LECTURE 15

REVIEW
SECOND LAW OF THERMODYNAMICS

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WE SAW LAST TIME

$$\frac{d}{dt}\left[K + \mathcal{E}\right] = P^{EXT} + \dot{Q}$$

HEAT POWER

KINETIC ENERGY

INTERNAL ENERGY

EXTERNAL POWER

$$\Rightarrow \quad \rho \dot{u} = \underline{\underline{\sigma}} : \underline{\underline{d}} + r - \underline{\nabla} \cdot \underline{q} \qquad \forall \underline{x} \in \Omega$$

$$\rho \dot{u} = \underline{\underline{P}} : \underline{\underline{\dot{F}}} + R - \underline{\nabla} \cdot \underline{Q} \qquad \forall \underline{X} \in \Omega$$

THE FIRST LAW OF THERMODYNAMICS IS A STATEMENT OF INTERCONVERTIBILITY OF HEAT & WORK MAINTAINING AN ENERGY BALANCE.

NO RESTRICTION IS PLACED ON THE DIRECTION. NAMELY HEAT IS NOT RESTRICTED FROM BEING CONVERTED INTO WORK

THE ABOVE RESTRICTION IS IMPOSED BY THE SECOND LAW OF THERMODYNAMICS THAT IN THE CONTEX OF CONTINUUM MECHANICS IS DESCRIBED BY THE CLAUSIUS-DUHEM INEQ.

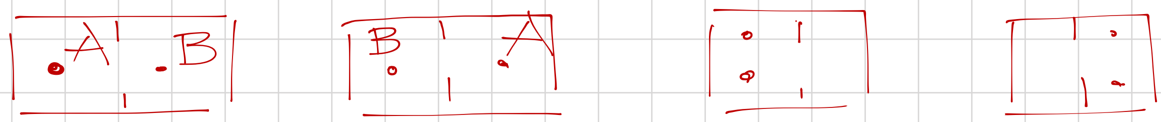
ENTROPY

NOTE THAT FOR A GIVEN TEMPERATURE THERE ARE SEVERAL CONFIGURATIONS & VELOCITIES OF OUR ENSAMBLE OF ATOMS THAT OUR COMPATIBLE WITH THE MACROSTATE

ENTROPY IS INTIMATELY RELATED TO THE NUMBER OF MICROSTATES CORRESPONDING TO THE MACROSTATE

OFTEN YOU'LL HEAR THAT ENTROPY IS A MEASURE OF DISORDER & THE SECOND LAW STATES THAT A PHYSICAL SYSTEM WILL FAVOR A STATE OF GREATER DISORDER

ONE WAY TO THINK ABOUT IT IS TO CONSIDER A BOX



(1, 1) (1, 1) (2, 0) (0, 2) ← MACROSTATE

MACROSTATE (1, 1) HAS HIGHER LIKELIHOOD HENCE IT'S THE FAVORED ONE

SECOND LAW OF THERMODYNAMICS IN CONTINUUM

LET US INTRODUCE THE SPECIFIC ENTROPY η

EFFECTIVELY η REPRESENTS THE ENTROPY OF THE ENSAMBLE OF ATOMS AT A POINT IN THE CONTINUM

THE TOTAL ENTROPY IN OUR CONTINUUM IS GIVEN BY

$$S(E \subseteq \Omega) = \int_E \rho \eta \, dv$$

LET $J(E)$ REPRESENT THE ENTROPY INPUT IN OUR SYSTEM

$$J(E \subseteq \Omega) = \int_E \tilde{r} \, dv + \int_{\partial E} h_{\underline{n}}(\underline{n}) \, ds$$

WHERE $h_{\underline{n}}(\underline{n})$ IS THE IN-FLUX OF ENTROPY & CAN SHOW

$$h_{\underline{n}}(\underline{n}) = -\underline{h} \cdot \underline{n}$$

SUCH THAT

$$J(E \subseteq \Omega) = \int_E \tilde{r} \, dv - \int_{\partial E} \underline{h} \cdot \underline{n} \, ds$$

AND THE NET PRODUCTION OF ENTROPY BECOMES

$$H = \frac{d}{dt} S - J$$

SINCE WE KNOW THAT THE MACROSTATE OF THE SYSTEM WILL TEND TO BE THE MOST LIKELY ONE HENCE ENTROPY WILL INCREASE

$H \geq 0 \Leftarrow$ SECOND LAW OF THERMODYNAMICS

A FUNDAMENTAL HYPOTHESIS THAT RELATES ENTROPY FLOW THROUGH HEAT FLOW STATES THAT $\exists\ \theta \in \mathbb{R}, \theta > 0$ (<u>THE ABSOLUTE TEMPERATURE</u>) SUCH THAT

$$\tilde{r} = \frac{r}{\theta} \quad , \quad \underline{h} = \frac{\underline{q}}{\theta}$$

