Timing techniques

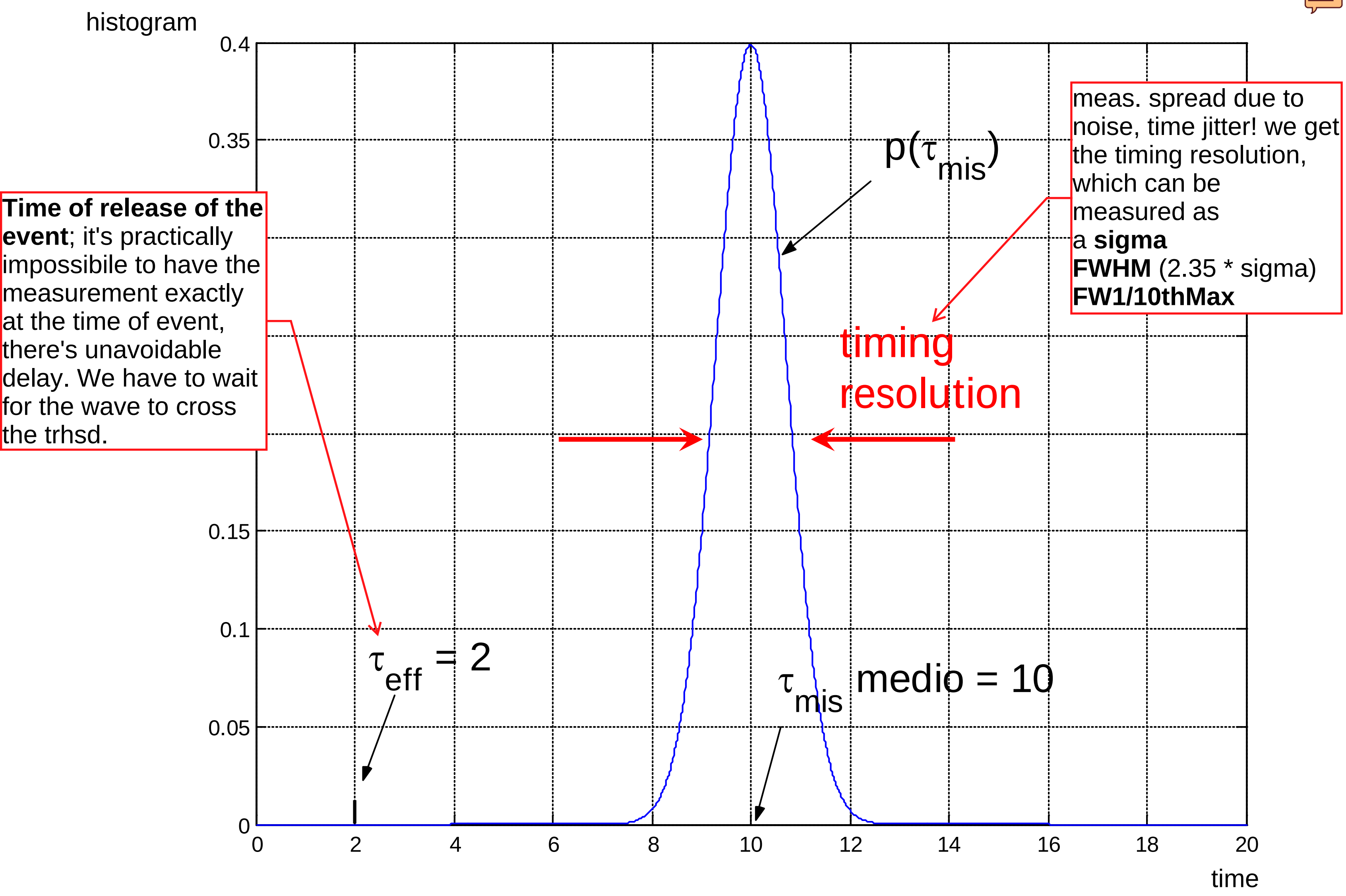
goal: to measure the time when a given event occurs (ex. detection of a γ in PET)

A timing system has to provide a measurement τ_{meas} of this time which has to differ as less as possible from the time τ_{eff} in which the event actually occurs. Generally, τ_{meas} is a statistical variable characterized by an average value $\overline{\tau_{meas}}$ and by a variance σ_τ^2.

$\overline{\tau_{meas}}$ is always different from τ_{eff} because no measuring instrument reacts instantaneously to the event to be detected and therefore the output signal is delayed with respect to τ_{eff}. This type of error is systematic and therefore can be corrected.

important in PET: we need to get a timestamp for the arrival. in conventional PET is used to **associate the 2 det. involved** in the detection. Also if timing is very good we can do **TOF PET**.

HISTOGRAM OF THE MEASURED TIME

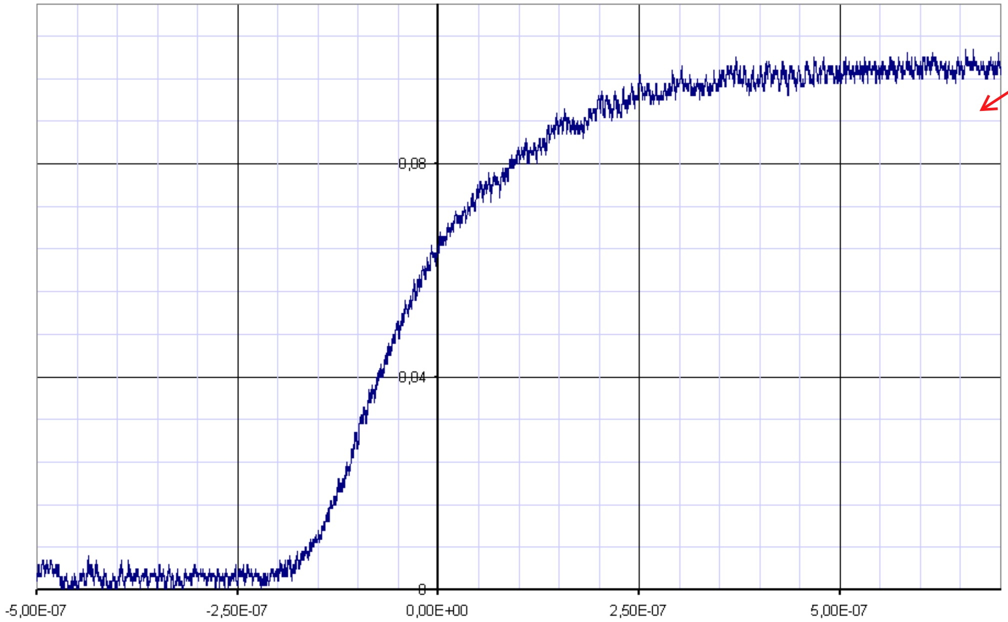


Timing techniques

- Leading edge triggering crossing of a threshold, dopo filter
- Crossover timing
- Constant Fraction timing

tagliato prima del decay
R? sì, è solo uno zoom,
la R c'è cmq, ma
tempistiche viene molto
dopo, il timing si gioca
subito

always this waveform.. which is NOT an heavyside step!!



