa Corporate Bond Yield and 10-Year Treasury rate	5	St. Louis Fed
TED spread	The difference between 3M Treasury bill and 3M LIBOR based on US dollars.	1	Stock and Watson (2016b)
Term Spread	The difference between 3M and 10Yr Treasury bill.	1	Stock and Watson (2016b)
Baa/10-yr T-rate spread	The spread between Moody's Seasoned Baa Corporate Bond and the 10Yr Treasury bill.	1	St. Louis Fed
Mortg-GS10 Spread	The spread btw 30-Year Conventional Mortgage Rate and 10Yr Treasury bill.	1	Stock and Watson (2016b)
CP3FM-TB3MS	The spread btw 3-Month AA Financial Commercial Paper Rate and 3M Treasury bill.	1	Stock and Watson (2016b)
Excess Bond Premium	The residual component of the GZ spread that reflects investor attitudes toward corporate credit risk.	1	S. Gilchrist Website
Intl. Spillover	Cross-country comovement in a set of macroeconomic variables.	5	Giglio et al. (2016)
CoVaR	Value at Risk (VaR) of the financial system conditional on institutions being under distress.	1	Giglio et al. (2016)
Delta CoVaR	Derives from the CoVaR.	1	Giglio et al. (2016)
Book lvg.	Book leverage for the 20 biggest institutions in the US.	1	Giglio et al. (2016)
Mkt. Lvg.	Market leverage for the 20 biggest instutotions in the US.	1	Giglio et al. (2016)
DCI	Number of Granger causal relationships between bank equity returns, as a measure of interconnectedness.	1	Giglio et al. (2016)
MES	A firms expected equity loss when market falls below a certain threshold over a given horizon.	1	Giglio et al. (2016)
MES-BE	Derives from the MES.	1	Giglio et al. (2016)
Volatility	Average volatility of the equity returns of the 20 biggest financial firms in the US.	1	Giglio et al. (2016)
Size conc.	The market equity Herfindal index of the 100 biggest financial firms in the US.	1	Giglio et al. (2016)
Turbulence	Covariance relative to a longer-term covariance estimate of top US financial firms.	1	Giglio et al. (2016)
PQR	Principal Components of a set of systemic risk measures.	1	Giglio et al. (2016)
Average DD.	Average Distance to Default measures the markets perception of the average risk of insolvency among major US banks.	1	Cleveland Fed
Portfolio DD.	Portfolio Distance to Default measures the markets perception of the systematic insolvency risk of the banking system as a whole.	1	Cleveland Fed
MRI CITI Index	CitiBank Risk Aversion Indicator, aggregates a set of indicators capturing risk aversion in the financial markets.	1	Bloomberg
CAPE	Cyclical Adjusted Price-to-Earnings ratio, calculated as the ratio of the current price of the S&P500 by its inflation adjusted historical earnings record over the past 10 years.	5	R. Shiller's Website
VXO	Implied volatility of near-the-money options of the S&P500.	1	Bloomberg
Sent. Index	Stock Market Investor Sentiment Index.	1	J. Wurgler Website
Credit-to-gdp gap	Difference between the credit-to-GDP ratio and its long-term trend given in percentage points.	1	BIS, Data Warehouse
Debt Service Ratio	The ratio of interest payments plus amortisations to income for the private non-financial sector.	1	BIS, Data Warehouse
Loan Supply	Net Percentage of Domestic Banks Tightening Standards for Commercial and Industrial Loans to Large and Middle-Market Firms as reported by Senior Loan Officers.	1	St. Louis Fed

Table 5: Data Description and Sources. The transformation codes (t-code column) are 1:levels; 2:1st dif; 5:1st dif of the logarithm.

Appendix B: Algorithm for Posterior Inference

The model is based on the priors for the regression parameters and their variance as defined in the body of the paper and outlined below

$$\beta_\tau|\gamma_\tau, \delta_\tau \sim N(0, \gamma_\tau\delta_\tau^2)$$
$$\delta_\tau^{-2} \sim Gamma(a_0, a_1)$$

These priors are common to all three methods presented to perform model selection and shrinkage. The key parameter to enforce sparcity is γ_τ and we define a hierarchical structure where the hyperprior distribution defined dictates which method from the three (spike-and-slab, bayesian lasso and ridge) is used. In what follows we present the conditional posteriors necessary to set up the Gibbs Sampler used for full posterior inference.

