

DIPARTIMENTO di ELETTRONICA, INFORMAZIONE e  
BIOINGEGNERIA

 POLITECNICO DI MILANO



## Lecture 7

# Conditioning of resistive sensors

# Conditioning of resistive sensors

General equation of a variable resistance :

$$R = R_0 f(x), \text{ con } f(0) = 1$$

For **linear** sensors we have:

$$R = R_0 (1+x)$$

The value x depends strongly on sensor type:

Linear potentiometers	from 0 to -1
Strain gauges	from $10^{-5}$ to $10^{-2}$
LDR, humistors	>1000
Gas sensors, liquid conductivity sensors	<100
PTC switching (above $T_{\text{critic}}$ )	>10000

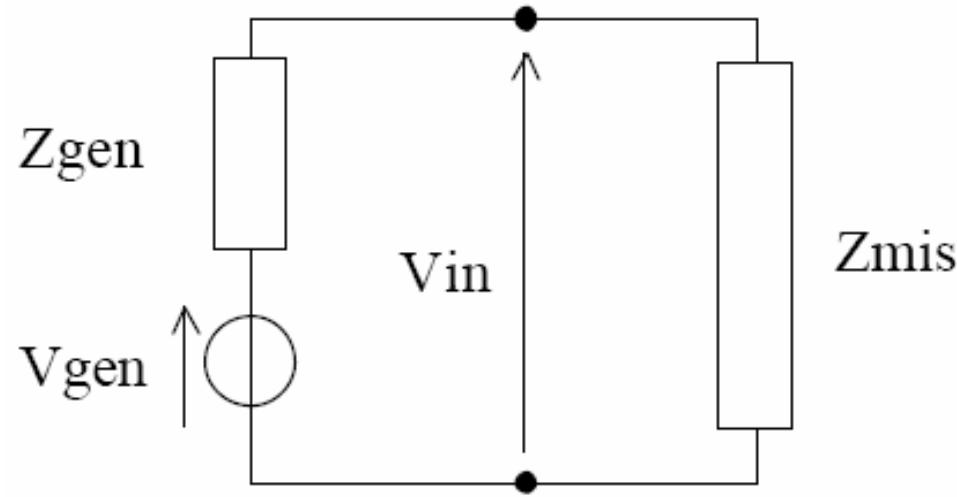
- they must drive the sensor with an electric voltage or current in order to obtain an output signal, because a change in resistance is not by itself a signal
- this supply, whose magnitude affects that of the output signal, is limited by sensor self-heating, which must be avoided unless the sensing principle uses sensor self-heating, as in some flowmeters and liquid level meters

## Alternative expressions for linear sensors

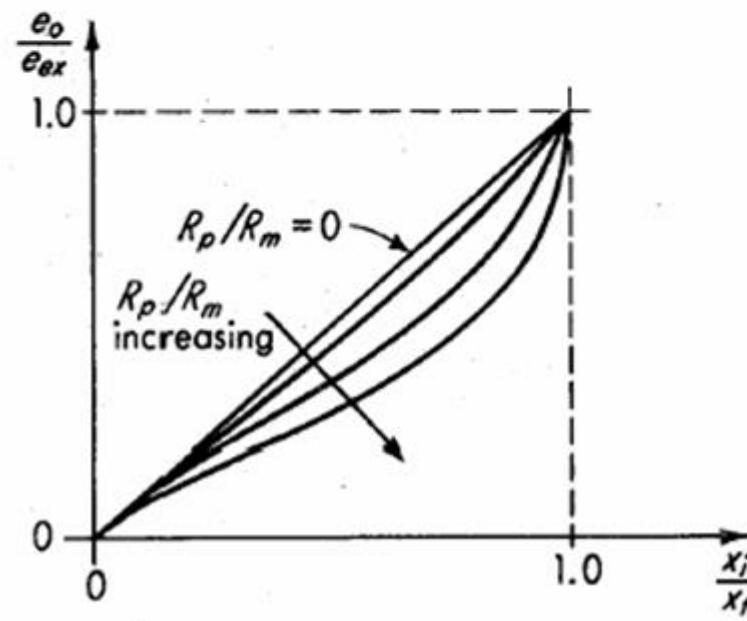
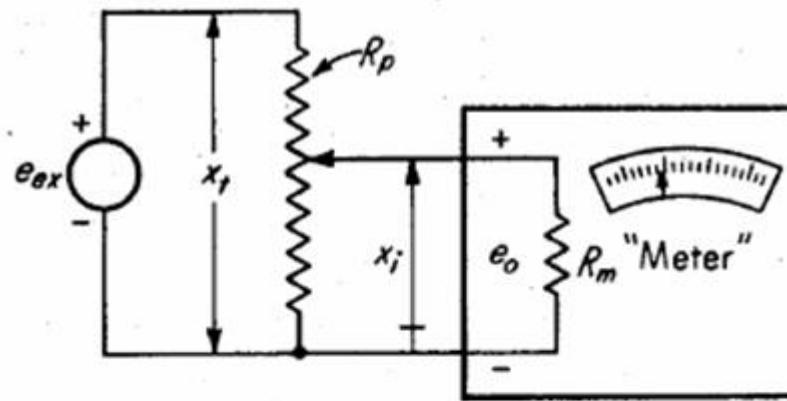
$$R = R_o (1+x) \Leftrightarrow R = R_o + \Delta R$$

$$\Rightarrow x \cdot R_o = \Delta R$$

$$\Rightarrow x = \Delta R / R_o$$

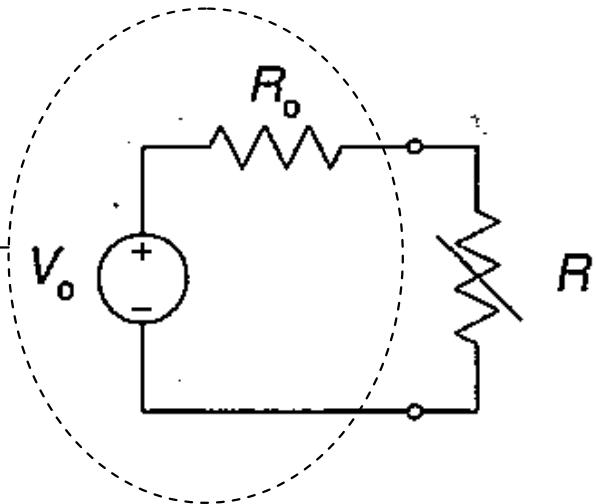


$$\frac{V_{in}}{V_{gen}} = \frac{1}{\frac{Z_{gen}}{Z_{mis}} + 1}$$



# Maximum power transfer in Resistive sensors

*Thevenin Equivalent Circuit (seen by the sensor)*



Power P absorbed by the sensor is :

$$P = \left( \frac{V_0}{R_0 + R} \right)^2 R$$

The Maximum value of P as a function of R is :

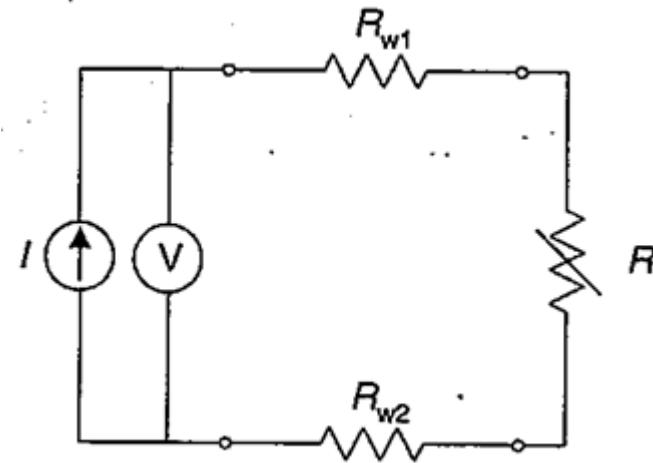
$$\frac{dP}{dR} = 2 \frac{V_0}{R_0 + R} \frac{-V_0}{(R_0 + R)^2} R + \left( \frac{V_0}{R_0 + R} \right)^2 = \left( \frac{V_0}{R_0 + R} \right)^2 \frac{R_0 - R}{R_0 + R} = 0$$

$$R = R_0; \quad P_{max} = \left( \frac{V_0}{R_0 + R_0} \right)^2 R_0 = \frac{V_0^2}{4R_0}$$

## Sensitivity to the length of connecting wires: two-wire circuit

Conditioners for remote sensors must be insensitive to connecting lead resistance or compensate for it.

$$V = I(R + R_{w1} + R_{w2})$$

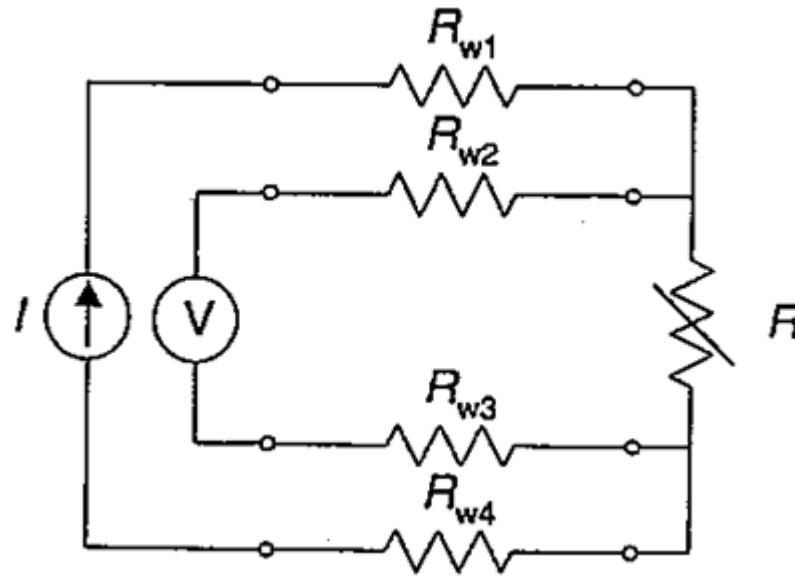


If  $R=0\Omega \Rightarrow V(0) \neq 0V$  (offset error)

(ex.: 10 m. of AWG wire n. 20 adds  $0.333\text{ m}\Omega$  to  $R$ , which is the variation in resistance of a Pt100 sensor for  $1^\circ\text{C}$ )

# Sensitivity to connecting lead resistance: four-wire Kelvin circuit

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- It is hypothesized that the voltage measurement circuit  $V$  has a very high input impedance
  - in such conditions,  $V$  is insensible to connecting lead resistance

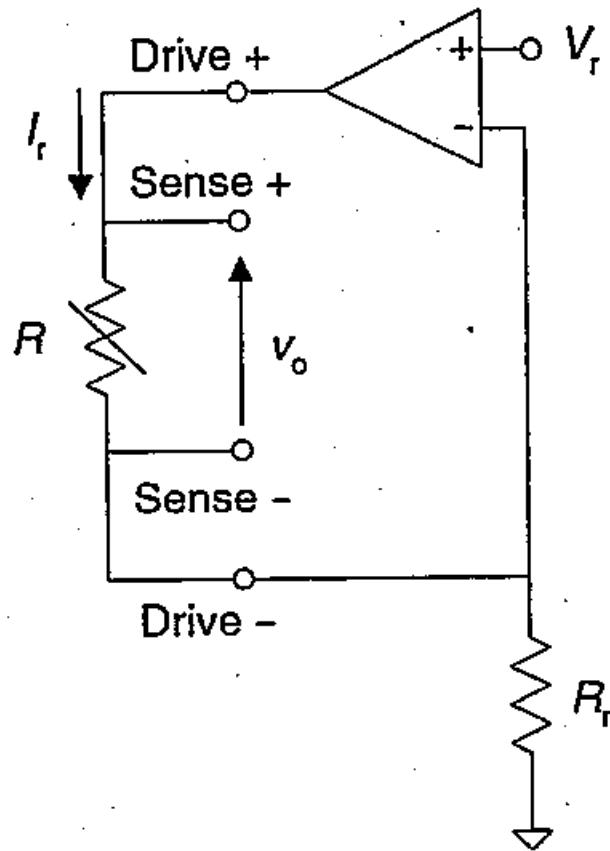
**TABLE 3.1 Copper Conductor Data<sup>a</sup>**

<b>AWG<sup>b</sup></b>	<b>Stranding</b>	<b>Diameter (mm)</b>	<b>Dc Resistance<sup>c</sup> (Ω/km)</b>
10	Solid	2.600	3.28
	37/26	2.921	3.64
	49/27	2.946	3.58
20	Solid	0.813	33.20
	10/30	0.899	33.86
	26/34	0.914	32.97
30	Solid	0.254	340.00
	7/38	0.305	338.58

<sup>a</sup>Solid wire data are from Spectra-Strip, and stranded conductor data are from Alpha Wire. There is additional information at <http://www.brimelectronics.com/AWGchart.htm>.

