Financial Condition Indexes in an Incomplete Data Environment

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Abstract

This paper proposes a mixed-frequency factor-augmented vector autoregressive model with time-varying coefficients and stochastic volatility to construct a financial conditions index (FCI). This framework is extended to allow for different unbalanced panel techniques based on probabilistic principal components. In an incomplete data setting, with up to 62% of data missing, the approach yields a less noisy FCI that tracks the movement of the underlying financial variables more accurately and delivers better macroeconomic forecasts, on average. Evidence may help reconcile opposing views in the literature with regards to the predictive power of financial indicators.

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1 Introduction

Financial conditions refer to *"the current state of financial variables that influence economic behavior and thereby the future of the economy"* Hatzius et al. (2010). Beyond their relevance in understanding monetary policy transmission (Adrian and Liang (2018)), recent research suggests financial conditions reflect downside risks to GDP because they capture frictions in credit markets and aggregate relevant information about future risks to the real economy (see Adrian et al. (2019) and Giglio et al. (2016)).

The importance of monitoring financial conditions has motivated the development of financial conditions indexes (FCI) that summarize commonalities in the movement of long-term interest rates, credit spreads, equity prices and exchange rates by a number of central banks, commercial banks and academics. Various closely-watched FCIs are currently disclosed on a regular basis[1] and are often viewed as an important barometer of credit conditions and financial imbalances in the economy. However, the development of a FCI is not a straightforward task. Financial conditions are unobserved in practice and therefore FCIs are subject to model uncertainty that should be taken into account during their construction.

FCIs summarize information from large panels of financial variables that are often unbalanced due to several reasons. One reason is that some time series are observed for longer periods that others. Another reason is related to the mixed-frequency nature of the data which implies that higher frequency variables are observed much more often than relatively lower frequency series. Lastly, some observations are missing at the end of the sample due to differences in publication lags, a feature of the data that is often referred to as a ragued edge problem.

Most FCI models avoid the problem of having to deal with incomplete datasets by restricting the sample size and aggregating higher frequency variables to a lower base frequency. Neglecting the mixed-frequency nature of the data leads to less informative indicators which may fail to capture potentially relevant movements in financial variables at high frequencies. An exception is Brave and Butters (2011) that employs Stock and Watson (2002) EM algorithm to estimate a Principal Component in a mixed-frequency setting. This allows the authors to estimate a FCI dating back to the early 1970s.

In another direction, most FCIs are built under the assumption of time-invariant parameters and volatilities. However, there are good reasons to believe that neither the importance of each financial variable in determining broad financial conditions

[1]See Hatzius et al. (2010) for a review of existing FCIs and GFSR (2017) for a discussion of cross-country FCIs.

nor the relation between financial conditions and the macroeconomy are constant over time. This point is particularly relevant when the intention is to estimate a FCI over long sample periods. The main contribution of Koop and Korobilis (2014) is to relax these assumptions and offer more flexibility in adjusting to the reality of stochastic volatilities, time-varying importance of financial variables driving financial conditions and changing macroeconomic relevance of financial conditions.

This paper constructs a financial conditions index that combines the flexibility of Koop and Korobilis (2014) with the comprehensive nature of Brave and Butters (2011) by allowing for mixed-frequency data. I use a mixed-frequency factor augmented vector autoregressive model with time-varying coefficients and stochastic volatility and extend this framework to allow for different unbalanced panel techniques based on probabilistic principal components, which leads to a more accurate measure of financial conditions in incomplete data environments. This approach inherits all the advantages of Koop and Korobilis (2014) and Brave and Butters (2011) by offering a flexible approach to measure financial conditions that allows for i) time-varying weights, which define the way each financial variable load into the FCI; ii) structural instability of the relationship between the FCI and the macroeconomy; iii) stochastic volatility and iv) joint modelling time series available at different frequencies.

Estimation follows closely the two-step procedure for dynamic factor models of Doz et al. (2011), applied to factor-augmented vector autoregressions, cast in state space form by Koop and Korobilis (2014). I extend the authors' work by allowing the model to be estimated with mixed-frequency data and by employing probabilistic PCA methods of Tipping and Bishop (1999) and Oba et al. (2003) with in-built regularization, particularly suitable for incomplete data.

The main purpose of the method proposed is to improve measurement of financial conditions. However, the versatility of the model allows the exploration of questions related to the macroeconomic relevance of financial conditions and how they changed over time. The framework also enables macroeconomic forecasting with mixed-frequency data. Estimation delivers a financial conditions index that yields better fit in incomplete data environments featuring high levels of sparsity. With up to 62 % of data missing, we manage to build a less noisy FCI that tracks the underlying financial variables more accurately. The in-sample Mean Squared Error (MSE) is estimated to reduce by as much as 50 % for the monthly base frequency when using probabilistic PCA as compared to standard PCA techniques commonly used in the literature. Moreover, evidence also suggests that the FCI based on such probabilistic based PCA methods delivers better forecasts for some pairs of frequency and time horizon, although this result is more discrete.

The exercise offers some insights on the relevance of financial variables in macroeconomic forecasting and may help reconcile opposing views in the literature on this subject. A number of papers have documented the unpredictability of GDP growth during the Great Moderation (see eg. D'Agostino et al. (2006); Rossi and Sekhposyan (2010)) and the fragile and unreliable predictive content of financial indicators (Stock and Watson (2003)). This evidence contrasts with some recent contributions documenting the relevance of financial conditions indexes and other financial indicators in predicting real activity (Hatzius et al. (2010); Koop and Korobilis (2014); Giglio et al. (2016); Adrian et al. (2019)). I find that FCIs estimated at a quarterly frequency seem to add value to forecasts of GDP. Whereas monthly FCIs do not. I interpret these results as a suggestion that the relevant cyclical behavior of financial variables is one occurring at lower frequencies. In theory, in a mixed-frequency environment the information set spanned by a monthly FCI will be greater. Nevertheless, the additional information in higher frequency fluctuations in financial variables appear to be too noisy and uninformative. Therefore, its inclusion does not add value to forecasting Output. Nevertheless, forecasts of interest rates and unemployment seem to improve with the inclusion of the FCI at a monthly frequency. In addition, the method used to estimate the FCI determines its macroeconomic predictive power. Parsimony seems to yield the best forecasting performance. The best performing model is one that does not account for any sort of time variation in its parameters.

The VIX emerges from our exercise as the most important driver of financial conditions since the early 1970s, which emphasizes the relevance of uncertainty and global risk appetite. The percentage of banks tightening lending standards, which can be interpreted as a measure of credit supply, the performance of below investment grade corporate debt assets and various measures of the term spread are also important determinants of financial conditions. I find that the sensitivity of GDP growth to shocks to financial conditions has reached unprecedented levels during the Great Recession, which is consistent with the fact that the crisis originated in the financial sphere of the economy. The impulse responses of GDP growth to financial condition shocks is more pronounced in the periods before and after the Great Moderation.

The remainder of the paper proceeds as follows. Section 2 outlines the model and explains the data and estimation methods. Section 3 discusses the results and section 4 concludes.

2 A Mixed-Frequency Financial Conditions Index with Structural Instabilities

2.1 MF-TVP-FAVAR

Consider a Mixed-Frequency Time-Varying Parameter Factor-Augmented VAR (MF-TVP-FAVAR), written as follows

$$X_t = \lambda_t^y Y_t + \lambda_t^F F_t + u_t, \tag{1}$$

$$\begin{bmatrix} Y_t \\ F_t \end{bmatrix} = B_{t,1} \begin{bmatrix} Y_{t-1} \\ F_{t-1} \end{bmatrix} + ... + B_{t,p} \begin{bmatrix} Y_{t-p} \\ F_{t-p} \end{bmatrix} + \varepsilon_t. \tag{2}$$

Where X_t is an $(N \times T)$ matrix of N mixed-frequency financial variables that load into the FCI denoted in this setting by F_t. λ_t^F are time-dependent loading parameters that define which financial variables x_{it} load into F_t at each point in time t. λ_t^Y denotes the coefficients of a set of observable mixed-frequency macroeconomic variables organized in Y_t, in the measurement equation (1). This term is key in ensuring that the variation in the financial variables that are attributable to changes in real economic conditions are purged from the F_t. The state equation (2) describes the relationship between macroeconomic variables and the financial condition index. The set of parameters that describe this relationship $B_{t,p}$ are also allowed to vary over time. All variables are reported at the highest frequency t amongst the data considered.

