DIPARTIMENTO di ELETTRONICA, INFORMAZIONE e BIOINGEGNERIA

POLITECNICO DI MILANO

Lecture 6
Temperature Resistive Sensors

Thermo-electric analogy

Thermal parameters			Electrical parameters		
	Parameter	Unit		Parameter	Unit
ΔT	Var Temperature	K	V	Voltage	V
Q	Heat flow	W	I	Current	A
R_{th}	Thermal Resistance	K/W	R	El. Resistance	Ω
α	Thermal Conductance	W/K	G	El. Conductance	$S=\Omega^{-1}$
C_{th}	Heat capacity	J/K	C	Capacitance	F
τ_{th}	Time constant	sec	τ	Time constant	Sec

$$C_{th} = \int \frac{Q dt}{\Delta T} \rightarrow Q = C_{th} \frac{dT}{dt}$$

$$C = \int \frac{I dt}{V} \rightarrow I = C \frac{dV}{dt}$$

Measurement by contact temperature sensors: thermal equivalent circuit

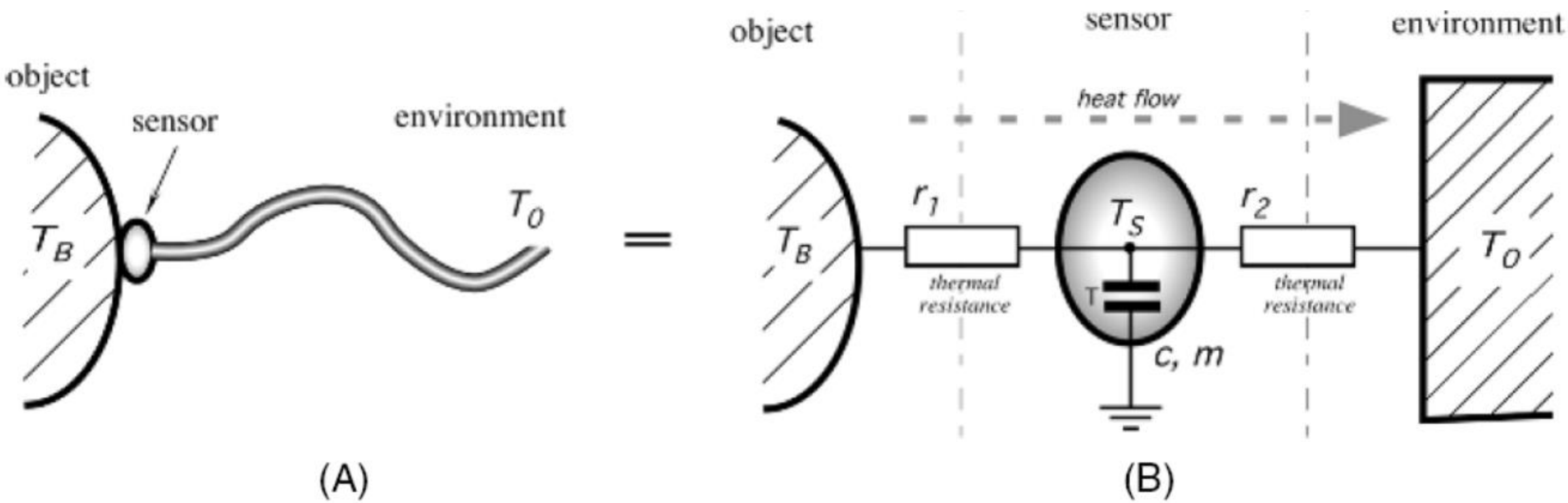


Fig. 16.1. A temperature sensor has thermal contacts with both the object and the connecting cable (A) and the equivalent thermal circuit (B).

$$\frac{T_B - T_S}{r_1} = \frac{T_B - T_0}{r_1 + r_2}$$

Temperature measurements: statics

$$\frac{T_B - T_S}{r_1} = \frac{T_B - T_0}{r_1 + r_2} \Rightarrow T_S = T_B - (T_B - T_0)\frac{r_1}{r_1 + r_2} =$$

$$= T_B - (T_B - T_0)\frac{1}{1 + \frac{r_2}{r_1}}$$

- the temperature of the sensor (T_S) is always different than that of the object (apart when $T_B - T_0 = 0$).
- T_S is more and more similar to T_B with $r_2 >> r_1$

The meaning is that to minimize the measurement error it is necessary to:

a) try to improve the thermal coupling between the object and the sensor

b) uncoupling the sensor from the environment

Temperature measurements: dynamics

$$\alpha_1 = 1/r_1\,, \qquad r_2 = \infty$$

transferred heat (body→sensor) $\quad Q = \alpha_1(T_B - T_S)$

heat absorbed by the sensor $\quad Q = mc\dfrac{dT}{dt}$

$$\alpha_1(T_B - T_S) = mc\frac{dT}{dt}$$

c =specific heat capacity [J kg^{-1} K^{-1}]

$$\tau_T = \frac{mc}{\alpha_1}$$

$$\Rightarrow \quad \frac{dT}{(T_B - T_S)} = \frac{dt}{\tau_T}$$

$$\Rightarrow \quad T_S = T_B - \left(T_B - T_0\right) e^{-t/\tau_T}$$

Temperature measurements: dynamics

$$T_S = T_B - \left(T_B - T_0\right) e^{-t/\tau_T} \quad \begin{cases} t = 0 \rightarrow T_S = T_0 \\ t \rightarrow \infty \rightarrow T_S = T_B \\ at\ any\ t,\ T_B - T_S = \left(T_B - T_0\right) \cdot e^{-t/\tau} \end{cases}$$

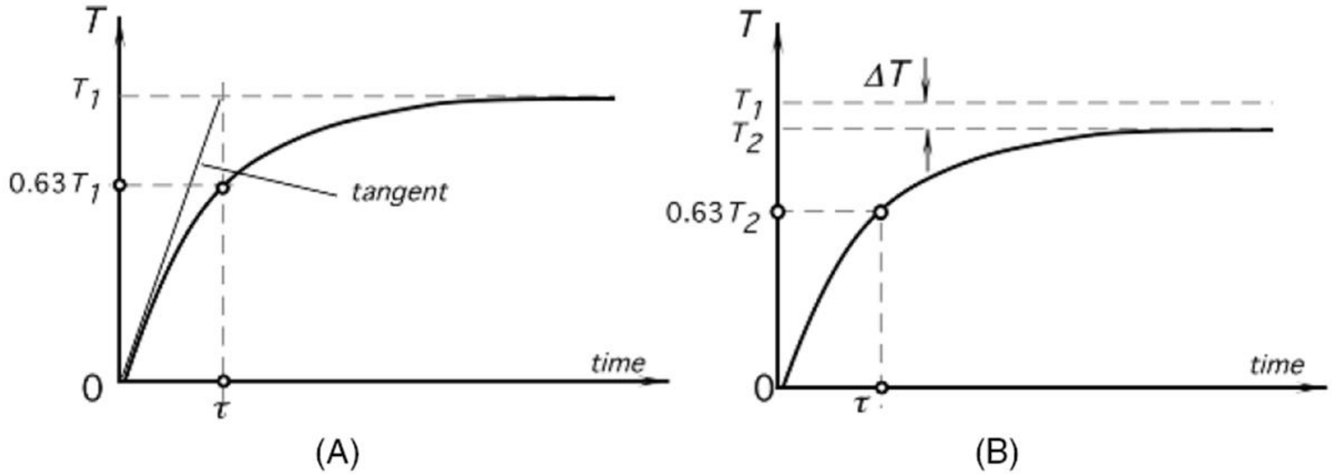


Fig. 16.2. Temperature changes of a sensing element: (A) the element is ideally coupled with the object (no heat loss); (B) the element has heat loss to its surroundings.

$$note\ that\ if\ \ r_2 \neq \infty \Rightarrow \tau_T = \frac{mc}{\alpha_1 + \alpha_2} = \frac{mcr_1}{1 + r_1/r_2}$$

Contact and thermal radiation sensors

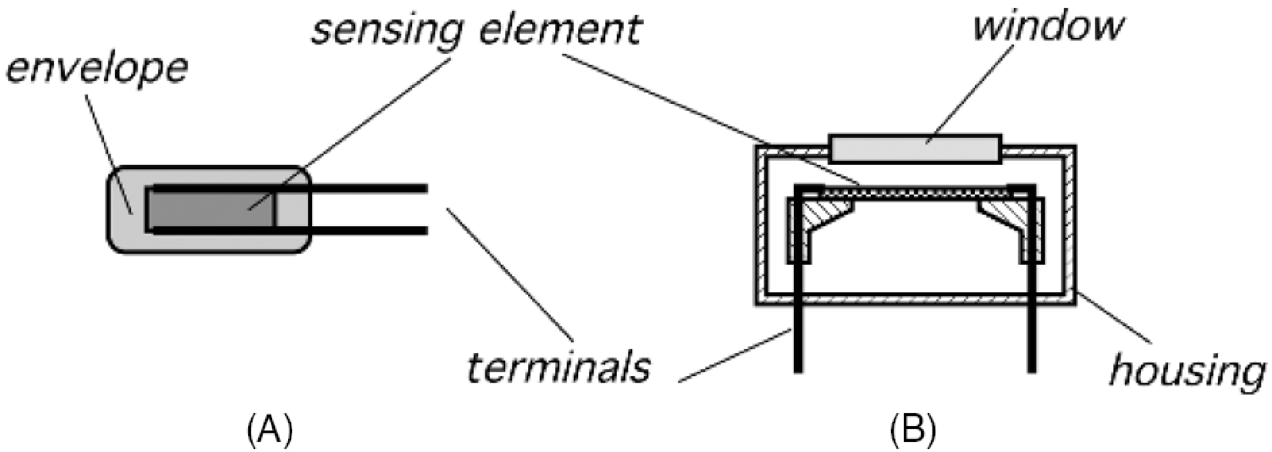


Fig. 16.3. General structures of temperature sensors: (A) contact sensor and (B) thermal radiation sensor (noncontact).

