Learn how to GRAPPA

FMRIB Graduate Course
Advanced Program

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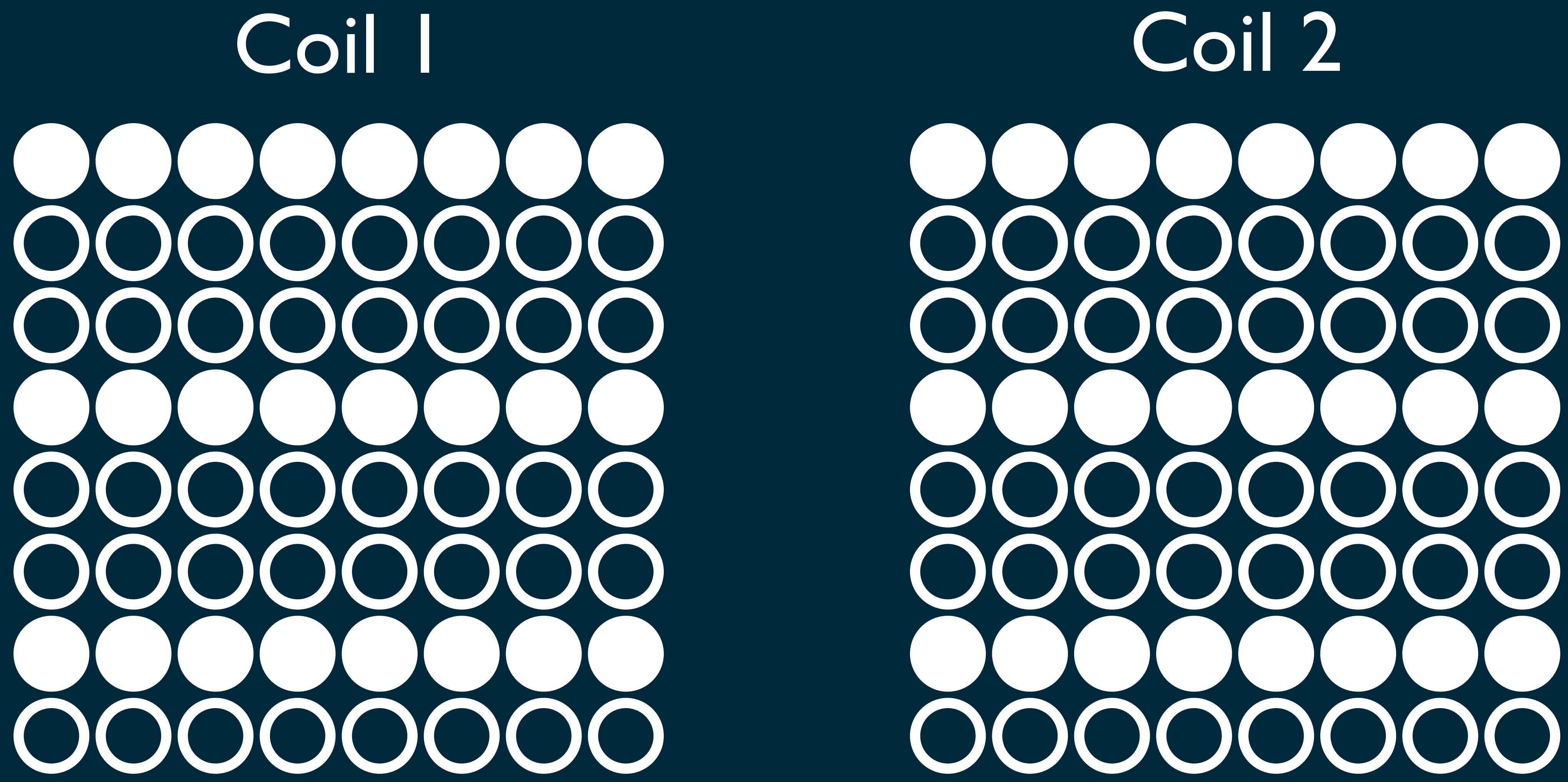
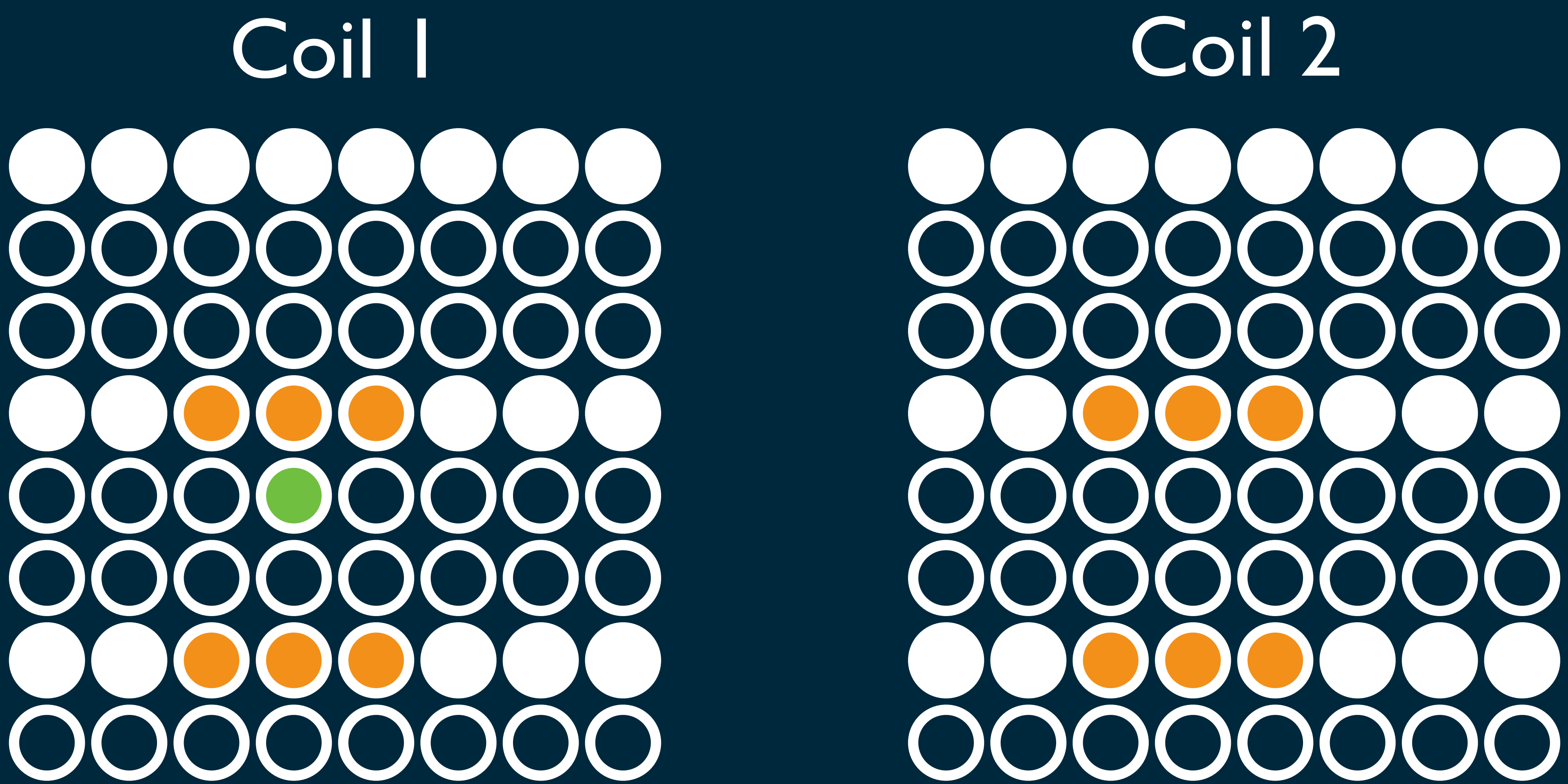
GRAPPA Geometry



