

COURSE OVERVIEW & INTRODUCTION

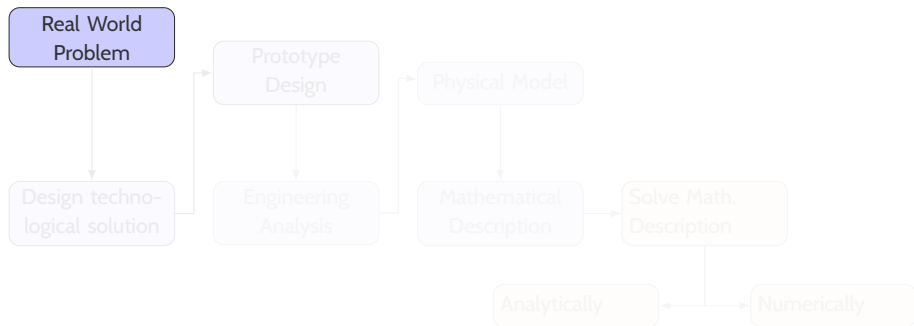
CEE 361/513: INTRO. TO FINITE ELEMENT METHODS

Maurizio M. Chiaramonte

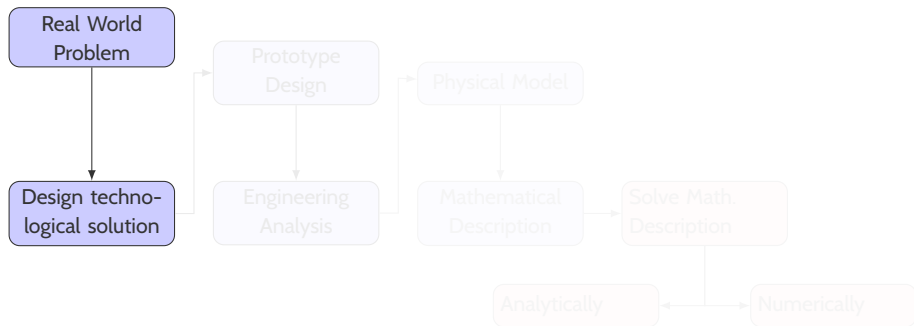
COURSE OVERVIEW



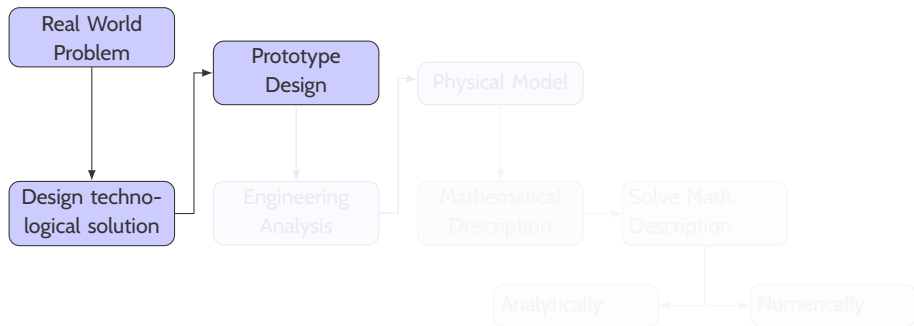
The role of the engineer



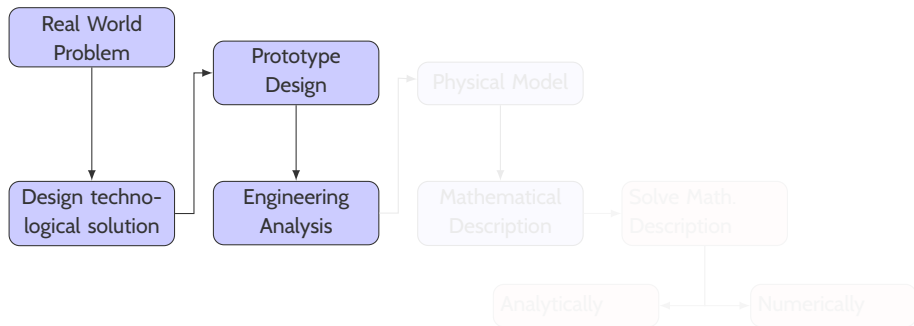
The role of the engineer



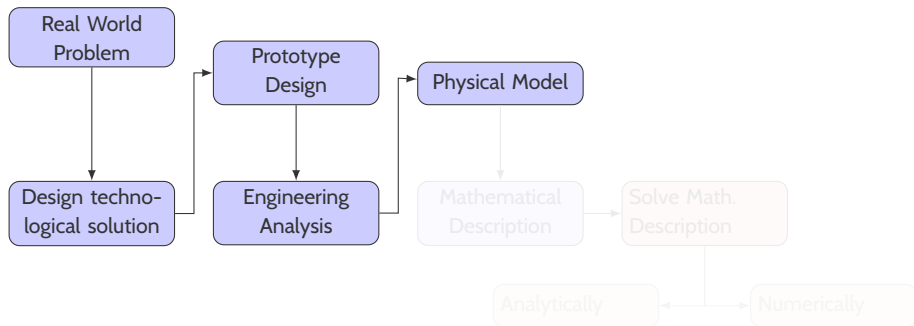
The role of the engineer



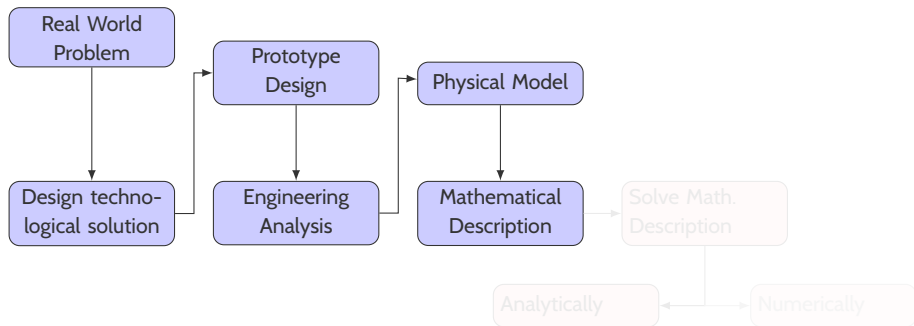
The role of the engineer