Elements of the contact sensors

1. **Sensible element** (low specific heat, small mass, high thermal conductivity, and strong and predictable temperature sensitivity)
2. **Contacts** (low thermal and electrical conductivity)
3. **Protecting sheathing or coating** (low thermal resistance, high electrical isolation, impermeable to moisture and other factors which may spuriously affect the sensing element)

Resistive temperature detectors (RTD)

RTDs are temperature sensors based on the change of electrical resistance in a metal. In a conductor, the number of electrons available to bring current does not depend on the temperature.

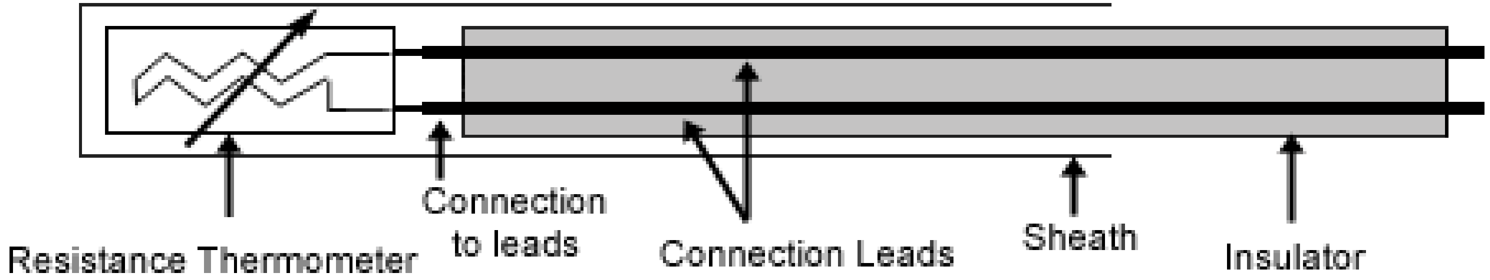
When temperature increases, however, atom vibrations around their equilibrium position increase, the dispersion of the electron increases, and their mean drift velocity reduces.

Resistive temperature detectors (RTDs)

1. **Thin-film RTDs** are often fabricated of a thin platinum or its alloys and deposited on a suitable substrate, such as a micromachined silicon membrane. The RTD is often made in a serpentine shape to ensure a sufficiently large length-to-width ratio.

2. **Wire-wound RTDs**, where the platinum winding is partially supported by a high temperature glass adhesive inside a ceramic tube. This construction provides a detector with the most stability for industrial and scientific applications.

Resistive temperature detectors (RTD)



For platinum, copper and nickel it is possible to write, at first approximation:

$$R = R_0\,[1 + \alpha_1\,(T - T_0)\,]$$

TABLE 2.3 Specifications for Some Different Resistance Temperature Detectors

Parameter	Platinum	Copper	Nickel
Resistivity at 20°C, $\mu\Omega \cdot$cm	10.6	1.673	6.844
α, $\Omega/\Omega/K$	0.00385	0.0043	0.00681
R_0, Ω at 0°C	25, 50, 100, 200, 500, . . .	10 (20°C)	50, 100, 120
Range	−200°C to +850°C	−200°C to +260°C	−80°C to +320°C

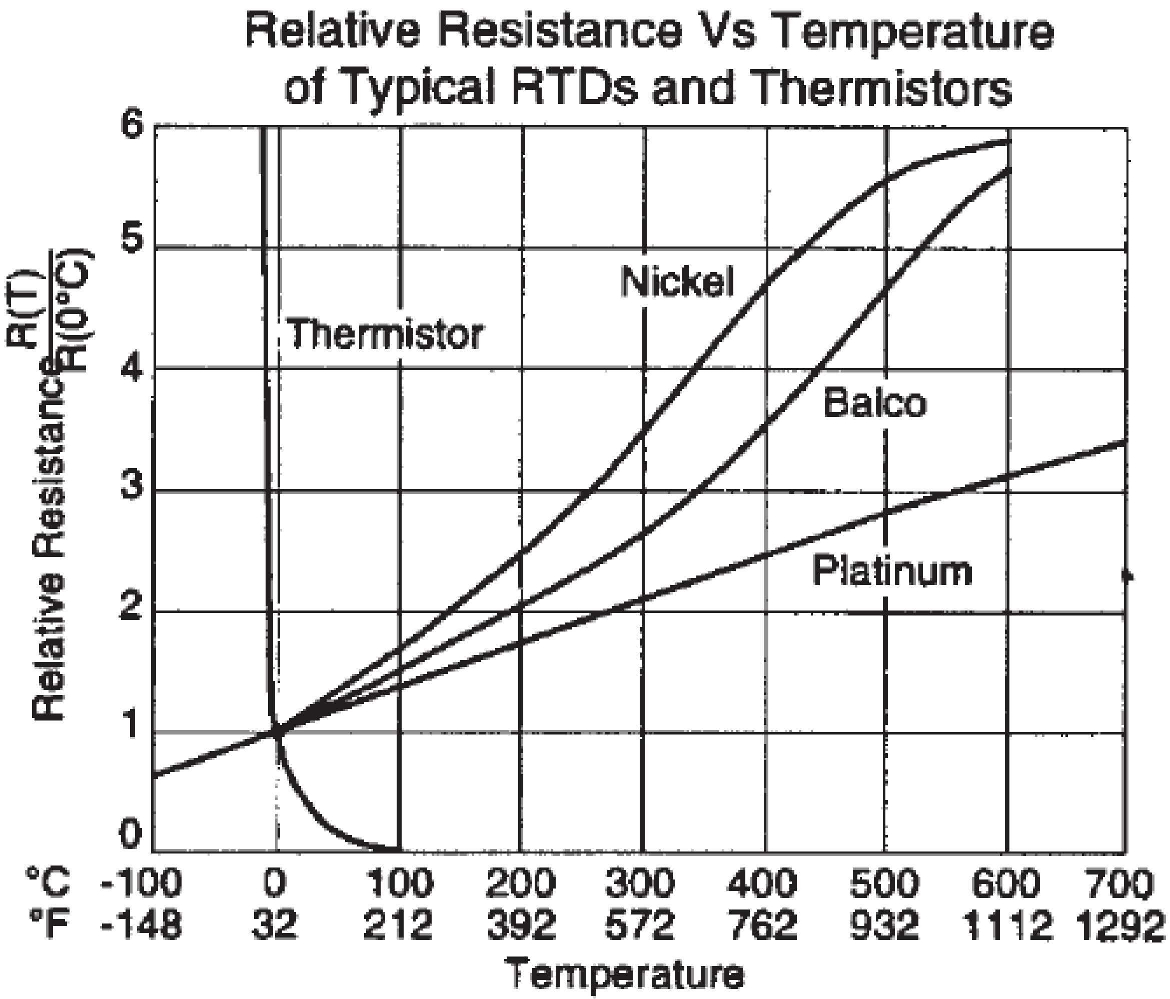
The relationship between temperature and resistance is given by the of Callendar-Van Dusen:

$$R_T = R_0\left[1 + AT + BT^2 + CT^3(T-100)\right] \;(-200\,^\circ\mathrm{C} < T < 0\,^\circ\mathrm{C}),$$
$$R_T = R_0\left[1 + AT + BT^2\right] \;(0\,^\circ\mathrm{C} \le T < 850\,^\circ\mathrm{C}).$$

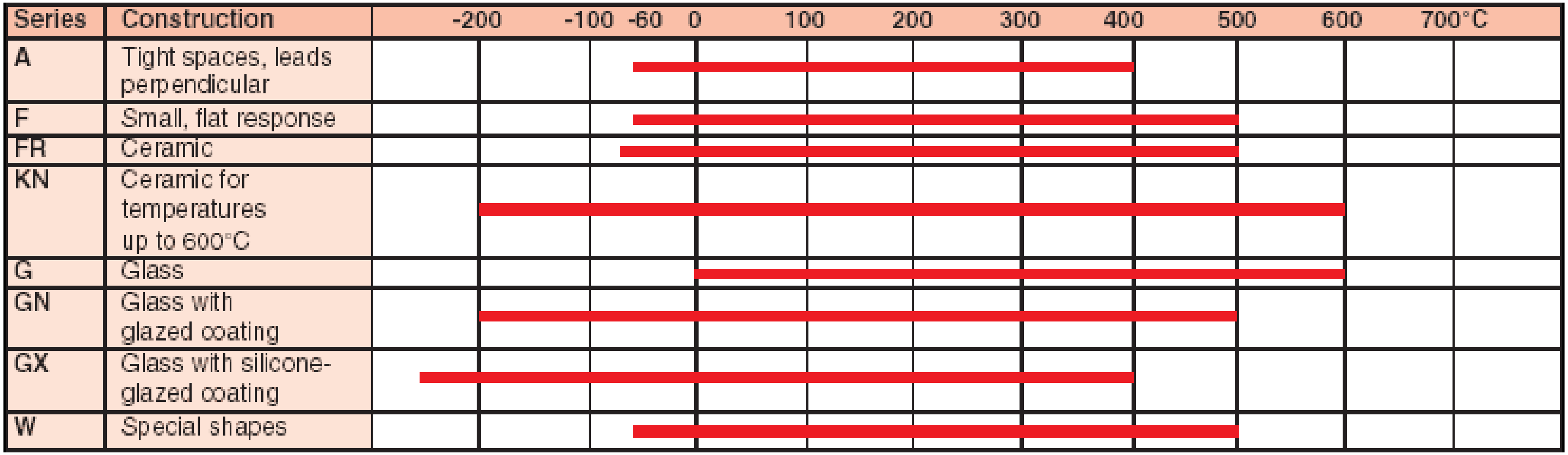
where R_T is the resistance at temperature T, R_0 is the resistance at 0 °C, and the costants (for α=0.00385 platinum RTD) are:

$$A = 3.9083 \times 10^{-3}\ ^\circ\mathrm{C}^{-1}$$
$$B = -5.775 \times 10^{-7}\ ^\circ\mathrm{C}^{-2}$$
$$C = -4.183 \times 10^{-12}\ ^\circ\mathrm{C}^{-3}.$$

Because B e C coefficients are relatively small, the resistance varies almost linearly with temperature.


