Advanced camera calibration methods

Fundamental Matrix estimation
8-point algorithm

$\mathbf{x} = (u, v, 1)^T, \quad \mathbf{x}' = (u', v', 1)^T$

$$(u, v, 1)\begin{pmatrix} F_{11} & F_{12} & F_{13} \\ F_{21} & F_{22} & F_{23} \\ F_{31} & F_{32} & F_{33} \end{pmatrix}\begin{pmatrix} u' \\ v' \\ 1 \end{pmatrix} = 0$$

$$(uu', uv', u, vu', vv', v, u', v', 1)\begin{pmatrix} F_{11} \\ F_{12} \\ F_{13} \\ F_{21} \\ F_{22} \\ F_{23} \\ F_{31} \\ F_{32} \\ F_{33} \end{pmatrix} = 0$$

$$\begin{pmatrix} u_1u'_1 & u_1v'_1 & u_1 & v_1u'_1 & v_1v'_1 & v_1 & u'_1 & v'_1 \\ u_2u'_2 & u_2v'_2 & u_2 & v_2u'_2 & v_2v'_2 & v_2 & u'_2 & v'_2 \\ u_3u'_3 & u_3v'_3 & u_3 & v_3u'_3 & v_3v'_3 & v_3 & u'_3 & v'_3 \\ u_4u'_4 & u_4v'_4 & u_4 & v_4u'_4 & v_4v'_4 & v_4 & u'_4 & v'_4 \\ u_5u'_5 & u_5v'_5 & u_5 & v_5u'_5 & v_5v'_5 & v_5 & u'_5 & v'_5 \\ u_6u'_6 & u_6v'_6 & u_6 & v_6u'_6 & v_6v'_6 & v_6 & u'_6 & v'_6 \\ u_7u'_7 & u_7v'_7 & u_7 & v_7u'_7 & v_7v'_7 & v_7 & u'_7 & v'_7 \\ u_8u'_8 & u_8v'_8 & u_8 & v_8u'_8 & v_8v'_8 & v_8 & u'_8 & v'_8 \end{pmatrix}\begin{pmatrix} F_{11} \\ F_{12} \\ F_{13} \\ F_{21} \\ F_{22} \\ F_{23} \\ F_{31} \\ F_{32} \end{pmatrix} = -\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

merit function to minimize

Minimize:

$$\sum_{i=1}^{N} (x_i^T F x'_i)^2$$

under the constraint

$$|F|^2 = 1$$

Fundamental Matrix estimation
8-point algorithm

$\sum_{i=1}^{N}(x_i^T F x_i')^2$ is the squared-sum of the distances between points x_i and relative epipolar lines $\mathbf{F}x_i'$ multiplied by a scale factor

✓ Poor numerical conditioning (example):

250906.36	183269.57	921.81	200931.10	146766.13	738.21	272.19	198.81
2692.28	131633.03	176.27	6196.73	302975.59	405.71	15.27	746.79
416374.23	871684.30	935.47	408110.89	854384.92	916.90	445.10	931.81
191183.60	171759.40	410.27	416435.62	374125.90	893.65	465.99	418.65
48988.86	30401.76	57.89	298604.57	185309.58	352.87	846.22	525.15
164786.04	546559.67	813.17	1998.37	6628.15	9.86	202.65	672.14
116407.01	2727.75	138.89	169941.27	3982.21	202.77	838.12	19.64
135384.58	75411.13	198.72	411350.03	229127.78	603.79	681.28	379.48

$$\begin{pmatrix} F_{11} \\ F_{12} \\ F_{13} \\ F_{21} \\ F_{22} \\ F_{23} \\ F_{31} \\ F_{32} \end{pmatrix} = - \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

requires data recentering and rescaling with T and T′ normalising transformations and estimation of $T^T F T'$

Focal length estimation from F

- Pre-requisite:
  - no skew
  - principal point known and centered in image plane

$$\mathbf{K} = \begin{bmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{bmatrix} = diag(f, f, 1)$$

$$\mathbf{K}' = diag(f', f', 1)$$

- By rotating the image reference systems such as epipoles lie on positive image x-axes thus assuming co-ordinates e=[e_1,0,e_3] and e'=[e_1',0,e_3'] (absolute conic geometric interpretation), matrix F simplifies in the form:

thus we can write the foundamental matrix in this way

$$F \sim \begin{bmatrix} e'_3 & & \\ & 1 & \\ & & -e'_1 \end{bmatrix} \begin{bmatrix} a & b & a \\ c & d & c \\ a & b & a \end{bmatrix} \begin{bmatrix} e_3 & & \\ & 1 & \\ & & -e_1 \end{bmatrix}$$

these are.. [the z coordinate of the epipole of camera2]

real numbers

with *a*, *b*, *c*, *d*, real numbers. With this representation direct formulae can be computed for the two focal lengths :

$$f^2 = \frac{-ace_1^2}{ace_3^2 + bd}, \quad f'^2 = \frac{-abe'^2_1}{abe'^2_3 + cd}$$

I can obtain numerical equations with real numbers. And I can obtain focal length. It's squared so I dont know if it's positive or negative (1st choice I'll have to make.