Estimate missing points from acquired points

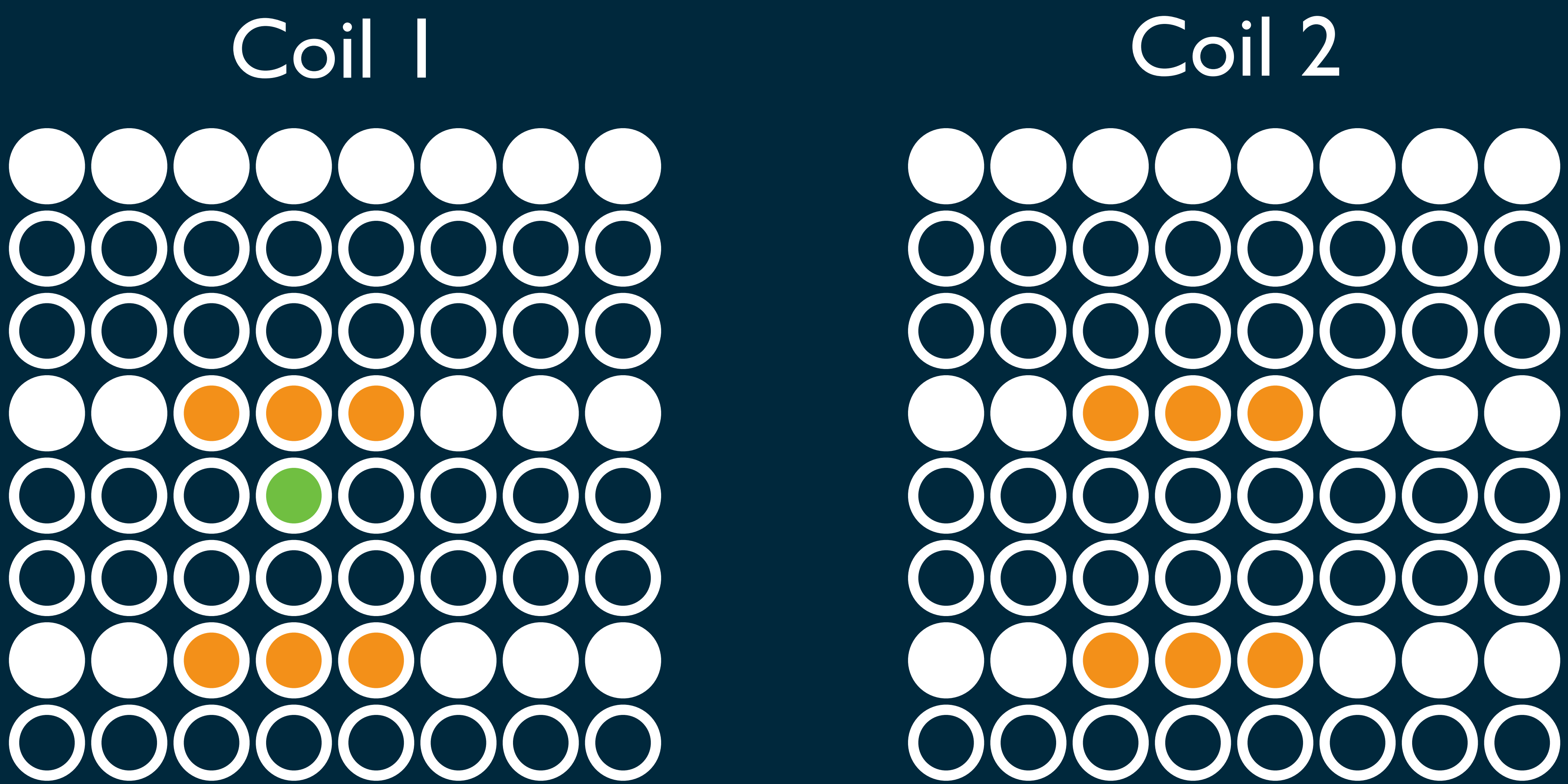
GRAPPA Geometry



Choose a kernel geometry

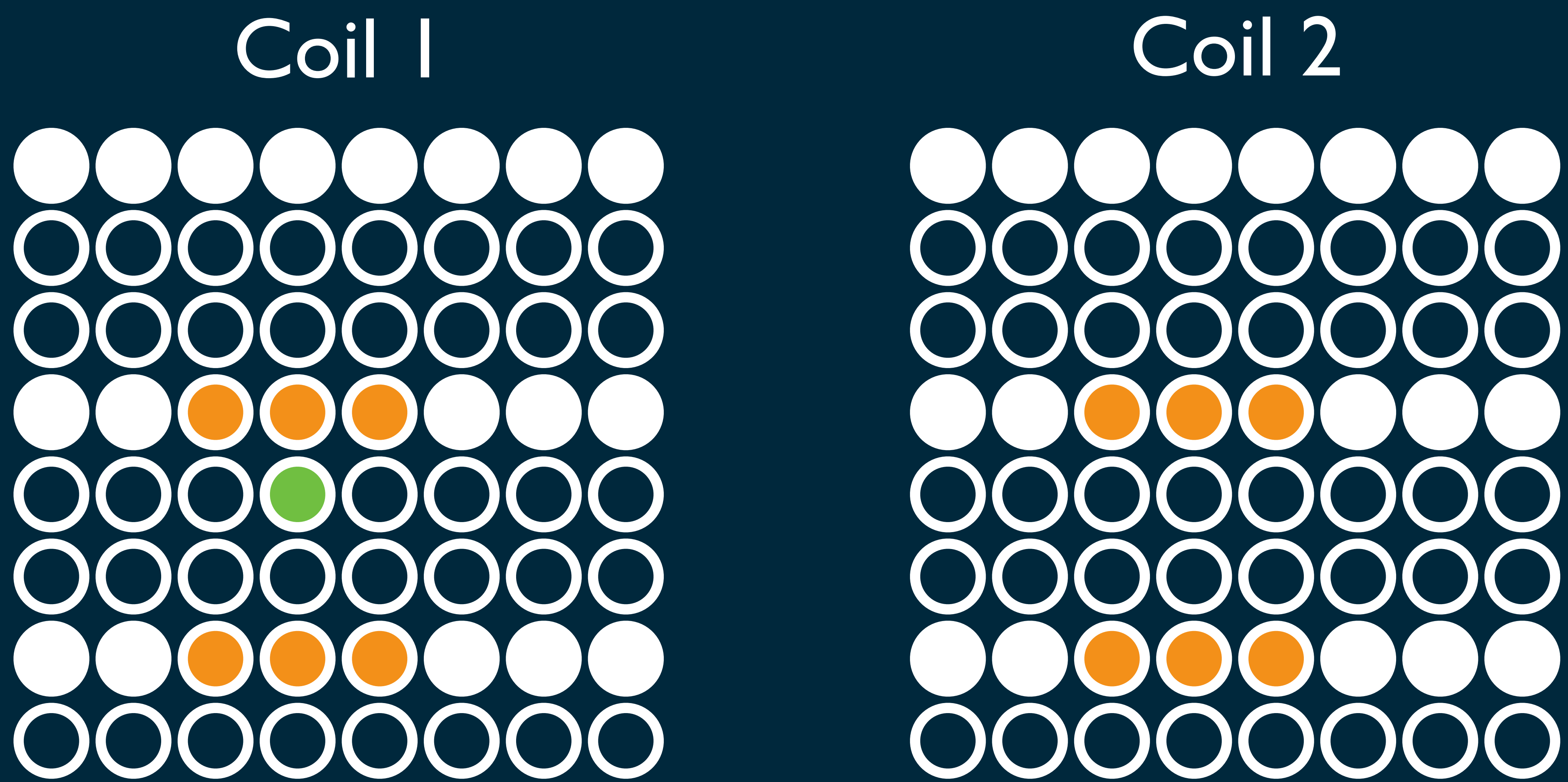
This dictates the spatial relationship between source and target points

GRAPPA Geometry



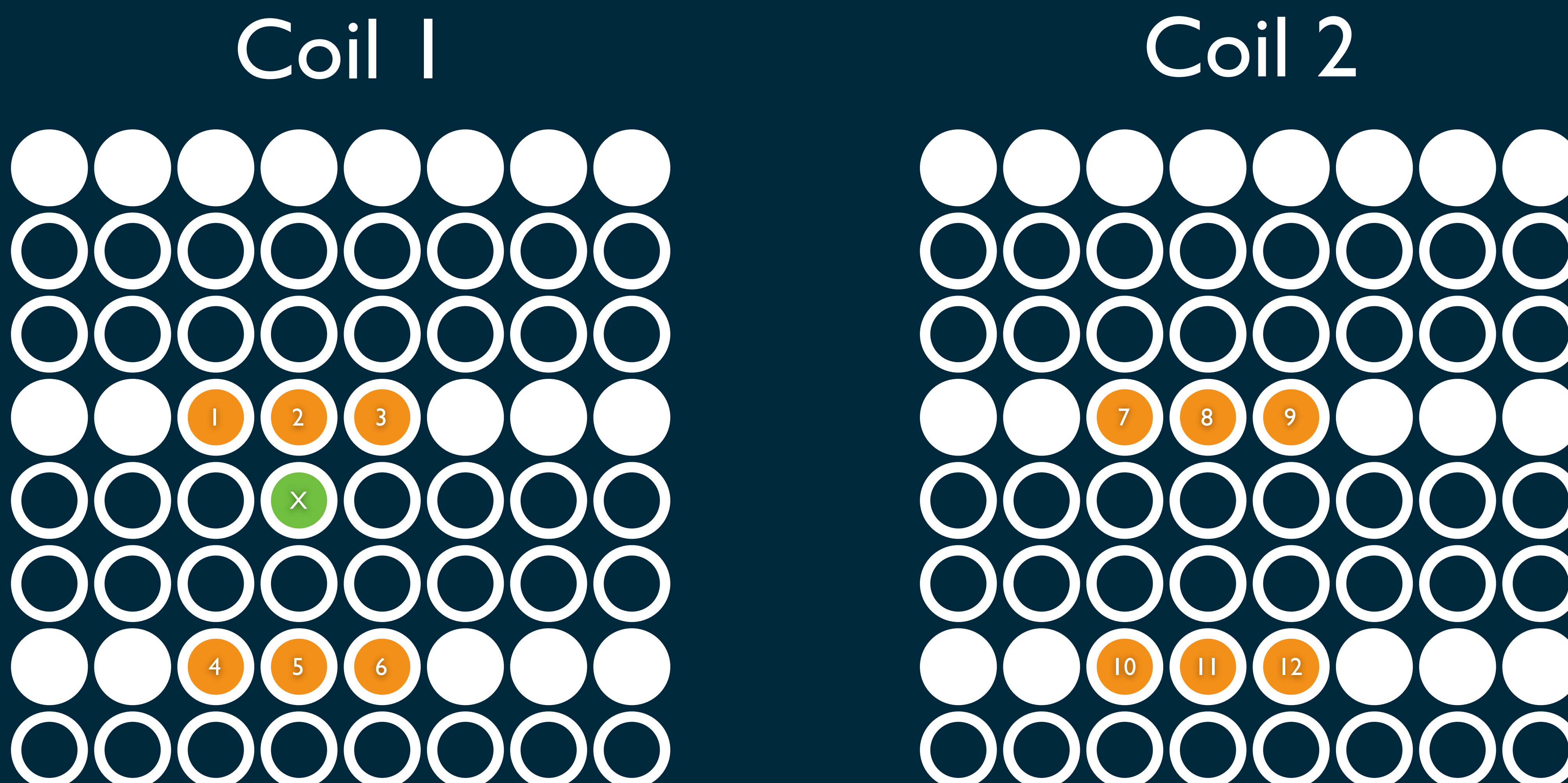
This is a [3,2] kernel geometry
3 points in x, 2 in y

GRAPPA Geometry



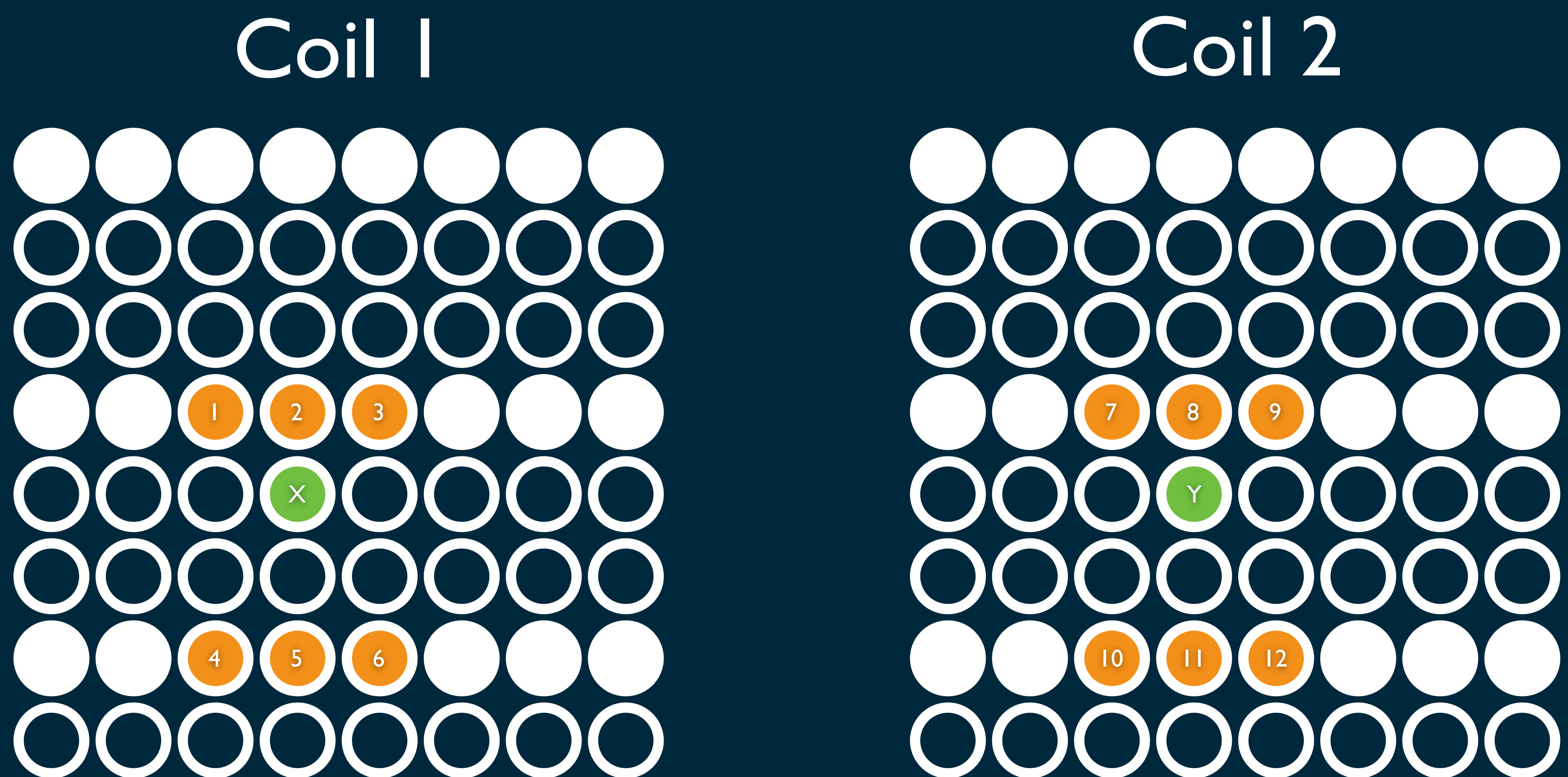
The kernel weights relate the source points from ALL coils to a target point in ONE coil