Relative Resistance Vs Temperature
of Typical RTDs and Thermistors
6
5
4
3
2
1
0
Relative Resistance
R(T)
R(0°C)
Thermistor
Nickel
Balco
Platinum
°C -100 0 100 200 300 400 500 600 700
°F -148 32 212 392 572 762 932 1112 1292
Temperature

Temperature ranges



Resistive temperature detectors (RTD)

ADVANTAGES

- Sensitivity (the greatest is that of Nickel)
- Linearity (the highest is that of Platinum)
- Repeatability, long term stability and accuracy (for Platinum)
- Low cost (for copper and Nickel)

DISADVANTAGES

- It is not possible to work at too high (i.e., close to the melting point) temperatures;
- It is necessary to avoid auto-heating in the material
- Presence of mechanical deformations that can determine variation of the resistance
- It is necessary a current source
- Response time
- Instability

Temperature sensors based on metal oxides or semiconductors

NTC = negative temperature coefficient

PTC = positive temperature coefficient

They are based on the dependance of the resistance on the variation of the number of available charge carriers.

In a semiconductor, with increasing temperature, the number of charge carriers increases and this is equivalent to a decrease of electrical resistance (NTC).

This dependence varies with the doping level, that when is very high determines properties very similar to that of a metal.

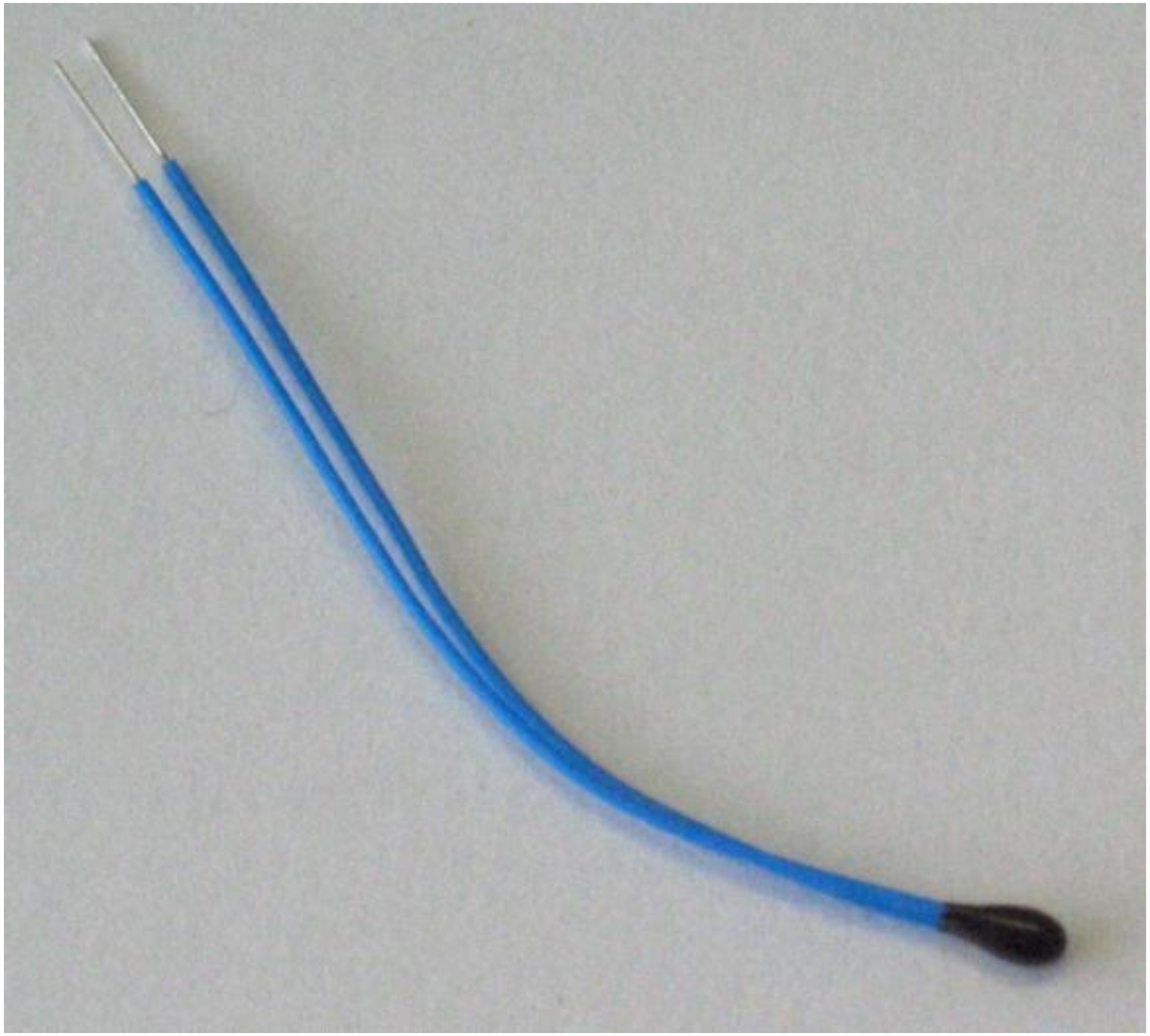
In the NTC sensors, the dependance of the resistance on the temperature can be considered exponential:

$$R_T = R_0 \exp\left[B\left(\frac{1}{T} - \frac{1}{T_0} \right) \right]$$

R_0 = reference resistance (e.g. 25°)

T_0 = 273 + 25 = 298 K

Thermistor



B (o β) = characteristic temperature of the material (~4000K), that in reality increases with increasing temperature:

$B(T_C) = B[1+\gamma\ (T_C-100)]$

R_0 = resistance at 25°

T_C = T- 273 (temperature in Celsius degrees)

$\gamma = 2.5 \times 10^{-4}$/K for $T_C > 100$°C

$\gamma = 5 \times 10^{-4}$/K for $T_C < 100$°C

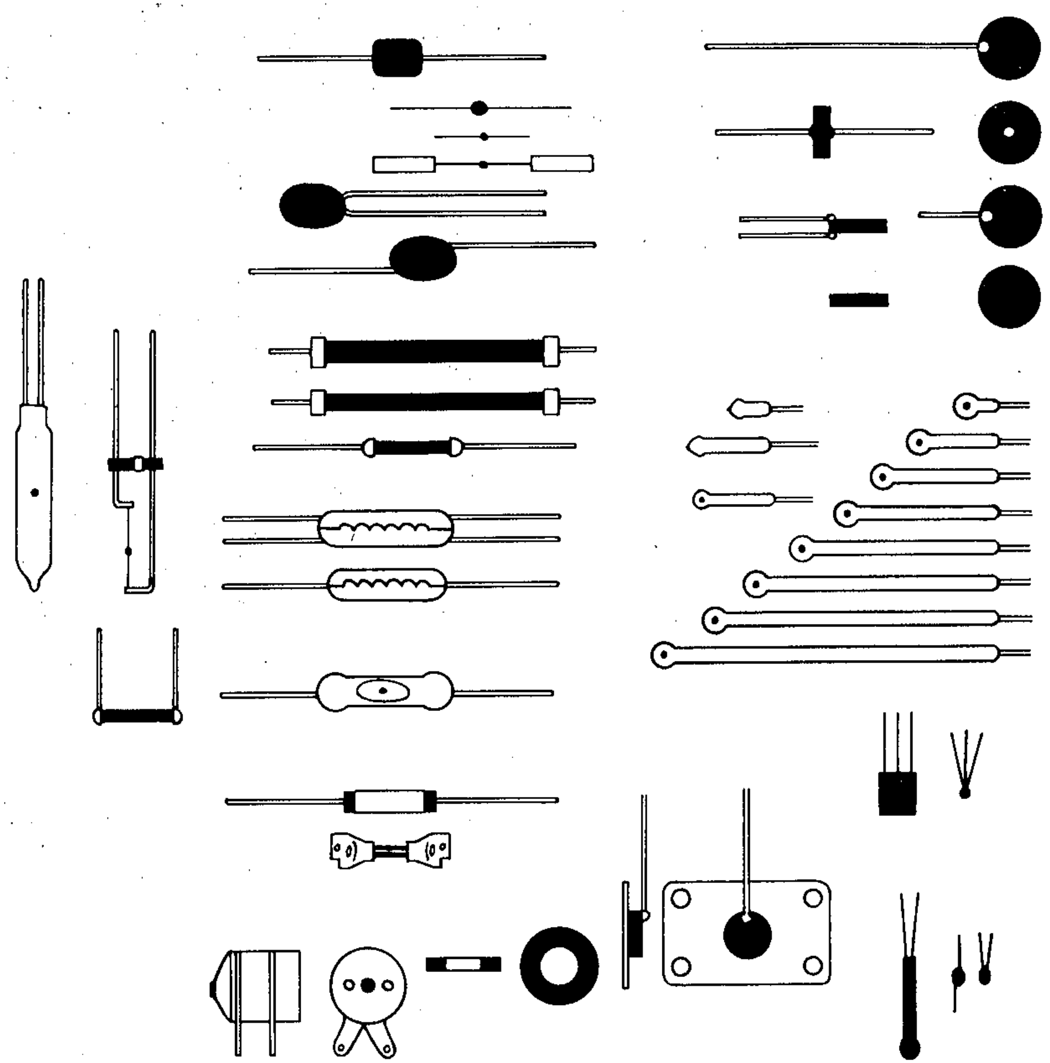
Sensitivity:

$$\alpha = \frac{dR_T / dT}{R_T} = \frac{-B}{T^2}$$

α is not constant and therefore it indicates the non linearity of the thermistor.

