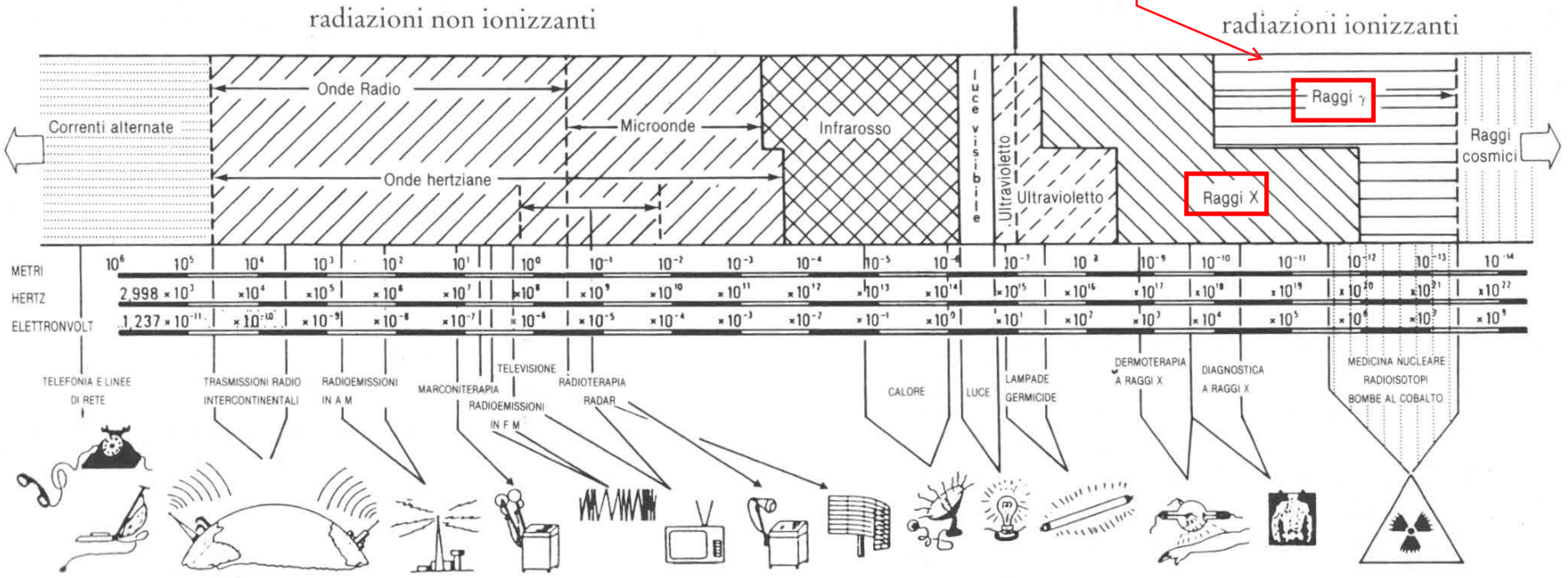
The radiation used in radiography, SPECT and PET: X rays and γ rays

overlap, depending on the ORIGIN we classify differently



xrays and gamma rays: difference in the position of the source. Xrays => source -> body -> det. (classico). [discorso assorbimento, bones assorbe molto, clean spot sull'immagine]. Muovendo source/det. => C.T.
G-rays: different. Combination of an emitter "something that emits g-rays" connected with a molecule that binds specifically with someting in the body. You inject this compound inside the body: (RADIOTRACER + molecule), injected in the body goes around and binds with the pathology. Once it binds it concentrate all the radiotracer inside it, so the external detector can take an image and identify the precise location of pathology (eg tumor).

X rays

- Energy: 10 eV $\rightarrow$ 1keV $\rightarrow$ 100-300 keV
- origin: fluorescence from atoms, Bremsstrahlung
- applications in medical diagnostics: radiography

(conventional, mammography, densitometry, ..., CT)

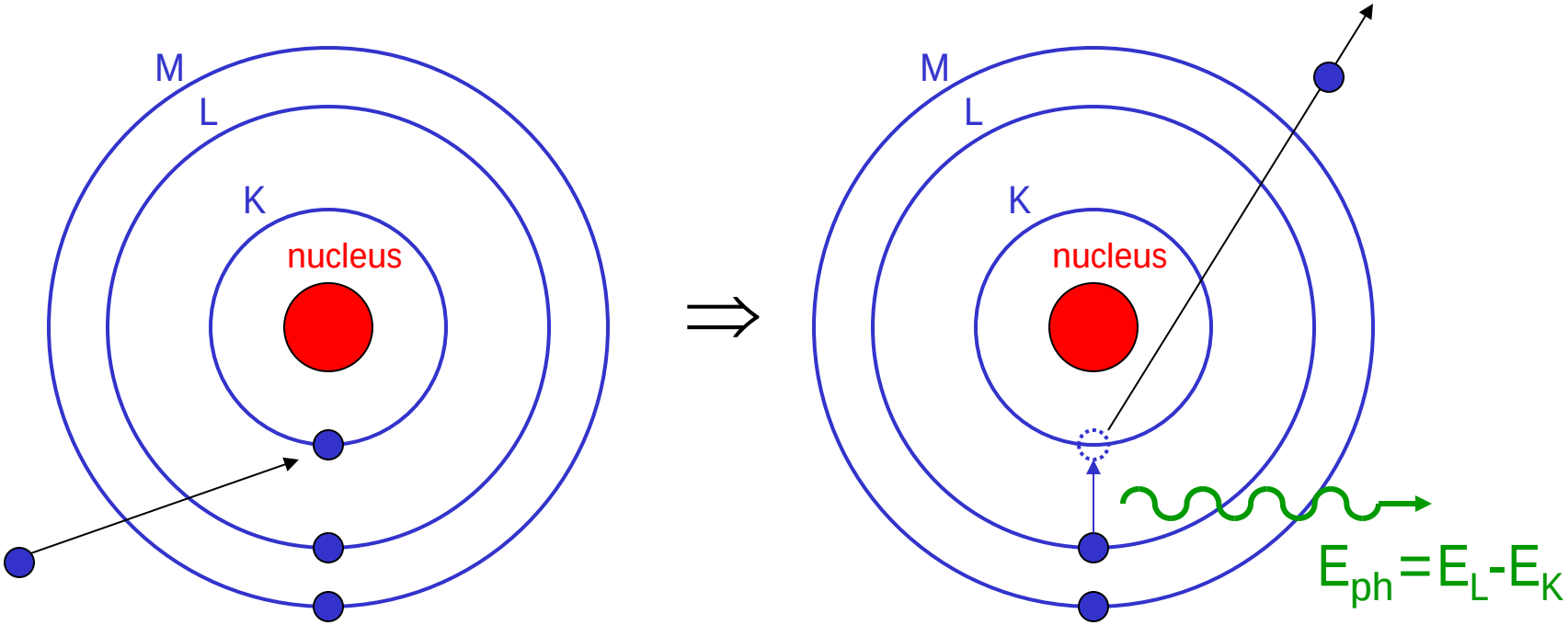
γ rays

- Energy: 10 keV $\rightarrow$ 100keV $\rightarrow$ 10MeV
- origin: nuclear emission, annihilation of positrons
- applications in medical diagnostics: SPECT, PET

(in red: energy range used in medicine)

Difference is not only on EnLevel but on the ORIGIN!. XRAY-> caused by transitions of el. from one level to another of the atom. G-Rays-> originate from NUCLEAR reaction that takes place inside the nucleai of the atom. Both rel. to atom but in diff. parts. There's an overlap in terms of enery. In the same range we can have both xray and grays.

Origin of X rays: 1) fluorescence

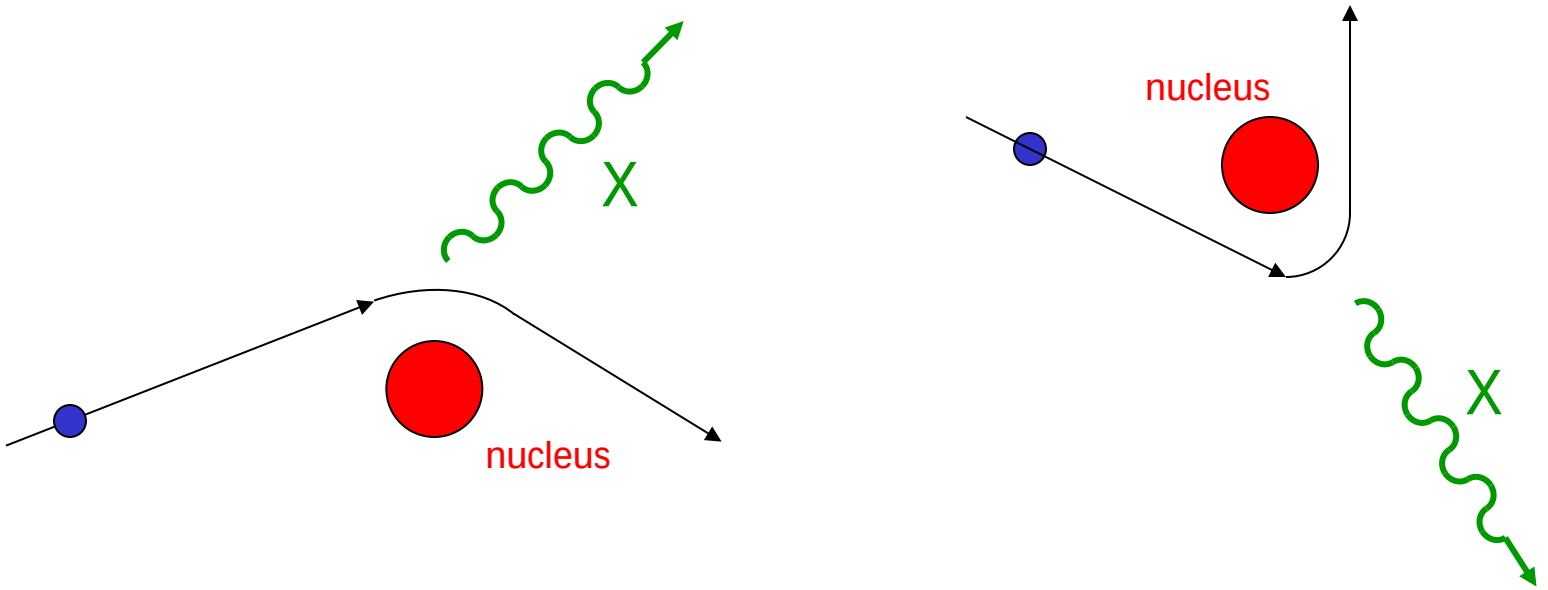


accelerated electron
(E ~ 1-100keV)

- the energy of the accelerated electron produces the emission of an electron belonging to an internal shell (K,L,M)
- an electron of a next shell takes the place of the ejected electron
- the process produces an X photon of definite energy, equal to the difference between the two energy levels

3 mechanisms to produce xrays. 1) Fluorescence. Accelerated particles hits an electron in the shell, which is kicked out. We have a vacancy -> unstable. The eltrs. on the other shell jump to fill this vacancy, in the most closed level (k). Once an el. does this loses En, which is emitted as an xray. The En of the xrays is the difference between the 2 orbitals. Very well defined, each atom of the periodic table has different conf. of shell, so for each atom this transition produces energy of different levels (basic of spectroscopy).