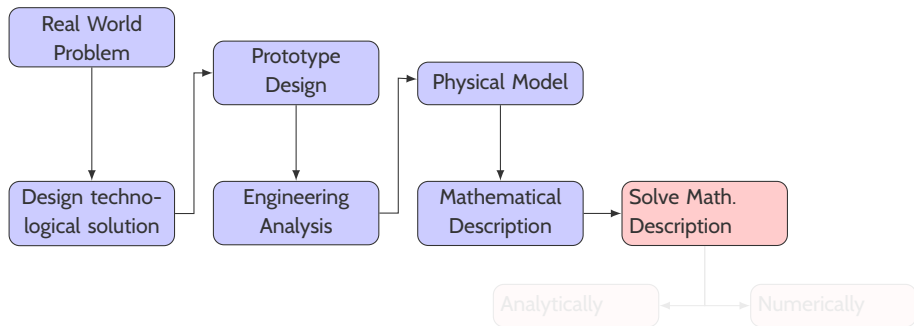
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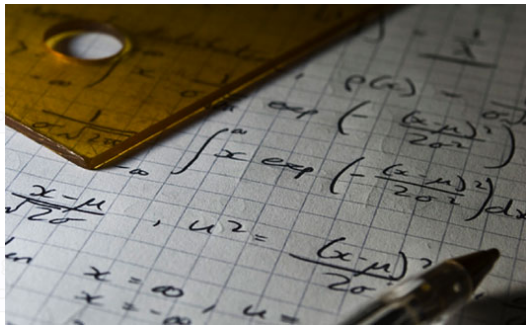
The role of the engineer



The role of the engineer



The role of the engineer

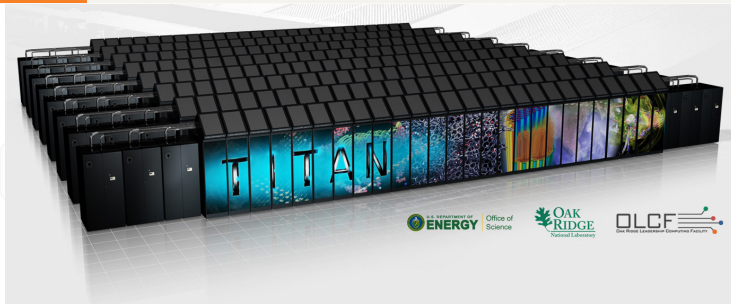


Solve Math.
Description

Analytically

Numerically

The role of the engineer



Design techno-
logical solution

Engineering
Analysis

Mathematical
Description

Solve Math.
Description

Analytically

Numerically



What is the course about ?