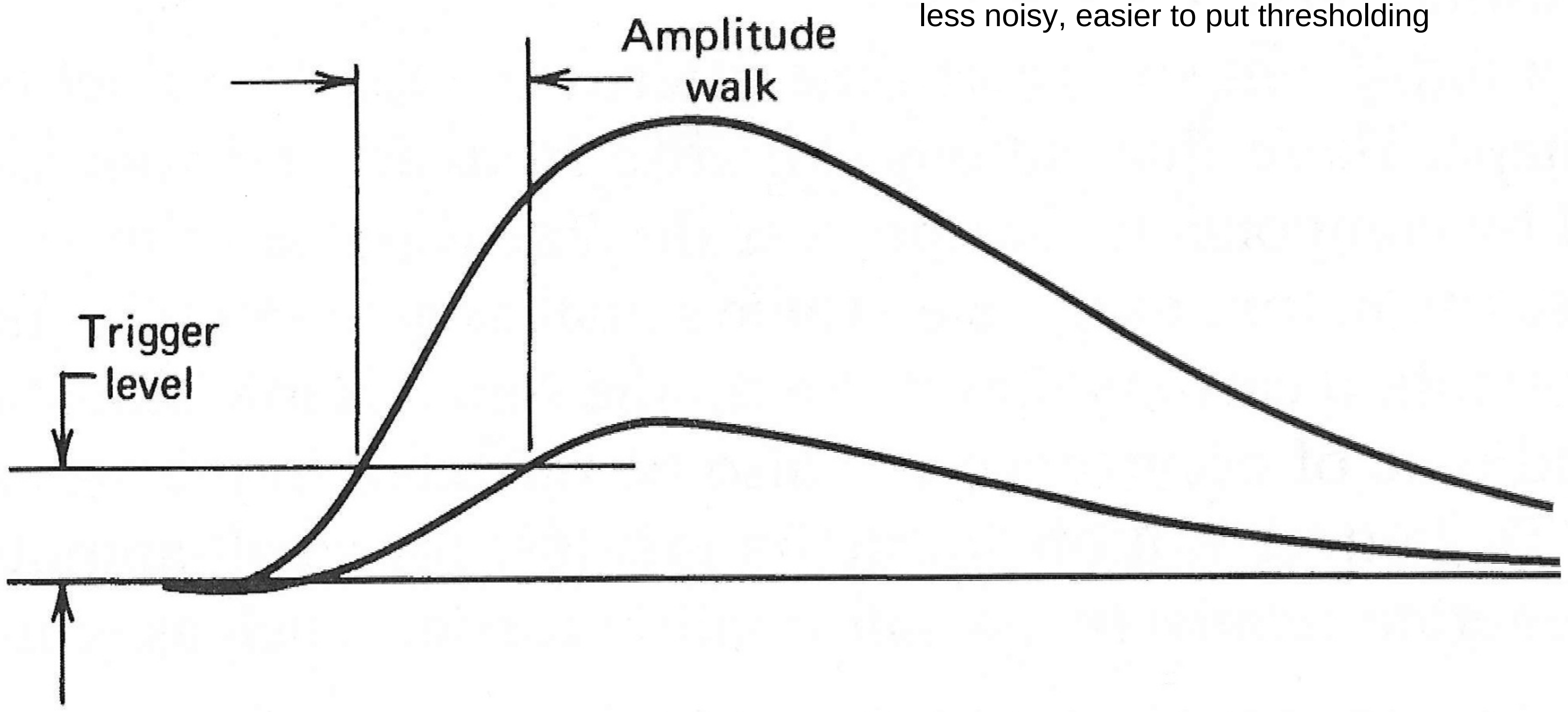
pulse at the output of the preamplifier common to the 3 techniques considered here)

For all technique will be common the **PREAMPLIFIER (the output signal of the charge preamp).** in principle we could put a threshold directly on the output of preamp. We don't have an haevyside! but a risetime. this is due to limit bw of preamplifier, but ALSO TO THE TIME CONSTANT OF SCINTILLATOR.

Leading Edge Triggering

goal: to detect the time in which the pulse cross a given threshold

FILTERED PULSE, NOT PREAMPLIFIER ONE! b/c it's less noisy, easier to put thresholding



limitation: *amplitude walk*, variation of the *timing* signal with the signal amplitude

We have a pulse, and look with a comporator if it overocomes a trigger level. PROBLEM: amplitude walk: if the detector is supplying pulses with different amplitude, we'll have trigger at different moments! shift just due to the fact that the amplitude is different! (*l'evento arriva sempre allo stesso momento). we introduce a delay which is signal dependant :(bad. next: quantify the problem

quantify amplitude walk, supposing a LINEAR pulse, not exp.

The effect is smaller larger is the slope of the rising section of the signal

Supposing a linear rise of the signal, if its amplitude changes around an average value $\bar{y}$, the variation of τ_{meas} as a function of variations of the amplitude δy are given by:

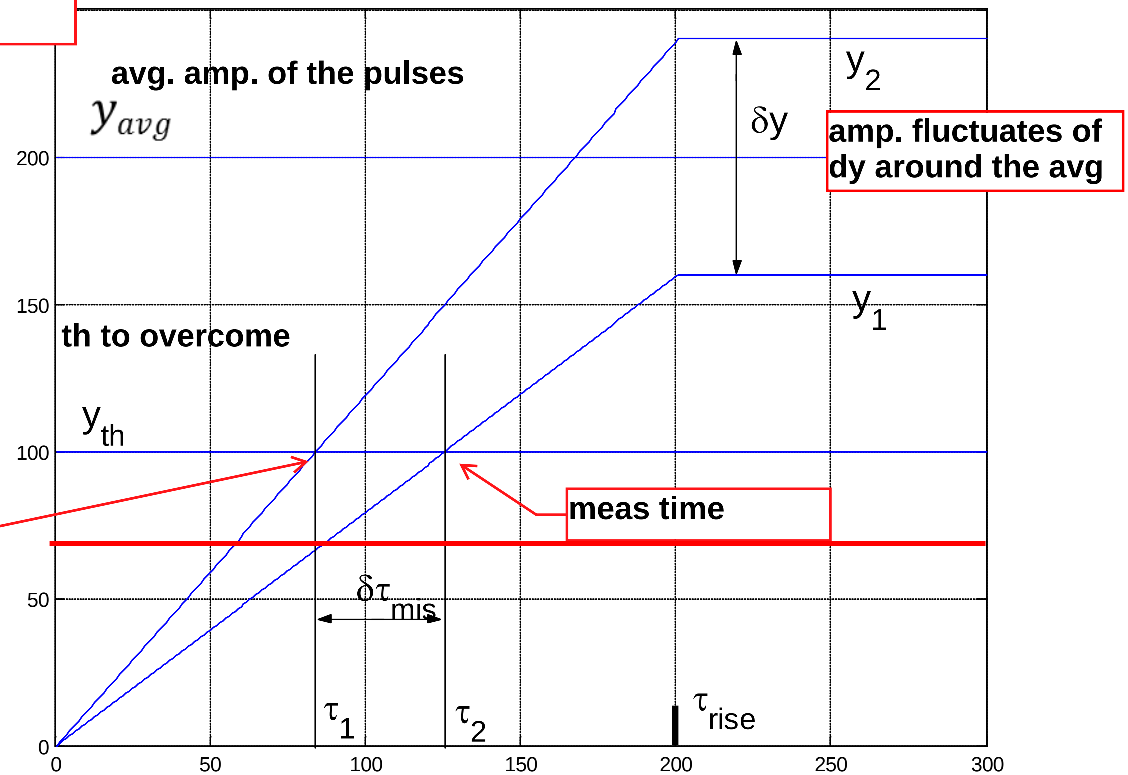
lower the trhs. smaller the time fluctuation. We don't put the thrs. to 0 b/c of the noise.. at a given point we are triggering on noise spikes

time fluctuation

$$\delta\tau_{meas} = -\frac{y_{th}\tau_{rise}}{y^2_{avg}}\delta y$$

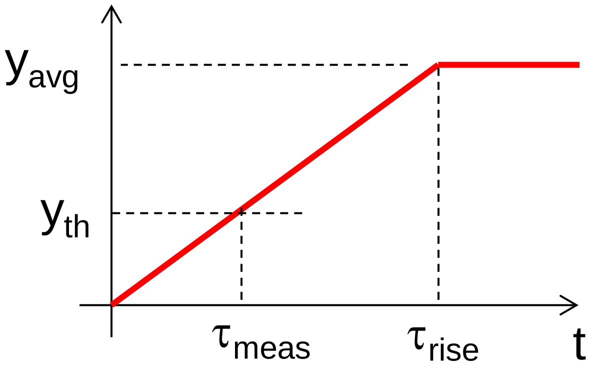
where y_{th} is the level at which the threshold is placed

meas. time



Due to the fluctuation the crossin time will change, and we obtain the deterministic fluctuation due to difference in amplitude. We can see that the time fluctuation is equal to THRESHOLD * RISE TIME * AMPLITUDE FLUCTUATION / SQ(avg). btw we dont place the timing on the output of step of preamp and we use already the filtering pulse. b/c the preamp is MORE NOISY! we use the filter, but is more sensitive to amplitude walk!!! [AUDIO!!!]

Calculation of the timing resolution in the leading edge



$$y = \frac{y_{avg}}{\tau_{rise}} t$$

$$y_{th} = \frac{y_{avg}}{\tau_{rise}} \tau_{meas}$$

$$\tau_{meas} = \frac{y_{th}}{y_{avg}} \tau_{rise}$$

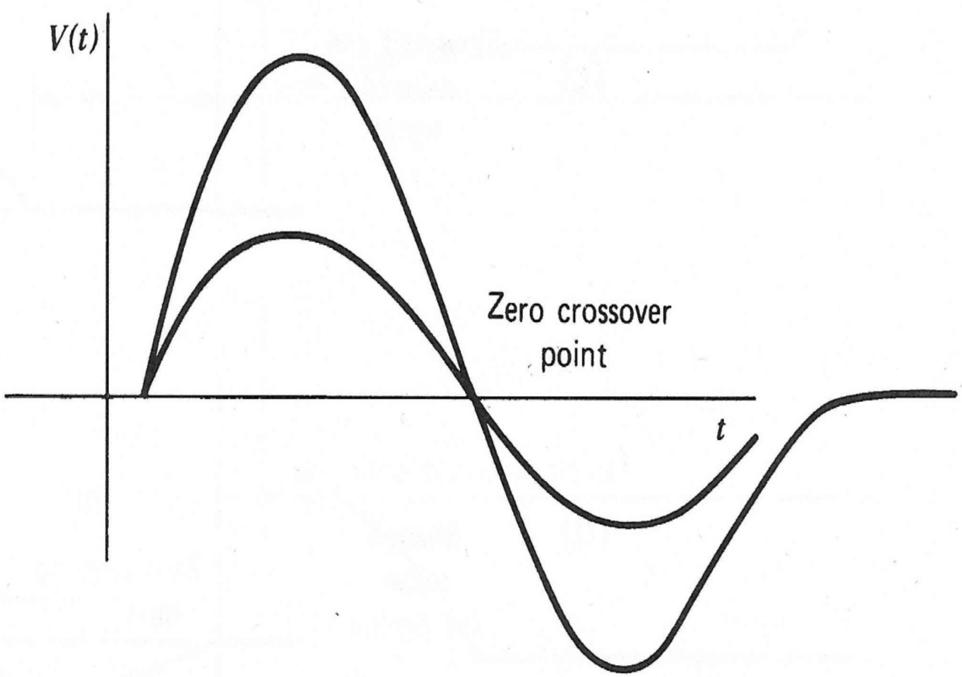
DIFFERENTIAL OF MEASRUMENT TIME, is the differential of [xx]

$$\boxed{\delta\tau_{meas} = -\frac{y_{th}\tau_{rise}}{y^2{}_{avg}} \delta y}$$

another way to eliminate effect of amplitude walking

Crossover timing AKA ZERO-CROSSING TIMING

A methode to eliminate the dependence τ_{meas} from the signal amplitudes is to filter the input signal to obtain a bipolar pulse and then to detect the time of zero crossing of this pulse.



