LECTURE 8:

TOPICS:

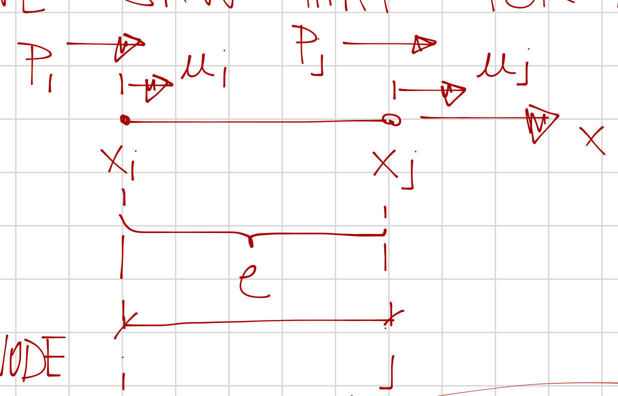
- REVIEW
- CONSTRAINTS
- 2D TRUSSES

LOGISTICS:

- HW 1 GRADED
- HW 2 DUE FRIDAY

REVIEW

WE SAW THAT FOR A GENERAL TRUSS ELEMENT ⓔ



STIFFNESS MATRIX

$$\begin{Bmatrix} P_i \\ P_j \end{Bmatrix} = \begin{bmatrix} k^e & -k^e \\ -k^e & k^e \end{bmatrix} \begin{Bmatrix} u_i \\ u_j \end{Bmatrix}$$

ELEMENT DEGREES OF FREEDOM

WHERE P_i AND P_j ARE THE LOADS APPLIED AT NODE i AND j THAT GIVE THE DISPLACEMENTS u_i, u_j. k^e IS THE ELEMENT STIFFNESS $k^e = \frac{AE}{e}$.

HOW DID WE ARRIVE AT THIS SYSTEM OF EQ?

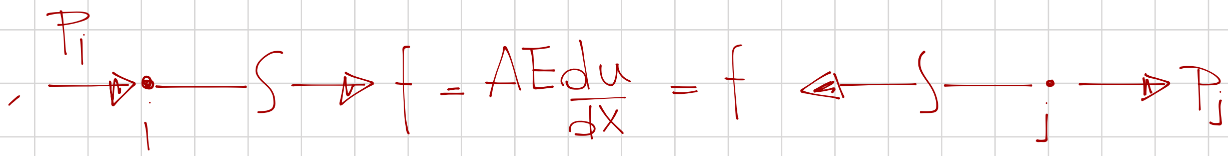
1) WE STARTE WITH THE PROBLEM

$$? \, u : [x_i, x_j] \to \mathbb{R} \;|\; AE \frac{d^2 u}{dx^2} = 0 \quad \forall x, \; u(x_i) = u_i, \; u(x_j) = u_j$$

2) WE SOLVE FOR u TO FIND THAT

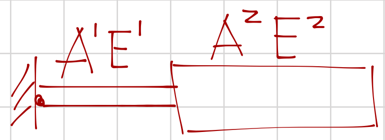
$$u(x) = \frac{(u_j - u_i)}{(x_j - x_i)} (x - x_i) + u_i$$

3) WE LOOKED AT NODE i & NODE j AND WITH THE GIVEN DISPLACEMENTS, WE WROTE THE SUM OF INTERNAL & EXTERNAL FORCES.

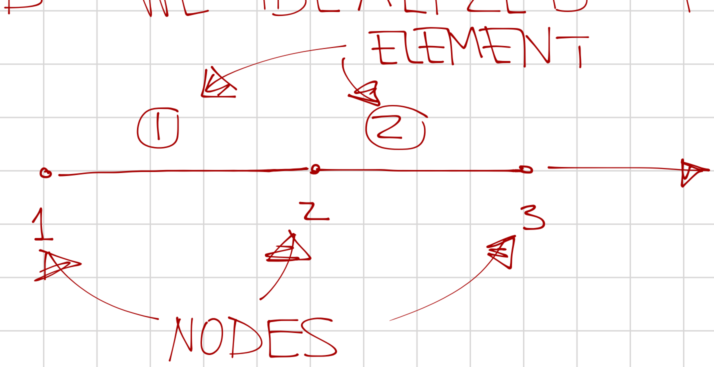


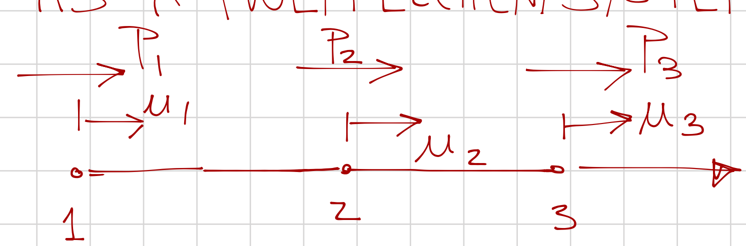
$$\begin{Bmatrix} P_i \\ P_j \end{Bmatrix} = \begin{Bmatrix} -f \\ f \end{Bmatrix} = \begin{bmatrix} k & -k \\ -k & k \end{bmatrix} \begin{Bmatrix} u_i \\ u_j \end{Bmatrix}$$