What is the course about ?

Finite Element Methods

What is the course about ?

*A numerical method for the solution of
Partial Differential Equations*

Examples of Partial Differential Equations

Examples of Partial Differential Equations

Partial Differential Equations describe phenomena all around us:

- Solid & Structural Mechanics - Cauchy mom. balance

$$\nabla \cdot \boldsymbol{\sigma} = \mathbf{0}$$

- Fluid Mechanics - Navier-Stokes

$$\frac{\partial \mathbf{v}}{\partial t} + \nabla \mathbf{v} \cdot \mathbf{v} + \nu \Delta \mathbf{v} = -\nabla p$$

$$\nabla \cdot \mathbf{v} = 0$$

- Electromagnetism - Maxwell

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

- Finance - Black-Scholes

$$\frac{\partial V}{\partial t} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS \frac{\partial V}{\partial S} - rV = 0$$

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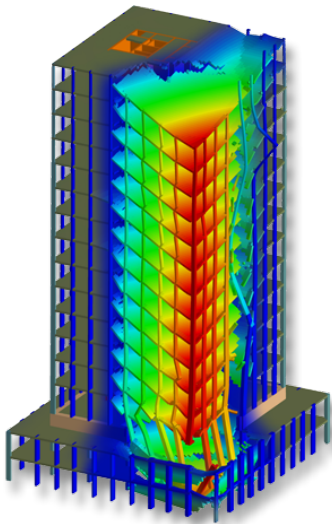
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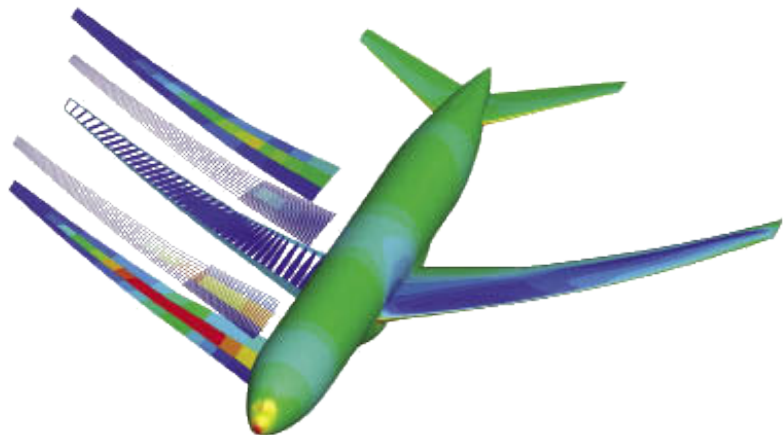
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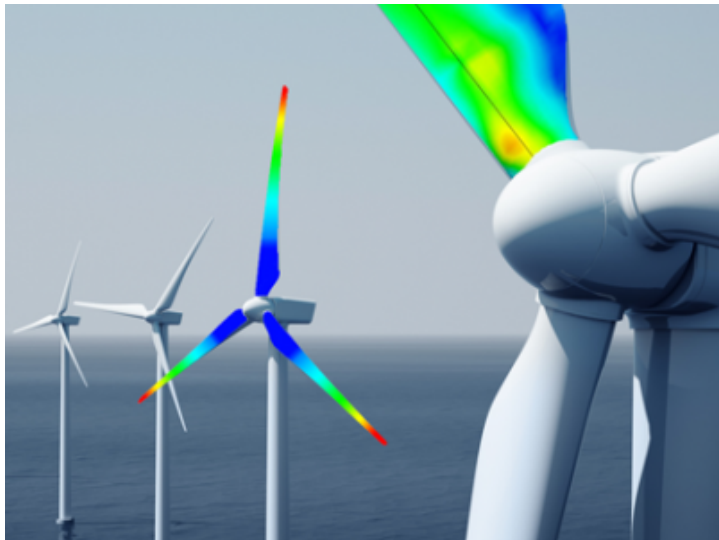
Applications of finite elements