Extraction of external parameters from E

- Once K and K’ are estimated from F, SVD is applied on E to extract T and R up to the sign and a scale factor. It holds:

tranlsation vector, baseline, positive or negative

essential matrix can be written as compositition of these

- $E=[UWV^T]$
- $T=-V^3$ (translation vector is parallel to the eigenvector associated to the smallest eigenvalue of E)
- $T=V^3$ is also possible
- $R=UZV^T$ or $R=UZ^TV^T$ (180° rotation difference) with $Z=\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

estimation of rotation matrix, wich can be direct or inverse, 180 shifts depending on the solution. I must choose the right combination. With right rotation and right trans. I have the poitns in front of the matrix and not behind! Numerically indeed I can have different solutions, and I must choose the right one geometrically!

- Among the four possible combinations, the correct T and R are the ones which ensure the 3D control points are in front of the TVcameras

Human motion Virtualization

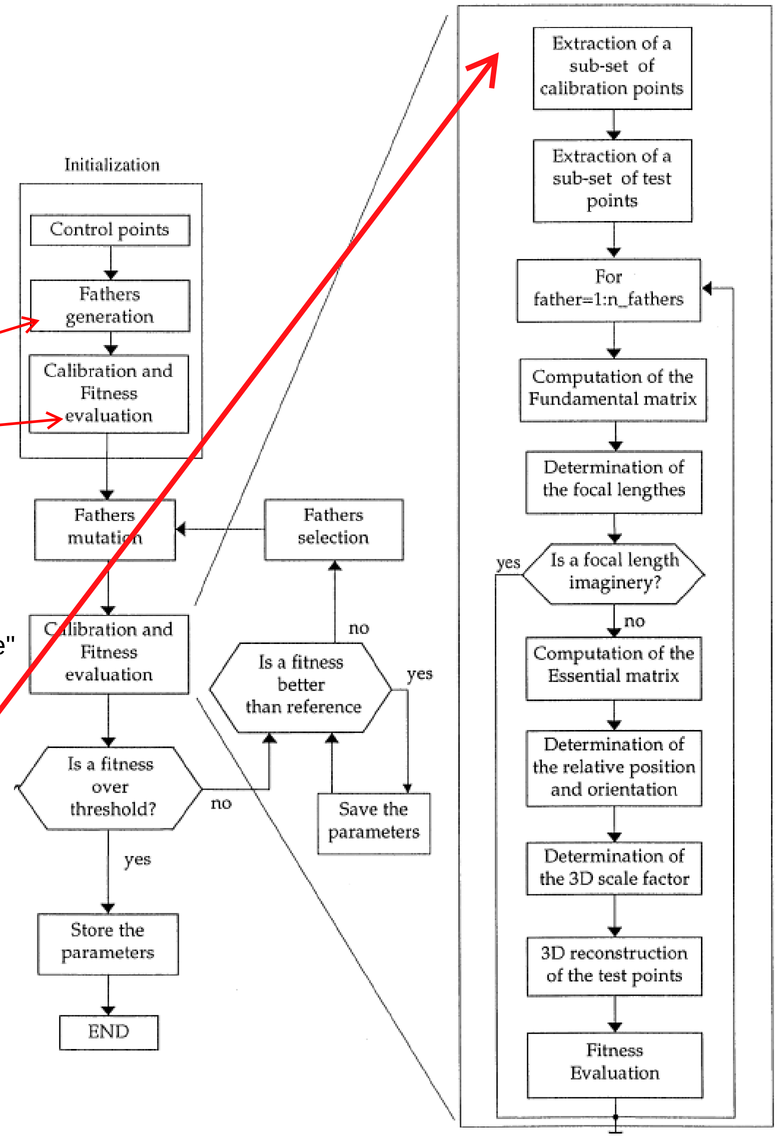
Self calibration algorithm example

- ✓ Large scale optimization (genetic algorithm) is applied for principal point estimation

  (from Borghese and Cerveri, 2000)

initialization step: all the points are acquired. Let's say I know pp, I can calibrate.

how good the system is performing?



what to remember:
rational of epipolar, constrain of it, "why should I buy an epipolar geom software"
how can numerical instability can be resolved, how works the total algrm for calibration,
which are the characterists of this algrm, what about the principal point stuff?

how is it possible to derive the fondamental equation of epipolar geom from the planar constrain (At least an idea)
btw matlab has a toolbox.

how fitness is evaluated, that's why we need many points: a subset for calibrating and a subset for testing