The model is completed by the last 2 state equations bellow.

$$\beta_t = \beta_{t-1} + \eta_t, \quad \eta_t \sim N(0, R_t), \tag{3}$$

$$\lambda_t = \lambda_{t-1} + v_t, \quad v_t \sim N(0, W_t), \tag{4}$$

which allow the parameters to smoothly change over time.

The model defined by equations (1) - (4) differs from that proposed by Koop and Korobilis (2014) in one important dimension. The time series that are included in Y_t and X_t need not have the same frequency. The model adjusts to the higher frequency amongst the variables included. Therefore, if we were to estimate the model with the data listed in Table 3, the base frequency would be daily, since that is the highest frequency amongst all variable considered [2]. However, variables

[2]Although it is possible to estimate the model at a daily frequency we set a weekly frequency as our high frequency benchmark. This is because in this particular applications the majority of variables considered are available at lower frequencies.

that are reported at other frequencies such as inflation which is observed monthly or GDP which is disclosed quarterly will still feature in the model, organized in a sparse matrix with many missing values which highlight the dates for which there are no releases/observations for the respective variables.

2.2 Data and Estimation

The financial variables used to construct the FCI are listed in Table 3 and include a broad range of variables that characterize the financial system. The observable sector of the model includes GDP growth, inflation, unemployment and the Federal Funds Rate as a measure of the stance of monetary policy. The model defined by equations (1)-(4) can be cast in state space form and is estimated through a Bayesian Kalman filtering and smoothing routine which allows for mixed-frequency data (see Appendix A for a detailed explanation of the algorithm).

The estimation algorithm extends the one derived in Koop and Korobilis (2014) to a mixed-frequency data environment and involves the following steps:

- **Step 1:** Estimate F_t using PCA methods which accommodate missing values.
- **Step 2:** Conditional on $\tilde{F}_t$, estimate the parameters in the MF-TVP-FAVAR by applying Kalman filtering and smoothing.
- **Step 3:** Conditional on the parameters estimated in Step 2 estimate $\hat{F}_t$, which is used as our FCI, by applying Kalman filtering and smoothing.

The mixed frequency nature of the model gives rise to sparsity, which requires proper treatment. This represents a challenge because a sparse dataset implies that at some points across the time dimension there are very few observations. This is particularly relevant for early periods in the sample for which a small subset of all variables considered are observed. One of the great advantages of estimation through Kalman filtering techniques is that it is straightforward to deal with missing values. The algorithm of Koop and Korobilis (2014) (Steps 2 and 3) is modified in accordance to Koopman and Commandeur (2008) by simply setting the Kalman Gain term to zero in each updating equation, whenever an observation is missing. This guarantees that in absence of any new information regarding a variable (ie, when it is missing), our best guess for the current state of the respective parameter remains unchanged.

Step 1 consists in using an algorithm to pre-estimate the factor F_t which accounts missing values, that is then used as an input to step 2. In this step, the challenge is to reconstruct the unbalanced panel of data X_t which contains missing observations. We discuss in detail the treatment of missing values next. In Step 3, a Kalman filter/smoother is passed-through the pre-estimated factor yielding our final estimate of the FCI. Steps 2 and 3 are similar the method proposed by Doz et al. (2011).

2.2.1 Mixed-Frequency adjustments and treatment of Missing Values

The missing values in our dataset occur for two reasons. First, not all variables are observed at the same frequency. Those that are observed at lower frequencies (say quarterly) will configure sparse columns in the dataset since they are not observed in many periods. Second, some time series start sooner than others or are missing at the end of sample giving rise to the so called "ragged-edge" problem. For instance, while GDP is observed throughout the sample period considered, that is not true for most of financial variables which are observed only for varying subsamples.

There are many alternatives in the literature to deal with mixed-frequency data [3]. We follow the methodology by Doz et al. (2011) that relies on a two-step estimation previously explained that combines principal components with the Kalman filter and smoother. First the parameters are estimated conditional on the estimated factors, where the latter are obtained through principal components calculated on a balanced version of the dataset. Then, the Kalman smoother is used to update the estimate of the signal variable on the basis of the full unbalanced dataset. As noted by Foroni and Marcellino (2013) the framework can be interpreted as a large bridge model whereby a large number of variables bridge monthly time series with forecasts of the quarterly variables.

Missing values in the measurement equation (1) and state equation (2) are treated by modifying a Kalman filter as explained above and detailed in section A.1. However, the existence of missing values in X_t need to be taken into account in Step 1 to pre-estimate F_t. The missing values in X_t can be dealt with using a variety of unbalanced panel methods. The following section discusses various alternatives available.

2.2.2 Principal Component Analysis in the presence of missing values

One of the most popular approaches to PCA is based on single value decomposition (SVD) of the covariance matrix of the data

$$C = N^{-1} X^T X = UDU^T, \tag{5}$$

[3] Foroni and Marcellino (2013) and Schorfheide and Song (2015) provide a comprehensive review of the different techniques.

where D contains the eigenvalues and U the eigenvectors of the covariance matrix C, normalized to have unit-length. This approach is only valid when all the observations are non-missing, that is, when the panel is balanced. In presence of missing values, Least-Square techniques are still available but need to be modified. In this paper I explore the following four alternative solutions to finding principal components in incomplete datasets.

i) *Standard PCA with missing values set to zero (PCA)*
ii) *Least Squares Expectation-Maximization PCA (EM-PCA)*
iii) *Probabilistic PCA (PPCA)*
iv) *Variational Bayesian PCA (VBPCA)*

These methods vary in several important ways [4]. The standard PCA removes missing values by setting them to zero. Because the variables in X are standardized, this is equivalent to setting each missing value to its unconditional mean. Next, the Expectation-Maximization PCA (EM-PCA) as proposed by Stock and Watson (2002), results in an imputed matrix X where the missing values are reconstructed such that the Least-Squares Likelihood is maximized. In each iteration any missing points are in-filled with their conditional mean.

The PCA and EM-PCA belong to the family of Least-Squares methods since they work by minimizing the squared residuals of the PC regression. Whereas the probabilistic PCA and the Variational Bayes PCA approach the PC regression from a probabilistic point of view. A probabilistic formulation of PCA offers a number of benefits, including a well-founded treatment of missing values, extendability and regularization. The PPCA is based on the work of Tipping and Bishop (1999). Estimation in an incomplete data setting is discussed in Ilin and Raiko (2010) which use an EM algorithm treating the latent factors as hidden variables. Although the PPCA performs some regularization (ie, the gaussian priors set on the factors penalize large values in F), this might be insufficient if the data is very sparse. An extension that allows for more strict penalization is proposed by Oba et al. (2003). This approach which I label Variational Bayesian PCA (VBPCA) consists in imposing priors over the remaining parameters in the model that penalize more complex explanations of the data. The main advantage of the last method discussed is that it allows automatic selection of the right number of components needed for PCA analysis.

To summarize, estimation of the Financial Condition Index proposed is based on the MF-TVP-FAVAR which is estimated in three steps. In the first step, the data of

[4] Here I discuss the main idea and intuition for each method. More details are provided in Appendix A

a large number of financial variables X is compressed into a common factor by using one of the four methods suggested in this section. Second, taking this factor as given, a Kalman filter and smoother is applied to estimate time-varying parameters and stochastic volatilities. Third, conditional on the parameters obtained the Kalman filter and smoother is applied to the factors resulting in the FCI.

3 Results

Financial Conditions are unobserved and therefore the measurement of FCIs is subject to model uncertainty related to a number of dimensions. The main virtue of our approach is its versatility in adjusting to different model specifications. First, the frequency at which FCIs are estimated may differ. High frequency financial conditions indices tend to capture rapid movement in the underlying financial variables. Figure 1 shows financial conditions indices estimated for three different base frequencies using the method previously proposed. As expected, the FCI measured at a weekly base frequency is more volatile than the index estimated at lower frequencies. This comes as no surprise because at higher frequency, the index contains a richer information set that includes high frequency movements in the underlying variables. In particular, the unbalanced panel from which we extract the principal component, includes 10 variables that are available at daily and weekly frequencies. Therefore, working at higher base frequencies yields a more volatile FCI. However, It is unclear whether the information content at high frequencies is meaningful or simply noisy. We will examine this point later and study the forecasting performance of FCIs estimated at each base frequency.