At 25°C, with B=4000K, α=-4.5%

(>10 times greater than a sensor Pt100)



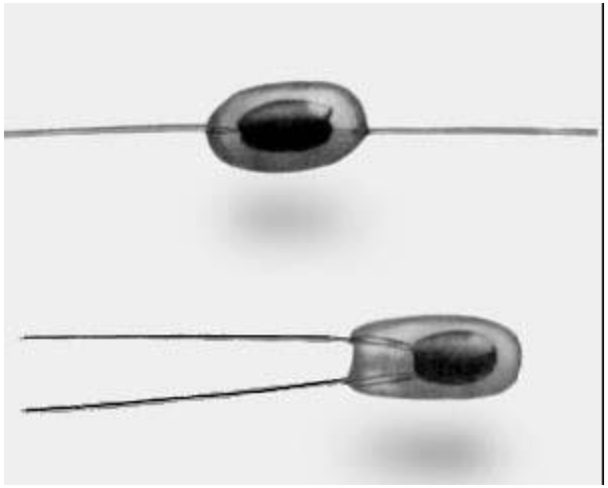


FIGURE 2.16 Different shapes for NTC thermistors (from Fenwal Electronics).

Bead-type thermistors. The beads may be bare or coated with glass epoxy, or encapsulated into a metal jacket. All of these beads have platinum alloy lead wires which are sintered into the ceramic body. When fabricated, a small portion of a mixed metal oxide with a suitable binder is placed onto parallel lead wires, which are under slight tension. After the mixture has been allowed to dry or has been partly sintered, the strand of beads is removed from the supporting fixture and placed into a tubular furnace for the final sintering. The metal oxide shrinks onto the lead wires during this firing process and forms an intimate electrical bond. Then, the beads are individually cut from the strand and are given an appropriate coating.

Chip thermistors. Surface contacts for the lead wires. Usually, the chips are fabricated by a tape-casting process, with subsequent screenprinting, spraying, painting, or vacuum metallization of the surface electrodes. The chips are either bladed or cut into the desired geometry. If desirable, the chips can be ground to meet the required tolerances.

The third type of thermistor is fabricated by the **depositing semiconductive materials** on a suitable substrate, such as glass, alumina, silicon, and so forth. These thermistors are preferable for integrated sensors and for a special class of thermal infrared detectors

Steinhart-Hart Equation

For accurate measurements of temperature, the resistance – temperature curve is described by the third-order equation :

$$\frac{1}{T} = a + b\,\ln(R) + c\,\ln^3(R)$$

Where *a, b, c* are called Steinhart-Hart parameters and are specific for each sensor.

T is temperature (in kelvin) and R is resistance (in ohm).

To express the resistance as a function of temperature, the equation must be re-arranged in the following way:

$$R = e^{\left(\beta - \frac{\alpha}{2}\right)^{\frac{1}{3}} - \left(\beta + \frac{\alpha}{2}\right)^{\frac{1}{3}}}$$

$$\alpha = \frac{a - \frac{1}{T}}{c} \qquad \beta = \sqrt{\left(\frac{b}{3c}\right)^3 + \frac{\alpha^2}{4}}$$

In Steinhart-Hart equation, the temperature measurement error is generally lower than 0.02°C.

As an example, typical values of a thermistor with a resistance equal to 3000 Ω at ambient temperature (25°C = 298.15 K) are:

$$a = 1.40 \times 10^{-3}$$

$$b = 2.37 \times 10^{-4}$$

$$c = 9.90 \times 10^{-8}$$

Resistance-temperature curve

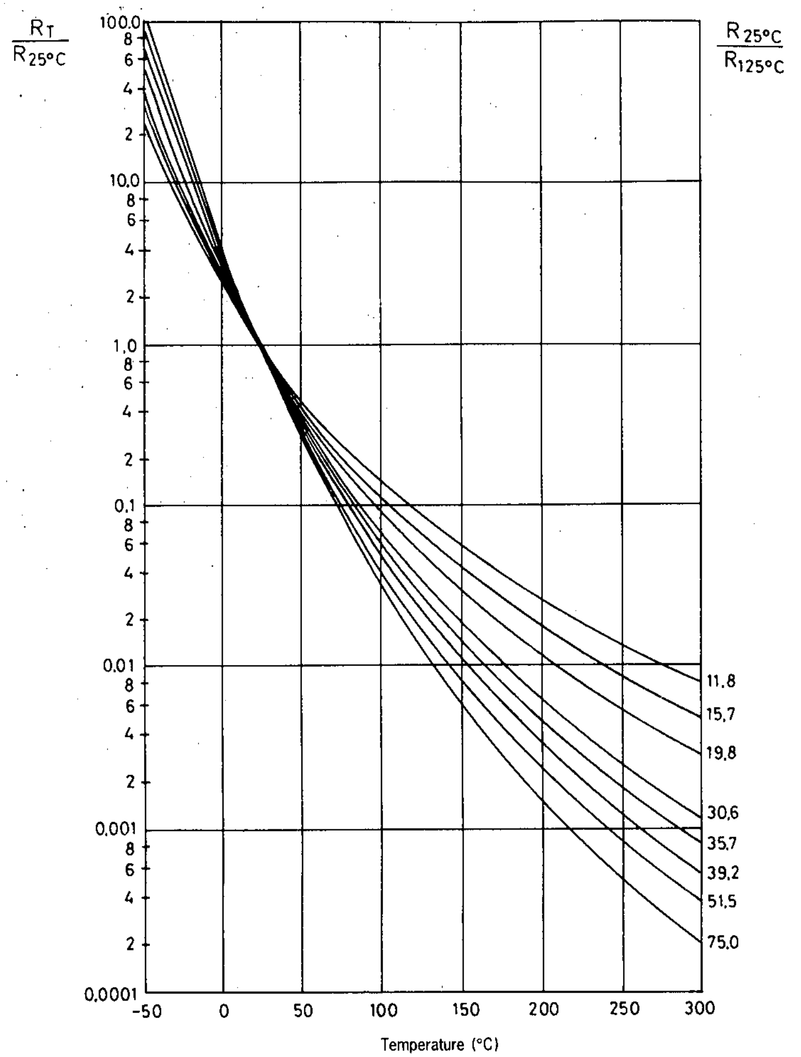


FIGURE 2.11 Resistance-temperature curve for several NTC thermistors (from Thermometrics).

Thermistors
PTC (Positive Temperature Coefficient)

In a semiconductor, with increasing temperature the number of charge carriers in the conduction band increase, thus the resistance decreases.

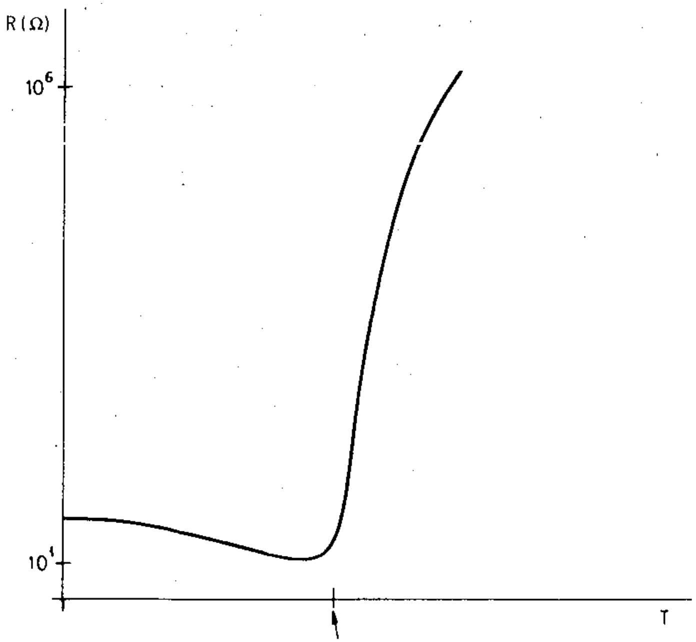
This dependence of resistance on temperature, however, depends on the number of impurities.

When doping is very high the semiconductor assume properties which are very similar to those of a metal and a positive coefficient of temperature.

PTC (Positive Temperature Coefficient) thermistors

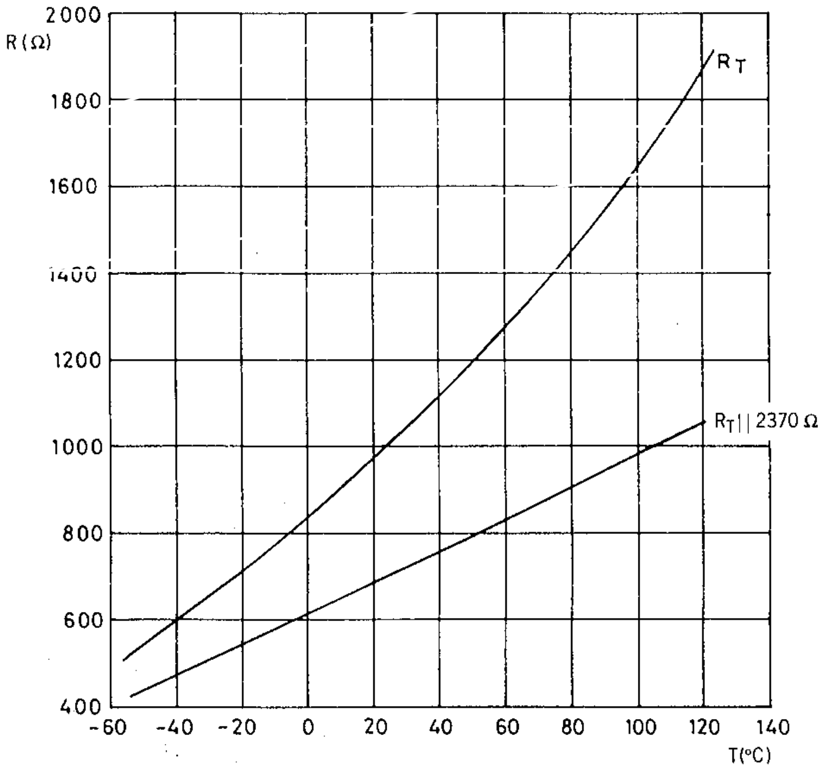
POSISTORS *("switching")*

- **Ceramic material**
- Sudden increase of R at Curie* temperature



SILISTORS

- Doped Silicon



**temperature at which a material loses ferromagnetic properties (i.e., it has an intrinsic magnetic moment) and becomes paramagnetic. In other words, is the point at which a material's permanent magnetism changes to induced magnetism*

Characteristics of PTC sensors

1. Zero power resistance, R_{25}*, at 25◦C,* where self-heating is negligibly small.
2. Minimum resistance *Rm:* is the value on the curve where the thermistor changes its TCR from positive to negative value

3. Transition temperature T_T is the temperature where resistance begins to change rapidly. It coincides approximately with the Curie point of the material. A typical range for the transition temperatures is from−30°C to +160°C

4. TCR is defined in a standard form

$$\alpha = \frac{1}{R}\frac{\Delta R}{\Delta T}$$

The coefficient changes very significantly with temperature and often is specified at point *x* (i.e., at its highest value), which may be as large as 2/°C (meaning the change in resistance is 200% per °C)

5. Maximum voltage *E*max is the highest value the thermistor can withstand at any temperature.

6. Thermal characteristics are specified by a thermal capacity, a dissipation *δ* (specified under given conditions of coupling to the environment), and a thermal time constant (defines speed response under specified conditions).

