FINITE ELEMENTS IN N-D

COMPUTING ELEMENT ARRAYS

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Recall that

$$\mathbf{K} = \mathbb{A}_{e=1}^{n_{el}} \, \mathbf{k}^{e}, \quad \mathbf{F} = \mathbb{A}_{e=1}^{n_{el}} \, \mathbf{f}^{e}$$

where \mathbb{A} is the assembly operator, and

$$\begin{split} \boldsymbol{k}_{ab}^{e} &= \boldsymbol{\alpha}^{e}(\phi_{a}^{e}, \phi_{b}^{e}) = \int_{\Omega^{e}} \nabla_{\boldsymbol{x}} \phi_{a}^{e}(\boldsymbol{x}) \cdot \nabla_{\boldsymbol{x}} \phi_{b}^{e} d\Omega = \\ &= \int_{\hat{\Omega}} [(\nabla_{\xi} \hat{\boldsymbol{x}}^{e}(\xi))^{-\top} \nabla_{\xi} \hat{\phi}_{a}(\xi)] \cdot [(\nabla_{\xi} \hat{\boldsymbol{x}}^{e}(\xi))^{-\top} \nabla_{\xi} \hat{\phi}_{b}(\xi)] \hat{\boldsymbol{j}}(\xi) d\hat{\Omega} \\ &\approx \sum_{(\tilde{\xi}_{O}, \omega_{O}) \in \mathcal{Q}} [(\nabla_{\xi} \hat{\boldsymbol{x}}^{e}(\tilde{\xi}_{O}))^{-\top} \nabla_{\xi} \hat{\phi}_{a}(\tilde{\xi}_{O})] \cdot [(\nabla_{\xi} \hat{\boldsymbol{x}}^{e}(\tilde{\xi}_{O}))^{-\top} \nabla_{\xi} \hat{\phi}_{b}(\tilde{\xi}_{O})] \hat{\boldsymbol{f}}^{e}(\tilde{\xi}_{O}) \omega_{O}(\xi) \\ &= (\tilde{\xi}_{O}, \omega_{O}) \in \mathcal{Q} \end{split}$$

where $\hat{j}^e(oldsymbol{\xi}) = \det(
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where $\hat{j}(\boldsymbol{\xi}) = \det(\nabla_{\boldsymbol{\xi}}\hat{\boldsymbol{x}}^{e}(\boldsymbol{\xi}))$

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