Continuum Mechanics and Thermodynamics

Overview

The course covers the fundamentals of the mechanics and thermodynamics of continua. It begins by reviewing concepts of tensor analysis on manifolds and tensor calculus. It then proceed by developing the fundamental concepts of the kinematics of a deforming continuum. The notion of stress is then introduced and various measures of stress, with their respective energy-conjugate strain measures, will be discussed. Balance laws will be presented discussing conservation of mass, balance of momentum and moment of momentum, as well as energy. Balance of energy in thermodynamics will also be discussed alongside the restrictions of the second law on constitutive theories. Constitutive theories will be discussed and specific examples will be explored. Variational principles will be presented and the Euler-Lagrange equations will be re-connected with balance laws.

Audience:	Graduate students only
Prerequisite:	Knowledge of multivariable calculus, partial differential equations, and fluid and/or solid mechanics
Instructor:	Maurizio M. Chiaramonte E324 Engineering Quad chiaramonte@princeton.edu
Course website:	continuummechanics.princeton.edu The course website will contain homework, notes and other relevant material.
Grading:	Letter grades A-F. Grades will be based on homework assignments ($4 \times 7.5\% = 30\%$), a midterm & final exam ($2 \times 25\% = 50\%$) and a project (20%).
Homework:	There will be bi-weekly homework assignments focusing on theory and applications of continuum mechanics. The homework should be typeset in LaTeXand are to be submitted in Blackboard's dropbox. Homework will generally be posted on Friday and due in two weeks.
Exams:	The exams will be closed-book and closed-notes. Dates TBD.

Project: The project will consist in a written review and a brief presentation of a relevant manuscript.

Textbooks: No required textbook. The following are good references for the course.

M. E. Gurtin, E. Fried, and L. Anand. *The Mechanics and Thermodynamics of Continua*. Cambridge: Cambridge University Press, 2010. ISBN: 9780511762956. DOI: 10.1017/CBO9780511762956. URL: http://ebooks.cambridge.org/ref/id/CBO9780511762956.

G. A. Holzapfel. *Nonlinear Solid Mechanics: A Continuum Approach for Engineering*. Wiley, 2000. ISBN: 0471823198.

L. E. Malvern. *Introduction to the Mechanics of a Continuous Medium*. 1969. ISBN: 134876032. DOI: 10.1115/1.3625016.

Course Outline

1. Review of tensor analysis:
   (a) Vector and tensor algebra
   (b) Tensor analysis on manifolds
   (c) Tensor calculus and integral theorems
2. Kinematics - Motion:
   (a) Configuration and motion
   (b) Displacement
   (c) Velocity and acceleration
   (d) Material and space derivatives
3. Kinematics - Deformation:
   (a) Stretch
   (b) Deformation gradient
   (c) Polar decomposition
   (d) Metric tensors

4. Kinematics - Strain:
    (a) Strain tensors
    (b) Spherical and deviatoric decomposition
    (c) Principal stretch/strain
    (d) Lie derivative
5. Concept of stress:
    (a) Traction vector and stress tensor
    (b) Stress tensors
    (c) Principal stresses
6. Balance principles - Mass and momentum
    (a) Balance of mass
    (b) Reynolds' transport theorem
    (c) Balance of linear and angular momentum
    (d) Cauchy's first equation of motion
7. Balance principles - Energy and entropy
    (a) Kinetic and internal energy
    (b) Balance of mechanical energy
    (c) Balance of energy in continuum thermodynamics
    (d) Entropy inequality
    (e) Master balance principle
8. Aspects of objectivity
    (a) Change of observer and objectivity
    (b) Superimposed rigid body motion
    (c) Objective rates
    (d) Invariance
9. Constitutive theory
    (a) Thermodynamic potentials
    (b) Second law of thermodynamics

(c) Strain energy function

(d) Stress and elasticity tensors

(e) Concept of internal variables

10. Variational Principles

(a) Review of Calculus of Variations

(b) Stationarity of Potential Energy

(c) Two- and three-field Variational Principles

Tentative Schedule:

Monday	Wednesday
Feb 5th **1** Course introduction and overview **Tensor Analysis:** Vector and tensor algebra	7th **2** **Tensor Analysis:** Vector and tensor algebra
12th **3** **Tensor Analysis:** Vector and tensor calculus, generalized coordinates and a brief overview of differential geometry concepts	14th **4** **Kinematics:** Configuration, motion, and conditions for admissibility
19th **5** **Kinematics:** Velocities, accelerations, material and spatial derivatives	21st **6** **Kinematics:** Deformation gradient, metric tensor, and metric changes
26th **7** **Kinematics:** Polar decomposition, push forward and pull back, and covariance and contravariance	28th **8** **Kinematics:** Strain tensors, Euler-Almansi, Green-Lagrange etc.
Mar 5th **9** **Stress:** Forces in continuum mechanics, traction vector, Cauchy's tetrahedron theorem and $\exists$ of stress tensor	7th **10** **Stress:** Different stress measures, Cauchy's, first and second Piola-Kirchhoff stress tensors

Monday	Wednesday
12th **11** **Balance Principles:** Balance of mass and Reynold's transport theorem	14th **12** **Balance Principles:** Balance of momentum and moment of momentum
19th Spring Recess	21st Spring Recess
26th **13** **Midterm exam**	28th **14** **Balance Principles:** Kinetic and internal energy, balance of mechanical energy
Apr 2nd **15** **Balance Principles:** Continuum thermodynamics	4th **16** **Balance Principles:** Balance of energy in continuum thermodynamics
9th **17** **Balance Principles:** Entropy and the Clausius-Duhem inequality	11th **18** **Balance Principles:** Master balance principle
16th **19** **Objectivity:** Change of observers, objective fields, and objective rates	18th **20** **Constitutive Theories:** General remarks, Coleman-Noll exploitation, examples of strain energy functions
23rd **21** **Constitutive Theories:** Simple damage models, thermodynamics of materials	25th **22** **Variational Principles:** Review of the calculus of variations, principle of minimum of potential energy, virtual work
30th **23** **Variational Principles:** Two- and three- field variational principles (Final Project Presentations)	May 2nd **24** Final Project Presentations

Monday	Wednesday
7th Reading Period	9th Reading Period
14th Reading Period	16th Final Examination Period
21st Final Examination Period	23rd Final Examination Period