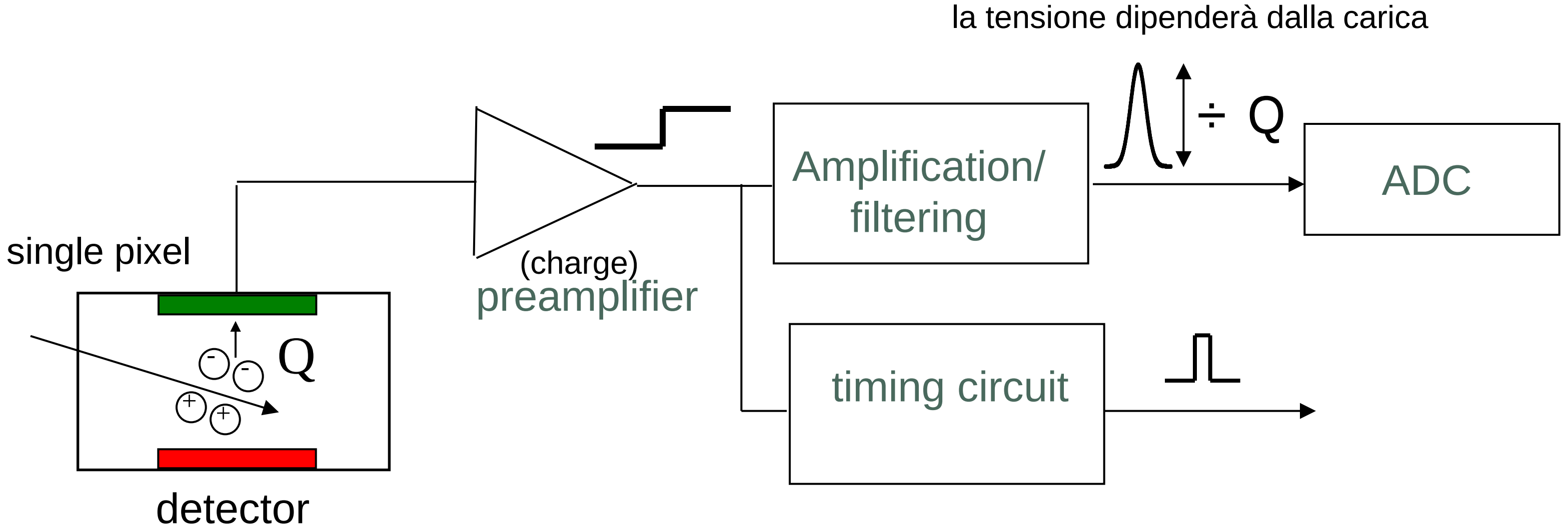
Electronics for the readout and filtering of signals from radiation detectors

[SINGLE PHOTON MEASUREMENT]



Measurement of the charge and/or of the time of occurrence

$\Rightarrow$ Measurement of the energy released by the radiation

$\Rightarrow$ Measurement of the arrival time of the event

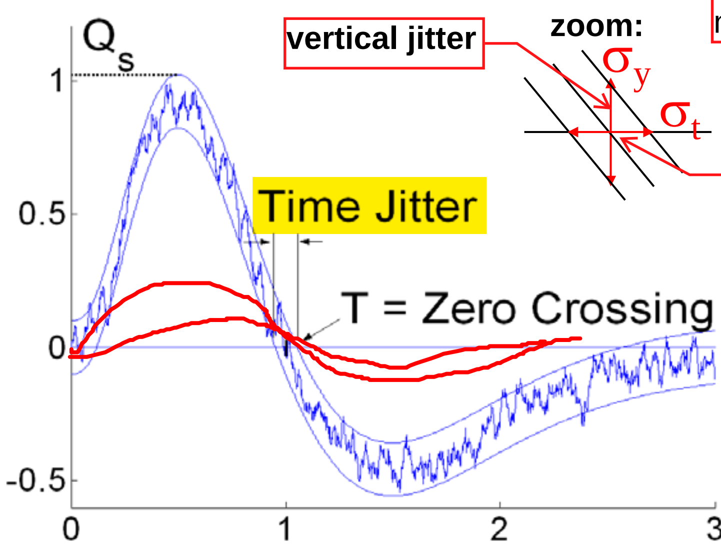
The role of the electronics noise associated to the detector and to the readout electronics circuit

$$(S/N) = \frac{N_s}{\sqrt{(N_s F + ENC^2/M^2)}}$$

Amplitude measurement

electronic noise has to be minimized

we can use Multiplications.. but it brings F :(*moltiplicazione fatta nel sensore



time jitter proportional to voltage jitter depending on the slope (the larger, the lower the effect)

how does el.noise influcences timing?