Origin of X rays: 2) Bremsstrahlung

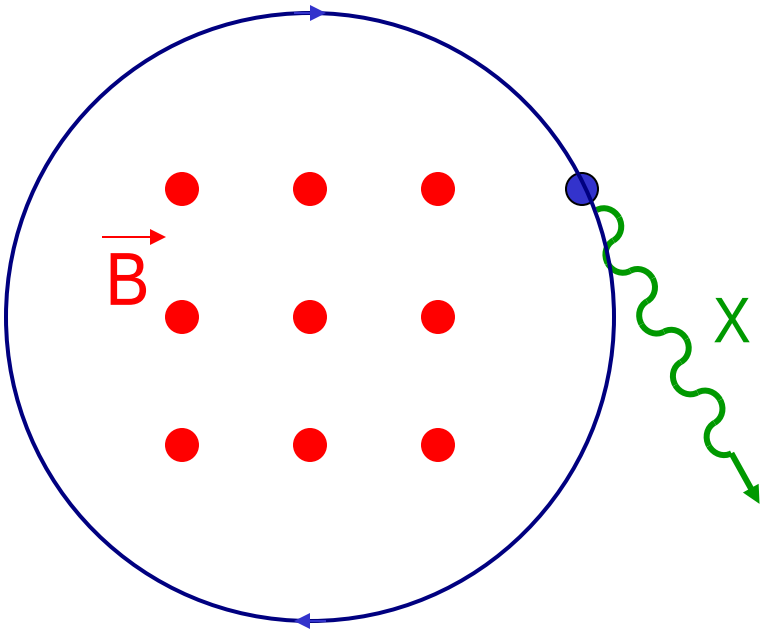


- the Bremsstrahlung radiation is produced when an electron changes its velocity following the Coulombian interaction with the atomic nucleus
- the emission is larger more energetic is the electron and larger is Z of the absorbing material
- the energy emitted as X rays has a continuous spectrum from 0 to Emax (the kinetic energy of the incoming electron, in the case it is fully absorbed in the interaction)

El travelling, the velocity is reduced (deceleration) the energy is reduced (kinetic energy). This energy is dissipated in the form of emission of xray. DIFFERENCE with flourescence: in that case the energy is discreate in peaks corresponding all the possible transitions here can take any value, up to the case where it's completely stopped (max energy). Between 0 and this max we can have all the energy, so spectrum continuous. Tipically at low en higher probabilty of emission of energy to respect higher en.

Origin of X rays: 3) synchrotron light

b/c it's tipically obtained in a synchrotron



an electron forced to run a circular trajectory loses energy in the form of X rays (synchrotron radiation)

In summary:

an e- decelerated loses part of its energy in the form of X rays in the following ways:

1) the deceleration changes its tangential velocity → bremsstrahlung
2) the deceleration changes its normal velocity → synchrotron light

Tipically done inside a syncrotron: particles goes in a circular trajectory, taken and accelerated by magnetic means. The NORMAL velocity is changing! b/c it's turning direction. (Tg velocity and angular the same). Thus you have emission of some energy in the form of xray.

Generation of X rays: X-ray tubes

Tipical source; chamber in vacume

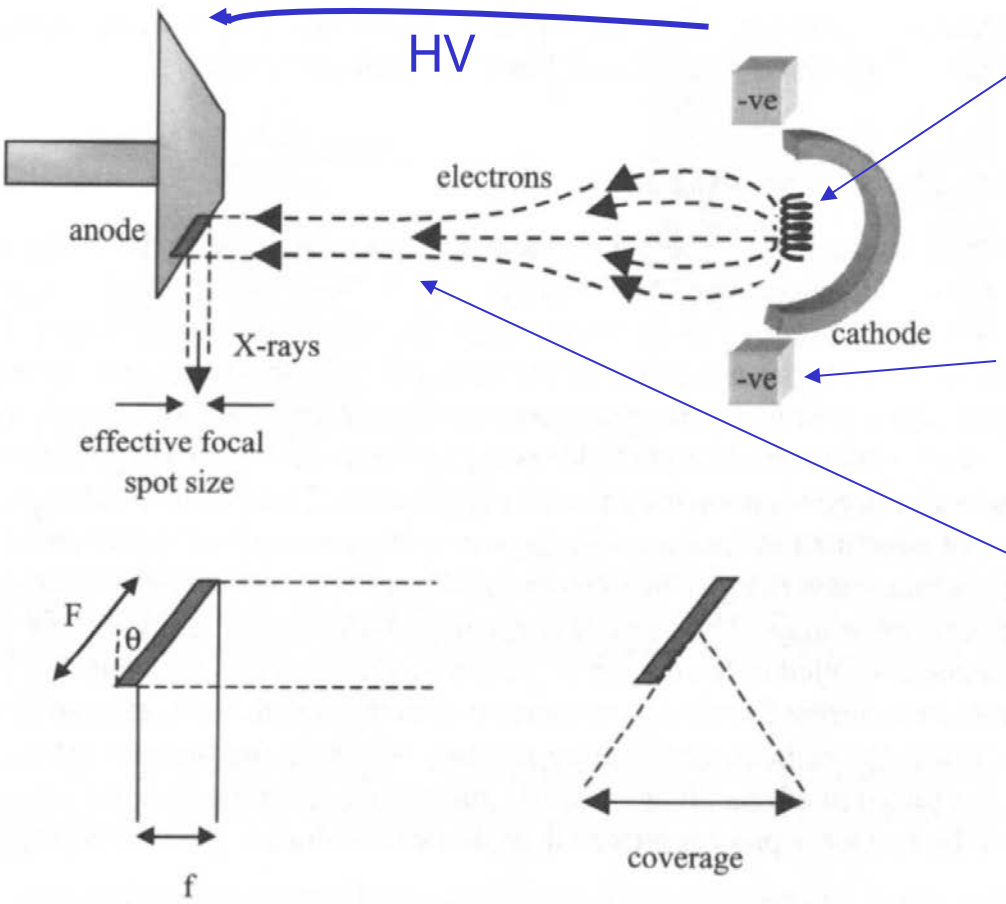


FIGURE 1.4. *(Top) A negatively charged focusing cup within the X-ray cathode produces a tightly focused beam of electrons and increases the electron flux striking the tungsten anode. (Bottom) The effect of the anode bevel angle θ on the effective focal spot size f and the X-ray coverage.*

the electrons are emitted by a filament which is heated by means of current flow

filament:ince tel

suitably biased electrodes focus the beam of electrons emitted by the cathode

the electrons are accelerated by the high voltage applied between anode and cathode and let collide on the anode where they produce X rays from Bremsstrahlung and fluorescence

Filament heated by flow of current, at a certain point it emits electrodes. The electrons (free in the vacum) are accelerated towards the anode (strong ddp that accel. els). When the elcs. hit the anode => **2 phenomena**: bresstralung + fluorescence(of atom of the anode). We have a shielding with only a little window that allows the irradiation only in one direction

The architecture of the X ray tube (with rotating anode)

ddp: 100kV + additional electrode to help direction

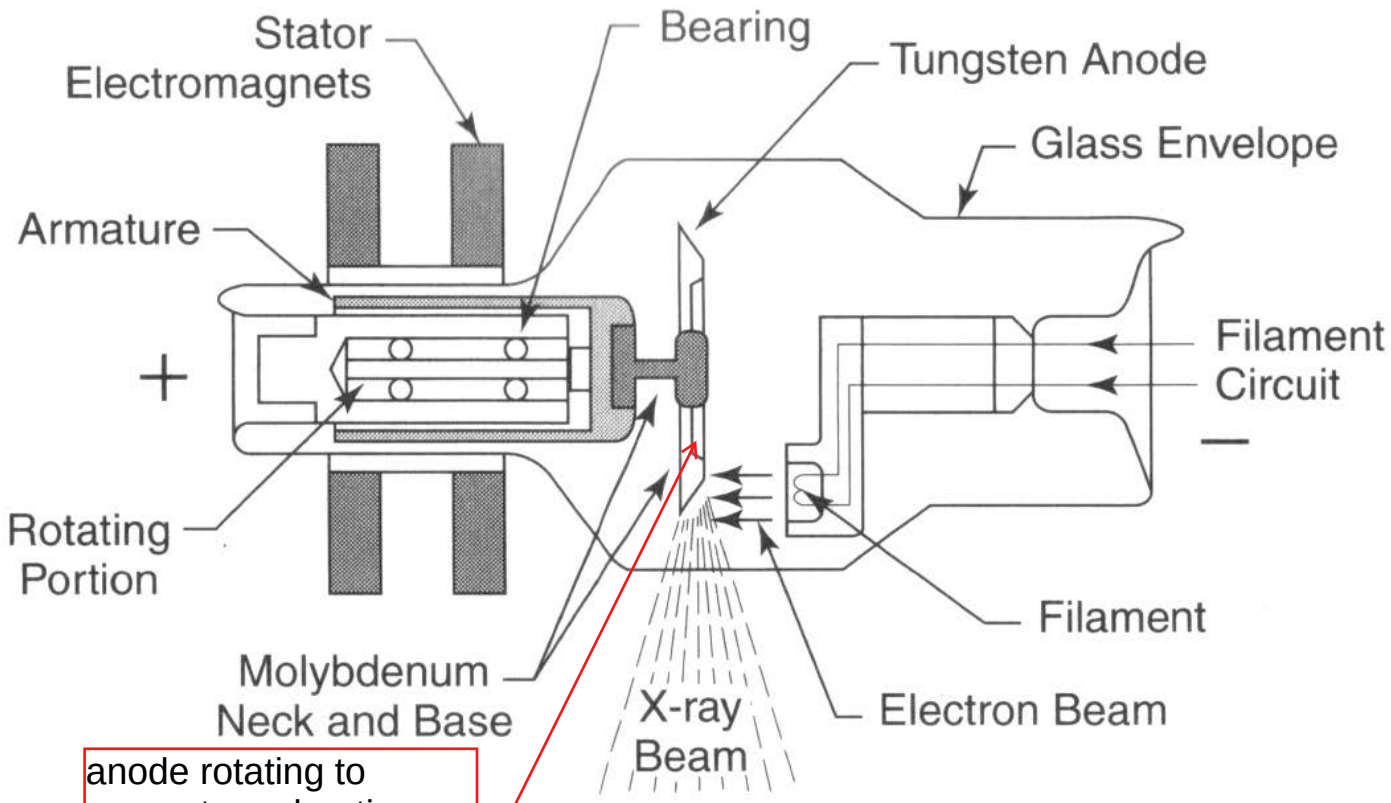


FIGURE 5-1
Simplified x-ray tube with a rotating anode and a heated filament.