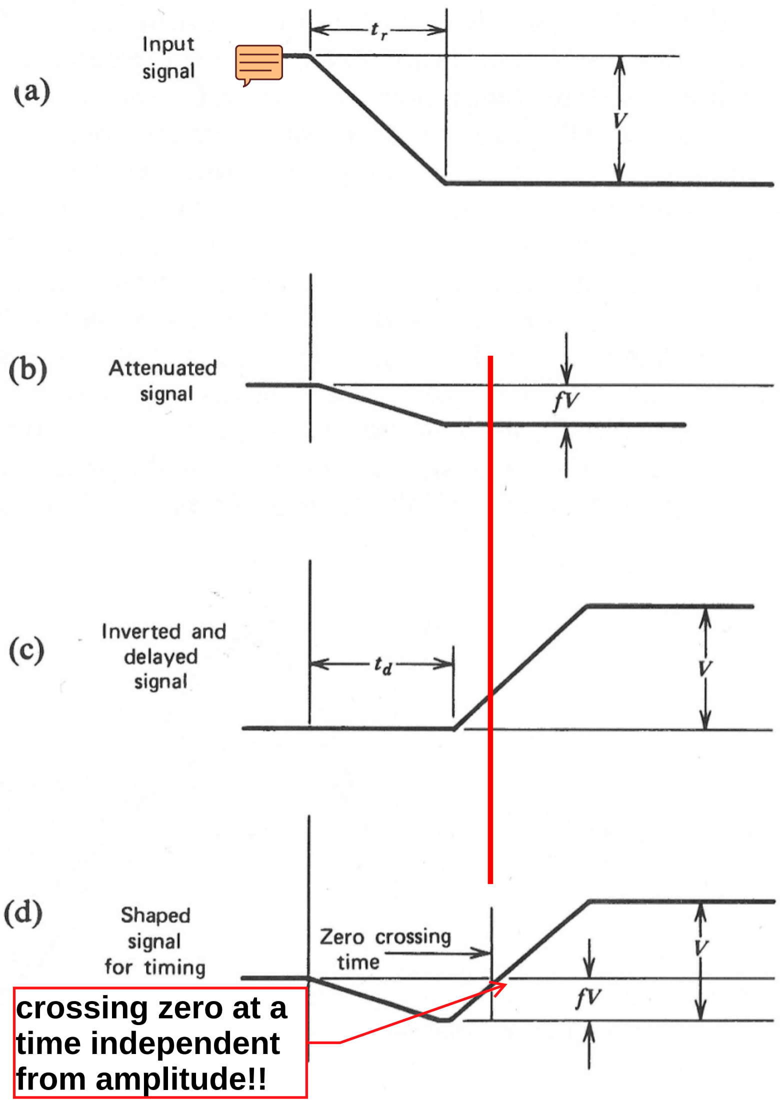
The measured time does not change, even if the two signals have different amplitudes

To obtain the bipolar pulse, different types of shaping may be employed, among them a semigaussian signal suitably derived. These types may also offer the advantage to improve the S/N ratio at the output of the filter.

We transform unipolar pulse -> bipolar pulse. We construct a BIPOLAR FILTER, and consider as a time stamp not the overcoming of a threshold, but when it crosses 0. aka ZERO-CROSSING TIMING. We need a comparator. On one side we put the bipolar pulse and on the other 0.It can be demonstrated that if we have bipolar pulses generated by diff. energies, and diff. amplitude, the zero crossing point is always the same. So the zerocrossing point is BY DEFINITION indepedent from the amplitude. dimostrazione sul quad.

we can do the same concept implemented in a different technique: doesn't imply any filtering, much simpler, starting already from the signal amplifier.. smart manipulation of preamplifier pulse. BETTER THAN DERIVATIVE FILTER!!

Constant fraction timing



It is similar to the crossover timing, except for the shaping of the bipolar pulse, $y_{bip}(t)$.

The input signal $y_{riv}(t)$ is taken, attenuated by a suitable factor f, and it is again added to an inverted and replica of $y_{riv}(t)$ delayed by a time τ_d .

Usually the best attenuation factor is ranging from 0,1 and 0,2, while τ_d has to be larger than the rise time τ_r of the pulse.

It is obtained:

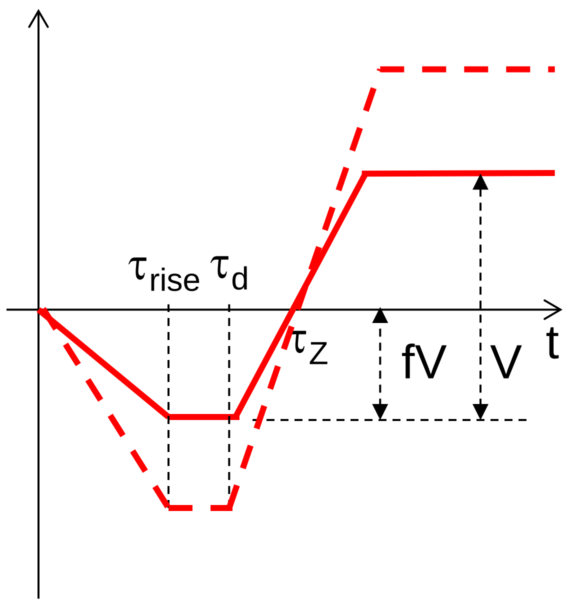
""bipolar"" input att. fipping delayed wv.f

$$y_{bip}(t) = fy_{riv}(t) - y_{riv}(t - \tau_d)$$

The zero-crossing time results independent from the amplitude of the signal

Which one is better? before we had to spend money in the components for the filter and derivator, thus more complicated and expensive, but it has an advantage: the shaping (gaus, e poi bipolar) is better for the noise. Instead in this method we dont apply any filtering!! it's just signal manipulation, we dont apply any suppression of noise! the technique is "somehow" simpler, but can be used only if you don't have a lot of electronic noise (which is the case of photodetectors with moltipication: pmts, avd .., indeed the ENC is killed.) So this techninuqe is common for pmts, and with PD w/o it's used the other.

Changing amplitude V, the zero crossing point is always the same!!



$$y_{bip}(t) = f y_{riv}(t) - y_{riv}(t - \tau_d)$$

we set it for amplitude zero...

