LECTURE 14

THERMODYNAMICS

CONSIDER THE KINETIC ENERGY

$$dK^{CONT} = \rho \frac{|\underline{V}|^2}{2} dV = \frac{\sum_{i=1}^{N} m_i |\underline{V}|^2 \cancel{dV}}{\cancel{dV}}$$

AND

$$dK^{EX} = \sum_{i=1}^{N} \frac{m_i}{2} \underline{v}_i \cdot \underline{v}_i$$

$$dK^{CONT} - dK^{EX} = \sum_{i=1}^{N} m_i \left(|\underline{V}|^2 - |\underline{v}_i|^2 \right)$$

$$= \sum_{i=1}^{N} m_i \left(|\underline{V}|^2 - |(\underline{v}_i - \underline{V}) + \underline{V}|^2 \right)$$

$$= \sum_{i=1}^{N} m_i \left(|\underline{V}|^2 - |\Delta \underline{v}_i|^2 - |\underline{V}|^2 - 2 \quad \underline{v}_i \cdot \underline{V}_i \right)$$

$$= \sum_{i=1}^{N} \frac{m_i}{2} |\Delta \underline{v}_i|^2 - V \cdot \underbrace{\sum_{i=1}^{N} m_i}_{= 0 \text{ (OSCILLATIONS ABOUT CM)}}$$

$$= \sum_{i=1}^{N} \frac{m_i}{2} |\Delta \underline{v}_i|^2$$

SO THERE IS EXCESS KINETIC ENERGY WHICH IS NOT ACCOUNTED FOR BY A PURELY MECHANISTIC CONTINUUM APPROACH

THIS EXCESS KINETIC ENERGY IS INTIMATELY RELATED TO THE TEMPERATURE OF THE SYSTEM.

A RESULT THAT DERIVES FROM THE EQUIPARTITION THEOREM WHICH STATES THAT THE ENERGY IS SHARED EQUALLY AMONGST ALL ENERGETICALLY ACCESSIBLE DEGREES OF FREEDOM OF A SYSTEM.

BEYOND SCOPE OF CLASS BUT A GOOD REFER STATISTICAL MECHANICS OF ELASTICITY. WEINER

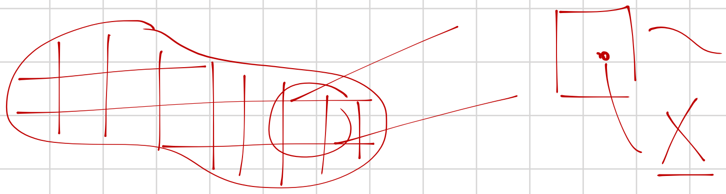
THIS EXCESS KINETIC ENERGY IN CONTINUUM MECHANICS IS DEALT VIA THE INTRODUCTION OF TWO NEW MACROSCOPIC QUANTITIES.

- TEMPERATURE

  (AN AVERAGE MEASURE OF KINETIC ENERGY)

- INTERNAL ENERGY

ONE WAY TO THINK ABOUT THIS IS THAT EVERY CONTINUUM MATERIAL POINT IS A THERMODYNAMIC SYSTEM INTERACTING WITH ITS NEIGHBORS



THERMODYNAMIC SYSTEM WITH INTERNAL ENERGY

A CONTINUUM THAT POSSESES BOTH MECHANICAL & THERMAL ENERGY IS TERMED A THERMODYNAMIC CONTINUUM

ALL DEGREES OF FREEDOM THAT CHARACTERIZE THE STATE OF A SYSTEM ARE CALLED THERMODYNAMIC STATE VARIABLES

ANY EQUATION THA INTERRELATES STATE VARIABLES IS CALLED A CONSTITUTIVE EQUATION

ZEROTH LAW OF THERMODYNAMICS

IF A IN EQ WITH B & C THEN B & C IN THERMAL EQ

FIRST LAW OF THERMODYNAMICS

ROUGHLY SPEAKING DISCUSSION

STATEMENT OF CONSERVATION OF ENERGY

TEMPERATURE IS A MACROSCOPIC VARIABLE WHICH CAN REPRESENT AN AVERAGE KINETIC ENERGY OF THE ATOMS

(SIDE NOTE: IF THE AVERAGE KINETIC ENERGY INCREASES YOU HAVE VOLUME EXPANSION. THIS IS HOW WE MEASUR TEMPERATURE WITH THERMOMETER: MERCURY EXPANDS OR CONTRACTS)