Typical spectrum of the X-ray tube

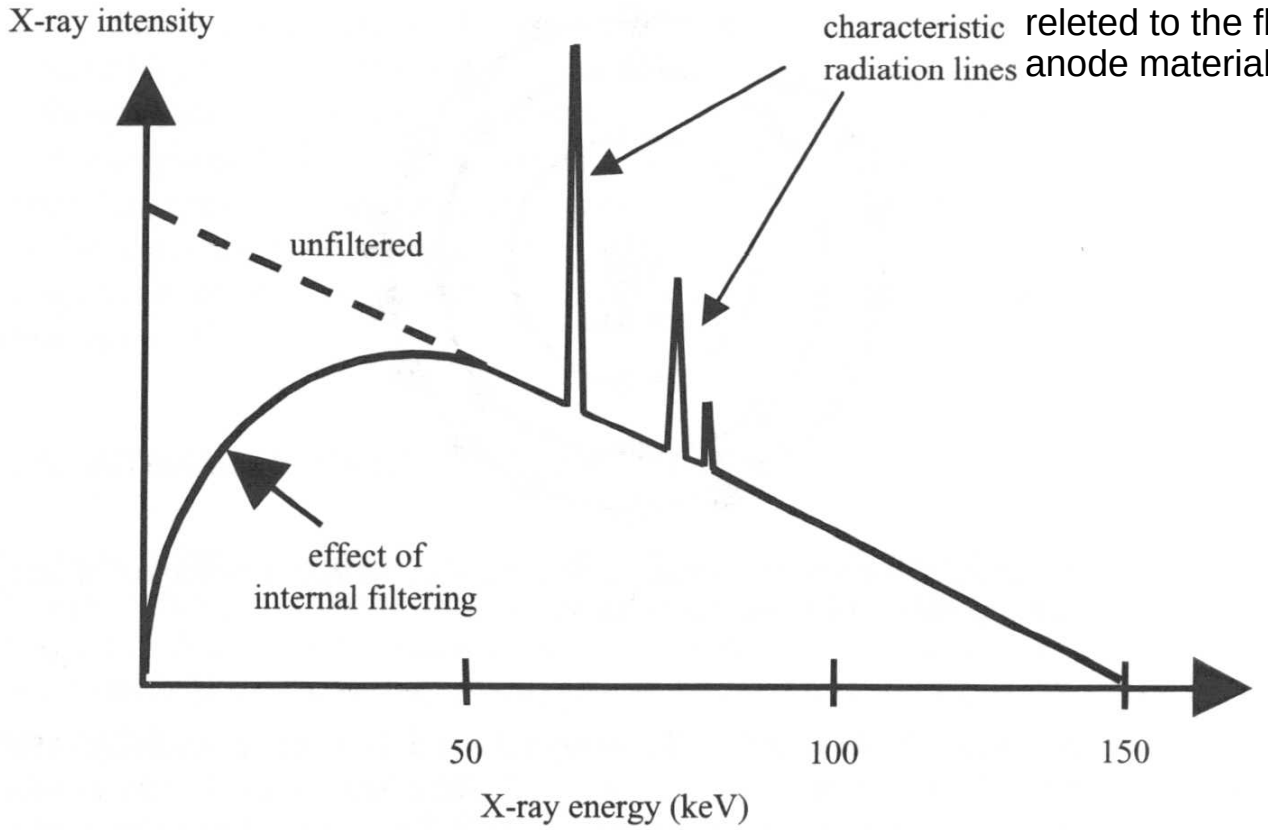


FIGURE 1.5. *A typical X-ray energy spectrum produced from a tube with a kV_p value of 150 keV, using a tungsten anode. Low-energy X-rays (dashed line) are absorbed by the components of the X-ray tube itself. Characteristic radiation lines from the anode occur at approximately 60 and 70 keV.*

The spectrum is continous, highest energy (eg here 150keV) corresponding to an acceleration ddp of 150kV [eV => energy associated with an electrone associated with a ddp]. At very low en. we are not following the ideal brsmSpectrum, b/c we have some absorption at low en. from the window and air. NOT OPTIMAL TO ANALYZE, it's too spread! for radiography to it's fine, in the end the detector has a broad en. window .

we are in the nucleous of the atom X^a _z (z: atomic number, number of protons. [in neutral atom == n° of elctr.]) A: mass number=sum of number of protons + neutrons (number of particles in the nucleous).

Origin of γ rays: the radioactive decay

ACTIVITY [1/s]: number of decay per seconds [or curie, or becquerel]

$$A = -\frac{dN}{dt} = \lambda N$$

n. of decay/s

proportional to total number of nucleai

A: activity of the radionuclide
(Curie,Ci=3.7×10^{10} Bq)
(Becquerel=disintegrations/s)
N: number of nuclei
λ: decay constant

$$N = N_0 \exp(-\lambda t)$$

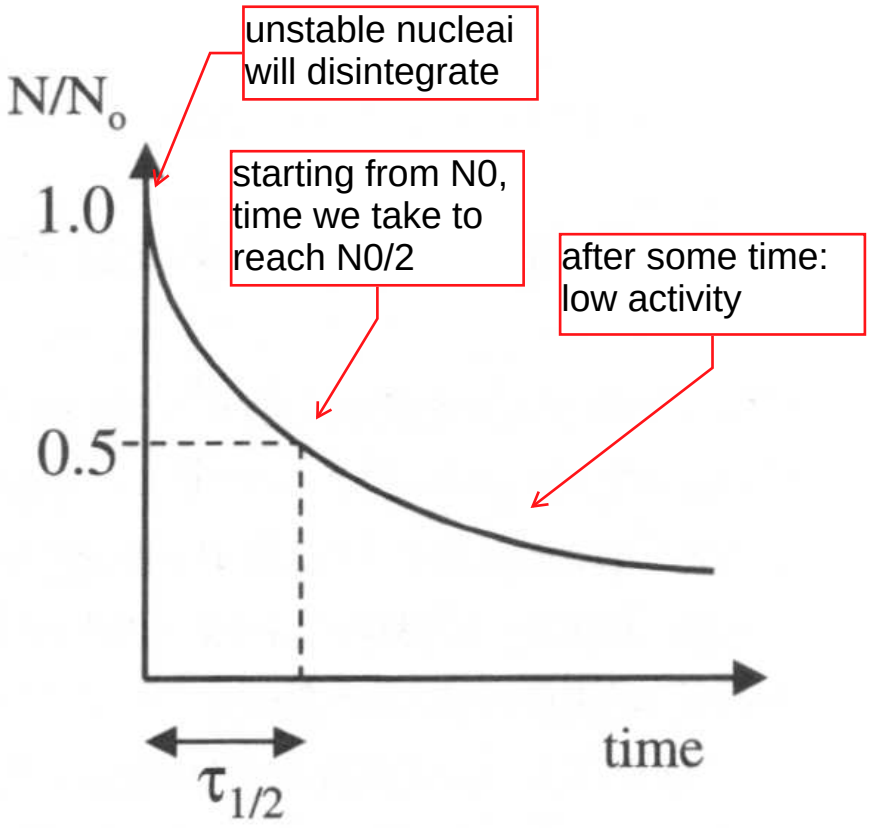
N_0: number of nuclei at t=0

$$\tau_{1/2} = \frac{\ln 2}{\lambda}$$

diff. material are characterized by this

$\tau_{1/2}$: half-life constant

radioactivity: emission of radiation (α,β,γ) following the spontaneous change of the nucleus composition in instable isotopes

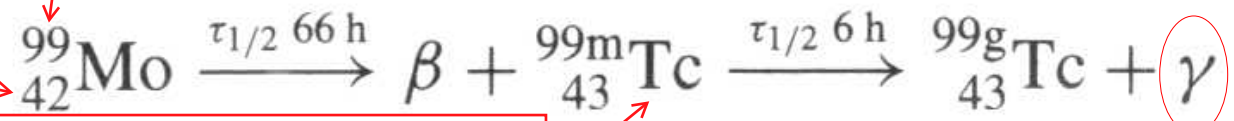


Produced by radioactive materials. Some atoms are not stable and their nucleous is changing over time. these reaction over time produces radiations in different ways (alpha: nucleum of He (2 prot 2 neutr), beta: electron (these two so particle emission) gamma: photons). How to characterize radioactivity? The atom is the same, but different isotopes differs b/c particles in the nucleous changes (nucleo (*neutron?) -> proton). We define the frequency of reaction with that equation

One popular γ-ray emitter for medicine: Tecnetium 99
(used in 90% of the diagnostic analyses in nuclear medicine)

*il numero di protoni cambia

il numero di massa è lo stesso



it's not stable, 99 should also be changed (m=> metastable)

enough to take the measurement!

- ^{99m}Tc is metastable state ($\tau_{1/2}$ = 6h, useful for diagnostics)
- Energy γ = 140 keV (good energy for diagnostics)

BOTH NUMBER ARE CHOSEN ACCORDING TO A TRADEOFF

Few general notes on radionuclides for diagnostics:

- $\tau_{1/2}$ too short ⇒ short time between injection and diagnostics
- $\tau_{1/2}$ too long ⇒ low activity, patient radioactive for too long
- Eγ too low ⇒ few rays reach the detector absrbd by the body
- Eγ too high ⇒ patient 'transparent', difficult detection

keeps emitting radioactivty :(

Molibdenum99: 42 protons, 99=42+neutrons. It's radioactive, it undergoes a transition producing 1 beta particle (el) and 1 proton. Mo99 -> B +43Tc (tecnetium). Tc99 is the radiotracer that we use in the body, b/C it's easy to attach this atom to other molecoles to use as marker to identify specific pathology. It produces a gammaray. Egamma too high: also det. become transparent! :((patient transparent is what we need).

Other nuclides radioactive for electron capture

(capture by the nucleus of an electron from shell K or L, followed by an emission of a γ ray and possible X-ray fluorescence):

$$^{123}_{53}\mathrm{I} + {}^{0}_{-1}e \xrightarrow{13\,\mathrm{h}} {}^{123}_{52}\mathrm{Te} + \gamma \quad (159\ \mathrm{keV})$$

e- +p → n

$$^{201}_{81}\mathrm{Tl} + {}^{0}_{-1}e \xrightarrow{73\,\mathrm{h}} {}^{201}_{80}\mathrm{Hg} + \gamma \quad (167\ \mathrm{keV}) + \text{X-rays} \quad (68\text{–}82\ \mathrm{keV})$$

$$^{111}_{49}\mathrm{In} + {}^{0}_{-1}e \xrightarrow{67\,\mathrm{h}} {}^{111}_{48}\mathrm{Cd} + \gamma \quad (171\ \mathrm{keV})$$

TABLE 2.1. Properties of Common Radionuclides Used in Nuclear Medicine

Radionuclide	Half-life	γ-ray Energy (keV)
^{99m}Tc	6.02 h	140
^{67}Ga	3.2 d	93, 185, 300, 394
^{201}Tl	3.0 d	68–82 (X-rays)
^{133}Xe	5.3 d	81
^{111}In	2.8 d	171, 245
^{131}I	8 d	364
^{123}I	13 h	159

Tecnicium is not the only one. Also Ga, Ti, etc. Iodine is quite similar to Tc in terms of half-life. If the detector is able to discriminate different energy we can take an image with multiple energies. we can inject the body with 2 markers (a: Tc, b: I123). One marker goes to one pathology, the other addresses another pathology. We can distinguish the two [not yet common, basically we cannot just count the rays, we must measure the spectrum]. The detector should have good energy resolution. 19keV is challenge as res :((usually like 40keV)

Generation of ^{99}Tc in hospital starting from ^{99}Mo

$$\frac{dN_{Mo}}{dt} = -\lambda_{Mo}N_{Mo} \qquad \frac{dN_{Tc}}{dt} = \lambda_{Mo}N_{Mo} - \lambda_{Tc}N_{Tc} \qquad A_{Tc} = +\lambda_{Tc}N_{Tc}$$