1) Spike-and-Slab

The full posterior of the form

$$p(\beta_\tau, \delta_\tau^{-2}, \gamma, \pi_0, z|x, y, \tau) \propto$$
$$p(y|x, \tau, \beta_\tau, \delta_\tau^{-2}, \gamma, \pi_0, z)p(\beta_\tau, \delta_\tau^{-2}, \gamma, \pi_0, z) \propto$$
$$p(y|x, \tau, \beta_\tau, \delta_\tau^{-2}, \gamma, \pi_0, z)p(\beta_\tau|\delta_\tau^{-2}, \gamma, \pi_0, z)p(\delta_\tau^{-2}|\beta_\tau, \gamma, \pi_0, z)p(z|\beta_\tau, \delta_\tau^{-2}, \gamma, \pi_0)p(\gamma|\pi_0)p(\pi_0),$$

results from a straightforward application of the Bayes rule which states that it is proportional to the product of the likelihood and the priors for all parameters in the model. We define a grid that sets the quantiles considered τ from 5 to 95 with increments of 5. In practice, each quantile defined gives rise to an additional regression to estimate. Thus parameters vary across quantiles τ. The following points describe a full pass-throw of the Gibbs Sampler, for each quantile τ and each point a single block in the Sampler.

- The conditional posterior of β_τ is given by

$$p(\beta_\tau|\delta_\tau^{-2}, \gamma, \pi_0, z) \sim N(\bar{\beta}, \bar{V}),$$
$$with \quad \bar{\beta} = \bar{V}\Big[\sum_{t=1}^{T} \tilde{x}_t(y_t - \theta z_t)/\phi^2 z_t\Big], \bar{V} = \Big[\sum_{t=1}^{T} \frac{\tilde{x}_t{}'\tilde{x}_t}{\tau^2 z_t} + diag(\gamma_\tau\delta_\tau^{-2})\Big].$$

- The conditional posterior of δ_τ^{-2} is given by

$$p(\delta_\tau^{-2}|\beta_\tau,\gamma,\pi_0,z) \sim Gamma(\bar{a_0},\bar{a_1}),$$
$$with \quad \bar{a_0} = a_0 + 1/2 \quad and \quad \bar{a_1} = \beta_\tau^2/2 + a_1.$$

- The conditional posterior of z_t is given by

$$p(z_t|\beta_\tau,\gamma,\delta_\tau^{-2},\pi_0,) \sim GIG(\frac{1}{2},\bar{\kappa_0},\bar{\kappa_1}),$$
$$with \quad \bar{\kappa_0} = \sum_{t=1}^{T}(y_t - x_t\beta_\tau)/\phi \quad and \quad \bar{\kappa_1} = \sqrt{2+\theta^2}/\phi.$$

- The conditional posterior of each element of γ_τ is given by

$$p(\gamma_\tau|\beta_\tau,\delta_\tau^{-2},\pi_0,z) \sim Bernoulli(\pi_0).$$

- The conditional posterior of π_0 is given by

$$p(\pi_0|\beta_\tau,\gamma_\tau,\delta_\tau^{-2},z) \sim Beta(\bar{b_0},\bar{b_1}),$$
$$with \quad \bar{b_0} = 1 + b_0 \quad and \quad \bar{b_1} = n - 1 + b_1.$$

2) Bayesian Lasso

The full posterior of the form

$$p(\beta_\tau,\delta_\tau^{-2},\gamma,\pi_0,z|x,y,\tau) \propto$$
$$p(y|x,\tau,\beta_\tau,\delta_\tau^{-2},\gamma,\pi_0,z)p(\beta_\tau,\delta_\tau^{-2},\gamma,\pi_0,z) \propto$$
$$p(y|x,\tau,\beta_\tau,\delta_\tau^{-2},\gamma,\pi_0,z)p(\beta_\tau|\delta_\tau^{-2},\gamma,\pi_0,z)p(\delta_\tau^{-2}|\beta_\tau,\gamma,\pi_0,z)p(z|\beta_\tau,\delta_\tau^{-2},\gamma,\pi_0)p(\gamma|\lambda)p(\lambda),$$

has been proposed by Park and Casella (2008) to describe the Bayesian Lasso and requires very few alterations to the Gibbs sampler previously characterized. The descriptions below entails the blocks of the spike-and-slab algorithm that should be changed in order to achieve this specification.

