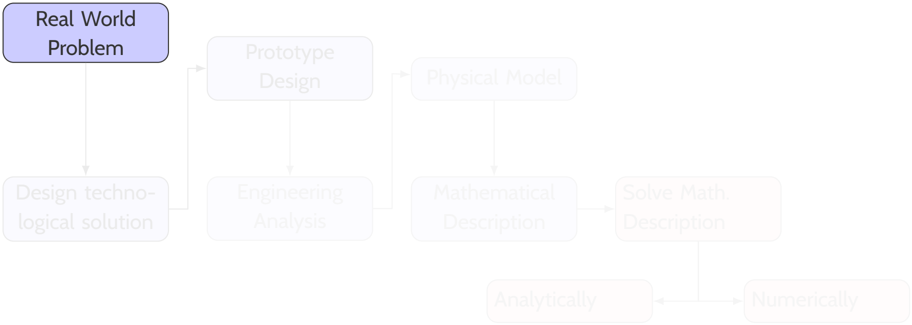
Course Overview & Introduction

CEE 361/513: Intro. to Finite Element Methods

Maurizio M. Chiaramonte

Course Overview

The role of the engineer



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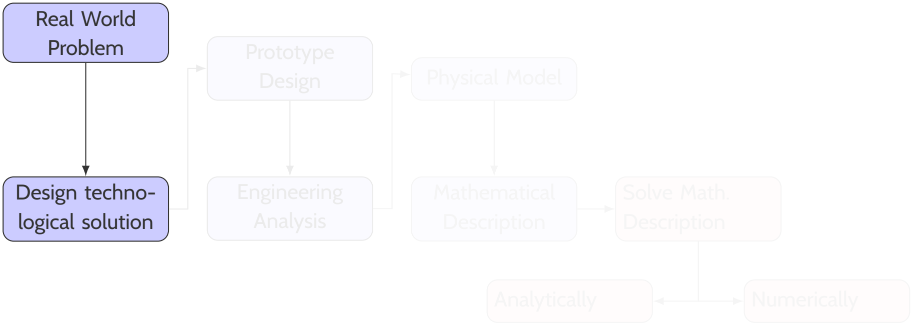

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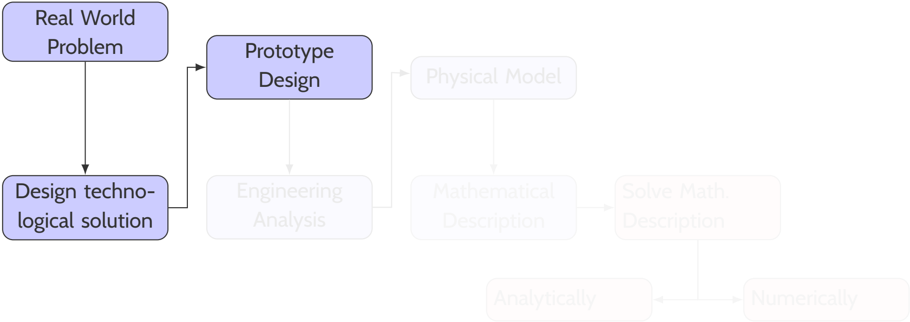

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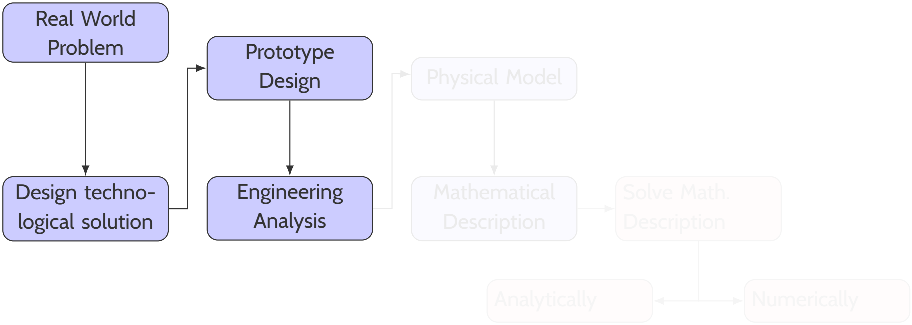