$\tau_{1/2\ Mo} = 66h$

$\tau_{1/2\ Tc} = 6h$

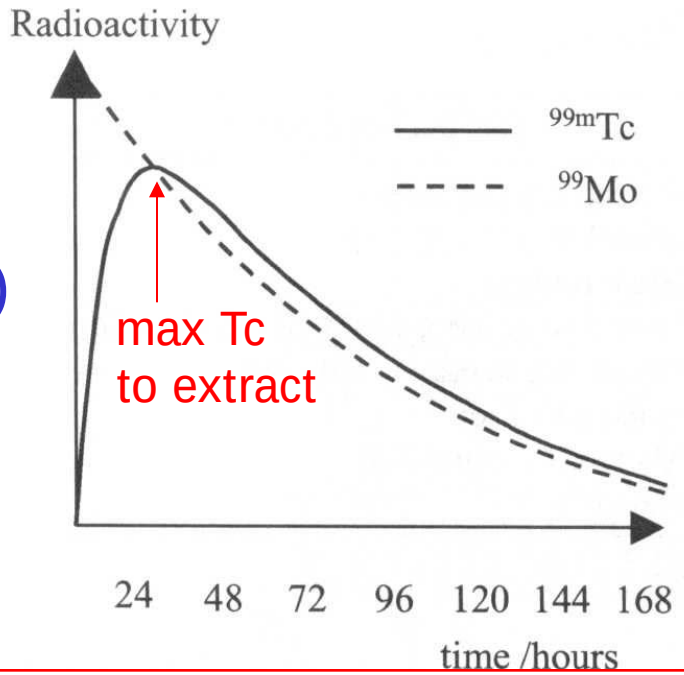
$$N_{Mo}(t) = N_{Mo}(0)e^{-\lambda_{Mo}t} \qquad N_{Tc}(t) = N_{Mo}(0)\lambda_{Mo}/(\lambda_{Tc}-\lambda_{Mo})(e^{-\lambda_{Mo}t} - e^{-\lambda_{Tc}t})$$

$$A_{Mo} = -\frac{dN_{Mo}}{dt} = \lambda_{Mo}N_{Mo}(0)e^{-\lambda_{Mo}t}$$

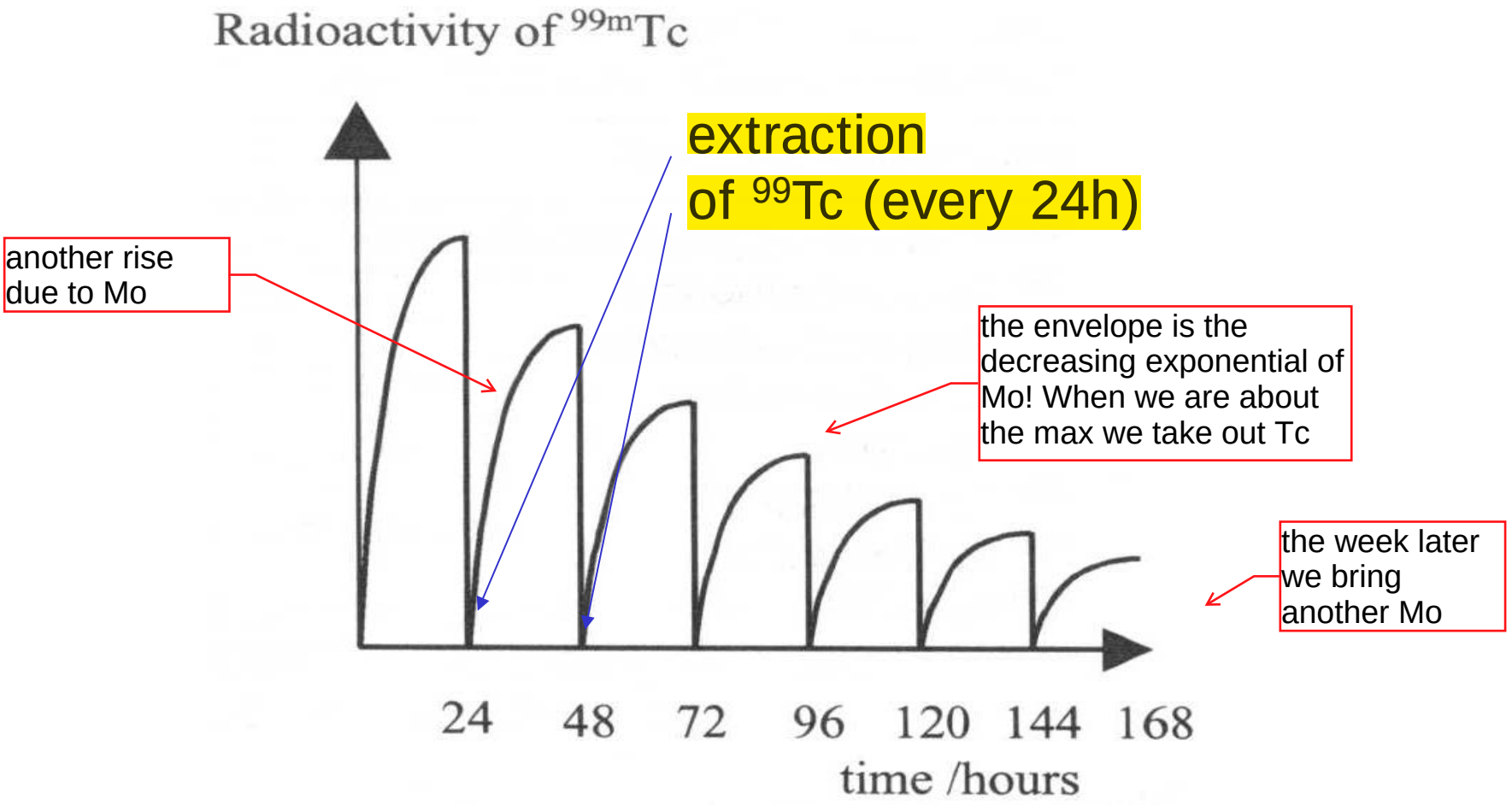
$$A_{Tc} = N_{Mo}(0)\frac{\lambda_{Mo}\lambda_{Tc}}{(\lambda_{Tc}-\lambda_{Mo})}(e^{-\lambda_{Mo}t} - e^{-\lambda_{Tc}t})$$

~ 1

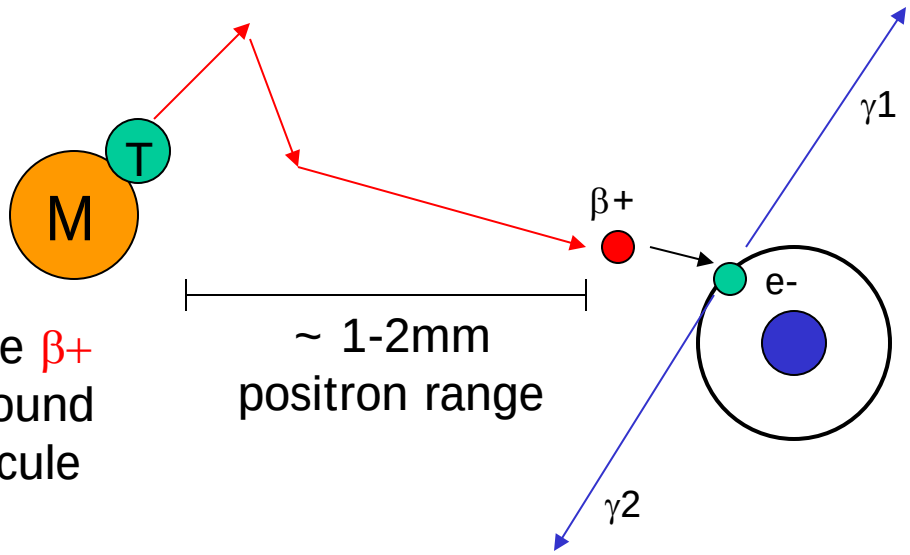
per $t >> 0$ $\qquad A_{Tc} \sim A_{Mo}$



(1)molidenum -> Tc taken and injected. We have 2 radioactive reactions. We start with a given amount of Mo (created inside an accelerator, tipically a cyclotron, taken every week to the hospital). So it goes down (dashed line). Tc: at the beginning it's 0, then Mo start to transform in Tc so it goes up. At a certain point also Tc start to disintegrate, with its own decrease


Radioactivity of ^{99m}Tc
extraction
of ^{99}Tc (every 24h)
another rise
due to Mo
the envelope is the
decreasing exponential of
Mo! When we are about
the max we take out Tc
the week later
we bring
another Mo
24
48
72
96
120
144
168
time /hours

Radionuclides β+ emitter for PET



1) β+ emission:

neutrino

$$ {}^{A}_{Z}X \rightarrow {}^{A}_{Z-1}X + \beta^{+} + \upsilon $$

2) annihilation:

$$ \beta^{+} + e^{-} \rightarrow \gamma + \gamma $$

β+ emitter isotopes typically used:
^{11}C, ^{15}O, ^{18}F, ^{13}N

TABLE 2.3. Properties of the Most Common Radionuclides Used for PET

Radionuclide	Half-life (min)
^{11}C	20.4
^{15}O	2.07
^{13}N	9.96
^{18}F	109.7

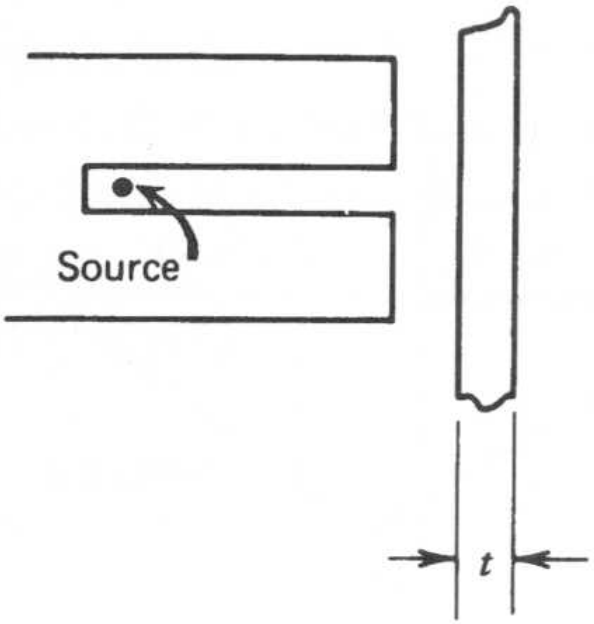
no

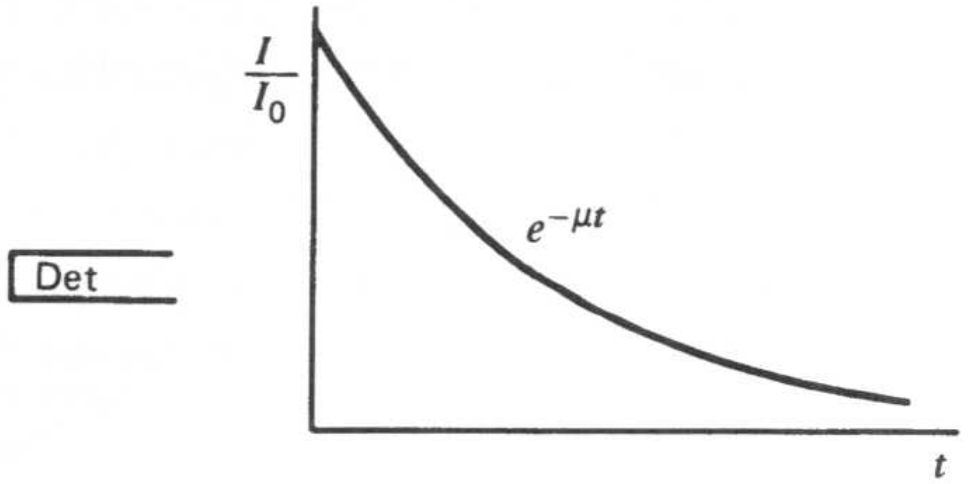
- β+ radionuclides are produced in a cyclotron (directly in hospital) by means of irradiation with protons (~10MeV) or deuterium (~5MeV)
- they are bound to the molecule by means of chemical synthesis

$$^{12}C \xrightarrow{\text{proton}} {}^{11}C + \text{proton} + \text{neutron}$$
$$^{14}N \xrightarrow{\text{proton}} {}^{11}C + {}^{4}_{2}He$$
$$^{16}O \xrightarrow{\text{proton}} {}^{11}N + {}^{4}_{2}He$$
$$^{13}C \xrightarrow{\text{proton}} {}^{13}N + \text{neutron}$$
$$^{15}N \xrightarrow{\text{proton}} {}^{15}O + \text{neutron}$$
$$^{14}N \xrightarrow{\text{deuterium}} {}^{15}O + \text{neutron}$$
$$^{18}O \xrightarrow{\text{deuterium}\ \not\text{proton}} {}^{18}F + \text{neutron}$$
$$^{20}Ne \xrightarrow{\text{deuterium}} {}^{18}F + {}^{4}_{2}He$$