[Figure 1 about here]

Second, FCIs can be regarded as a principal component of a large set of financial variables. In an incomplete, mixed-frequency data environment it is not trivial to understand which signal extraction technique is more appropriate to measure this common component. Figures B1 show the FCI measured with different signal extraction methods considered in section 3. The tendency of some methods to overfit is evident, particularly at high frequencies. Therefore, an important outstanding question is to understand which method performs better in and out of sample. We will carefully consider this question later by comparing the in and out of sample performance of each competing technique.

Third, assumptions related to how parameters in the model are allowed to vary over time may differ. We have explored the options of allowing for heteroscedasticity and smooth changes in the parameters describing the relationship between the macroeconomy, financial conditions and the way the many financial variables determine financial conditions (ie, the loadings). The forecasting performance of these different model specifications has been examined by Koop and Korobilis (2014) at a quarterly frequency in a single frequency setting. We re-examine this point in a mixed frequency environment later in this section.

Once the best FCI measure is established, we can examine the driving forces and macroeconomic relevance of Financial Conditions. Figure 2 shows the average importance of each underlying financial variable in driving the FCI, throughout the period from 1971-2018.

[Figure 2 about here]

The VIX was the most important driver of financial conditions, emphasizing the relevance of uncertainty and global risk appetite. The percentage of banks tightening lending standards, which can be interpreted as a measure of credit supply, the performance of below investment grade corporate debt assets and various measures of the term spread are also important determinants of financial conditions.

We now focus on the macroeconomic relevance of FCIs and how it changed over time. The autoregressive, time-varying nature of the model offers the possibility of computing dynamic impulse responses that vary throughout the sample. Figure 3 describes the reaction of GDP growth to a shock to financial conditions equivalent to 1 standard deviation. The shock to financial conditions is identified recursively via Cholesky decomposition where the FCI is ordered last. The main messages from this figure are twofold: first, the sensitivity of real activity to financial condition shocks varies through time, having reached unprecedented levels following the Great Recession. Second, disruptions to real activity that result from financial shocks are persistent, lasting at least 20 months.

[Figure 3 about here]

3.1 In-sample fit of different signal extraction methods

Let us start by examining the in-sample performance of the various signal extraction methods proposed.

[Figure 4 about here]

Figure 4 reports the in-sample Mean Squared error calculated as follows

$$MSE = \frac{1}{N} \sum_{ij \in O} (x_{ij} - \hat{x}_{ij})^2, \tag{6}$$

where O includes all indice for which x_{ij} is observed and $\hat{x}_{ij}$ results from the projection of X_t on the factors F_t in equation 1, permitting the reconstruction of the incomplete data set of financial variables that loan onto the FCI. Figure 4 suggests two key points. First, higher frequencies involve greater reconstruction error. This suggests that there is a cost of working at higher frequencies that tend to capture short-run movements in financial variables. The information content gained from the consideration of the additional observations of variables observed at a higher frequencies is crowded-out by greater noise. All methods, except the PPCA exhibit poorer in-sample fit at higher frequencies.

Second, some methods deal with noise better than others. In particular the PPCA and the VBPCA suffer less when transitioning from low to high frequency estimation as their in-sample fit mildly deteriorates. This is not surprising given the in-built regularization that these methods enjoy, which seem to help in the signal extraction process. This result is consistent with Figure B1 that also suggests that the PPCA and VBPCA methods deliver less noisy FCIs.

3.2 Forecasting with Financial Conditions Indices

One of the most attractive properties of a Financial Condition Index is its forward looking ability. We study the pseudo out-of-sample performance of our FCI across the following dimensions

- **Signal extraction method:** PCA, EMPCA, PPCA and VBPCA.
- **Specification:** Time-varying parameters and stochastic volatility.

Each alternative FCI is evaluated by comparing relative MSFE of the mixed-frequency VAR (in short MF-VAR) model that results from suppressing the factor sector and time-variation of all parameters from the model discussed in section 2.1. We study the alternative FCIs produced for two different frequencies (monthly and quarterly).

3.2.1 Signal extraction method

First, we examine the out-of-sample forecasting performance of each FCI, computed under alternative signal extraction methods. Table 1 summarizes the relative MSFE of a MF-VAR that includes key macroeconomic variables plus each alternative FCI, relative to the MF-VAR bechmark that does not include the FCI. The first row in the table, corresponding to each macroeconomic variable, reports the MSFE and the following rows the relative MSFE. If the inclusion of a particular FCI improves macroeconomic forecasting of a particular variable, the relative MSFE should be lower than 1. We observe that, at a monthly frequency none of the FCIs considered improve forecasting of GDP growth and inflation significantly. Wheres, they seems to add value to forecasts of unemployment and interest rates. The FCI based on a simple Principal Component (PCA) seems to mildly improve the baseline forecast of GDP, but this predictability does not last beyond one month horizon. On the contrary, relative MSFE of unemployment and interest rates forecasts seem to decrease significantly with the inclusion of the FCIs. The FCI computed with a Variational Bayes PCA method (VBPCA) seems to beat the benchmark significantly, with its relative MSFE at 0.85 for forecasts of interest rates 1 month ahead. Indeed, it is the Variational Bayes PCA (VBPCA) that delivers the best overall forecasting performance.

Table 1: Summary of the relative MSFE of the FCIs computed under alternative PCA methods at a monthly base frequency.

horizon (months)	1	3	6	12	24
GDP growth					
MF-VAR (no fci)	0.8475	0.7062	0.7711	0.8519	0.8499
EMPCA	1.06	1.18	1.10	1.18	1.19
PCA	0.94	1.05	1.06	1.27	1.27
PPCA	1.16	1.30	1.45	1.42	1.21
VBPCA	0.99	1.04	1.00	1.17	1.01
Inflation					
MF-VAR (no fci)	1.0608	1.2899	1.2338	1.2247	1.2154
EMPCA	0.99	1.04	1.02	1.04	1.00
PCA	1.01	1.02	1.04	1.08	1.07
PPCA	0.99	1.03	1.01	1.01	0.98
VBPCA	1.00	1.03	1.01	1.05	1.06
Unemployment					
MF-VAR (no fci)	0.7971	0.7885	0.8806	1.0648	1.1725
EMPCA	0.98	1.04	0.94	0.92	0.89
PCA	0.98	0.99	0.96	0.97	1.00
PPCA	0.97	1.04	1.04	1.04	0.92
VBPCA	0.95	0.97	0.91	0.88	0.87
Interest rates					
MF-VAR (no fci)	1.0550	0.8841	0.8417	0.7935	0.7130
EMPCA	0.92	0.95	1.00	1.08	1.04
PCA	0.93	0.96	0.93	0.98	1.00
PPCA	0.89	0.97	1.05	1.17	1.05
VBPCA	0.85	0.92	0.95	0.98	0.96

Notes: Relative Mean-Squared Forecast Error (MSFE) statistics refer to the out-of-sample MSFE of the model estimated with different signal extraction methods (EMPCA, PCA, PPCA and VBPCA) relative to the baseline MF-VAR model which results from suppressing the factor section and time variation of the more general MF-TVP-FAVAR discussed. Values less than 1 suggest better performance relative to the baseline.

To understand how these results change when the base frequency is set to quarterly, we recompute these values and report the results in Table B2. The quarterly FCI contains different information, related to the lower frequency dynamics of the underlying financial variables which seem to be relevant in forecasting GDP and unemployment but do not seem to help predict interest rates and inflation.

Taken together, higher frequency movements in financial variables do not seem to improve benchmark forecasts on GDP and inflation, while forecasts of interest rates and unemployment improve with the inclusion of the FCI at the monthly frequency. On the contrary, at a quarterly frequency the inclusion of the FCI in the baseline MF-VAR, delivers better forecasts of GDP and unemployment while inflation and interest rate predictions deteriorate. Noticeable, the quarterly frequency FCI based on a PPCA algorithm improves baseline predictions by 21 per cent.

3.2.2 Specification

Next, we study the forecasting performance of the different variants of the model presented in section 2, using the VBPCA as the elected signal extraction method [5].