Time measurement

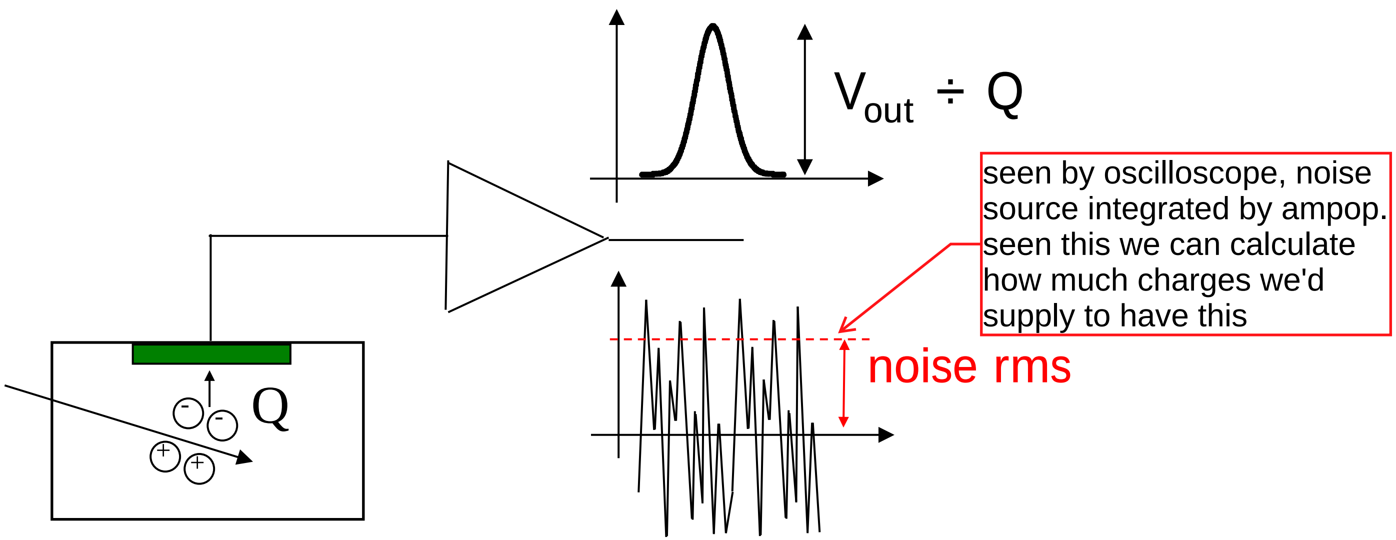
$$(N/S) \div \frac{ENC}{Q_s}$$

larger signal => larger vertical slope [audio!!!]

e.g. of technique to make timing: ZERO CROSSING. We measure the timing of the event as the time the pulse crosses the 0. It can be demonstrated that if we simply put in a comparator the bipolar pulse, and we check when it crosses 0, this time is INDEPENDENT from the amplitude of the event (varie onde di ampiezza diverse che passano per 0 tutte nello stesso momento) so **NO TIME JITTER DUE TO CHANGE IN AMPLITUDE**, which we'd have if had performed a simple threshold crossing. Ofc it's not really jitter free.. there's electronic noise: random fluctuation of my waveform, this causes a change in the exact time.. zooming we actually have multiple z-crossing :(

again: noise defined as ENC

The electronics noise: ENC - Equivalent noise charge



$$\boxed{Q = ENC \quad \Rightarrow \quad S/N=1}$$

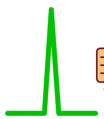
ENC: the charge to be release in the detector to produce a S/N at the output of the system equal to 1

Readout scheme with voltage preamplifier

(with charge preamplifier the conclusions are the same)