Interaction of X and γ rays with matter: the absorption in materials

VALIDO PER ENTRAMBI





$$I = I_0 \exp(-\mu t)$$

absorption coefficient

μ: coefficient of linear attenuation

λ=1/μ: free mean path (absorption length)

$$I = I_0 \exp(-\mu t)$$

μ depends not only on the material composition but also on its density ρ

$\Rightarrow \mu' = \mu/\rho$ coefficient of mass absorption

$$I = I_0 \exp(-\mu' \rho t)$$

μ' depends on three absorption meccanisms:

1) photoelectric absorption
2) Compton absorption
3) production of e-/e+ pairs

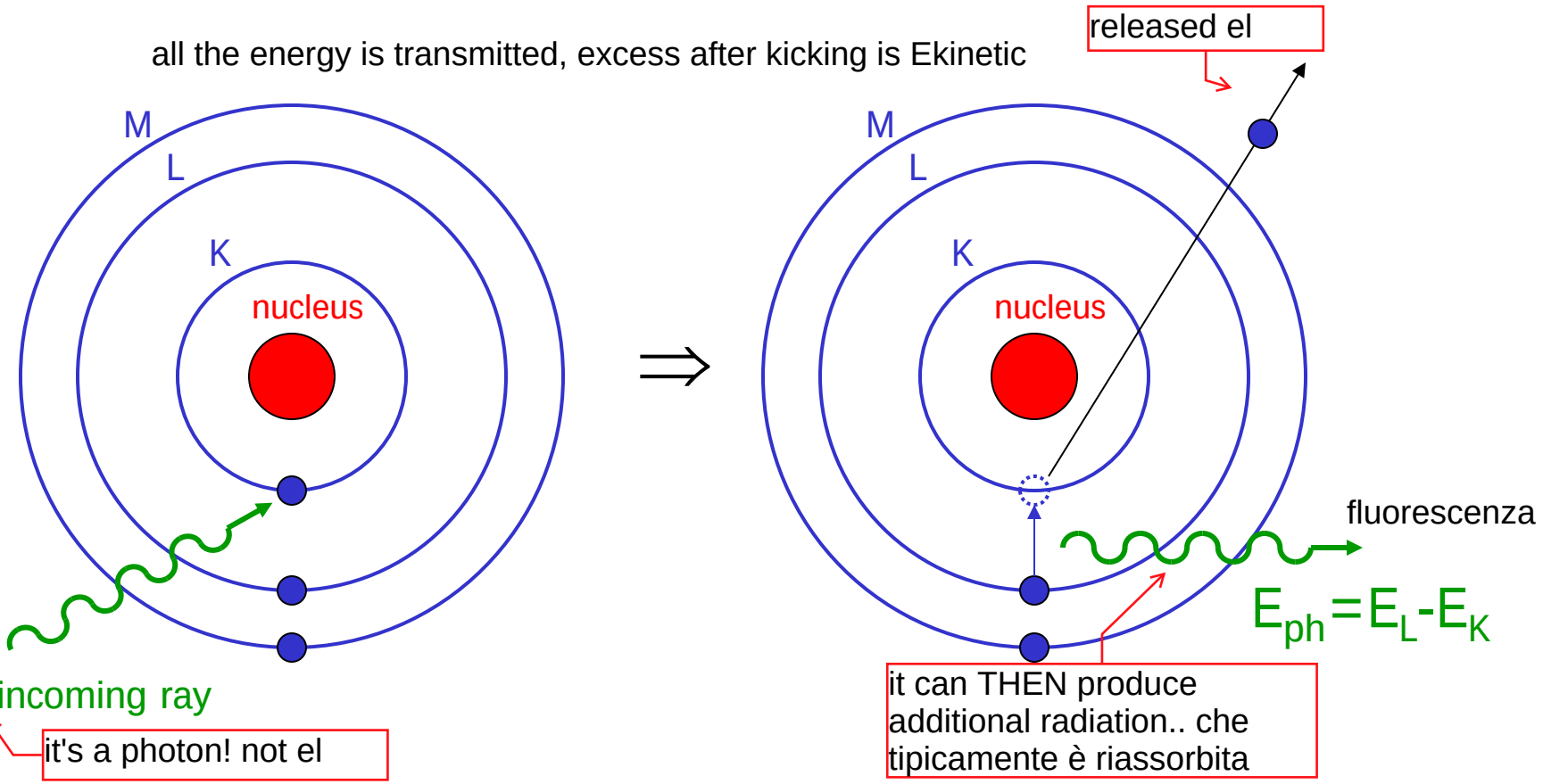
change En-> change absorp.

each one strongly depends on energy

$$\mu' = \mu'_{photoelectric} + \mu'_{Compton} + \mu'_{pair}$$

Photoelectric absorption*

all the energy is transmitted, excess after kicking is Ekinetic



the incoming radiation succeeds to extract an electron belonging to an internal shell (K,L,M), an electron of the next shell fills the empty position and an X-ray is emitted

*already see as mechanism for the production of X rays

from the energy point of view:

*l'energia cinetica è l'energia in più a quella che serviva per staccarlo

energia per staccarlo

$$E_{e-} = h\upsilon - E_b$$

energy of photoelectron (e- emitted)

energy of incoming X ray

binding energy of the e- in the shell

- the energy of the photoelectron is the following absorbed by means of next ionizations and hits in the materials

- if the incoming X ray is sufficiently energetic and is absorbed in depth, the X fluorescence photon (which has low E) is re-absorbed in the material
  $\Rightarrow$ the whole energy of the incoming photon is absorbed in the material

The photon could be [audio]

dependance

u IN PHOTOELECTRIC CASE

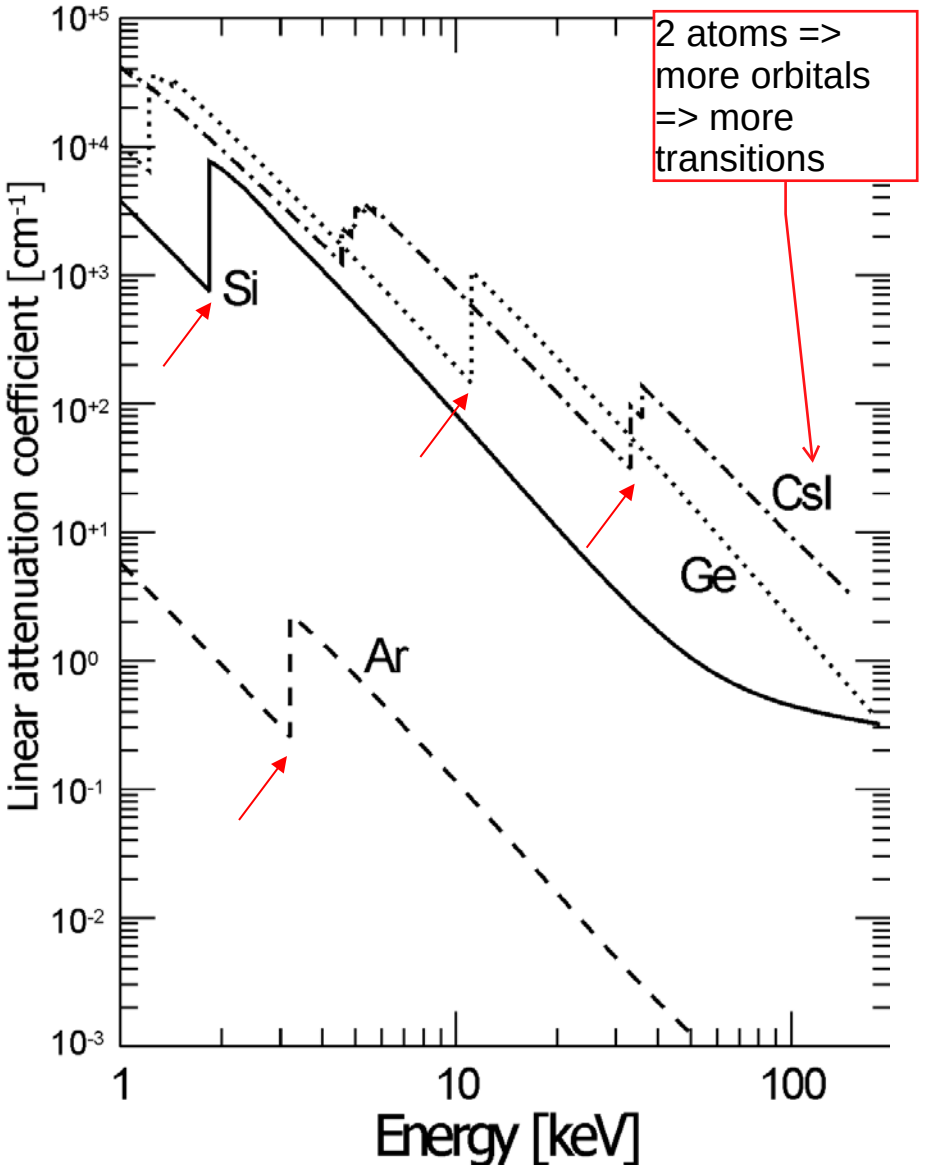
$\mu'_{photoelectric} \div Z^n/E^3_\gamma$

n: 4-5

⇒ high photoelectric absorption at low E and for materials at high Z

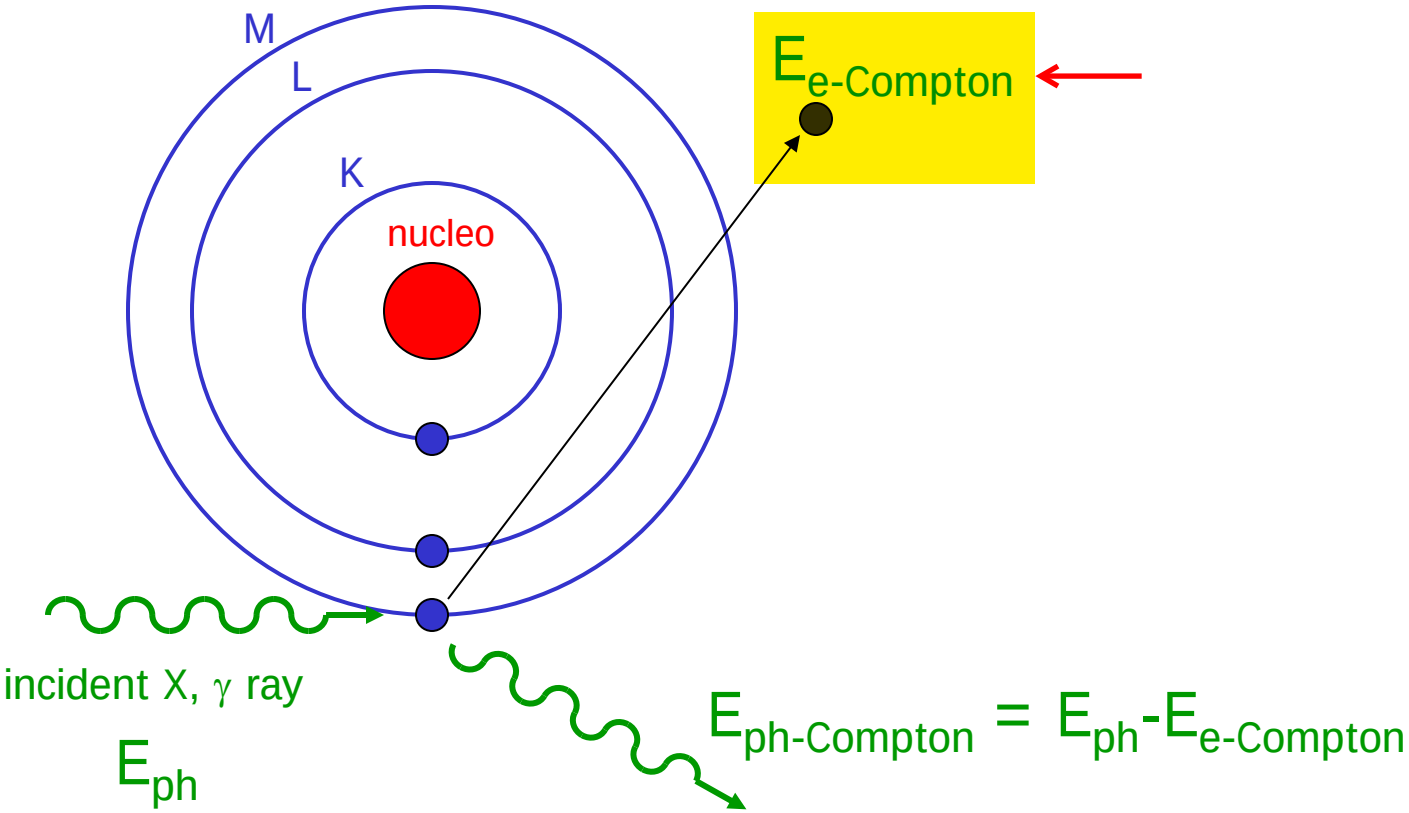
in addition, the absorption is strongly conditioned by "edges" (thresholds of energies sufficient to eject an e- from a given shell K-L-M)