- The conditional posterior γ_τ is given by

$$p(\gamma_\tau|\beta_\tau,\delta_\tau^{-2},\lambda,z) \sim GIG(\mu,\lambda),$$
$$with \quad \mu = \sqrt{\lambda\delta^{-2}\beta^{-2}}.$$

- The conditional posterior of λ is given by

$$p(\lambda|\beta_\tau, \gamma_\tau, \delta_\tau^{-2}, z) \sim Gamma(\bar{c_0}, \bar{c_1}),$$
$$with \quad \bar{c_0} = r + p \quad and \quad b_2 = \gamma/2 + \Delta.$$

Where Δ and r are hyperparameters that are set to 200 and 1 respectively.

3) Ridge Regression

A Bayesian interpretation of the Ridge regression is licit and has been discussed in the literature (see Kapetanios et al. (2018); Giannone et al. (2017)). In particular, Giannone et al. (2017) highlights that the Ridge regression can be viewed as a particular case of the spike-and-slab model. The authors highlight that the Ridge regression can be obtained by simply setting $\pi_0 = 1$. This alteration is effortless and does not require any change of the spike-and-Slab Gibbs Sampler described above. The full posterior is now given by

$$\begin{aligned} p(\beta_\tau, \delta_\tau^{-2}, z|x, y, \tau) &\propto \\ p(y|x, \tau, \beta_\tau, \delta_\tau^{-2}, z)p(\beta_\tau, \delta_\tau^{-2}, z) &\propto \\ p(y|x, \tau, \beta_\tau, \delta_\tau^{-2}, z)p(\beta_\tau|\delta_\tau^{-2}, z)p(\delta_\tau^{-2}|\beta_\tau, z)p(z|\beta_\tau, \delta_\tau^{-2}). \end{aligned}$$

This is simply the posterior of Normal-Inverse Gamma model.

