LECTURE 13:

- REVIEW
- BALANCE OF MECHANICAL ENERGY

BALANCE OF MECHANICAL ENERGY

NOTE THAT BALANCE OF MECHANICAL ENERGY, WHEN OTHER SOURCES OF ENERGY (THERMAL, CHEMICAL ...) ARE NEGLECTED, IS NOT AN ADITIONAL CONDITION BUT A CONSEQUENCE OF BALANCE OF LINEAR MOMENTUM.

CONSIDER THE EXTERNAL POWER

$$P^{EXT} = \int_\Theta \underline{b} \cdot \underline{v} \, dv + \int_{\partial\Theta} \underline{t} \cdot \underline{v} \, ds =$$

$$= \int_\Theta \underline{b} \cdot \underline{v} + \int_\Theta \underline{v} \cdot \underline{\underline{T}} \, \underline{n} \, ds$$

$$= \int_\Theta \underline{b} \cdot \underline{v} + \int_\Theta \nabla \cdot (\underline{\underline{T}} \, \underline{v}) \, dv = \int_\Theta \underline{b} \cdot \underline{v} + \nabla \cdot \underline{\underline{T}} \cdot \underline{v} + \int_\Theta \underline{\underline{T}} : \nabla \underline{v}$$

$$= \int_\Theta \underline{v} \cdot (\underline{b} + \nabla \cdot \underline{\underline{T}}) + \int_\Theta \underline{\underline{T}} : \nabla \underline{v}$$

$$= \int_\Theta \underline{v} \cdot (\rho \dot{\underline{v}}) + \int_\Theta \underline{\underline{T}} : \nabla \underline{v} = \frac{D}{Dt} \int_\Theta \frac{1}{2} \rho \underline{v} \cdot \underline{v} + \int_\Theta \underline{\underline{T}} : \nabla \underline{v}$$

$$\Rightarrow \underbrace{\int_\Theta \underline{b} \cdot \underline{v} \, dv + \int_{\partial\Theta} \underline{t} \cdot \underline{v} \, ds}_{\substack{P^{EXT} \\ \uparrow \\ \text{EXTERNAL POWER}}} = \underbrace{\frac{D}{Dt} \int_\Theta \frac{1}{2} \rho \underline{v} \cdot \underline{v} \, dv}_{\substack{\frac{D}{Dt} K \\ \uparrow \\ \text{KINETIC ENERGY}}} + \underbrace{\int_\Theta \underline{\underline{T}} : \nabla \underline{v} \, dv}_{\substack{P^{INT} \\ \uparrow \\ \text{STRESS WORK}}}$$

$$\frac{D}{Dt} K + P^{INT} = P^{EXT}$$

NOTE THAT KINETIC ENERGY IS NOT CONSERVED

IF $P^{EXT} = 0$ WE HAVE FREE VIBRATIONS

IF $\frac{D}{Dt} K = 0 \Rightarrow \rho \underline{\dot{v}} = 0$ WE HAVE A QUASI-STATIC PROB.

NOTE THAT IN GENERAL WE CANNOT WRITE

$$\int_\Theta \underline{\underline{T}} : \nabla \underline{v} \quad \text{AS} \quad \frac{D}{Dt} \int_\Theta e(\underline{x}, t) \, dv$$

(AS WE WILL SEE WE COULD HAVE DISSIPATION)

IF THE MATERIAL IS ELASTIC

$$\int_\Theta \underline{\underline{T}} : \nabla \underline{v} = \frac{D}{Dt} \int_\Theta e \, dv$$

$$\Rightarrow \frac{D}{Dt} \left[\underbrace{\int_\Theta \frac{1}{2} \rho \underline{v} \cdot \underline{v} \, dv + \int_\Theta e \, dv}_{\text{TOTAL INTERNAL ENERGY}} \right] = P^{EXT}$$

ALSO NOTE THAT THE STRESS WORK IS GIVEN

$$\underline{\underline{T}} : \nabla \underline{v} = \underline{\underline{T}} : \nabla^S \underline{v} = \underline{\underline{T}} : \underline{\underline{L}}$$

$\underline{\underline{T}}$ IS SAID TO BE POWER CONJUGATE TO $\underline{\underline{L}}$