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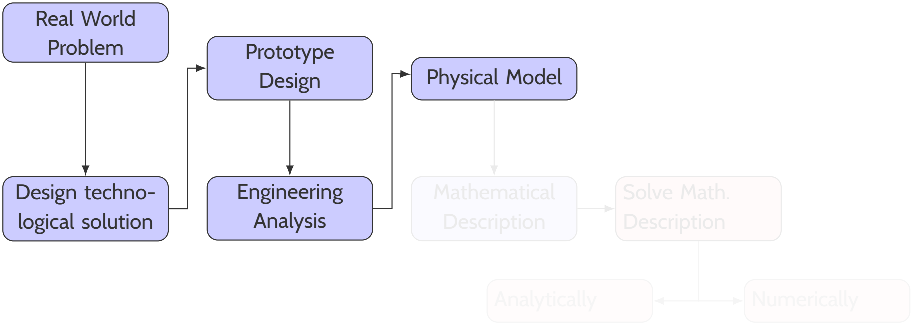

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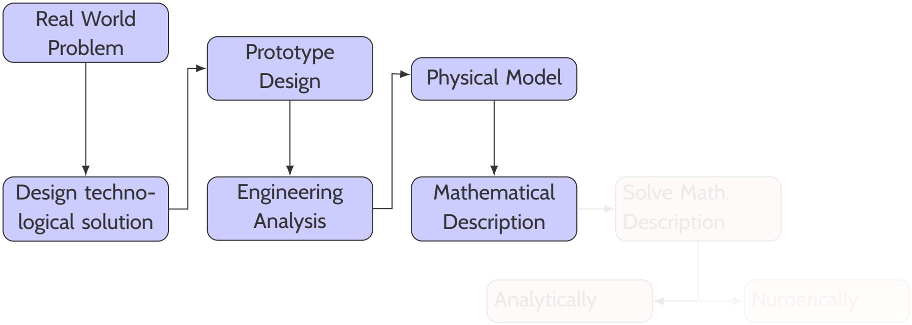

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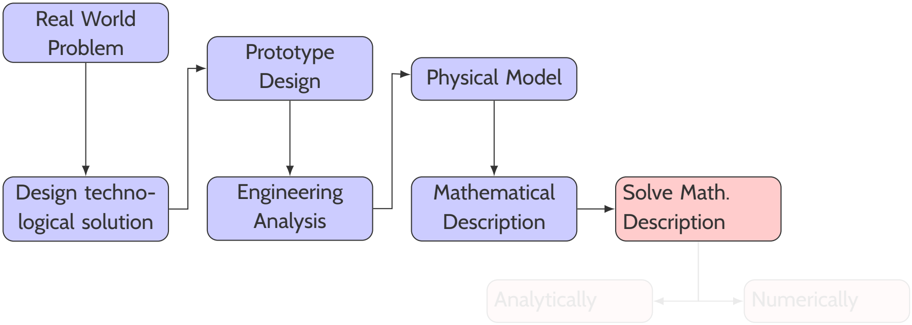

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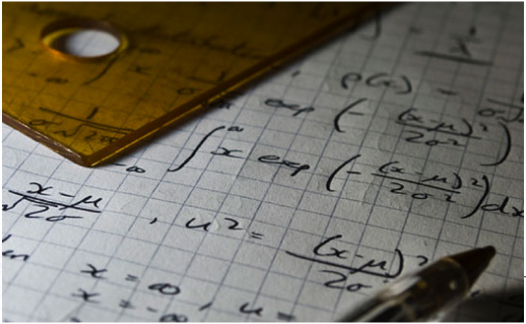