Pulse after .. we are interested in the peak amplitude

very fast! semplification

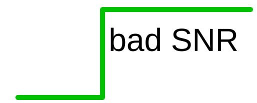


$$i_S(t) = Q\delta(t)$$

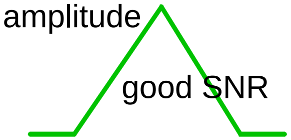
detector releasing a charge q in a delta like

Voltage step on transistor

$$v_G(t) = \frac{Q}{C_D + C_G} u(t)$$

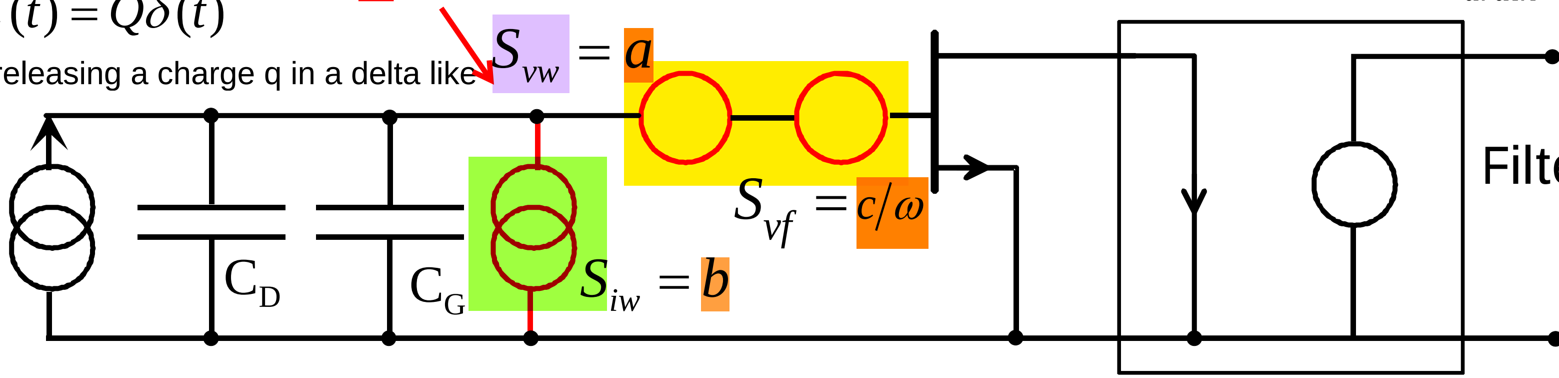


$$i_{drain}(t) = g_m v_G(t)$$



$$v_{out}(s) = i_{drain}(s) T(s)$$

$$S_{vw} = a$$



$$S_{vf} = c/\omega$$

$$S_{iw} = b$$

SNR AT THE OUTPUT:

"max output dato da 1 carica, per le cariche"

PEAK AT OUTPUT

$$\frac{S}{N} = \frac{v_{so\,peak}}{\sqrt{\langle v_{no}^2 \rangle}} = \frac{Q \cdot Max\left[v_{so\,u\delta}(t) \right]}{\sqrt{\langle v_{no}^2 \rangle}}$$

$v_{so\,u\delta}(t)$ Output amplitude in correspondence of Q=1

VARIANCE of the noise measured at the putout (the noise rms we use for ENC)

$$\Rightarrow \quad ENC = \frac{\sqrt{\langle v_{no}^2 \rangle}}{Max\left[v_{so\,u\delta}(t) \right]}$$

u(t): Heaviside

(main) noise sources (mathematical spectral noise density)

$$S_{vw}=\alpha\frac{2kT}{g_m}=\alpha\frac{2kT}{C_G}\frac{1}{\omega_T}$$

white noise:

thermal noise of the input FET

? [audio] $\omega_T=g_m/C_G$

α ~ 2/3

thermally produced current: shot noise (2qI)

$$S_{iD}=qI_D \quad \textbf{(2)}$$

shot noise of the leakage current of the detector we can reduce it cooling down the detector

$$S_{iG}=qI_G \quad \textbf{(2)}$$

shot noise of the gate current of the FET

(4)

$$S_{iR}=qI_{R\,eq}=\frac{2kT}{R}$$

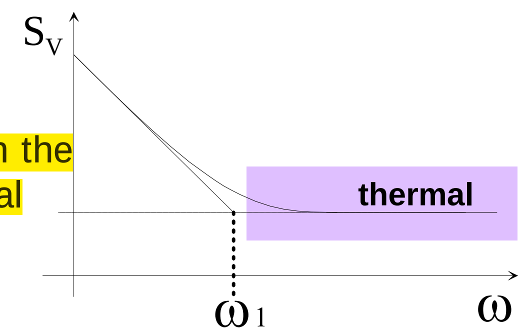
thermal noise of a possible resistance connected at the input for the discharge of the signal and of the leakage current

