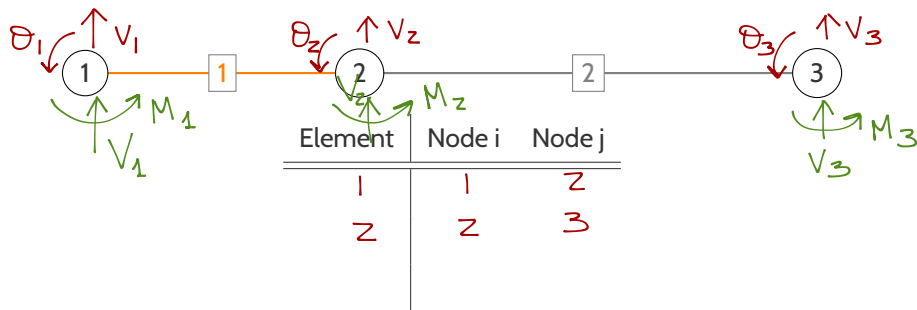


GLOBAL ASSEMBLY



Global Assembly of Beams



Global Assembly of Beams

Diagram illustrating the global assembly of beams for a structure with three nodes (1, 2, 3) and two beam elements.

The structure consists of three nodes: Node 1 (left), Node 2 (middle), and Node 3 (right). A beam element connects Node 1 and Node 2, and another beam element connects Node 2 and Node 3.

The stiffness matrix for the beam element connecting Node 1 and Node 2 is shown as a 4x4 matrix:

$$\begin{bmatrix} V_i \\ M_i \\ V_j \\ M_j \end{bmatrix} = \begin{bmatrix} k_{fv} & k_{f\theta} & -k_{fv} & k_{f\theta} \\ k_{mv} & k_{m\theta} & -k_{mv} & \hat{k}_{m\theta} \\ -k_{fv} & -k_{f\theta} & k_{fv} & -k_{f\theta} \\ k_{mv} & \hat{k}_{m\theta} & -k_{mv} & k_{m\theta} \end{bmatrix} \begin{bmatrix} v_i \\ \theta_i \\ v_j \\ \theta_j \end{bmatrix}$$

The global assembly of the beam is shown as a 6x6 matrix:

$$\begin{bmatrix} V_1 \\ M_1 \\ V_2 \\ M_2 \\ V_3 \\ M_3 \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} k_{fv} & k_{f\theta} & -k_{fv} & k_{f\theta} \\ k_{mv} & k_{m\theta} & -k_{mv} & \hat{k}_{m\theta} \\ -k_{fv} & -k_{f\theta} & k_{fv} & -k_{f\theta} \\ k_{mv} & \hat{k}_{m\theta} & -k_{mv} & k_{m\theta} \end{bmatrix} & \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \\ \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} & \begin{bmatrix} k_{fv} & k_{f\theta} & -k_{fv} & k_{f\theta} \\ k_{mv} & k_{m\theta} & -k_{mv} & \hat{k}_{m\theta} \\ -k_{fv} & -k_{f\theta} & k_{fv} & -k_{f\theta} \\ k_{mv} & \hat{k}_{m\theta} & -k_{mv} & k_{m\theta} \end{bmatrix} \end{bmatrix} \begin{bmatrix} v_1 \\ \theta_1 \\ v_2 \\ \theta_2 \\ v_3 \\ \theta_3 \end{bmatrix}$$

Red arrows indicate the assembly process, showing how the element stiffness matrix is added to the global matrix at the appropriate positions.

Global Assembly of Beams

Diagram illustrating the global assembly of beams for a structure with three nodes (1, 2, 3). The nodes are connected by beam elements. Node 1 is highlighted with an orange box and label, and Node 2 is highlighted with a gray box and label.

The global stiffness matrix equation is shown as:

$$\begin{Bmatrix} V_i \\ M_i \\ V_j \\ M_j \end{Bmatrix} = \begin{bmatrix} k_{fv} & k_{f\theta} & -k_{fv} & k_{f\theta} \\ k_{mv} & k_{m\theta} & -k_{mv} & \hat{k}_{m\theta} \\ -k_{fv} & -k_{f\theta} & k_{fv} & -k_{f\theta} \\ k_{mv} & \hat{k}_{m\theta} & -k_{mv} & k_{m\theta} \end{bmatrix} \begin{Bmatrix} v_i \\ \theta_i \\ v_j \\ \theta_j \end{Bmatrix}$$