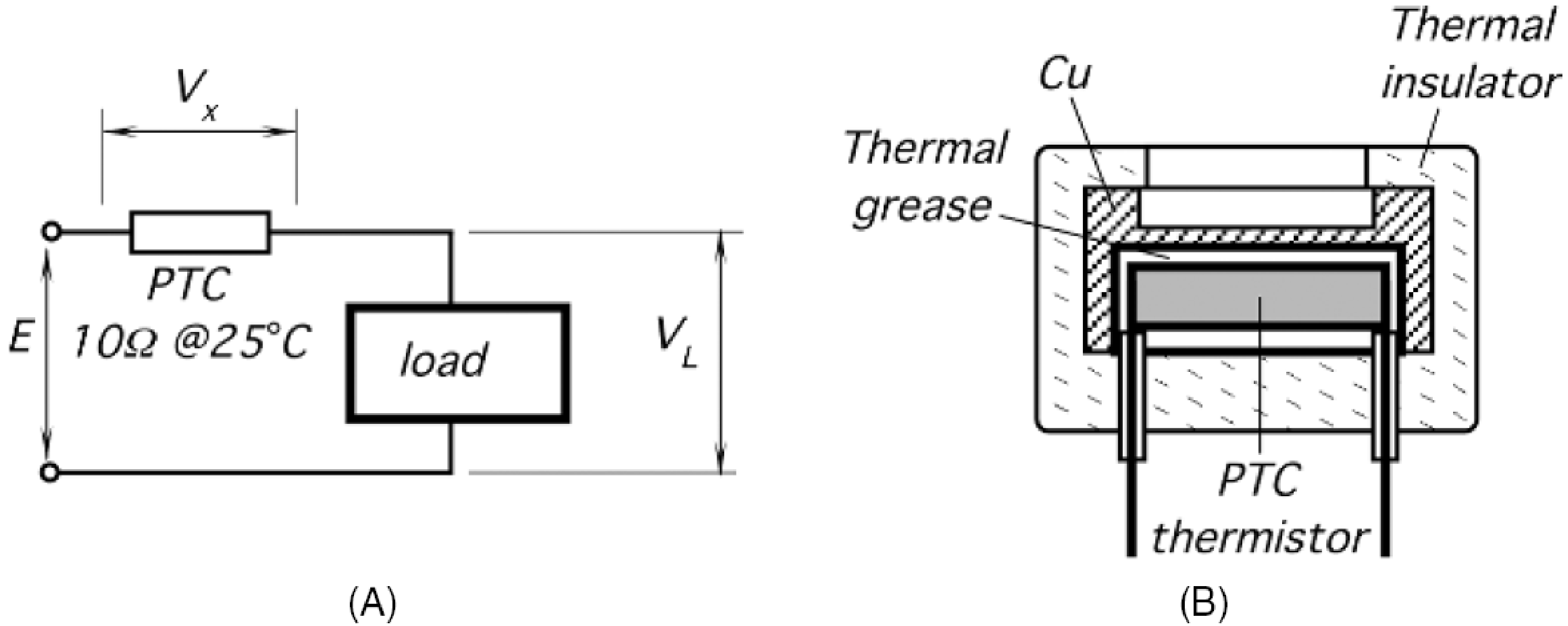


Fig. 16.14. Applications of PTC thermistors: (A) current-limiting circuit; (B) microthermostat.

Comparison thermistors / RTD

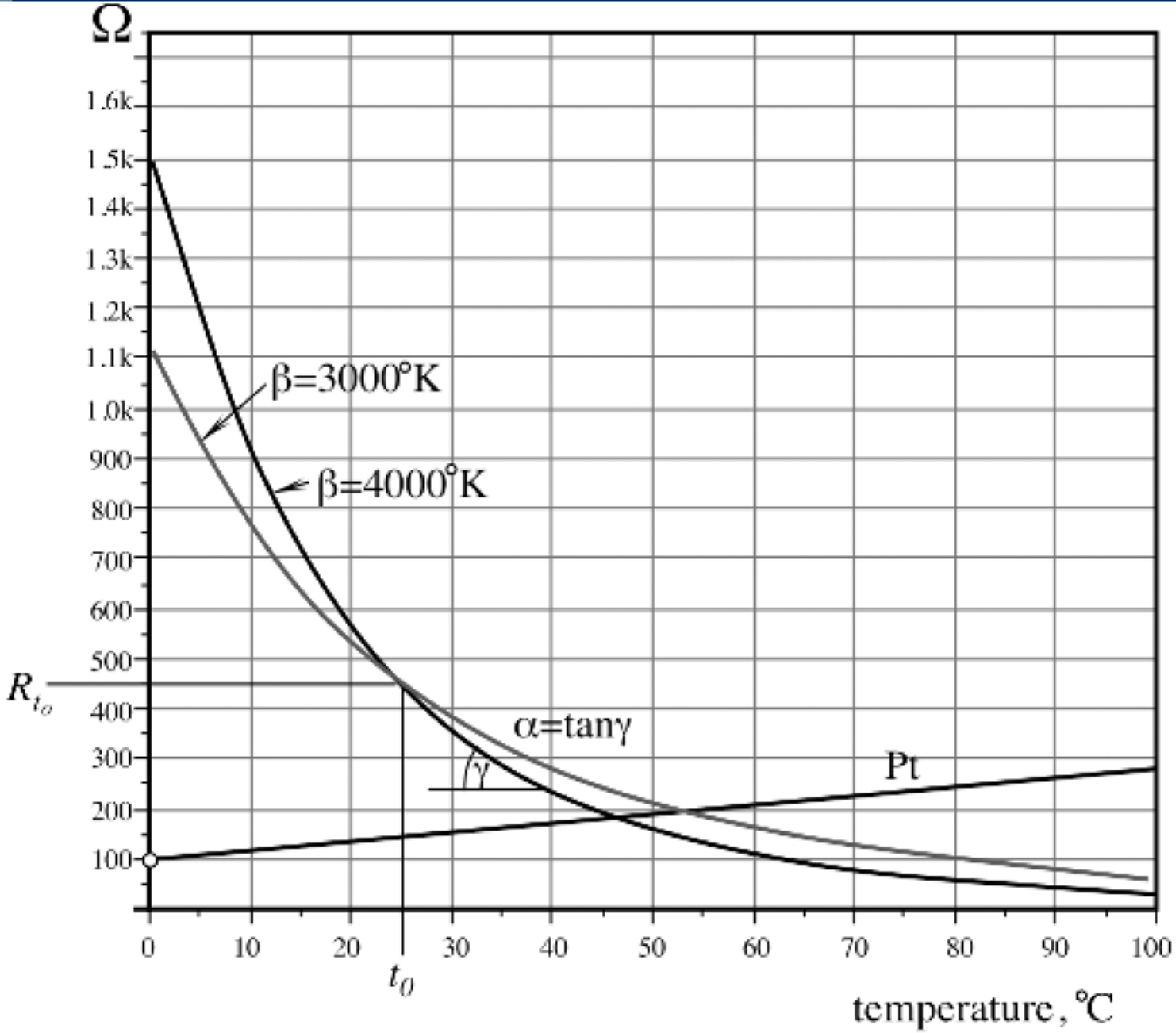
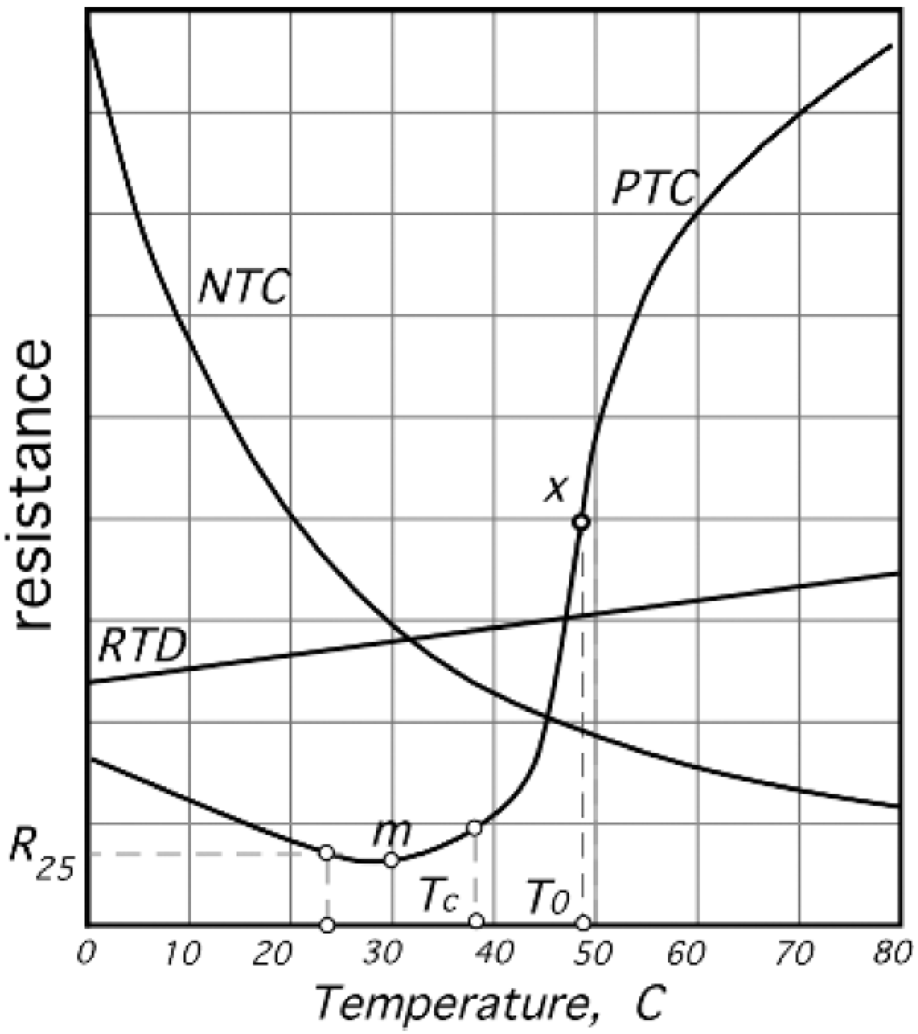
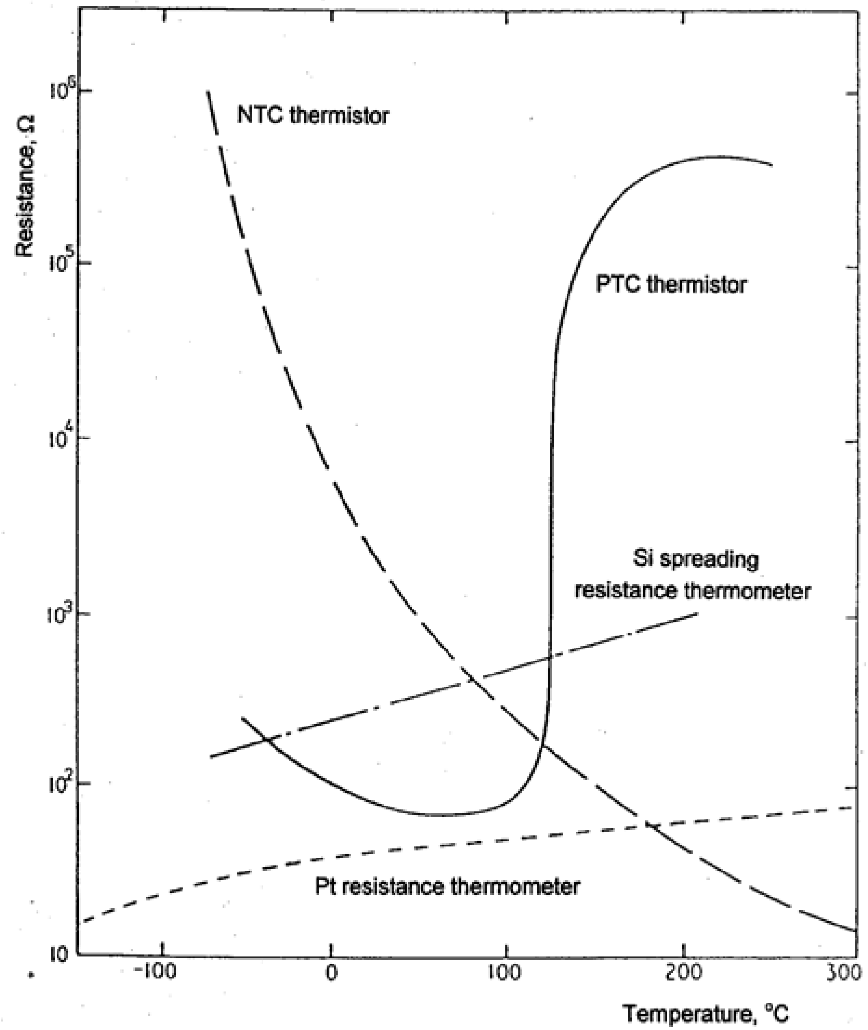


