Frame Elements

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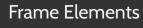
$$
\left\{ \begin{array}{c} V_i \\ M_i \\ V_j \\ M_j \end{array} \right\} = \left[\begin{array}{cccc} k_{fv} & k_{f\theta} & -k_{fv} & k_{f\theta} \\ k_{mv} & k_{m\theta} & -k_{mv} & \hat{k}_{m\theta} \\ -k_{fv} & -k_{f\theta} & k_{fv} & -k_{f\theta} \\ k_{mv} & \hat{k}_{m\theta} & -k_{mv} & k_{m\theta} \end{array} \right] \left\{ \begin{array}{c} v_i \\ \theta_i \\ v_j \\ \theta_j \end{array} \right\}
$$

$$
\left\{ \begin{array}{c} P_i \\ P_j \end{array} \right\} = \left[\begin{array}{cc} k_p & -k_p \\ -k_p & k_p \end{array} \right] \left\{ \begin{array}{c} u_i \\ u_j \end{array} \right\}
$$

where

$$
k_{fv} = \frac{12EI}{\ell^3}, \quad k_{mv} = k_{f\theta} = \frac{6EI}{\ell^2}, \quad k_{m\theta} = \frac{4EI}{\ell}, \quad \hat{k}_{m\theta} \frac{2EI}{\ell}, \quad k_p = \frac{AE}{\ell}.
$$

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$$
\left\{ \begin{array}{c} P_i \\ V_i \\ M_i \\ P_j \\ V_j \\ M_j \end{array} \right\} = \left[\begin{array}{cccccc} k_p & 0 & 0 & -k_p & 0 & 0 \\ 0 & k_{fv} & k_{f\theta} & 0 & -k_{fv} & k_{f\theta} \\ 0 & k_{mv} & k_{m\theta} & 0 & -k_{mv} & \hat{k}_{m\theta} \\ -k_p & 0 & 0 & k_p & 0 & 0 \\ 0 & -k_{fv} & -k_{f\theta} & 0 & k_{fv} & -k_{f\theta} \\ 0 & k_{mv} & \hat{k}_{m\theta} & 0 & -k_{mv} & k_{m\theta} \end{array} \right] \left\{ \begin{array}{c} u_i \\ v_i \\ \theta_i \\ u_j \\ v_j \\ \theta_j \end{array} \right\}
$$

Frame Elements

$$\left\{ \begin{array}{c} \boldsymbol{F}_i \\ M_i \\ \boldsymbol{F}_j \\ M_j \end{array} \right\} = \left[\begin{array}{cccc} \boldsymbol{K}_{fw} & \boldsymbol{k}_{f\theta} & -\boldsymbol{K}_{fw} & \boldsymbol{k}_{f\theta} \\ \boldsymbol{k}_{mw} & k_{m\theta} & -\boldsymbol{k}_{mw} & \hat{k}_{m\theta} \\ -\boldsymbol{K}_{fw} & -\boldsymbol{k}_{f\theta} & \boldsymbol{K}_{fw} & -\boldsymbol{k}_{f\theta} \\ \boldsymbol{k}_{mw} & \hat{k}_{m\theta} & -\boldsymbol{k}_{mw} & k_{m\theta} \end{array} \right] \left\{ \begin{array}{c} \boldsymbol{w}_i \\ \theta_i \\ \boldsymbol{w}_j \\ \theta_j \end{array} \right\}$$

Where

$\boldsymbol{F}_{i,j} \in \mathbb{R}^2$ is a vector

$\boldsymbol{w}_{i,j} \in \mathbb{R}^2$ is a vector

$M_{i,j} \in \mathbb{R}$ is a scalar

$\theta_{i,j} \in \mathbb{R}$ is a scalar

$\boldsymbol{K}_{fw} \in \mathbb{R}^2 \times \mathbb{R}^2$ is a tensor

$\boldsymbol{k}_{f\theta} \in \mathbb{R}^2$ is a vector

$\boldsymbol{k}_{Mw} \in \mathbb{R}^2$ is a vector

$k_{m\theta}, \hat{k}_{m\theta} \in \mathbb{R}$ is a scalar

$$\boldsymbol{K}_{fw} = \frac{AE}{\ell} \boldsymbol{n} \otimes \boldsymbol{n} + \frac{12EI}{\ell^3} \boldsymbol{s} \otimes \boldsymbol{s}, \quad k_{m\theta} = \frac{4EI}{\ell}, \quad \hat{k}_{m\theta} = \frac{2EI}{\ell}, \quad \boldsymbol{k}_{mw} = \boldsymbol{k}_{f\theta} = \frac{6EI}{\ell^2} \boldsymbol{s}$$