HEAT IS THE WORK PERFORMED BY ALL MICROSCOPIC FORCES (IE ALL FORCES THAT ARE NOT MACROSCOPIC)

EG ELECTROMAGNETIC, KINETICS ETCS

OR YOU CAN PUT TWO THERMODYNAMIC SYSTEMS IN CONTACT $\rightarrow$ TRANSFER OF THERMAL FLUCTUATIONS

THE FIRST LAW STATES THAT $\exists\ \mathcal{E}$ OF THE THERMODYNAMIC STATE s S.T.

$$\underbrace{\mathcal{E}(s_2)-\mathcal{E}(s_1)}_{\text{INTERNAL ENERGY OF THE SYSTEM}} = \underbrace{W_{1\to 2}}_{\text{WORK (MACROSCOPIC FORCES)}} + \underbrace{Q_{1\to 2}}_{\text{HEAT (MICROSCOPIC FORCES)}}$$

WITH THE ABOVE THEN THE RATE OF THERMAL WORK (THERMAL POWER)

$$\dot{Q} = \int_{\partial E} \underbrace{q_n(\underline{n})}_{\text{HEAT FLUX}}\, ds + \int_E \underbrace{r}_{\text{HEAT SOURCE}}\, dv \qquad E \subseteq \Sigma$$

SIMILARLY TO TRACTIONS & STRESS TENSOR YOU CAN SHOW THAT q DEPENDS ONLY ON THE NORMAL & FURTHER q DEPENDS LINEARLY ON $\underline{n}$. HENCE $\exists\ \underline{q}$ SUCH THAT

$$q_n = -\underline{q}\cdot\underline{n}$$

THE ABOVE FOLLOWS FROM STOKE'S HEAT FLUX THEOREM OR THE HAMMEL-NOLL THM

NOW WE CAN POSTULATE THE EXISTANCE OF A SPECIFIC (PER UNIT MASS) INTERNAL ENERGY u SUCH THAT

$$\mathcal{E} = \int_E \rho u \, dv$$

& HENCE THE FIRST LAW

$$\frac{d\mathcal{E}}{dt} = P^{INT} + \dot{Q}$$

STRESS POWER → DEFORMATION POWER

$$\frac{d}{dt}\int_E \rho u \, dv = \int_E \underline{\underline{\nabla}} : \underline{\underline{d}} \, dv + \left[\int_E r \, dv + \int_{\partial E} -\underline{q} \cdot \underline{n} \, ds\right]$$

$$\int_E \left(\dot{\rho} u + \rho \dot{u} + \rho u \underline{\nabla} \cdot \underline{v}\right) dv = \int_E \underline{\underline{\nabla}} : \underline{\underline{d}} \, dv + \left[\int_E r - \underline{\nabla} \cdot \underline{q} \quad dv\right]$$

↓ CONS OF MASS

$$\int_E \rho \dot{u} \, dv = \int_E \left[\underline{\underline{\nabla}} : \underline{\underline{d}} + r - \underline{\nabla} \cdot \underline{q}\right] dv$$

$$\Rightarrow \int_E \left[\rho \dot{u} - \underline{\underline{\nabla}} : \underline{\underline{d}} - r + \underline{\nabla} \cdot \underline{q}\right] dv = 0$$

$$\Rightarrow \boxed{\rho \dot{u} - \underline{\underline{\nabla}} : \underline{\underline{d}} - r + \underline{\nabla} \cdot \underline{q} = 0 \quad \forall \, \underline{x} \in \Omega}$$

EXAMPLE: ASSUME WE HAVE NO DEFORMATION OR HEAT SOURCE. $u = \theta$ ← TEMPERATURE AND FOLLOWING FOURIER'S LAW

$$\underline{q} = -k \nabla \theta$$

$$\rho \frac{d\theta}{dt} = k \Delta \theta \quad \leftarrow \text{THE HEAT EQUATION !!!}$$