Solve Math. Description

Analytically

Numerically

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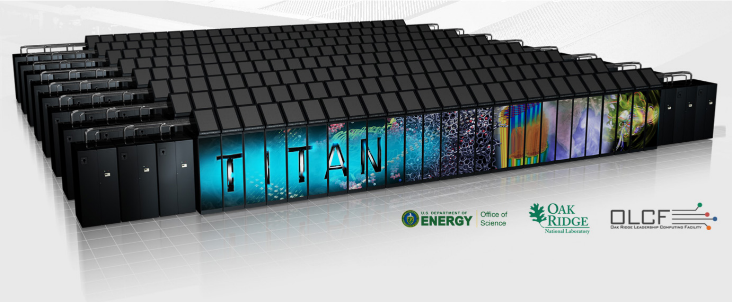

Design techno-logical solution → Engineering Analysis → Mathematical Description → Solve Math. Description

Solve Math. Description → Analytically

Solve Math. Description → Numerically

What is the course about ?

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Finite Element Methods

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A *numerical method* for the solution of *Partial Differential Equations*

Examples of Partial Differential Equations

Examples of Partial Differential Equations

Partial Differential Equations describe phenomena all around us:

- Solid & Structural Mechanics - Cauchy mom. balance

$$\nabla \cdot \boldsymbol{\sigma} = 0$$

- Fluid Mechanics - Navier-Stokes

$$\frac{\partial \boldsymbol{v}}{\partial t} + \nabla \boldsymbol{v} \cdot \boldsymbol{v} + \nu \Delta \boldsymbol{v} = -\nabla p$$

$$\nabla \cdot \boldsymbol{v} = 0$$

- Electromagnetism - Maxwell

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$$

- Finance - Black-Scholes

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS\frac{\partial V}{\partial S} - rV = 0$$

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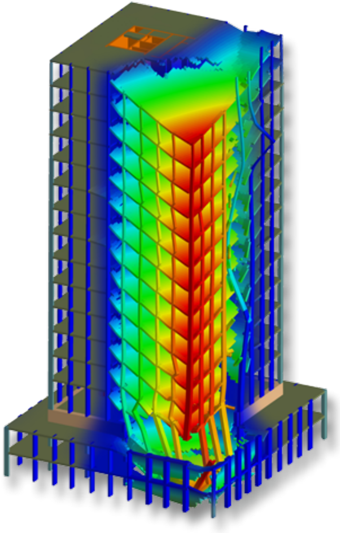
Applications of finite elements



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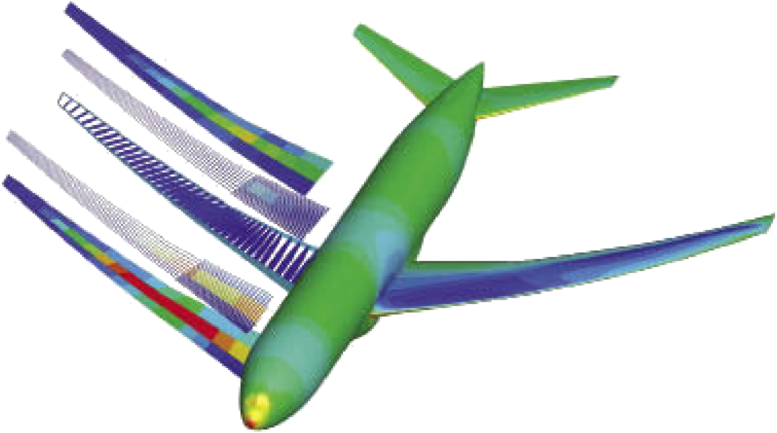

