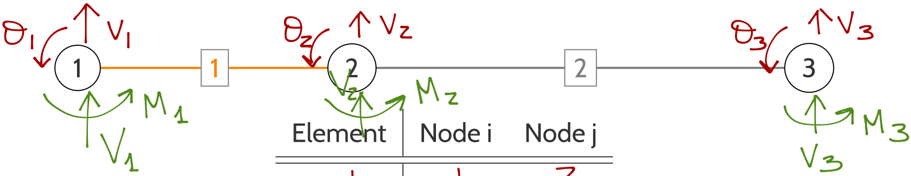
Global Assembly

Global Assembly of Beams



Element	Node i	Node j
1	1	2
2	2	3

Global Assembly of Beams

$$\begin{Bmatrix} V_i \\ M_i \\ V_j \\ M_j \end{Bmatrix} \begin{matrix} V_1 \\ M_1 \\ V_2 \\ M_2 \end{matrix} = \begin{array}{c} \begin{bmatrix} k_{fv} & k_{f\theta} & -k_{fv} & k_{f\theta} \\ k_{mv} & k_{m\theta} & -k_{mv} & \hat{k}_{m\theta} \\ -k_{fv} & -k_{f\theta} & k_{fv} & -k_{f\theta} \\ k_{mv} & \hat{k}_{m\theta} & -k_{mv} & k_{m\theta} \end{bmatrix} \\ \begin{matrix} V_1 & \theta_1 & V_2 & \theta_2 \end{matrix} \end{array} \begin{Bmatrix} v_i \\ \theta_i \\ v_j \\ \theta_j \end{Bmatrix}$$

$$\begin{Bmatrix} V_1 \\ M_1 \\ V_2 \\ M_2 \\ V_3 \\ M_3 \end{Bmatrix} = \begin{bmatrix} & & & & 0 & 0 \\ & & & & 0 & 0 \\ & & & & & \\ & & & & & \\ 0 & 0 & & & & \\ 0 & 0 & & & & \end{bmatrix} \begin{Bmatrix} v_1 \\ \theta_1 \\ v_2 \\ \theta_2 \\ v_3 \\ \theta_3 \end{Bmatrix}$$

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$$\begin{Bmatrix} V_i \\ M_i \\ V_j \\ M_j \end{Bmatrix} = \begin{bmatrix} k_{fv} & k_{f\theta} & -k_{fv} & k_{f\theta} \\ k_{mv} & k_{m\theta} & -k_{mv} & \hat{k}_{m\theta} \\ -k_{fv} & -k_{f\theta} & k_{fv} & -k_{f\theta} \\ k_{mv} & \hat{k}_{m\theta} & -k_{mv} & k_{m\theta} \end{bmatrix} \begin{Bmatrix} v_i \\ \theta_i \\ v_j \\ \theta_j \end{Bmatrix}$$

$$\begin{Bmatrix} V_1 \\ M_1 \\ V_2 \\ M_2 \\ V_3 \\ M_3 \end{Bmatrix} = \begin{bmatrix} k^1_{fv} & k^1_{f\theta} & -k^1_{fv} & k^1_{f\theta} & 0 & 0 \\ k^1_{mv} & k^1_{m\theta} & -k^1_{mv} & \hat{k}^1_{m\theta} & 0 & 0 \\ -k^1_{fv} & -k^1_{f\theta} & k^1_{fv} & -k^1_{f\theta} & & \\ k^1_{mv} & \hat{k}^1_{m\theta} & -k^1_{mv} & k^1_{m\theta} & & \\ 0 & 0 & & & & \\ 0 & 0 & & & & \end{bmatrix} \begin{Bmatrix} v_1 \\ \theta_1 \\ v_2 \\ \theta_2 \\ v_3 \\ \theta_3 \end{Bmatrix}$$

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$$

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$$

Enforce hinge constraint

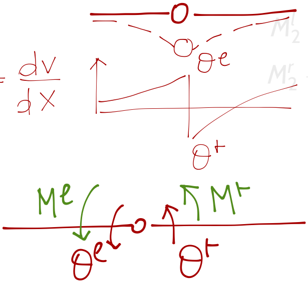
1. Duplicate $\theta_2 \to \theta_2^l, \theta_2^r$
2. Write equilibrium equations for each side of the hinge

! NO MOMENT TRASFER!
& NO ROTATION !!

$$M_2^l - m_j^1 = 0$$

$$M_2^r + m_i^2 = 0$$

$\theta = \frac{dv}{dx}$



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$$\begin{Bmatrix} V_1 \\ M_1 \\ V_2 \\ ? \\ ? \\ V_3 \\ M_3 \end{Bmatrix} = \begin{bmatrix} k_{fv}^1 & k_{f\theta}^1 & -k_{fv}^1 & ? & ? & 0 & 0 \\ k_{mv}^1 & k_{m\theta}^1 & -k_{mv}^1 & ? & ? & 0 & 0 \\ -k_{fv}^1 & -k_{f\theta}^1 & k_{fv}^1 + k_{fv}^2 & ? & ? & -k_{fv}^2 & k_{f\theta}^2 \\ ? & ? & ? & ? & ? & ? & ? \\ ? & ? & ? & ? & ? & ? & ? \\ 0 & 0 & -k_{fv}^2 & ? & ? & k_{fv}^2 & -k_{f\theta}^2 \\ 0 & 0 & k_{mv}^2 & ? & ? & -k_{mv}^2 & k_{m\theta}^2 \end{bmatrix} \begin{Bmatrix} v_1 \\ \theta_1 \\ v_2 \\ \theta_2^l \\ \theta_2^r \\ v_3 \\ \theta_3 \end{Bmatrix}$$

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Apply boundary conditions

$$v_1 = 0, \quad v_3 = 0, \quad \theta_3 = 0$$

$$\begin{Bmatrix} V_1 \\ M_1 \\ V_2 \\ M_2^l \\ M_2^r \\ V_3 \\ M_3 \end{Bmatrix} = \begin{bmatrix} k_{fv}^1 & k_{f\theta}^1 & -k_{fv}^1 & k_{f\theta}^1 & 0 & 0 & 0 \\ k_{mv}^1 & k_{m\theta}^1 & -k_{mv}^1 & \hat{k}_{m\theta}^1 & 0 & 0 & 0 \\ -k_{fv}^1 & -k_{f\theta}^1 & k_{fv}^1+k_{fv}^2 & -k_{f\theta}^1 & k_{f\theta}^2 & -k_{fv}^2 & k_{f\theta}^2 \\ k_{mv}^1 & \hat{k}_{m\theta}^1 & -k_{mv}^1 & k_{m\theta}^1 & 0 & 0 & 0 \\ 0 & 0 & k_{mv}^2 & 0 & k_{m\theta}^2 & -k_{mv}^2 & \hat{k}_{m\theta}^2 \\ 0 & 0 & -k_{fv}^2 & 0 & -k_{f\theta}^2 & k_{fv}^2 & -k_{f\theta}^2 \\ 0 & 0 & k_{mv}^2 & 0 & \hat{k}_{m\theta}^2 & -k_{mv}^2 & k_{m\theta}^2 \end{bmatrix} \begin{Bmatrix} v_1 \\ \theta_1 \\ v_2 \\ \theta_2^l \\ \theta_2^r \\ v_3 \\ \theta_3 \end{Bmatrix}$$

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