**AWG** stands for **American Wire Gauge**, also known as B&S gauge (from Brown and Sharp), which is a diameter specification. Metric gauge numbers correspond to 10 times the diameter in millimeters. For example, a metric wire No. 6.0 is 0.6 mm in diameter. AWG numbers do not correspond to diameter sizes proportionally; and the higher the gauge number, the smaller the diameter. Typical household wiring is AWG number 12 or 14. Telephone wire is usually 22, 24, or 26. Strand number n/m means that taking n strands of wire number m makes the desired wire number. For example, 10 strands of 30AWG or 26 strands of 34AWG make a 20AWG wire.

# Signal conditioner for a resistive sensor based on current excitation



$$v_0 = I_r R = \frac{V_r}{R_r} R_0 (1 + x) \quad (\text{linear relationship})$$

If  $R_r = R_0$ , subtracting the drop in voltage across  $R_r$  from  $v_0$ , yields

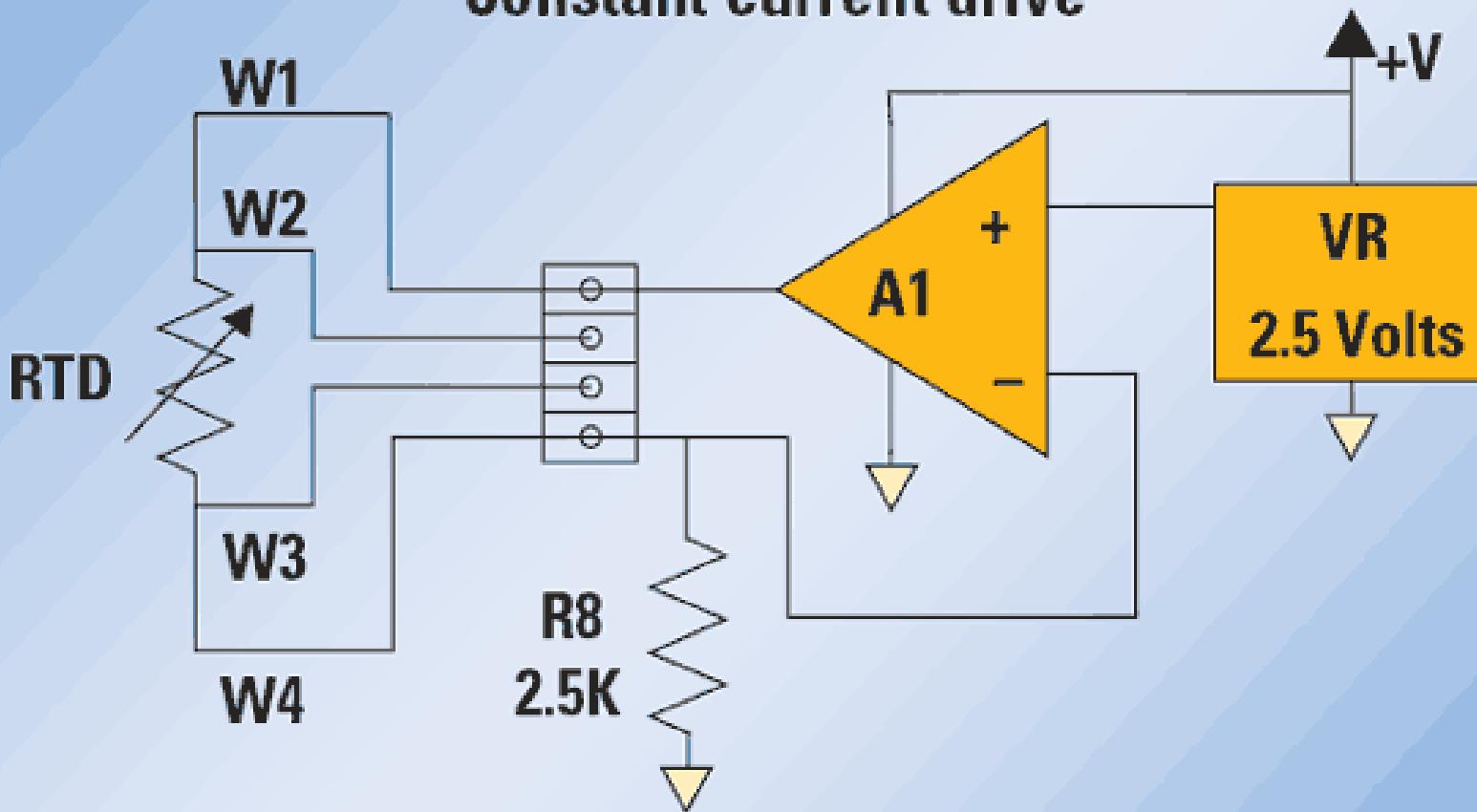
$$v_s = v_0 - I_r R_r = V_r (1 + x) - V_r = V_r x$$

Which is 0 for  $x=0$

# Example: conditioning of a RTD sensor (National Semiconductor)

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## Constant current drive

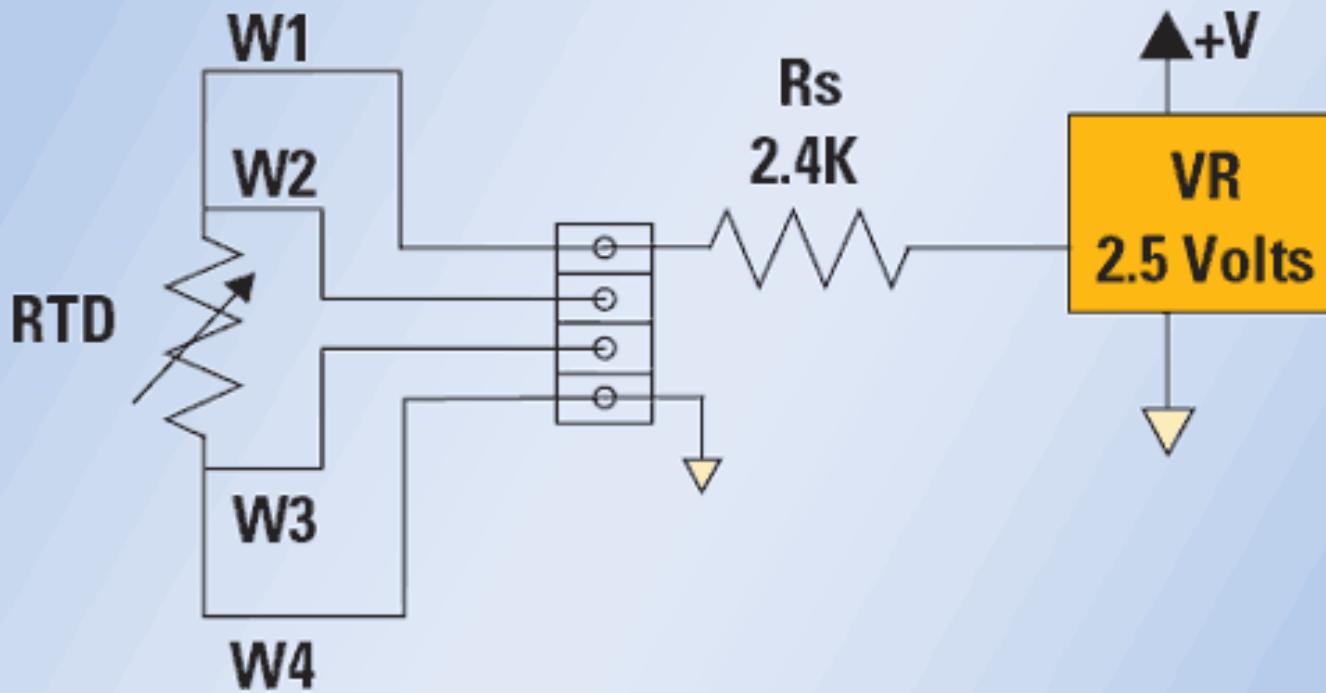


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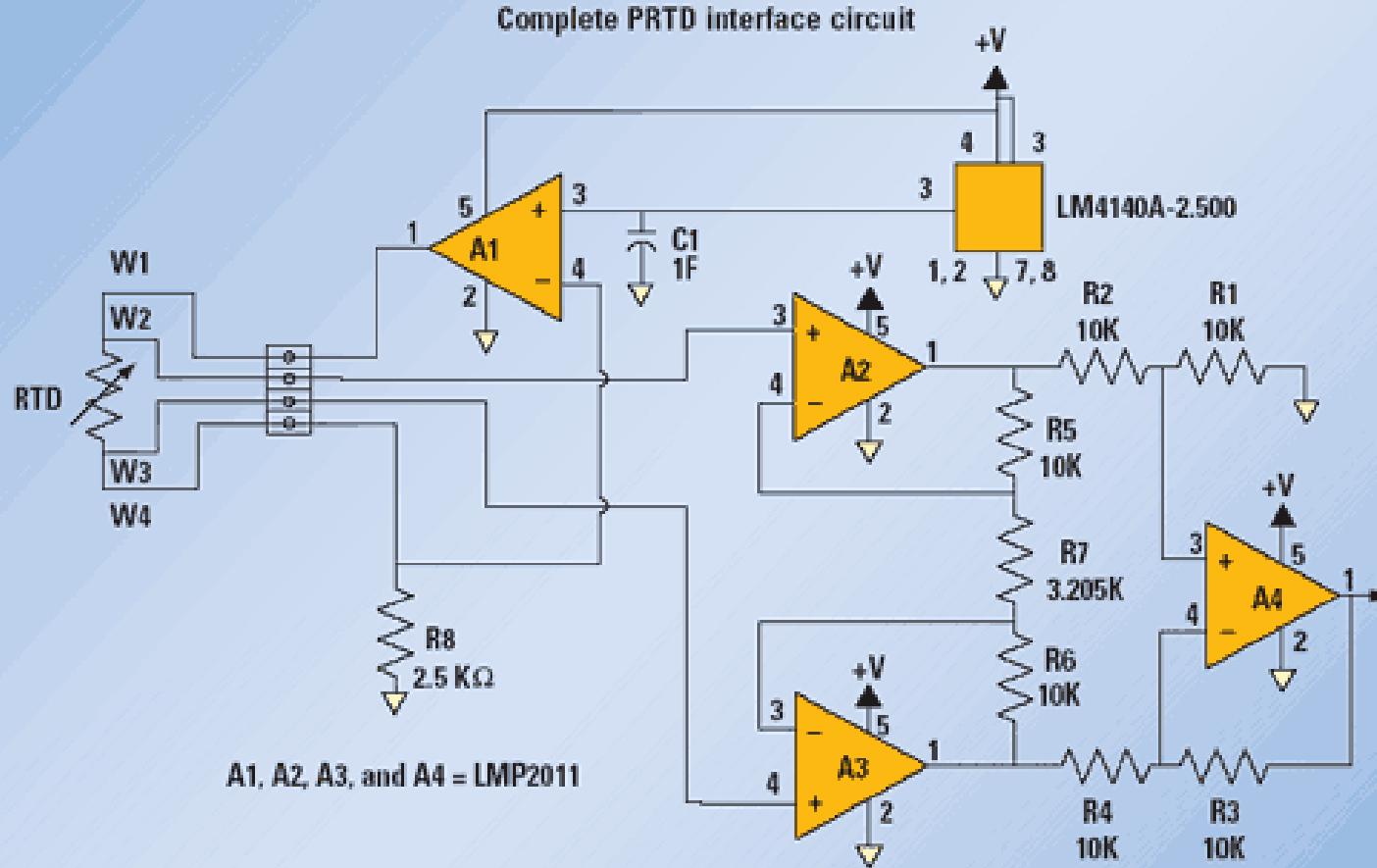
# Example: conditioning of a RTD sensor (National Semiconductor)