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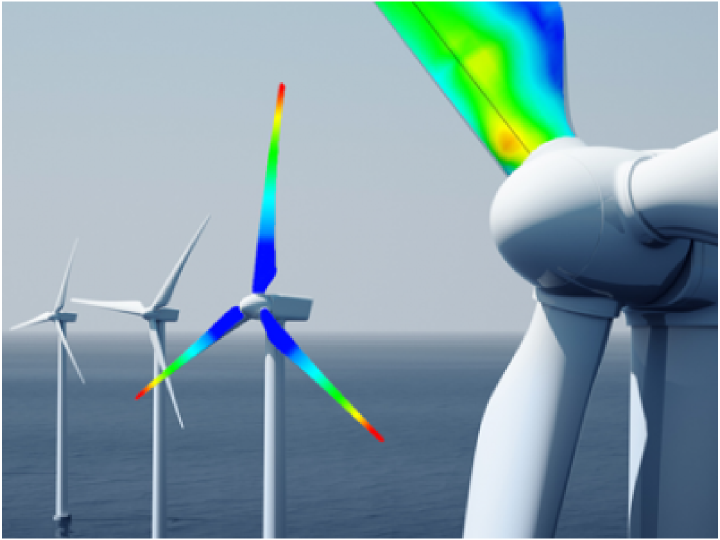

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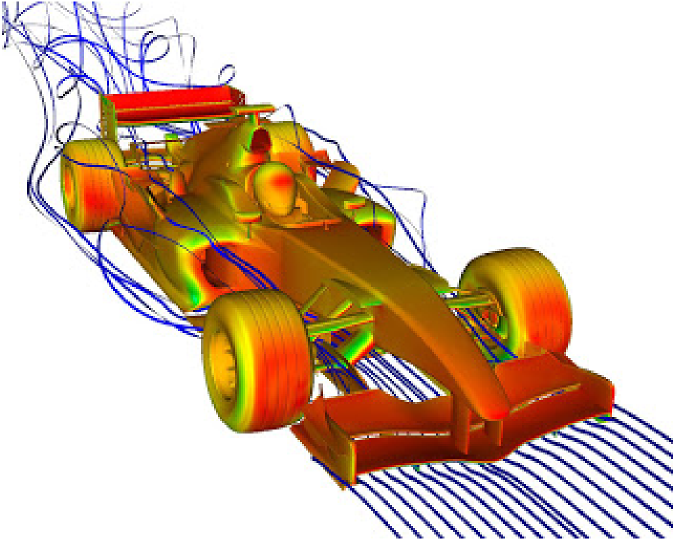

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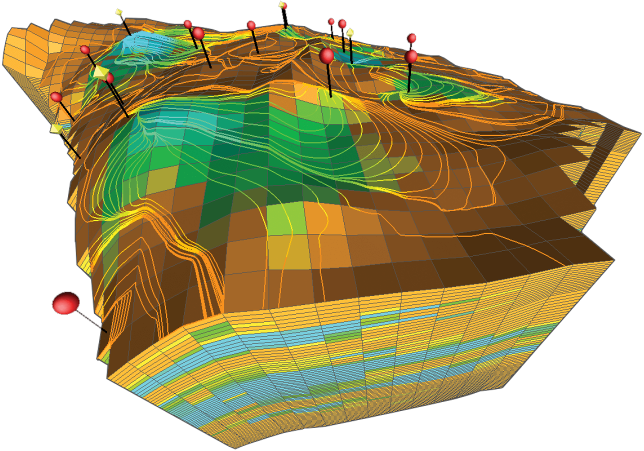

Applications of finite elements

Course Objective

What are we going to talk about

We will learn about finite element methods.

The focus will be on:

- FORMULATING finite element methods.

$$\text{PDEs} \mapsto \text{numerical discretization}$$

- IMPLEMENTING finite element methods

$$\text{numerical discretization} \mapsto \text{computer implementation}$$

Course Logistics

Course Outline

1. Math Preliminaries
   1.1 Tensor algebra and tensor calculus
   1.2 Review of partial differential equations
2. Direct Stiffness Methods
   2.1 Truss Equation
   2.2 Beam Equation
   2.3 Global Assembly
   2.4 Boundary conditions
3. One-dimensional Finite elements
   3.1 Intro to the Calculus of variations
   3.2 Strong and weak form
   3.3 Galerkin approximation
   3.4 Matrix form
   3.5 Shape functions
   3.6 Numerical integration
   3.7 Error analysis