NOW RECALL THAT

$$\int_E \underline{b}\cdot\underline{v}\,dV + \int_{\partial E} \underline{t}\cdot\underline{v}\,ds = \int_E \underline{\underline{T}} : \underline{\underline{d}}\,dv + \frac{d}{dt}\int_E \frac{1}{2}\rho\underline{v}\cdot\underline{v}\,dv$$

$$\Rightarrow \int_E \underline{\underline{T}} : \underline{\underline{d}}\,dv = \int_E \underline{b}\cdot\underline{v}\,dv + \int_{\partial E} \underline{t}\cdot\underline{v}\,ds - \frac{d}{dt}\int_E \frac{1}{2}\rho\underline{v}\cdot\underline{v}\,dv$$

$$\frac{d}{dt}\int_E \left(\frac{1}{2}\rho\underline{v}\cdot\underline{v} + \rho u\right)dv = \int_E (r + \underline{b}\cdot\underline{v})\,dv + \int_{\partial E}(\underline{q}\cdot\underline{n} + \underline{t}\cdot\underline{v})\,ds$$

$$\frac{d}{dt}K + \frac{d}{dt}\mathcal{E} = P^{EXT} + Q$$

NOW SIMILARLY WE CAN WRITE

$$Q = \int_E r\,dv + \int_{\partial E} \underline{q}\cdot\underline{n}\,ds = \int_{E_0} rJ\,dV_0 + \int_{\partial E_0} \underline{q}\cdot J\underline{\underline{F}}^{-T}\underline{N}\,dS_0 = \int_{E_0} R\,dV_0 + \int_{\partial E_0} \underline{Q}\cdot\underline{N}\,dS_0$$

$$R = Jr, \quad \underline{Q} = J\underline{q}\,\underline{\underline{F}}^{-T}$$

AND

$$\mathcal{E} = \int_E \rho u\,dV = \int_{E_0} \rho_0 u\,dV_0$$

$$\frac{d}{dt}\int_{E_0} \rho_0 u\,dV_0 = \int_{E_0} \underline{\underline{P}} : \dot{\underline{\underline{F}}}\,dV_0 + \int_{E_0} R\,dV_0 - \int_{E_0} \underline{Q}\cdot\underline{N}\,dS_0$$

$$\boxed{\rho_0\dot{u} = \underline{\underline{P}} : \dot{\underline{\underline{F}}} - \nabla_{\underline{X}}\cdot\underline{Q} + R \quad \forall\ \underline{X} \in \Omega}$$

REPLACING

$$\int_{E_0} \underline{P} : \underline{\underline{F}} \, dV_0 = \int_{E_0} \underline{B} \cdot \underline{V} + \int_{\partial E_0} \underline{T} \cdot \underline{V} \, dS_0 + \frac{d}{dt} \int \frac{1}{2} \rho_0 \underline{V} \cdot \underline{V} \, dV_0$$

$$\frac{d}{dt} \int_{E_0} \left(\frac{1}{2} \rho_0 \underline{V} \cdot \underline{V} + \rho_0 u \right) dV_0 = \int_{E_0} \left(R + \underline{B} \cdot \underline{V} \right) dV_0 + \int_{\partial E_0} \left(\underline{T} \cdot \underline{V} + \underline{Q} \cdot \underline{N} \right) dS_0$$

NOTE THAT EFFECTIVELY THE ABOVE STATES THAT ENERGY CAN BE EXCHANGED BETWEEN MECHANICAL & THERMAL ENERGY.

THIS TOTAL INCONVERTIBILITY DOES NOT HOLD TRUE FOR IRREVERSIBLE PROCESSES.

IE. MECHANICAL WORK CAN BE TRANSFERED THROUGH FRICTION IN HEAT BUT THE VICEVERSA IS NOT TRUE.

THE ABOVE IS THE MOTIVATION FOR THE SECOND LAW OF THERMODYNAMICS.