[5] As previously discussed, the VBPCA delivers the best out-of-sample forecasting power and therefore we adopt this method as the benchmark in our exercise

Table 2: Summary of the relative MSFE of the FCIs computed under alternative model specifications, at monthly base frequency.

horizon (months)	1	3	6	12	24
GDP growth					
MF-VAR (no fci) baseline	0.8475	0.7063	0.7711	0.8519	0.8499
MF-TVP-FAVAR	0.99	1.05	1.01	1.18	1.01
MF-SV-FAVAR	1.14	1.27	1.31	1.08	0.90
MF-TVL-FAVAR	0.99	1.05	1.01	1.18	1.01
MF-FAVAR	1.28	1.03	1.03	1.20	1.10
Inflation					
MF-VAR (no fci) baseline	1.0608	1.2900	1.2339	1.2248	1.2154
MF-TVP-FAVAR	1.00	1.03	1.01	1.06	1.07
MF-SV-FAVAR	0.97	1.00	0.97	0.99	0.98
MF-TVL-FAVAR	1.00	1.03	1.01	1.06	1.07
MF-FAVAR	0.94	1.07	1.06	1.08	1.07
Unemployment					
MF-VAR (no fci) baseline	0.7972	0.7886	0.8806	1.0649	1.1726
MF-TVP-FAVAR	0.95	0.98	0.92	0.88	0.88
MF-SV-FAVAR	0.97	1.01	0.89	0.86	0.82
MF-TVL-FAVAR	0.95	0.98	0.92	0.88	0.88
MF-FAVAR	0.86	0.97	0.96	0.94	0.96
Interest rates					
MF-VAR (no fci) baseline	1.0550	0.8842	0.8418	0.7935	0.7130
MF-TVP-FAVAR	0.86	0.93	0.96	0.99	0.97
MF-SV-FAVAR	0.88	0.96	0.98	1.03	0.98
MF-TVL-FAVAR	0.86	0.93	0.96	0.99	0.97
MF-FAVAR	0.84	0.91	1.00	1.00	0.99

Notes: Relative Mean-Squared Forecast Error (MSFE) statistics refer to the out-of-sample MSFE of the model estimated with different specifications relative to the baseline MF-VAR model which results from suppressing the factor section and time variation of the more general MF-TVP-FAVAR discussed. The MF-SV-FAVAR arises by suppressing time-variation in all coefficients yet allowing for stochastic volatility (SV); the MF-TVL-FAVAR allows exclusively for time varying loadings (TVL) and stochastic volatility; the MF-FAVAR suppresses time variation in all coefficients

The results presented in Table 2 above summarize the relative MSFE of four different specifications that are nested in the model described in section 3. The MF-TVP-FAVAR indicates the general specification. The MF-SV-FAVAR results from the general specification where the time-variation in the parameters is shut down and only the volatilities are allowed to vary. Thirdly, the MF-TVL-FAVAR shuts down the time-variation in the parameters of the state equation 2, while allowing the loadings to vary over time. Finally, the MF-FAVAR is the most parsimonious model since it does not allow for any kind of time-variation in the parameters and therefore is a constant parameter model without stochastic volatility.

Table B3 summarizes MSFE statistics for the forecasting exercise with different model specifications, where the FCI is estimated at quarterly frequency. We observe that the MF-FAVAR is the best performing specification when forecasting GDP and unemployment. In contrast, at a monthly frequency we see the more complex specification do not generally underperform the MF-FAVAR, suggesting that it can be advantageous to use models that provide more flexibility at higher frequencies. However, the gains in predictive power associated to time-varying models are not expressive. Therefore, the general result that it is difficult to beat parsimony still holds since no alternative specification consistently delivers better forecasts than the MF-FAVAR.

4 Conclusion

Recent research has highlighted the importance of monitoring financial conditions. The questions of how the information in financial conditions indexes can inform monteary policy conduct and macroeconomic forecasting are subject to debate. One of the common critiques to FCIs is related to measurement error that the construction of such indicators involves.

This paper's main focus is on advancing the measurement of Financial Conditions. Some of the challenging aspects of constructing a financial conditions index are related to the incompleteness of the data. Traditional econometric techniques cope poorly with the mixed-frequency nature of the data, proliferation of missing values and the need to build indexes at high frequencies. The factor augmented VAR proposed estimated through a combination of probabilistic PCA methods and Kalman filtering and smoothing routines offer a solution to cope with those features of the data. The FCI obtained shows a better fit and an improved forecasting ability. These improvements are driven by the novel signal extraction methods explored. In particular, probabilistic PCA methods (PPCA and VBPCA) deliver the best in-sample fit. The predictive content of the FCI for forecasting GDP and unemployment is found at a quarterly frequency. Whereas, the FCI estimated on a monthly basis does not add value to forecasting GDP. However, the monthly FCI improved baselines forecasts of unemployment and interest rates.

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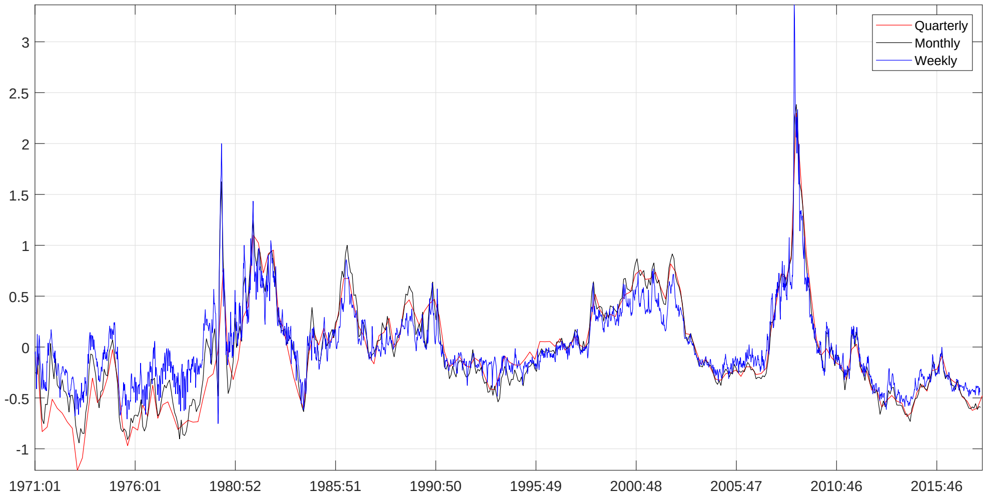
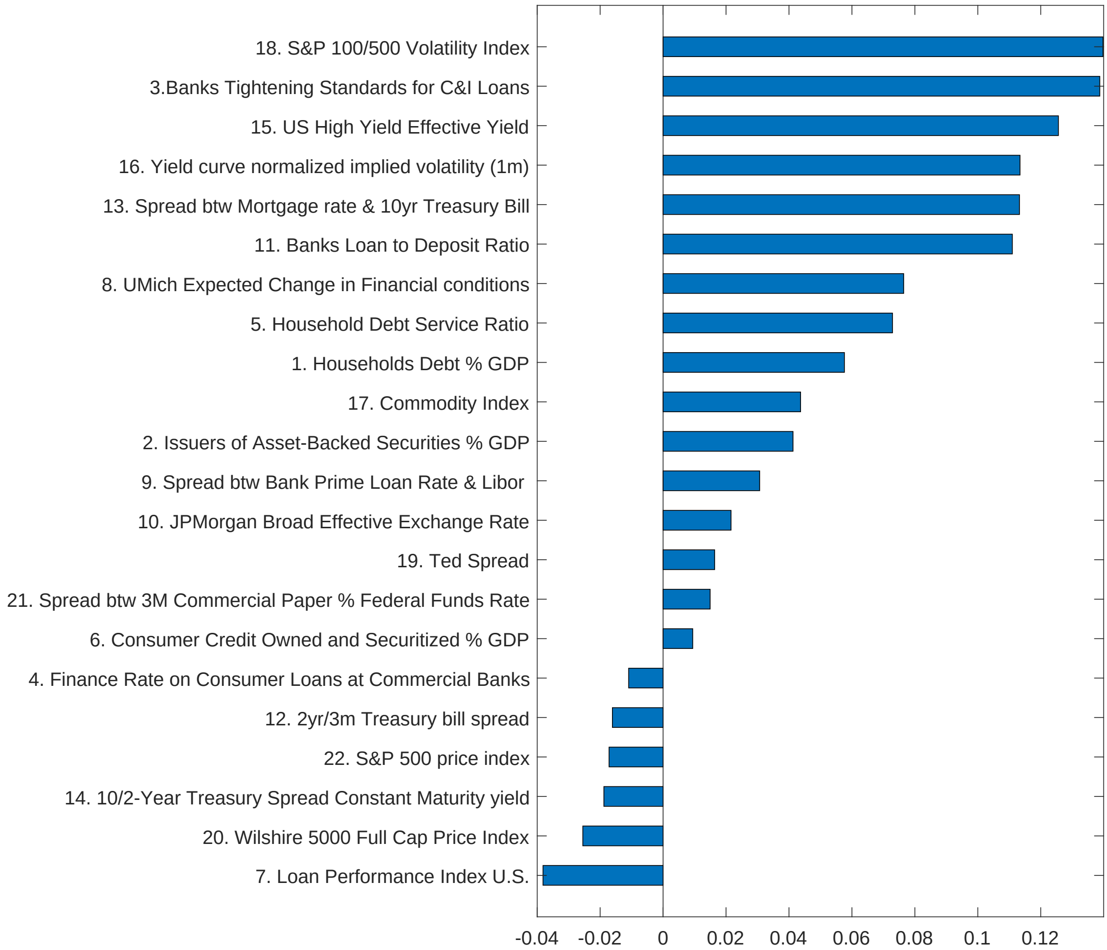
5 Main Figures

Figure 1: Financial Conditions Indices estimated at quarterly, monthly and weekly frequencies.



