LECTURE 12:

TOPICS:
- REVIEW
- BEAMS IN 2D
- FRAME ELEMENTS

LOGISTICS:

- EXAM ON THURSDAY

REVIEW:

LAST TIME WE STARTED FROM

$$? \; w: \quad EI \frac{d^4 w}{dx^4} = 0 \qquad \forall x \in (0, l)$$

$$w(0) = w_i \;, \; w(\ell) = w_j$$

$$\frac{dw}{dx}(0) = \theta(0) = \theta_i \;, \; \frac{dw}{dx}(\ell) = \theta(\ell) = \theta_j$$

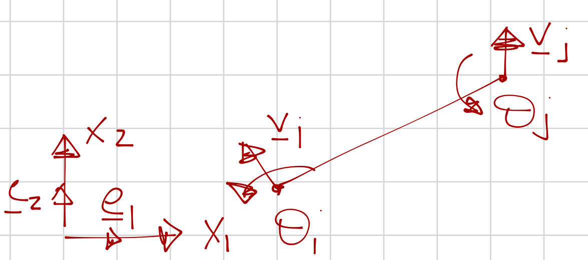
AND SOLVED IT ANALYTICALLY AS

$$w(x) = w_i N_1(x) + \theta_i N_2(x) + w_j N_3(x) + \theta_j N_4(x)$$

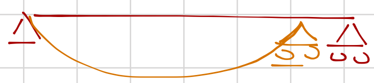
LOOKING FOR BALANCE OF FORCES AND MOMENTS @ NODES WE ARRIVED AT

$$\begin{Bmatrix} V_i \\ M_i \\ V_j \\ M_j \end{Bmatrix} = \begin{bmatrix} K_{fv} & K_{f\theta} & -K_{fv} & K_{f\theta} \\ K_{mv} & K_{m\theta} & -K_{mv} & \hat{K}_{m\theta} \\ -K_{fv} & K_{f\theta} & K_{fv} & K_{f\theta} \\ K_{mv} & \hat{K}_{m\theta} & K_{mv} & K_{m\theta} \end{bmatrix} \begin{Bmatrix} w_i \\ \theta_i \\ w_j \\ \theta_j \end{Bmatrix}$$

TODAY'S QUESTION IS WHAT IF



START BY LOOKING AT BEAM-COLUMS



THE PROBLEM BECOMES FIND THE HORIZONTAL DISPLACEMENT u & VERTICAL DISPLACEMENT w SUCH THAT

$\exists\ u, w:$

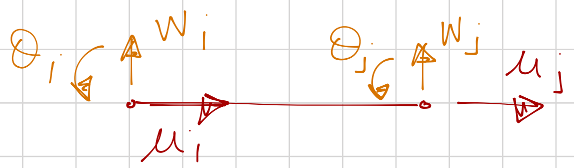
$$AE \frac{d^2 u}{dx^2} = 0 \qquad \forall x \in (0, \ell)$$

$$EJ \frac{d^4 w}{dx^4} = 0 \qquad \forall x \in (0, \ell)$$

$$u(0) = u_i, \quad u(\ell) = u_j \qquad w(0) = w_i, \quad w(\ell) = w_j$$

$$\frac{dw}{dx}(0) = \theta(0) = \theta_i, \quad \frac{dw}{dx}(\ell) = \theta(\ell) = \theta_j$$

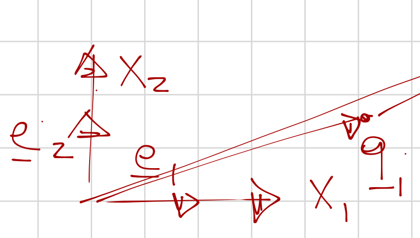
LOOKING AT BALANCE OF VERTICAL & HORIZONTAL FORCES AS WELL AS MOMENTS

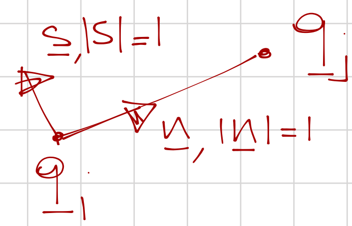


$$\begin{Bmatrix} P_i \\ V_i \\ M_i \\ P_j \\ V_j \\ M_j \end{Bmatrix} = \begin{bmatrix} K & & \\ 0 & K_{fv} & K_{f\theta} \\ 0 & & \\ K & & \\ 0 & & \\ 0 & & \end{bmatrix}$$

SIMILARLY TO TRUSSES, WE ARE ONLY INTERESTED IN THE COMPONENT OF $\underline{v}$ ALONG THE NORMAL OF OUR BEAM.

SUPPOSE THE COORDINATES OF OUR BEAM ARE GIVEN BY $\underline{q}_i$, $\underline{q}_j$





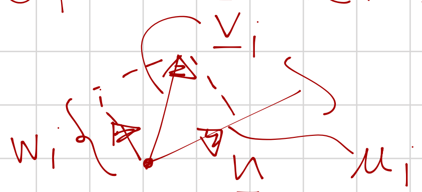
LET $\underline{n}$ BE THE UNIT VECTOR ALONG OUR BEAM AND $\underline{s}$ THE UNIT NORMAL VECTOR TO $\underline{n}$ ROTATED COUNTERCLOCKWISE.

