PRECEPT 10

CEE 361-513: Introduction to Finite Element Methods

Monday Dec. 4

In this precept we look at the convergence of poisson's equation on a circular domain to demonstrate the use of curved elements. First we create the manufactured solution to study the convergence.

```
# Create the mesh of a circle
w = 0.99 # The radius

# Manufacture a solution
x,y = sp.symbols('x y')
r = sp.sqrt( x**2 + y**2 )
ue = sp.exp(1. + 1./(r**2 - 1.) )*sp.cos( r*3 ) + r**2
sp.latex(ue)
_f = sp.lambdify((x,y),sp.diff(ue,x,2) + sp.diff(ue,y,2)  )
print(sp.latex(_f))
f = lambda x: _f(x[0],x[1] )
```

Next we make a coarse mesh. Since we are only using straight lines as element boundaries, we start with a octagon to approximate a circle.

```
coordinates = np.array([ (0.,0.), (w,0), (w/np.sqrt(2),w/np.sqrt(2)),\
      (0.,w), (-w/np.sqrt(2),w/np.sqrt(2)), (-w,0.),\
       (-w/np.sqrt(2),-w/np.sqrt(2)), (0.,-w), (w/np.sqrt(2),-w/np.sqrt(2))
])
```

We use linear interpolation function to solve the problem first, i.e.order of polynomial = 1

The solution with the initial mesh looks the following:

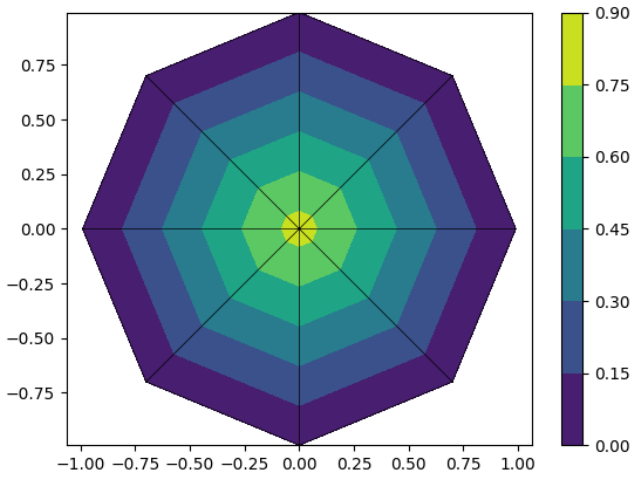

Figure 1: Solution on the initial mesh

The error in the solution and the solution surface are:

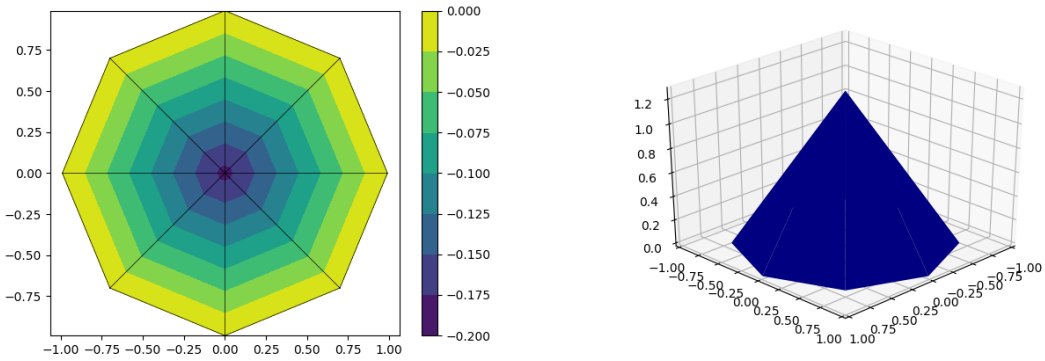

Figure 2: Error and solution surface on the initial mesh

Clearly it is not a converged solution. Refining it 4 times we obtain the following results

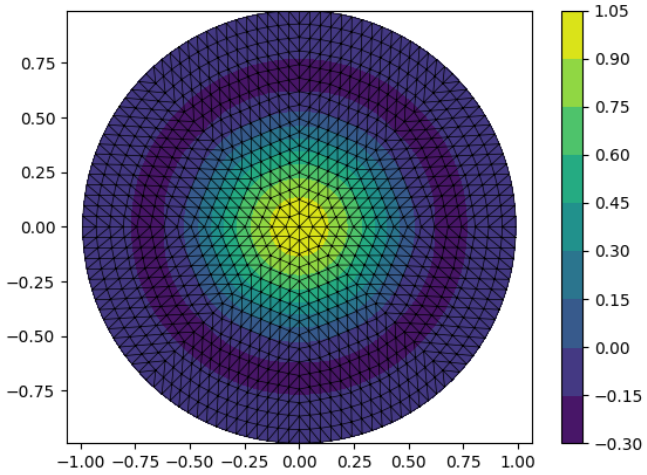

Figure 3: Converged Solution on the refined mesh

The error in the solution and the solution surface are:

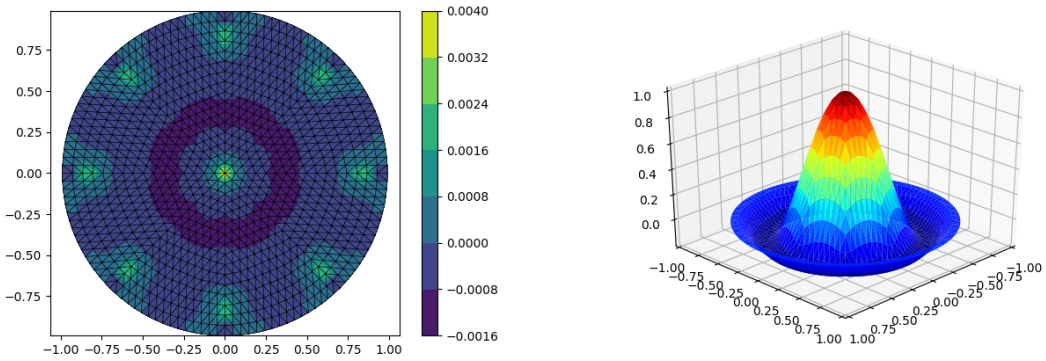

Figure 4: Error and solution surface on the refined mesh

Next we perform convergence study for the linear interpolating function using $L2 - norm$ of the error.

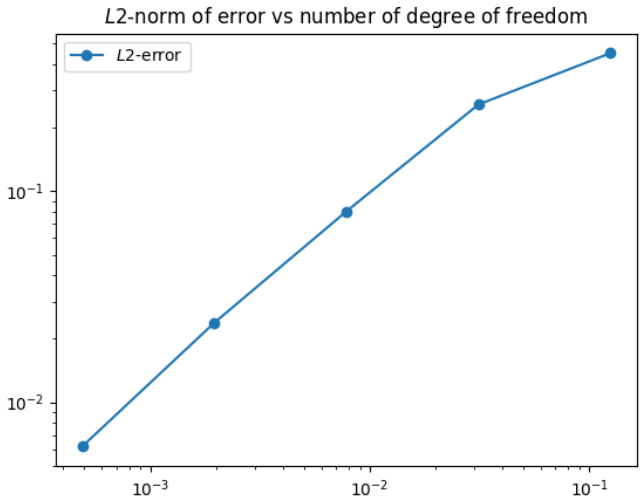

Figure 5: Convergence plot for $L2 - norm$ of error

The rate of convergence is $1.95 \approx 2$

Now we solve the same problem with polynomial order 2. The solution with the initial mesh looks the following:

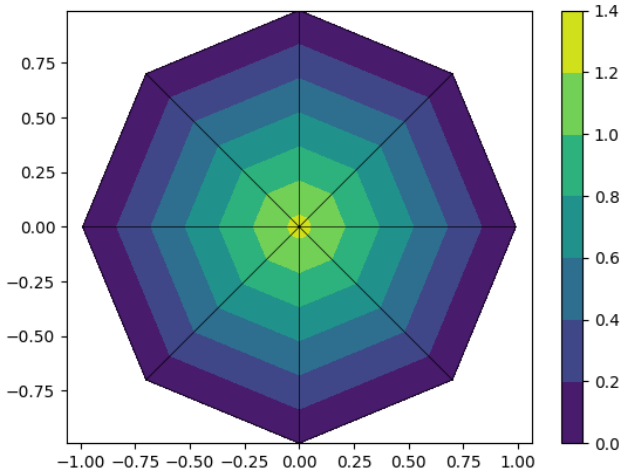

Figure 6: Solution on the initial mesh

The error in the solution and the solution surface are:

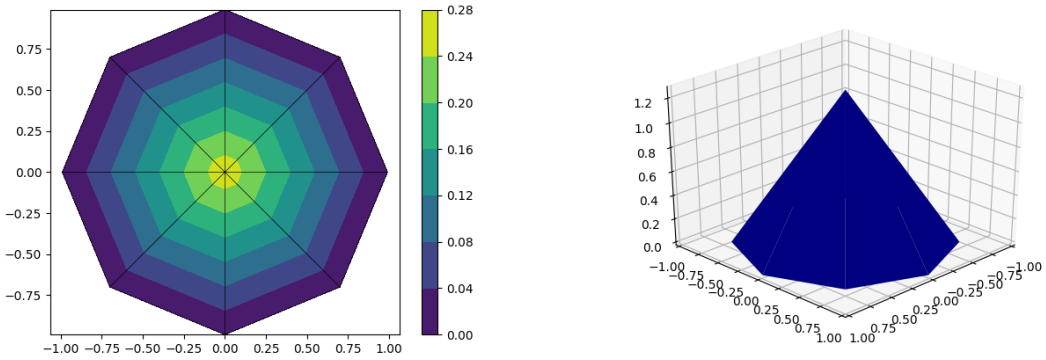

Figure 7: Error and solution surface on the initial mesh

Clearly it is not a converged solution. Refining it 4 times we obtain the following results

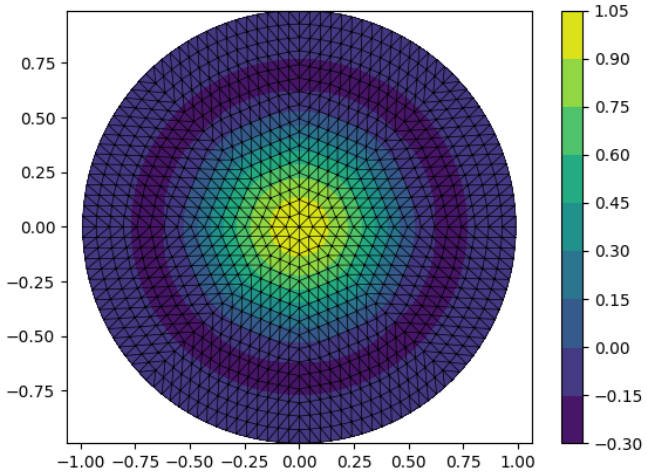

Figure 8: Converged Solution on the refined mesh

The error in the solution and the solution surface are:

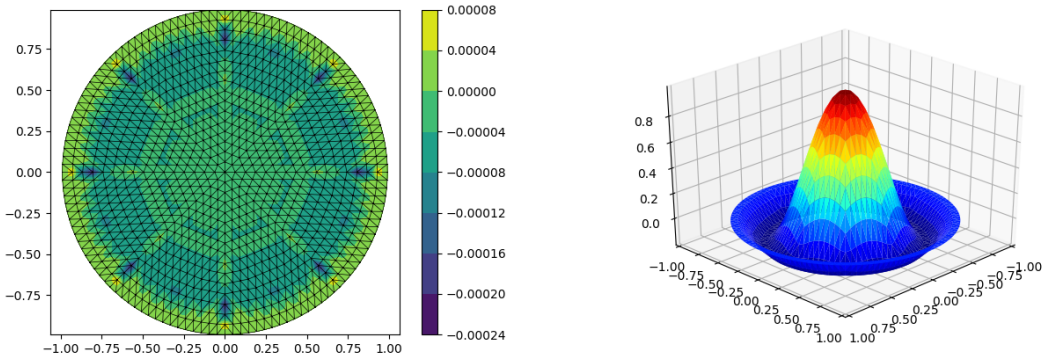

Figure 9: Error and solution surface on the refined mesh

Next we perform convergence study for the linear interpolating function using $L2 - norm$ of the error.

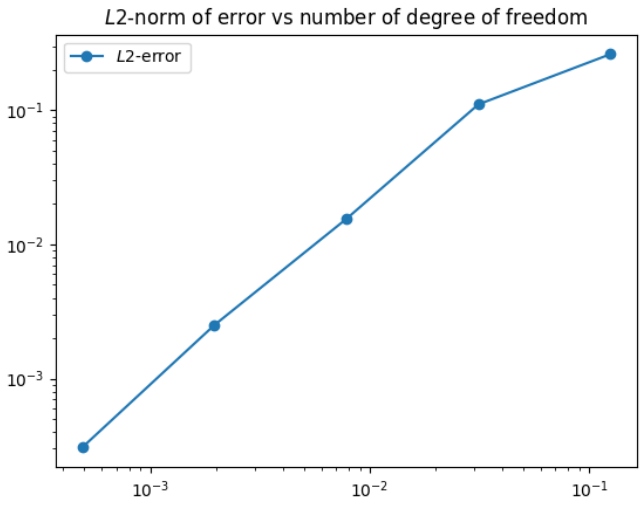

Figure 10: Convergence plot for $L2 - norm$ of error

The rate of convergence is 3.