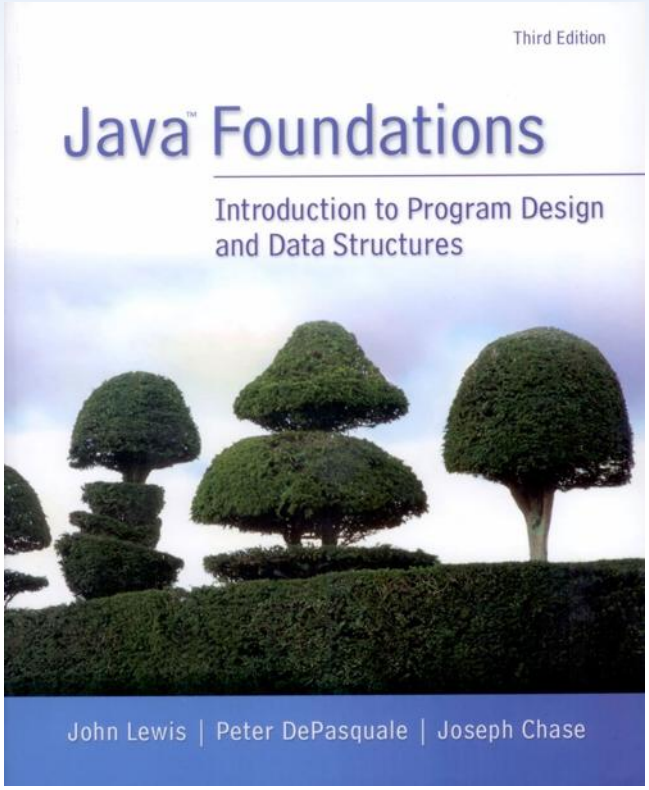


Chapter 23

Multi-Way Search Trees

Chapter Scope

- Examine 2-3 and 2-4 trees
- Introduce the concept of a B-tree
- Example specialized implementations of B-trees

Combining Tree Concepts

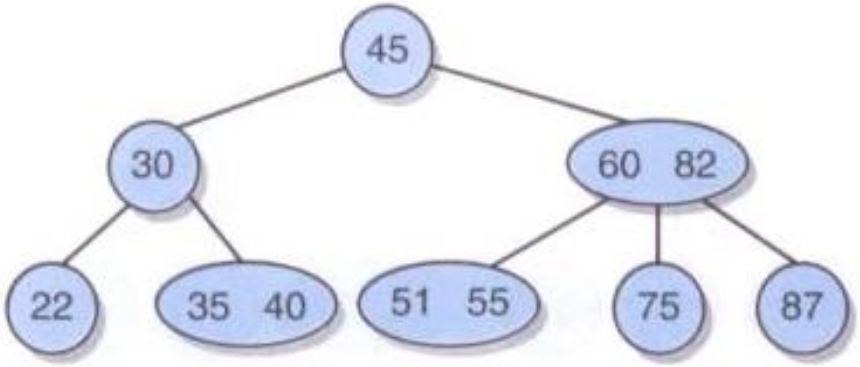
- We've seen:
  - general trees, with multiple children per node
  - binary search trees, with a relationship among the elements but only two children per node
- A *multi-way search tree* combines these elements
- Each node might have more than two children with a specific relationship among the elements

2-3 Trees

- In a 2-3 tree, each node has two or three children
- A 2-node contains one element, and a 3-node contains two elements
- A 2-node can has either two children or no children
- A 3-node has either three children or no children