Course Outline

4. Two and three space dimensions
   4.1 Review of tensors calculus
   4.2 Extension of the notions of strong, weak, and matrix forms
   4.3 Hexahedral elements
   4.4 Simplicial elements
   4.5 Isoparametric interpolations
5. C^0 approach to beams and shells

   If Time Permits:
6. Time dependent problems
   6.1 Generalized θ method
   6.2 Stability of the θ-method
7. Analysis of Finite Element Methods
   7.1 Best approximation and error estimates
   7.2 Consistency & stability
   7.3 Locking issues
8. Constraints
   8.1 Lagrange multipliers
   8.2 Penalty methods

Grading

Grading Breakdown

Grades will be based:

- Homework assignments (40%)
- A midterm & final exam (30%)
- A final project (30%).

Grading

Grading Computation

The final grade for the class is computed with both an absolute and relative (curve) scale. If you get above or equal to:

- 98% of the maximum score you get an A+
- 90% of the maximum score you get an A
- 85% of the maximum score you get an A-
- 80% of the maximum score you get an B+
- 75% of the maximum score you get an B
- 70% of the maximum score you get an B-
- 65% of the maximum score you get an C+
- 60% of the maximum score you get an C
- 55% of the maximum score you get an C-
- 50% of the maximum score you get an D+
- 45% of the maximum score you get an D

At least 25% of the class will get an A, at least 50% of the class will get a B or higher, and at least 75% of the class will get a C or higher, as long as the minimum of 45% of the maximum total score is achieved. This distribution will be achieved by uniformly shifting everyone's grade up by the same amount. (Separately for 500 and 300 level)

Homeworks

Homework details and policies

- 6-10 homework
- Homework will be posted on Thursday and due the next Thursday @ midnight
- Homework will be submitted electronically through `https://blackboard.princeton.edu` dropbox
- No late homework will be accepted
- The lowest of your homework grades will be dropped
- All homework will be weighted the same unless specified
- A mixture of theory and computations
- CEE 513 homework will be typeset in LaTeX (will post the *pdf and *tex)
- Non-513 students if homework is typeset in LaTeX get an extra 5%

Exams

Exams details and policies

- Closed book & closed notes
- Focus on theory
- Midterm exam tentative date is Oct 26.

Project

Project plan

- Identify a topic of interest within the realm of finite element method
  - Applications of finite elements to a specific project
  - Development of techniques for finite element methods
- Submit a project proposal after fall recess
- Submit a mid-term project report on Dec 7th
- Submit a final project report due Jan 16th (Dean's date)

Tools for the course

Textbook

Required textbook:

Hughes, T. J. R. (1987).
The Finite Element Method: Linear Static and Dynamic Finite Element Analysis.

Suggested readings:

Prévost, J. H. and S. Bagrianski (2017).
An Introduction to Matrix Structural Analysis and Finite Element Methods.

Tools

- `Python`
- `FEniCS`

Lecture and Precepts

Lecture

- T,Th Friend 008 11:00-12:20
- Will mostly cover theory
- Must come to lecture to take notes
- Please do not eat in class
- Be courteous to other classmates (i.e. no talking, etc …)
- Be engaged and ask questions (if you have any …)!

Precepts

- M 7:30pm-8:30pm
- Will cover sample problems
- Interactive environment

Getting Help

Office hours

Maurizio M. Chiaramonte

Office Hours: Wednesday 1:00pm - 3:00pm, EQuad E324

chiaramonte@princeton.edu

Precept + Office hours

Vivek Kumar

Time: Monday, Precept 7:30pm - 8:30pm Office hours: 9:30pm- 10:30pm

vivekk@princeton.edu

Piazza

`http://piazza.com/princeton/fall2017/cee361mae325mse331cee513f17`

Course Website

Course Website

`http://finiteelements.princeton.edu`

username: `fea`

password: `galerkin`

`Syllabus`

Additional Resources

Princeton Institute for Computational Science and Engineering (PICSciE)

PICSciE