1/f noise

$$S_{vf}(\omega)=\boxed{\frac{1}{2}\frac{A_f}{|f|}}=\frac{\pi A_f}{|\omega|}=\cancel{\alpha\frac{2kT}{C_G}\frac{\omega_1}{\omega_T}}\frac{1}{|\omega|}$$

1/f noise of the FET

(ω_1: angular frequency at which the 1/f noise is equel to the thermal noise) **noise corner freq**



Equivalent noise sources

white, series: thermal noise of the input JFET

fast, not commented

$$S_{vw} = a = \alpha \frac{2kT}{C_G} \frac{1}{\omega_T}$$

Series white noise

white, parallel: overall shot e termal

$$S_{iw} = b = S_{iD} + S_{iG} + S_{iR} = q(I_D + I_G + I_{Req}) = qI_L$$

Parallel white noise

$$S_{vf}(\omega) = c/\omega = \alpha \frac{2kT}{C_G} \frac{\omega_1}{\omega_T} \frac{1}{|\omega|}$$

1/f series noise

note: C_D contains possible parasitic capacitances associated to the connections between detector and electronics and the possible feedback capacitance of the charge preamplifier.

***dopo** lo stadio del filtro, com'è l'ENC?

final formula, calculation not asked!!!!!!!!

ENC as function of the noise sources and of the filter

square of noise, b/c it can be simply divided by 3 contribs

thermal noise input FET

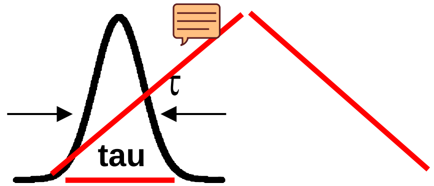
1/f noise coefficient

cumulative shot noise (detect, gate current, disc. R)

$$ENC^2 = \underbrace{(C_D + C_G)^2}_{\text{sum of cap}} a \frac{1}{\tau} A_1 + (C_D + C_G)^2 c A_2 + b \tau A_3$$

tau&A: depends on time and duration of the filter.

τ : shaping time (characteristic time related to the width of the pulse at the output of the filter)



A triangular filter is characterized by a set of A1,A2,A3, and a GIVEN time duration. **SHAPING TAU. For the representation of the noise, we have split the filter property in SHAPE (identified in A1,2,3, and DURATION rapresented by TAU). B/c we may have chosen a given filter shape but with different duration! Or we could have different shape (più arrotondato, meno arrotondato). sharing the same duration tau.**

A_1, A_2, A_3 : coefficients that depends only on the "shape" of the pulse at the output of the filter and not from its width
(note: the coefficients depends on the definition of τ)