2-3 Trees

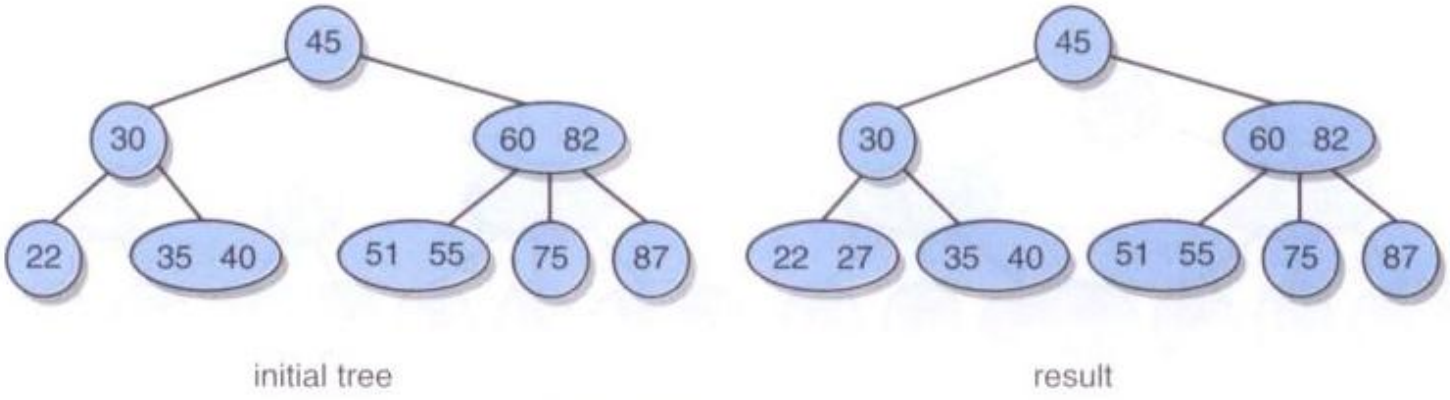
- A 2-3 tree:



- The relationship among the parents and children reflect the contents

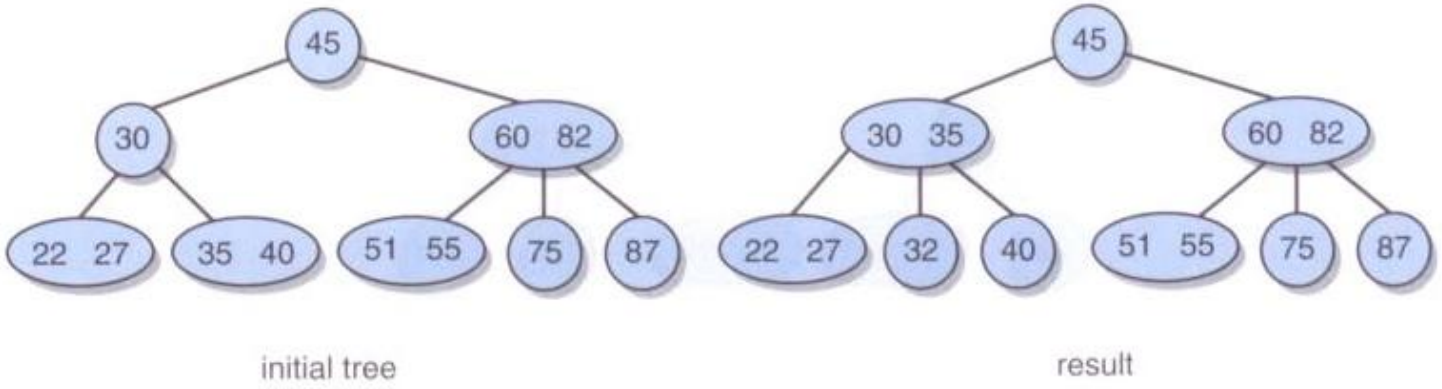
2-3 Trees

- Inserting an element may simply add an element to a leaf node
- Adding 27:



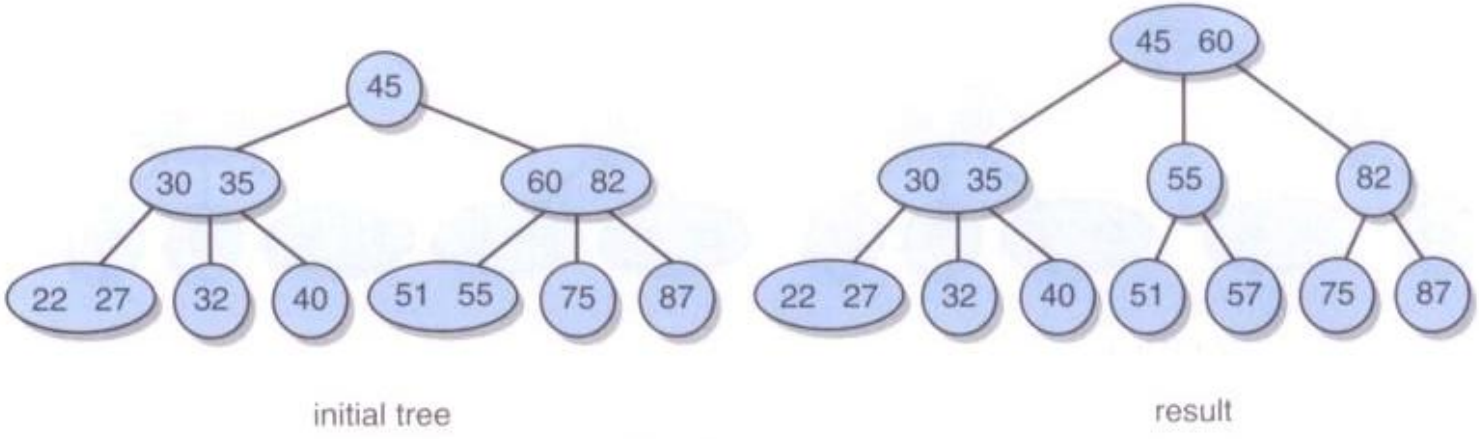
2-3 Trees

- Inserting an element may split a 3-node and move an element up
- Adding 32:



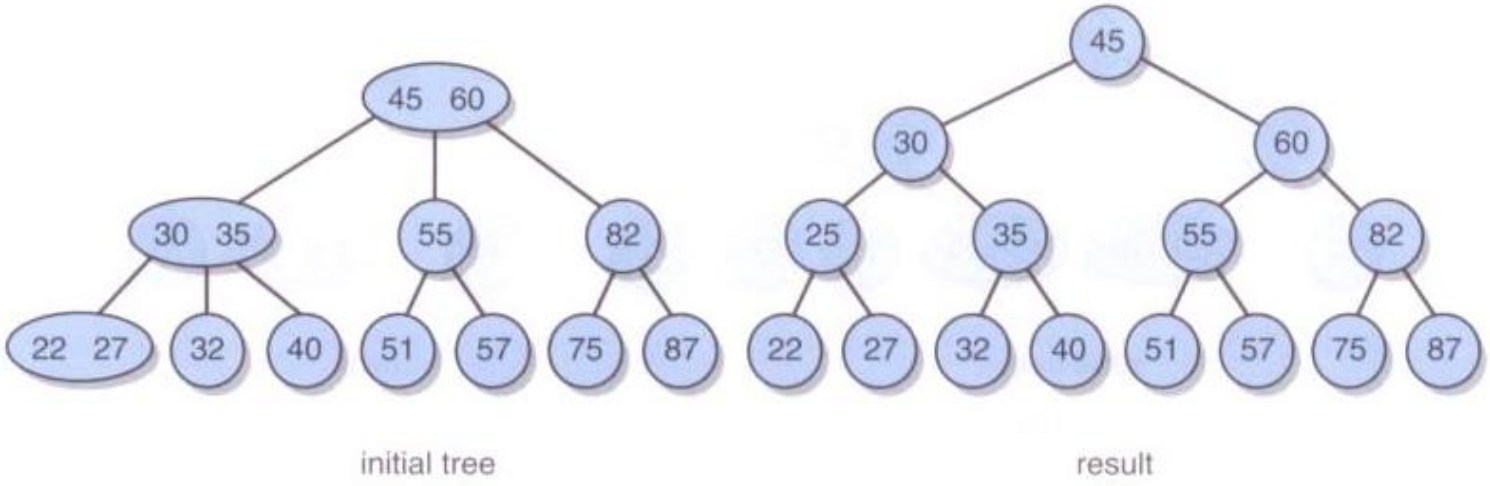
2-3 Tree

- Splitting a 3-node whose parent is already a 3-node causes ripple effects
- Adding 57:



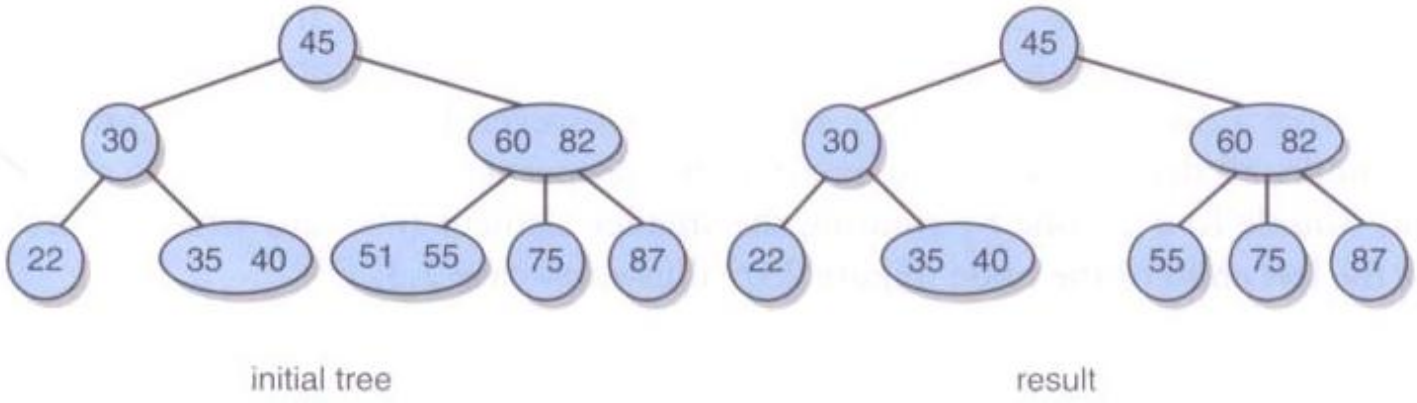
2-3 Trees

- If a ripple effect propagates all the way to the root, a new 2-node is created
- Adding 25:



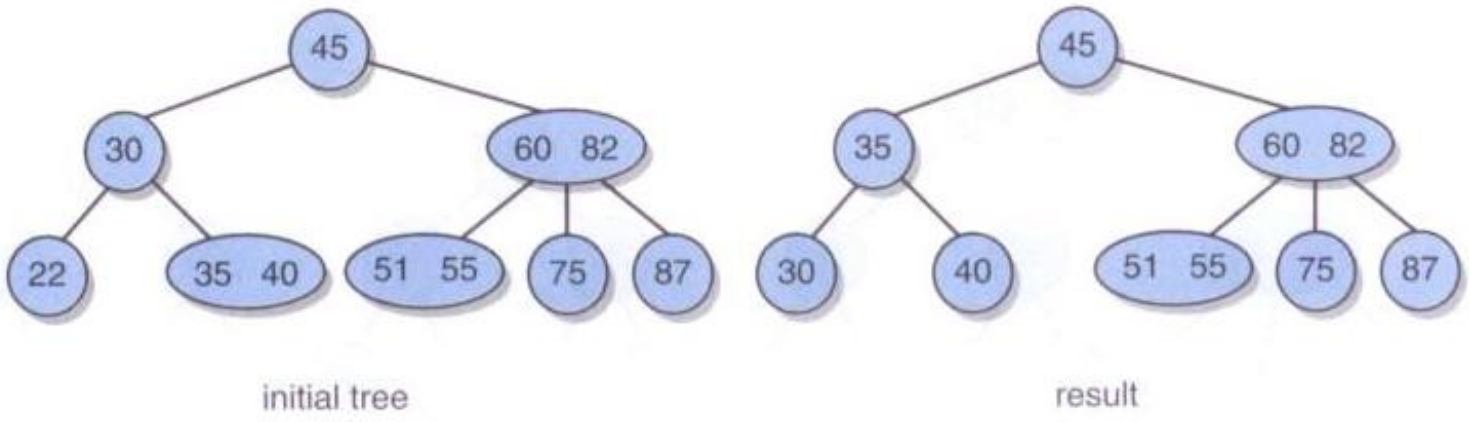
2-3 Trees

- Removing an element may convert a 3-node into a 2-node
- Removing 51:



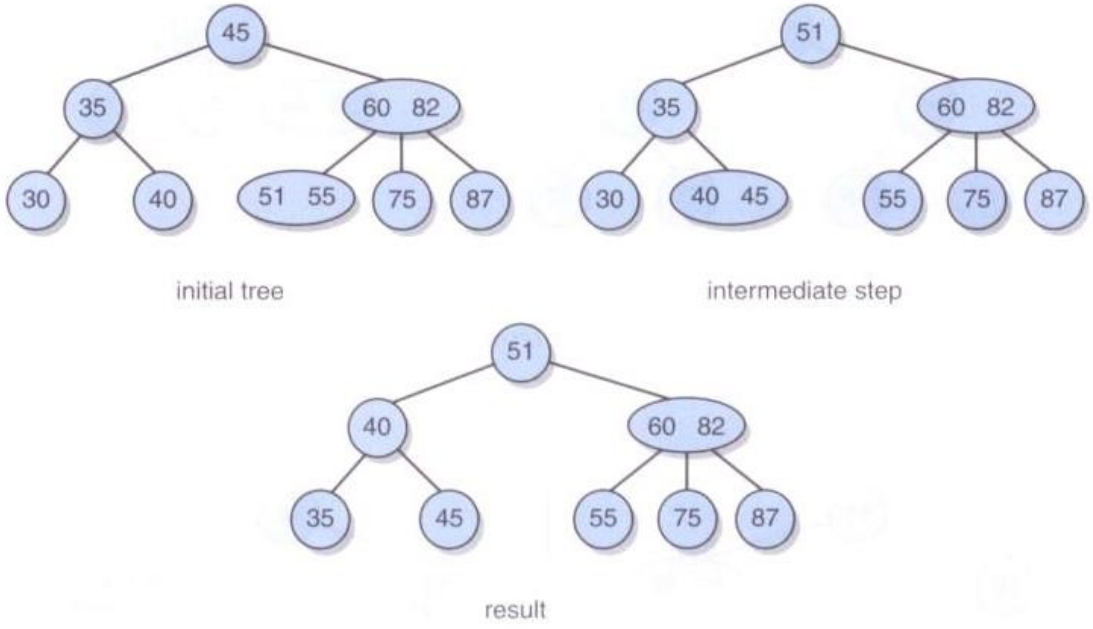
2-3 Trees

- Removing a 2-node leaf causes an *underflow* and requires a rotation
- Removing 22:



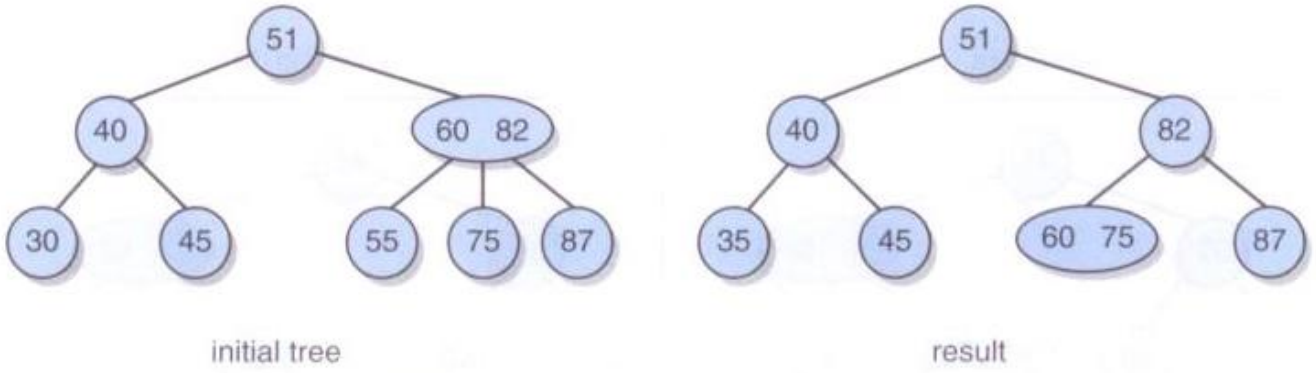
2-3 Trees

- Underflows may require multiple rotations
- Removing 30:



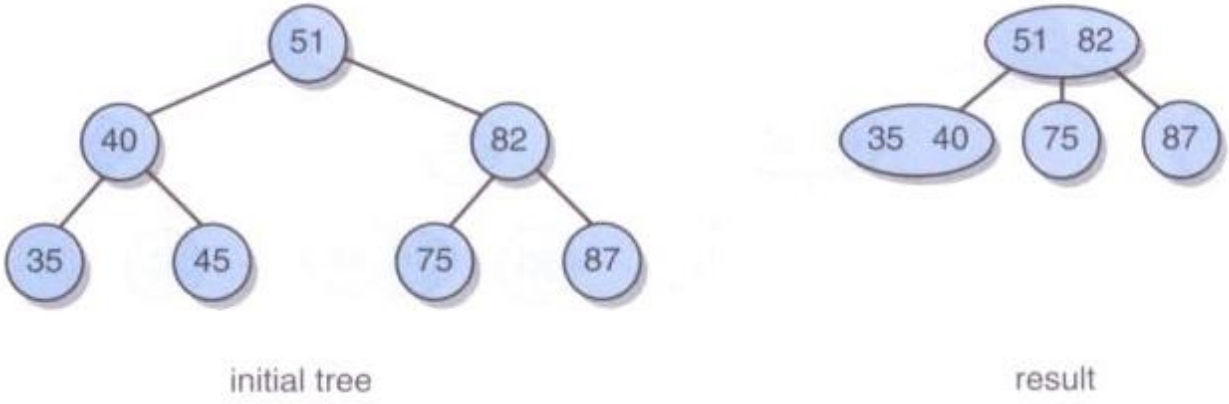
2-3 Trees

- Removing 55:



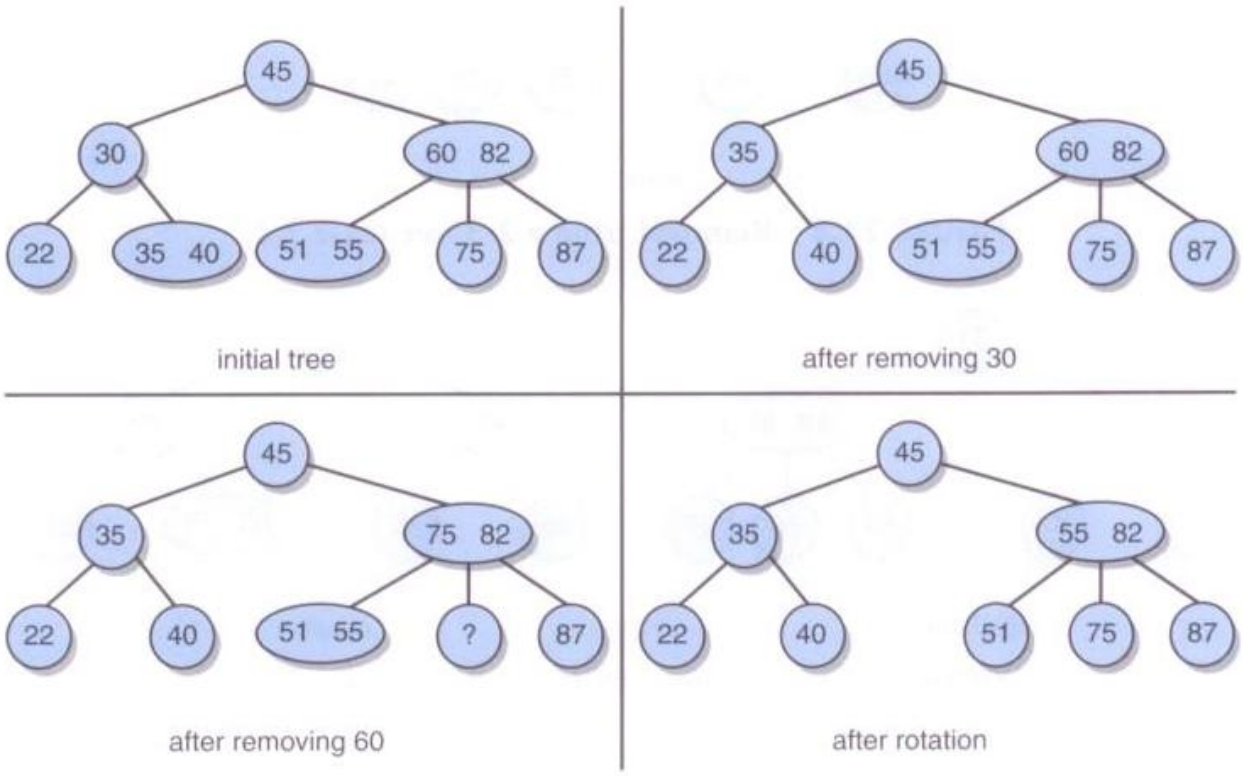
2-3 Trees

- Rotations may not solve everything – the height of the tree may have to be reduced
- Removing 45:



2-3 Trees

- Removing internal nodes 30, then 60:

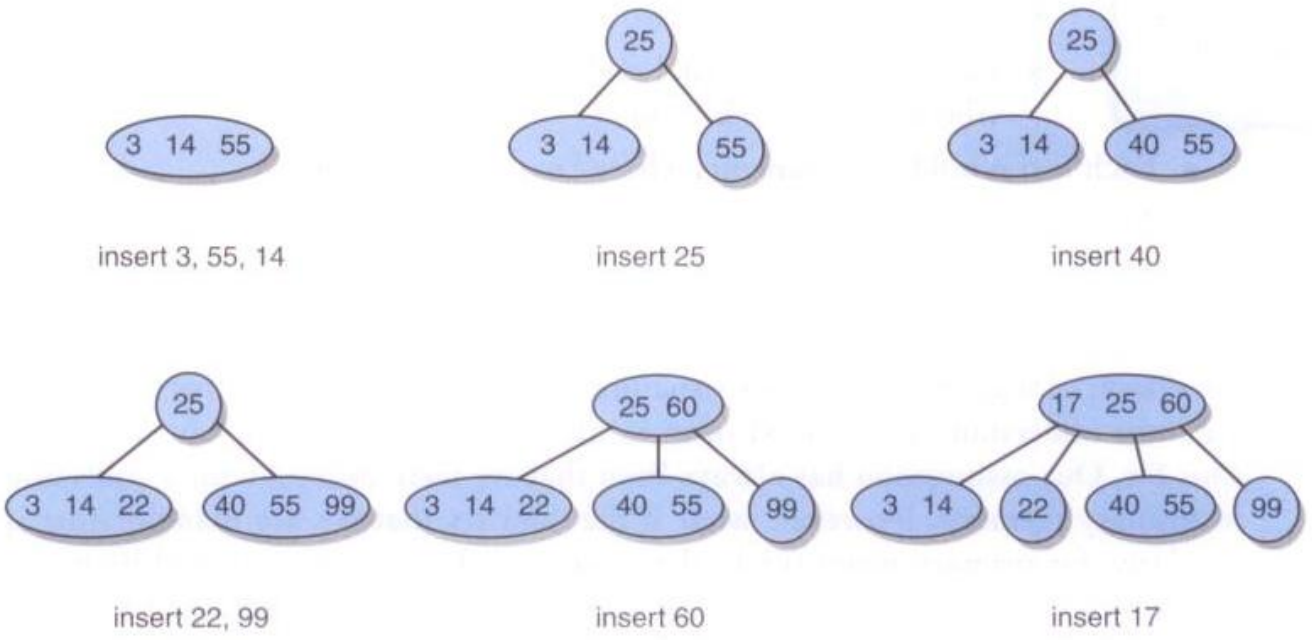


2-4 Trees

- A *2-4 tree* is similar to a 2-3 tree, with nodes that can contain three elements
- A 4-node has either four children or no children
- The same ordering rules apply
- Similar cases govern insertions and removals

