HOMEWORK 9

CEE 361-513: Introduction to Finite Element Methods

Due: Friday Dec. 15 @ Midnight

NB: Students taking CEE 513 must complete all problems. All other students will not be graded for problems marked with $\star$, but are encourage to attempt them anyhow.

PROBLEM 1:

1. Read and summarize Sections 4.1 - 4.4.1 (excluded)

PROBLEM 2: Incompressible Elasticity

The problem looks at 2-D incompressible elasticity. The problem reads : find $\boldsymbol{u} : \Omega \to \mathbb{R}^2$ and $p : \Omega \to \mathbb{R}$ such that :

$$\nabla \cdot \boldsymbol{\sigma}(\nabla \boldsymbol{u}, p) = \boldsymbol{f}, \quad \forall \boldsymbol{x} \in \Omega$$
$$\nabla \cdot \boldsymbol{u} + \frac{p}{\lambda} = 0, \quad \forall \boldsymbol{x} \in \Omega$$

and

$$\boldsymbol{u} = \boldsymbol{g} \quad \text{on } \Gamma_\mathrm{D}$$
$$\boldsymbol{\sigma}(\nabla \boldsymbol{u}, p) \cdot \boldsymbol{n} = \boldsymbol{t} \quad \text{on } \Gamma_\mathrm{N}$$

where Γ_D and Γ_N are the Dirichlet and Neumann boundaries respectively. Further,

$$\boldsymbol{\sigma}(\nabla \boldsymbol{u}, p) = -p\mathbf{1} + 2\mu \nabla^S \boldsymbol{u}$$

The specific problem we are are looking at is called "cook-membrane" problem. The basic problem configuration is summarized in the image below (1). A beam of specific dimensions is fixed at one end and a uniform traction load is applied at the other end such that the total force acting on this surface totals 1 N. Further, plane strain condition is assumed.

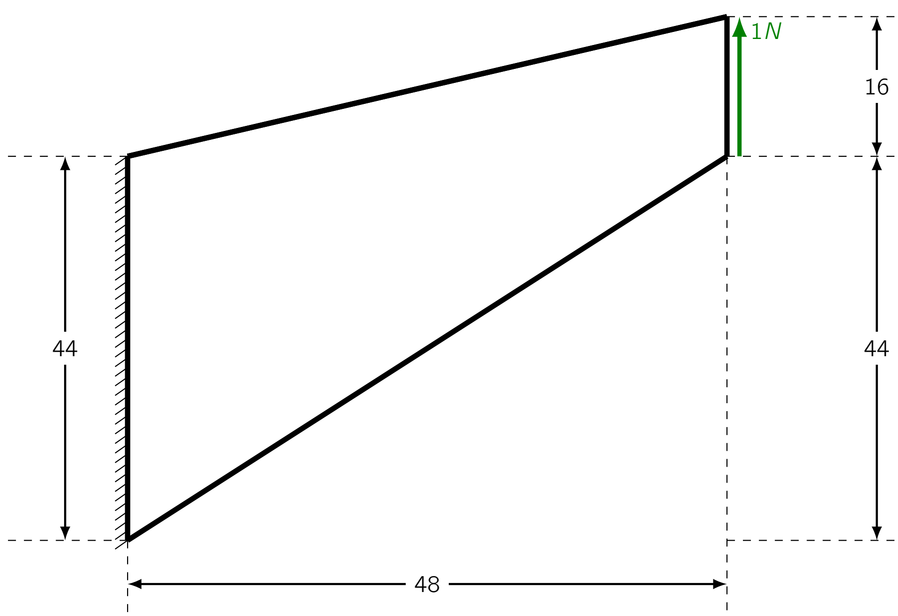

Figure 1: Cook Membrane

1. First we try to solve the problem using standard elasticity in the compressible region ($\nu = 0.3$). Modify the code provided `standard_elasticity.py` to solve the above problem. You would have to perform the following steps:

   (a) Redefine the Dirichlet and Neumann boundaries on lines# 70,75

   (b) Modify the forcing functional on line # 101

   What is the maximum y-displacement?
   Plot the deformed shape.

2. Now we would solve the same problem using standard elasticity equations but in near incompressible region. How could you modify the code to achieve near incompressible condition? What is the maximum y-displacement. Plot your deformed shape. Comment on your observation.

3. Next, we solve the problem using the `incompressible_elasticity.py`. But before you could run the code, you need to perform the following:

   (a) Derive the weak form for the problem.

   (b) Define a function for $\boldsymbol{\sigma}$, basically complete the function sigma(u, p) on line # 68

   (c) Use the Dirichlet and Neumann boundaries from the previous problem on lines# 76, 81

   (d) Fill the bilinear form on line # 97 and the forcing functional on line # 103

   Now run the code and:

   (a) Report the maximum y-displacement

   (b) Plot the deformed shape.

   (c) Comment on your observations.

   Now we modify the type of element used.

   (a) Modify the type of displacement element to use polynomial order 2 and the pressure to use polynomial order 1.

   (b) Plot the deformed shape.

   (c) Why did it change the solution? [Hint : Refer Sec 4.3 of Hughes book]