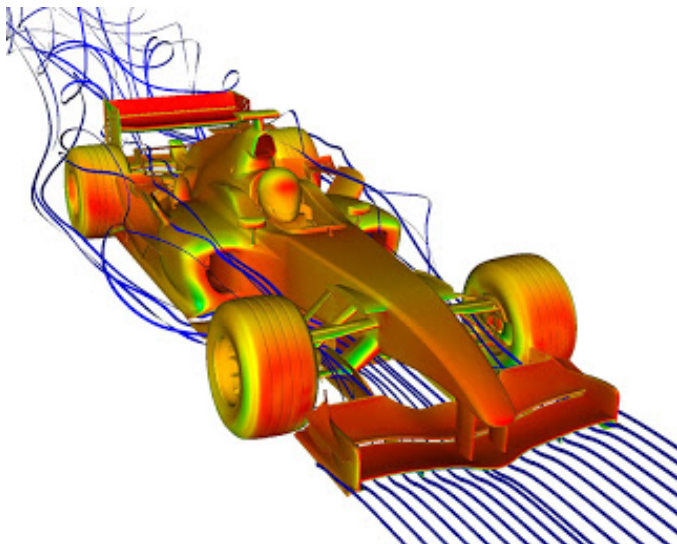
Applications of finite elements



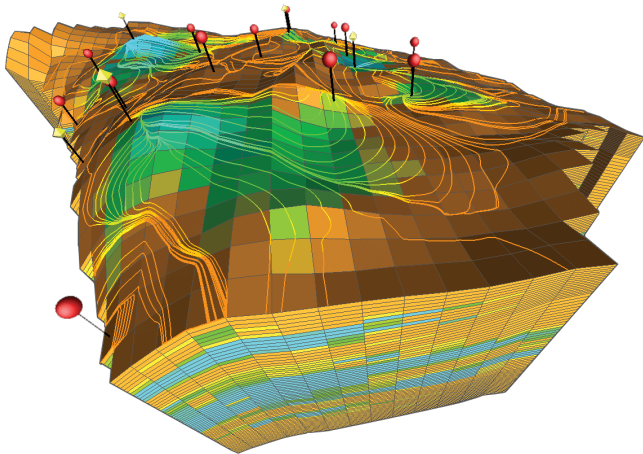
Applications of finite elements



Applications of finite elements



Applications of finite elements



Course Objective

What are we going to talk about

We will learn about finite element methods.

The focus will be on:

- **Formulating** finite element methods.

PDEs \mapsto numerical discretization

- **Implementing** finite element methods

numerical discretization \mapsto computer implementation

COURSE LOGISTICS

Course Outline

1. Math Preliminaries
 - 1.1 Tensor algebra and tensor calculus
 - 1.2 Review of partial differential equations
2. Direct Stiffness Methods
 - 2.1 Truss Equation
 - 2.2 Beam Equation
 - 2.3 Global Assembly
 - 2.4 Boundary conditions
3. One-dimensional Finite elements
 - 3.1 Intro to the Calculus of variations
 - 3.2 Strong and weak form
 - 3.3 Galerkin approximation
 - 3.4 Matrix form
 - 3.5 Shape functions
 - 3.6 Numerical integration
 - 3.7 Error analysis

Course Outline

- 4. Two and three space dimensions
 - 4.1 Review of tensors calculus
 - 4.2 Extension of the notions of strong, weak, and matrix forms
 - 4.3 Hexahedral elements
 - 4.4 Simplicial elements
 - 4.5 Isoparametric interpolations
- 5. C^0 approach to beams and shells
If Time Permits:
- 6. Time dependent problems
 - 6.1 Generalized θ method
 - 6.2 Stability of the θ -method
- 7. Analysis of Finite Element Methods
 - 7.1 Best approximation and error estimates
 - 7.2 Consistency & stability
 - 7.3 Locking issues
- 8. Constraints
 - 8.1 Lagrange multipliers
 - 8.2 Penalty methods

Grading

Grading Breakdown

Grades will be based:

- Homework assignments (50%)
- A midterm (20%)
- A final project (30%).