The element stiffness matrix equation is shown as:

$$\begin{Bmatrix} V_1 \\ M_1 \\ V_2 \\ M_2 \\ V_3 \\ M_3 \end{Bmatrix} = \begin{bmatrix} k_{fv}^1 & k_{f\theta}^1 & -k_{fv}^1 & k_{f\theta}^1 & 0 & 0 \\ k_{mv}^1 & k_{m\theta}^1 & -k_{mv}^1 & \hat{k}_{m\theta}^1 & 0 & 0 \\ -k_{fv}^1 & -k_{f\theta}^1 & k_{fv}^1 & -k_{f\theta}^1 & 0 & 0 \\ k_{mv}^1 & \hat{k}_{m\theta}^1 & -k_{mv}^1 & k_{m\theta}^1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} v_1 \\ \theta_1 \\ v_2 \\ \theta_2 \\ v_3 \\ \theta_3 \end{Bmatrix}$$

Red arrows indicate the mapping of the element stiffness matrix coefficients to the global stiffness matrix coefficients:

- k_{fv}^1 maps to the k_{fv} term in the first row of the element matrix.
- $k_{f\theta}^1$ maps to the $k_{f\theta}$ term in the first row of the element matrix.
- $-k_{fv}^1$ maps to the $-k_{fv}$ term in the third row of the element matrix.
- $k_{f\theta}^1$ maps to the $k_{f\theta}$ term in the third row of the element matrix.
- k_{mv}^1 maps to the k_{mv} term in the second row of the element matrix.
- $k_{m\theta}^1$ maps to the $k_{m\theta}$ term in the second row of the element matrix.
- $-\hat{k}_{m\theta}^1$ maps to the $-\hat{k}_{m\theta}$ term in the second row of the element matrix.
- $\hat{k}_{m\theta}^1$ maps to the $\hat{k}_{m\theta}$ term in the fourth row of the element matrix.

Global Assembly of Beams

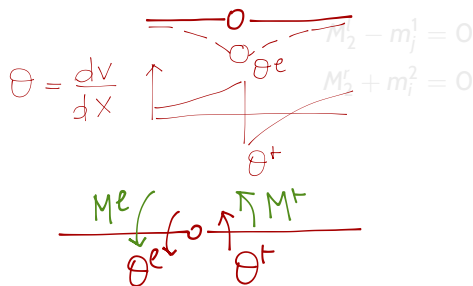
$$\begin{Bmatrix} V_i \\ M_i \\ V_j \\ M_j \end{Bmatrix} = \begin{bmatrix} k_{fv} & k_{f\theta} & -k_{fv} & k_{f\theta} \\ k_{mv} & k_{m\theta} & -k_{mv} & \hat{k}_{m\theta} \\ k_{fv} & -k_{f\theta} & k_{fv} & -k_{f\theta} \\ k_{mv} & \hat{k}_{m\theta} & -k_{mv} & k_{m\theta} \end{bmatrix} \begin{Bmatrix} v_i \\ \theta_i \\ v_j \\ \theta_j \end{Bmatrix}$$

$$\begin{Bmatrix} V_1 \\ M_1 \\ V_2 \\ M_2 \\ V_3 \\ M_3 \end{Bmatrix} = \begin{bmatrix} k_{fv}^1 & k_{f\theta}^1 & -k_{fv}^1 & k_{f\theta}^1 & 0 & 0 \\ k_{mv}^1 & k_{m\theta}^1 & -k_{mv}^1 & \hat{k}_{m\theta}^1 & 0 & 0 \\ -k_{fv}^1 & -k_{f\theta}^1 & k_{fv}^1 + k_{fv}^2 & -k_{f\theta}^1 + k_{f\theta}^2 & -k_{fv}^2 & k_{f\theta}^2 \\ k_{mv}^1 & \hat{k}_{m\theta}^1 & -k_{mv}^1 + k_{mv}^2 & k_{m\theta}^1 + k_{m\theta}^2 & -k_{mv}^2 & \hat{k}_{m\theta}^2 \\ 0 & 0 & -k_{fv}^2 & -k_{f\theta}^2 & k_{fv}^2 & -k_{f\theta}^2 \\ 0 & 0 & k_{mv}^2 & \hat{k}_{m\theta}^2 & -k_{mv}^2 & k_{m\theta}^2 \end{bmatrix} \begin{Bmatrix} v_1 \\ \theta_1 \\ v_2 \\ \theta_2 \\ v_3 \\ \theta_3 \end{Bmatrix}$$

Global Assembly of Beams

Enforce hinge constraint

1. Duplicate $\theta_2 \rightarrow \theta_2^l, \theta_2^r$! NO MOMENT TRANSFER!
2. Write equilibrium equations for each side & NO ROTATION !!



