PRECEPT 11

CEE 361-513: Introduction to Finite Element Methods

Monday Dec. 11

PROBLEM 1: Incompressible Elasticity

Today we look at the issue of locking in incompressible solids and how could we alleviate the problem. For the Strong and Weak form of the problem refer to lecture notes and the homework-9. The basic problem configuration is summarized in the image below (1). A plate of specific dimensions is fixed at left and bottom edges and a point load is applied at the top right corner. Further, plane strain condition is assumed.

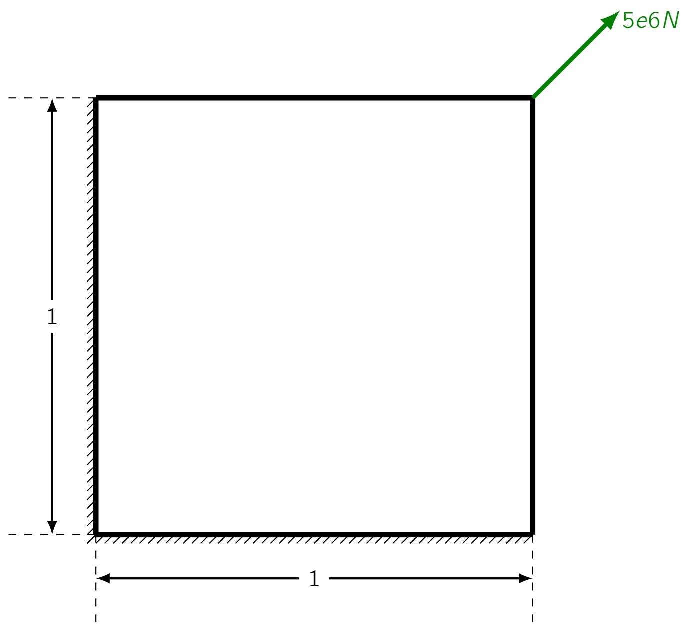

Figure 1: Geometry

1. First we try to solve the problem using standard elasticity in the compressible region ($\nu = 0.2$). We modify the code provided on the website under homework 9.
   The mesh is generated as below:

```
height = 1.
length = 1.
# Create the mesh object
mesh = RectangleMesh(Point(0.0, 0.0), Point(length,height),10,10,'right')
```

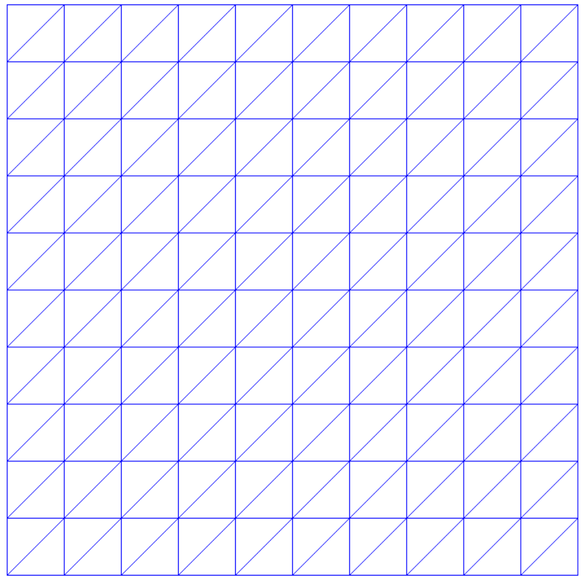

Figure 2: Mesh

Next we apply the point load. In FEniCS point load is applied in a very specific manner. You can read about it in the documentation.

```
# Define the variational form
a = inner(sigma(u),grad(v))*dx
L = dot(Constant((0.0,0.0)),v)*dx

A, b = assemble_system(a, L, bc)

# Point load
delta_0 = PointSource(V.sub(0), Point(length, height), 3.535e6)
delta_1 = PointSource(V.sub(1), Point(length, height), 3.535e6)

delta_0.apply(b)
delta_1.apply(b)

# Compute the soultion
uh = Function(V)

solve(A, uh.vector(), b)
```