⇒



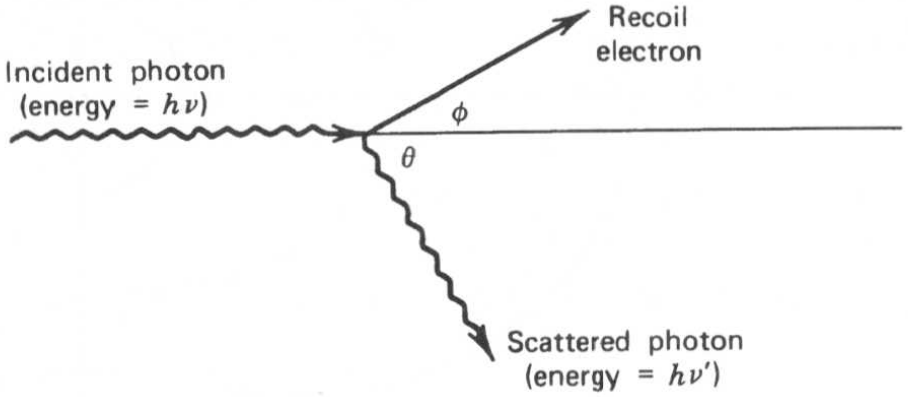
Low Z (Atomic number)=> more transparent, the opposite is true, that's why shielding material is typically lead. The HIGHER the energy of the radiation, the lower the absorption! If it's very energetic it goes through the material with less absorption. The jumps are due to the fact that at a certain energy you have the transition from the binding energy of a shell to another, you reach exactly the energy to kick an electron.

Compton Scattering



Compton scattering takes place when an incoming photon hits an electron weakly bound to the atom and produces a free electron and a photon deflected of lower energy given by the difference between the initial energy and the one of the free electron (note: also momentum is conserved)

Gammaray travels, hit the electron, the en is not completely transferd to the electron, some it is. the photon is deviated and keeps a lower energy. (this is generally speaking scattering, in our case inelastic one). Most common one.



$$h\nu' = \frac{h\nu}{1 + \frac{h\nu}{m_0 c^2}(1 - \cos\theta)}$$

Energy of emitted photon

not important to remember

$$\frac{d\sigma}{d\Omega} = Zr_0^2 \left(\frac{1}{1 + \alpha(1 - \cos\theta)}\right)^2 \left(\frac{1 + \cos^2\theta}{2}\right) \left(1 + \frac{\alpha^2(1 - \cos\theta)^2}{(1 + \cos^2\theta)[1 + \alpha(1 - \cos\theta)]}\right)$$

fraction of emitted photons at a given angle θ (Klein-Nishina)

r_0: e- radius
$\alpha = h\upsilon/m_0c^2$

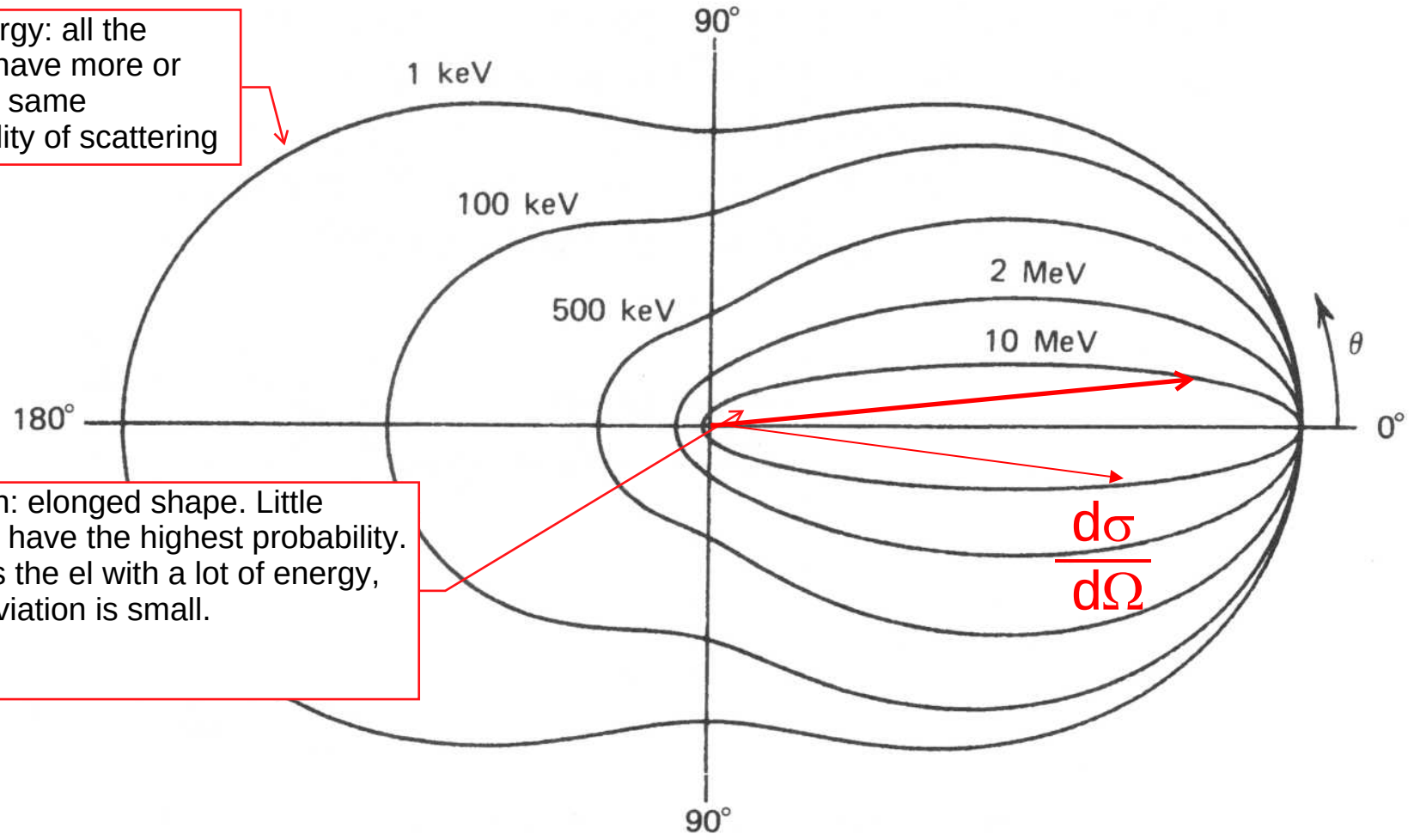
$\mu'_{Compton} \div Z$

(the probability depends on the number of electrons available for the hits and is therefore proportional to Z)

the interaction will have angles of deviations. We are interested in the scattered photon. m0 => mass of electron. The absoprtion related to scattering is proportional to Z. AGAIN, THE BIGGER THE ATOM THE HIGHER THE PROBABILITY OF THE INTERACTION.

Radial plot which relates different angles with the energy and probability of scattering. Vector module => probability that the scattering will happen, at that angle. Each curve correspond to different energies.

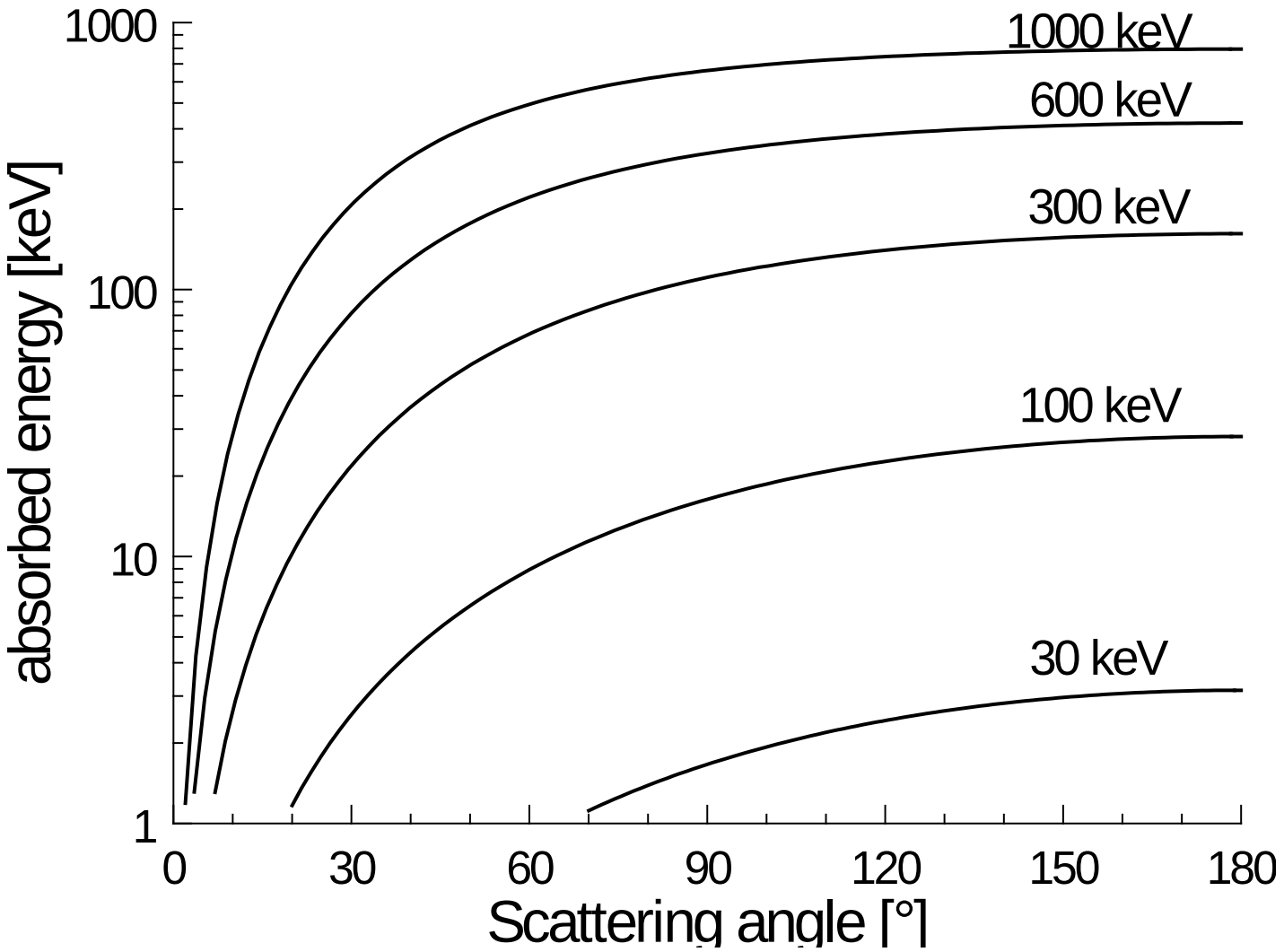
low energy: all the angles have more or less the same probability of scattering



high en: elonged shape. Little angles have the highest probability. If it hits the el with a lot of energy, the deviation is small.