GRAPPA Geometry



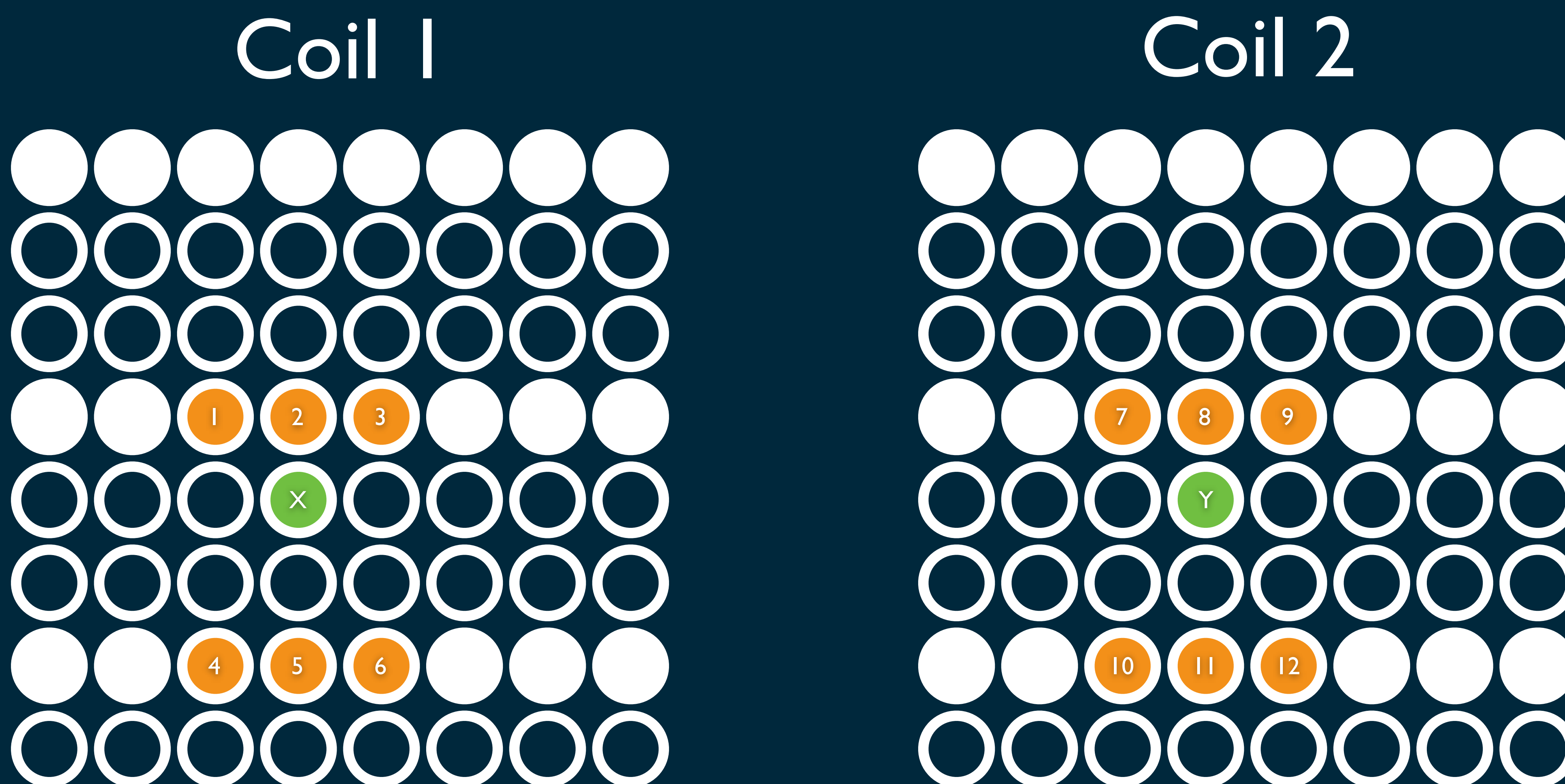
$$X = w_{1,1} \times 1 + w_{1,2} \times 2 + \ldots w_{1,12} \times 12$$

GRAPPA Geometry



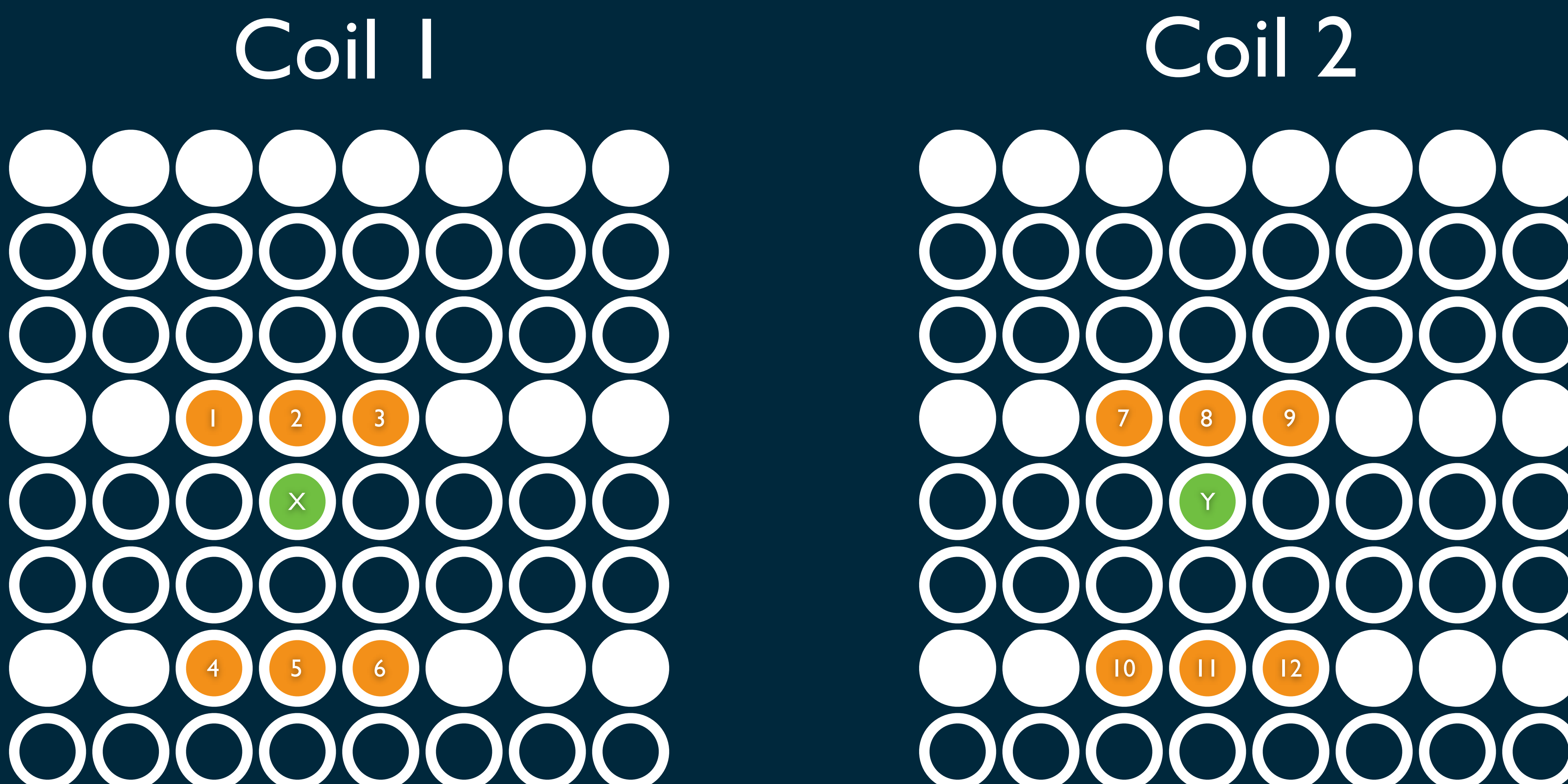
$$Y = w_{2,1} \times 1 + w_{2,2} \times 2 + \ldots w_{2,12} \times 12$$

GRAPPA Geometry



For this kernel, the all the weights can form a matrix
Each row corresponds to a different target coil
And one column per source point