Appendix C: Additional Figures

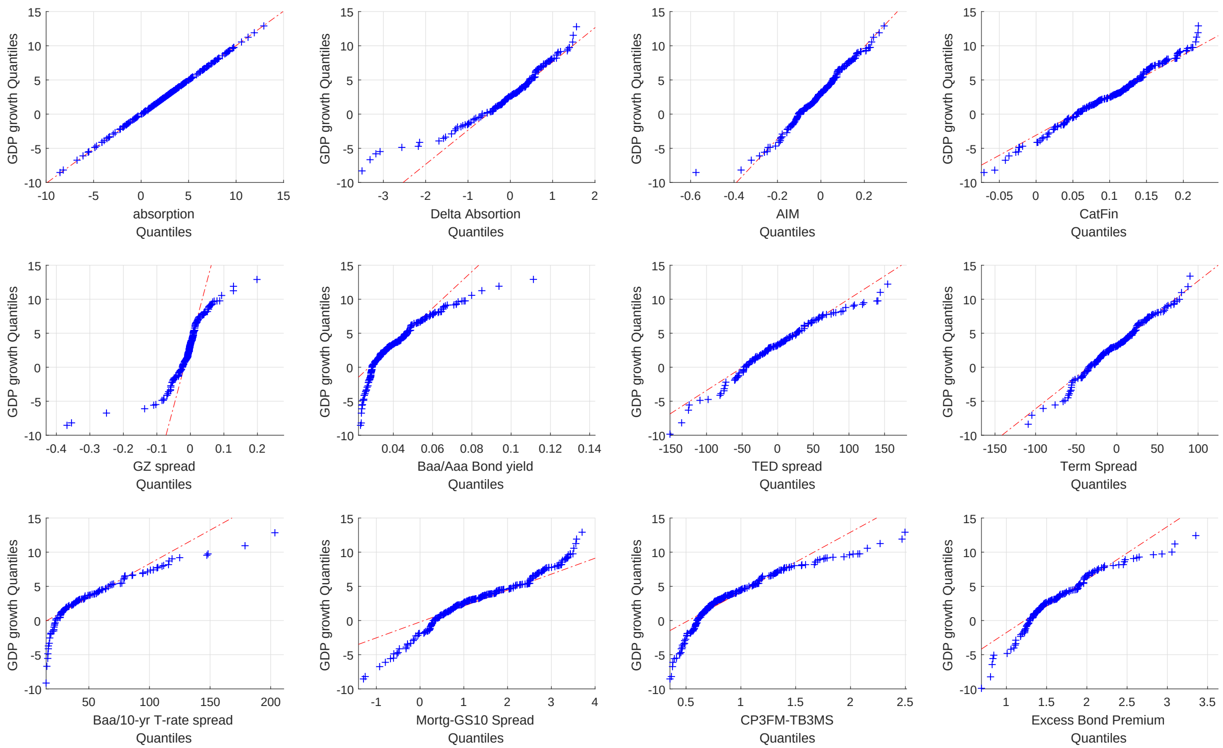

Figure 3: Q-Q (quantile-quantile) plot (I) of the GDP growth distribution (y-axis) against the distribution of each individual Systemic Risk measure (x-axis).

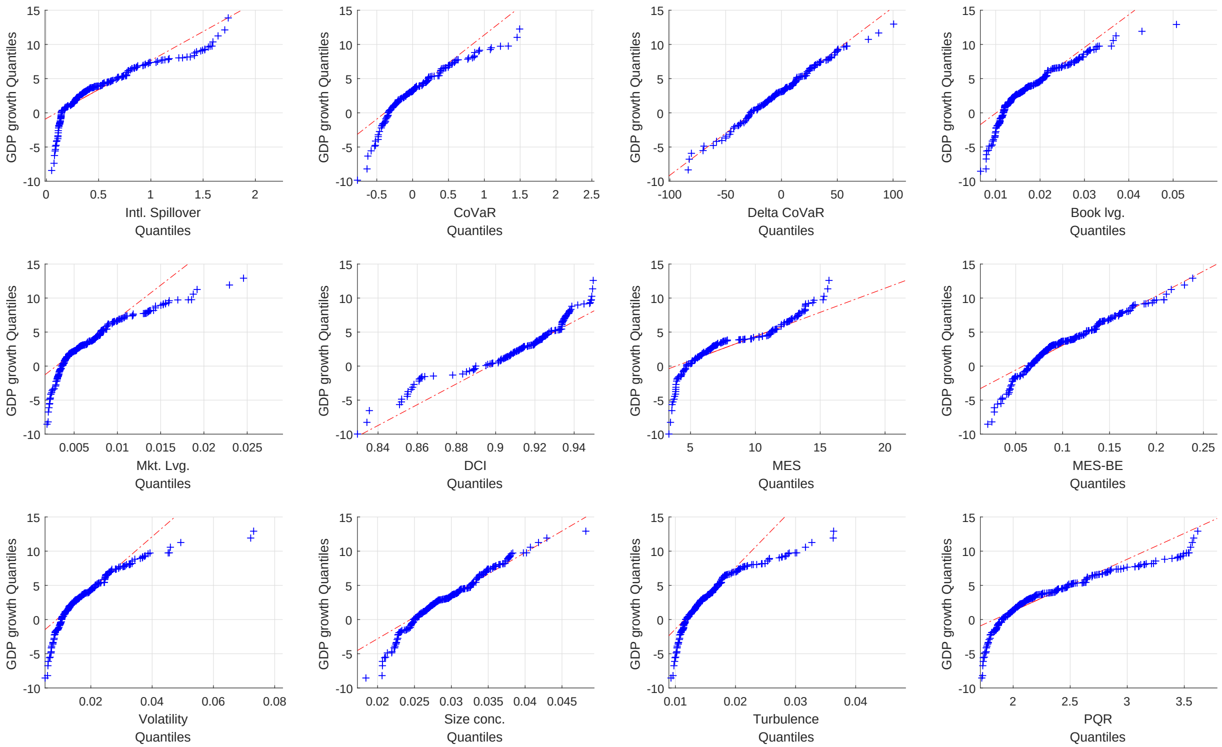

Figure 4: Q-Q (quantile-quantile) plot (II) of the GDP growth distribution (y-axis) against the distribution of each individual Systemic Risk measure (x-axis).