SIMILARLY

$$P^{EXT} = \int_{\Theta} \underline{b} \cdot \underline{v} \, dv + \int_{\partial\Theta} \underline{t} \cdot \underline{v} \, ds =$$

$$= \int_{\Theta_0} \underline{B} \cdot \underline{V} \, dV + \int_{\partial\Theta} \underline{V} \cdot \underline{\underline{P}} \, \underline{N} \, dS_0 = \ldots + \int_{\Theta} \underline{V} \cdot \underline{\nabla}_X \underline{\underline{P}} + \int_{\Theta_0} \underline{\underline{P}} : \underline{\nabla}_X \underline{V}$$

$$= \int_{\Theta_0} \underline{V} \, \rho_0 \dot{\underline{V}} + \int_{\Theta_0} \underline{\underline{P}} : \dot{\underline{\underline{F}}}$$

$$\Rightarrow \int_{\Theta_0} \underline{B} \cdot \underline{V} + \int_{\partial\Theta_0} \underline{V} \cdot \underline{T} \, dS_0 = \frac{D}{Dt} \int \frac{1}{2} \rho_0 \underline{V} \cdot \underline{V} + \int_{\Theta_0} \underline{\underline{P}} : \dot{\underline{\underline{F}}}$$

$\underline{\underline{P}}$ IS POWER CONJUGATE TO $\dot{\underline{\underline{F}}}$

$$\underline{\underline{P}} : \dot{\underline{\underline{F}}} = (\underline{\underline{F}}\,\underline{\underline{F}}^{-1})\underline{\underline{P}} : \dot{\underline{\underline{F}}} = \underline{\underline{F}}^{-1}\underline{\underline{P}} : (\underline{\underline{F}})^T \dot{\underline{\underline{F}}} = \underline{\underline{S}} : \underline{\underline{F}}^T \dot{\underline{\underline{F}}} = \underline{\underline{S}} : \dot{\underline{\underline{E}}}$$

$$\underline{\underline{E}} = \frac{1}{2}(\underline{\underline{C}} - \underline{\underline{1}}) = \frac{1}{2}(\underline{\underline{F}}^T\underline{\underline{F}} - \underline{\underline{1}}) \Rightarrow \dot{\underline{\underline{E}}} = \frac{1}{2}\dot{\underline{\underline{C}}} = \frac{1}{2}(\dot{\underline{\underline{F}}}^T\underline{\underline{F}} + \underline{\underline{F}}^T\dot{\underline{\underline{F}}})$$

$$SYM(\underline{\underline{F}}^T\dot{\underline{\underline{F}}}) = \frac{1}{2}(\underline{\underline{F}}^T\dot{\underline{\underline{F}}} + \dot{\underline{\underline{F}}}^T\underline{\underline{F}})$$

$\Rightarrow \underline{\underline{S}}$ IS POWER CONJUGATE TO $\dot{\underline{\underline{E}}}$

$$\underline{\underline{\tau}} : \nabla^S \underline{v} = J \underline{\underline{\sigma}} : \frac{\nabla^S \underline{v}}{J} = \underline{\underline{\tau}} : J^{-1} \nabla \underline{v}$$

$$\underline{\nabla} \underline{v} = \frac{d\underline{v}}{d\underline{x}} = \frac{d\underline{v}}{d\underline{X}} \frac{d\underline{X}}{d\underline{x}} = \underline{\nabla}_X \underline{V} \, \underline{\underline{F}}^{-1} = \dot{\underline{\underline{F}}}\,\underline{\underline{F}}^{-1}$$

CONTINUUM THERMODYNAMICS

GENERALIZATION OF THERMO TO CONTINUUM

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EACH MATERIAL POINT IS AN OPEN SYSTEM IN THERMODYNAMIC EQUILIBRIUM EXCHANGING HEAT ETC WITH NEIGHBORS

EACH MATERIAL POINT REPRESENTS A COLLECTION OF ATOMS

MACROSCOPIC QUANTITIES STATISTICALLY MEASURABLE

MICROSCOPIC QUANTITIES MOTIONS OF EACH IND ATOM

STATISTICAL MECHANICS FROM MICRO TO MACRO

THERMODYNAMICS RELATES MACROSCOPIC QUANTITIES