GRAPPA Geometry



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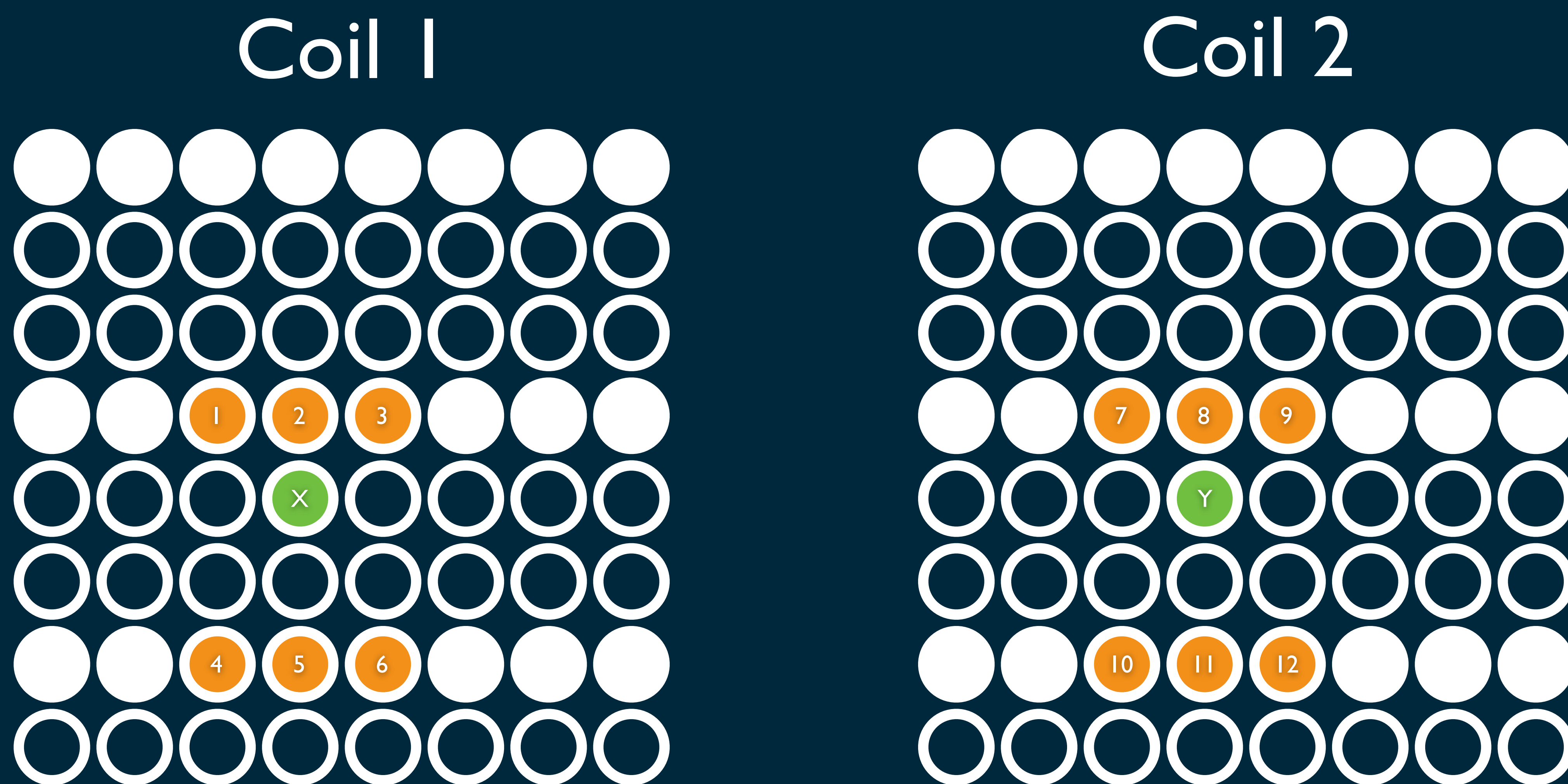
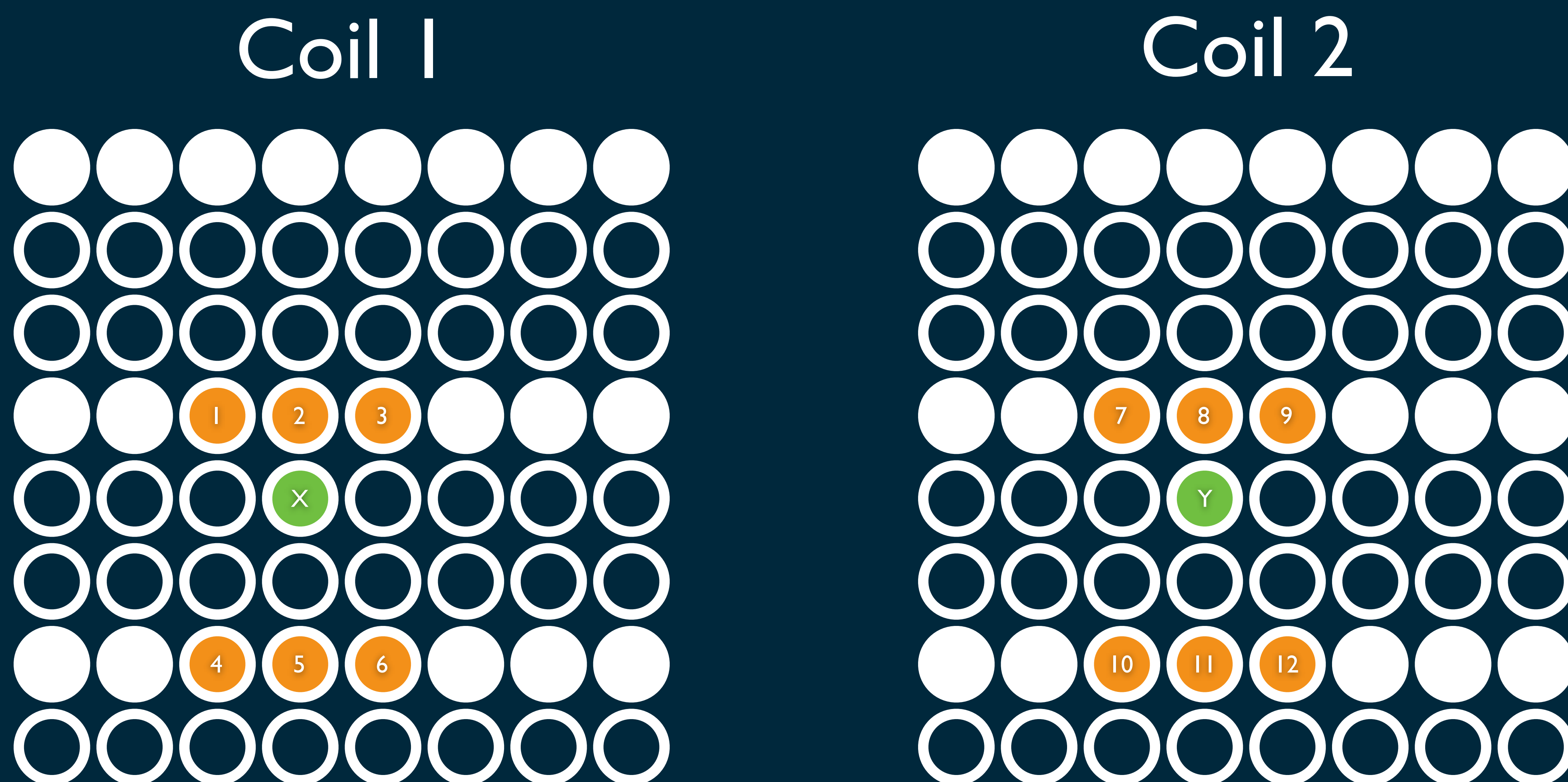


$$\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} w_{1,1} & w_{1,2} & \cdots & w_{1,12} \\ w_{2,1} & w_{2,2} & \cdots & w_{2,12} \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ \vdots \\ 12 \end{bmatrix}$$

GRAPPA Geometry



$$\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} W^{(1)}{}_{1,1} & W^{(1)}{}_{1,2} & \cdots & W^{(1)}{}_{1,12} \\ W^{(1)}{}_{2,1} & W^{(1)}{}_{2,2} & \cdots & W^{(1)}{}_{2,12} \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ \vdots \\ 12 \end{bmatrix}$$