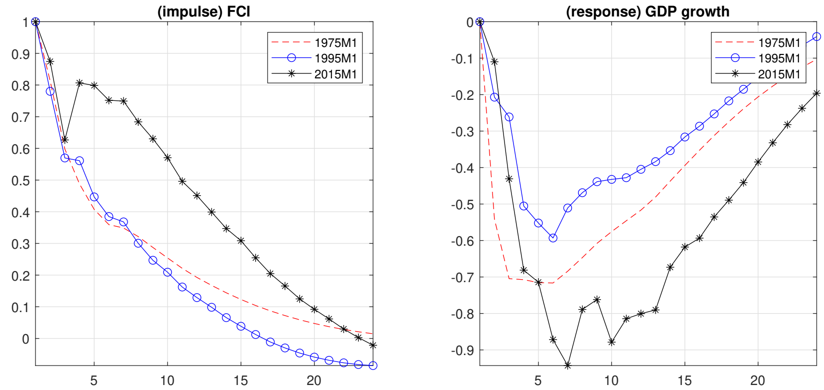
Notes: The Financial Conditions Index in this figure is estimated for the period 1971-2018 with the Variational Bayes variant of our algorithm for the three frequencies considered.

Figure 2: Average weights of individual financial variables loading into the Financial Conditions, 1971-2018.



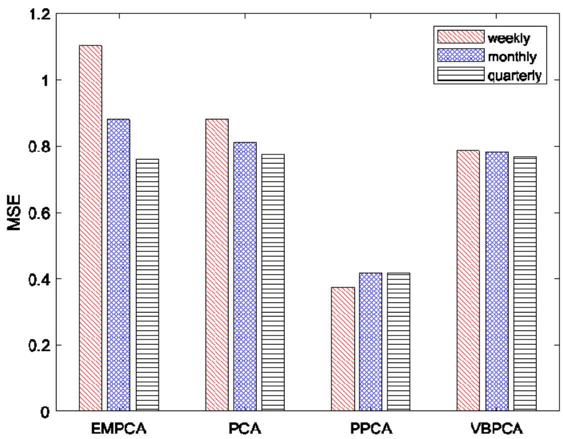
Notes: The weights represent the average time-varying loadings of each financial indicator into the FCI for the full sample period 1971-2018.

Figure 3: Impulse response function of GDP growth to shocks to the FCI, measured in standard deviations.



Notes: Impulse responses of GDP growth to a one standard deviation shock in the FCI. Identification results from a Cholesky decomposition the variance covariance matrix of the reduced form innovations of the factor-augmented vector autoregression which includes GDP growth, inflation, unemployment, the Federal Funds Rate and the FCI which is ordered last.

Figure 4: Reconstruction Mean Squared Error across four alternative PCA methods



Notes: Mean-squared error statistics are calculated in-sample for the observable section of the standardized unbalanced panel of financial indicators across the different signal extraction methods.

Appendix

A Econometric Methods

A1 Mixed-Frequency Bayesian Kalman Filter with Forgetting Factors

The model defined in (10)-(13) configures a MF-TVP-FAVAR and can be written compactly, in state space form as follows

$$X_t = Z_t \Lambda_t + u_t, \quad u_t \sim N(0, V_t), \tag{A1}$$
$$Z_t = Z_{t-1}\beta_t + \varepsilon_t, \quad \varepsilon_t \sim N(0, Q_t), \tag{A2}$$
$$\beta_t = \beta_{t-1} + \eta_t \quad \eta_t \sim N(0, R_t), \tag{A3}$$
$$\Lambda_t = \Lambda_{t-1} + v_t \quad v_t \sim N(0, W_t). \tag{A4}$$

Where $\Lambda_t = [\lambda_t^y, \lambda_t^f]'$, $Z_t = \begin{bmatrix} y_t \\ F_t \end{bmatrix}$. Note that Z_t depends on the latent factor F_t that is taken as data [6]. Let $\theta_t = \{\Lambda_t, \beta_t\}$ denote the parameter set and $D_t = \{X_t, Z_t\}$ the data for $t = \{1, ..., T\}$. Assuming that we know the posterior of θ at time $t-1$, Bayesian filtering/smoothing is based on the equations below

$$p(\theta_t, \theta_{t-1}|D_{t-1}) = p(\theta_t|\theta_{t-1}, D_{t-1})p(\theta_{t-1}|D_{t-1}), \tag{A5}$$
$$p(\theta_t|D_{t-1}) = \int_\Omega p(\theta_t|\theta_{t-1}, D_{t-1})p(\theta_{t-1}|D_{t-1})d\theta_{t-1}, \tag{A6}$$

where Ω is the support of θ_{t-1}. The prediction step is given by the Chapman-Kolmogorov equation A6.

Next, at each iteration t, the prior $p(\theta_t|D_{t-1})$ gets updated according to equation A6 and the measurement likelihood $p(D_t|\theta_t)$ is augmented by an additional observation of D_t. Hence the posterior distribution is updated according to the Bayes rule

$$p(\theta_t|D_t) = \frac{1}{H_t} p(D_t|\theta_t, D_{t-1})p(\theta_t|D_{t-1}). \tag{A7}$$

Where $H_t = \int p(D_t|\theta_t)p(\theta_t|D_{t-1}))$ is the normalizing constant. Equation A7 is refered to as the updating step. To summarize, the algorithm extends the one

[6] This quantity is the first principal component calculated in Step 1 by employing one of the four algorithms discussed in the body of the paper and detailed in section A2 to reconstruct the incomplete data matrix X_t

derived in Koop and Korobilis (2014) to a mixed-frequency environment where D_t is allowed to contain missing values. It consists in 2 steps, iterating through prediction (A6) and updating (A7) after the system is initialized. These two main steps are repeated for $t = \{1, ..., T\}$.

1. Kalman Filter

1.1 Initialization (Priors) All quantities are initialized according to their priors which are chosen following the diffuse choices of Koop and Korobilis (2014): $f_0 \sim N(0, 10)$, $\Lambda_0 \sim N(0, I)$, $\beta_0 \sim N(0, I)$. The variances of the innovations in A1-A4 can be seen as hyperparameters and are set to $\hat{V}_0 = 0.1 * I$, $\hat{Q}_0 = 0.1 * I$, $\hat{R}_0 = 10^{-5} * I$ and $\hat{W}_0 = 10^{-5} * I$. However, in this setting the hyperparameter are allowed to smoothly change over time following a Exponentially Weighted Moving Average (EWMA).

1.2 Prediction

$$\Lambda_t \sim N(\Lambda_{t|t-1}, \Sigma^{\Lambda}_{t|t-1}), \tag{A8}$$
$$\beta_t \sim N(\beta_{t|t-1}, \Sigma^{\beta}_{t|t-1}). \tag{A9}$$

Where $\Lambda_{t|t-1} = \Lambda_{t-1|t-1}$, $\beta_{t|t-1} = \beta_{t-1|t-1}$ and

$$\Sigma^{\beta}_{t|t-1} = \Sigma^{\beta}_{t-1|t-1} + \hat{R}_{t|t-1}, \tag{A10}$$
$$\Sigma^{\Lambda}_{t|t-1} = \Sigma^{\Lambda}_{t-1|t-1} + \hat{W}_{t|t-1}. \tag{A11}$$

The state covariances in the equations above are estimated by

$$\hat{R}_{t|t-1} = \frac{1}{\kappa_3^*} \hat{R}_{t-1|t-1}, \tag{A12}$$
$$\hat{W}_{t|t-1} = \frac{1}{\kappa_4^*} \hat{W}_{t-1|t-1}. \tag{A13}$$

[7] where κ_3^* and κ_4^* are forgetting factors that define the law of motion of the parameters. With mixed-frequency data, we set these quantities to 1 for the periods and variables for which data is missing and to $\kappa_3 = \kappa_4 = 0.99$ otherwise [8].