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## Constant voltage drive

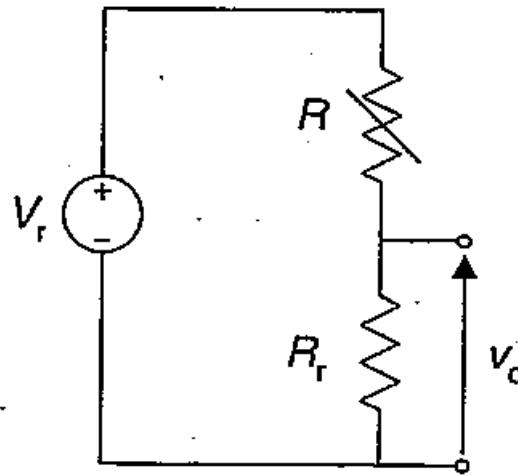


<http://www.national.com/nationaledge/dec04/article.html>



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Are typically used to measure high values of resistance

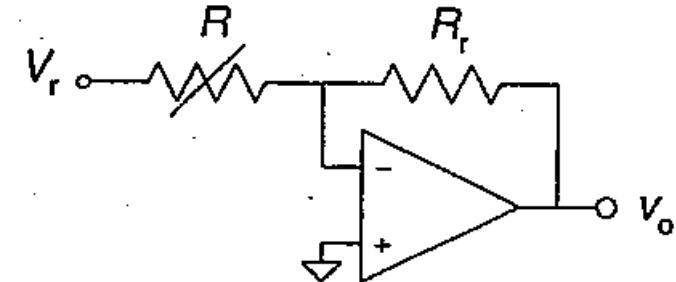


$$v_o = \frac{V_r}{R_r + R} R_r$$

From this relationship it is obtained the unknown resistance  $R$ :

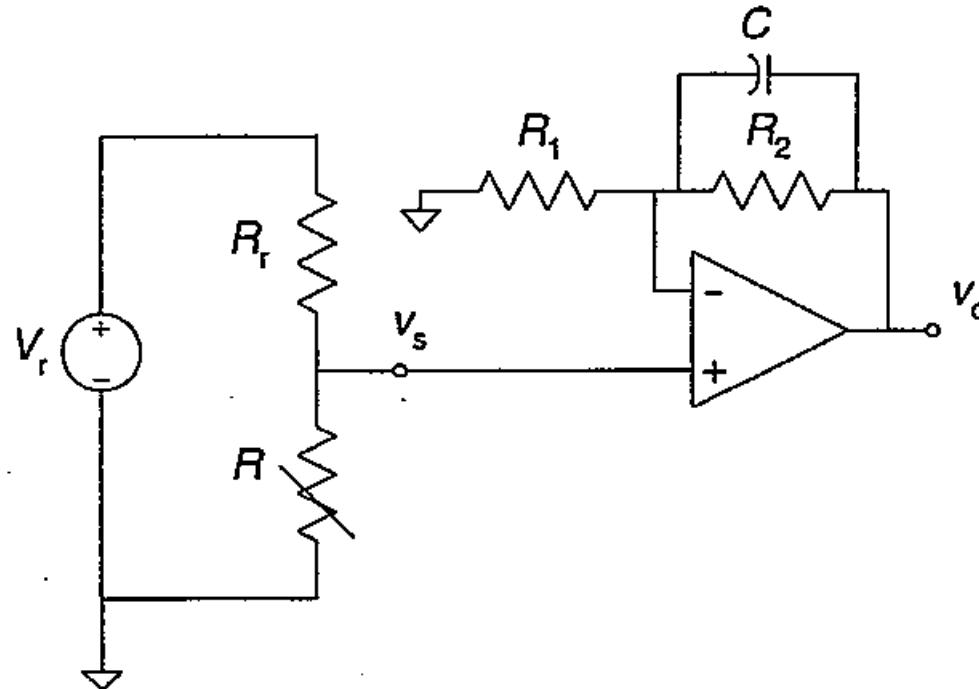
$$R = R_r \frac{V_r - v_o}{v_o}$$

By adding an op-amp to the voltage divider we obtain a circuit in which the output voltage is inversely proportional to the unknown resistance.



Voltage dividers need voltage measurement systems with high input impedance.

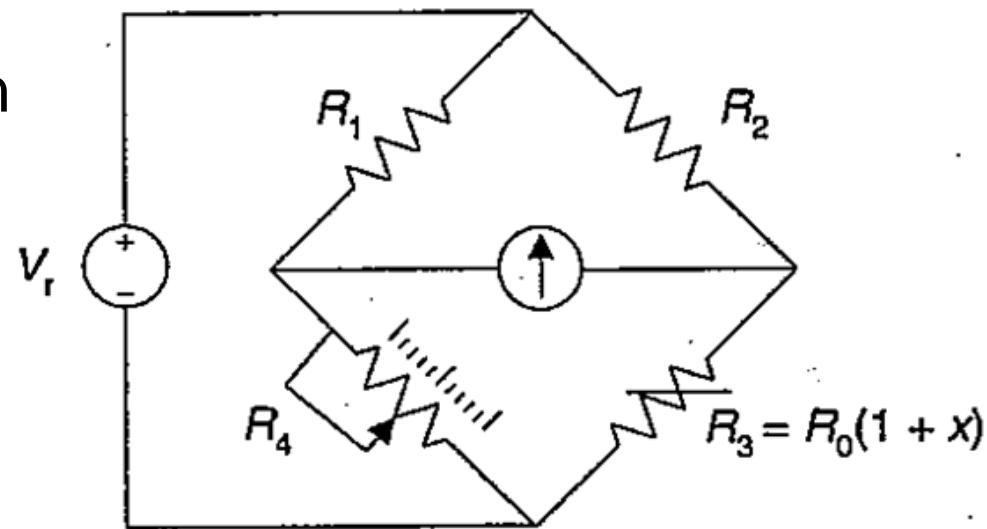
The following example shows a non-inverting amplifier with high-impedance input and a gain equal to  $1 + R_2/R_1$



The Wheatstone bridge measurement method was first proposed by S. H. Christie in 1833 and reported by Sir Charles Wheatstone to the Royal Society (London) in 1858 as a method to measure small resistances. It is based on a feedback system, either electric or manual, in order to adjust the value of a standard resistor until the current through the galvanometer or other null indicator is zero.

Once the balance condition has been achieved we have

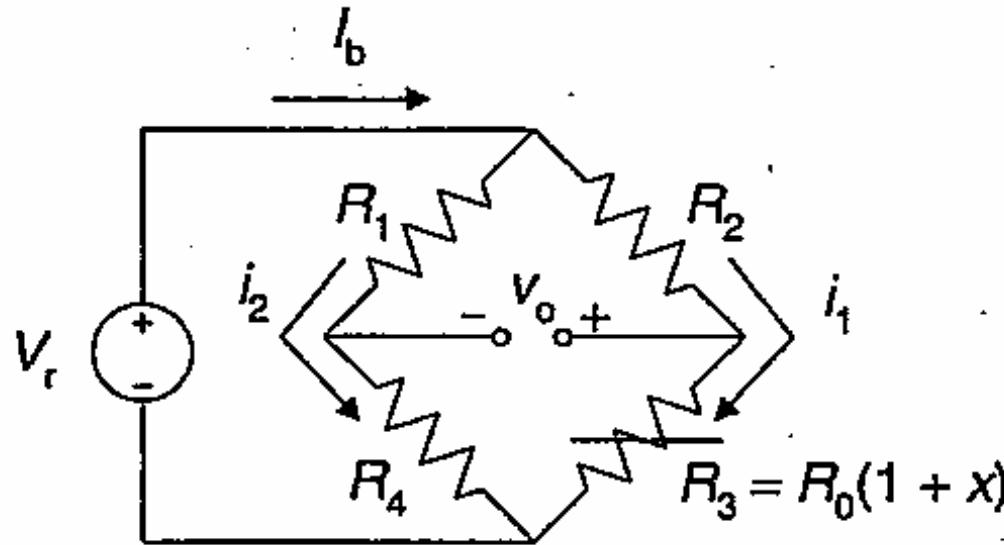
$$R_3 = R_4 \frac{R_2}{R_1}$$



That is, changes in  $R_3$  are directly proportional to the corresponding changes we have to produce in  $R_4$  in order to balance the bridge.

This measurement method can be also used as a polarity detector because the output is positive or negative, depending on whether  $x$  is greater or less than a given threshold.

Wheatstone bridges are often used in the deflection mode. Instead of measuring the action needed to restore balance on the bridge, this method measures the voltage difference between both voltage dividers or the current through a detector bridging them.



If the bridge is balanced when  $x=0$ , which is the usual situation, we define a parameter

$$k = \frac{R_1}{R_4} = \frac{R_2}{R_0}$$

The voltage difference between both branches is

$$v_0 = V_r \left( \frac{R_3}{R_2 + R_3} - \frac{R_4}{R_1 + R_4} \right) = V_r \frac{kx}{(k+1)(k+1+x)} \quad (1)$$

Thus the output voltage is proportional to the changes in  $R_3$  only when  $x \ll k+1$ ; that is, the sensitivity depends on  $x$  (and  $k$  and  $V_r$ )

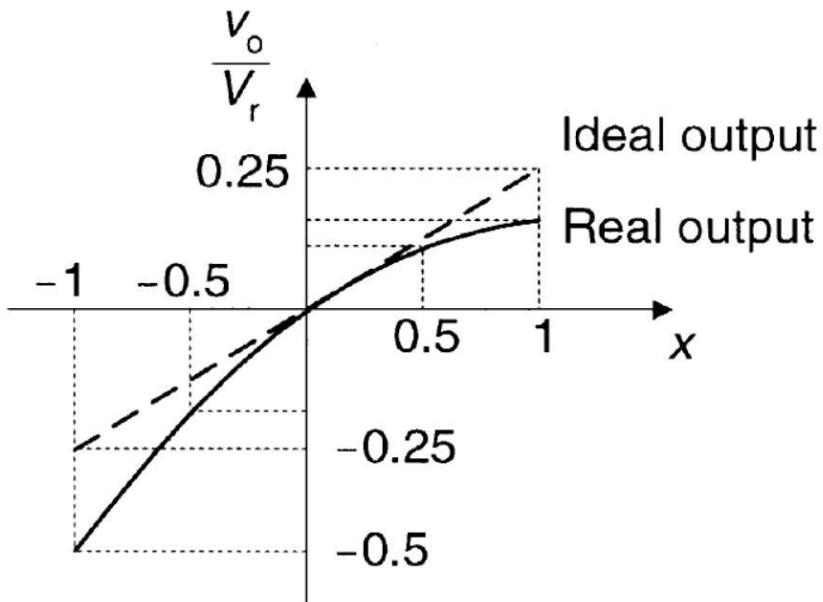
For  $x=0$ , the sensitivity is

$$S_0 = \frac{dv_o}{d(xR_0)} \Big|_{x=0} = \frac{V_r k}{R_0} \frac{1}{(k+1)^2} \quad (2)$$

The maximal sensitivity as a function of  $k$  is obtained by setting  $dS_0/dk = 0$ , which yields  $k=1$ . By calculating the second derivative we can verify that this point is a maximum.

On the other hand, from the relationship 1 (see above) we infer that  $k=1$  yields a nonlinear output unless  $x \ll 2$ .