Figure 2.19 A polar plot of the number of photons (incident from the left) Compton scattered into a unit solid angle at the scattering angle θ. The curves are shown for the indicated initial energies.

same thing


1000
100
10
1
absorbed energy [keV]
1000 keV
600 keV
300 keV
100 keV
30 keV
0
30
60
90
120
150
180
Scattering angle [°]

energy of the scattered photon is the same of the incoming photon, we just change the trajectory

Elastic scattering (Rayleigh)

- It is also a hit between the incoming photon and an electron weakly bound to the atom

- Differently from the Compton scattering, the hit is elastic, i.e. the photon is deflected without energy transfer to the electron

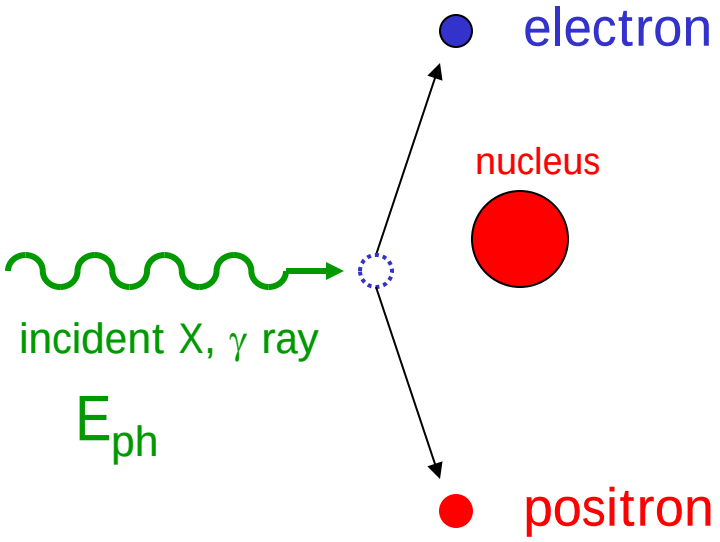
- Consequently, Rayleigh scattering does not involve energy absorption by the matter but. However, it is important because it can change significantly the flight direction of the photon
(ex. photons emitted by a region of the patient may be not collected by detectors placed along the flight direction as they are deviated; on the contrary, photons arriving from other regions can be deviated on the perpendicular direction of the detectors and they can be misunderstood as photons originated from a source aligned along this direction)

It's more "detrimental" the image we are doing. Nella inelastica, ho emissione -> scattering -> punto fittizio; però l'energy è < di quella generata.. quindi nello "spettro dell'energia" posso considerare solo quelli nella photonpeak, ignorando quelli che hanno low energy, perché sono dovuti solo ad interazioni con la materia che ne cambiano la direzione. So Compton non corrected but minimized impact by filtering. With rayleigh if it's equal to the source energy we cant! and it's taking a wrong trajectory :(. Luckily less common!

Generation of pairs

loool 1.022/2 => 511kEv ;)

one photon with energy larger than the double of energy of the electron at rest ($2m_0c^2$=1.022MeV) in the Coulombian field of a nucleus has a given probability to create an electron-positron pair



1MeV is a lot not common for medical imaging

- the Coulombian field of the nucleus increases the probability to create an *e-*/*e+* because it contributes to keep energy and momentum
- (Eph – 1.022MeV) is divided between *e-* and *e+* (less the absorb. in the nucleus)

Opposite reaction of annihilation [particle electron+positron => disappearance => release of energy]. We start from energy (in the form of gammaray radiation) and this produces two particles. we should be "in the right place" (*close to the nucleus). Condition => Eph = 2* en. of electron at rest, Einst equation. If the energy is exactly this we create the two and the photon disappear.

Sum of all mechanism

Dependence of the absorption probabilities for the three mechanisms as function of the energy and Z

low energy, gen. of pair is impossibile, compton is less relevant. predominance of photoelectric, which increases with Z.

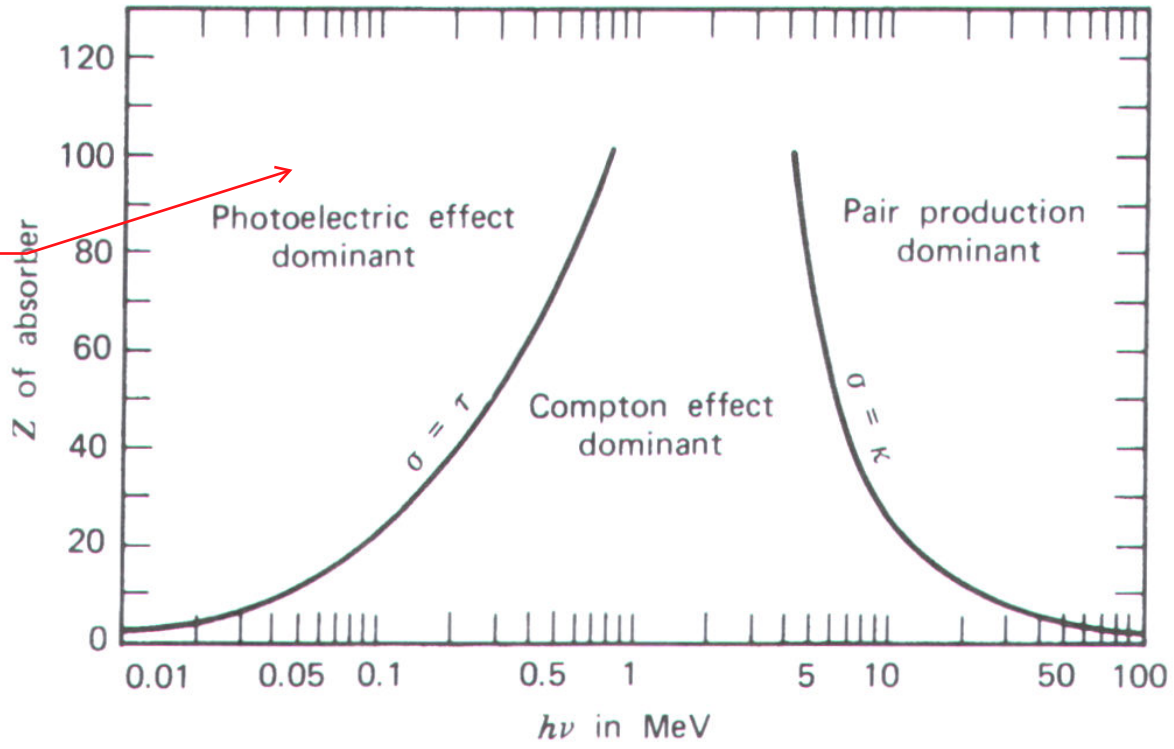
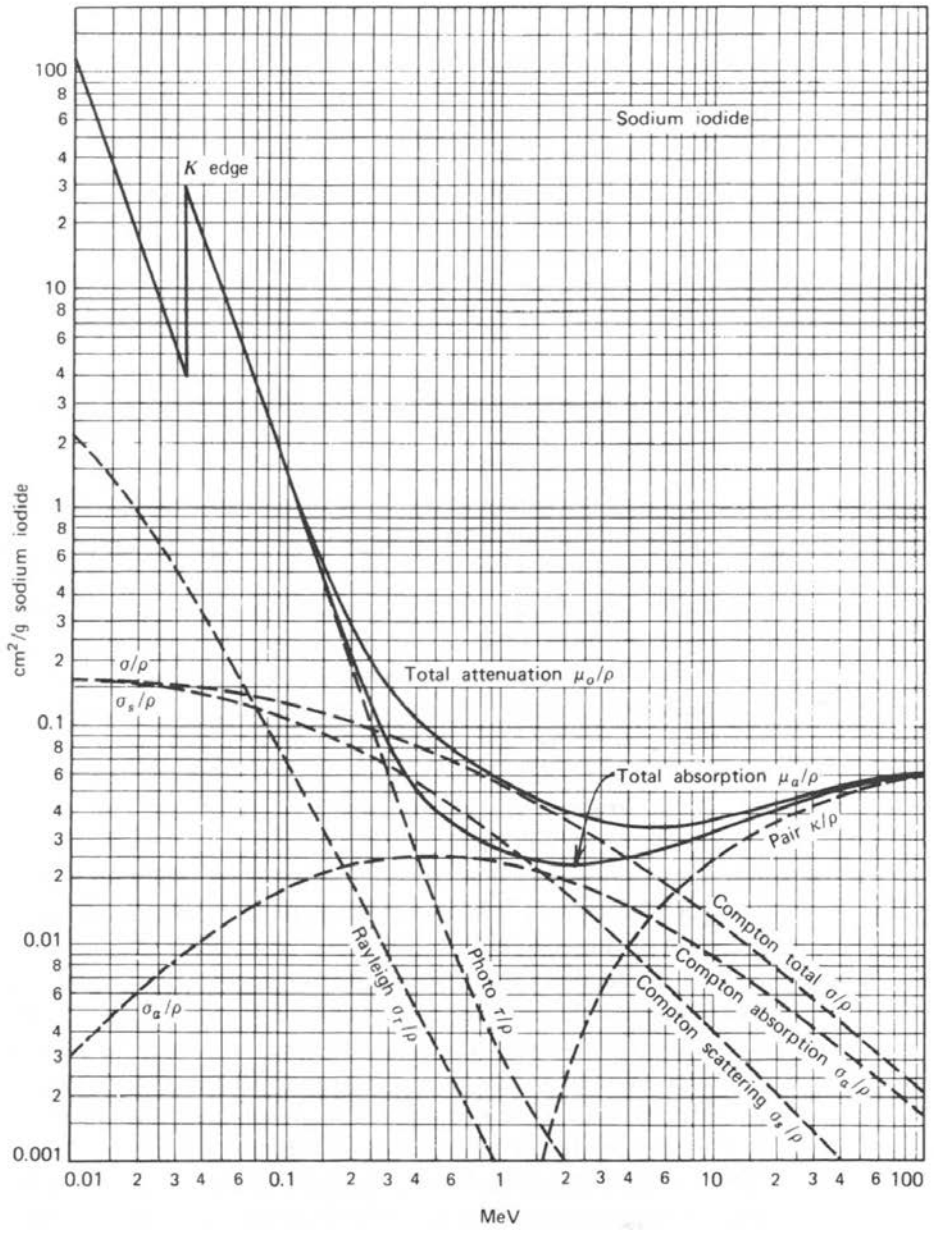


Figure 2.20 The relative importance of the three major types of gamma-ray interaction. The lines show the values of Z and $h\nu$ for which the two neighboring effects are just equal. (From *The Atomic Nucleus* by R. D. Evans. Copyright 1955 by the McGraw-Hill Book Company. Used with permission.)



ideally no interaction between photons and air/body, while instead completely to the detector, with the simplest travel. all the material in between should be minimized (*penso si riferisca al PET/SPEC)

absorption
in the sodium iodide
(scintillator)

all is true also for xray (ofc at lower energy)