AFTERWARDS WE LOOKED AT MORE COMPLICATED SYSTEMS



AND WE IDEALIZED IT AS A MULTI-ELEMENT SYSTEM





WE CONSTRUCTED THE CONNECTIVITY ARRAY

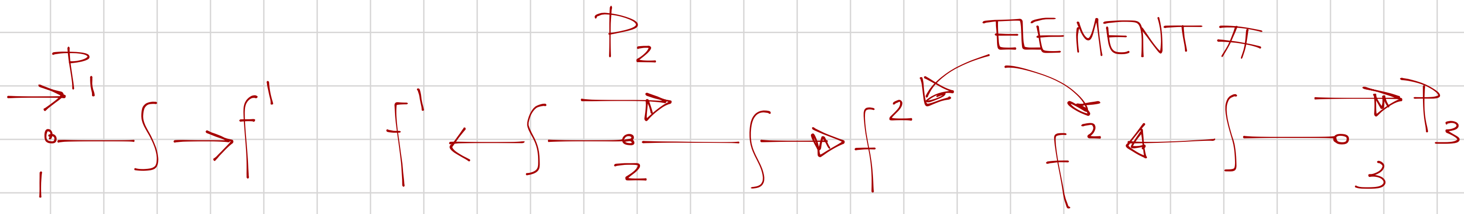
ELEMENT	NODES i	NODES j
1	1	2
2	2	3

WITH THE CONNECTIVITY ARRAY WE CONSTRUCTED A LOCAL DOF TO GLOBAL DOF MAP (L2G MAP)

THE L2G MAP TAKE AN ELEMENT NUMBER, A LOCAL NODE LABLE (i, j) AND RETURNS THE CORRESPONDING GLOBAL DOF.

EG: $L2G(1, i) = 1$, $L2G(1, j) = 2$, ...

LASTLY WE ASSEMBLED THE SYSTEM OF EQUATIONS THAT REPRESENT FORCE BALANCE AT EACH OF THE GLOBAL NODES



TO ASSEMBLE THE GLOBAL STIFFNESS MAT WE USE THE L2G MAP

NAMELY WE ADD THE GLOBAL STIFFNESS THE CONTRIBUTION FROM EACH ELEMENT.

$$\begin{array}{c|c} & \begin{array}{cc} i & j \end{array} \\ \hline \begin{array}{c} i \\ j \end{array} & \begin{bmatrix} k^e & -k^e \\ -k^e & k^e \end{bmatrix} \end{array} \quad M(e,i), M(e,j)$$

$$\begin{Bmatrix} P_1 \\ P_2 \\ P_3 \end{Bmatrix} = \begin{array}{c|c} & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \hline \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix} \end{array}$$

APPLYING DISPLACEMENTS OR BC

SUPPOSE WE HAVE THE FOLLOWING STIFFNESS MATRIX

$$\begin{Bmatrix} P_1 \\ P_2 \\ P_3 \end{Bmatrix} = \begin{bmatrix} K_1 & -K_1 & 0 \\ -K_1 & K_1+K_2 & -K_2 \\ 0 & -K_2 & K_2 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix}$$

SUPPOSE WE HAVE DIRICHLET CONDITIONS SUCH THAT $u_1 = \hat{u}_1$

IN THIS CASE u_1 IS NO LONGER UKNOWN.

SO WHAT WE DO IS REPLACE THE FIRST EQ ($P_1 = K_1 u_1 - K_1 u_2$) WITH THE NEW EQUATION $u_1 = \hat{u}_1$

SO WE

1) ZERO OUT THE ROW OF K THAT CORRESPONDS TO THE DEGREE OF FREEDOM

$$\begin{Bmatrix} P_1 \\ P_2 \\ P_3 \end{Bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ -K_1 & K_1+K_2 & -K_2 \\ 0 & -K_2 & K_2 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix}$$

2) WE PLACE 1 ON THE DIAGONAL

$$\begin{Bmatrix} P_1 \\ P_2 \\ P_3 \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -K_1 & K_1+K_2 & -K_2 \\ 0 & -K_2 & K_2 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix}$$

3) WE REPLACE THE ROW IN LOAD VECTOR WITH THE VALUE OF THE PRESCRIBED DISPL

$$\begin{Bmatrix} \hat{u}_1 \\ P_2 \\ P_3 \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -K_1 & K_1+K_2 & -K_2 \\ 0 & -K_2 & K_2 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix}$$

$$\begin{array}{ccccccc} P_2 & = & -K_1 & K_1+K_2 & -K_2 & & u_2 \\ P_3 & & 0 & -K_2 & K_2 & & u_3 \end{array}$$

EFFECTIVELY WE ARE REPLACING THE FIRST EQUATION

$$P_1 = K_1 u_1 - K_2 u_2$$

WITH

$$\hat{u}_1 = u_1$$