The actual output departs from a straight line through the origin for  $k \neq 1$ :



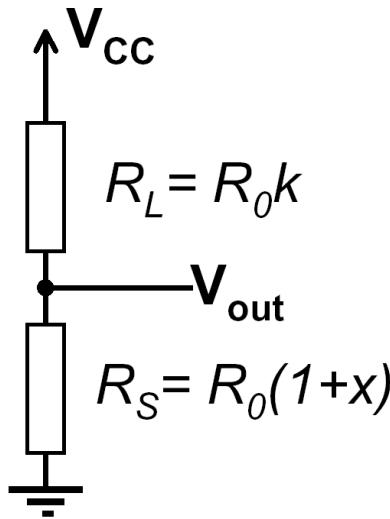
If the bridge is supplied by a constant current  $I_r$ , the output voltage is

$$v_o = I_r R_0 \frac{kx}{2(k+1) + x}$$

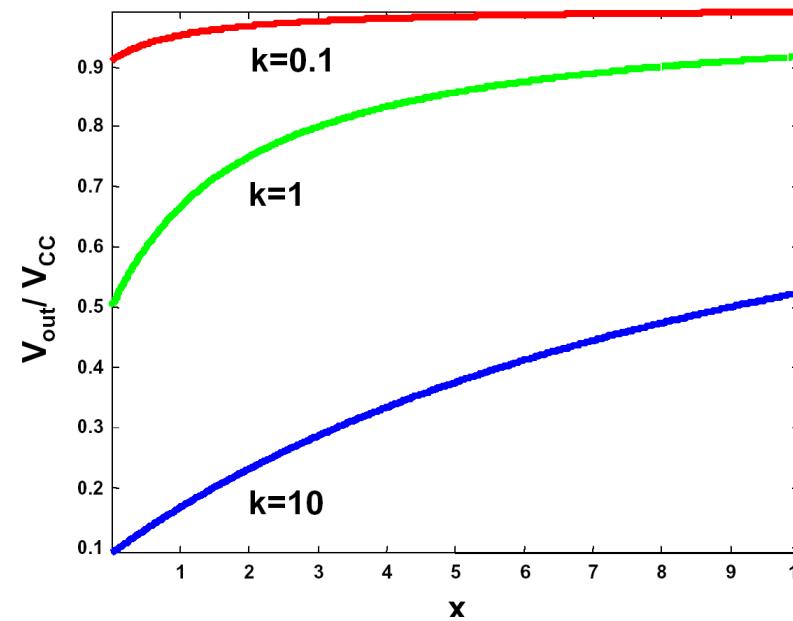
To have an approximately linear output we need  $x < 2(k+1)$ , and  $x < 4$  when  $K=1$ . Linearity is not necessary to achieve good accuracy. What matters is the repeatability of the results. But the output is easily interpreted when it is proportional to the measurand.

For metal strain gages,  $x$  seldom exceeds 0.02. Therefore, usually  $k=1$  to improve sensitivity and, unless a very high linearity is desired, the presence of  $x$  in the denominator of equations (1) and (2) is ignored. Alternatively, the actual  $x$  can be calculated from the output voltage or current by solving for  $x$  in those equations. For example, for a bridge supplied by constant voltage and  $k=1$ , from (1) we obtain

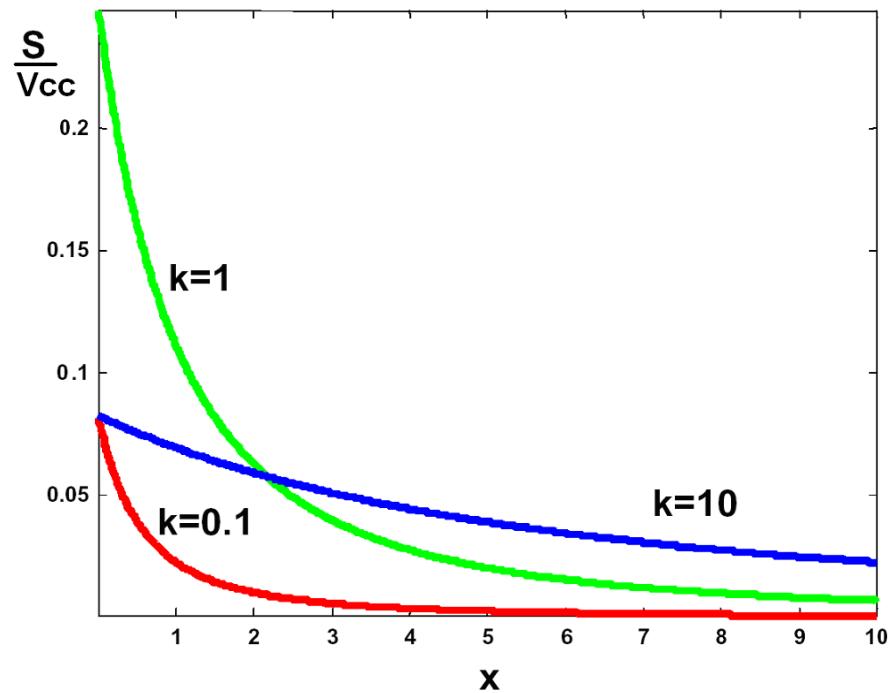
$$x = \frac{4v_o}{V_r} \frac{1}{1 - \frac{2v_o}{V_r}}$$



$$\begin{aligned}
 V_{\text{out}} &= V_{\text{CC}} \frac{R_S}{R_S + R_L} = \\
 &= V_{\text{CC}} \frac{R_0(1+x)}{R_0(1+x) + R_0 k} = V_{\text{CC}} \frac{1+x}{1+x+k}
 \end{aligned}$$



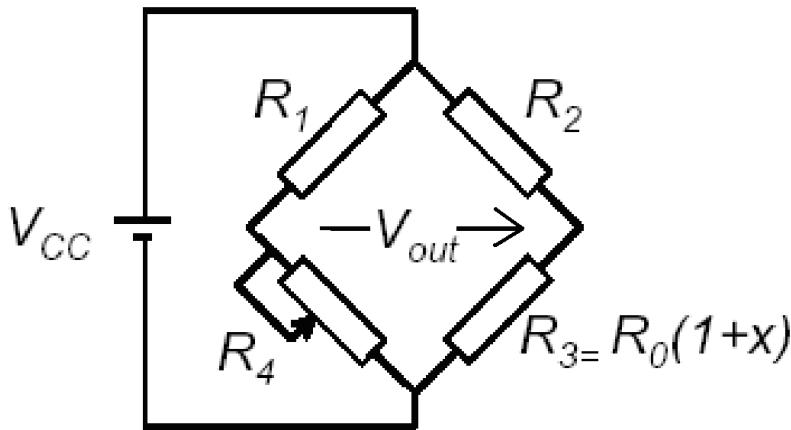
$$\begin{aligned}
 S &= \frac{dV_{out}}{dx} = \frac{d}{dx} \left( V_{CC} \frac{1+x}{1+x+k} \right) = \\
 &= V_{CC} \frac{(1+x+k) - (1+x)}{(1+x+k)^2} = \\
 &= V_{CC} \frac{k}{(1+x+k)^2}
 \end{aligned}$$



$$\frac{dS}{dk} = 0 \Rightarrow \frac{d}{dk} \left( V_{CC} \frac{k}{(1+x+k)^2} \right) = 0$$

$$\Rightarrow \frac{(1+x+k)^2 - k2(1+x+k)}{(1+x+k)^4} = 0 \Rightarrow k = 1+x$$

Maximal sensitivity for  $R_L = R_S$



By operating close to the equilibrium condition :

therefore:

“deflection” mode:

$$V_{out} = V_{cc} \left( \frac{R_3}{R_2 + R_3} - \frac{R_4}{R_1 + R_4} \right)$$

niente deve essere uguale a niente, dobbiamo avere questo rapporto uguale quando siamo in equilibrio, se vogliamo che sia in equilibrio nell'intorno di  $x=0$  allora consideriamo come  $R_3 = R_0$ . Questo per quanto riguarda l'uscita =0 per  $x=0$ . Ed infatti togliamo l'offset. Se poi vogliamo anche che intorno a  $x=0$  ci sia la massima sensitività allora dobbiamo impostare questo rapporto = 1. Ciò non significa che  $R_1$  e  $R_4$  debbano essere uguali a  $R_0$ . Ma sicuramente  $R_2$  sì.

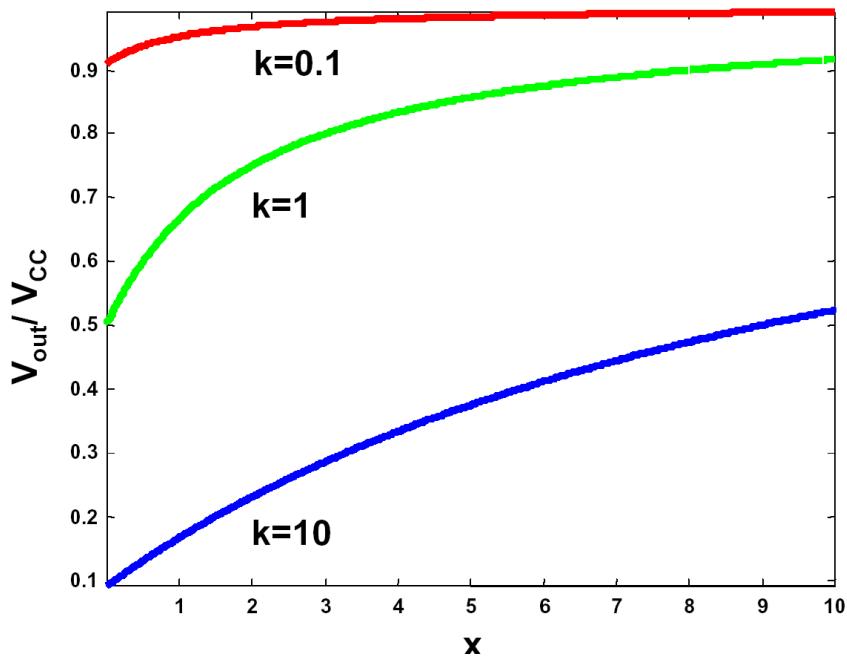
$$k = \frac{R_1}{R_4} = \frac{R_2}{R_0}$$

$$\begin{aligned} V_{out} &= V_{cc} \left( \frac{R_0(1+x)}{R_0 k + R_0(1+x)} - \frac{R_4}{R_4 k + R_4} \right) = \\ &= V_{cc} \left( \frac{(1+x)}{k + (1+x)} - \frac{1}{k+1} \right) = V_{cc} \frac{kx}{(1+k)(1+k+x)} \end{aligned}$$

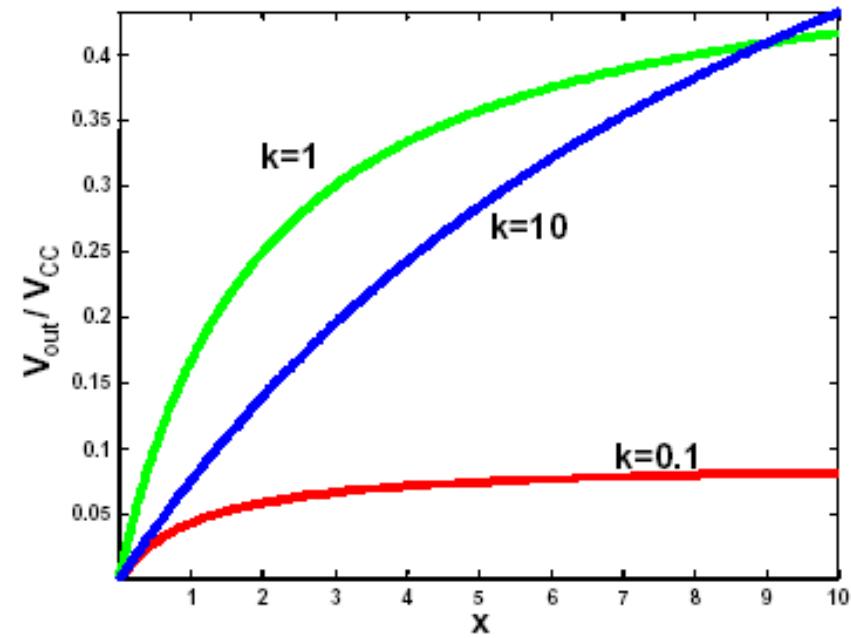
# Voltage divider vs Wheatstone bridge: output