[7] In practice these two equations are approximations of $\hat{R}_{t|t-1} = \hat{R}_{t-1|t-1} + \hat{\eta}_{t-1}\hat{\eta}'_{t-1}$ and $\hat{W}_{t|t-1} = \hat{W}_{t-1|t-1} + \hat{\upsilon}_{t-1}\hat{\upsilon}'_{t-1}$in the standard Kalman Filter (see Koop and Korobilis (2013) and Raftery et al. (2010))

[8] by setting the decay terms to 1 when there is missing data we suppress the movement of the parameters. By doing so, we implicitly assume that high frequency variables are fast moving relative to lower frequency series

The prediction step allows us to compute measurement equation prediction errors that are necessary inputs for the updating step and computed as

$$\hat{u}_t = x_t - \hat{x}_{t|t-1}, \tag{A14}$$

$$\hat{\varepsilon}_t = z_t - \hat{z}_{t|t-1}. \tag{A15}$$

Where $\hat{x}_{t|t-1} = z_t \Lambda_{t|t-1}$ and $\hat{z}_{t|t-1} = z_{t-1}\beta_{t|t-1}$. With missing data, we simple set to zero the error corresponding to variables missing at a given point in time t.

1.3 Update

Update each Λ_{it} for $i = 1, ..., n$ and β_t

$$\Lambda_{it} \sim N(\Lambda_{it|t}, \Sigma^{\Lambda}_{ii,t|t}), \tag{A16}$$

$$\beta_t \sim N(\beta_{t|t}, \Sigma^{\beta}_{t|t}). \tag{A17}$$

The terms in (A16) are calculated as

$$\Lambda_{it|t} = \Lambda_{it|t-1} + \Sigma^{\Lambda}_{ii,t|t-1} z_t'(\hat{V}_t + z_t \Sigma^{\Lambda}_{ii,t|t-1} F_t')^{-1} \hat{\varepsilon}_t, \tag{A18}$$

$$\Sigma^{\Lambda}_{ii,t|t} = \Sigma^{\Lambda}_{ii,t|t-1} - \Sigma^{\Lambda}_{ii,t|t-1} z_t'(\hat{V}_t + z_t \Sigma^{\Lambda}_{ii,t|t-1} z_t')^{-1} z_t \Sigma^{\Lambda}_{ii,t|t-1}. \tag{A19}$$

Where the term $\Sigma^{\Lambda}_{ii,t|t-1} z_t'(\hat{V}_t + z_t \Sigma^{\Lambda}_{ii,t|t-1} F_t')^{-1}$ is the Kalman Gain for each time period t and is set to zero for the variables missing at each step.

The terms in (A17) are calculated as

$$\beta_{t|t} = \beta_{t|t-1} + \Sigma^{\beta}_{t|t-1} z_t'(\hat{Q}_t + z_{t-1} \Sigma^{\beta}_{t|t-1} z_{t-1}')^{-1}(z_t - z_{t-1}\hat{\beta}_{t-1}) \tag{A20}$$

$$\Sigma^{\beta}_{t|t} = \Sigma^{\beta}_{t|t-1} - \Sigma^{\beta}_{t|t-1} z_t'(\hat{Q}_t + z_t \Sigma^{\beta}_{t|t-1} z_t')^{-1} z_t \Sigma^{\beta}_{t|t-1} \tag{A21}$$

Where the term $\Sigma^{\beta}_{t|t-1} z_t'(\hat{Q}_t + z_{t-1} \Sigma^{\beta}_{t|t-1} z_{t-1}')^{-1}$ is the Kalman Gain for each time period t and is set to zero for the variables missing at each step.

The only outstanding terms that need defining are the measurement equation error covariance matrices that can be obtained using EWMA as follows

$$\hat{V}_t = \kappa_1^* \hat{V}_{t-1} + (1 - \kappa_1^*)\hat{u}_t \hat{u}_t', \tag{A22}$$

$$\hat{Q}_t = \kappa_2^* \hat{Q}_{t-1} + (1 - \kappa_2^*)\hat{\varepsilon}_t \hat{\varepsilon}_t'. \tag{A23}$$

where κ_1^* and κ_2^* are forgetting factors that define the law of motion of the idiosyncratic volatilities in the measurement equation and the volatilities of the observable variables and the factors in the state equation. With mixed-frequency data, we set these quantities to 1 for the periods and variables for which data is missing and to

$\kappa_1 = \kappa_2 = 0.99$ otherwise[9].

2. Kalman Smoother

The kalman filter algorithm described in (1.1)-(1.3) above works by forward recursion and outputs estimates of $E(\theta|D^t)$ for all parameters $\theta = [\hat{V}_t, \hat{Q}_t, \hat{R}_t, \hat{W}_t, \hat{\beta}_t, \hat{\Lambda}_t]$ in the model using the data available D^t for $t = 1, ..., t$. However, we are ultimately interested in an estimate of $E(\theta|D^T)$ which yields the parameter states conditional on the entire sample $t = 1, ..., T$.

Therefore, the kalman smoother is applied to the output of the kalman filter working by backward recursion. Given that the Kalman filter takes into account missing data, no alteration is necessary to the Kalman smoother.

3. Kalman Smoother/filter for Factors

The full three step procedure described in section 5.2 is complete by applying the Kalman filter and smoother algorithm to the factors, which are initialized with a PCA estimate. The algorithm and mixed frequency extensions are analogous to that previously described.

[9] by setting the decay terms to 1 when there is missing data we suppress the movement of volatilities. By doing so, we implicitly assume that high frequency variables are move volatile relative to lower frequency series.

A2 PCA algorithms for Incomplete Data Environments

A2.1 Least-Squares PCA in the presense of missing data

Several simple ways to deal with missing values in a classical LS PCA framework consist in setting missing values to zero or using an Expectation Maximization PCA (EM PCA) [Dempster et al. 1977, Rubin and Thayer, 1982].

The later technique is used by Stock and Watson (2002) and consists of an iterative procedure that alternates between imputing missing values in X (E-step) and applying standard PCA to the pseudo-balanced panel of data (M-step) until convergence is reached. To summarize the algorithm proceeds as follows:

- **E-step:** Reconstruct X_t by filling in its missing values:

$$X_t^* = \begin{cases} X_t & \text{for observed values} \\ \hat{\Lambda}^k \hat{F}_t^k & \text{for missing values.} \end{cases} \tag{A24}$$

- **M-step:** Perform standard PCA by SVD on the infilled matrix X_t^* and obtain new values for $\{\hat{\Lambda}^k, \hat{F}_t^k\}$.

The algorithm alternates between the E-M steps until convergence is reached in which case new estimates for the parameters in iteration $k-1$ do not improve the Least-squares minimization problem solved in iteration k.

A2.2 Variation view of the Expectation Maximization (EM) algorithm for incomplete data environments

Consider the standard PC regression

$$X_t = \Lambda F_t + \xi_t, \quad \xi_t \sim N(0, v_x I). \tag{A25}$$

Where $\theta = \{\Lambda, F_t, v_y\}$ are model parameters and a subset X_{mis} of the data matrix is missing and treated as hidden variables.

The variational view of the EM algorithm (see Neal and Hinton 1999 and Attias 1999) consists in minimizing the objective function

$$V(\theta, p(X_{mis})) = \int p(X_{mis}) log \frac{p(X_{mis})}{p(X|\theta)} dX_{mis} = \tag{A26}$$

$$\int p(X_{mis}) log \frac{p(X_{mis})}{p(p(X_{mis}|\theta)} dX_{mis} - logp(X_{obs}|\theta), \tag{A27}$$

wrt the model parameters θ and the density over the missing data $p(X_{mis})$. X_{obs} denotes the observed data such that $X = X_{mis} \bigoplus X_{obs}$.

E-step. Equation A26 is the Kullback-Leibler divergence between the pdfs over observable and unobservable data. The minimization of this expression wrt $p(X_{mis})$, given θ is shown to yield

$$p(X_{mis}|\theta) = \prod_{ij \in O} N(\hat{x}(\theta)_{ij}, v_x). \tag{A28}$$

where O is the set of indices for which observation x_{ij} is observed, $\hat{x}(\theta)_{ij}$ result from the reconstruction of the incomplete data matrix X from expression A24, for a given θ. This procedure is refered to as the E-step of the algorithm.