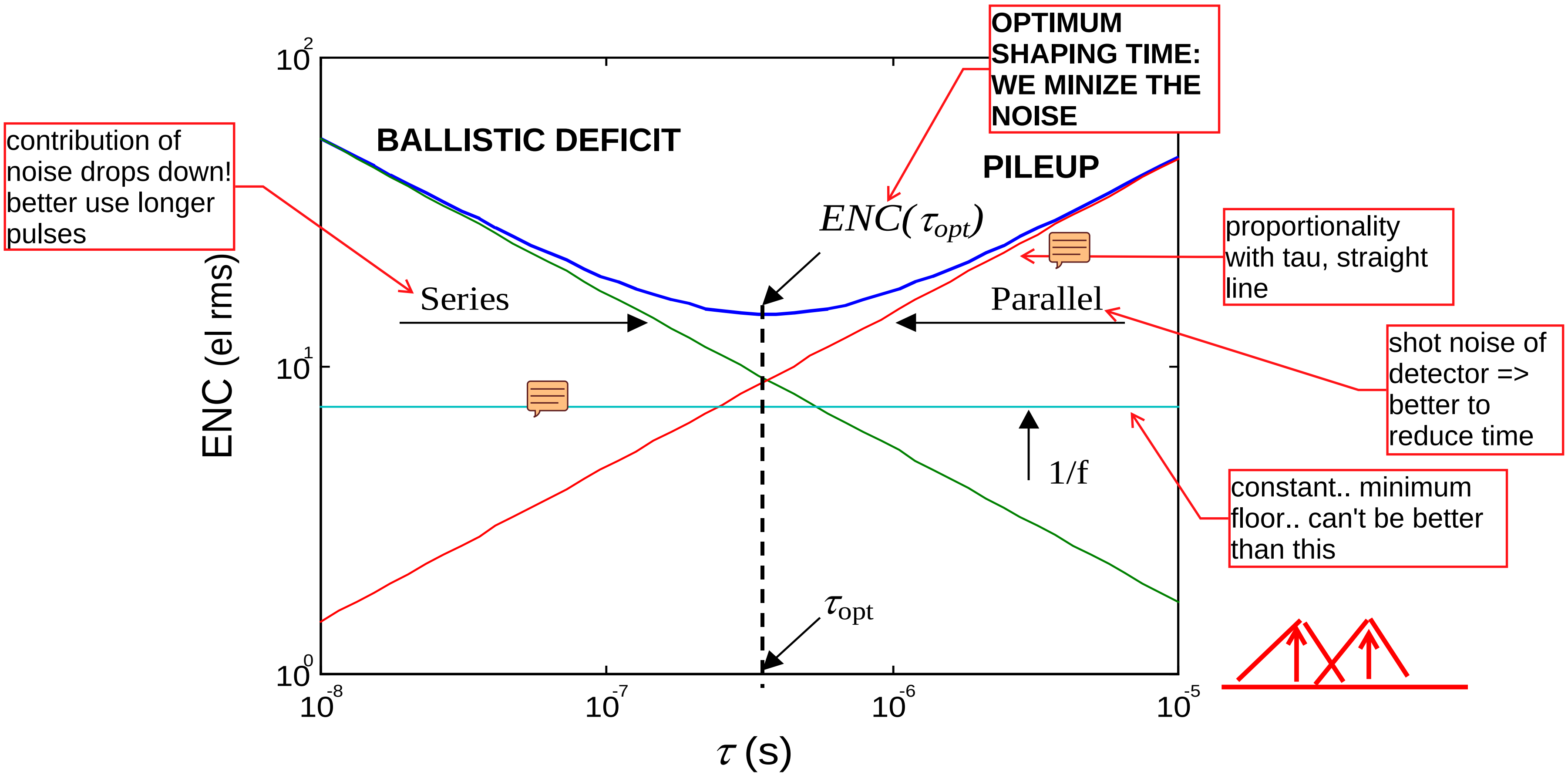
btw shaping time is an arbitrary definition... base duration, FWHM, peaking time..

So we insert parameters, and the 3 different pieces of the formula, associated with different part of noise, have a different contribution with shaping and time. And Time.

1/f: independent from time
shotnoise: proportional

so we can plot

Choice of the optimum shaping time



We can't really operate at this tau :(we may be forced to choose one or the other.
1st eg.) **shorter**? when **pileup** appears! We'd have a mess, we are forced to use shorter shaping time. We pay the price of having a worse noise :(

2nd eg) **longer**?typical case of scintillators. The current at the input of chain, given by scint., is not a delta but an exponential pulse. So if use a deltaQ and put in a filter shaping with a triangle we get a nice triangle. If we have the same charge Q distributed in an exponential (not delta..) and we make the convolution with the same shape-filter (response to the filter to a delta ;) we obtain un triangolo smussato, con l'amplitude ridotta. this penalty in amplitude is called **BALLISTIC DEFICIT.** [loss of amplitude if we process a signal with some duration]. eg.2.2: det. with collection time very long, common problem. amplitude reduced :(=> we worsen the noise [b/c ENC is calculated with SNR = 1... the signal has been penalized.. the noise no!.. so we'd need more equivalent charge to match the ratio!] We solve this using a longer tau => it's less sensitive to duration of the signal. "larger the pulse, more the exp. lookslike delta"

57.00, very quickly calculation, no details, just guide [da qui in poi fatto tutto in 7 minuti]

ENC calculation

simply parallel b/w the two.. we can take all the noise generator and calculate the laplace output.