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Partitore di tensione



Ponte di Wheastone

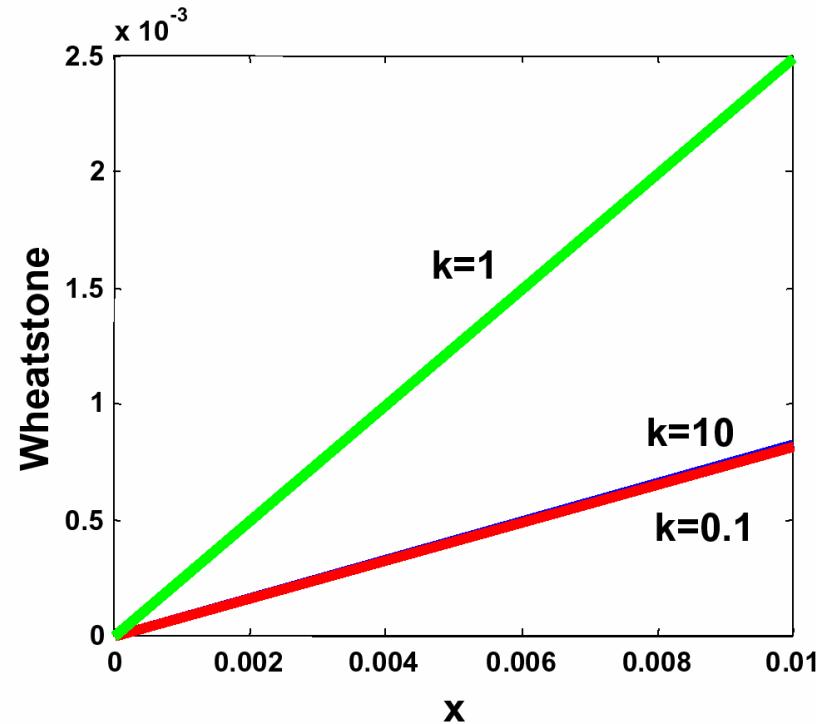
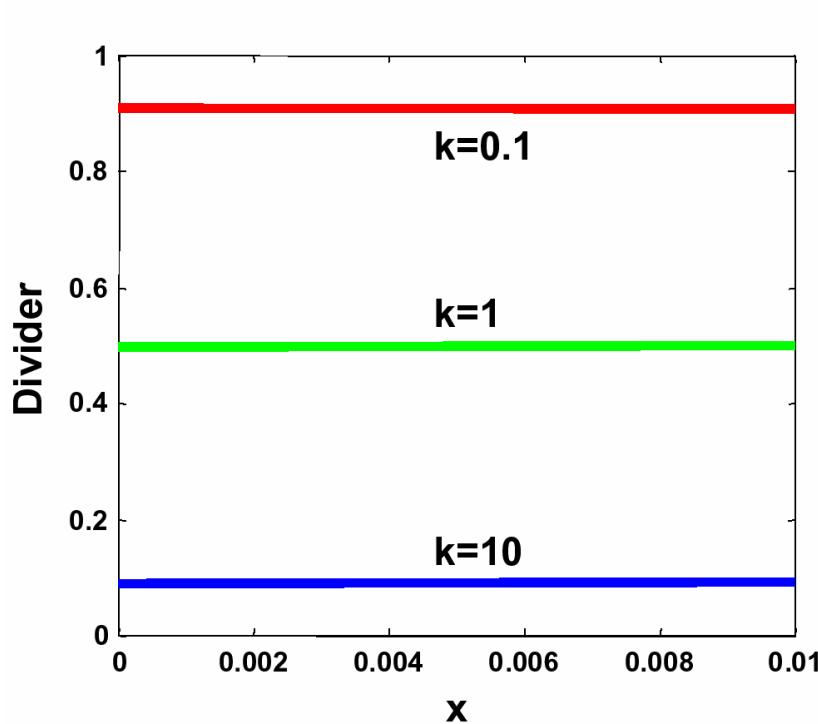


# Wheatstone bridge: sensitivity

$$\begin{aligned} S &= \frac{dV_{\text{out}}}{dx} = V_{\text{cc}} \frac{d}{dx} \left( \frac{kx}{(1+k)(1+k+x)} \right) = \\ &= V_{\text{cc}} \frac{k(1+k)(1+k+x) - kx(1+k)}{(1+k)^2(1+k+x)^2} = \\ &= V_{\text{cc}} \frac{k}{(1+k+x)^2} \end{aligned}$$

# Voltage divider vs Wheatstone bridge: sensitivity

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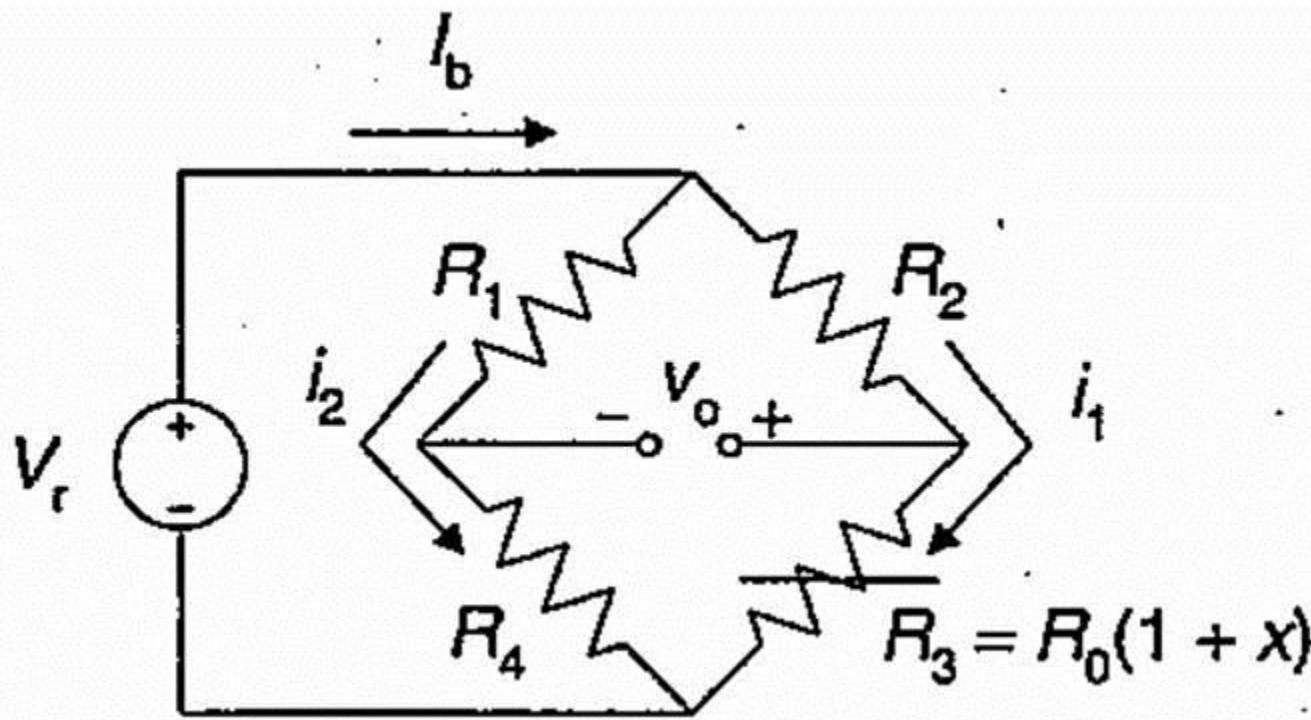


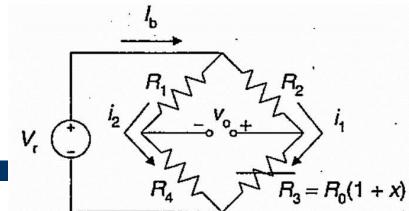
The voltage divider has a great offset compared with voltage changes, and this makes the curves 'flat' close to  $x=0$ .

The sensitivity of the two circuits is the same, but the sensitivity of the Wheatstone bridge can be increased by an amplifier stage

# Wheatstone bridge: different configurations (“quarter bridge”, “half bridge”, “full bridge”)

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 $R_1$  $R_2$  $R_3$  $R_4$  $V_0$   
(const.  $V_r$ ) $V_0$   
(const. I)

$R_0$	$R_0$	$R_0(1+x)$	$R_0$	$V_r \frac{x}{2(2+x)}$	$IR_0 \frac{x}{4+x}$
$R_0(1+x)$	$R_0$	$R_0(1+x)$	$R_0$	$V_r \frac{x}{2+x}$	$IR_0 \frac{x}{2}$
$R_0$	$R_0$	$R_0(1+x)$	$R_0(1-x)$	$V_r \frac{2x}{(4-x^2)}$	$IR_0 \frac{x}{2}$
$R_0$	$R_0(1-x)$	$R_0(1+x)$	$R_0$	$V_r \frac{x}{2}$	$IR_0 \frac{x}{2}$
$R_0(1-x)$	$R_0$	$R_0(1+x)$	$R_0$	$V_r \frac{-x^2}{(4-x^2)}$	$IR_0 \frac{-x^2}{4}$
$R_0(1+x)$	$R_0(1-x)$	$R_0(1+x)$	$R_0(1-x)$	$V_r x$	$IR_0 x$

- ⇒ Bridges supplied by constant current are generally more linear than bridges supplied by constant voltage
- ⇒ “full bridge” configurations are much used with strain gauges.

## Thermal compensation by Wheatstone bridge

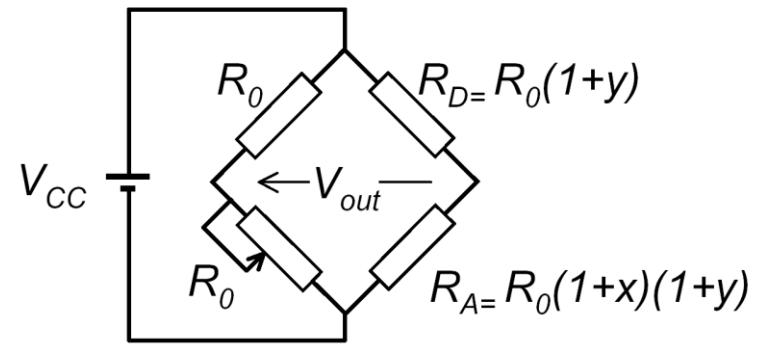
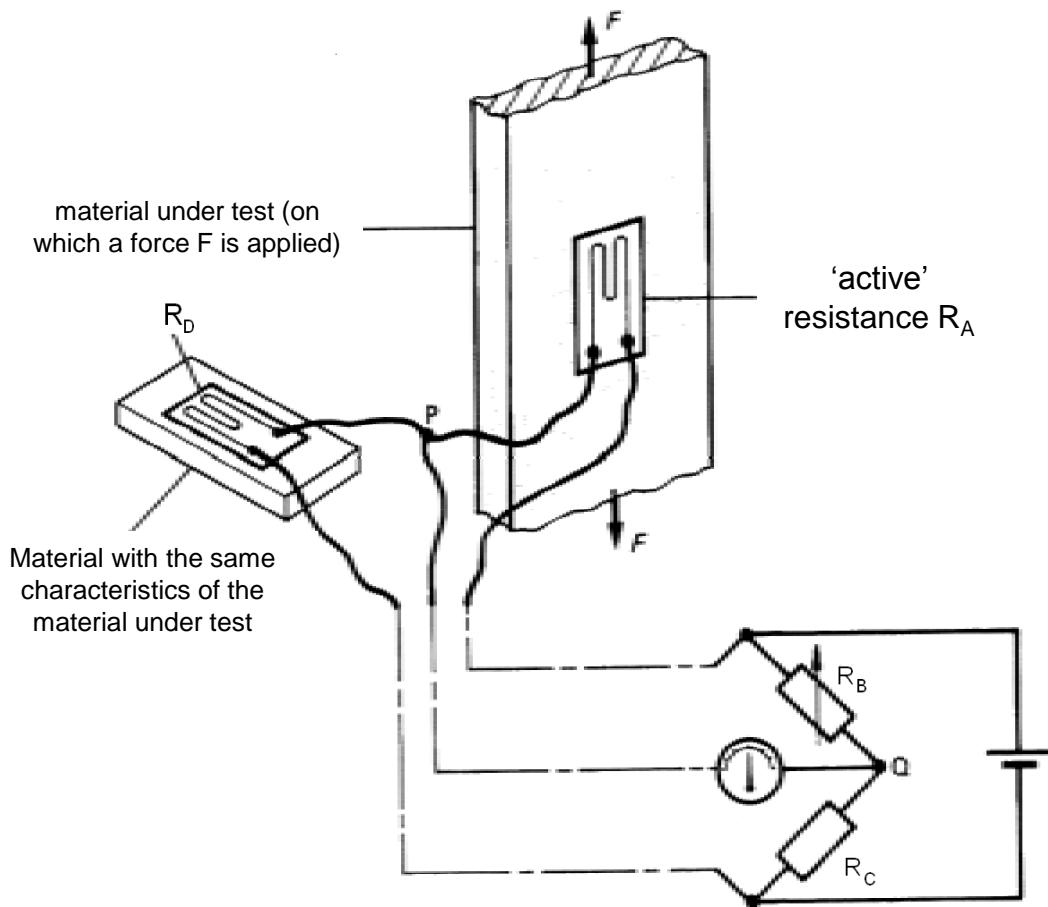
Strain gauges are sensible also to temperature changes.

