LECTURE 17:

TOPICS:

- FINAL REVIEW 1-D FINITE ELEMENTS
- STRUCTURE OF FE CODE
- n-D FINITE ELEMENTS

LOGISTICS

- HW # 7 DUE DEC 7

REVIEW

SEE SLIDES REVIEW FEM 1-D

SEE SLIDES STRUCTURE OF FEM CODE

SHOW CODE

ND FINITE ELEMENTS:

SIMILARLY TO BEFORE (S) $\Longleftrightarrow$ (W) $\Longrightarrow$ (G) $\Longleftrightarrow$ (M)

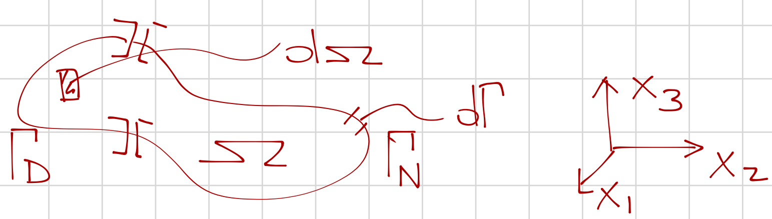
WE START WITH MODEL PROBLEM

STRONG FORM (S)

$$? \, u : \Omega \to \mathbb{R} : \quad \alpha \Delta u = f \qquad \forall \, \underline{x} \in \Omega$$

$$u\big|_{\Gamma_D} = g$$

$$\alpha \underline{\nabla} u \cdot \underline{n}\big|_{\Gamma_N} = h$$



WEAK FORM (W):

$$S = \{ u \in H^1(\Omega) \mid u|_{\Gamma_D} = g \}, \quad V = \{ v \in H^1(\Omega) \mid v|_{\Gamma_D} = 0 \}$$

$$R(x) = \alpha \Delta u - f \qquad (d\Omega = dx_1 dx_2 dx_3)$$

$$? \, u \in S : \quad \int_\Omega (\alpha \Delta u - f) v \, d\Omega = 0 \qquad \forall \, v \in V$$

$$\int_\Omega \alpha \underline{\nabla} \cdot (\underline{\nabla} u) v \, d\Omega = \int_\Omega \underline{\nabla} \cdot (\alpha \underline{\nabla} u \, v) d\Omega - \int_\Omega \alpha \underline{\nabla} u \underline{\nabla} v \, d\Omega =$$

$$\underline{\nabla} \cdot (\underline{\nabla} u \, v) = (\underline{\nabla} \cdot \underline{\nabla} u) v + \underline{\nabla} u \cdot \underline{\nabla} v \Rightarrow \underline{\nabla} \cdot \underline{\nabla} u \, v = \underline{\nabla} \cdot (\underline{\nabla} u \, v) - \underline{\nabla} u \cdot \underline{\nabla} v$$

$$= \int_\Gamma \alpha v \underline{\nabla} u \cdot \underline{n} \, d\Gamma - \int_\Omega \alpha \underline{\nabla} u \cdot \underline{\nabla} v \, d\Omega = 0 \qquad \forall \, v \in$$

NOW RECALL THAT $\Gamma = \Gamma_D \cup \Gamma_N$ AND THUS

$$= \int_{\Gamma_D} \cancel{\alpha v} \nabla u \cdot \underline{n} \, d\Gamma + \int_{\Gamma_N} v \underbrace{\alpha \nabla u \cdot \underline{n}}_{= h} d\Gamma - \int_{\Omega} \alpha \nabla u \cdot \nabla v \, d\Omega$$

$$= \int_{\Gamma_N} v \, h \, d\Gamma - \int_{\Omega} \alpha \nabla u \cdot \nabla v \, d\Omega = \int_{\Omega} \alpha \nabla \cdot (\nabla u) \, v \, d\Omega$$

NOW GOING BACK TO THE STATEMENT OF WEIGHTED RESIDUALS WE HAVE

$$\int_{\Omega} \alpha \nabla u \cdot \nabla v \, d\Omega = \int_{\Gamma_N} v h \, d\Omega - \int_{\Omega} f v \, d\Omega \quad \forall v \in V$$

THUS THE WEAK FORM BECOMES

(W) FIND $u \in S$ S.T.

$$a(u, v) = F(v) \qquad \forall \; v \in V$$

$$a(u,v) = \int_{\Omega} \alpha \nabla u \cdot \nabla v \, d\Omega$$

$$F(v) = \int_{\Gamma_N} h v \, d\Gamma - \int_{\Omega} f v \, d\Omega$$

AT THIS POINT WE PERFORM THE GALERKIN APPROXIMATION

(G) $S^h \subseteq S, \; V^h \subseteq V$

$$\exists ! \, u^h \in S^h : \quad a(u^h, v^h) = F(v^h) \qquad \forall v^h \in V^h$$

SIMILARLY TO BEFORE THE WAY WE CONSTRUCT S^h IS BY SUBDVIDING THE DOMAIN

$$\Omega = \bigcup_{e=1}^{Ne} \Omega^e$$


Ω^e

THEN OVER EACH ELEMENT WE APPROXIMATE THE FUNCTION WITH COMPLETE POLYNOMIALS OF ORDER P.

THEN OVER EACH ELEMENT WE CONSTRUCT BASIS THAT SPAN THE SPACE OF POLYNOMIALS OF ORDER UP TO p

TO DO SO WE HAVE DIFFERENT CHOICES FOR THE SUBDIVISION OF OUR DOMAIN

HEXAHEDRAL ELEMENTS VS SIMPLICIAL ELEMENTS