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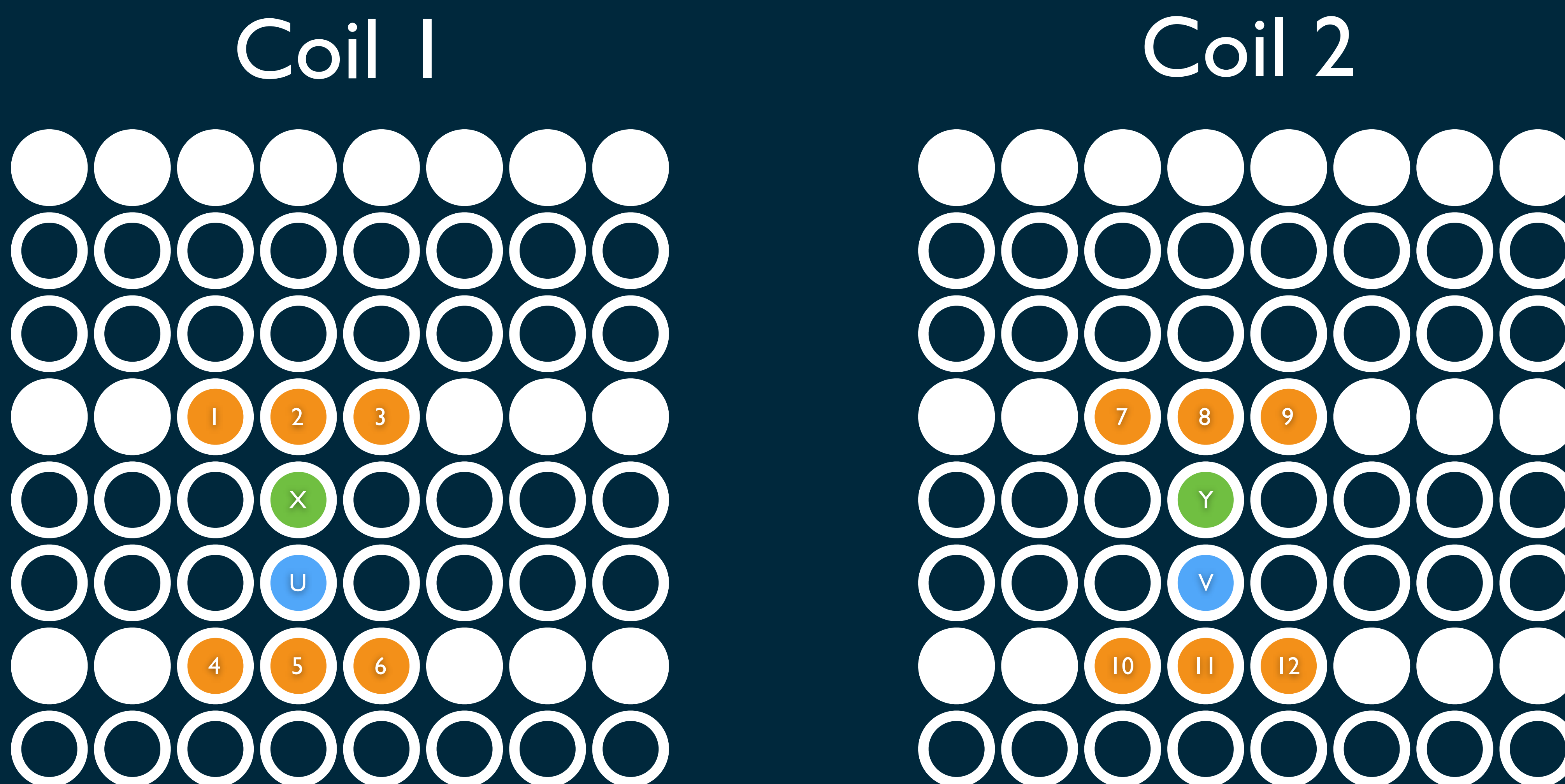
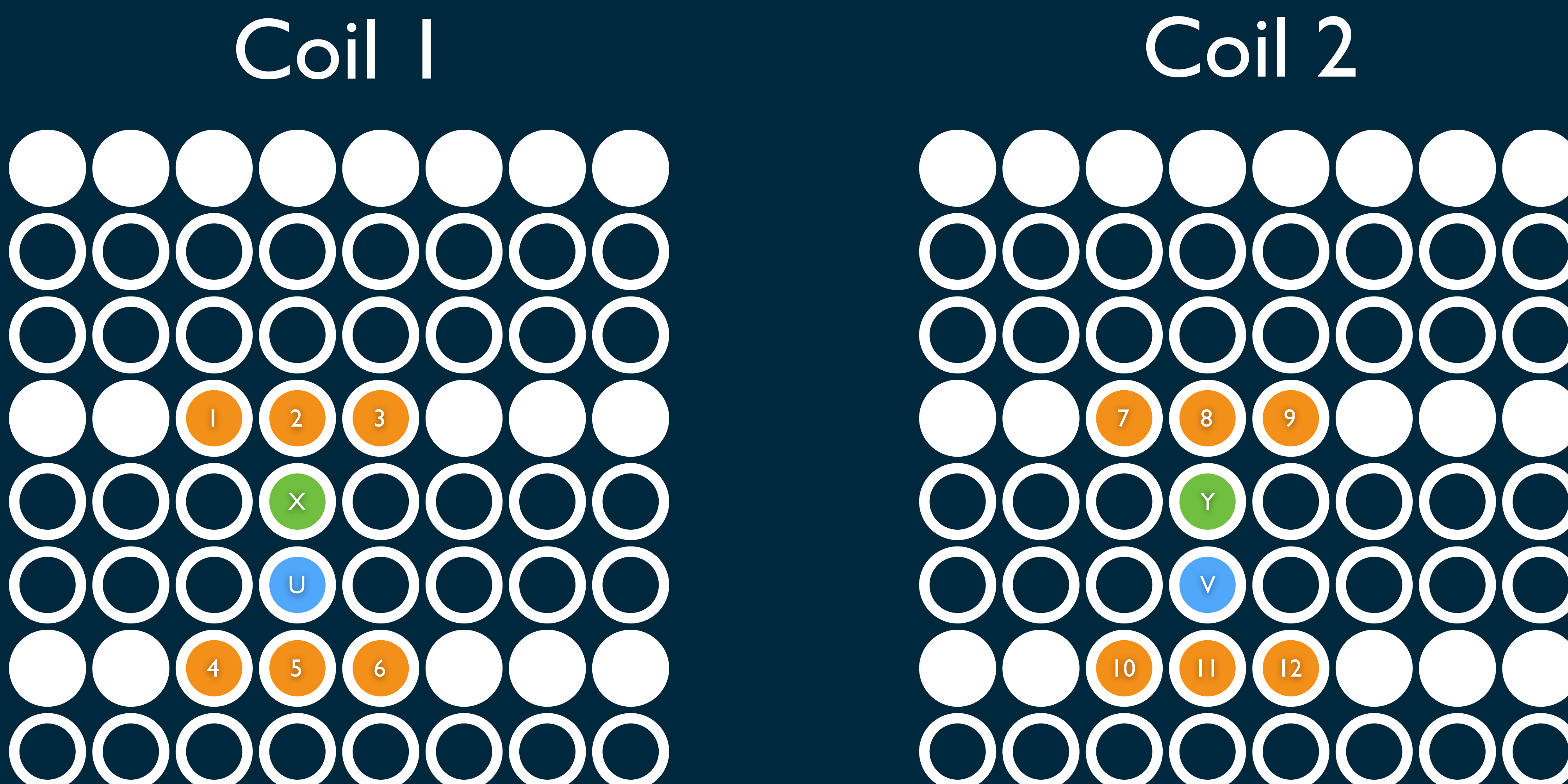


$$\begin{bmatrix} U \\ V \end{bmatrix} = \begin{bmatrix} W^{(2)}{}_{1,1} & W^{(2)}{}_{1,2} & \cdots & W^{(2)}{}_{1,12} \\ W^{(2)}{}_{2,1} & W^{(2)}{}_{2,2} & \cdots & W^{(2)}{}_{2,12} \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ \vdots \\ 12 \end{bmatrix}$$

GRAPPA Geometry



$$\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} W^{(1)}_{1,1} & W^{(1)}_{1,2} & \cdots & W^{(1)}_{1,12} \\ W^{(1)}_{2,1} & W^{(1)}_{2,2} & \cdots & W^{(1)}_{2,12} \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ \vdots \\ 12 \end{bmatrix}$$

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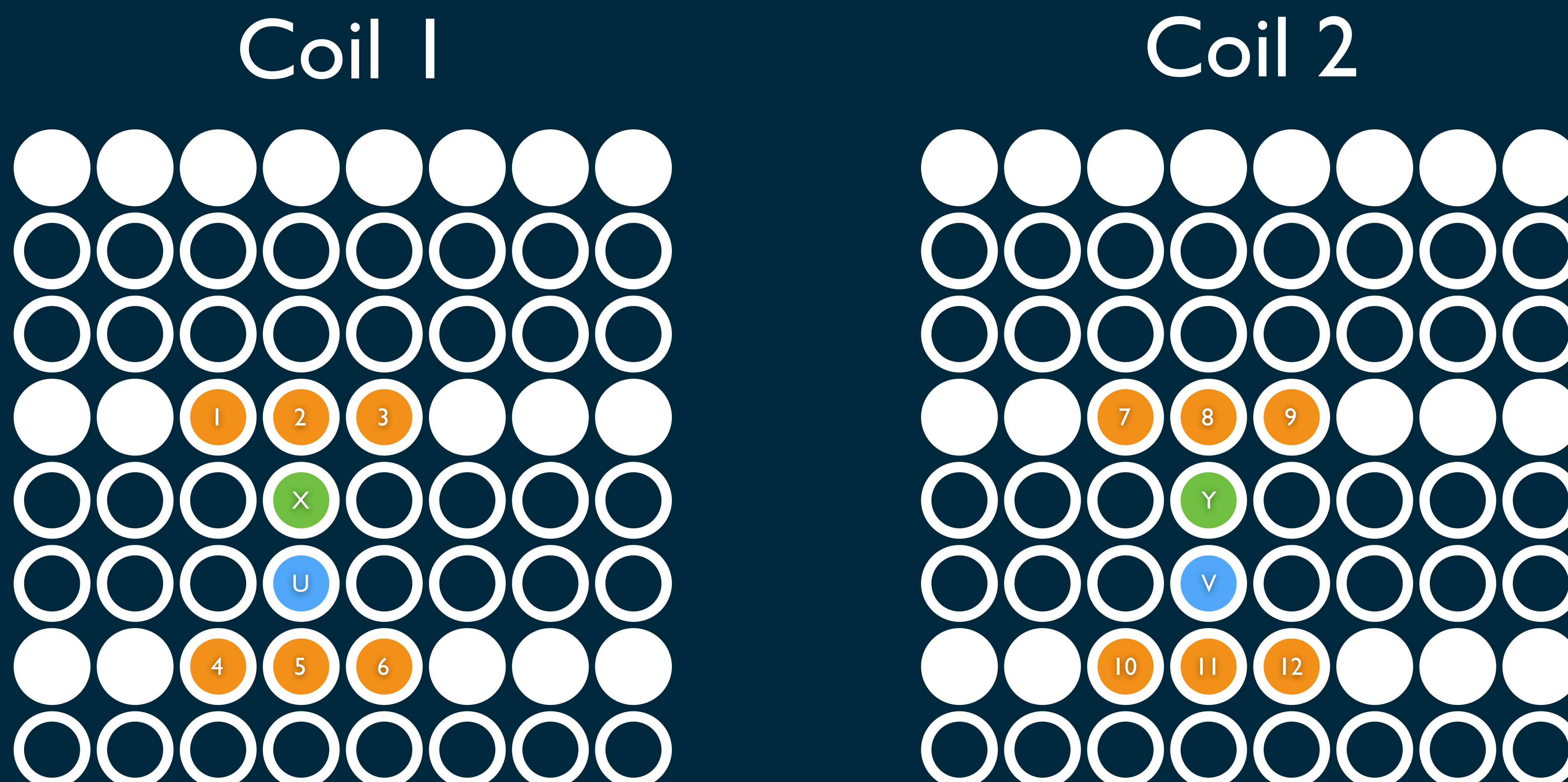

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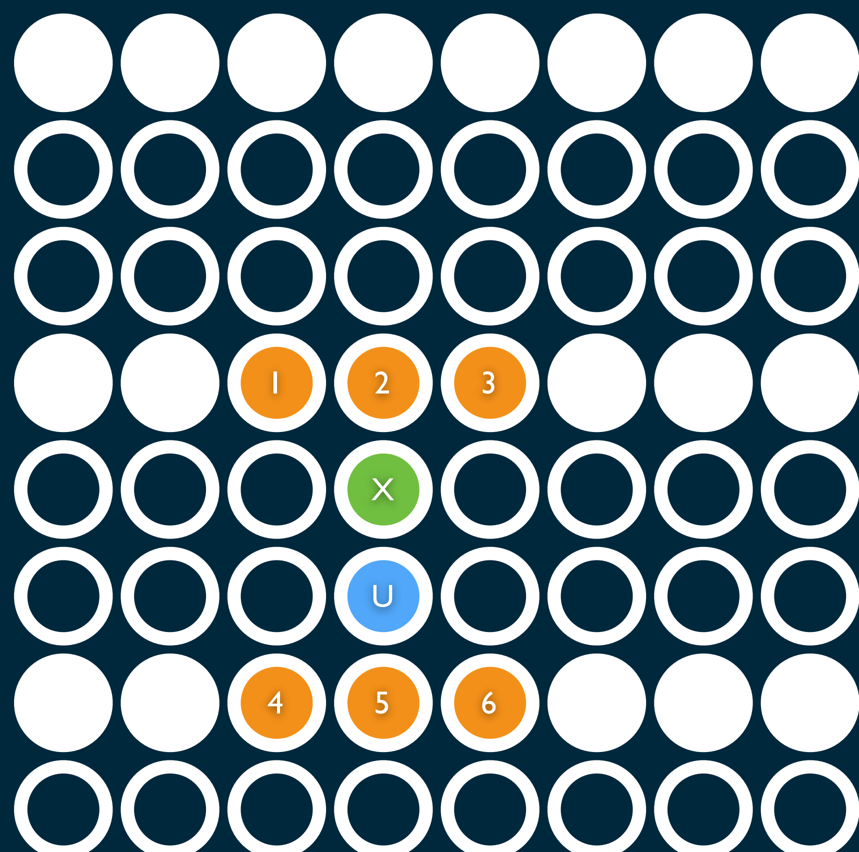
GRAPPA Geometry

