A simplified energy-balance model for ice melt below debris

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This model simulates ice melt below debris at the surfaces of debris-covered glaciers using a simplified energy-balance approach.

It solves heat conduction through a debris layer as such:

$$\rho_d c_d \frac{\partial T_d}{\partial t} = \frac{\partial}{\partial z}\left(k_d \frac{\partial T_d}{\partial z}\right) \tag{1}$$

where ρ_d is debris density, c_d is debris specific heat capacity, k_d is debris thermal conductivity, t is time, z is depth within the debris, and T_d is debris temperature.

The boundary condition at the debris surface is a simplified debris-surface energy balance:

$$S\downarrow(t) - S\uparrow(t) + c_1[T_a(t) - T_d(0,t)] + c_2 - k_d \frac{\partial T_d(0,t)}{\partial z} = 0 \tag{2}$$

where $S\downarrow$ is incoming shortwave radiation, $S\uparrow$ is outgoing shortwave radiation, T_a is air temperature, and c_1 and c_2 are free parameters to be calibrated. This equation has a similar form to the simplified debris-free energy balance of Oerlemans (2001). Surface temperature is determined iteratively for each model timestep using Newton's method, following Reid and Brock (2010).

The boundary condition at the ice surface is the temperature of melting ice, T_i (273.15 K):

$$T_d(h,t) = T_i \tag{3}$$

where h is debris thickness.

Melt rate, M, is computed as follows:

$$M(t) = \frac{\Delta t}{\rho_i L_f} k_d \frac{\partial T_d(h,t)}{\partial z} \tag{4}$$

where Δt is the model timestep, ρ_i is ice density, and L_f is the latent heat of fusion of water.

References

Oerlemans, J. (2001). Glaciers and climate change. CRC Press.

Reid, T. D., & Brock, B. W. (2010). An energy-balance model for debris-covered glaciers including heat conduction through the debris layer. Journal of Glaciology, 56(199), 903-916.