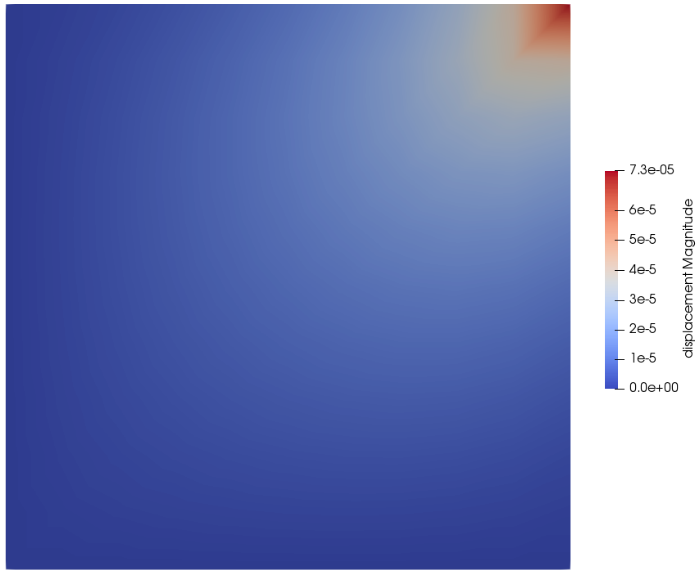

Figure 3: Solution for $\nu = 0.2$

2. Now we would solve the same problem using standard elasticity equations but in near incompressible region. We modify the code by changing ν to ≈ 0.5 to achieve near incompressible condition

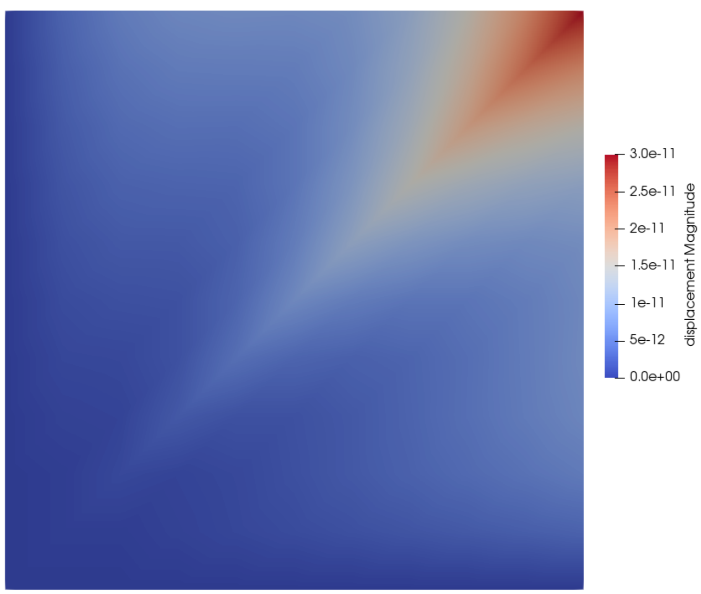

Figure 4: $\nu \approx 0.5$

We clearly see that the standard elasticity cannot solve the problem correctly

3. Next, we solve the problem using the `incompressible_elasticity.py`.
The first type of element we solve the problem with is one in which displacement is linear and continuous while the pressure is constant and discontinuous across elements. The displacement and pressures are

plotted. We can clearly see that the solution is not representative of the real condition. This is because we have more constraints than degrees of freedom and we experience locking phenomenon.