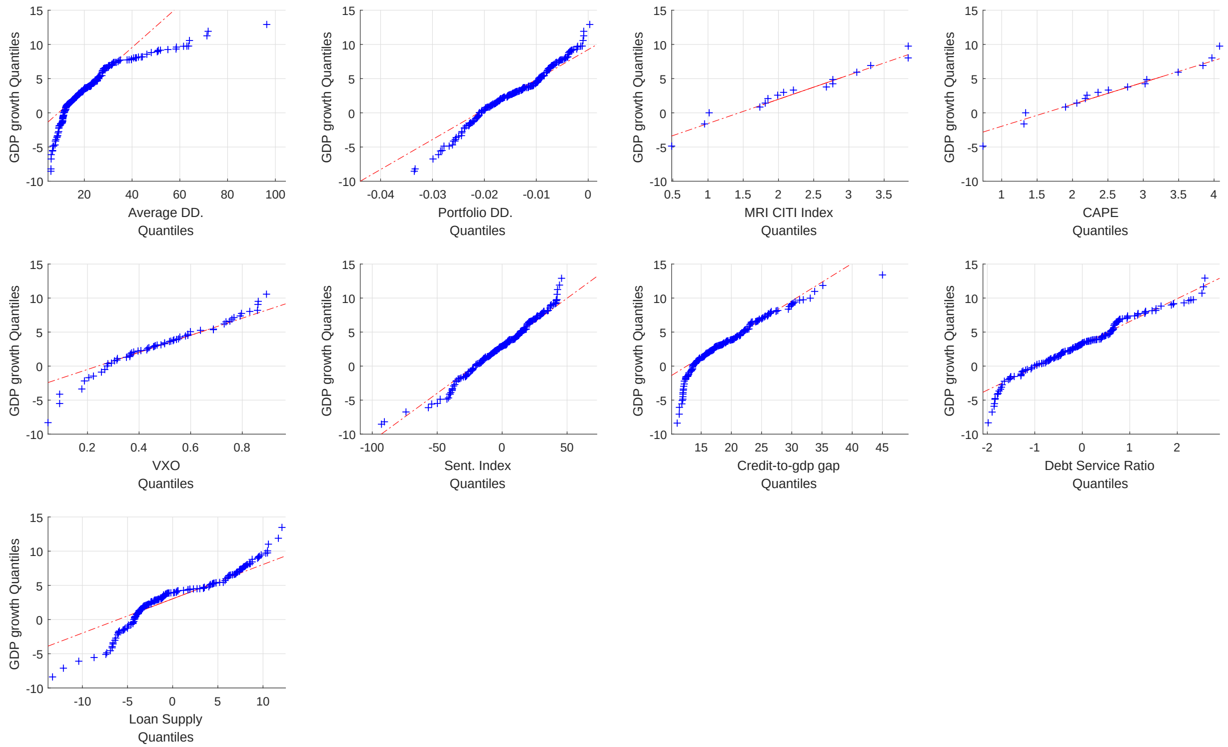

Figure 5: Q-Q (quantile-quantile) plot (III) of the GDP growth distribution (y-axis) against the distribution of each individual Systemic Risk measure (x-axis).