$$y_{bip}(\tau_Z) = 0$$

we look for tz where ybip = 0

at this time the amplitude is just fV

$$0 = -fV + \frac{V}{\tau_r}(\tau_Z - \tau_d)$$

how much it has risen from the zeroc., vedi immagine precedente e linea verticale

result

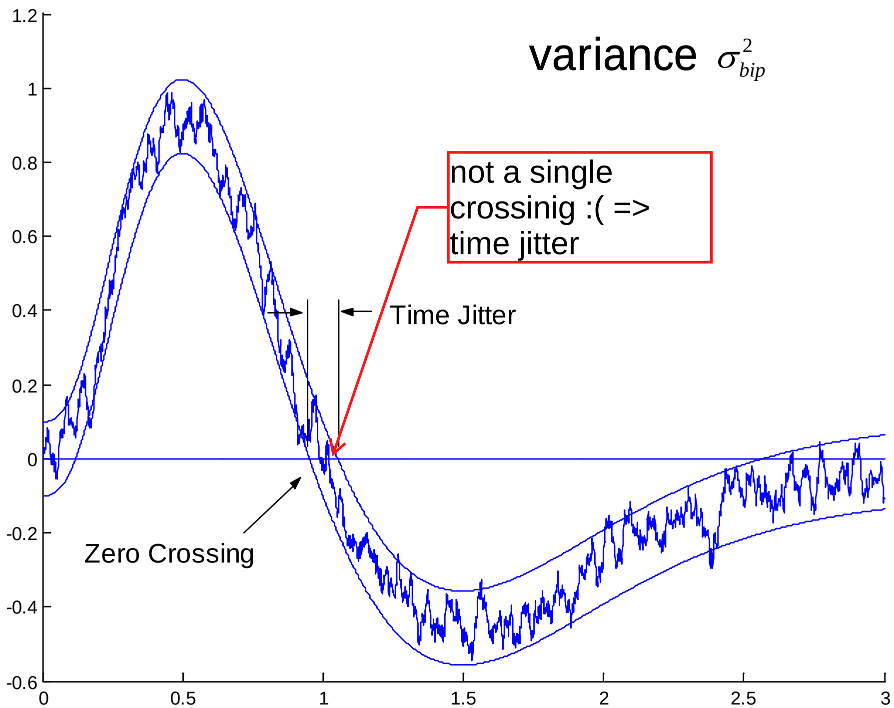
$$\boxed{\tau_Z = f\tau_r + \tau_d}$$

independent from the amplitude V of the signal

NOW: NOISE ON TIMING, on CROSSOVER TIMING (b/c it means we have decided we have a noisy system**nel senso che il metodo si usa di per se quando c'è del noise, infatti andiamo a fare un semigaussiano e poi un bip.filter che lo riducono. altrimenti const.Frac.)

The resolution of the zero-crossing (crossover and constant fraction)

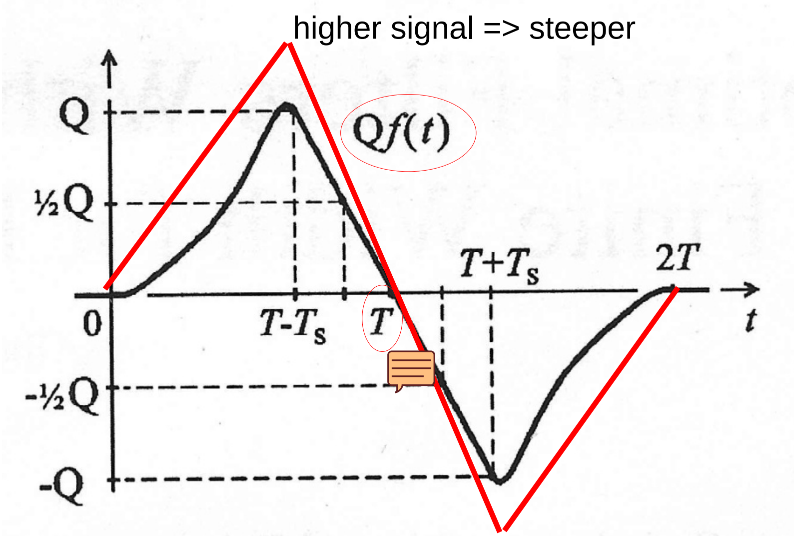
One can subdivide the bipolar signal where to find the zero crossing into the sum of a deterministic signal and of a statistical fluctuation due to the noise, characterized by a variance σ^2_{bip}



$$y_{bip}(t) = y_{\det}(t) + n(t)$$

The presence of $n(t)$ generates also statistical fluctuations on the measured variable τ_{meas}. These have zero average level and can be described by their variance σ^2_{τ}.

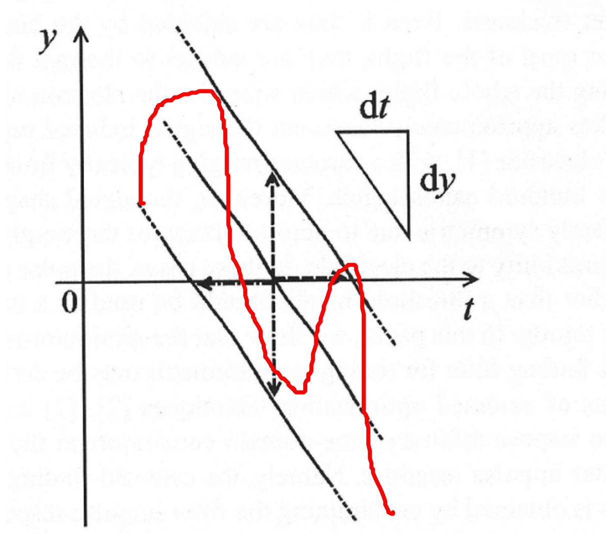
time jitter: incertezza nello zero crossing. si vedrà che il tipo del filtro implementato avrà un ruolo su questo.



The noise can be held as a statistical fluctuation of the signal which can translate high or low in a random way the waveform $y_{\mathrm{det}}(t)$.

As first approximation, around the zero crossing point, it exists a linear relationship between amplitude and timing fluctuations (jiiter):

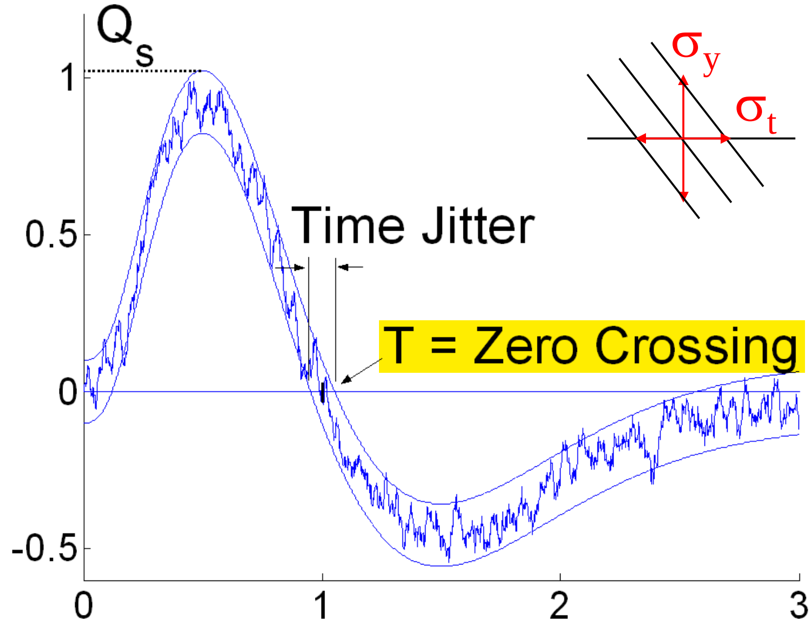
$$\sigma_\tau = \frac{\sigma_{bip}}{y'_{riv}\big|_{t=T}}$$