Grading

Grading Computation

The final grade for the class is computed with both an absolute and relative (curve) scale. If you get above or equal to:

- 98% of the maximum score you get an A+
- 90% of the maximum score you get an A
- 85% of the maximum score you get an A-
- 80% of the maximum score you get an B+
- 75% of the maximum score you get an B
- 70% of the maximum score you get an B-
- 65% of the maximum score you get an C+
- 60% of the maximum score you get an C
- 55% of the maximum score you get an C-
- 50% of the maximum score you get an D+
- 45% of the maximum score you get an D

At least 25% of the class will get an A, at least 50% of the class will get a B or higher, and at least 75% of the class will get a C or higher, as long as the minimum of 45% of the maximum total score is achieved. This distribution will be achieved by uniformly shifting everyone's grade up by the same amount. (Separately for 500 and 300 level)

Homeworks

Homework details and policies

- 10 homework
- Homework will be posted on Friday and due the next Friday @ midnight
- Homework will be submitted electronically through <https://blackboard.princeton.edu> dropbox
- No late homework will be accepted
- The lowest of your homework grades will be dropped
- All homework will be weighted the same unless specified
- A mixture of theory and computations
- CEE 513 homework will be typeset in \LaTeX (will post the *.pdf and *.tex)
- Non-513 students if homework is typeset in \LaTeX get an extra 5%

Exams details and policies

- Closed book & closed notes
- Focus on theory
- Midterm exam tentative date is Oct 25th.

Project

Project plan - CEE 361

- Open ended longer homework assignment
- Can be done in groups of up to three students (different expectations)
- More details will follow

Project plan - CEE 513

- Identify a topic of interest within the realm of finite element method
 - Applications of finite elements to a specific project
 - Development of techniques for finite element methods

Project

Project plan - CEE 361

- Open ended longer homework assignment
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- More details will follow

Project plan - CEE 513

- Identify a topic of interest within the realm of finite element method
 - Applications of finite elements to a specific project
 - Development of techniques for finite element methods

Tools for the course

Textbook

Suggested readings:

Hughes, T. J. R. (1987).

The Finite Element Method: Linear Static and Dynamic Finite Element Analysis.

Prévost, J. H. and S. Bagrianski (2017).

An Introduction to Matrix Structural Analysis and Finite Element Methods.

Tools

○ Python

○ FEniCS

Lecture and Precepts

Lecture

- T,Th Friend 008 11:00-12:20
- Will mostly cover theory
- Must come to lecture to take notes
- Please do not eat in class
- Be courteous to other classmates (i.e. no talking, use laptops, etc ...)
- Be engaged and ask questions (if you have any ...)!

Precepts

- W 7:30pm-8:30pm EQuad E225
- Will cover sample problems
- Interactive environment

Getting Help

Office hours

Maurizio M. Chiaramonte

Office Hours: Wednesday 1:30pm - 3:30pm, EQuad E324

chiaramonte@princeton.edu

Precept + Office hours

Isabel Morris

Time: W, Precept 7:30pm - 8:30pm

Office hours: 9:30pm- 10:30pm

imorris@princeton.edu

Piazza

<http://piazza.com/princeton/fall2018/cee361mae325mse331cee513f18>

Homework # 0

Go and sign up on Piazza!

Course Website

Course Website

`http://finiteelements.princeton.edu`

username: bubnov

password: galerkin

Syllabus

Additional Resources

Princeton Institute for Computational Science and Engineering (PICSciE)

PICSciE

Python Minicourse:

1. Section I - Two Mondays, Sept. 24 and Oct. 1, 3:30-5:00 pm
2. Section II - Two Tuesdays, Sept. 25 and Oct. 2, 3:30-5:00 pm
3. Section III - Two Wednesdays, Sept. 26 and Oct. 3, 3:30-5:00 pm
4. Section IV - Two Thursdays, Sept. 27 and Oct. 4, 3:30-5:00 pm

If you attend the full series you will receive an extra 5 points in the final project.