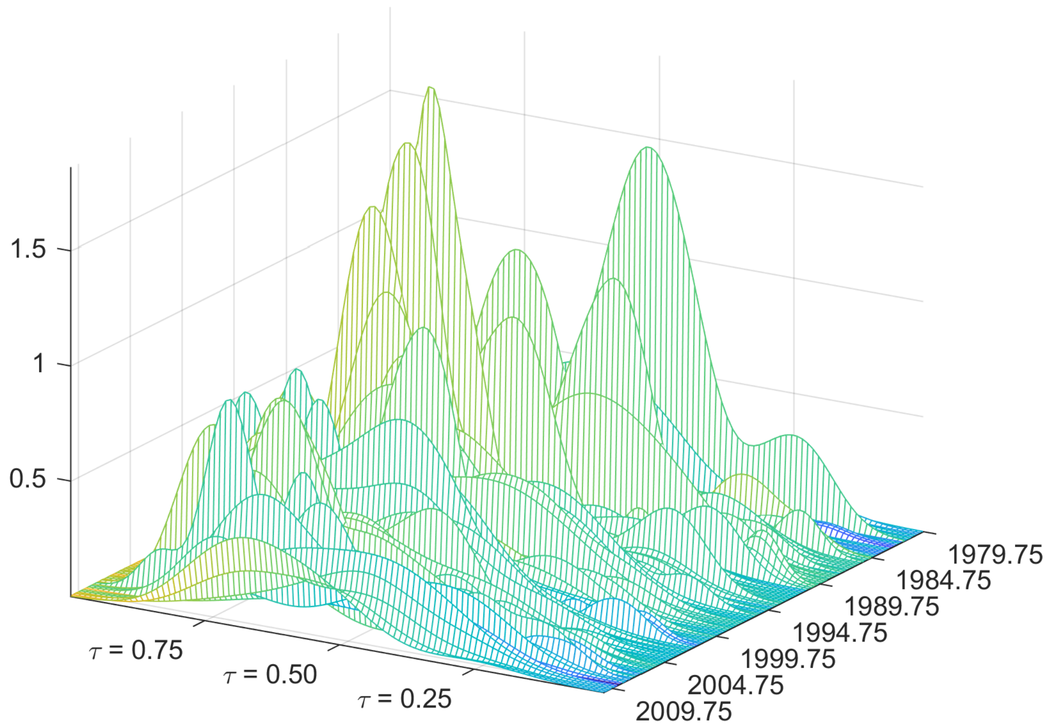

Figure 6: Quantile regression predictive densities, 4 quarters ahead, from 1979Q1 to 2011Q4.