HOW TO ROTATE?

$$\underline{\underline{R}} = -\underline{e}_1 \otimes \underline{e}_2 + \underline{e}_2 \otimes \underline{e}_1$$

$$\underline{s} = n_1 \underline{e}_2 - n_2 \underline{e}_1$$

$\underline{n}, \underline{s}$ BECOME BASIS VECTORS ATTACHED TO OUR ELEMENT SUCH THAT



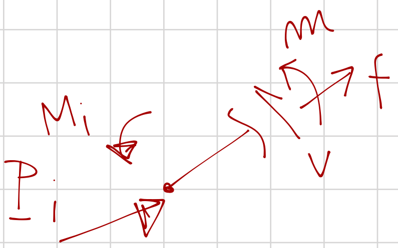
$$\underline{v}_i = u_i \underline{n} + w_i \underline{s}$$

NOW WE CAN RE-WRITE THE NORMAL COMPONENT OF THE DISPL AS

$$w(\xi) = \underline{v}(\xi) \cdot \underline{s}$$

NOW LET'S LOOK AT BALANCE OF FORCES

AT i NODE



THE INTERNAL FORCE $\underline{q}(\xi)$ AT NODE i IS

$$\underline{q}(0) = f(0)\,\underline{n} + V(0)\,\underline{s}$$

WHERE

$$f(0) = AE\frac{du}{d\xi}(0) = \frac{AE}{\ell}(u_j - u_i) = \frac{AE}{\ell}\,\underline{n}\cdot(\underline{v}_j - \underline{v}_i)$$

AND

$$V(0) = EI\frac{d^3w}{dx^3} = k_{fv}(w_i - w_j) + k_{f\theta}(\theta_i + \theta_j) =$$

$$= k_{fv}\,\underline{s}\cdot(\underline{v}_i - \underline{v}_j) + k_{f\theta}(\theta_i + \theta_j)$$

$$= \frac{12EI}{\ell^3}\,\underline{s}\cdot(\underline{v}_i - \underline{v}_j) + \frac{6EI}{\ell^2}(\theta_i + \theta_j)$$

THEREFORE

$$\underline{q}(0) = \left(-\frac{AE}{\ell}\,\underline{n}\otimes\underline{n} + \frac{12EI}{\ell^3}\,\underline{s}\otimes\underline{s}\right)(\underline{v}_i - \underline{v}_j) + \frac{6EI}{\ell^2}\,\underline{s}(\theta_i + \theta_j)$$

AND

$$\underline{P}_i + \underline{q}(0) = 0 \Rightarrow \underline{P}_i = \left(\frac{AE}{\ell}\,\underline{n}\otimes\underline{n} + \frac{12EI}{\ell^3}\,\underline{s}\otimes\underline{s}\right)(\underline{v}_i - \underline{v}_j)$$

$$+ \frac{6EI}{\ell^2}\,\underline{s}$$

$$\underline{F}_i = \{ \underline{\underline{K}}_{fv} \quad \underline{K}_{f\theta} \quad -\underline{\underline{K}}_{fv} \quad \underline{K}_{f\theta} \} \begin{Bmatrix} \underline{v}_i \\ \theta_i \\ \underline{v}_j \\ \theta_j \end{Bmatrix}$$

$$\underline{\underline{K}}_{fv} \quad \frac{AE}{\ell}\, \underline{n} \otimes \underline{n} + \frac{12EI}{\ell^3}\, \underline{s} \otimes \underline{s}$$

$$\underline{K}_{f\theta} = 6EI\,\underline{s}$$

SUM OF MOMENTS

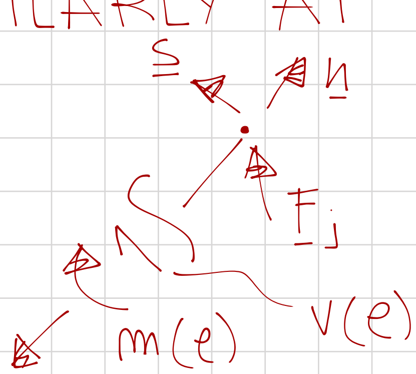
$$M_i + m(0) = 0 \Rightarrow M_i = -m(0) = -EI \frac{d^2 w}{dx^2}(0)$$

$$= k_{mv}(w_i - w_j) + k_{m\theta}\,\theta_i + \hat{k}_{m\theta}\,\theta_j =$$

$$= k_{mv}\,\underline{s} \cdot (\underline{v}_i - \underline{v}_j) + k_{m\theta}\,\theta_i + \hat{k}_{m\theta}\,\theta_j =$$

$$= \{ \underline{k}_{mv} \quad k_{m\theta} \quad -\underline{k}_{mv} \quad \hat{k}_{m\theta} \} \begin{Bmatrix} \underline{v}_i \\ \theta_i \\ \underline{v}_j \\ \theta_j \end{Bmatrix}$$

SIMILARLY AT NODE j



$$\underline{q}(\ell) = -f(\ell)\,\underline{n} + v(\ell)\,\underline{s}$$

$$\underline{F}_j + \underline{q}(\ell) = 0$$