time domain | Laplace

$$i_D(t) = Q\delta(t) \quad \rightarrow \quad I_D(s) = Q$$

unitary step

$$i_{DRAIN}(t) = \frac{Q}{C_D + C_G} g_m u(t) \quad \rightarrow \quad I_{DRAIN}(s) = \frac{1}{s}\frac{Q}{C_D + C_G} g_m$$

$u(t)$: Heaviside

we can go back
simply with the reverse Laplace

amplitude

heavyside

$$v_{so}(t) = \frac{Q}{C_D + C_G} g_m \cdot \mathrm{L}^{-1}\left[\frac{1}{s}T(s)\right] \quad \leftarrow \quad V_{so}(s) = \frac{Q}{C_D + C_G} g_m \frac{1}{s} T(s)$$

output voltage: Input * Transfer function of the filter

T(s): transfer function of the filter

L^{-1}: inverse operator of the Laplace transform noise spectral densities

tdf^2 of the filter

$$S_{no}(\omega) = \left(a + \frac{2\pi a_f}{|\omega|} + \frac{b}{\omega^2 (C_D + C_G)^2} \right) g_m^2 \left| T(j\omega) \right|^2$$

Power noise spectrum at the filter output

tdf^2 del mosfet

$$\left\langle v_{no}^2 \right\rangle = \frac{1}{2\pi} \int_{-\infty}^{+\infty} S_{no}(\omega)\, d\omega$$

rms noise at the filter output (*rms*²)

$$\Downarrow$$

and then we apply the definition.

fast

per ottenere l'ENC^2 divido per la voutmax con 1 charge

$$ENC^2 = \frac{\left\langle v_{no}^2 \right\rangle}{Max^2 \left(v_{so\, u\delta} \right)} = \frac{\frac{1}{2\pi} \int_{-\infty}^{+\infty} \left(a + \frac{2\pi a_f}{|\omega|} + \frac{b}{\omega^2 (C_D + C_G)^2} \right) g_m^2 \left| T(j\omega) \right|^2 d\omega}{\frac{g_m^2}{(C_D + C_G)^2} Max^2 \left(L^{-1} \left[\frac{1}{s} T(s) \right] \right)}$$

and then we integrate, but we change the variables and instead of w we define x=w*duration of pulse

we assume this equal to 1

The noise is the noise calculated at the output of the filter. We perform the same laplace calculations, but for the noise gen. We take them and calculate the laplace at the output. We take the noise sepctral density and we multiply by the transfer function squared of the filter.

Considerations:

NO

1) ENC does not depend on the filter gain
(which changes signal and noise in the same way)

$\Rightarrow$ let us assume $Max\left\{L^{-1}\left[T(s)/s\right]\right\}$ = 1

2) we represent the frequency response of the filter as function of the dimensionless variable $x = \omega\tau$

where τ is a characteristic time associated to the time width of the pulse

fast

$$ENC^2 = (C_D + C_G)^2 \, a \frac{1}{\tau}\frac{1}{2\pi}\int_{-\infty}^{+\infty} |T(x)|^2 \, dx +$$

$$+ (C_D + C_G)^2 \, 2\pi a_f \frac{1}{2\pi}\int_{-\infty}^{+\infty} \frac{1}{|x|}|T(x)|^2 \, dx +$$

$$+ b\tau \frac{1}{2\pi}\int_{-\infty}^{+\infty} \frac{1}{x^2}|T(x)|^2 \, dx$$

A1,A2,A3: integral of dimensionless variable, depending only on the shape T and not its duration

We define the shaping factors A_1, A_2, A_3 :

$$A_1 = \frac{1}{2\pi}\int_{-\infty}^{+\infty} |T(x)|^2 \, dx$$

$$A_2 = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \frac{1}{|x|}|T(x)|^2 \, dx$$

$$A_3 = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \frac{1}{x^2}|T(x)|^2 \, dx$$

Note:

These factors depend only on the "shape" of the filter and not on the width of its response, as the integrals are calculated as function of the dimensionless variable x

now the integrals are function of x which is a dimensionless value

So A1, A2, A3 are integral to respect a dimensionless variables, depending just on the shape of the filter T and not on its duration.
FINAL COMMENT: factors A1, A2, A3 are based on a specific choise of tau, for istance the base of triangle. Choosing a different def. of tau (e.g. FWHM) the shaping factor are different, but we simply use transformation formulas.