Coil 1



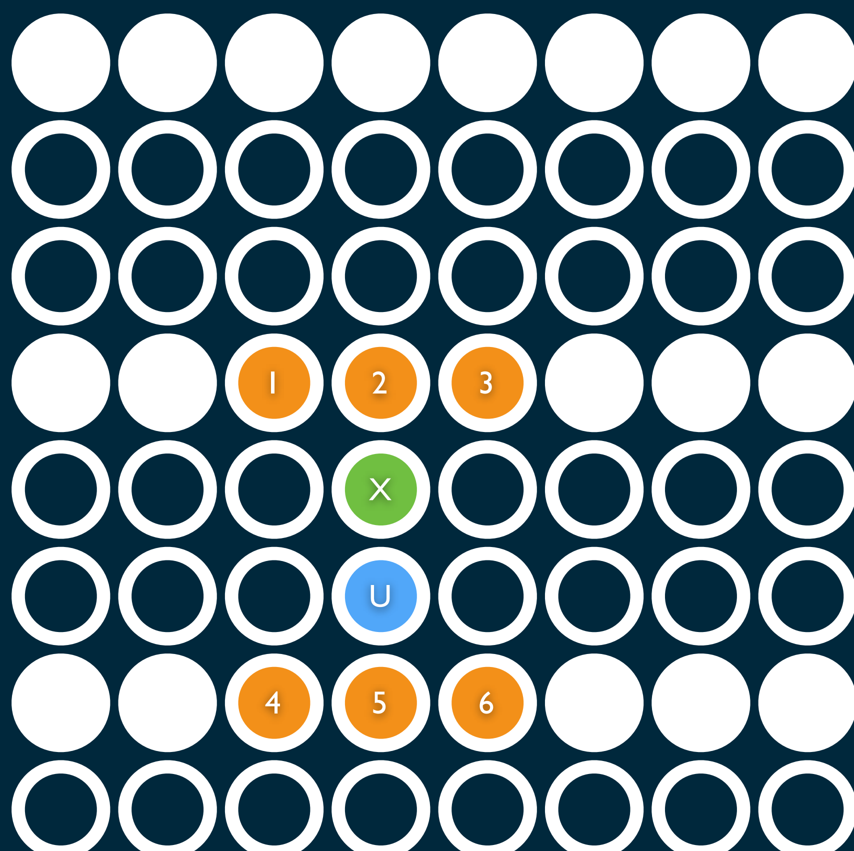
In general, there are R-1 different kernel types

Every missing point should fit into one of these classes

$$\begin{bmatrix} X \\ Y \\ U \\ V \end{bmatrix} = \begin{bmatrix} W^{(1)}_{1,1} & W^{(1)}_{1,2} & \cdots & W^{(1)}_{1,12} \\ W^{(1)}_{2,1} & W^{(1)}_{2,2} & \cdots & W^{(1)}_{2,12} \\ W^{(2)}_{1,1} & W^{(2)}_{1,2} & \cdots & W^{(2)}_{1,12} \\ W^{(2)}_{2,1} & W^{(2)}_{2,2} & \cdots & W^{(2)}_{2,12} \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ \vdots \\ 12 \end{bmatrix}$$

GRAPPA Geometry

Coil 1



So in total, for a system with L coils, kernel size [kx, ky], and undersampling factor R

You need an :

`L*(R-1) by L*kx*ky`

matrix of weights

$$\begin{bmatrix} X \\ Y \\ U \\ V \end{bmatrix} = \begin{bmatrix} w^{(1)}_{1,1} & w^{(1)}_{1,2} & \cdots & w^{(1)}_{1,12} \\ w^{(1)}_{2,1} & w^{(1)}_{2,2} & \cdots & w^{(1)}_{2,12} \\ w^{(2)}_{1,1} & w^{(2)}_{1,2} & \cdots & w^{(2)}_{1,12} \\ w^{(2)}_{2,1} & w^{(2)}_{2,2} & \cdots & w^{(2)}_{2,12} \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ \vdots \\ 12 \end{bmatrix}$$

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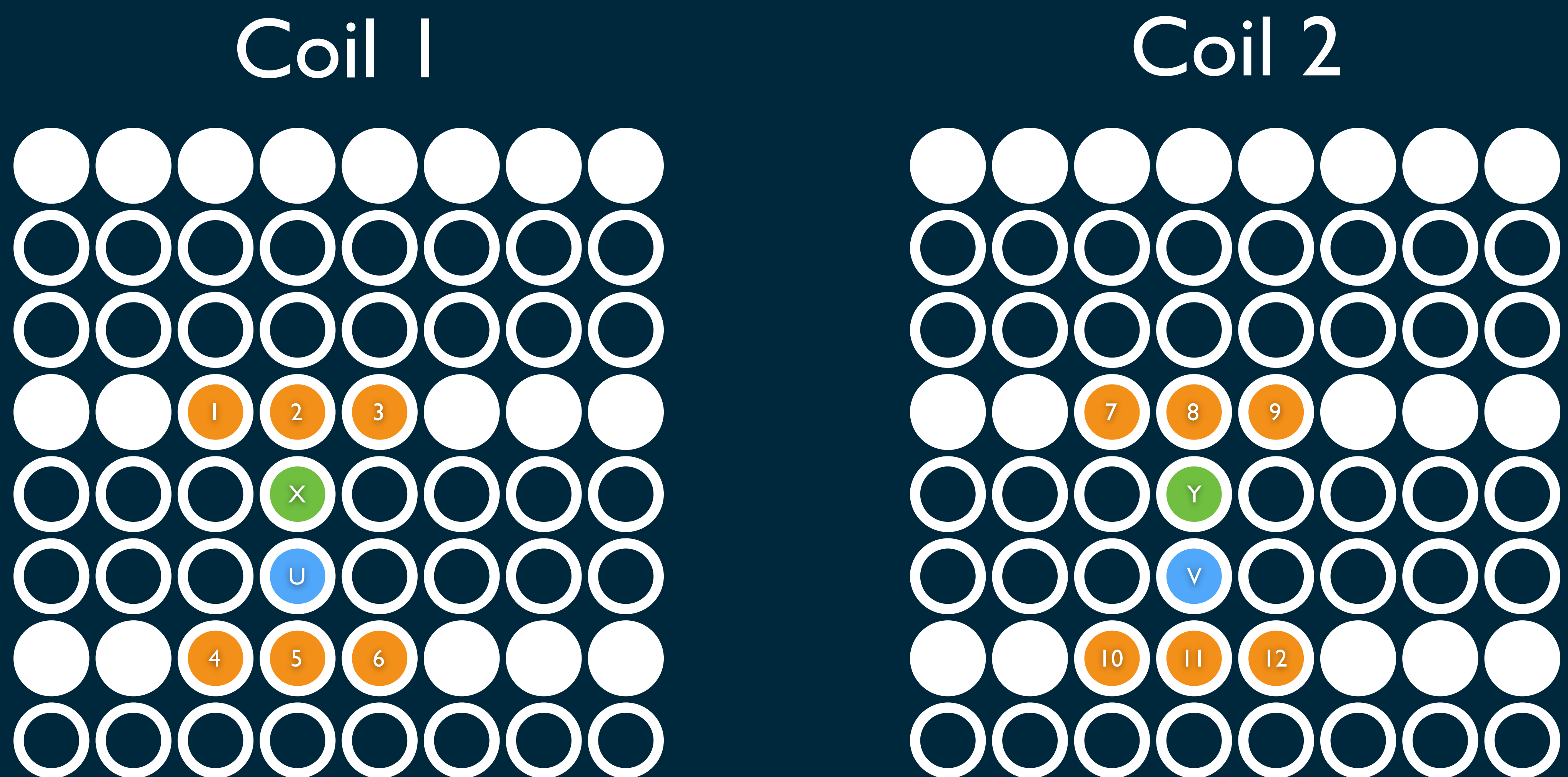
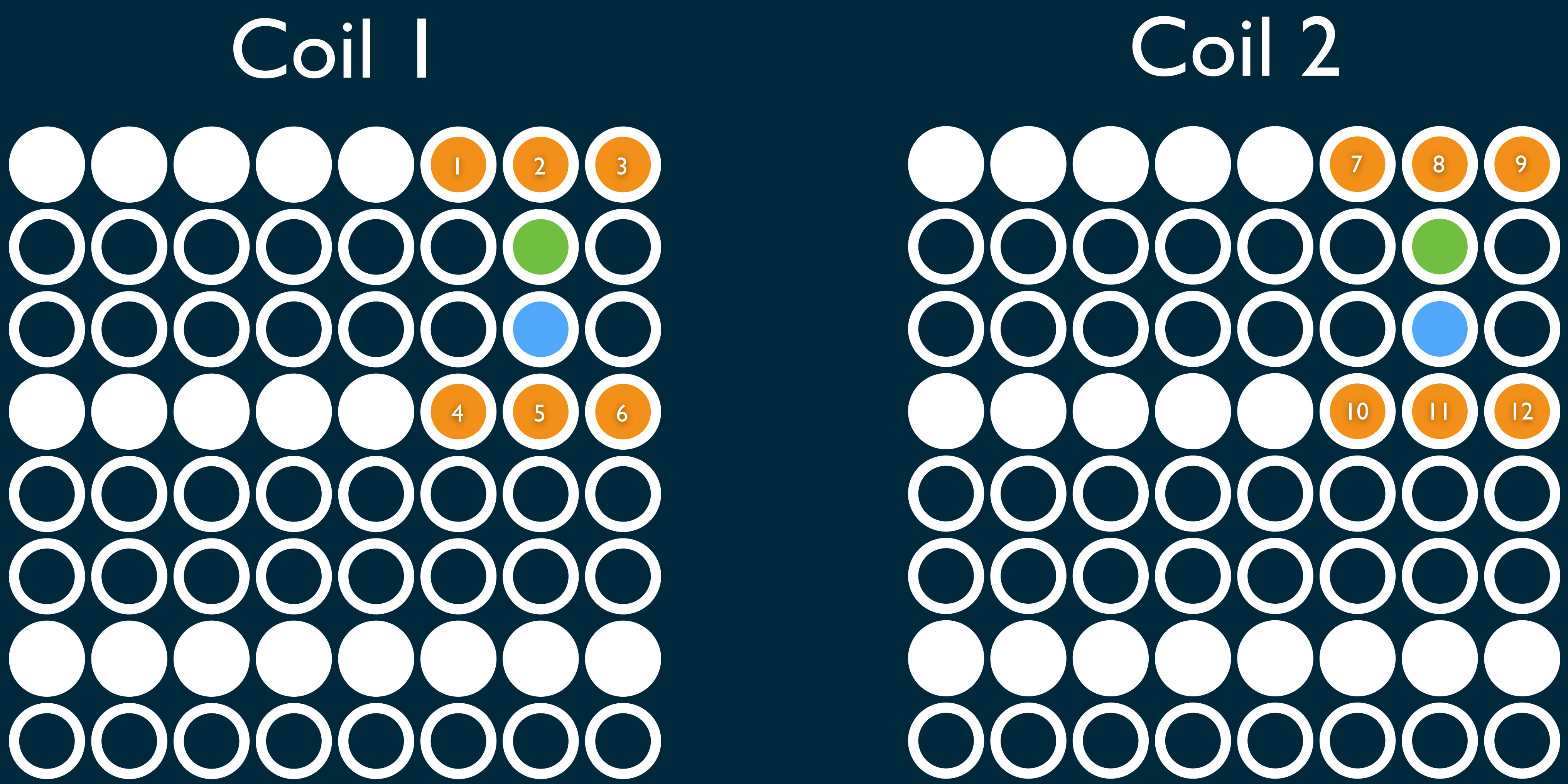