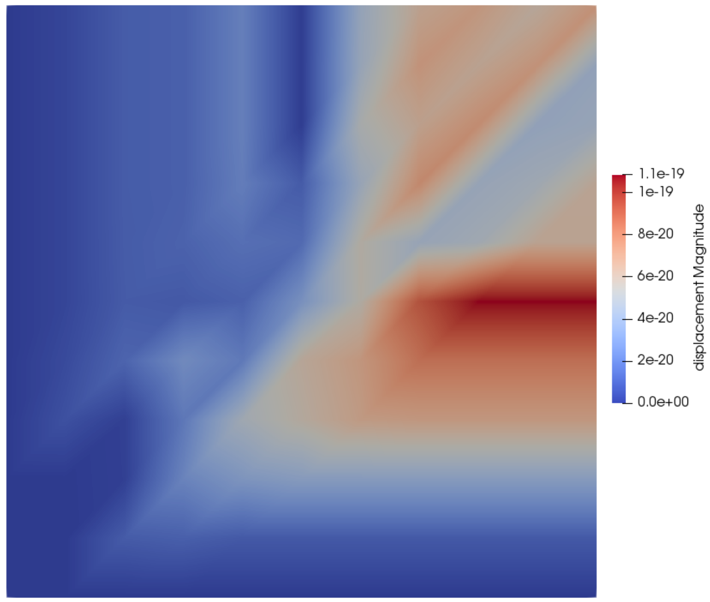

Figure 5: Deformation for 1st type of elements $\nu \approx 0.5$

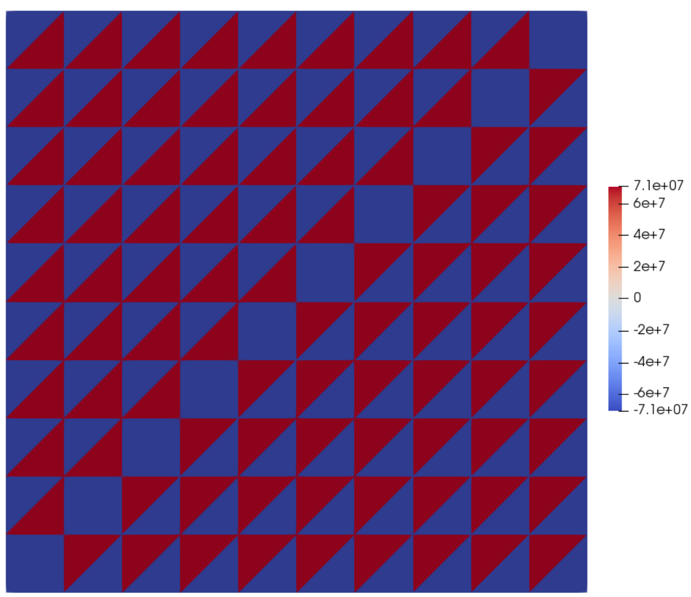

Figure 6: Checkerboard pattern of pressure

Next we change the elements by changing the order of interpolation to 2 and 1 for displacement and pressure respectively and we can see that the exact solution is recovered

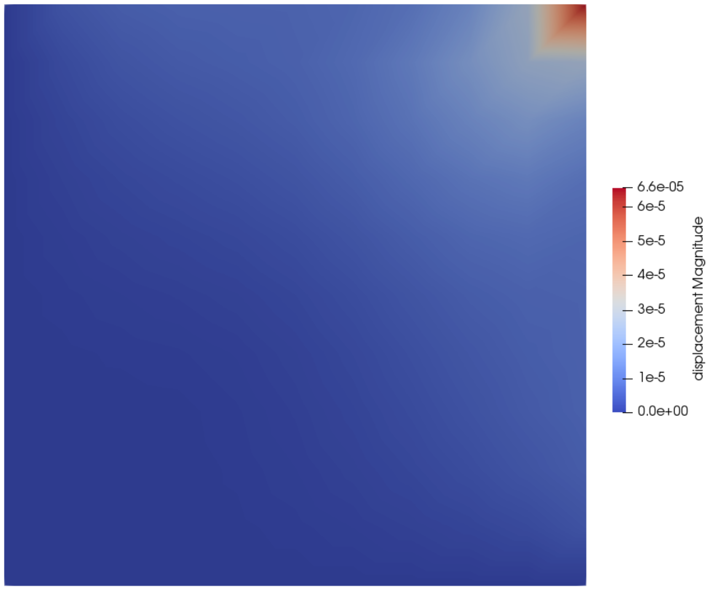

Figure 7: Deformation with quadratic interpolation of displacement and linear for pressure for $\nu \approx 0.5$

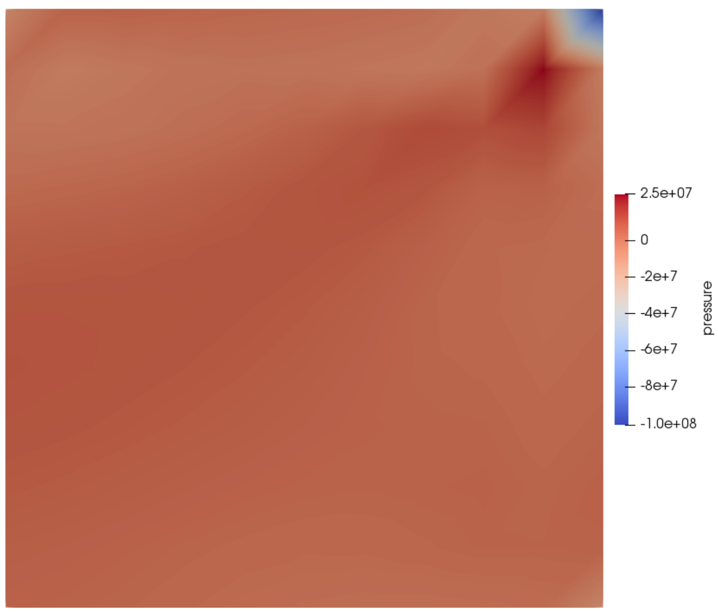

Figure 8: Pressure with quadratic interpolation of displacement and linear for pressure for $\nu \approx 0.5$