Fig. 3.18. Resistance–temperature characteristics for two thermistors and Pt RTD ($R_0 = 1k$); thermistors are calibrated at $t_0 = 25°C$ and RTD at $0°C$.

Comparison NTC-PTC thermistors / RTD





Self-heating in NTC sensors

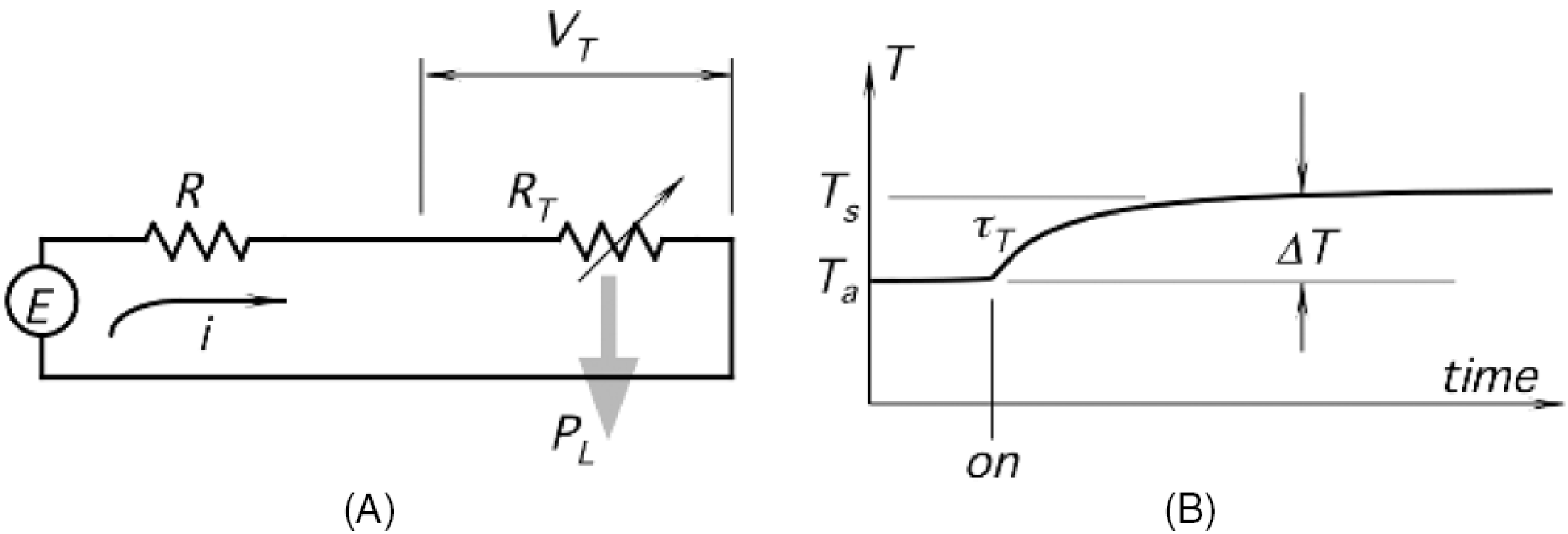


Fig. 16.10. (A) Current through thermistor causes self-heating; (B) temperature of thermistor rises with thermal time constant τ_T. P_L is the thermal power lost to the surroundings.

When electric power P is applied to the circuit, the rate at which energy is supplied to the thermistor must be equal the rate at which energy H_L is lost plus the rate at which energy H_S is absorbed by the thermistor body. The absorbed energy is stored in the thermistor's thermal capacity C. The power balance equation is:

$$\frac{dH}{dt} = \frac{dH_L}{dt} + \frac{dH_S}{dt}$$

According to the law of conservation of energy, the rate at which thermal energy is supplied to the thermistor is equal to the electric power delivered by the voltage source *E*:

$$\frac{dH}{dt} = P = \frac{V_{T^2}}{R} = V_T i$$

The rate at which thermal energy is lost from the thermistor to its surroundings is proportional to the temperature gradient *T* between the thermistor and surrounding temperature *Ta*:

$$P_L = \frac{dH_L}{dt} = \delta \Delta T = \delta (T_S - T_a)$$

where *δ* is the so-called *dissipation factor* which is equivalent to a thermal conductivity from the thermistor to its surroundings. It is defined as the ratio of dissipated power and a temperature gradient (at a given surrounding temperature).

The rate of heat absorption is proportional to thermal capacity of the sensor assembly:

$$\frac{dH_S}{dt} = C\frac{dT_S}{dt}$$

Therefore the differential equation that describes the thermal behaviour of the thermistor is :

$$\frac{dH}{dt} = P = Ei = \delta(T_S - T_a) + C\frac{dT_S}{dt} \qquad (*)$$

whose solution is:

$$\Delta T = (T_S - T_a) = \frac{P}{\delta}\left[1 - e^{-\delta/Ct}\right]$$

Upon waiting sufficiently long to reach a steady-state level T_S, the rate of change in equation (*) becomes equal to zero *(dT_S/dt =0)*; then, the rate of heat loss is equal to supplied power::

$$\delta(T_S - T_a) = \delta \Delta T = V_T i$$

If by selecting a low supply voltage and high resistances, the current *i* is made very low, the temperature rise *T* can be made negligibly small, and self-heating is virtually eliminated. The differential equation that describes the thermal behaviour of the thermistor thus becomes:

$$\frac{dT_S}{dt} = -\frac{\delta}{C}(T_S - T_a)$$

The solution of this differential equation yields an exponential function, which means that the sensor responds to the change in environmental temperature with time constant τ_T

$$T_S = T_B - \Delta T e^{-t/\tau_T}$$

1. **Resistance vs temperature characteristic**. In most of the applications based on this characteristic, the self heating effect is undesirable. Thus, the nominal resistance R_{T0} *of the thermistor* should be selected high and its coupling to the object should be maximized (increase in δ)*. The characteristic is primarily used for sensing and measuring temperature.* Typical applications are contact electronic thermometers, thermostats, and thermal breakers.
2. **Current vs time characteristic** (or resistance versus time).
3. **Voltage vs current characteristic** is important for applications where the self-heating effect is employed, or otherwise cannot be neglected

$$\delta(T_S - T_a) = \delta \Delta T = V_T i$$

In equilibrium conditions electrical power and P_L are equal:

$$\delta(T_S - T_a) = \delta \Delta T = V_T i$$

where T_S is thermistor temperature which is function of its resistance $R(T)$, T_a is the surrounding ambient temperature and δ is the dissipation coefficient (expressed in mW/°C).