:((((

$$\Rightarrow \quad ENC^2 = (C_D + C_G)^2 a \frac{1}{\tau} A_1 + (C_D + C_G)^2 2\pi a_f A_2 + b\tau A_3$$

variare questa varia anche gm e quindi il noise

fast

Note:

Given a filter, if we change the definition of τ,

also the factors A_1, A_2, A_3 changes according to the formula:

transformation formulas

$$\tau'' = k\tau'$$

A1,A2,A3 in the new duration

$$A_1(\tau'') = k A_1(\tau')$$

useful to compare filters for the same duration

$$A_2(\tau'') = A_2(\tau')$$

$$A_3(\tau'') = \frac{1}{k} A_3(\tau')$$

... btw se semplicemente fai la formula inversa e poi sostituisci nella formula generale viene..

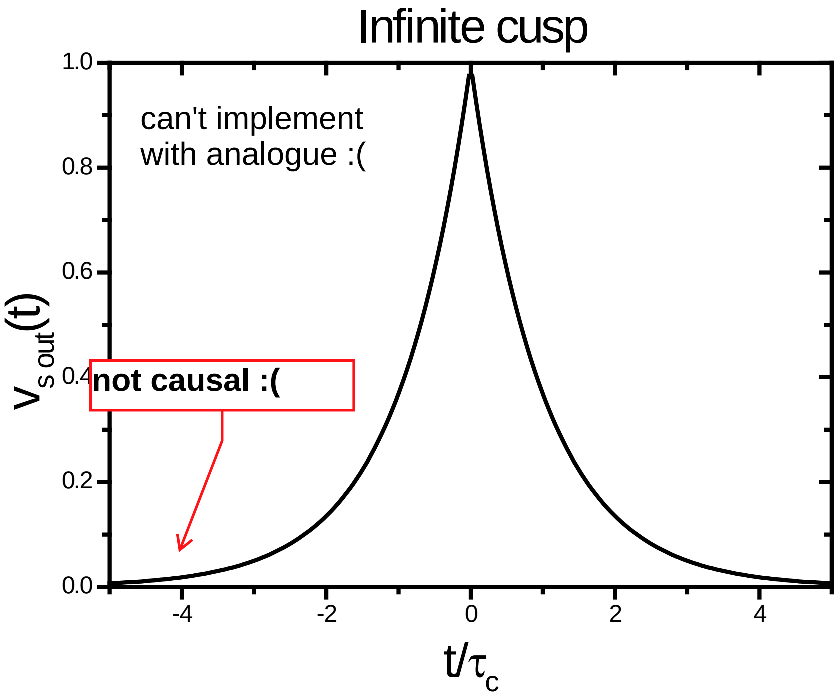
.. basically: it's important to make integral calculation for a given shape only once. se cambiamo to respect a different duration def. trasformiamo.

Given the noise sources, it is possible to choose an “optimum” filter

$\Rightarrow$ Theory of the optimum filter

Example: optimum filter in presence of white noise generators only (1/f assumed negligible)

..it can be demonstrated that this filter minimize the noise



$$v_{so}(t) = \exp\left(-\frac{|t|}{\tau_c}\right)$$

$$\tau_c = (C_D + C_G)\sqrt{\frac{a}{b}}$$

THEORY OF OPTIMUM FILTER: given noise sources, we can always synth. a filter that provides the best noise performance [supplying A1A2A3 which minimize the noise formula].