- The shaping realized with the Constant Fraction method leaves almost unchanged the value of σ_{bip} and is generally used for detectors in which the electronics noise is negligible, typically in those with internal gain $>>1$, like PMTs.
- With a suitable bipolar filter and the crossover, it is possible to optimize the noise σ_{bip}.
- The derivative in T covers a role in both cases
- Note: the statistical fluctuation of Q does not count

Geometrical consideration. Guardando lo zoom del bipolar pulse intorno allo zero. Carica Q, straight line, è come se avessimo una constant slope (un po' esagerato ..) è chiaramente più steep più larga la Q, e più steep shorter is T! (derivative of the crossing) The derivative is important, because, supposing we have a given noise, we have a given dy fluctuation. the dt fluctuation due to dy fluct. is given by geometrical consideration: vertical fluct / derivative. this is why we concentrate on the derivative => steepest it is, smaller the time jitter for a given vertical jitter. steeper? for the same voltage jitter, the horizontal jitter is less!!. According to this, the best bipolar pulse (we'll see it's not true) is the vertical one, dove ogni fluttuazione verticale non varierebbe lo zero crossing.

calculate time jitter according to the derivative



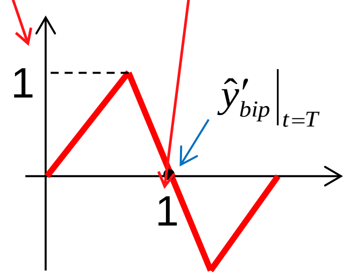
vertical jitter

derivative calculated at zero crossing time

$$\sigma_\tau = \frac{\sigma_{bip}}{y'_{bip}\big|_{t=T}} = \frac{\sigma_{bip}}{y_{MAX}} \frac{1}{\dfrac{y'_{bip}\big|_{t=T}}{\dfrac{T}{y_{MAX}}}} T = \sigma^{\%}_{bip} \frac{1}{\hat{y}'_{bip}\big|_{t=T}} \cdot T$$

amplitude 1

zero crossing 1



$$\hat{y}'_{bip}\big|_{t=T}$$

$$\sigma_\tau = \frac{ENC}{Q_s} \frac{1}{\hat{y}'_{bip}\big|_{t=T}} T$$

$$\frac{ENC}{Q_s}$$ = S/N

$$\hat{y}'_{bip}\big|_{t=T}$$ = derivative normalized in time and amplitude

I can say it's the signal in charge.. I could also say signal in mV it's the same

zero crossing time. for the same bipolar filter and same amplitude, try to get T as short as possible

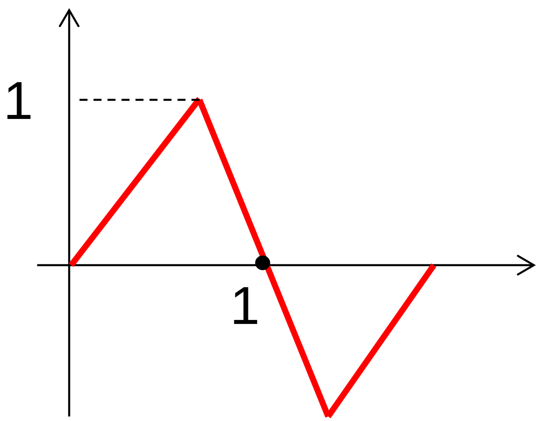
$$\sigma_\tau = \frac{ENC^{(T,A)}}{Q_s} \frac{1}{\hat{y}'_{bip}\big|_{t=T}} T$$

CONCLUSION

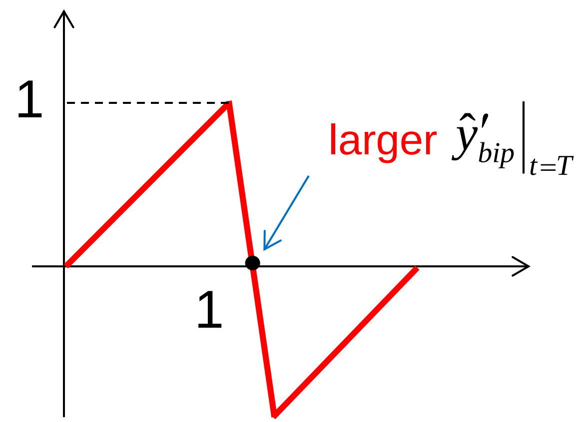
time jitter reduced with:

- small ENC (but look to dependency from T)
- large Qs
- small T (but be careful to preamp rise time)
- large $\hat{y}'_{bip}\big|_{t=T}$

increasing $\hat{y}'_{bip}\big|_{t=T}$ to minimize time jitter?



(for a given T)



but increasing $\hat{y}'_{bip}\big|_{t=T}$ has an effect also on ENC through the filtering factors

=> *infatti.. ENC aveva dipendenza da T e da SHAPING factor. Cambiare la derivata significa CAMBIARE shaping factor! avevo una situazione in cui erano migliori. se aumento la derivata può essere che peggiorino. Stessa cosa per T, se la riduco devo andare a valutarla.

Ele
Pro

noise formula, parte series

$$ENC^2{}_{series} = aC_T{}^2 1/2\pi \int_{-\infty}^{+\infty} |T(j\omega)|^2 d\omega =$$

$H(j\omega)$

is the overall output response (preamp + filter T(jω))

$$= aC_T{}^2 1/2\pi \int_{-\infty}^{+\infty} |H(j\omega)|^2 \omega^2 d\omega =$$

integral of square of H(w) = int. of time response of h(w), che dalle proprietà della trasf. ... h(w) <-> h(t) allora h(w) * w <-> h'(t) (h derivato)

$$= [Parseval\ theorem] = aC_T{}^2 \int_{-\infty}^{+\infty} [h'(t)]^2 dt$$

NEL TEMPO!!