Thermistor current and voltage will depend on the specific configuration of the electric circuit in which the sensor is included. If, by example, a constant voltage is applied at equilibrium, according to Ohm's law, we have $\boldsymbol{i=V_T/R(T)}$**,** therefore

$$\delta\left(T_S - T_0\right) = \frac{V_T^{\ 2}}{R(T)}$$

and therefore:

$$T_0 = T_S - \frac{V_T^{\ 2}}{\delta \cdot R(T)}$$

$$T_0 = T_S - \frac{{V_T}^2}{\delta \cdot R(T)}$$

The actual ambient temperature is that measured by the thermistor minus the term of the self heating.

As an example, if a thermistor has a dissipation coefficient of 1,5 mW/°C and, at a given temperature, it dissipates 1 mW, the measured temperature will be 0,67 °C greater than that of the environment in which it is immersed.

The greater is the dissipation coefficient (δ), the lower will be the difference between the measured temperature and the ambient temperature (T0).

The dissipation coefficient is therefore a measure of the thermal connection between the thermistor and the surrounding environment.

Usually, it is given for a thermistor in still air or in well agitated oil.

Typical values for small thermistors encapsulated in glass are 1,5 mW/°C in still air and 6,0 mW/°C in agitated oil.

If the ambient temperature is known, it is possible to determine the dissipation coefficient of the thermistor

Exercise

A given PRT has 100 Ω and δ= 6 mW/K when immersed in air and δ= 100 mW/K when immersed in still water. Calculate the maximal current through the sensor to keep the self-heating error below 0.1 C.

The temperature increment above ambient temperature when dissipating a power PD will be

$$\Delta T = \frac{P_D}{\delta} = \frac{I^2 R}{\delta}$$

Hence, the maximal current for a given temperature increment is

$$I = \sqrt{\frac{\Delta T \times \delta}{R}}$$

When the sensor is immersed in air, we obtain

$$I = \sqrt{\frac{0.1°C \times 0.006\, W/K}{100\ ohm}} = 2.4\ mA$$

When the sensor is immersed in water, we have

$$I = \sqrt{\frac{0.1°C \times 0.1\, W/K}{100\ ohm}} = 10\ mA$$

A dissipation factor more than 15 times higher in water than in air permits us to use a current just four times higher.

Voltage-current characteristic

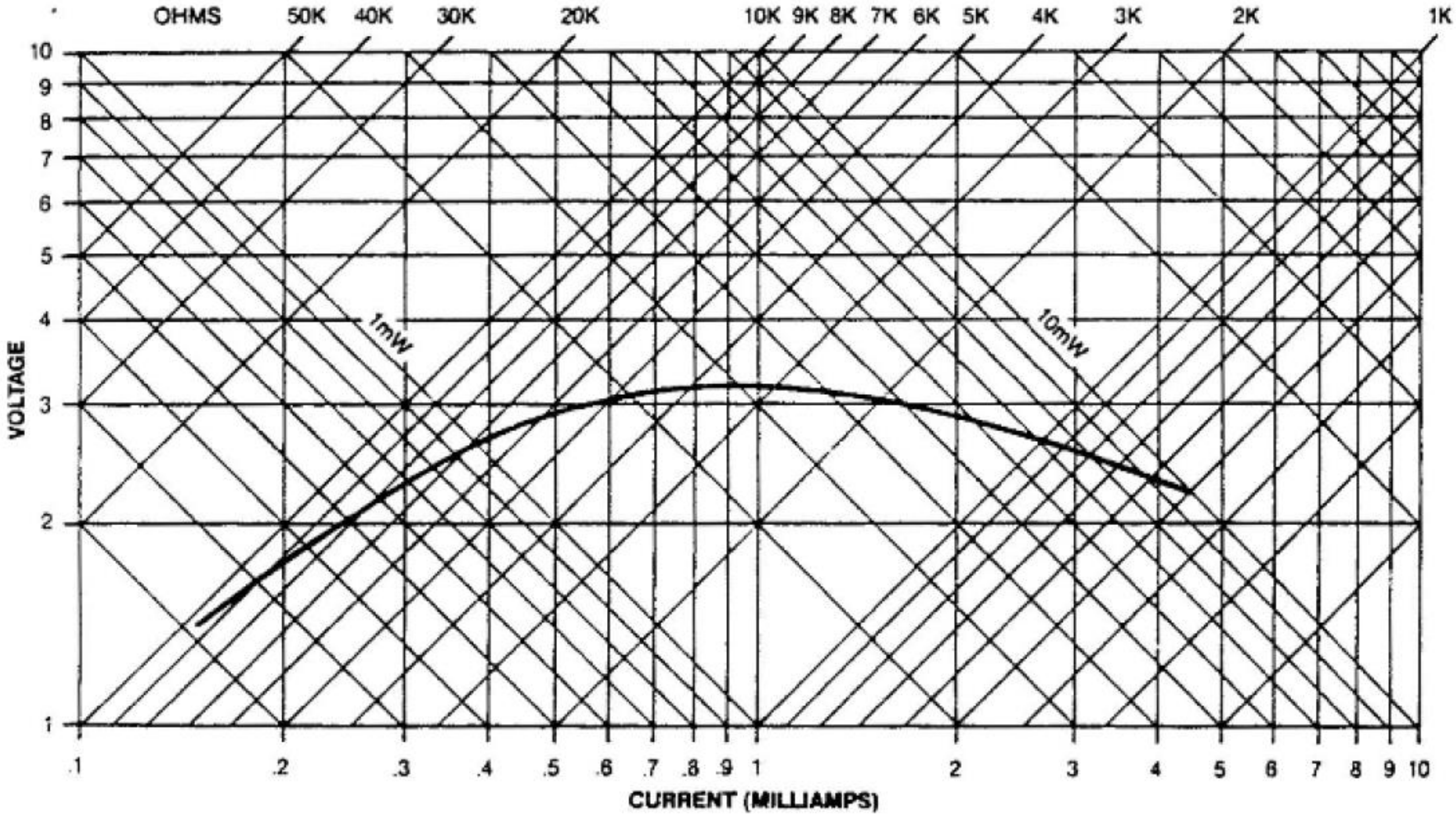


Fig. 16.11. Voltage–current characteristic of an NTC thermistor in still air at 25°C; the curvature of the characteristic is due to the self-heating effect.

Ambient heated mode

When used in this mode, the temperature of the NTC and thus the resistance depend on the ambient temperature. This requires that the power dissipation in the NTC is negligible and does not influence the temperature of the NTC. In this mode NTC resistors are used for

- **measurement purposes**: the temperature of air, a liquid or a surface is sensed and the resulting resistance is electronically processed for indication or registration purposes.

- **control purposes**: the temperature is sensed and the resulting resistance is used in control or servo circuits to control the temperature.

- **protection purposes**: the temperature is sensed and if it exceeds certain limits some control circuit will take counter measures.

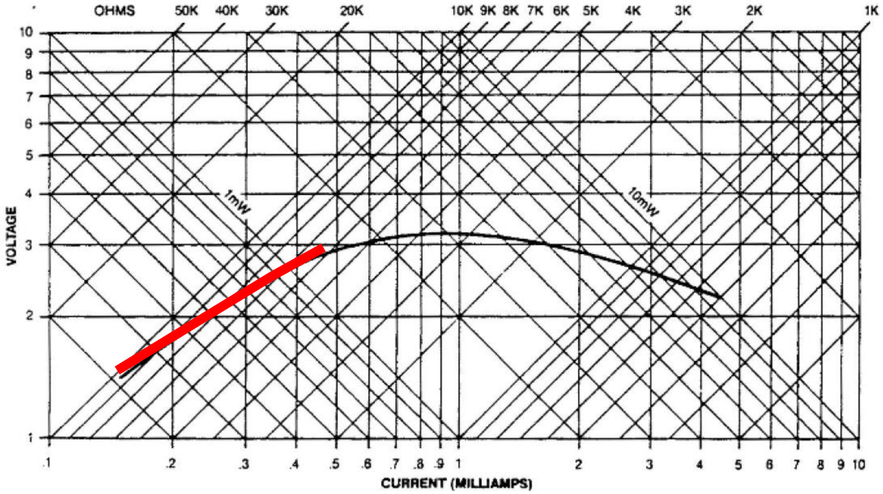


Fig. 16.11. Voltage–current characteristic of an NTC thermistor in still air at 25°C; the curvature of the characteristic is due to the self-heating effect.

Self-heated mode

In this mode the temperature of the NTC resistor depends basically on its own power dissipation. As the NTC resistor reduces its resistance with increasing temperature it tends to increase its current when getting hot. This is called "thermal run-away". If the NTC resistor is operated on a constant voltage the power dissipation will increase to very high values, finally destroying the element.

Therefore NTCs in the self-heating mode will always require some measures to limit the current to safe values.