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$$\boldsymbol{F}_i = \boldsymbol{K_{fw}}\boldsymbol{w}_i + \boldsymbol{k_{f\theta}}\,\theta_i - \boldsymbol{K_{fw}}\boldsymbol{w}_i + \boldsymbol{k_{f\theta}}\theta_j \qquad \text{Vector equation!}$$

$$M_i = \boldsymbol{k_{mw}} \cdot \boldsymbol{w}_i + k_{m\theta}\,\theta_i - \boldsymbol{k_{mw}} \cdot \boldsymbol{w}_i + \hat{k}_{m\theta}\theta_j \qquad \text{Scalar equation!}$$

$$\boldsymbol{F}_j = -\boldsymbol{K_{fw}}\boldsymbol{w}_i - \boldsymbol{k_{f\theta}}\,\theta_i + \boldsymbol{K_{fw}}\boldsymbol{w}_i - \boldsymbol{k_{f\theta}}\theta_j \qquad \text{Vector equation!}$$

$$M_j = \boldsymbol{k_{mw}} \cdot \boldsymbol{w}_i + \hat{k}_{m\theta}\,\theta_i + -\boldsymbol{k_{mw}} \cdot \boldsymbol{w}_i + k_{m\theta}\theta_j \qquad \text{Scalar equation!}$$

$$
\left\{
\begin{array}{c}
[\boldsymbol{F}_i] \\
M_i \\
[\boldsymbol{F}_j] \\
M_j
\end{array}
\right\}
=
\left[
\begin{array}{cccc}
[\boldsymbol{K_{fw}}] & [\boldsymbol{k_{f\theta}}] & -[\boldsymbol{K_{fw}}] & [\boldsymbol{k_{f\theta}}] \\
[\boldsymbol{k_{mw}}]^{\top} & k_{m\theta} & -[\boldsymbol{k_{mw}}]^{\top} & \hat{k}_{m\theta} \\
-[\boldsymbol{K_{fw}}] & -[\boldsymbol{k_{f\theta}}] & [\boldsymbol{K_{fw}}] & -[\boldsymbol{k_{f\theta}}] \\
[\boldsymbol{k_{mw}}]^{\top} & \hat{k}_{m\theta} & -[\boldsymbol{k_{mw}}]^{\top} & k_{m\theta}
\end{array}
\right]
\left\{
\begin{array}{c}
[\boldsymbol{w}_i] \\
\theta_i \\
[\boldsymbol{w}_j] \\
\theta_j
\end{array}
\right\}
$$

Frame Elements

$$
\left\{ \begin{array}{c} [\boldsymbol{F}_i]_{2\times 1} \\ M_i \\ [\boldsymbol{F}_j]_{2\times 1} \\ M_j \end{array} \right\} = \left[\begin{array}{cccc} [\boldsymbol{K}_{fw}]_{2\times 2} & [\boldsymbol{k}_{f\theta}]_{2\times 1} & -[\boldsymbol{K}_{fw}]_{2\times 2} & [\boldsymbol{k}_{f\theta}]_{2\times 1} \\ [\boldsymbol{k}_{mw}]_{1\times 2}^{\top} & k_{m\theta} & -[\boldsymbol{k}_{mw}]_{1\times 2}^{\top} & \hat{k}_{m\theta} \\ -[\boldsymbol{K}_{fw}]_{2\times 2} & -[\boldsymbol{k}_{f\theta}]_{2\times 1} & [\boldsymbol{K}_{fw}]_{2\times 2} & -[\boldsymbol{k}_{f\theta}]_{2\times 1} \\ [\boldsymbol{k}_{mw}]_{1\times 2}^{\top} & \hat{k}_{m\theta} & -[\boldsymbol{k}_{mw}]_{1\times 2}^{\top} & k_{m\theta} \end{array} \right] \left\{ \begin{array}{c} [\boldsymbol{w}_i]_{2\times 1} \\ \theta_i \\ [\boldsymbol{w}_j]_{2\times 1} \\ \theta_j \end{array} \right\}
$$