LECTURE 17

- OBJECTIVITY

FRAME INDIFFERENCE

EFFECTIVELY BALANCE LAWS GOVERN ALL CONTINUA AND CONSTITUTIVE LAWS ARE SPECIFIC TO MATERIALS

THE STATEMENT OF MATERIAL FRAME INDEFFERENCE STATES THAT THE MATERIAL THERMOMECHANICAL POTENTIAL SHOULD BE INDEPENDENT OF OBSERVER. I.E. INVARIANT UNDER TRANSLATION & ROTATION. TO FORMALIZE THE ABOVE WE NEED TO BUILD UP TO IT.

CONSIDER $\underline{x} = \underline{\varphi}(\underline{X}, t)$

S.T. $\underline{x}_0 = \underline{\varphi}(\underline{X}, t_0)$, $\underline{x}_1 = \underline{\varphi}(\underline{X}, t_1)$

NOW LET $(\underline{x}_i, t_i) \mapsto (\underline{x}_i^+, t_i^+)$ SUCH THAT DISTANCE $|\underline{x}_0 - \underline{x}_1|$ & TIME ELAPSED ARE PRESERVED.

ONE SUCH MAP IS

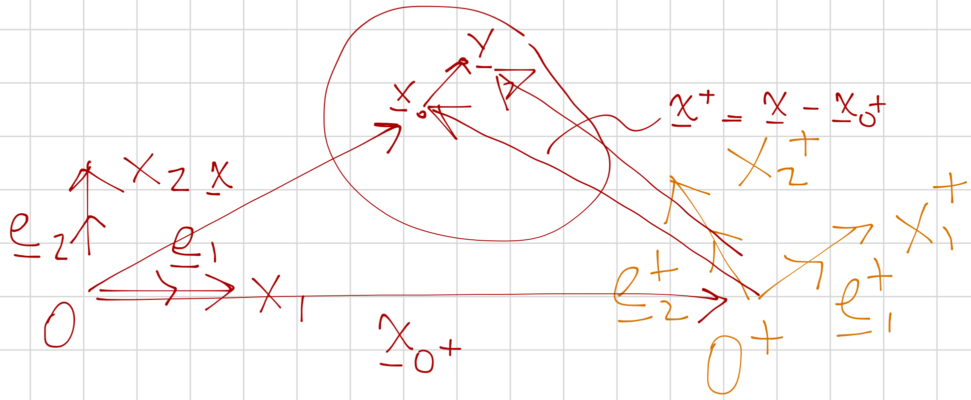
$$\underline{x}^+ = \underline{c}(t) + \underline{\underline{Q}}(t)\underline{x} \quad , \quad t^+ = t + \alpha$$

WHERE $\underline{\underline{Q}} \in SO(3)$.

MAPPING OF THE ABOVE FORM ARE KNOWN AS EUCLEDIAN TRANSFORMATIONS

NOW THINK OF TWO OBSERVERS O & O^+

TO EACH OBSERVER WE ATTACH A COORDINATE SYSTEM & A BASIS



O & O^+ ARE MOVING RELATIVE TO ONE ANOTHER $\dot{\underline{x}}_{0^+} \neq 0$

LET TWO POINTS BE EXPRESSED AS $\underline{x}, \underline{y}$

$$\underline{x}^+ = \underline{\underline{Q}}\,\underline{x} + \underline{c}, \quad \underline{y}^+ = \underline{\underline{Q}}\,\underline{y} + \underline{c}$$

THEN LET

$\underline{u} = \underline{y} - \underline{x}$

$$\underline{u}^+ = \underline{y}^+ - \underline{x}^+ = \underline{\underline{Q}}(\underline{y} - \underline{x}) = \underline{\underline{Q}}\,\underline{u}$$

EFFECTIVELY A CHANGE OF OBSERVER CAN BE EXPRESSED BY THE SUPERPOSITION OF A RIGID BODY MOTION, NAMELY A ROTATION & A TRANSLATION.