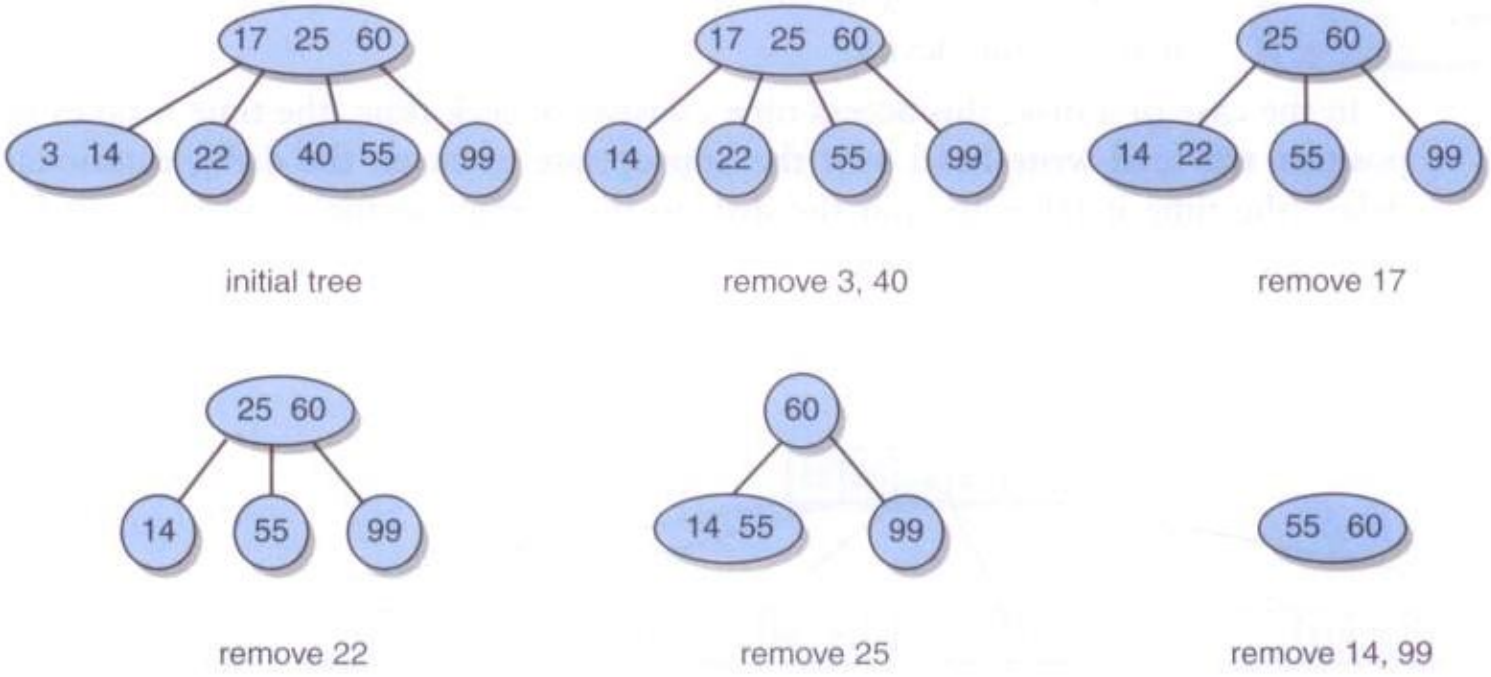
2-4 Trees

- Insertions into a 2-4 tree:



2-4 Trees

- Removals from a 2-4 tree:

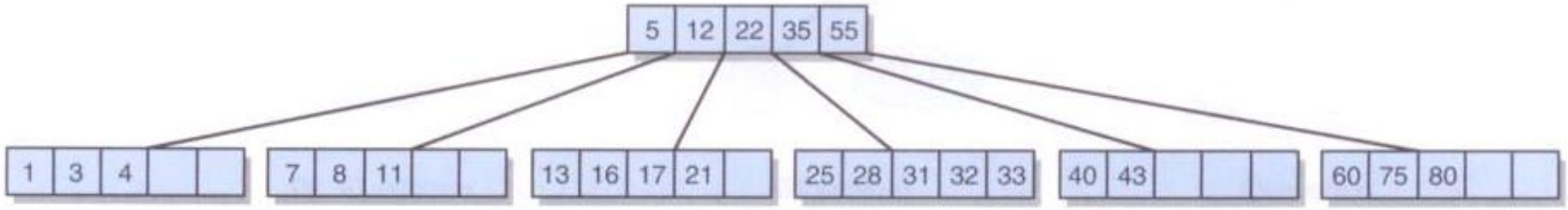


B-trees

- 2-3 and 2-4 trees are examples of a larger class of multi-way search trees called *B-trees*
- The maximum number of children of each node is called the order of the B-tree
- Thus, a 2-3 tree is a B-tree of order 3
- A 2-4 tree is a B-tree of order 4

B-Trees

- A B-tree of order 6:



Variations on B-Trees

- A *B***-tree* is a B-tree that guarantees that each non-root node is at least two-thirds full
- This avoids the problem of the B-tree being half empty
- In a *B*$^{+}$*-tree*, each element appears in a leaf, even if it also appears in an internal node
- This improves the sequential access to all elements of the tree

Variations on B-trees

- A B^+-tree of order 6: