LECTURE 22:

TOPICS:

- INCOMPRESSIBLE ELASTICITY

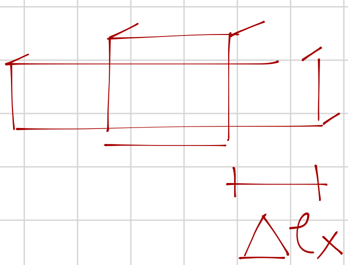
LOGISTICS:

- HW # 8 DUE MONDAY
- FINAL PROJECT DUE JANUARY 15th

NEARLY INCOMPRESSIBLE

RECALL THE POISSON RATIO ν RELATES STRAINS IN DIFFERENT DIRECTIONS

$$\frac{d\varepsilon_y}{d\varepsilon_x} = -\nu$$



$V = 1$

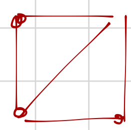
$$\hat{V} = (1+\Delta e_x)(1-\nu\Delta e_x)^2$$
$$= (1+\Delta e_x)(1+\nu^2\Delta e_x^2 - 2\nu\Delta e_x)$$
$$= 1 - 2\nu\Delta e_x + \Delta e_x + O(\Delta e_x^2)$$
$$\approx 1 + (1-2\nu)\Delta e_x$$

if $\nu = 0 \Rightarrow V = \hat{V} \Rightarrow$ MATERIAL IS INCOMPRESSIBLE

$$\lambda = \frac{2\nu\mu}{1-2\nu} \qquad \text{if } \nu \to 0.5, \ \lambda \to \infty$$

NOW THINK OF A MESH

IF



THE ONLY WAY TO PRESERVE AREA

EFFECTIVELY TWO CONSTRAINTS ONE DOF

SHOW CODE

INCOMPRESSIBLE

IN THE CASE $\nu = 0.5$ THEN $\lambda = \infty$

WE CHANGE THE PROBLEM

$$\lambda \underline{\nabla} \cdot \underline{u} = -p \Longrightarrow \underline{\underline{\sigma}}(\underline{\nabla}\, \underline{u}, p) = -p \underline{\underline{1}} + 2\mu \underline{\underline{\varepsilon}}$$

$\underline{\nabla} \cdot \underline{u} = -p/\lambda$ BUT if $\lambda \to \infty$ $\underline{\nabla} \cdot \underline{u} = 0$

(S) FIND $p, \underline{u}$ SUCH THAT

$$\underline{\nabla} \cdot (\underline{\underline{\sigma}}(\underline{\nabla}\, \underline{u}, p)) = \underline{b} \qquad \forall\ \underline{x} \in \Omega$$

$$\underline{\nabla} \cdot \underline{u} = 0 \qquad \forall\ \underline{x} \in \Omega$$

AND

$$\underline{u} = \underline{g} \quad \text{ON} \quad \Gamma_D$$

$$\underline{\underline{\sigma}}\, \underline{n} = \underline{t} \quad \text{ON} \quad \Gamma_N$$

(W)

$S_{\underline{u}} = \{ \underline{u} \in \text{SMOOTH} \mid \underline{u} = \underline{g} \text{ ON } \Gamma_D \}$

$S_p = \{ p \in \text{SMOOTH} \}$

$V_{\underline{u}} = \{ \underline{v} \in \text{SMOOTH} \mid \underline{v} = 0 \text{ ON } \Gamma_D \}$

$V_p = \{ q \in \text{SMOOTH} \}$

$$\int_\Omega (\underline{\nabla} \cdot \underline{\underline{\sigma}} - \underline{b}) \cdot \underline{v} \, d\Omega + \int_\Omega (\underline{\nabla} \cdot \underline{u}) q \, d\Omega = 0 \quad \forall \underline{v} \in V_{\underline{u}}, \forall q \in V_p$$

$$\int_\Omega \underline{\underline{\sigma}} : \underline{\nabla}\, \underline{v} \, d\Omega - \int_{\Gamma_N} \underline{t} \cdot \underline{v} \, d\Gamma + \int_\Omega \underline{b} \cdot \underline{v} \, d\Omega + \int_\Omega \underline{\nabla} \cdot \underline{u} \, q \, d\Omega = 0$$

$$\forall \underline{v} \in V_{\underline{u}}, \forall q \in V_p$$

$$\underline{\underline{\nabla}} : \underline{\nabla}\underline{v} = (-p\underline{\underline{1}} + 2\mu\underline{\underline{\varepsilon}}) : \underline{\nabla}\underline{v} = -p\underline{\nabla}\cdot\underline{v} + 2\mu\underline{\underline{\varepsilon}} : \underline{\nabla}\underline{v}$$

$$\int_\Omega 2\mu\,\underline{\underline{\varepsilon}} : \underline{\nabla}\underline{v}\, d\Omega - \int_\Omega p\,\underline{\nabla}\cdot\underline{v}\, d\Omega = \int_{\Gamma_N} \underline{t}\cdot\underline{v}\, d\Gamma - \int_\Omega \underline{b}\cdot\underline{v}\, d\Omega \quad \forall\, \underline{v}$$

$$\int_\Omega \underline{\nabla}\cdot\underline{u}\, q\, d\Omega = 0 \qquad \forall\, p$$

$$?\ \underline{u} \in S_{\underline{u}},\ p \in S_p$$

$$a(\underline{v}, \underline{u}) - (\underline{\nabla}\cdot\underline{v}, p) = F(\underline{v}) \qquad \forall\ \underline{v} \in V_{\underline{u}}$$

$$(q, \underline{\nabla}\cdot\underline{u}) = 0 \qquad \forall\ p \in V_p$$

(G) P^k ← SPACE OF POLYNOMIALS OF ORDER UP TO k

$$S^h_{\underline{u}} = \{\underline{u}^h \in S_{\underline{u}} \mid \underline{u}^h|_{\Omega^e} \in P^{k_u}\}$$

k_u ← DEGREE OF POLYNOMIAL APPROXIMANT FOR DISPLACEMENT FIELD $\underline{u}$

$$S^h_p = \{p \in S_p \mid p^h|_{\Omega^e} \in P^{k_p}\}$$

k_p ← DEGREE OF POLYNOMIAL APPROXIMANT FOR PRESSURE

$$V^h_{\underline{u}} = \{\underline{v}^h \in V_{\underline{u}} \mid \underline{v}^h|_{\Omega^e} \in P^{k_u}\}$$

$$V^h_p = \{p^h \in V_p \mid p^h|_{\Omega^e} \in P^{k_p}\}$$

$$?\ \underline{u}^h \in S^h_{\underline{u}},\ p^h \in S^h_p$$

$$a(\underline{v}^h, \underline{u}^h) - (\underline{\nabla}\cdot\underline{v}^h, p^h) = F(\underline{v}^h) \qquad \forall\ \underline{v} \in V_{\underline{u}}$$

$$(q^h, \underline{\nabla}\cdot\underline{u}^h) = 0 \qquad \forall\ p \in V_p$$

(M) LET $\{u\}$, $\{p\}$ THE VECTORS OF UNKNOWN DISPL & PRESS

$$K_{ij} u_j - Q_{is} p_s = F_i \qquad \forall\ i = 1 \dots N_{DOF_u}$$

$$Q_{ti} u_i = 0 \qquad \forall\ t = 1 \dots N_{DOF_p}$$

$$K_{ij} = \int_\Omega 2\mu \, \underline{\nabla} \phi_i \otimes \underline{\nabla} \phi_j \, d\Omega \qquad Q_{is} = \int_\Omega \underline{\nabla} \phi_i \, \psi_s \, d\Omega$$

DISPL BASIS

PRESSURE BASIS FUNCTIONS

THE WAY YOU MAY THINK OF THIS IS SIMPLY THE ELASTICITY PROBLEM WITH N_{Dof_u} DEGREES OF FREEDOM & N_{Dof_p} CONSTRAINTS

IF WE HAVE TOO MANY CONSTRAINTS IN RESPECT TO DOFs THE PROBLEM IS OVER CONSTRAINED $\Longrightarrow$ LOCKING

NAMELY

$$\frac{N_{Dof_u}}{N_{DOF_p}} < d \longrightarrow \text{LOCKING}$$

PROBLEM DIM

IF YOU CHOOSE $K_p = K_u - 1 \longrightarrow$ SAFE