You solve for all the missing points by applying the same relative kernel weights

GRAPPA Geometry



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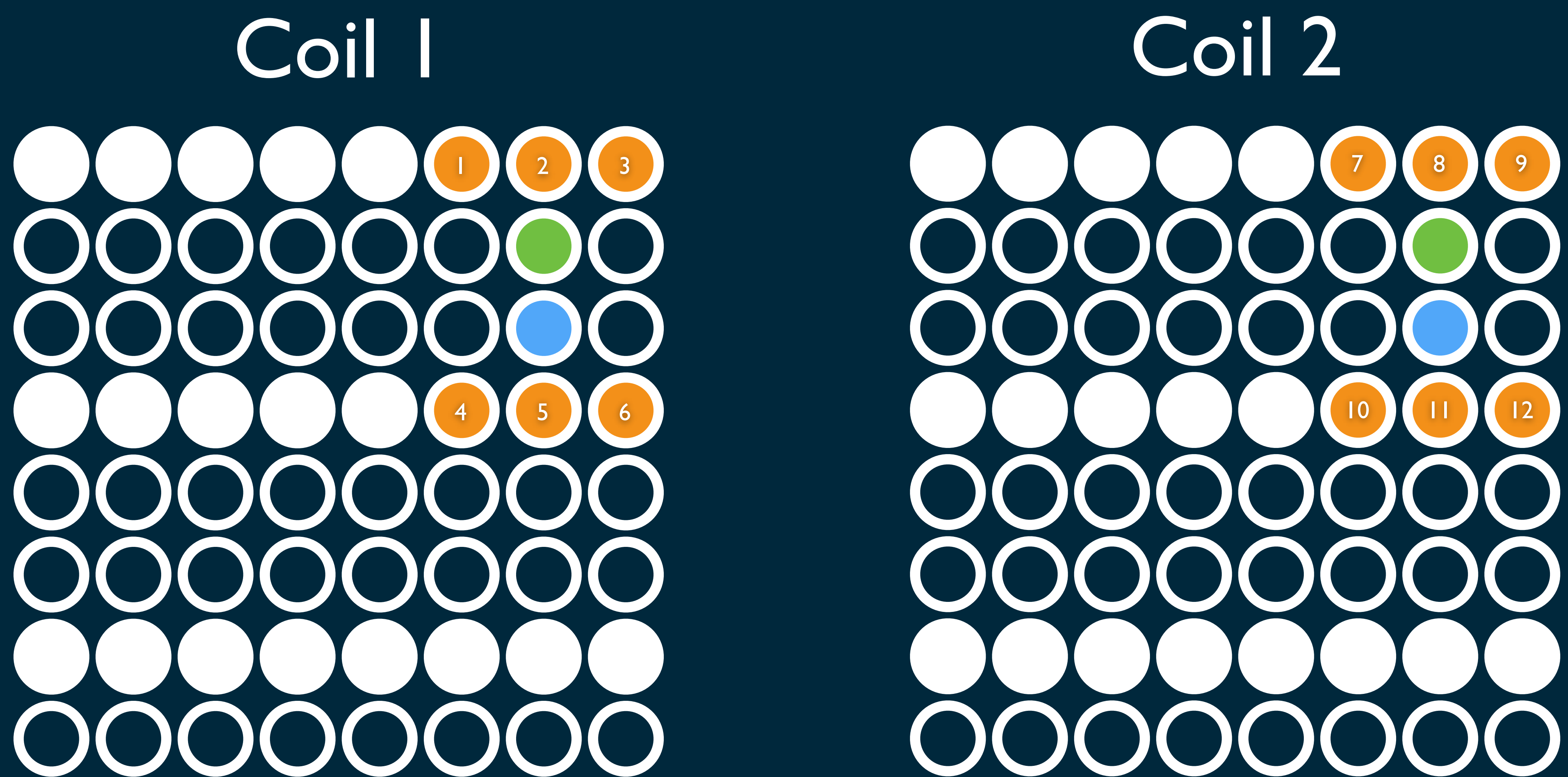

They way you find the weights, is that you solve the exact same problem

GRAPPA Geometry

$$
\begin{array}{ccccc}
\text{target} & & \text{weights} & & \text{source} \\
\begin{bmatrix} X \\ Y \\ U \\ V \end{bmatrix} & = &
\begin{bmatrix}
w^{(1)}_{1,1} & w^{(1)}_{1,2} & \cdots & w^{(1)}_{1,12} \\
w^{(1)}_{2,1} & w^{(1)}_{2,2} & \cdots & w^{(1)}_{2,12} \\
w^{(2)}_{1,1} & w^{(2)}_{1,2} & \cdots & w^{(2)}_{1,12} \\
w^{(2)}_{2,1} & w^{(2)}_{2,2} & \cdots & w^{(2)}_{2,12}
\end{bmatrix} & &
\begin{bmatrix} 1 \\ 2 \\ \vdots \\ 12 \end{bmatrix}
\end{array}
$$

They way you find the weights, is that you solve the exact same problem

But now, instead of the GRAPPA synthesis problem where x, y, u, v are unknown

GRAPPA Geometry

$$\begin{array}{cccc} \text{target} & & \text{weights} & \text{source} \\ \begin{bmatrix} X \\ Y \\ U \\ V \end{bmatrix} & = & \begin{bmatrix} w^{(1)}_{1,1} & w^{(1)}_{1,2} & \cdots & w^{(1)}_{1,12} \\ w^{(1)}_{2,1} & w^{(1)}_{2,2} & \cdots & w^{(1)}_{2,12} \\ w^{(2)}_{1,1} & w^{(2)}_{1,2} & \cdots & w^{(2)}_{1,12} \\ w^{(2)}_{2,1} & w^{(2)}_{2,2} & \cdots & w^{(2)}_{2,12} \end{bmatrix} & \begin{bmatrix} 1 \\ 2 \\ \vdots \\ 12 \end{bmatrix} \end{array}$$

You require some fully sampled calibration data

Where x, y, u, v are known, and you solve for w
The shift-invariance of the kernel weights
allows you to fit this over a large set of different
source and target points

GRAPPA Practical

- `https://psg.fmrib.ox.ac.uk/mchiew/grappa-practical/`
- `git clone https://psg.fmrib.ox.ac.uk/mchiew/grappa-practical.git`
- `https://psg.fmrib.ox.ac.uk/mchiew/grappa-practical/repository/archive.zip`

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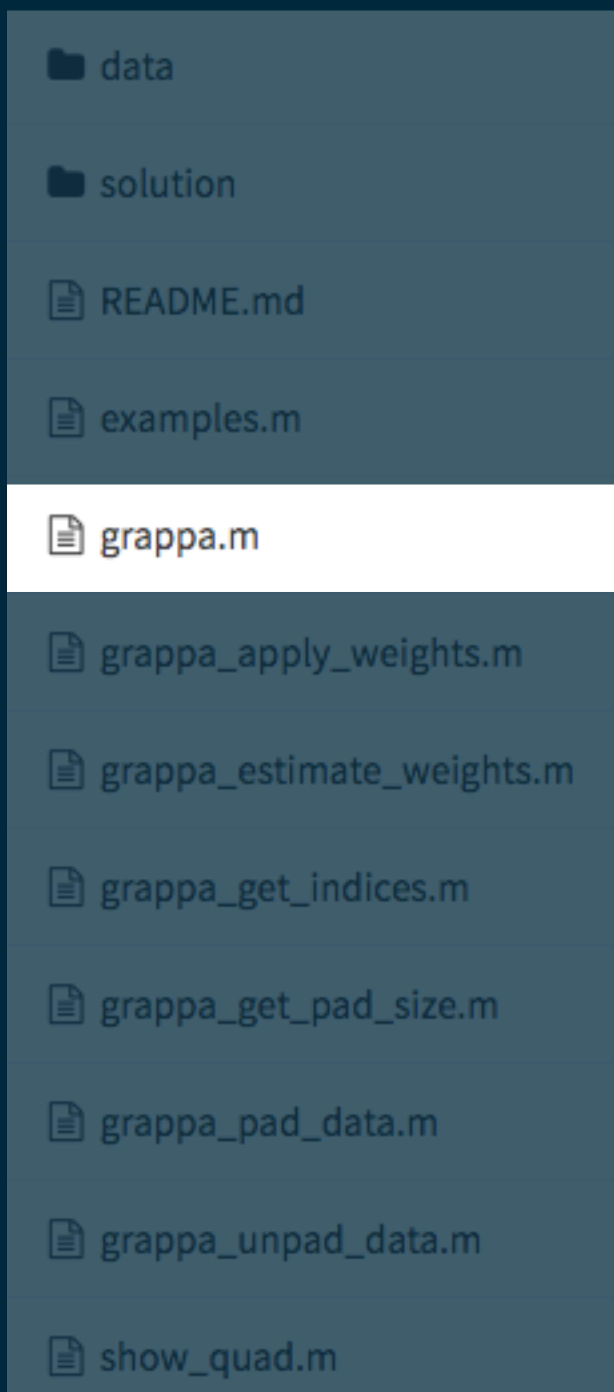