Wheatstone bridge configuration can be useful to compensate this effect.

Assume the following setup:

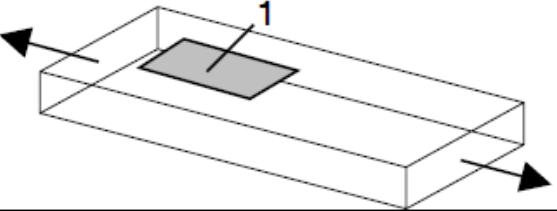
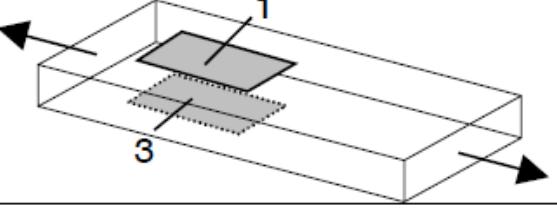
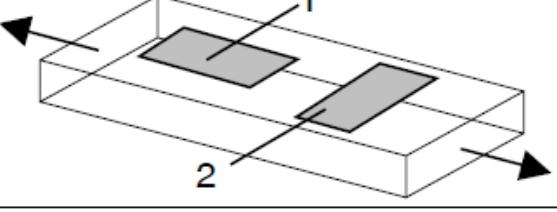
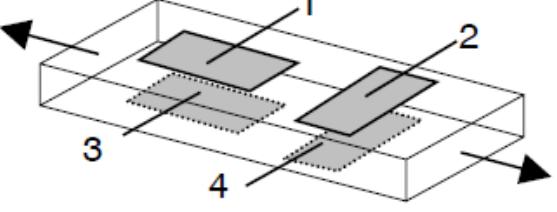
- An “active” resistance  $R_A$  submitted to both temperature (y) and mechanical (x) variations;
- A ‘dummy’ resistance  $R_D$ , positioned close to the active one, submitted ONLY to temperature variations;

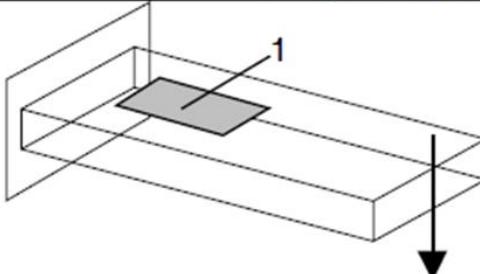
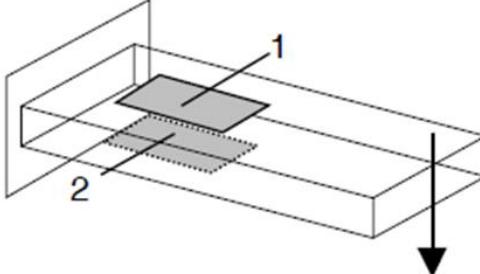
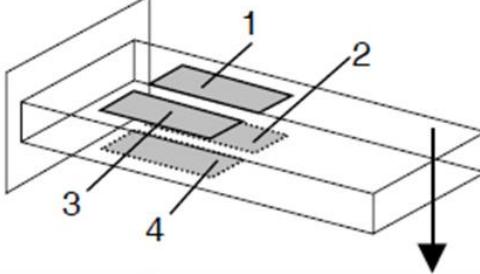
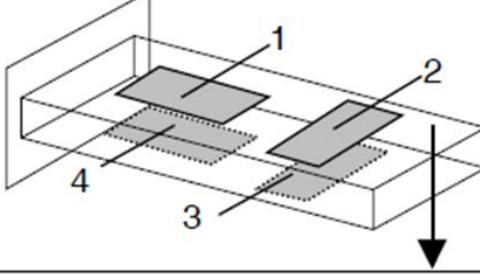
# Thermal compensation by Wheatstone bridge

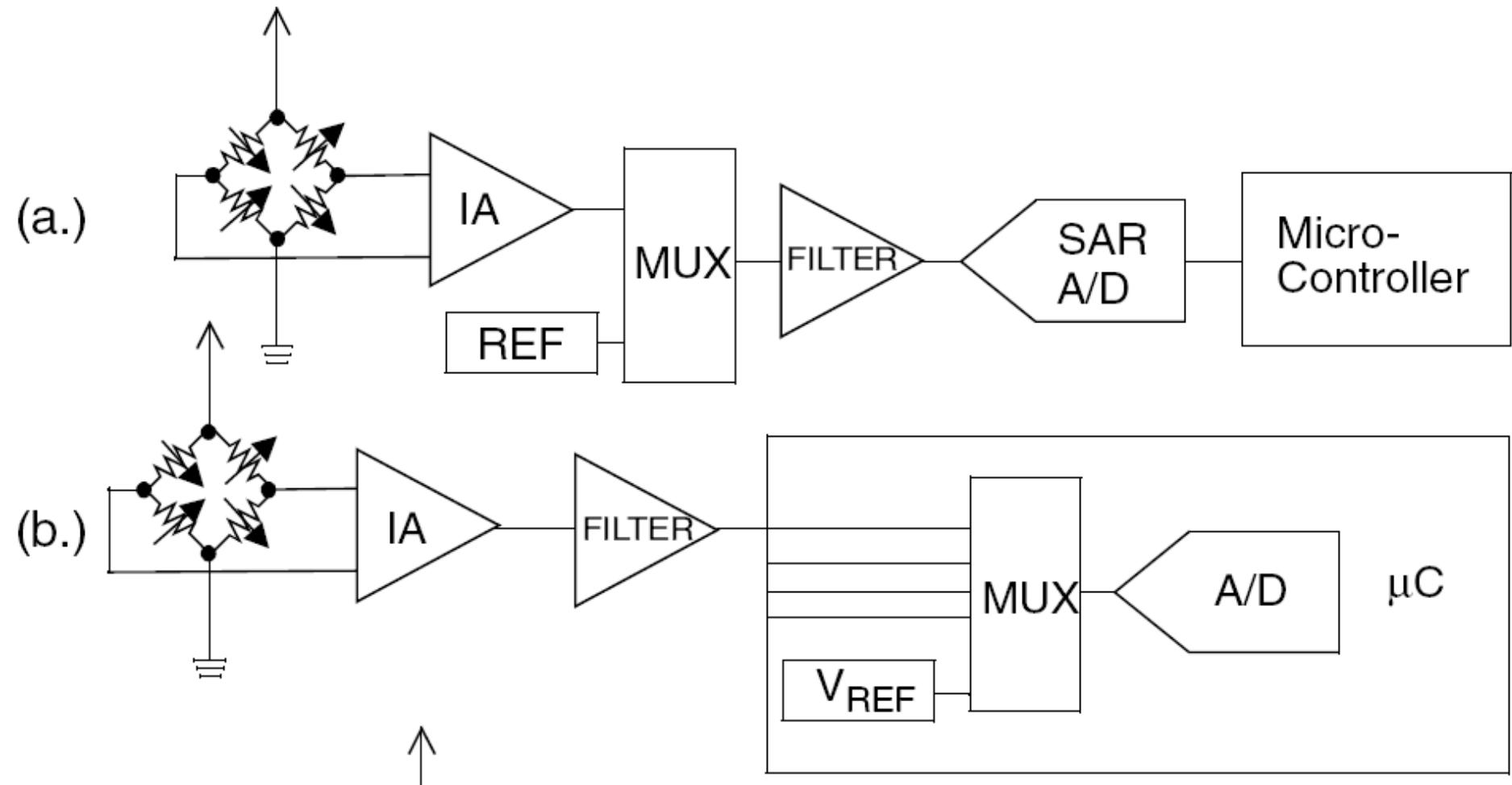


⇒ The effect of  $(1+y)$  is cancelled



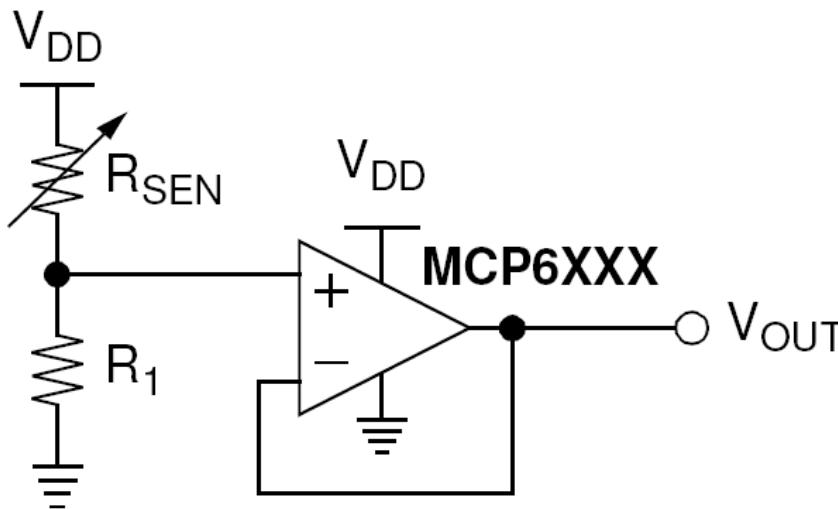
No.	K	Configuration	Notes
A-1	1		Must use dummy gage in an adjacent arm (2 or 4) to achieve temperature compensation
A-2	2		Rejects bending strain but not temperature compensated; must add dummy gages in arms 2 & 4 to compensate for temperature.
A-3	(1+v)		Temperature compensated but sensitive to bending strains
A-4	2(1+v)		<b>Best:</b> compensates for temperature and rejects bending strain.

No.	K	Configuration	Notes
F-1	1		Also responds equally to axial strains; must use dummy gage in an adjacent arm (2 or 4) to achieve temperature compensation
F-2	2		Half-bridge; rejects axial strain and is temperature compensated; dummy resistors in arms 3 & 4 can be in strain indicator.
F-3	4		<b>Best:</b> Max sensitivity to bending; rejects axial strains; temperature compensated.
F-4	$2(1+\nu)$		Adequate, but not as good as F-3; compensates for temperature and rejects axial strain.



# 1. Resistance-to-voltage conversion:

## a) voltage divider



The voltage divider ( $R_{SEN}$  and  $R_1$ ) converts the sensor resistance to a voltage. The op-amp buffers the voltage divider for further signal processing.

### Advantages

- Simplicity
- Ratiometric output\* (with an Analog-to-Digital Converter (ADC) using  $V_{DD}$  as its reference voltage)
- Detection of open sensor (failure)

### Disadvantages

- Poor common mode noise rejection
- Voltage is a non-linear function of resistance

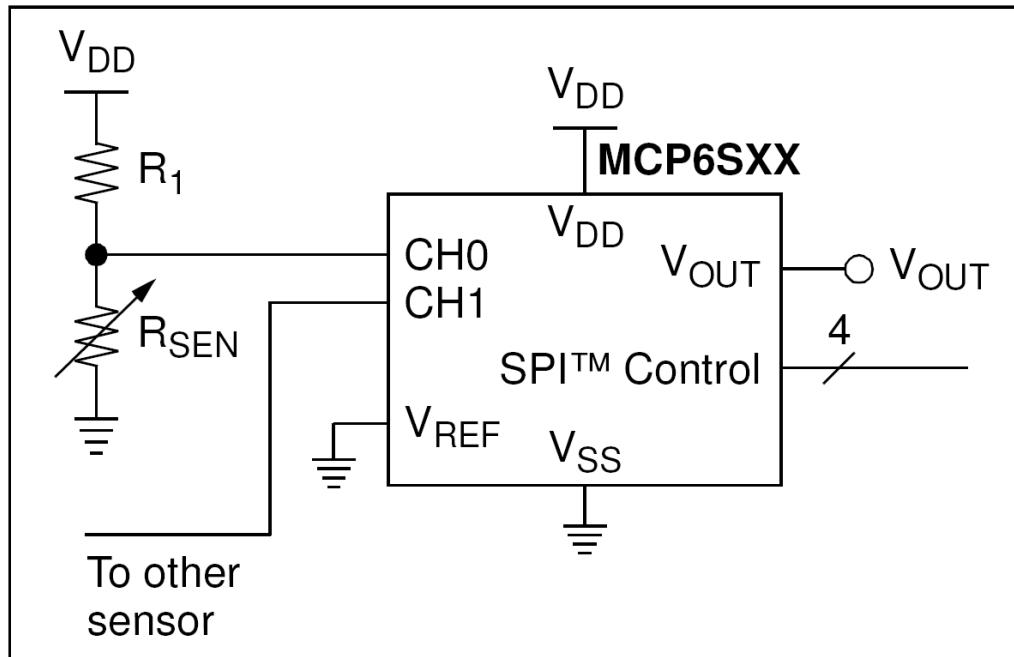
### Sensor Examples

- Thermistor
- RTD
- Magneto-resistive sensor

\* the output is proportional to the supply voltage

# 1. Resistance-to-voltage conversion:

## b) Voltage Divider and Variable Gain



*The voltage divider ( $R_{SEN}$  and  $R_1$ ) converts the sensor resistance to a voltage. The MCP6Sxx (a PGA, programmable gain amplifier) buffers the voltage divider for further signal processing and can be set to different gains when the sensor is non-linear.*