$$\underline{x} = \underline{\varphi}(\underline{X}, t)$$

$$\underline{x}^+ = \underline{\varphi}^+(\underline{X}, t) = \underline{\xi}(\underline{\varphi}(\underline{X}, t), t), \quad \underline{\xi}(\underline{x}, t) = \underline{\underline{Q}}(t)\underline{X} + \underline{c}(t)$$

$$\underline{x}^+ = \underline{\underline{Q}}(t)\underline{x} + \underline{c}$$

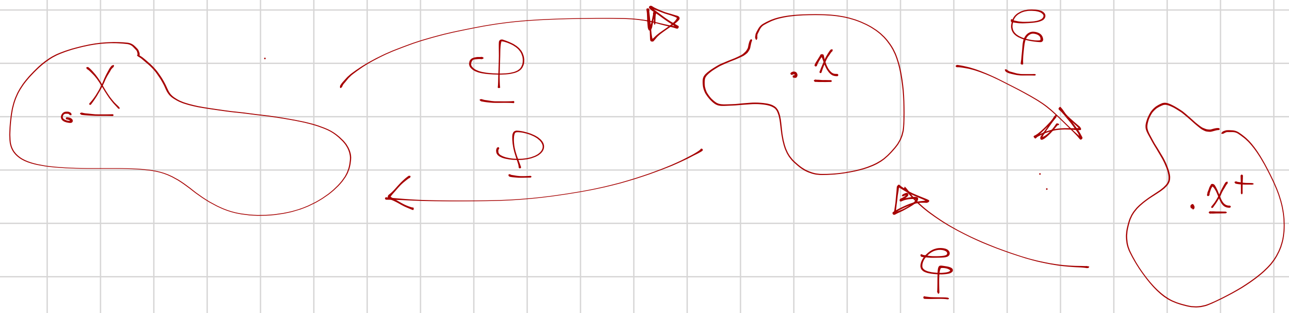
NOW CONSIDER $\underline{X}, \underline{Y}$, $\underline{x}, \underline{y}$, $\underline{x}^+, \underline{y}^+$

A VECTOR FIELD $\underline{w} = \underline{y} - \underline{x}$ IS SAID TO BE OBJECTIVE IF

$$\underline{w}^+ = \underline{y}^+ - \underline{x}^+ = \underline{\underline{Q}}(t)\underline{w} = \underline{\underline{Q}}(t)(\underline{y} - \underline{x}) = (\underline{\underline{Q}}\,\underline{y} + \underline{c}) - (\underline{\underline{Q}}\,\underline{x} + \underline{c}) = \underline{y}^+ - \underline{x}^+$$

NAMELY IT PRESERVE MAGNITUDE AND RELATIVE DIR

NOTE $\underline{x} = \underline{\xi}^{-1}(\underline{x}^+, t) = \underline{\underline{Q}}^T(\underline{x}^+ - \underline{c})$



$$\underline{v} = \dot{\underline{X}}, \quad \underline{v} = \overline{\left(\underline{\underline{Q}}^T(\underline{x}^+ - \underline{c})\right)}^{\cdot} = \dot{\underline{\underline{Q}}}(\underline{x}^+ - \underline{c}) + \underline{\underline{Q}}^T(\underline{v}^+ - \dot{\underline{c}})$$

$$\underline{v}^+ = \underline{\underline{Q}}\,\underline{v} - \underline{\underline{Q}}\,\dot{\underline{\underline{Q}}}(\underline{x}^+ - \underline{c}) + \dot{\underline{c}} = \underline{\underline{Q}}\,\underline{v} + \underline{\underline{\Omega}}(\underline{x}^+ - \underline{c}) + \dot{\underline{c}}$$

NOTE

$$\overline{\underline{\underline{Q}}\,\underline{\underline{Q}}^T}^{\cdot} = 0 \Rightarrow \dot{\underline{\underline{Q}}}\,\underline{\underline{Q}}^T + \underline{\underline{Q}}\,\dot{\underline{\underline{Q}}}^T = 0 \Rightarrow \dot{\underline{\underline{Q}}}\,\underline{\underline{Q}}^T = -\underline{\underline{Q}}\,\dot{\underline{\underline{Q}}}^T$$

THUS $\underline{\underline{\Omega}} = \dot{\underline{\underline{Q}}}\,\underline{\underline{Q}}^T = -\underline{\underline{Q}}\,\dot{\underline{\underline{Q}}}^T = -\underline{\underline{\Omega}}^T$ ← SKEW SYMMETRIC

THUS THE SPATIAL VELOCITY FIELD IS NOT OBJECTIVE.

IT IS OBJECTIVE WITH RESPECT TO TIME INDEPENDENT RIGID TRANSFORMATIONS, NAMELY IF

$$\dot{\underline{\underline{Q}}} = 0, \quad \dot{\underline{c}} = 0$$

AKA THE OBSERVER IS NOT MOVING. SUCH THAT

$$\underline{v}^+ = \underline{\underline{Q}}\,\underline{v}$$

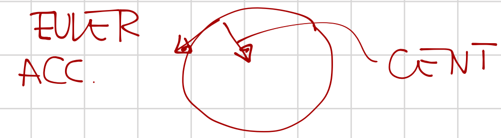
NOW IN TERMS OF ACCELLERATION

$$\underline{a}^+ = \dot{\underline{v}}^+ = \dot{\underline{\underline{Q}}}\,\underline{v} + \underline{\underline{Q}}\,\dot{\underline{v}} + \dot{\underline{\underline{\Omega}}}(\underline{x}^+ - \underline{c}) + \underline{\underline{\Omega}}(\underline{v}^+ - \dot{\underline{c}}) + \ddot{\underline{c}} =$$

