HOMEWORK 4

CEE 530: Continuum Mechanics and Thermodynamics

Due: April 30, 2018

PROBLEM 1

Assume a free energy function having the general form $\psi = \hat{\psi}(a, b, c, d)$ and a local Dissipation function of the form $\rho\mathcal{D}_{loc} = A\dot{a} - \rho\dot{\psi} - \rho B\dot{b} \geq 0$.

1. (10) Apply Coleman's Exploitation Method to obtain the constitutive equations for the variables A and B for a non-dissipative process.
2. (10) Based on the previous result, which variables from a to d can be excluded from the constitutive equation $\hat{\psi}(a, b, c, d)$?

PROBLEM 2

Consider a one dimensional material whose Helmholtz free energy is given by

$$\rho\hat{\psi}(\varepsilon, \theta) = \frac{1}{2}E\varepsilon^2 - E\alpha_t\varepsilon(\theta - \theta_0) + \rho c\left[(\theta - \theta_0) - \theta \ln\frac{\theta}{\theta_0}\right]$$

with $\varepsilon = \partial_x\varphi$ and

$$q(\partial_x\theta) = -\kappa\partial_x\theta, \quad \kappa > 0.$$

1. (10) Obtain in detail the explicit expression of the entropy.
2. (10) Obtain in detail the expression for stress.
3. (10) What restrictions do we have on E, k, c, α_t?

PROBLEM 3

Consider the Helmholtz free energy for a Neo-Hookean material

$$\psi(\boldsymbol{F}, \theta) = \frac{\mu(\theta)}{2}\boldsymbol{F} : \boldsymbol{F} + \frac{\lambda(\theta)}{2}(\log J)^2 - \mu(\theta)\log J$$

where μ and λ are material parameters that depend on the temperature.

1. (10) Obtain in detail the explicit expression of the entropy.
2. (10) Obtain in detail the expression for stress (first Piola-Kirhoff).