THEN THE CLAUSIUS-DUHEM INEQUALITY

$$\mathcal{H} = \frac{d}{dt}\int_E \rho\eta + \int_{\partial E} \frac{\underline{q}}{\theta}\, ds - \int_E \frac{r}{\theta}\, dv \geq 0$$

$$\Rightarrow \quad \rho\dot{\eta} + \underline{\nabla}\cdot\left(\frac{\underline{q}}{\theta}\right) - \frac{r}{\theta} \geq 0$$

NOTE THAT

$$\underline{\nabla}\cdot(\underline{q}/\theta) = \frac{1}{\theta}\underline{\nabla}\cdot\underline{q} - \frac{1}{\theta^2}\underline{q}\cdot\underline{\nabla}\theta$$

SPECIFIC ENTROPY PRODUCTION

$$\Rightarrow \rho\gamma = \rho\dot{\eta} + \frac{1}{\theta}\underline{\nabla}\cdot\underline{q} - \frac{r}{\theta} - \frac{1}{\theta^2}\underline{q}\cdot\underline{\nabla}\theta \geq 0$$

TRUESDELL & NOLL POSTULATED A STRONGER REQUI.

LET THE DISSIPATION $\mathcal{D}$ BE DECOMPOSED IN
(OR ENTROPY PRODUCT)
A LOCAL TERM

$$\mathcal{D}_{LOC} = \rho\dot{\eta} + \frac{1}{\theta}\underline{\nabla}\cdot\underline{q} - \frac{r}{\theta}$$

AND A CONDUCTION TERM

$$\mathcal{D}_{CON} = -\frac{1}{\theta^2}\underline{q}\cdot\underline{\nabla}\theta \geq 0$$

S.T.

$$\mathcal{D} = \mathcal{D}_{LOC} + \mathcal{D}_{CON}$$

THEY POSTULATED THAT BOTH

$$D_{LOC} \geq 0 \,, \quad D_{CON} \geq 0$$

NOTE, THE ABOVE IMPOSES CONSTRAINTS ON CONST LAWS AS WE WILL SEE. EG

$$D_{CON} = -\frac{1}{\theta^2}\underline{q}\cdot\underline{\nabla}\theta \geq 0$$

IF $\underline{q} = k\underline{\nabla}\theta \Rightarrow -\frac{1}{\theta^2}\|\underline{\nabla}\theta\|^2 k \geq 0 \Rightarrow k \leq 0$

$\Rightarrow \underline{q} = -|k|\underline{\nabla}\theta$ ← HEAT FLOWS FROM WARMER TO COLDER REGION

NOW RECALL

$$\rho\dot{u} = \underline{\underline{\nabla}} : \underline{\underline{d}} + r - \underline{\nabla}\cdot\underline{q} \Rightarrow \underline{\nabla}\cdot\underline{q} - r = \underline{\underline{\nabla}} : \underline{\underline{d}} - \rho\dot{u}$$

THEN

$$D_{LOC} = \rho\dot{\eta} + \frac{1}{\theta}\left(\underline{\nabla}\cdot\underline{q} - r\right) = \rho\dot{\eta} + \frac{1}{\theta}\left(\underline{\underline{\nabla}} : \underline{\underline{d}} - \rho\dot{u}\right)$$

$$\Rightarrow \rho\dot{\eta} + \frac{1}{\theta}\left(\underline{\underline{\nabla}} : \underline{\underline{d}} - \rho\dot{u}\right) - \frac{1}{\theta^2}\underline{q}\cdot\underline{\nabla}\theta \geq 0$$

SINCE $\theta > 0$

$$-\rho\underbrace{(\dot{u} - \theta\dot{\eta})}_{\dot\psi} + \underline{\underline{\nabla}} : \underline{\underline{d}} - \frac{1}{\theta}\underline{q}\cdot\underline{\nabla}\theta \geq 0$$

ψ ← HELMOLTZ FREE ENERGY

IF THE PROCESS IS ISOTHERMAL (θ = CONST)

$$\underline{\underline{\nabla}} : \underline{\underline{d}} - \rho\dot{\psi}$$

$$H = \frac{d}{dt}\int_E \rho \eta + \int_{\partial E} \frac{\underline{q}}{\theta} \cdot \underline{n}\, ds - \int_E \frac{r}{\theta}\, dv \geq 0$$

$$\Rightarrow \frac{d}{dt}\int_{E_0} \rho \eta J + \int_{\partial E_0} \frac{\underline{q}}{\theta} \cdot J \underline{\underline{F}}^{-T} \underline{N}\, dS_0 - \int_{E_0} \frac{R}{\theta}\, dv_0 \geq 0$$

$$\int_{E_0} \rho_0 \dot{\eta} + \underline{\nabla} \cdot \left(\frac{\underline{Q}}{\theta}\right) - \frac{R}{\theta} \geq 0$$