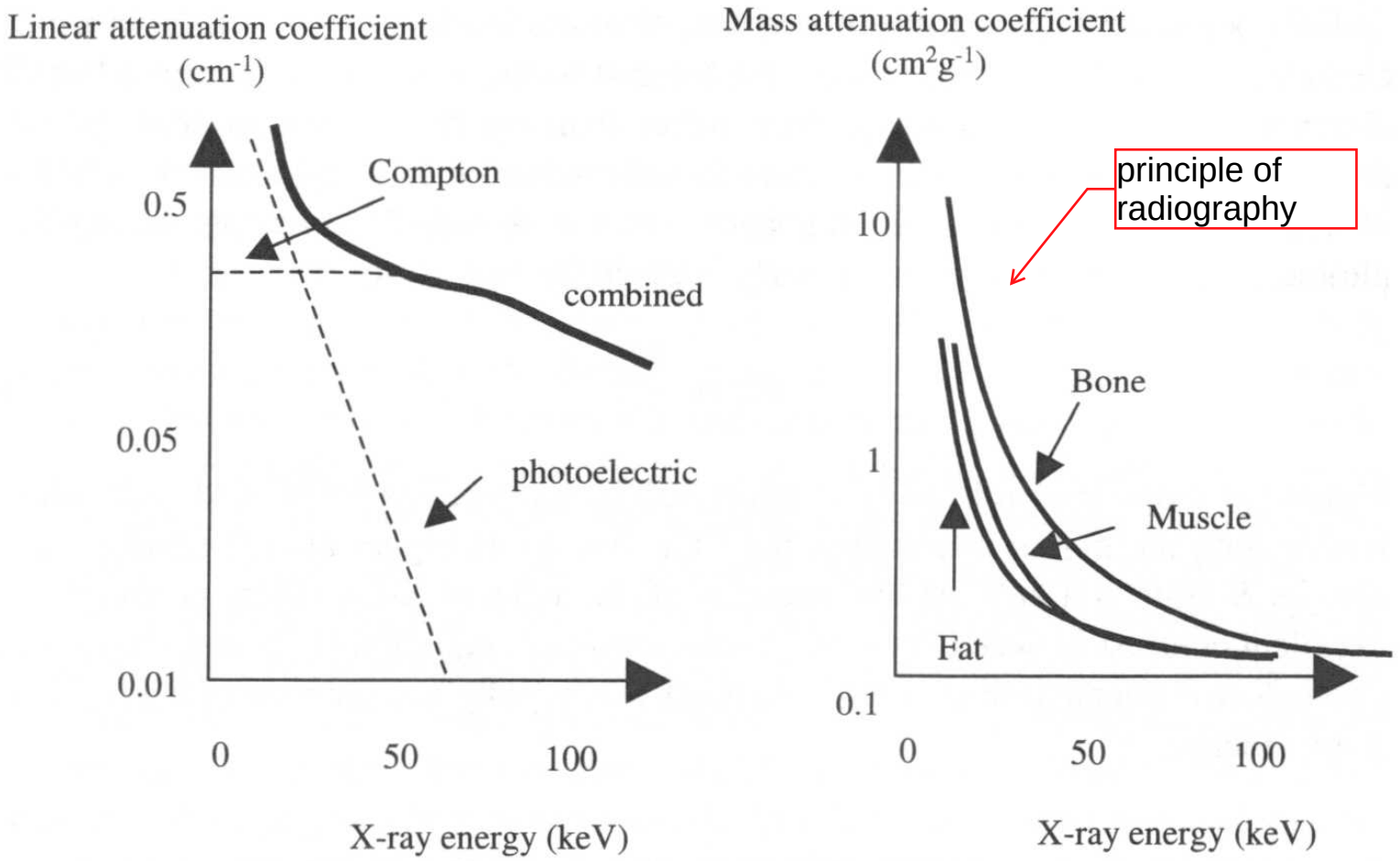
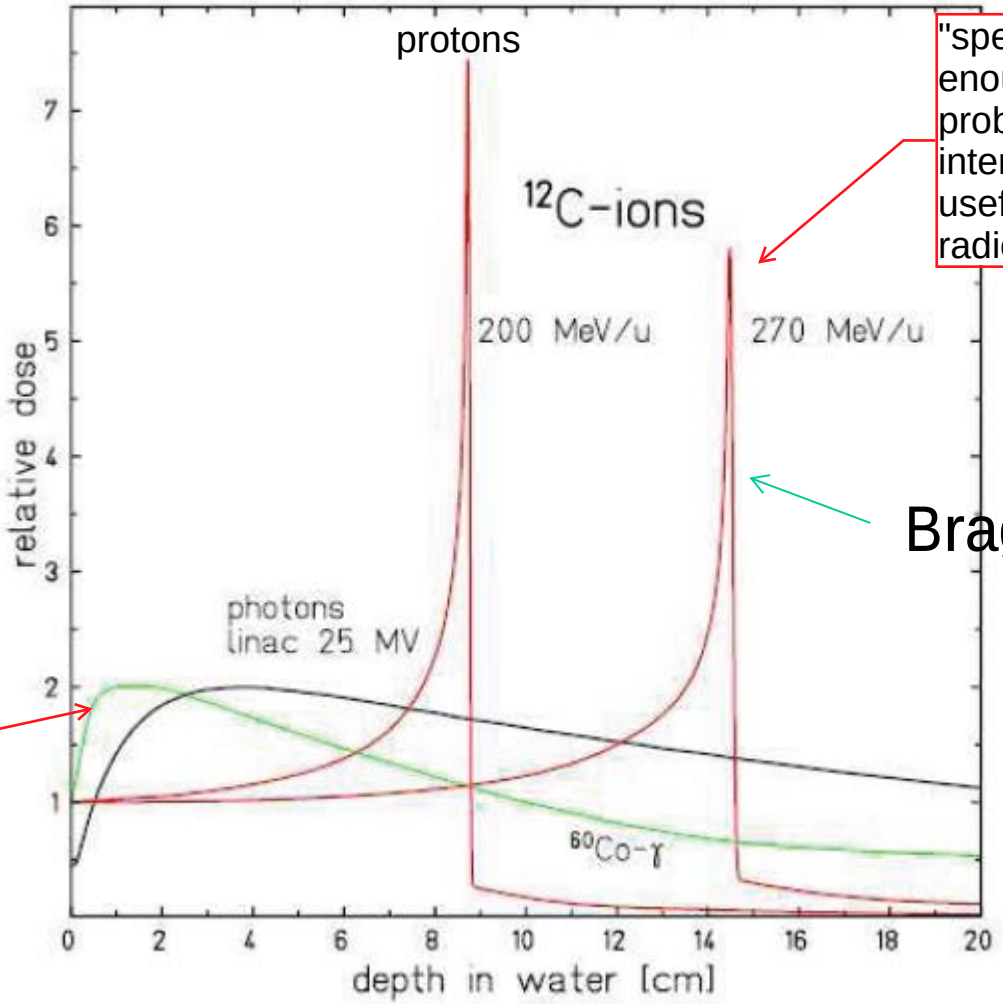


FIGURE 1.10. *(Left) The relative contributions from Compton scattering and photoelectric interactions to the linear attenuation coefficient in soft tissue as a function of the incident X-ray energy. The dashed lines represent straight-line approximations to the relative contributions, with the solid line representing actual experimental data corresponding to the sum of the contributions. (Right) The mass attenuation coefficient in bone, muscle, and fat as a function of X-ray energy.*

Comparison of interaction of charge particles vs. photons in materials



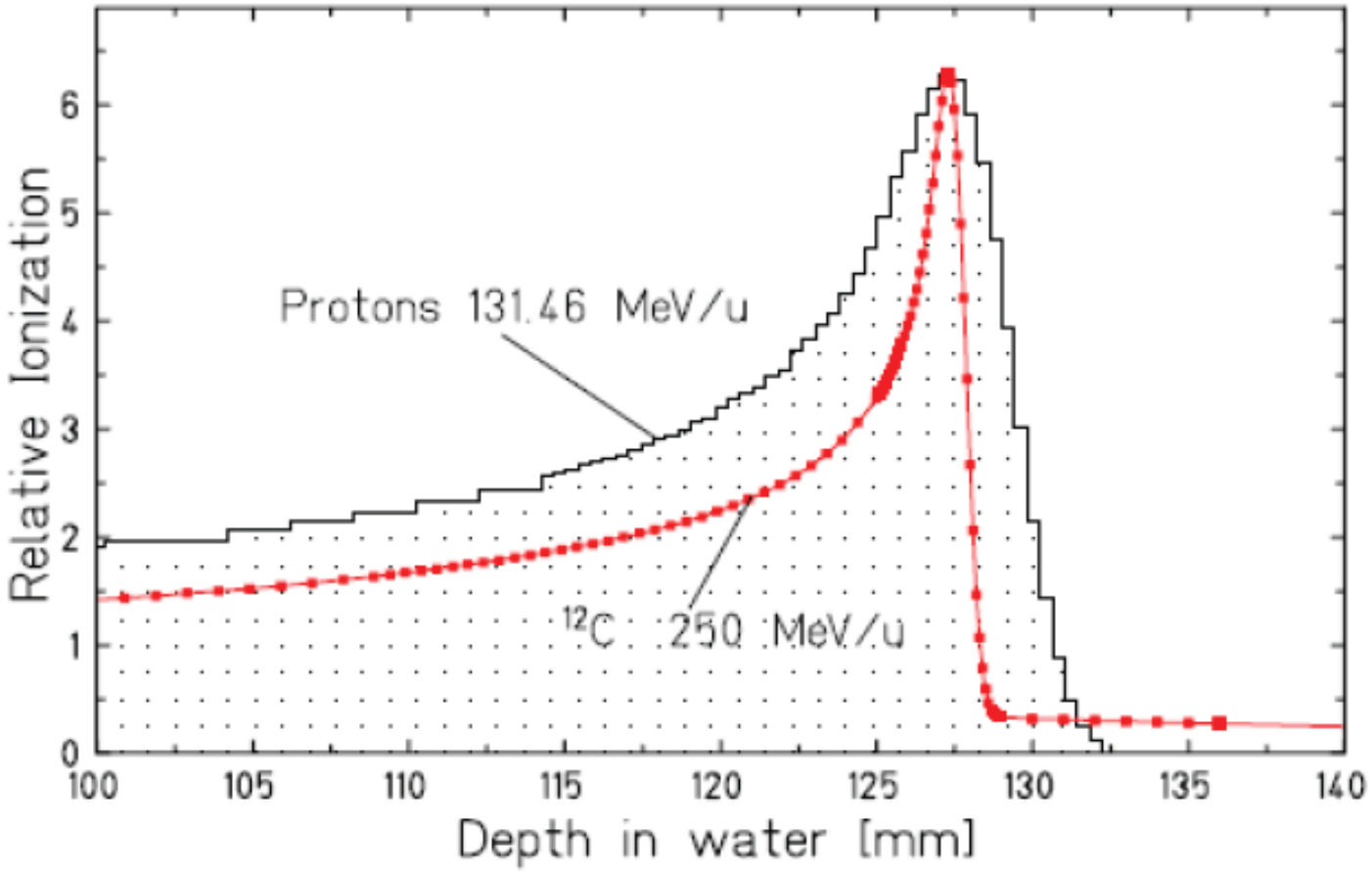
"speed" low enough => high probability of interaction. useful for radiotherapy

Bragg peak

high probability of getting absorbed

det. thickness should be matched with these lengths. Eg Si. long enough to detect all the photons. Btw when the proton is absorbed eg durante radioterapia viene emesso un gammaray che possiamo detectare per essere precisi nella delivery

no


Relative Ionization
Depth in water [mm]
Protons 131.46 MeV/u
12C 250 MeV/u
0
1
2
3
4
5
6
100
105
110
115
120
125
130
135
140