$$= \underline{\underline{Q}}\,\underline{a} + \dot{\underline{\underline{Q}}}\left(\dot{\underline{\underline{Q}}}^T(\underline{x}^+ - \underline{c}) + \underline{\underline{Q}}^T(\underline{v}^+ - \dot{\underline{c}})\right) + \dot{\underline{\underline{\Omega}}}(\underline{x}^+ - \underline{c}) + \underline{\underline{\Omega}}(\underline{v}^+ - \dot{\underline{c}}) + \ddot{\underline{c}} =$$

$$= \underline{\underline{Q}}\,\underline{a} + \ddot{\underline{c}} + (\dot{\underline{\underline{\Omega}}} - \underline{\underline{\Omega}}\,\underline{\underline{\Omega}})(\underline{x}^+ - \underline{c}) + 2\underline{\underline{\Omega}}(\underline{v}^+ - \dot{\underline{c}})$$

THUS IN ORDER FOR $\underline{a}^+$ TO BE OBJECTIVE WE MUST HAVE



$\underline{\underline{\Omega}}\,(\underline{x}^+ - \underline{c})$ ← EULER ACCELERATION

$-\underline{\underline{\Omega}}\,\underline{\underline{\Omega}}\,(\underline{x}^+ - \underline{c})$ ← CENTRIFUGAL ACCELERATION

$2\underline{\underline{\Omega}}\,(\underline{v}^+ - \underline{c})$ ← CORIOLIS

IF $\ddot{\underline{c}} = 0$, $\dot{\underline{\underline{Q}}} = 0$ THEN OUR NEW FRAME OF REFERENCE IS INERTIAL & THUS NEWTON'S LAWS OF MOTION STILL HOLD

A TRANSFORMATION THA SATISFIES THE ABOVE IS KNOWN AS A GALILEAN TRANSFORMATION

$$\underline{\varphi}(\underline{x}, t) = \underline{c}(t) + \underline{\underline{Q}}\,\underline{x} \qquad \ddot{\underline{c}} = 0,\ \dot{\underline{\underline{Q}}} = 0$$

IN THE ABOVE CASE

$$\underline{a}^+ = \underline{\underline{Q}}\,\underline{a}$$

HENCE THE ACCELERATION IS OBJECTIVE WITH RESPECT TO GALILEAN TRANFORMATIONS.

CONSIDER A TENSOR $\underline{\underline{G}}$ & TWO VECTOR FIELDS THAT ARE FRAME IND. S.T.

$$\underline{u} = \underline{\underline{G}}\,\underline{w} \quad \& \quad \underline{u}^+ = \underline{\underline{Q}}\,\underline{u},\ \underline{w}^+ = \underline{\underline{Q}}\,\underline{w}$$

THEN $\underline{\underline{G}}$ IS FRAME INDIFFERENT IF $\underline{\underline{G}}^+$

$$\underline{u}^+ = \underline{\underline{G}}^+ \underline{w}^+$$

TRANSFORMS AS $\underline{\underline{G}}^+ = \underline{\underline{Q}}\,\underline{\underline{G}}\,\underline{\underline{Q}}^T$.

IF $\underline{\underline{G}} = \underline{g}_1 \otimes \underline{g}_2 \Rightarrow \underline{\underline{G}}^+ = \underline{\underline{Q}}\,\underline{g}_1 \otimes \underline{\underline{Q}}\,\underline{g}_2 \Rightarrow \underline{\underline{G}}^+ = \underline{\underline{Q}}\left(\underline{g}_1 \otimes \underline{g}_2\right)\underline{\underline{Q}}^T$

EUCLEDIAN TRANSFORMATION OF KIN $\underline{Q}$

$$\underline{\underline{F}}^{+} = \frac{d\, \underline{\varphi} \circ \underline{\varphi}}{d \underline{X}} = \underline{\underline{Q}}\,\underline{\underline{F}}$$ ← STILL OBJECTIVE SINCE $\underline{\underline{F}}$ IS A TWO POINT TENSOR

$$\underline{\underline{F}}^{+} = \underline{\underline{Q}}\, \underline{g} \otimes \underline{G}$$

SINCE $\underline{\varphi}^{+}(\underline{X}, 0) = \underline{X}$