Global Assembly of Beams

Enforce hinge constraint

1. Duplicate $\theta_2 \rightarrow \theta_2^l, \theta_2^r$

2. Write equilibrium equations for each side of the hinge

$$M_2^l - m_j^1 = 0$$

$$M_2^r + m_j^2 = 0$$

$$\begin{Bmatrix} V_1 \\ M_1 \\ V_2 \\ ? \\ ? \\ V_3 \\ M_3 \end{Bmatrix} = \begin{bmatrix} k_{fv}^1 & k_{f\theta}^1 & -k_{fv}^1 & ? & ? & 0 & 0 \\ k_{mv}^1 & k_{m\theta}^1 & -k_{mv}^1 & ? & ? & 0 & 0 \\ -k_{fv}^1 & -k_{f\theta}^1 & k_{fv}^1 + k_{fv}^2 & ? & ? & -k_{fv}^2 & k_{f\theta}^2 \\ ? & ? & ? & ? & ? & ? & ? \\ ? & ? & ? & ? & ? & ? & ? \\ 0 & 0 & -k_{fv}^2 & ? & ? & k_{fv}^2 & -k_{f\theta}^2 \\ 0 & 0 & k_{mv}^2 & ? & ? & -k_{mv}^2 & k_{m\theta}^2 \end{bmatrix} \begin{Bmatrix} v_1 \\ \theta_1 \\ v_2 \\ \theta_2^l \\ \theta_2^r \\ v_3 \\ \theta_3 \end{Bmatrix}$$

Global Assembly of Beams

Enforce hinge constraint

1. Duplicate $\theta_2 \rightarrow \theta_2^l, \theta_2^r$
2. Write equilibrium equations for each side of the hinge

$$M_2^l - m_j^1 = 0$$

$$M_2^r + m_j^2 = 0$$

$$\begin{Bmatrix} V_1 \\ M_1 \\ V_2 \\ M_2^l \\ M_2^r \\ V_3 \\ M_3 \end{Bmatrix} = \begin{bmatrix} k_{fv}^1 & k_{f\theta}^1 & -k_{fv}^1 & k_{f\theta}^1 & 0 & 0 & 0 \\ k_{mv}^1 & k_{m\theta}^1 & -k_{mv}^1 & \hat{k}_{m\theta}^1 & 0 & 0 & 0 \\ -k_{fv}^1 & -k_{f\theta}^1 & k_{fv}^1 + k_{fv}^2 & -k_{f\theta}^1 & k_{f\theta}^2 & -k_{fv}^2 & k_{f\theta}^2 \\ k_{mv}^1 & \hat{k}_{m\theta}^1 & -k_{mv}^1 & k_{m\theta}^1 & 0 & 0 & 0 \\ 0 & 0 & k_{mv}^2 & 0 & k_{m\theta}^2 & -k_{mv}^2 & \hat{k}_{m\theta}^2 \\ 0 & 0 & -k_{fv}^2 & 0 & -k_{f\theta}^2 & k_{fv}^2 & -k_{f\theta}^2 \\ 0 & 0 & k_{mv}^2 & 0 & \hat{k}_{m\theta}^2 & -k_{mv}^2 & k_{m\theta}^2 \end{bmatrix} \begin{Bmatrix} v_1 \\ \theta_1 \\ v_2 \\ \theta_2^l \\ \theta_2^r \\ v_3 \\ \theta_3 \end{Bmatrix}$$

Global Assembly of Beams

Apply boundary conditions

$$v_1 = 0, \quad v_3 = 0, \quad \theta_3 = 0$$

$$\begin{Bmatrix} V_1 \\ M_1 \\ V_2 \\ M_2^l \\ M_2^r \\ V_3 \\ M_3 \end{Bmatrix} = \begin{bmatrix} k_{fv}^1 & k_{f\theta}^1 & -k_{fv}^1 & k_{f\theta}^1 & 0 & 0 & 0 \\ k_{mv}^1 & k_{m\theta}^1 & -k_{mv}^1 & \hat{k}_{m\theta}^1 & 0 & 0 & 0 \\ -k_{fv}^1 & -k_{f\theta}^1 & k_{fv}^1 + k_{fv}^2 & -k_{f\theta}^1 & k_{f\theta}^2 & -k_{fv}^2 & k_{f\theta}^2 \\ k_{mv}^1 & \hat{k}_{m\theta}^1 & -k_{mv}^1 & k_{m\theta}^1 & 0 & 0 & 0 \\ 0 & 0 & k_{mv}^2 & 0 & k_{m\theta}^2 & -k_{mv}^2 & \hat{k}_{m\theta}^2 \\ 0 & 0 & -k_{fv}^2 & 0 & -k_{f\theta}^2 & k_{fv}^2 & -k_{f\theta}^2 \\ 0 & 0 & k_{mv}^2 & 0 & \hat{k}_{m\theta}^2 & -k_{mv}^2 & k_{m\theta}^2 \end{bmatrix} \begin{Bmatrix} v_1 \\ \theta_1 \\ v_2 \\ \theta_2^l \\ \theta_2^r \\ v_3 \\ \theta_3 \end{Bmatrix}$$

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