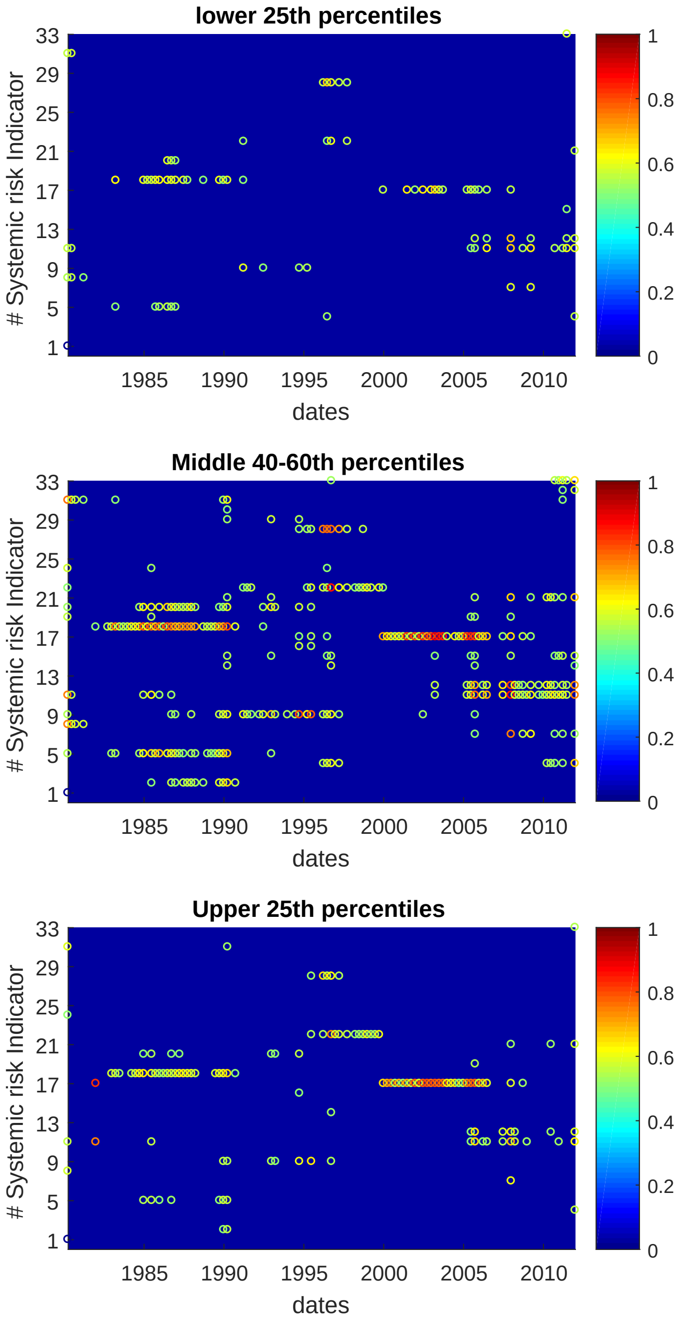

Figure 7: Systemic risk indicators automatically selected as predictors of the lower, middle and upper percentiles of GDP, 1 quarters ahead and respective probabilities of inclusion. Colour of the rings depict the probability of inclusion of each indicator, for each period.

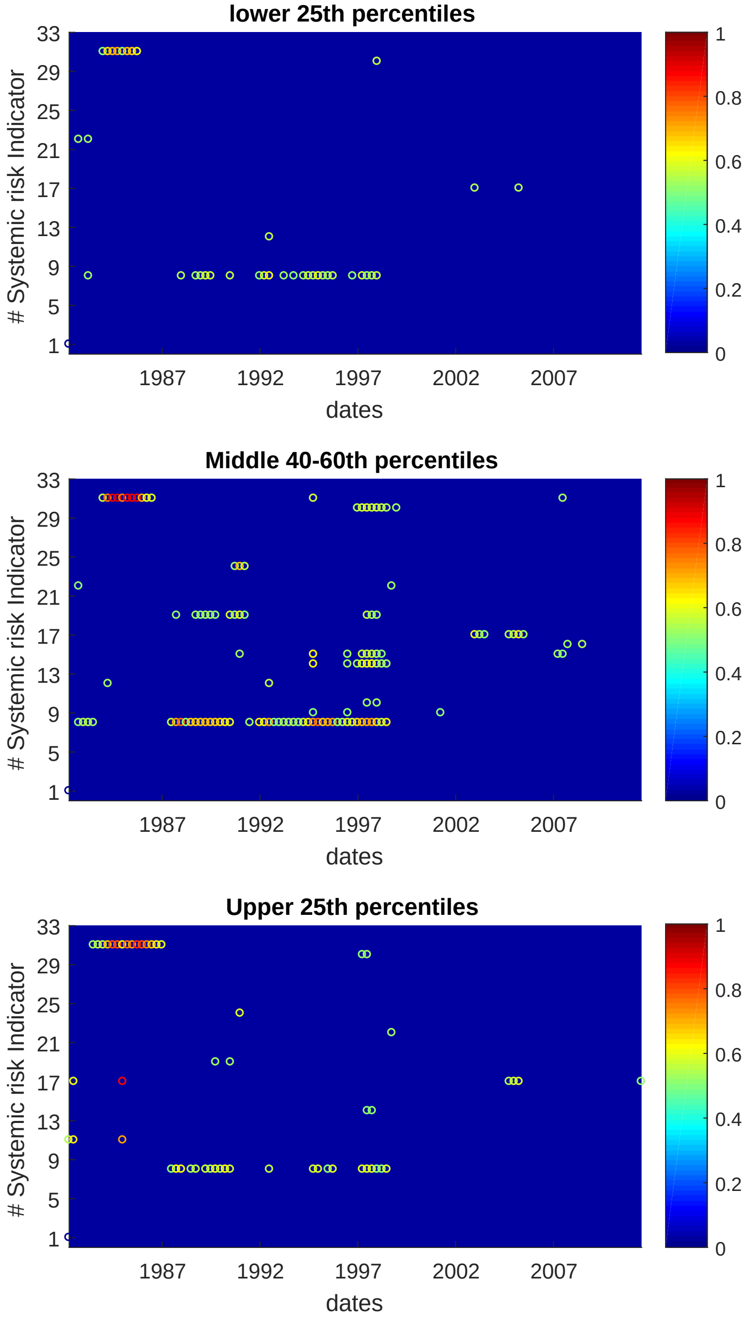

Figure 8: Systemic risk indicators automatically selected as predictors of the lower, middle and upper percentiles of GDP, 8 quarters ahead and respective probabilities of inclusion. Colour of the rings depict the probability of inclusion of each indicator, for each period.