M-step. Next, the proceedings from the E-step are substituted back into expression A26. The terms in the resulting expression which depend on θ are given by

$$-\int p(X_{mis}) logp(X|\theta) dX_{mis}. \tag{A29}$$

It can be shown that minimization of A28 wrt. θ is equivalent to minimizing the LS objective function in case of no missing data (see Neil and Hinton, 1999). Thus, the M-step of the algorithm consists in performing SVD decomposition to the imputed data matrix X. The algorithm alternatives between the E-M steps until convergence is reached (ie, when the reconstruction error stabilizes).

A2.3 Probabilistic PCA (PPCA)

A probabilistic PCA specification has been found to provide a good foundation to handle missing data Ilin and Raiko (2010). The probabilistic PCA set forth by Tipping and Bishop (1999) can be written as follows

$$X_t = m + \Lambda F_t + \xi_t, \tag{A30}$$

where both the principal component and the noise term are assumed normaly distributed as follows

$$p(F_t) \sim N(0, I_K), \tag{A31}$$
$$p(\xi_t) \sim N(0, \tau^{-1} I_N), \tag{A32}$$

where $\theta = \{m, \Lambda, \tau\}$ are model parameters, I_N and I_T denote identity matrices and τ is the scalar inverse variance of ξ. It can be shown that the Maximum Likelihood (ML) estimation of the PPCA is identical to PCA in the case of non-missing data. The great advantage of the PPCA is that, in case of incomplete data, it allows for regularization that arises naturally from the choice of Gaussian priors.

The model can then be estimation with a standard EM algorithm. The necessary extensions to handle missing data are discussed in Ilin and Raiko (2010). Below I summarize their procedure.

E-step. Estimate the conditional distribution of the hidden variables F given the data X and model parameters θ,

$$p(F|X,\theta) = \prod_{j=1}^{K} N(\bar{F}_j, \Sigma_{F_j}), \tag{A33}$$

based on the following updating rules

$$\Sigma_{F_j} = \tau^{-1}(\tau^{-1}I + \sum_{i\in O_j} \lambda_i \lambda_i^T)^{-1}, \tag{A34}$$

$$\bar{F}_j = \tau \Sigma_{F_j} \sum_{i\in O_j} \lambda_i (x_{ij} - m_i), j = 1, ..., K, \tag{A35}$$

$$m_i = \frac{1}{|O_i|} \sum_{j\in O_i} (x_{ij} - \lambda_i^T \bar{F}_j). \tag{A36}$$

M-step. re-estimate the model parameters as

$$\lambda_i = \big(\sum_{j\in O_i} \bar{F}_j \bar{F}_j^T + \Sigma_{F_j}\big)^{-1} \sum_{j\in O_i} \bar{F}_j (x_{ij} - m_i), i = 1, ..., N, \tag{A37}$$

$$\tau = \Big[\frac{1}{N} \sum_{ij\in O} \big[x_{ij} - \lambda_i^T \bar{x}_j - m_i\big]^2 + \lambda_i^T \Sigma_{F_j} \lambda_i\Big]^{-1}. \tag{A38}$$

where O_i, O_j and O denote the set of indices i, j for which x_{ij} is observed.

A2.4 Variational Bayesian PCA (VBPCA)

Some studies suggest that the standard PPCA is still vulnerable to over-fitting (see for example Ilin and Raiko (2010)). One possible reason that might lead to overfitted solution might be the nontrivial choice of the number of principal components to include in (16). Including a large number of common components F_t might cause the model to over-learn the data.

One possible solution to this problem consists in penalizing large values in the matrices Λ and F_t. The probabilistic pca model is flexible enough to allow for an automatic, data-driven selection of relevant common components by shrinking to zero the solutions λ_j that are small relative to the noise variance. This can be achieved through a variational bayesian PCA algorithm as explained below. We follow Oba et al. (2003).

Additional regularization can be imposed to penalize parameter values that yield more complex explanations of the data. Hence, in addition to (8)-(9) one can further impose

$$p(m|\tau) \sim N(0, (\gamma_{m0}\tau)^{-1} I_T), \tag{A39}$$

$$p(\lambda_j|\tau, \alpha_j) \sim N(0, (\alpha_j\tau)^{-1} I_T), \tag{A40}$$

$$p(\tau) \sim G(\tau|\bar{\tau}_0, \gamma_{\tau_0}), \tag{A41}$$

where $\psi = \{\gamma_{m0}, \gamma_{\tau_0}, \bar{\tau}_0, \alpha_j\}$ are hyperparameters and λ_j are the parameters in column j of the loadings matrix Λ that define the importance of each principal component F_j, $j = \{1, 2, ..., K\}$. The prior $p(\Lambda|\alpha, \tau)$, which has a hierarchical structure, is called an automatic relevance determination (ARD) prior. This structure plays a key role in guaranteeing parsimony of the model. Its variance $(\alpha_j\tau)^{-1}$ is determined by a hyperparameter α_j that becomes large when the euclidean distance $||\lambda_j||$ is small relative to the noise variance τ^{-1}.

Estimation of the model defined by (7)-(12) requires a variational EM algorithm as proposed by Attias (2000) to cope with the unknown analytical form of the posterior of the parameters $p(\Lambda, F_t, m|X, \psi)$, which invalidates the E-step of the standard EM algorithm. To overcome this difficulty, the author proposes an approximation to this this quantity by a simpler $q(\Lambda, F_t, m)$. More detailed are explained in appendix A.

As mentioned in the body of the paper, estimation of the model defined by (7)-(12) requires a variational EM algorithm as proposed by Attias (2000) to cope with the unknown analytical form of the posterior of the parameters $p(\theta|X)$.

Using a variational approach, the E-step is modified such that the objective function approximates $p(\theta|X)$ with a simpler pdf $p(\theta)$, written as follows

$$V(p(\theta),\psi)) = \int p(\theta) log \frac{p(\theta)}{p(X,\theta|\psi)} d\theta = \tag{A42}$$

$$\int p(\theta) log \frac{p(\theta)}{p(X,\theta|\psi)} d\theta - logp(X|\psi), \tag{A43}$$

E-step. Equation A33 is the Kullback-Leibler divergence between the true posterior and its approximation. In this step the approximation $p(\theta)$ is updated. This corresponds to minimizing this distance wrt $p(\theta)$.

M-step. Next, the approximation $p(\theta)$ is used as if it was the actual posterior $p(\theta|X,\psi)$ in order to increase $p(X|\psi)$. This consists in deriving the expression A33 wrt ψ.

The algorithm alternatives between the E-M steps until convergence is reached (ie, when the reconstruction error stabilizes).

B Additional Tables and Figures

#	mnemonic	description	frequency	t_code	Source
1	CMDEBT	Households and Nonprofit Organizations Debt % GDP	Q	5	St. Louis FRED
2	ABSITCMAHDFS	Issuers of Asset-Backed Securities % GDP	Q	5	St. Louis FRED
3	DRTSCILM	Net Percentage of Domestic Banks Tightening Standards for Commercial and Industrial Loans	Q	1	St. Louis FRED
4	TERMCBAUTO48NS	Finance Rate on Consumer Installment Loans at Commercial Banks, New Autos 48 Month Loan, Percent	Q	5	St. Louis FRED
5	TDSP	Household Debt Service Payments as a Percent of Disposable Personal Income	Q	1	St. Louis FRED
6	TOTALSL	Total Consumer Credit Owned and Securitized % GDP	M	1	St. Louis FRED
7	LOANHPI	Loan Performance Index U.S.	M	5	Bloomberg
8	CONSEXFI	UMich Expected Change in Financial conditions	M	1	Uni Michigan
9	BPLR	Bank Prime Loan Rate / Libor spread	M	1	St. Louis FRED
10	JPMNEER	JPMorgan Broad Nominal Effective Exchange Rate (2010=100)	M	5	Bloomberg
11	LDR	All Commercial Banks Loan to Deposit Ratio	M	1	Haver
12	2/3TBS	2yr/3m Treasury bill spread	M	1	St. Louis FRED
13	MORTGAGE30US	Mortgage rate / 10yr Treasury Bill spread	W	1	St. Louis FRED
14	T10Y2Y	10-Year Minus 2-Year Treasury Constant Maturity yield, Percent	D	1	St. Louis FRED
15	BAMLH0A0HYM2EY	ICE BofAML US High Yield Master II Effective Yield, Percent	D	1	Bloomberg
16	MOVE Index	Yield curve weighted index of the normalized implied volatility on 1-month Treasury options.	D	1	Bloomberg
17	CRY Index	Thomson Reuters/CoreCommodity CRB Commodity Index	D	1	Bloomberg
18	VXOVIX	Cboe S&P 100/500 Volatility Index	D	1	St. Louis FRED
19	BASPTDSP	Ted Spread	D	1	St. Louis FRED
20	WILL5000PRFC	Wilshire 5000 Full Cap Price Index	D	5	St. Louis FRED
21	CPFF	3-Month Commercial Paper Minus Federal Funds Rate, Percent, Daily, Not Seasonally Adjusted	D	1	St. Louis FRED
22	SP500	S&P 500 price index	D	5	St. Louis FRED

Notes: Mnemonic refers to the statistical reference with which the time series can be fetched from the source. t-code refers to transformation applied to each variable. 1:levels; 5: log-differences. Frequency is either Q: quarterly, M: monthly or W: weekly.