INTEGRAL OF DERIVATIVE SQUARED!!!, on absolute time scale, passiamo alla normalizzata time scale con t'

$$h(t) = L^{-1}[H(s)] = L^{-1}[T(s)/s]$$

$h(t)$ is the output response in the time domain, $h'(t)$ its derivative

using $t' = t/\tau$ as dimensionless variable, i.e. the time normalized to the shaping time τ :

conversions of derivative and time scales... (fasts)

$dt = dt' \cdot \tau$ $\qquad$ $h'(t) = h'(t') \cdot 1/\tau$

conclusion

dipende dallo square della derivative nel tempo normalizzato

$$ENC^2{}_{series} = aC_T{}^2 \frac{1}{\tau} \int_{-\infty}^{+\infty} [h'(t')]^2 dt'$$

introducing the shaping factor A_1 as done in a previous lesson:

NEW NICE DEFINITION OF A1 FACTOR!!

integral of square of derivative

$$A_1 = 1/2\pi \int_{-\infty}^{+\infty} |T(j\omega)|^2 d\omega = \int_{-\infty}^{+\infty} [h'(t')]^2 dt'$$

conclusion for the timing: steepest is the normalized derivative, larger A1 coeff. BE AWARE ABOUT TOO STEEP WAVEFORMS! the risk is that A1 coeff. may explode b/c of the derivative!!! so instead of increasing the denominator you are exploding ENC

⇒ increasing the derivative of the normalized response may worsen the series noise, through A_1, which may be the dominant noise contribution at short shaping times (as required for timing)

the same calc. can be done also for A3.. which is for parallel noise. in this case area of the square of h(t)

(Note: A_3 can be equivalently calculated as:

"ci sono delle slide che fanno questi passaggi, non volute" [audio in cui parla delle equazioni.. half derivative].. magari solo risultati finiti dei coefficienti?

$$A_3 = \int_{-\infty}^{+\infty} [h(t')]^2 dt'$$

and it increases with $h(t')$ total area)

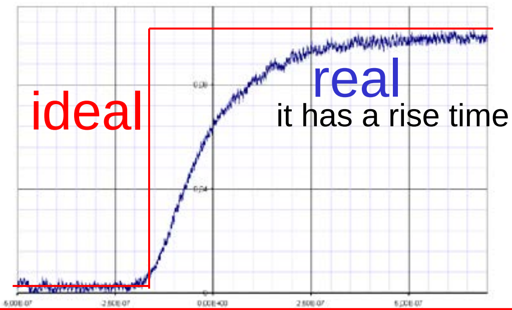
Reducing the shaping time vs. preamplifier rise time

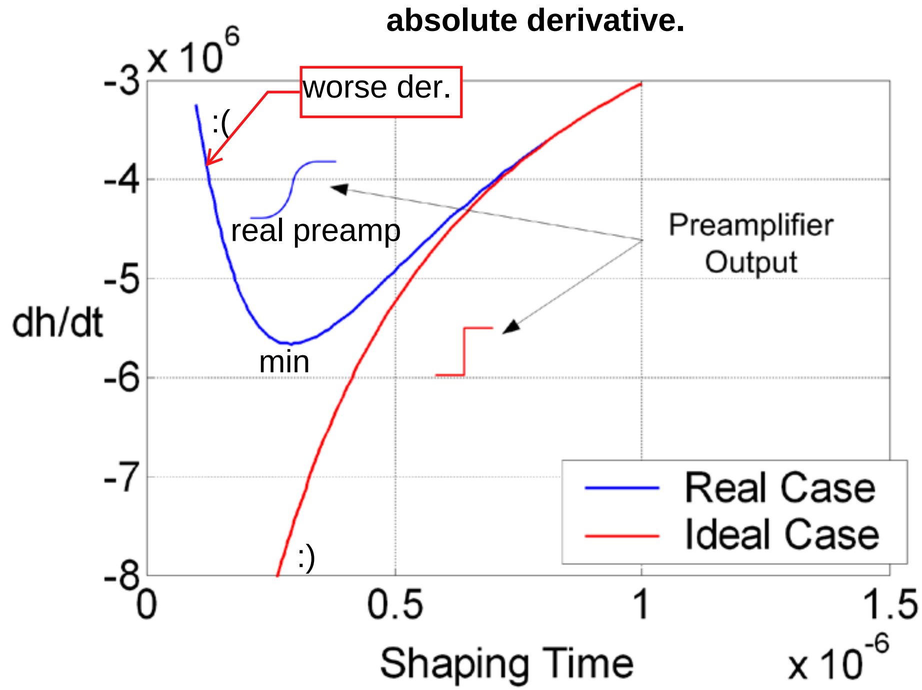
- Ideale Case: front-end rise time supposed to be zero

⇒ the derivative tends to $-\infty$ by reducing the zero-crossing time

- Real case: front-end rise time limited by three main effects:

  - detector collection time
  - rise time of the preamplifier bw
  - scintillator decay time





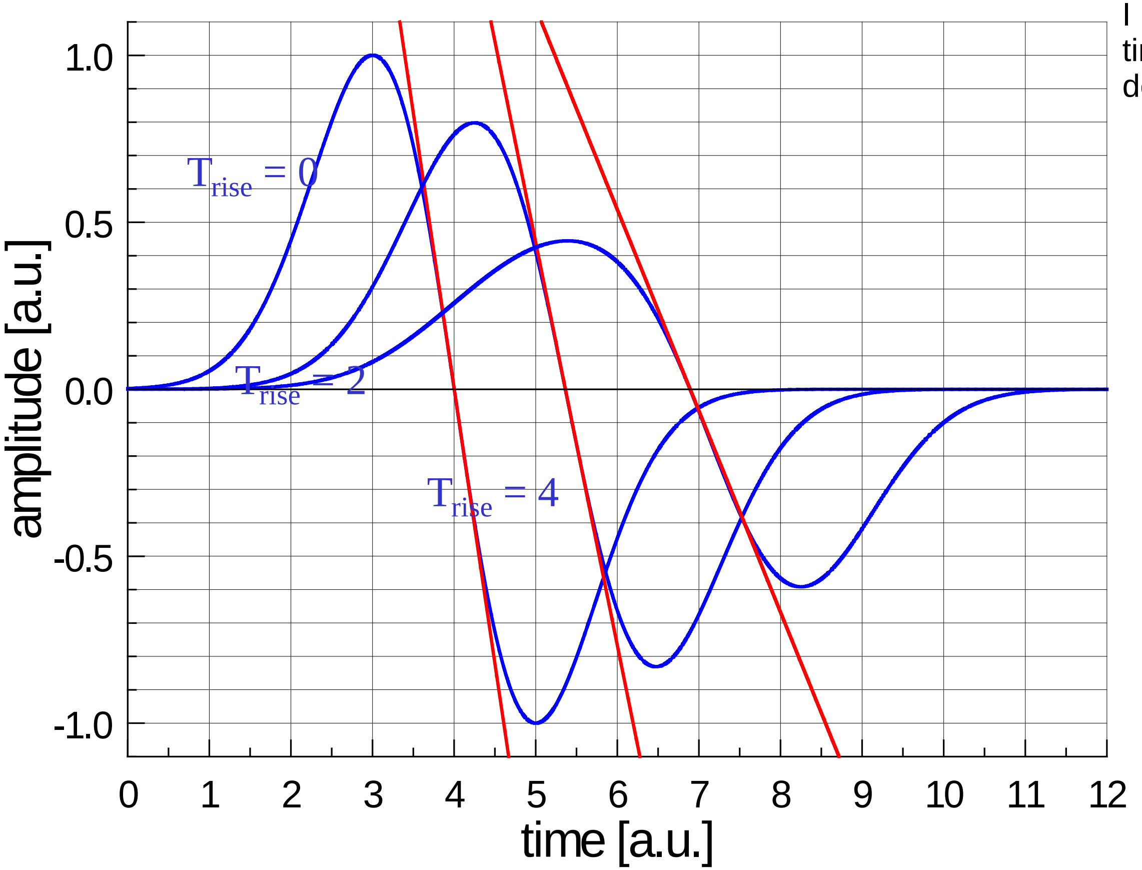
Pulse derivative at the Zero Crossing time vs. ZC time (<u>defined our shaping time</u>)