### Advantages

- Linearization of non-linear sensors
- Ratiometric output (with an ADC using  $V_{DD}$  as its reference voltage)
- Multiplexing several sensors
- Detection of open sensor(failure)

### Disadvantages

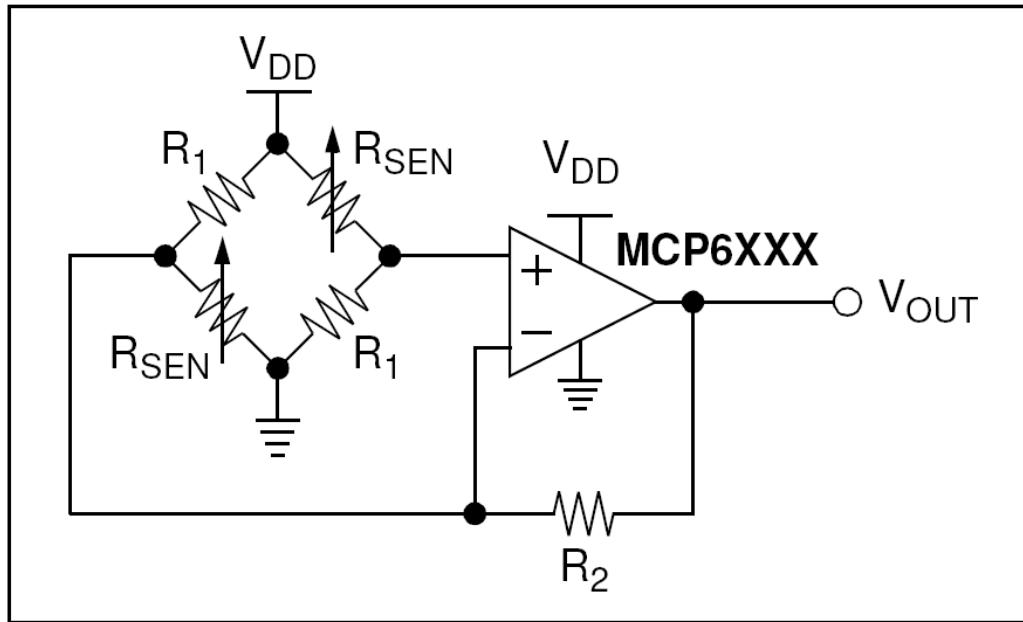
- Poor common mode noise rejection
- Voltage is a non-linear function of resistance
- Needs a controller and firmware

### Sensor Examples

- Thermistor

# 1. Resistance-to-voltage conversion:

## c) Wheatstone Bridge



A Wheatstone bridge converts a change in resistance to a change in differential voltage.  
The op amp amplifies the difference voltage.

### Advantages

- Good rejection of common mode noise
- Ratiometric output (with an ADC using  $V_{DD}$  as its reference voltage)
- Simplicity
- Detection of open sensor(failure)

### Disadvantages

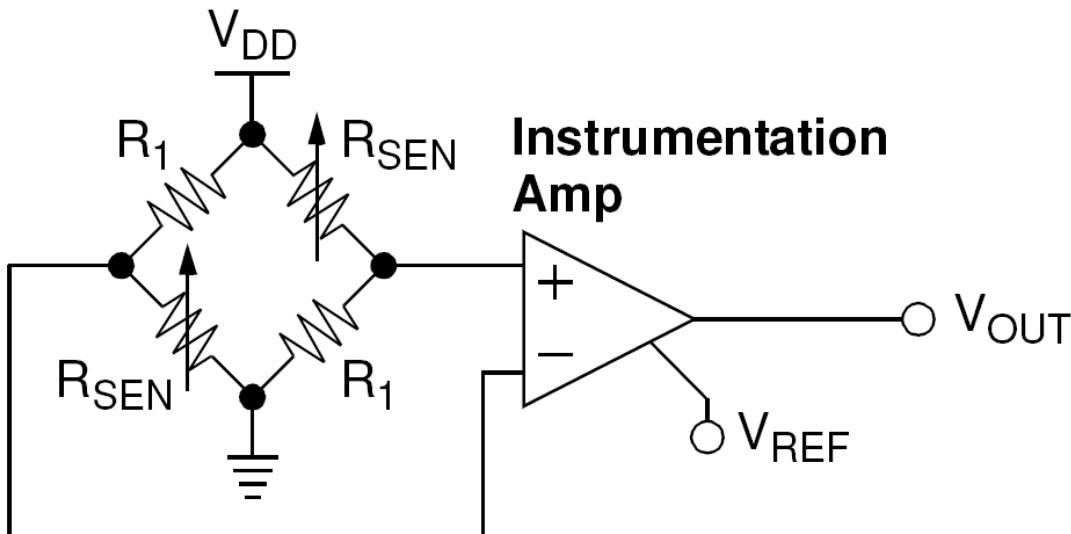
- Gain is a function of  $R_{SEN}$
- Needs a controller and firmware to correct
- Voltage is a non-linear function of resistance

### Sensor Examples

- Strain gage
- Pressure sensor
- Magneto-resistive sensor

# 1. Resistance-to-voltage conversion: d) Wheatstone Bridge and INA

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*In this Wheatstone bridge circuit the instrumentation amplifier amplifies the bridge's difference voltage and gives excellent rejection of common mode noise.*

## Advantages

- Excellent common mode noise rejection
- Ratiometric output (with an ADC using  $V_{DD}$  as its reference voltage)
- Detection of open sensor failure)

## Disadvantages

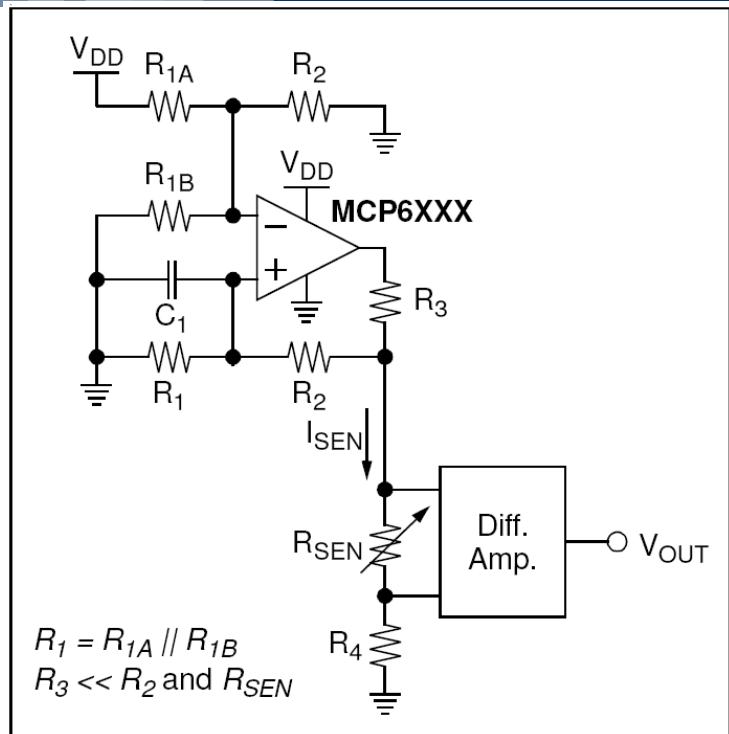
- Cost
- Voltage is a non-linear function of resistance

## Sensor Examples

- Strain gage
- Pressure sensor
- Magneto-resistive sensor

# 1. Resistance-to-voltage conversion:

## e) Floating Current Source



This circuit provides a current source ( $I_{SEN}$ ) that accurately converts resistance to voltage.  $R_{1A}$ ,  $R_{1B}$ ,  $R_1$ ,  $R_2$ ,  $R_3$  and the op amp form a current source (Howland current pump).  $C_1$  stabilizes this current source and reduces noise.  $R_4$  provides isolation from ground for remote sensors. The voltage across  $R_{SEN}$  is amplified by a difference amplifier which also rejects common mode noise. The voltage on top of  $R_4$  can be used to detect an open (failed) sensor.

### Advantages

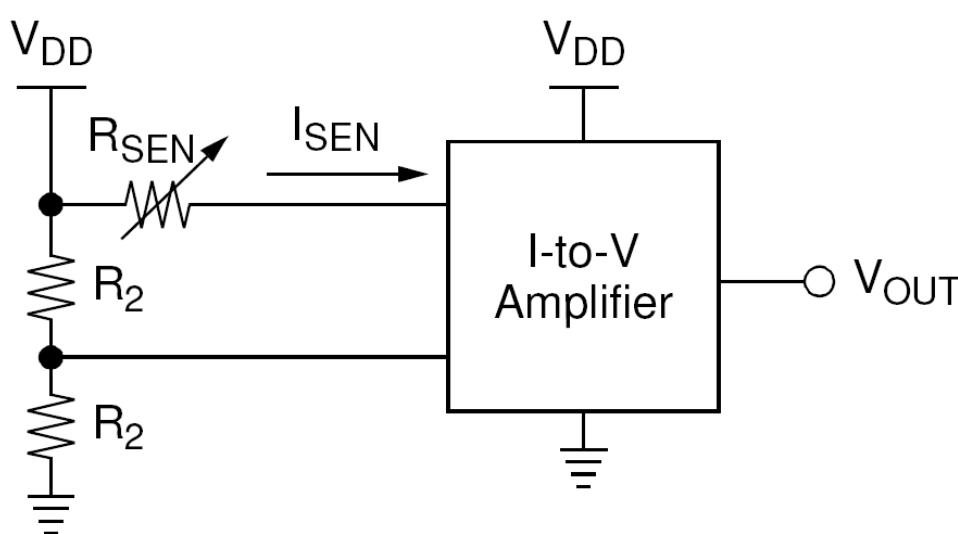
- Linearity of resistance to voltage conversion
- Ratiometric output (with an ADC using  $V_{DD}$  as its reference voltage)

### Disadvantages

- Cost
- Requires accurate resistors

### Sensor Examples

- Thermistor
- RTD
- Hot-wire anemometer



*The second strategy for conditioning a resistive sensor is to produce a current that is a function of the resistance. The basic strategy is to use a “I-to-V Amplifier” which can be a transimpedance amp or a logarithmic amp.*

### Advantages

- Linearity of resistance to voltage conversion
- Ratiometric output (with an ADC using  $V_{DD}$  as its reference voltage)