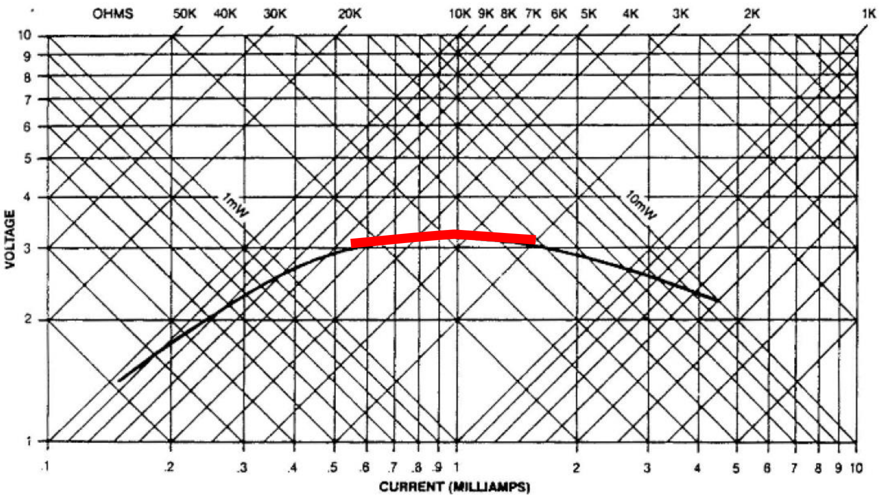
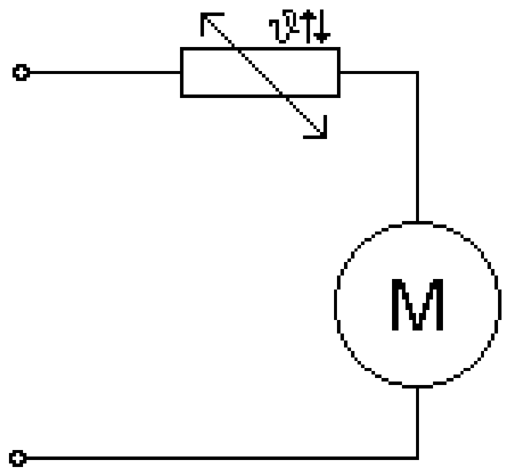


Fig. 16.11. Voltage–current characteristic of an NTC thermistor in still air at 25°C; the curvature of the characteristic is due to the self-heating effect.

Self-heated mode: applications

Starter NTC-resistor: the NTC resistor is in series with a load (e.g. a motor). When the load is off no current flows. The NTC is cold and has a high resistance. When the voltage is switched on the high resistance of the NTC will allow only little current and the major part of the power is dissipated in the NTC. The NTC will heat up, reducing its resistance, allowing more current to the load.

Finally a stable state will be reached, when the NTC reaches its final temperature with low resistance and the current being mainly limited by the load. Such circuits are used for "soft start" of motors and lamps.



Self-heated mode: applications

Cooling sense: The current is adjusted through the NTC so that it heats well over ambient temperature (e.g. 100°C). The NTC is mounted in the air stream of a cooling fan to be supervised. The air cooling will also cool the NTC, increasing its resistance. In case the cooling fan fails, the NTC will increase its temperature, reducing its resistance, which will be detected by a suitable electronic circuit giving an indication.

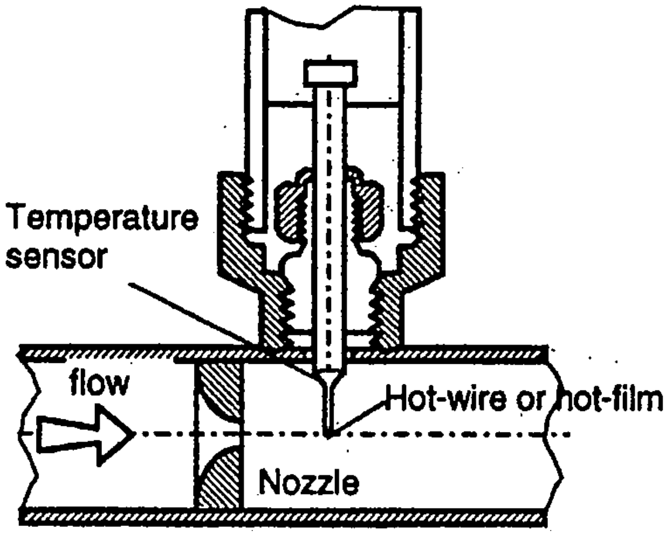
Media sense: A NTC in self heating mode will change its temperature and thus its resistance, if it comes into another media with different thermal conduction. E.g. a NTC can be mounted in a water reservoir to sense the water level. When the water reaches the self heated NTC it will cool the NTC, increasing its resistance, which can be used to give an indication.

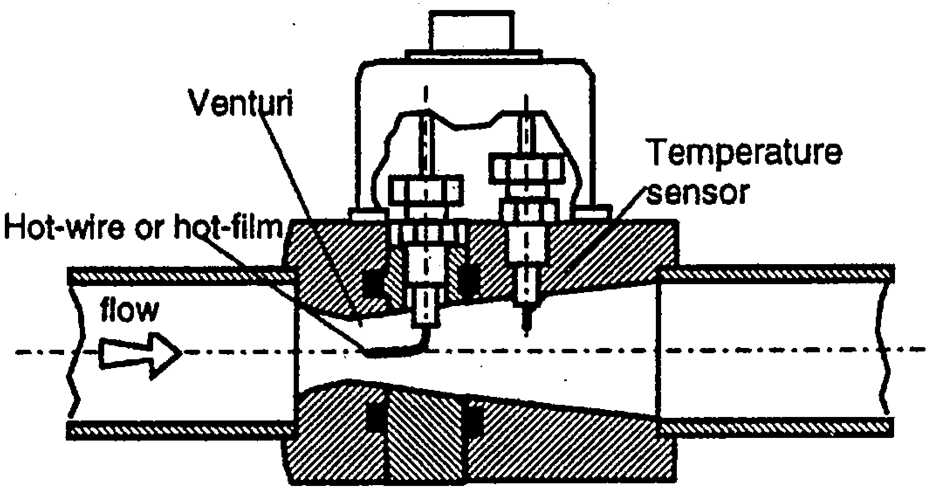
Application: measurement of a fluid velocity (hot-wire anemometer)

An electrical current I flows in a platinum wire of resistance R and dissipates for Joule effect a power = $\mathbf{I^2R}$ (“self-heating”).
Once immersed in a fluid, the wire cools down by convection and its resistance decreases.
The heat flow from wire to fluid is proportional to the thermal exchange area A, to the difference of temperature beween wire (w) and fluid (f) and to convection coefficient *h* (that on turn depends on wire dimensions) and depends on density, viscosity, specific heat and thermal conductivity of the fluid

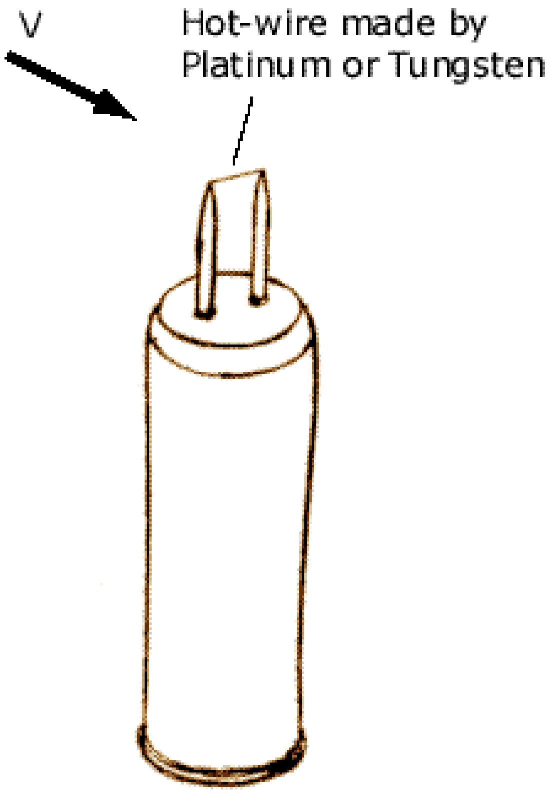
Measurement methods:
- Constant current in the probe, measurement of a change in resistance
- Measurement of the current needed to mantain the probe at constante temperature

Measurement of a fluid velocity (hot-wire anemometer)





Measurement of a fluid velocity (hot-wire anemometer)



I = current
R_w = wire resistance
R_{ref} = resistance at reference temperature T_{ref}
h = thermal convection coefficient of the wire
A_w = wire surface
T_w = wire temperature
T_f = fluid temperature
α = thermal coefficient of the wire
v_f = fluid velocity
a, b, c ($\approx$0.5) = coefficients

The wire, heated by the flowing electrical current, is in thermal equilibrium with the surrounding environment and the input electrical power is equal to the power lost by thermal convection (1):

$$I^2 R_w = h \cdot A_w \left(T_w - T_f \right)$$

Electrical resistance of the wire (2):

$$R_w = R_{\mathrm{Ref}} \left[1 + \alpha \left(T_w - T_{\mathrm{Ref}} \right) \right]$$

Thermal transfer coefficient *h* is function of fluid velecity v_f accordingly to the King's law (3):

$$h = a + b \cdot v_f^c$$

From relationships (1), (2), (3) it follows:

$$a + b \cdot v_f^{c} = \frac{I^2 R_{\mathrm{Ref}}\left[1 + \alpha\left(T_w - T_{\mathrm{Ref}}\right)\right]}{A_w\left(T_w - T_f\right)}$$

$$= g\left(T_w, T_f\right)$$

$$\Rightarrow \quad v_f = \left\{ \left[\frac{I^2 R_{\mathrm{Ref}}\left[1 + \alpha\left(T_w - T_{\mathrm{Ref}}\right)\right]}{A_w\left(T_w - T_f\right)} - a \right] \Big/ b \right\}^{1/c}$$

In the anemometers at **constant temperature**, the current is regulated in a way to mantain wire temperature and resistance (Tw and Rw) constant. Fluid velocity is function of input current and fluid temperature :

$$a + b \cdot v_f^c = \frac{I^2 R_w}{A_w \left(T_w - T_f\right)}$$

$$= f\left(I, T_f\right)$$

In the anemometers at **constant current**, fluid velocity is function of wire and fluid temperatures:

$$a + b \cdot v_f^c = \frac{I^2 R_{\mathrm{Ref}} \left[1 + a\left(T_w - T_{\mathrm{Ref}}\right)\right]}{A_w \left(T_w - T_f\right)}$$

$$= g\left(T_w, T_f\right)$$