- grappa.m is the main GRAPPA function
- It calls 6 different helper functions

GRAPPA Practical

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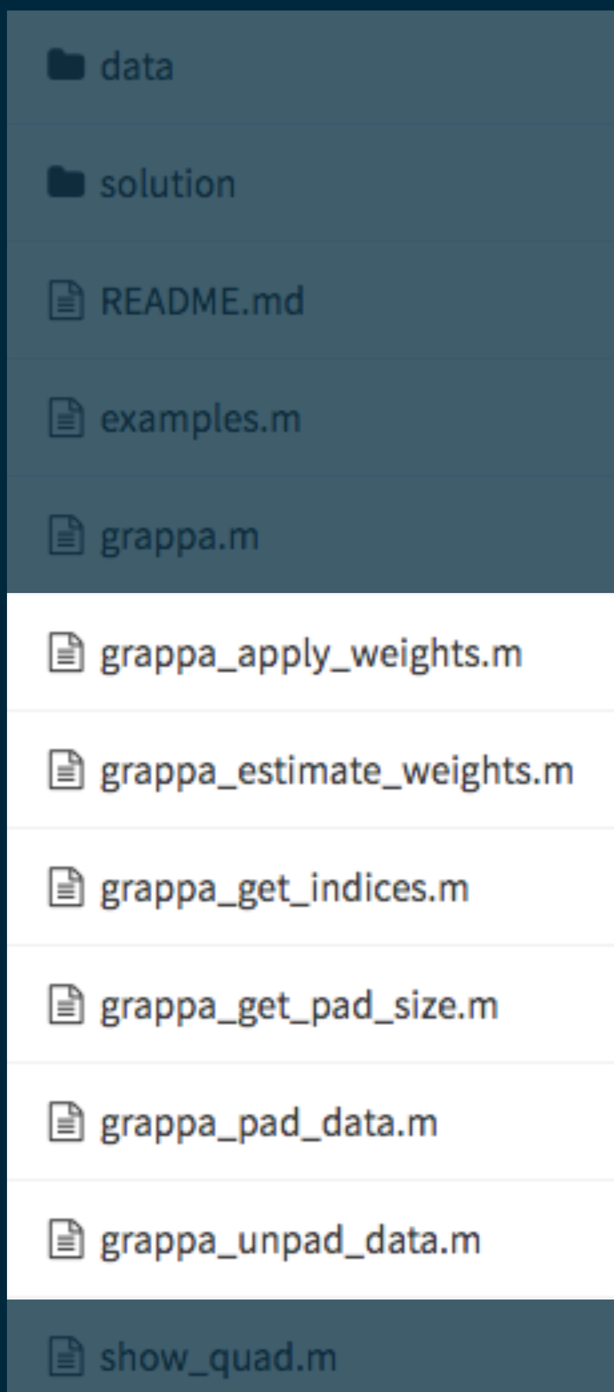

- These functions contain all the GRAPPA logic
- grappa_{apply_weights.m, estimate_weights.m} are the actual interesting bits
- The rest are housekeeping

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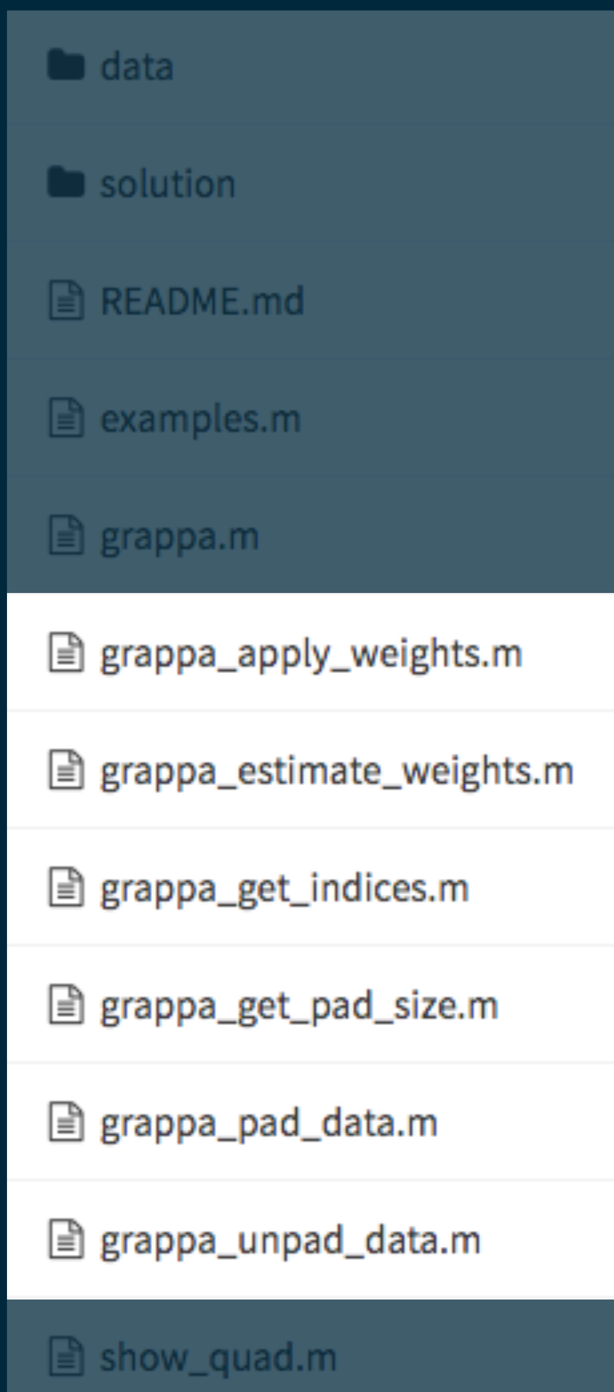

- Your job is to implement these internal bits
- The README has more detailed instructions on each component
- The files have function definitions and comments that should help you

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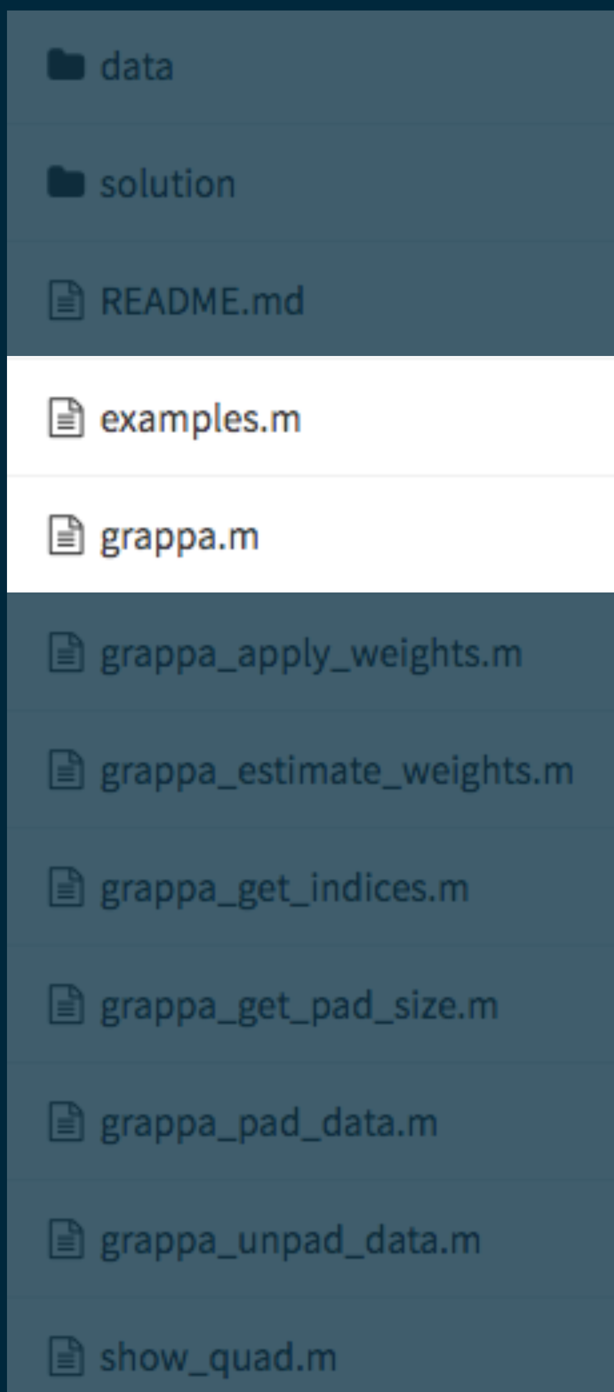

- Your goal is to be able to run grappa.m
- You can test this by running the examples.m script, which evaluates a few test cases

GRAPPA Practical

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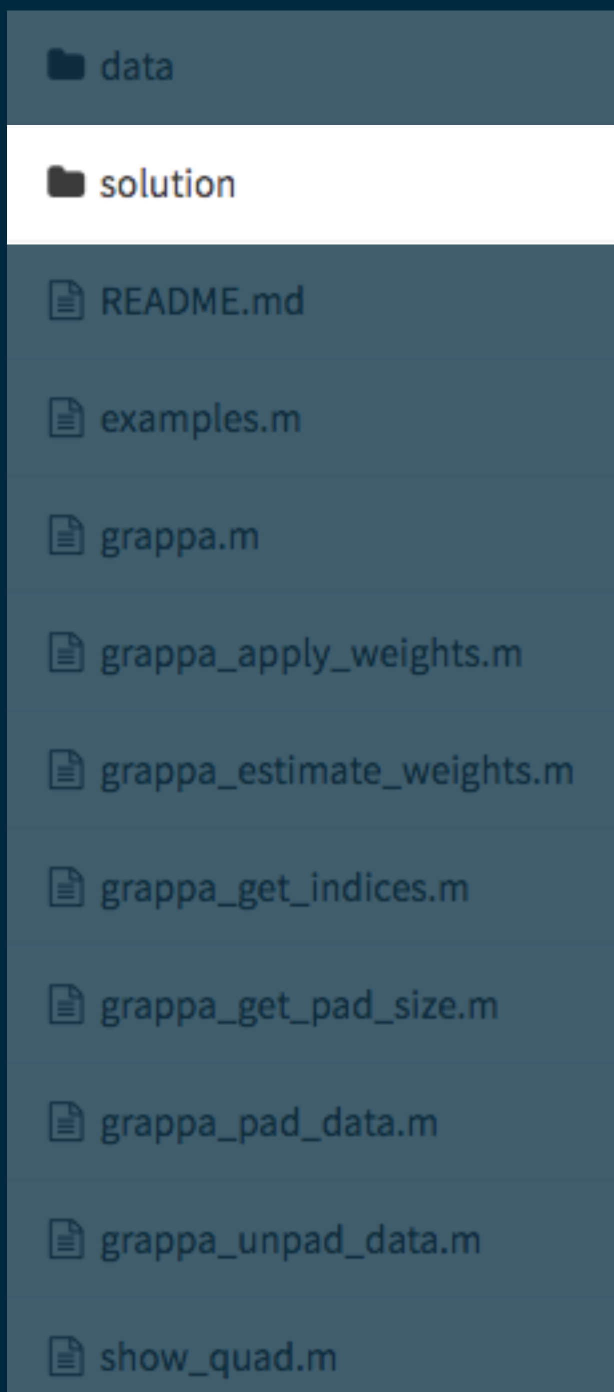

- A full working implementation for all the files can be found in the solution folder
- If you want to focus on the conceptually interesting aspects of GRAPPA, work on:
  - grappa_estimate_weights.m
  - grappa_apply_weights.m
- Copy the remaining files from solution