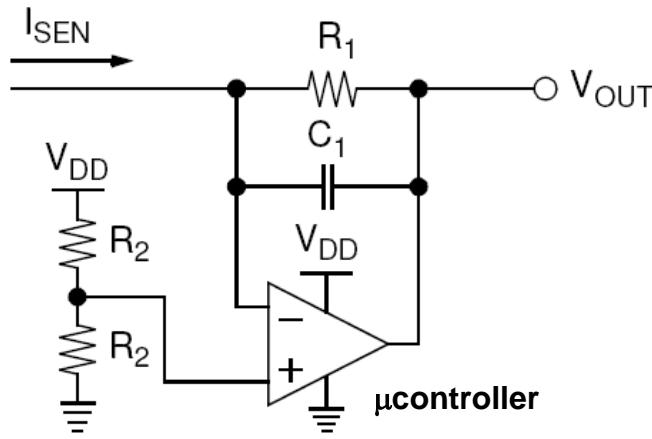
### Disadvantages

- Cost
- Requires accurate resistors

### Sensor Examples

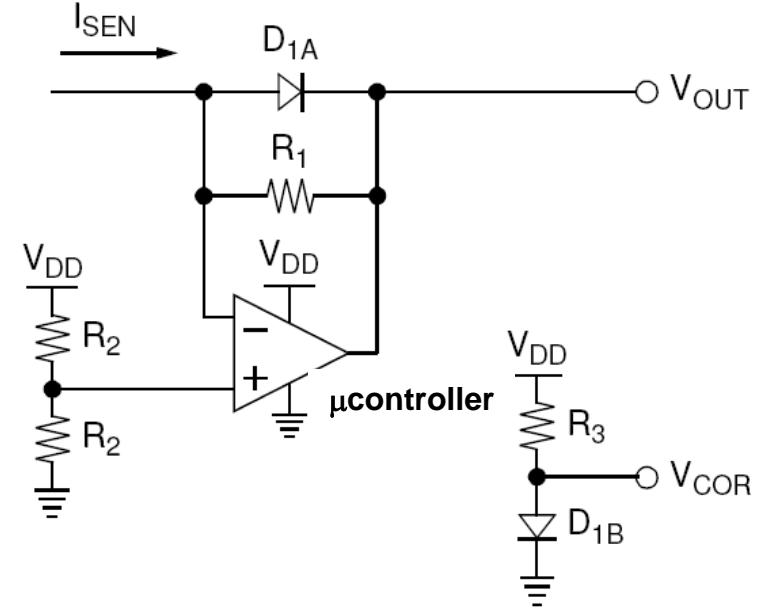
- Thermistor
- RTD
- Hot-wire anemometer

## Transimpedance Amplifier



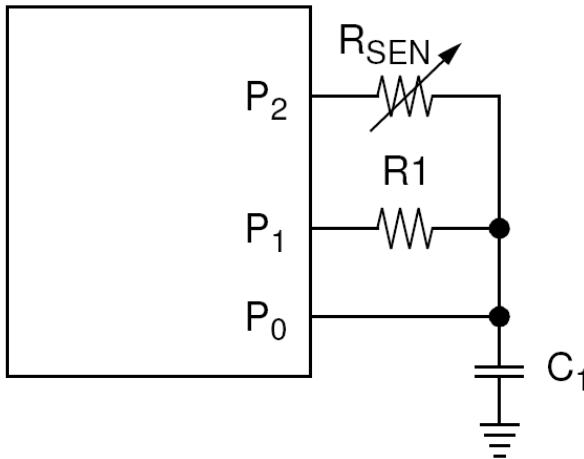
a transimpedance amplifier ( $R_1$  and the op amp) converts the sensor current ( $I_{SEN}$ ) to a voltage. The capacitor  $C_1$  is sometimes needed to stabilize the amplifier when the source has a large capacitance

## Logarithmic Amplifier



$D_{1A}$  and  $D_{1B}$  are a matched pair in the same package.

a logarithmic amplifier ( $D_{1A}$  and the op amp) converts the sensor current ( $I_{SEN}$ ) to a voltage proportional to the logarithm of the current.  $R_1$  maintains negative feedback when  $I_{SEN}$  is small or negative.  $D_{1B}$  is used to correct  $D_{1A}$  for temperature changes.

**μcontroller unit (MCU)**

*The third strategy for conditioning a resistive sensor is to produce a voltage with a RC decay (single pole response to a step). The time it takes for the voltage to decay to a threshold is a measure of the resistance. The figure shows a circuit using a MCU circuit that sets a ratiometric threshold (proportional to  $V_{DD}$ ). The time is measured for both  $R_1$  and  $R_{SEN}$  separately in order to correct for  $V_{DD}$ ,  $C_1$ , and temperature errors. The MCU provides the switching and control needed.*

**Advantages**

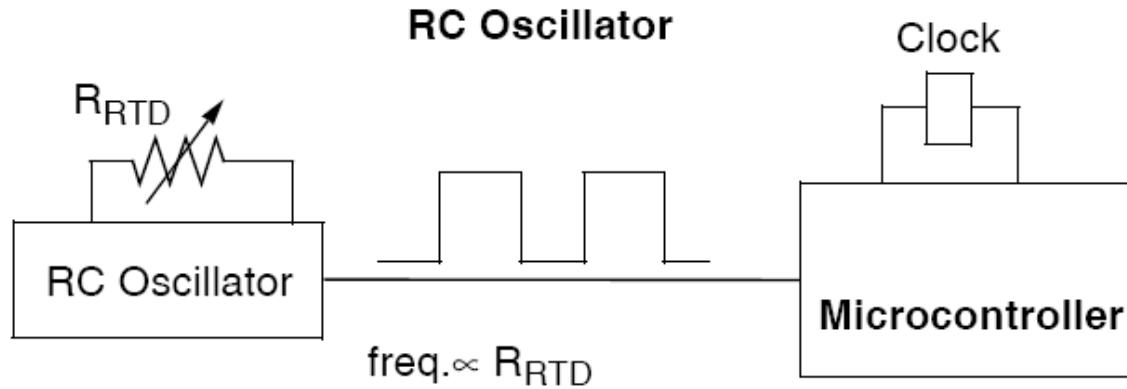
- Ratiometric correction of  $V_{DD}$ ,  $C_1$  and temperature errors
- Accurate
- Simple timing measurement

**Disadvantages**

- MCU timing resolution
- Digital noise
- Threshold must be ratiometric

**Sensor Examples**

- Thermistor



*The fourth strategy for conditioning a resistive sensor is to measure a change in oscillation frequency. Oscillator circuits can be used to provide accurate temperature measurement with Resistive Temperature Detector (RTD) sensors. Oscillators provide a frequency output that is proportional to temperature and are easily integrated into a microcontroller system.*

## Advantages

- Does not require ADC or  $V_{REF}$
- Excellent noise immunity
- Accuracy proportional to quality of microcontroller clock

## Disadvantages

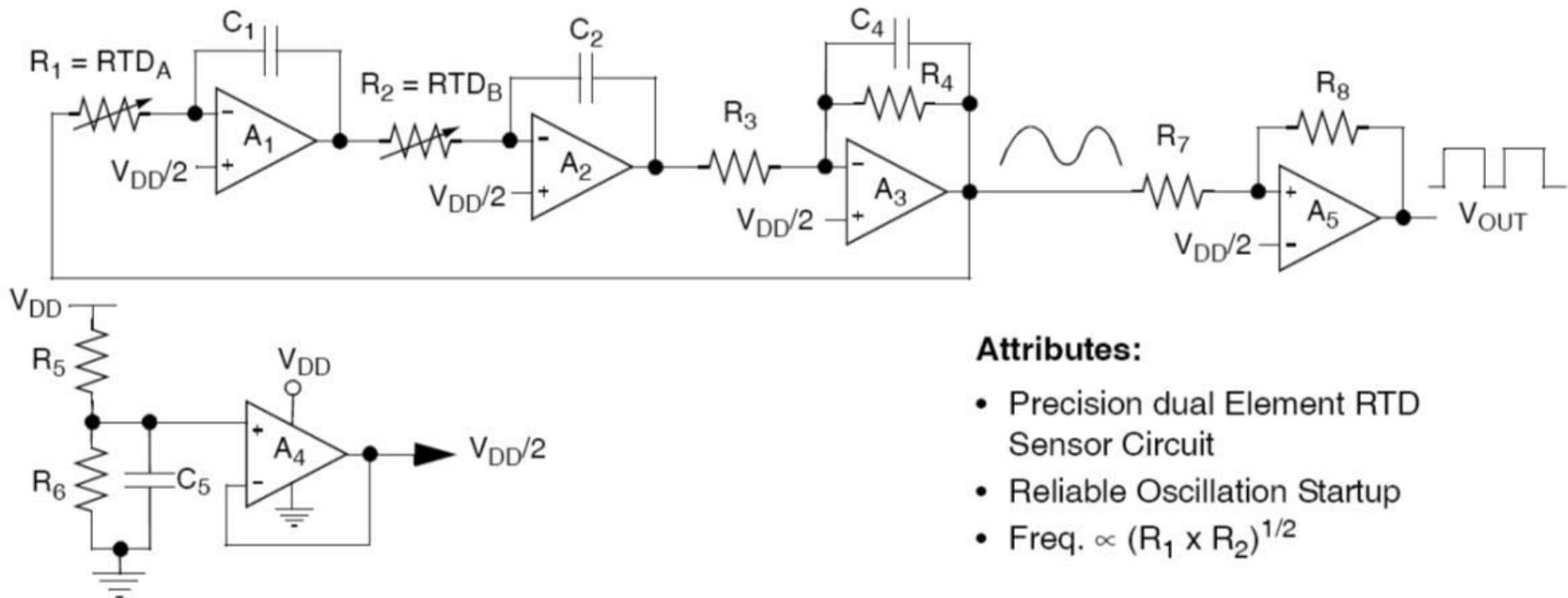
- Cost
- Design complexity

## Sensor Examples

- RTD
- hot-wire anemometer

## 4. Oscillator frequency: a) State variable oscillator

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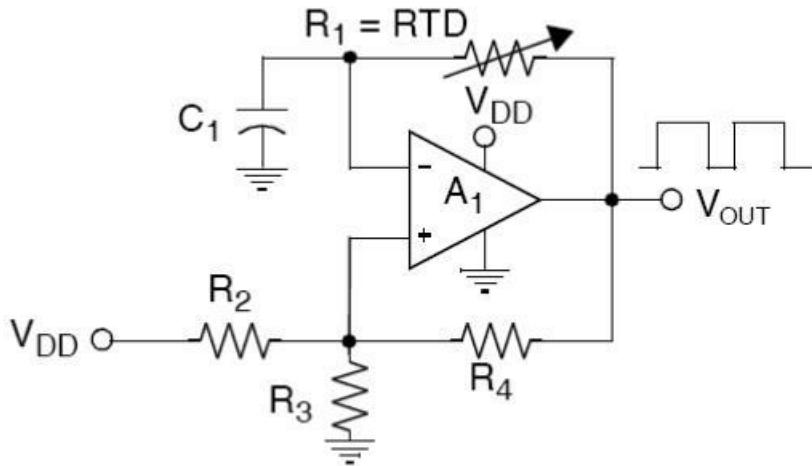


### Attributes:

- Precision dual Element RTD Sensor Circuit
- Reliable Oscillation Startup
- Freq.  $\propto (R_1 \times R_2)^{1/2}$

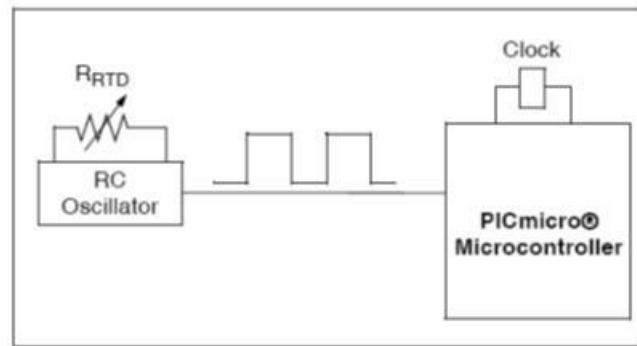
## 4. Oscillator frequency: b) Relaxation oscillator

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### Attributes:

- Low Cost Solution
- Single Comparator Circuit
- Square Wave Output
- Freq. =  $1 / (1.386 \times R_1 \times C_1)$

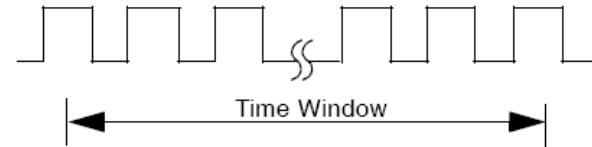


## Fixed time method

### Algorithm:

Count the number of clock pulses in a time window.

Oscillator  
Signal



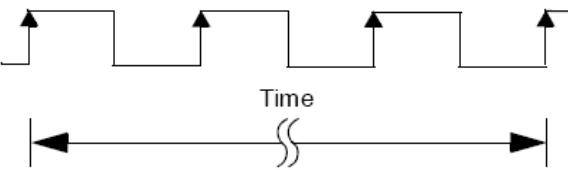
**Example:** Measure the number of oscillation pulses in a 100 ms window and multiply by 10 to determine the frequency.

## Fixed cycle method

### Algorithm:

Determine the time between a fixed number of oscillation pulses.

Oscillator  
Signal



**Example:** Measure time between four rising edges of the oscillation signal.