Comparison between filters

Table 1
Behaviour of A_1, A_2 and A_3 for different $h(f)$ functions

	Shaping	h (t) Function	A_2	$\sqrt{A_1 A_3}$	$\frac{A_2}{\sqrt{A_1 A_3}}$	A_1	A_3	$\sqrt{\frac{A_1}{A_3}}$
1	indefinite cusp	$e^{t/\tau}$, $e^{-t/\tau}$, O	0.64 $\left(\frac{2}{\pi}\right)$	1	0.64	1	1	1
2	truncated cusp	$k = \tau'/\tau$; $-\tau'$ $-\tau$ O τ τ'; k=1	0.77	1.04	0.74	2.16	0.51	2.06
		k=2	0.70	1.01	0.69	1.31	0.78	1.30
		k=3	0.67	1	0.67	1.31	0.91	1.10
3	triangular	O τ 2τ	0.88 $\left(\frac{4}{\pi}\ln 2\right)$	1.15 $\left(\frac{2}{\sqrt{3}}\right)$	0.76	2	0.67 $\left(\frac{2}{3}\right)$	1.73
4	trapezoidal	O τ 2τ 3τ	1.38	1.83	0.76	2	1.67	1.09
5	piecewise parabolic	O τ 2τ	1.15	1.43	0.80	2.67	0.77	1.86
6	sinusoidal lobe	O τ 2τ	1.22	1.57	0.78	2.47	1	1.57
7	RC-CR	O τ	1.18	1.85	0.64	1.85	1.85	1
8	semigaussian (n = 4)	O 4τ	1.04	1.35	0.77	0.51	3.58	0.38
9	gaussian	$e^{-1/2(t/\tau)^2}$, O	1	1.26	0.79	0.89	1.77	0.71
10	clipped approximate integrator	O 0.5τ	0.85	1.34	0.63	2.54	0.71	1.89
11	bipolar triangular	O τ 2τ	2	2.31	0.87	4	1.33	1.73

$$ENC^2 = A_1 (C_D + C_G)^2 S_{Vw} \frac{1}{\tau} + A_2 (C_D + C_G)^2 2\pi S_f f + A_3 S_{Iw} \tau$$

NON POSSIAMO comparare i numeri, dobbiamo moltiplicare 0.51 * 4 (conversion factor here is 4) and 0.51 ecomes 2... and we can compare it. 3.58 MUST BE divided by 4 instead..

CR-RC filter

made by a cascade of derivative + integration

$$v_{uso}(t) = \frac{t}{\tau_c}\exp\left(-\frac{t}{\tau_c}\right)$$

$$\tau_c = RC$$

$$\tau_p = \tau_c$$

tempo di picco

one zero in the origin (which compensates the pole of the preamplifier) + two poles

PRIMA POSSIBLITA'

semigaussian filter with real poles CR-RC^n

e.g. with 6: filter of the SEVENTH order.

made by a cascade of derivative + n integrations

$$v_{uso} = \frac{1}{720}\left(\frac{t}{\tau_c}\right)^6 \exp\left(-\frac{t}{\tau_c}\right)$$

es. filter of 7th order

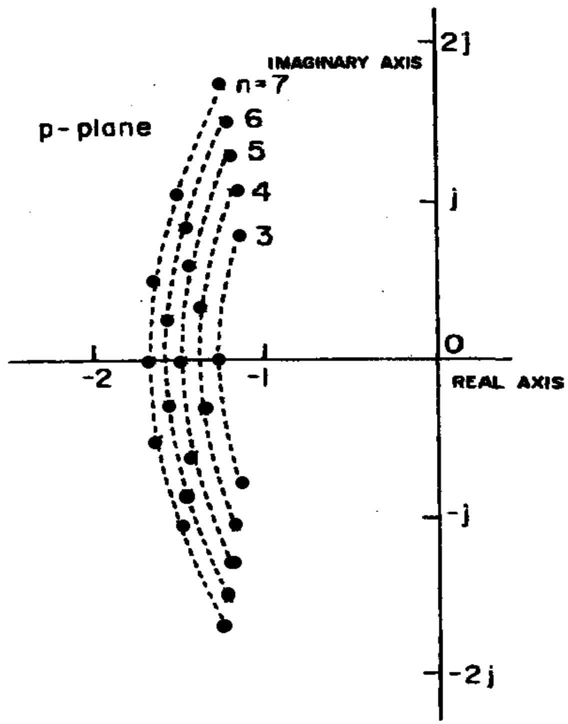
$$\tau_p = 6\tau_c$$

one zero in the origin (which compensates the pole of the preamplifier) + *(n+1)* poles

Semigaussian filter with conjugate complex poles

Approximation of the semigaussian filter, which is also an approximation of the “optimum” filter, but easy to implement

$$F := \frac{A_0\,(A_1{}^2 + W_1{}^2)\,(A_2{}^2 + W_2{}^2)\,(A_3{}^2 + W_3{}^2)}{(s + A_0)\,((s + A_1)^2 + W_1{}^2)\,((s + A_2)^2 + W_2{}^2)\,((s + A_3)^2 + W_3{}^2)}$$



Ai = real part of the pole
Wi = immaginary part of the pole