Table B1: Description of the series used to construct the Financial Conditions Index.

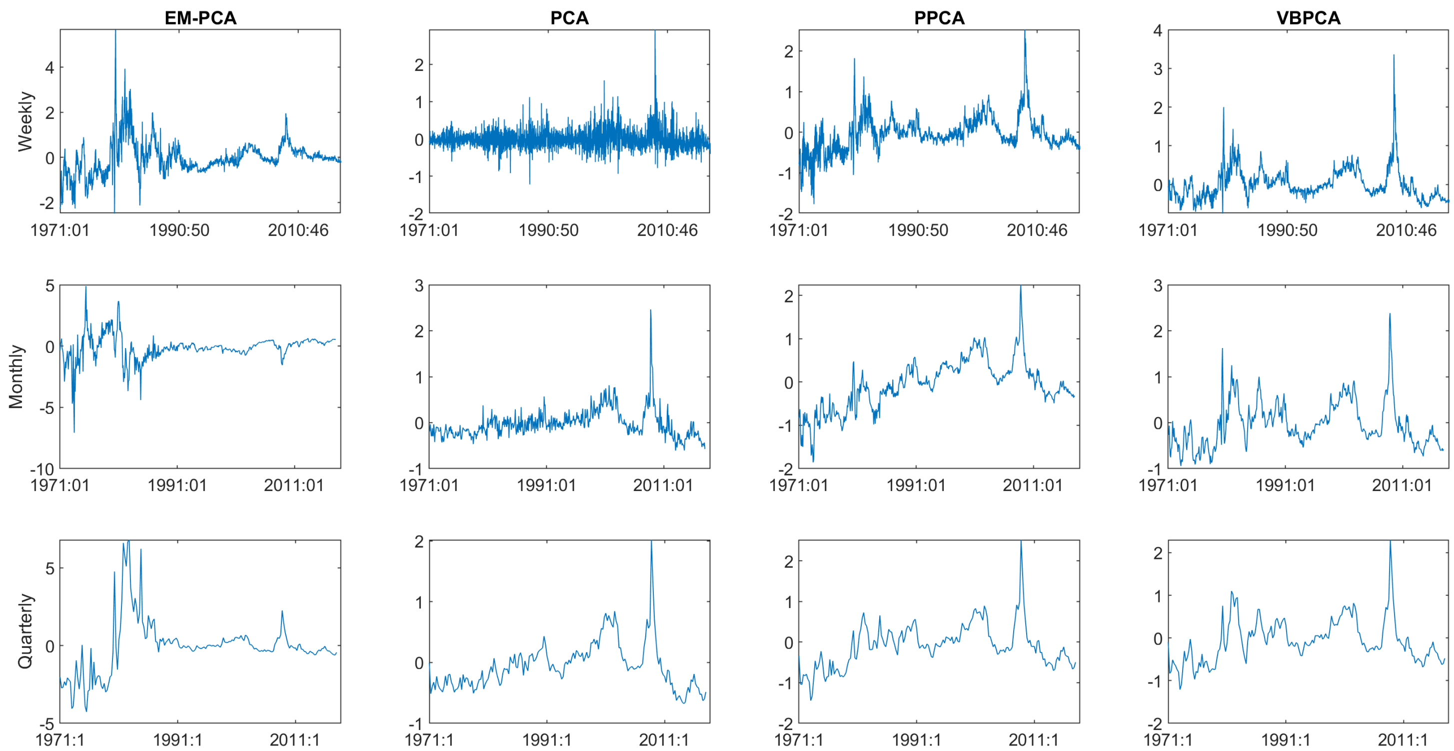


Figure B1: Common Components taken from the Mixed-Frequency financial variables dataset using four alternative methods for three base frequencies.

Table B2: Summary of the relative MSFE of the FCIs computed under alternative PCA methods, at a quarterly base frequency.

horizon (quarters)	1	2	3	4
GDP growth				
MF-VAR (no fci)	0.6311	0.7600	0.7839	0.7640
EMPCA	0.91	0.88	0.91	0.96
PCA	0.88	0.85	0.88	0.99
PPCA	0.79	0.78	0.91	1.04
VBPCA	0.84	0.85	0.97	1.05
Inflation				
MF-VAR (no fci)	2.4611	2.3938	2.1869	2.1594
EMPCA	1.00	1.00	1.00	1.00
PCA	1.02	1.02	1.03	1.01
PPCA	1.01	1.01	1.01	1.00
VBPCA	1.01	1.02	1.01	1.00
Unemployment				
MF-VAR (no fci)	0.8348	0.8804	0.8245	0.9613
EMPCA	0.97	0.97	1.00	1.01
PCA	0.71	0.68	0.82	0.80
PPCA	0.73	0.74	0.87	0.88
VBPCA	0.83	0.85	0.92	0.94
Interest rates				
MF-VAR (no fci)	0.2227	0.1101	0.0883	0.0805
EMPCA	1.26	1.13	1.08	1.09
PCA	1.39	1.49	1.09	1.25
PPCA	1.63	1.35	1.01	1.18
VBPCA	1.34	1.01	1.01	1.08

Notes: Relative Mean-Squared Forecast Error (MSFE) statistics refer to the out-of-sample MSFE of the model estimated with different signal extraction methods (EMPCA, PCA, PPCA and VBPCA) relative to the baseline MF-VAR model which results from suppressing the factor section and time variation of the more general MF-TVP-FAVAR discussed. Values less than 1 suggest better performance relative to the baseline.

Table B3: Summary of the relative MSFE of the FCIs computed under alternative model specifications, at quarterly base frequency.

horizon (quarters)	1	2	3	4
GDP growth				
MF-VAR (no fci)	0.6407	0.7833	0.7729	0.7569
MF-TVP-FAVAR	0.86	0.85	1.07	1.15
MF-SV-FAVAR	0.85	0.84	1.07	1.15
MF-TVL-FAVAR	0.86	0.85	1.07	1.15
MF-FAVAR	0.76	0.75	0.95	1.06
Inflation				
MF-VAR (no fci)	2.4394	2.3974	2.2154	2.1294
MF-TVP-FAVAR	1.01	1.02	1.01	1.01
MF-SV-FAVAR	1.01	1.02	1.01	1.01
MF-TVL-FAVAR	1.01	1.02	1.01	1.01
MF-FAVAR	1.02	1.03	1.00	1.00
Unemployment				
MF-VAR (no fci)	0.8773	0.9495	0.8774	1.0530
MF-TVP-FAVAR	0.79	0.84	0.96	0.99
MF-SV-FAVAR	0.78	0.83	0.97	1.00
MF-TVL-FAVAR	0.79	0.84	0.96	0.99
MF-FAVAR	0.57	0.67	0.83	0.88
Interest rates				
MF-VAR (no fci)	0.2031	0.1184	0.0804	0.0669
MF-TVP-FAVAR	2.00	1.60	1.53	1.02
MF-SV-FAVAR	1.99	1.59	1.50	1.02
MF-TVL-FAVAR	2.00	1.60	1.53	1.02
MF-FAVAR	2.07	1.56	1.52	1.02

Notes: Relative Mean-Squared Forecast Error (MSFE) statistics refer to the out-of-sample MSFE of the model estimated with different specifications relative to the baseline MF-VAR model which results from suppressing the factor section and time variation of the more general MF-TVP-FAVAR discussed. The MF-SV-FAVAR arises by suppressing time-variation in all coefficients yet allowing for stochastic volatility (SV); the MF-TVL-FAVAR allows exclusively for time varying loadings (TVL) and stochastic volatility; the MF-FAVAR suppresses time variation in all coefficients.