⇒ the abs. value of the derivative has a maximum and then its reduces to zero.

⇒ there is an optimum shaping time for the derivative

LAST PROBLEM OF THE EQ: we still think T->0 is good. Actually: the preamplifier is not a perfect STEP. with a rise time (due to finite collection time of detectors, scintillator decay, rise time of preamp (bw of preamp.)) This bring us to the conclusion that: if we reduce the T, due to limit of rise time, at a given point the derivative is deteriorated by rise time of preamp.... how can the derivative be high if the time of the rise time is slow... indeed see the plot. We crash at the limit due to this rising time. We have a minimum best zero crossing time plot to see how the rise time plays a role, dallo stesso zero crossing time, ho waveform che corrispondono a diversi rise time.. mi aspetterei fossero le stesse, invece

Worsening of the derivative because of the rise time

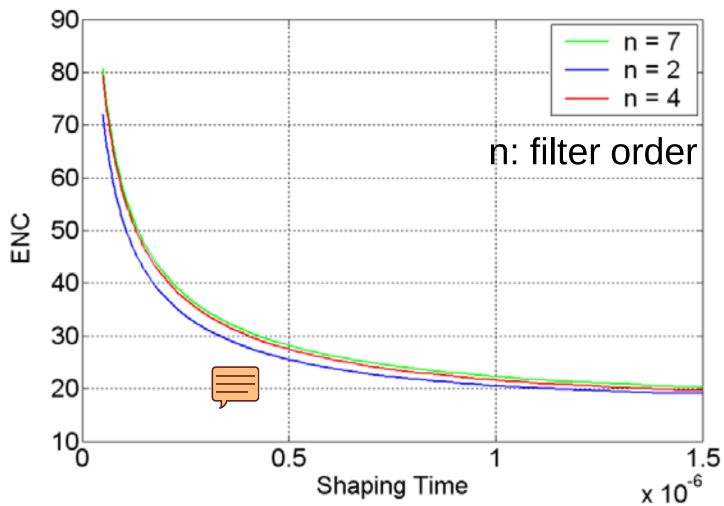


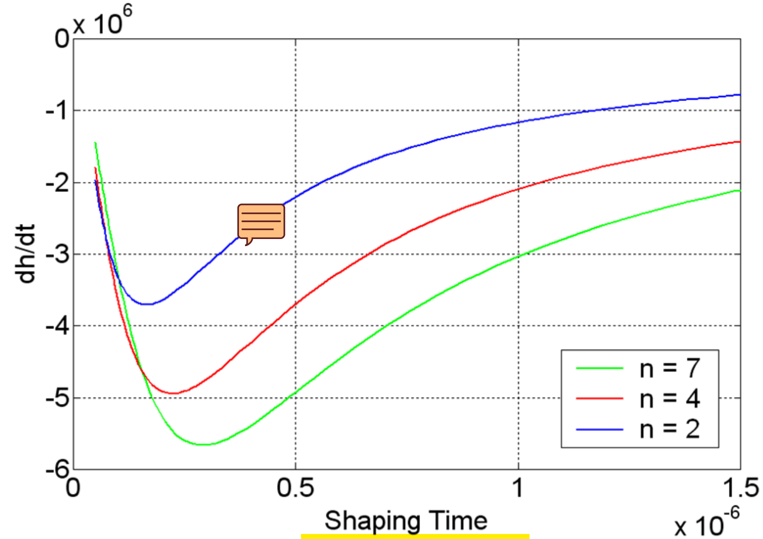
2 negative effect:
I increase the real zerocrossin
time, and a smoother
derivative.

in this graph the nominal zero-crossing time is kept constant, and the rise time of the preamplifier is increased ⇒ 1) the real zero-crossing time is increased and 2) the derivative is decreased (therefore the timing jitter is worsened)

Theoretical evaluation of the timing performances

electronic noise





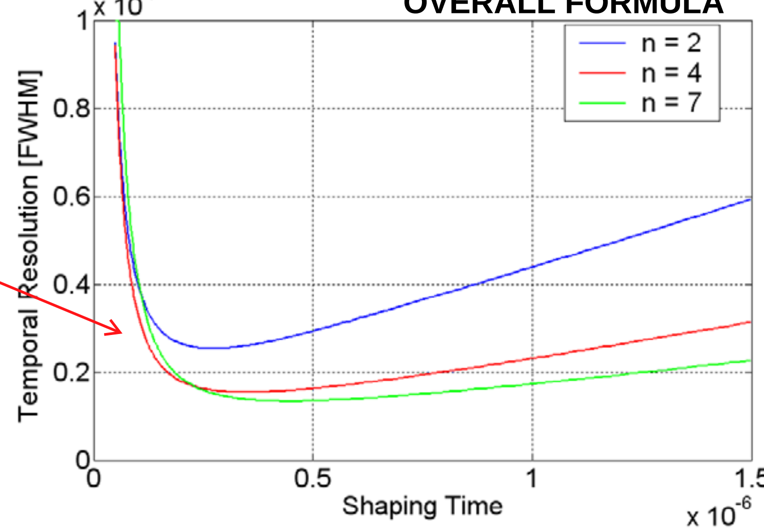
electronics noise (series, ~~1/f only~~)
*b(c lowT

derivative

TIME JITTER

OVERALL FORMULA

not true, the shortest the best. it starts divergence b/c of the derivative, time rise of preamp.



zero crossing time

timing resolution

$$\sigma_\tau = \frac{ENC}{Q_s} \frac{1}{\hat{y}'_{bip}\Big|_{t=T}} T$$