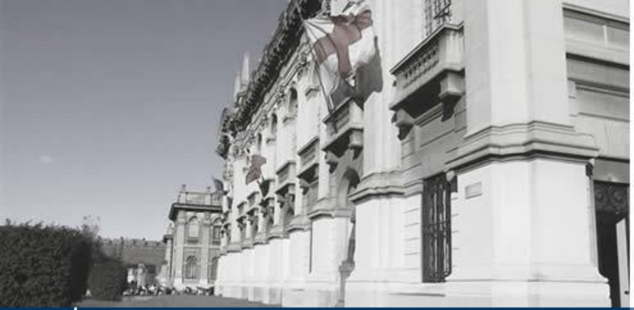
DIPARTIMENTO di ELETTRONICA, INFORMAZIONE e BIOINGEGNERIA



POLITECNICO DI MILANO





Lecture 9
Acceleration Sensors
Conditioning of capacitive sensors

Acceleration measurement (mono-axial accelerometer)

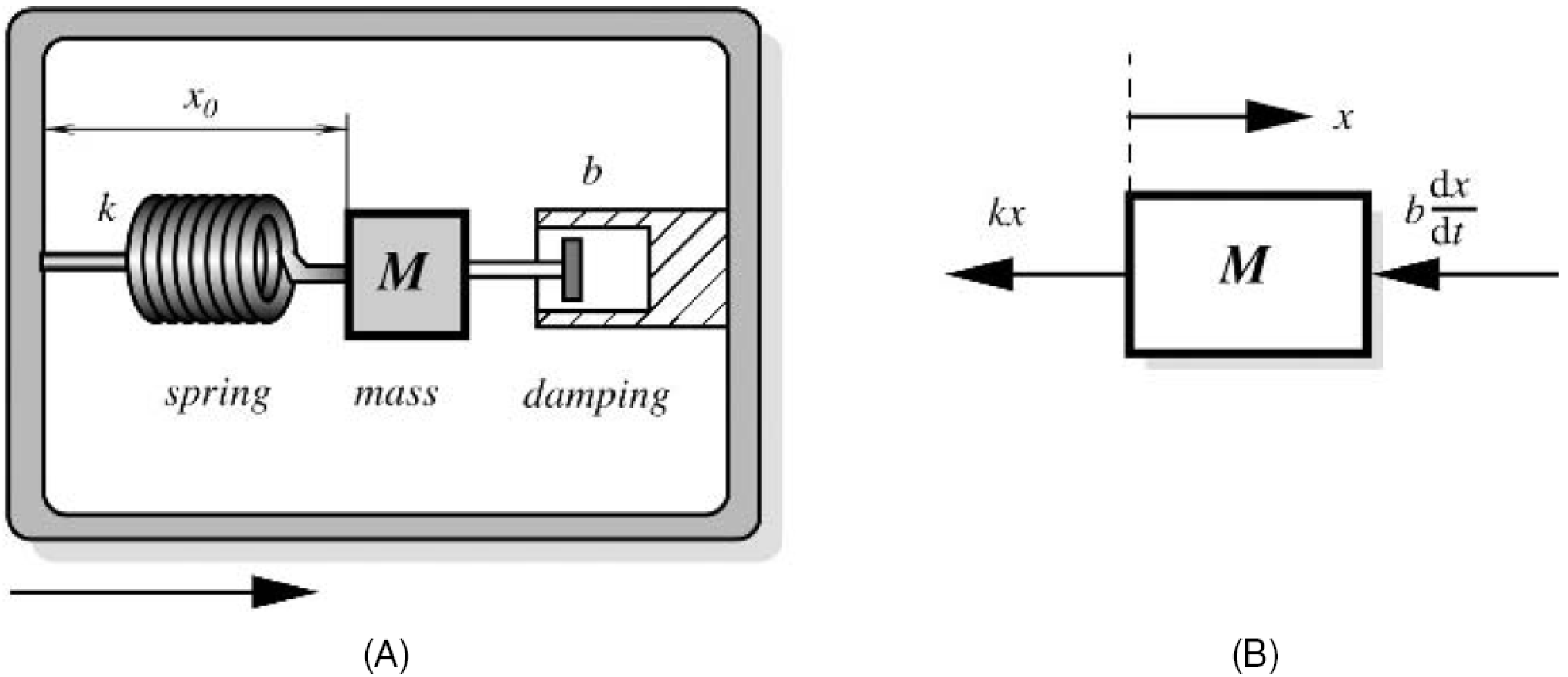


Fig. 3.49. Mechanical model of an accelerometer (A) and a free-body diagram of mass (B).

If the box is submitted to an acceleration equal to $\dfrac{d^2y}{dt^2}$

The output signal is proportional to displacement x of mass M

Acceleration measurement (mono-axial accelerometer)

By applying the Newton second law (with $x=x_0+cost$) we have:

$$Ma' = -kx - b\frac{dx}{dt}$$

where a' is the acceleration of mass M with respect to the Earth:

$$a' = \frac{d^2x}{dt^2} - \frac{d^2y}{dt^2}$$

Substituting a' we obtain the **equation of motion**:

$$M\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = M\frac{d^2y}{dt^2}$$

Acceleration measurement (mono-axial accelerometer)

In order to solve the differential equation of motion, it is possible to apply the Laplace transform :

$$Ms^2X(s)+bsX(s)+kX(s)=MA(s)$$

where $X(s)$ and $A(s)$ the Laplace transforms of $x(t)$ and $\frac{d^2y}{dt^2}$, respectively.

Solving, we obtain:

$$X(s)=\frac{MA(s)}{Ms^2+bs+k}$$

By introducing undamped angular frequency ω_0 and damping ratio ζ as

$$\omega_0=\sqrt{k/M} \quad and \;\; 2\xi\omega_0=b/M$$

We have

$$X(s)=\frac{A(s)}{s^2+2\xi\omega_0 s+\omega_0^2}$$

Step response of a second order system (e.g. mono-axial accelerometer)

$$G(s) = \frac{{\omega_n}^2}{s^2 + 2\zeta\omega_n s + {\omega_n}^2}$$

Underdamped (ζ<1): $x(t) \; = \; 1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}} \sin(\omega_d t + \psi)$

where $\omega_d = \omega_n\sqrt{1-\zeta^2}$ and $\psi = \tan^{-1}\frac{\sqrt{1-\zeta^2}}{\zeta}$

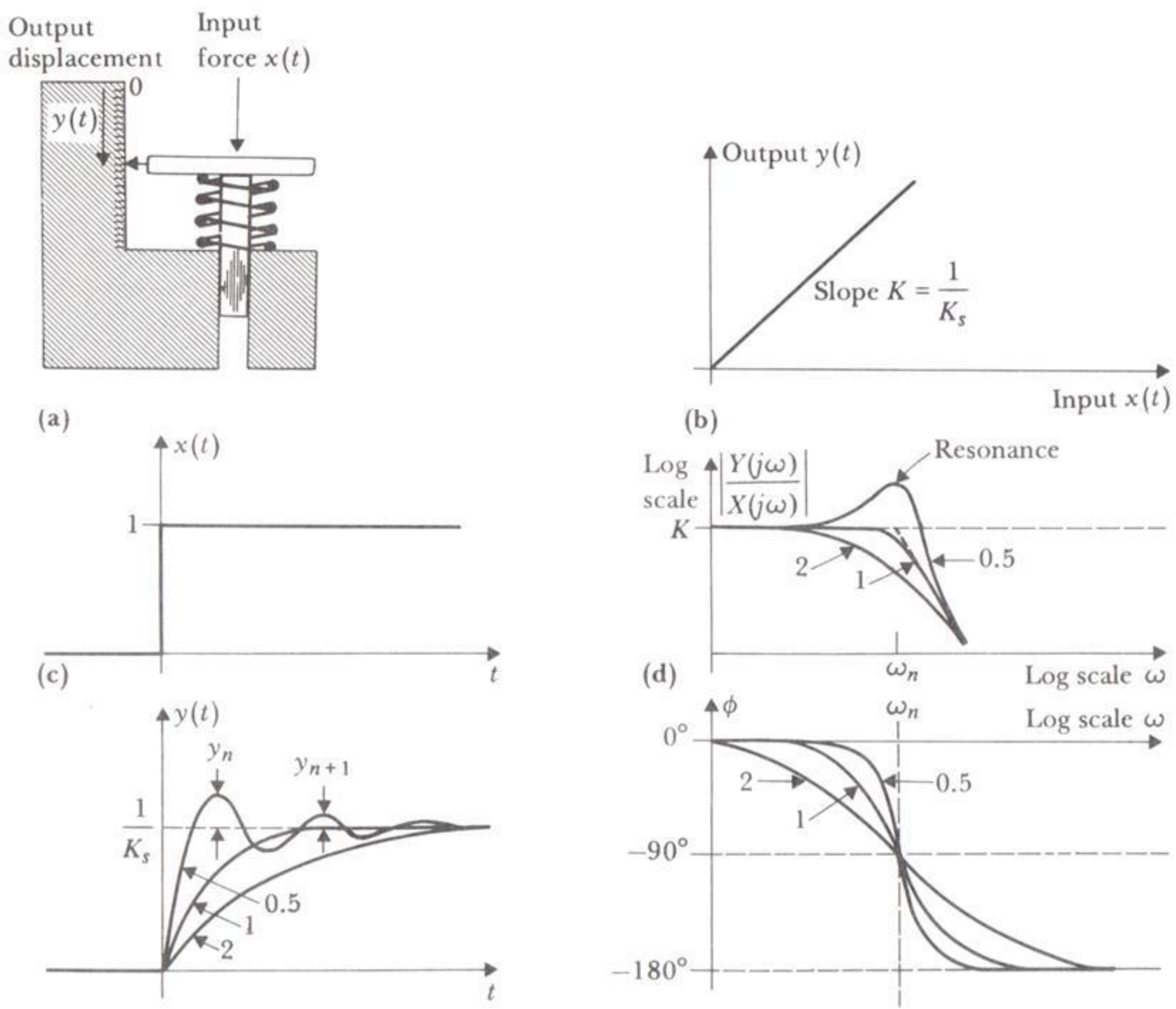
The system oscillates with the amplitude gradually decreasing to zero.

Critically damped (ζ=1): $x(t) = 1 - e^{-\omega_n t} - \omega_n t e^{-\omega_n t}$

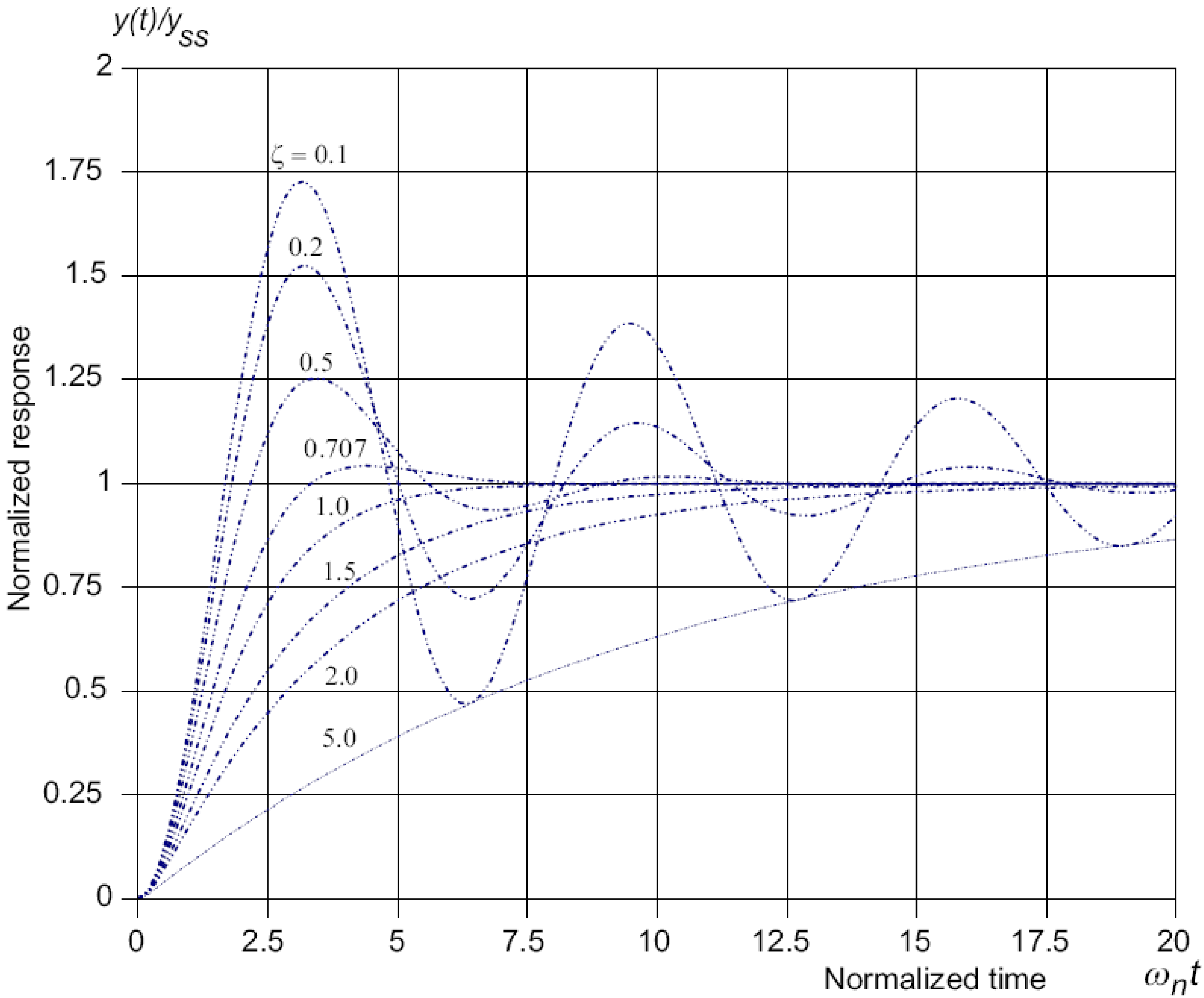
Overdamped (ζ>1): $x(t) \; = \; 1 - e^{-\omega_n t}\cosh\left(\omega_n\sqrt{\zeta^2-1}\right)t - \frac{\zeta}{\sqrt{\zeta^2-1}} e^{-\omega_n t}\sinh\left(\omega_n\sqrt{\zeta^2-1}\right)t$

The system returns (exponentially decays) to steady state without oscillating. Larger values of the damping ratio ζ return to equilibrium slower.

Second order systems



Step response



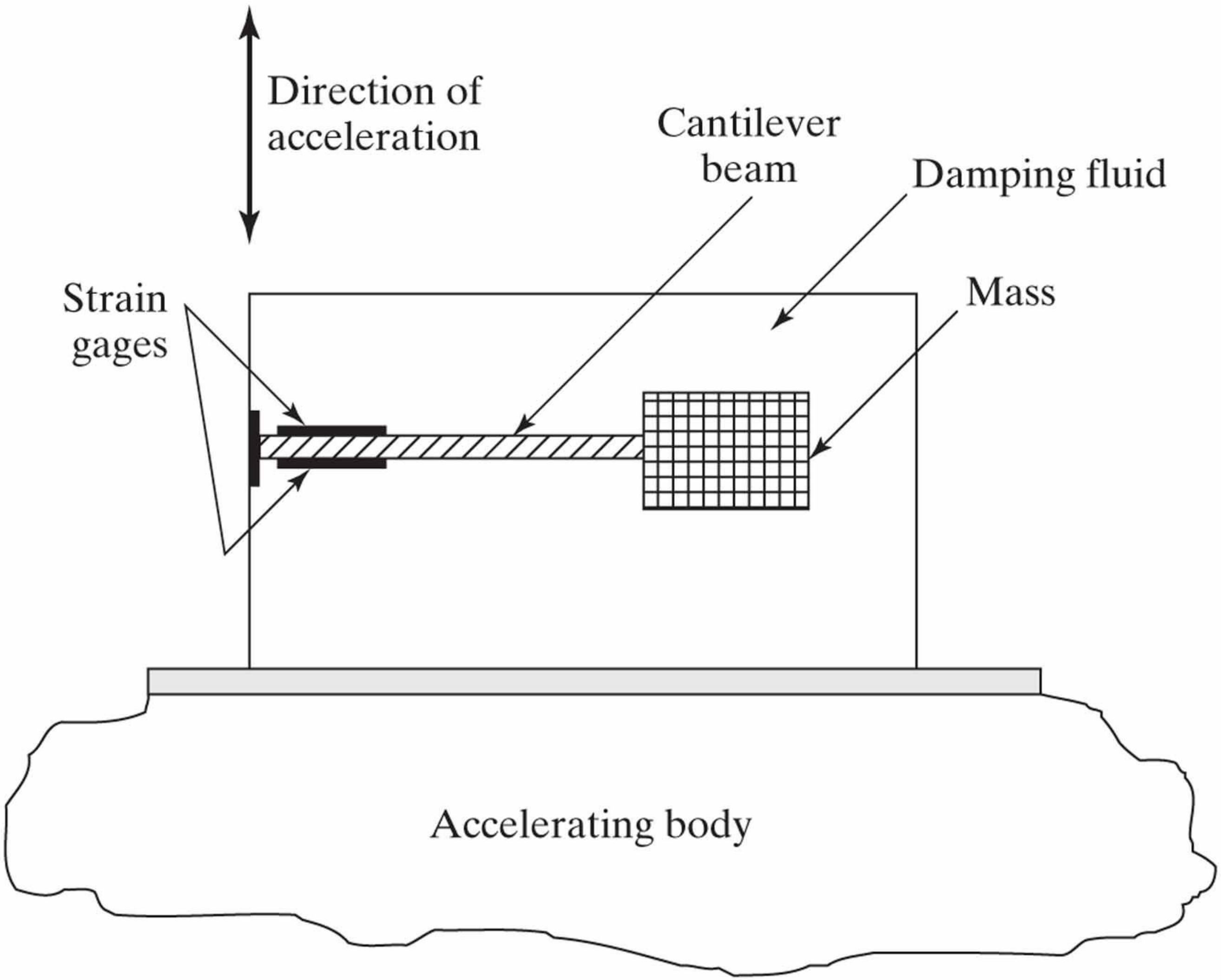
Acceleration sensors

An acceleration sensor is generally made of an element (“inertial mass” or “sismic element”) which, submitted to a given acceleration, moves with a given delay with respect to the housing.

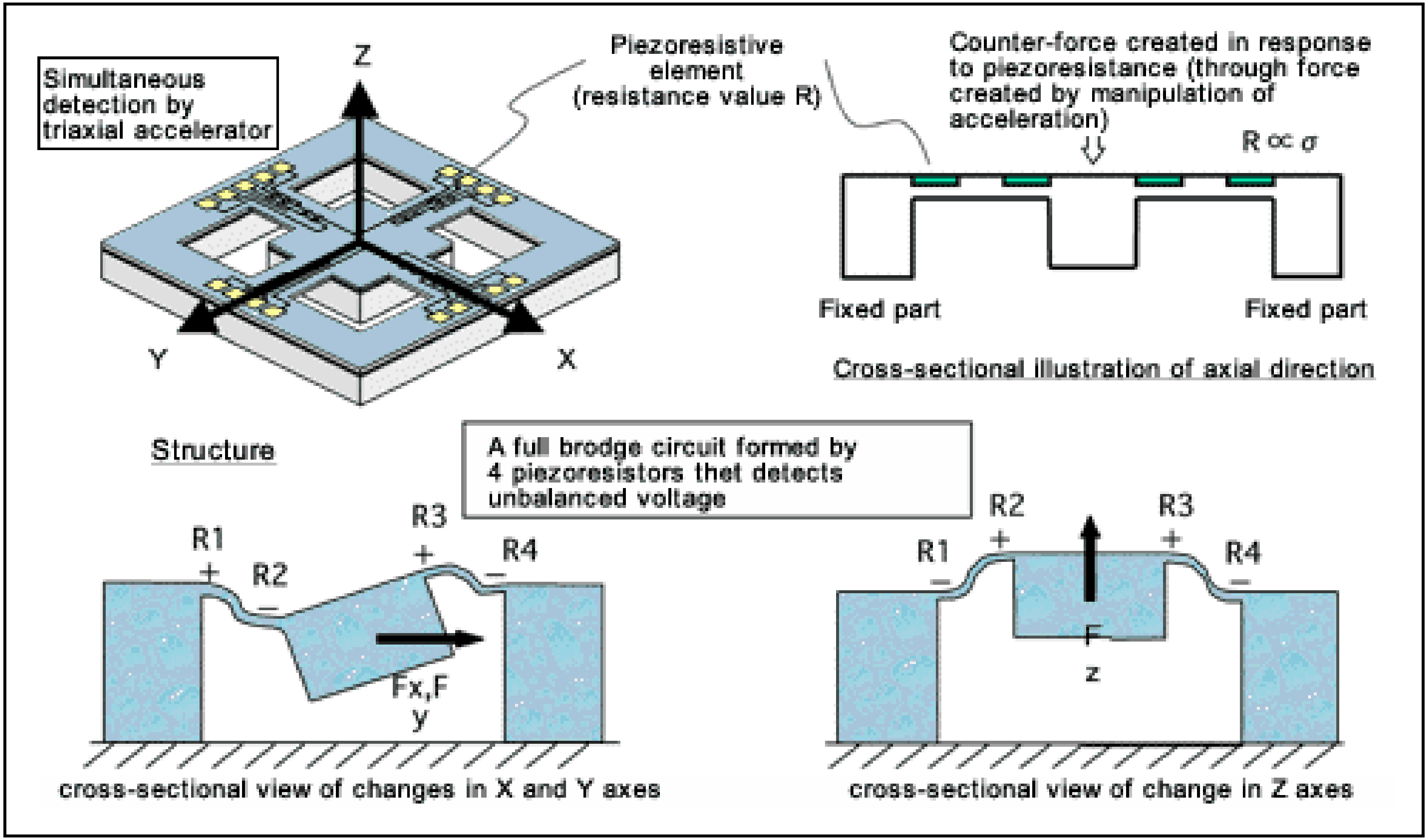
In other words, a given mass is displaced with respect to the housing.


ARRAY SPEEDS COMMUNICATIONS DEVELOPMENT
EVALUATING ASIC PROTOTYPES
FOR ENGINEERS AND ENGINEERING MANAGERS — WORLDWIDE
ELECTRONIC DESIGN
A PENTON PUBLICATION U.S. $5.00
AUGUST 8, 1991
COMBINING MICROMACHINING WITH ANALOG IC TECHNOLOGY
•SPECIAL SECTION: AUTOMOTIVE ELECTRONICS
•SPECIAL REPORT: RAMDACs FUEL GRAPHICS ADVANCES

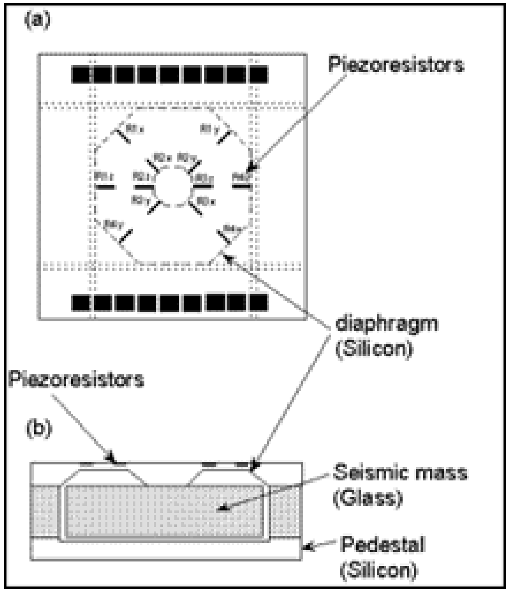
Piezoresistive acceleration sensors (1)



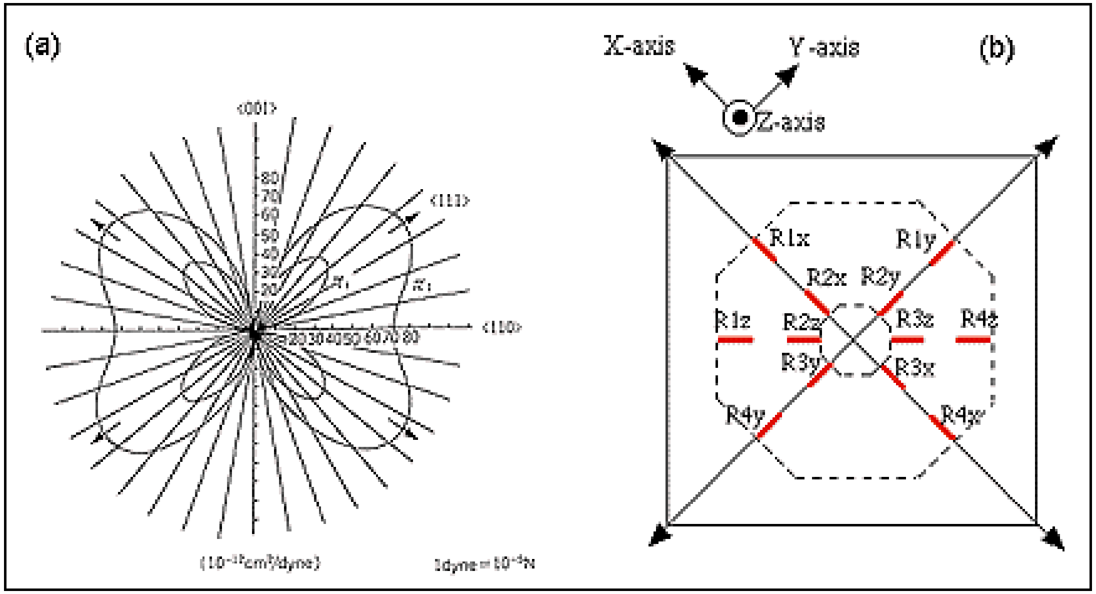
Piezoresistive acceleration sensors (2)



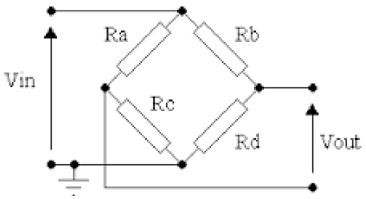
Piezoresistive accelerometer



The 3-axis silicon piezoresistive accelerometer is shown from the top (A) and in a cross-sectional view (B).



The piezoresistance coefficients in the P-type silicon (110) plane (A) lead directly to the schematic layout of the piezoresistors (B).



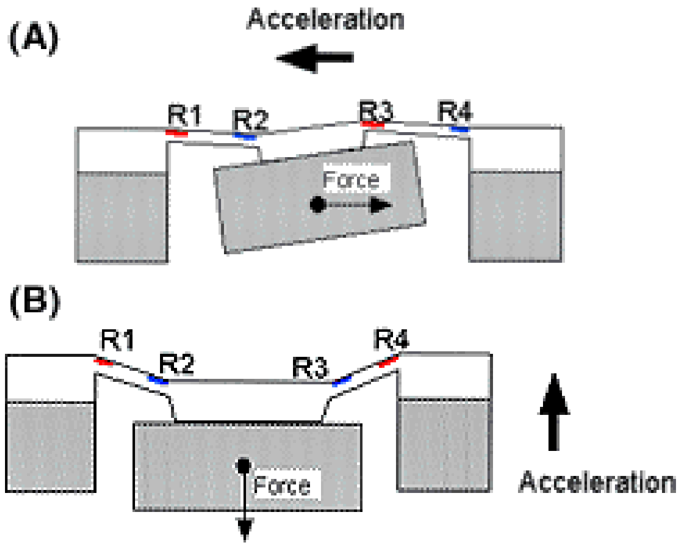
Three Wheatstone bridges:

Vout,x = [R4x/(R1x + R4x) - R3x/(R2x + R3x)] Vin

Vout,y = [R4y/(R1y + R4y) - R3y/(R2y+R3y)] Vin

Vout,z = [R3z/(R1z + R3z) - R4z/(R2z + R4z)] Vin

Change of each piezoresistance value in response to acceleration input along three axes



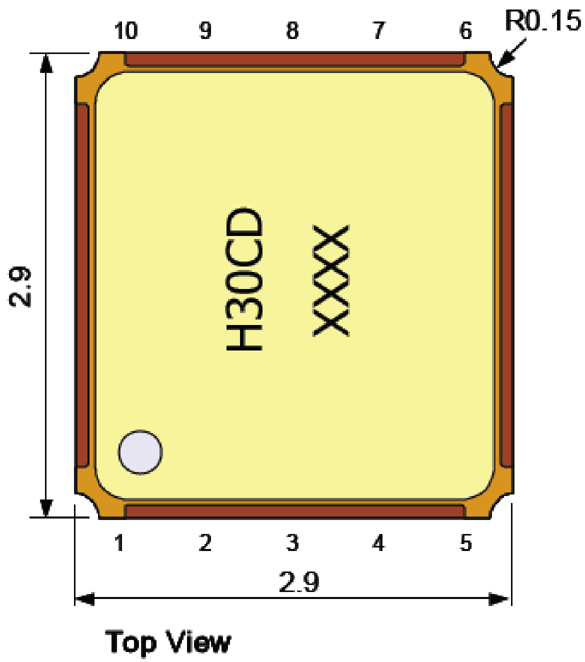
Acceleration applied along the X- or Y-axis causes the proof mass to incline (A), while acceleration along the Z-axis causes the mass to move in a downward direction (B).

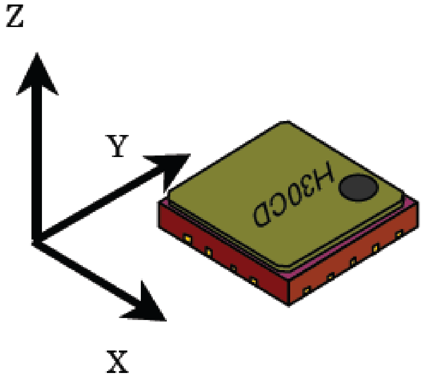
Acceleration direction	R1x	R2x	R3x	R4x	R1y	R2y	R3y	R4y	R1z	R2z	R3z	R4z
X-axis (horizontal)	+	-	+	-	0	0	0	0	+	-	+	-
Y-axis (horizontal)	0	0	0	0	+	-	+	-	+	-	+	-
Z-axis (vertical)	+	-	-	+	+	-	-	+	+	-	-	+

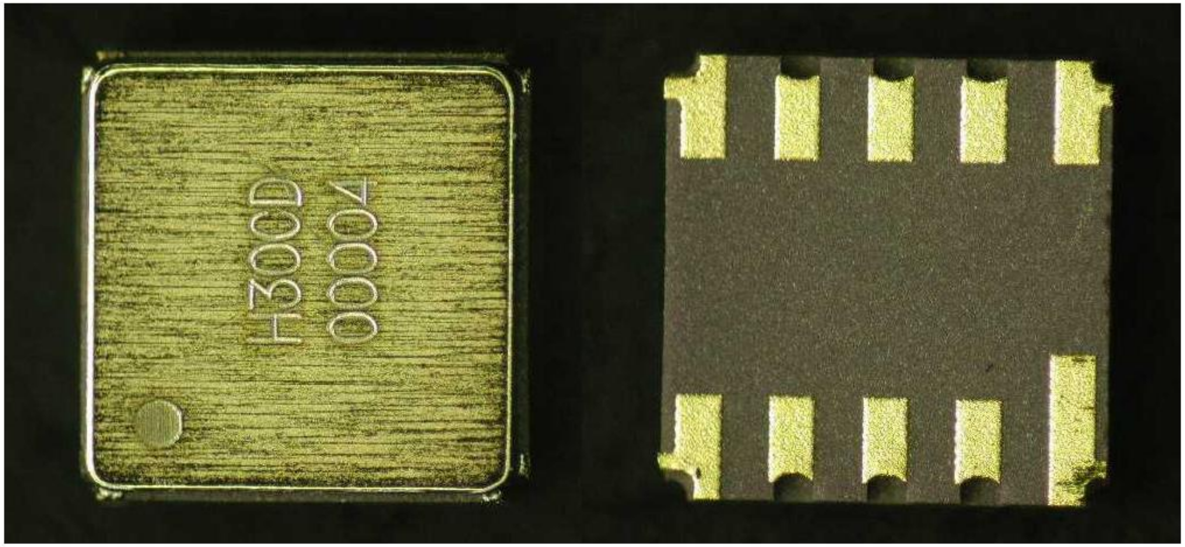
Note: + = increase; - = decrease; 0 = no change

http://archives.sensorsmag.com/articles/0299/0299_38/

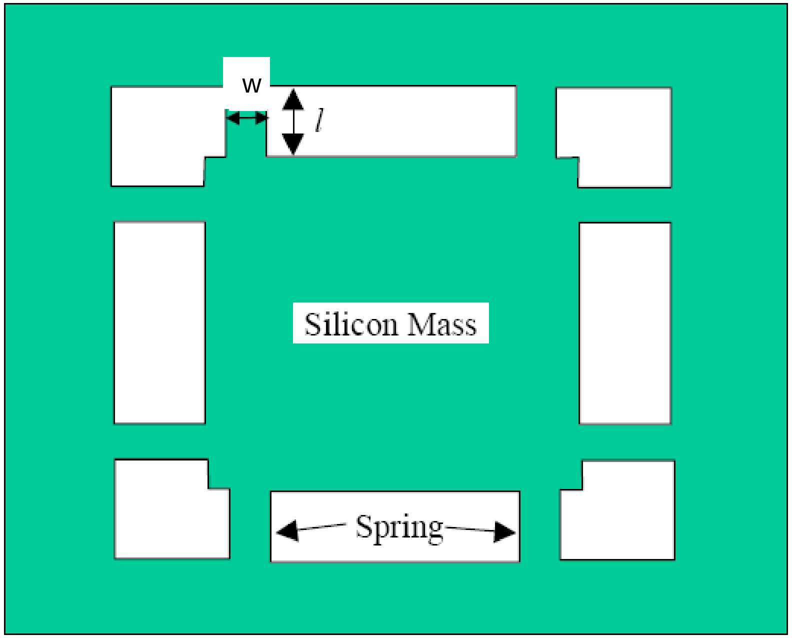
Piezoresistive acceleration sensors (3)



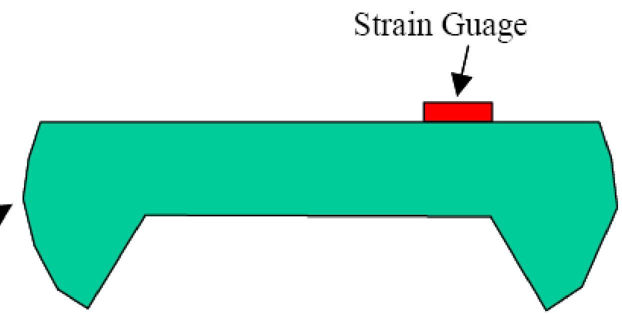


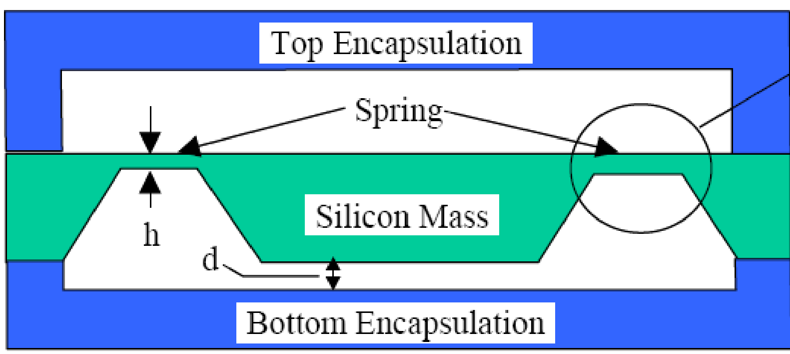


Silicon acceleration sensors



Top View





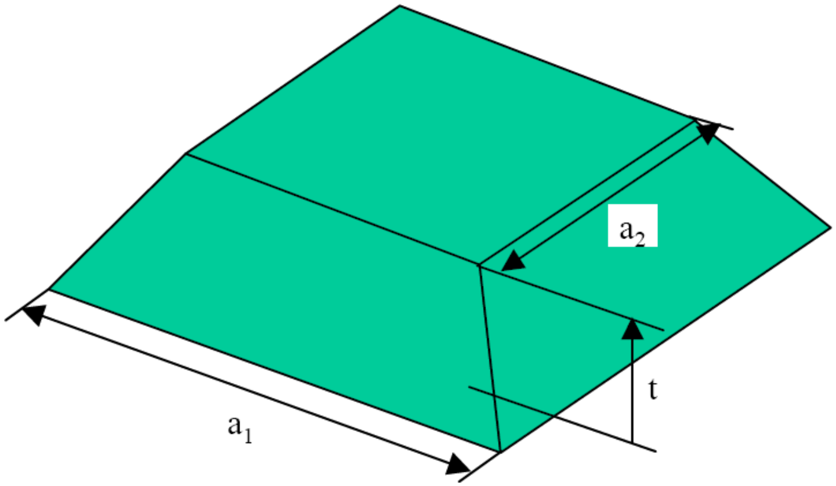
Cross-Sectional View

ELASTIC ELEMENT

$$k = \frac{E \cdot \left(wh^3\right)}{l^3}$$

$$for\ \ Si,\ E = 190 \times 10^9\ Nm^{-2}$$

Silicon acceleration sensors



INERTIAL ELEMENT

$$M = \frac{1}{3}\rho t \frac{\left(a_1^3 - a_2^3\right)}{\left(a_1 - a_2\right)}$$

$$for\ Si,\ \rho = 2300\ kg \cdot m^{-3}$$

RESISTIVE ELEMENT

$$b = \frac{0.42 \mu A^2}{d^3}$$

$$\mu = 1.85 \times 10^{-5}\ Nm^{-2}s^{-1}$$

(μ : air viscosity; $A = a_2^2$)

Capacitive accelerometer

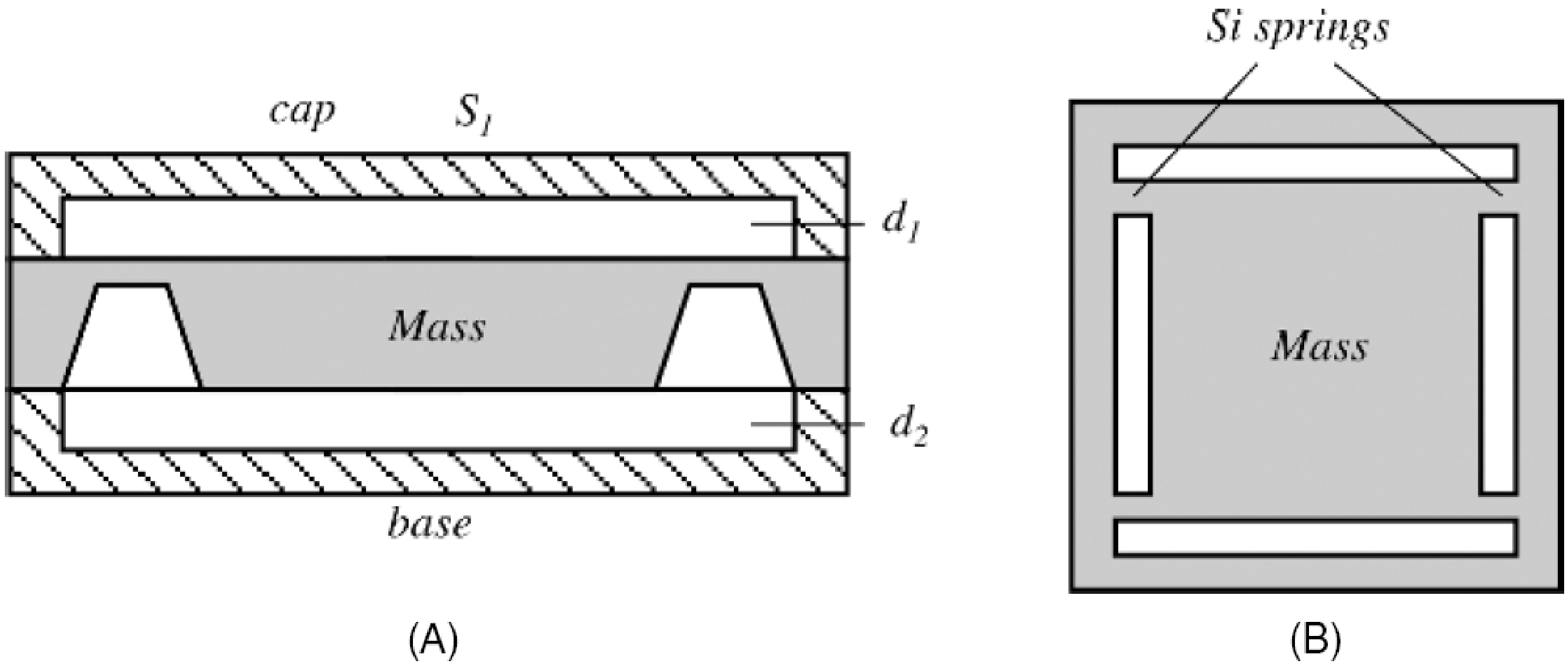


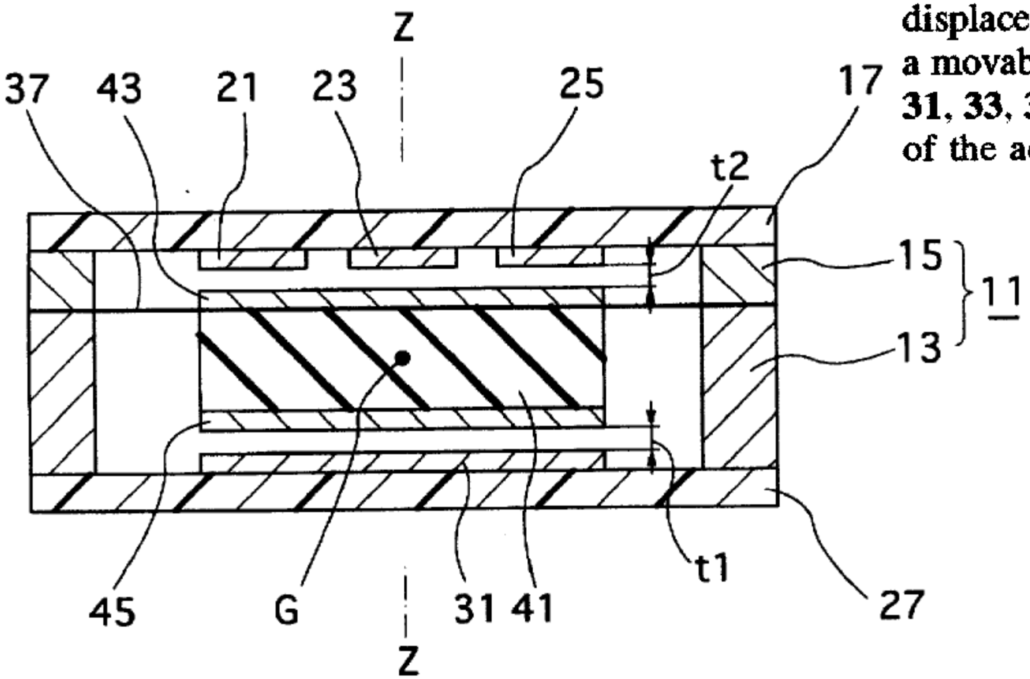
Fig. 8.3. Capacitive accelerometer with a differential capacitor: (A) side cross-sectional view; (B) top view of a seismic mass supported by four silicon springs.

Capacitive acceleration sensor (1)

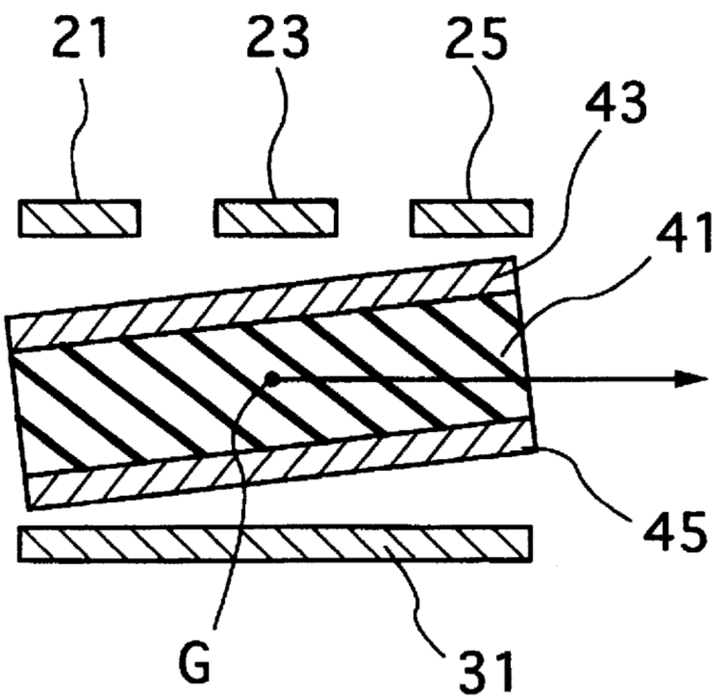
United States Patent [19] [11] **Patent Number: 5,719,336**

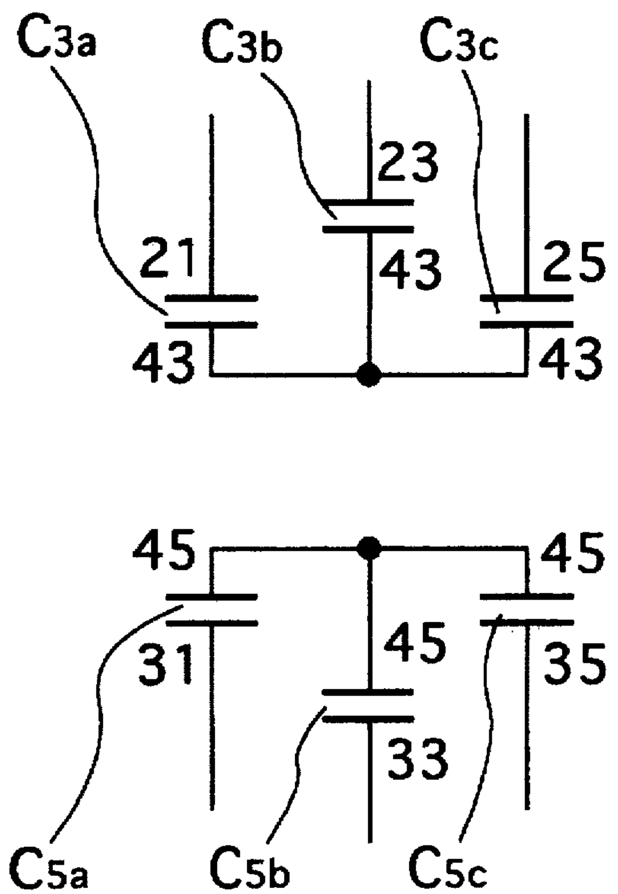
Ando et al. [45] **Date of Patent: Feb. 17, 1998**

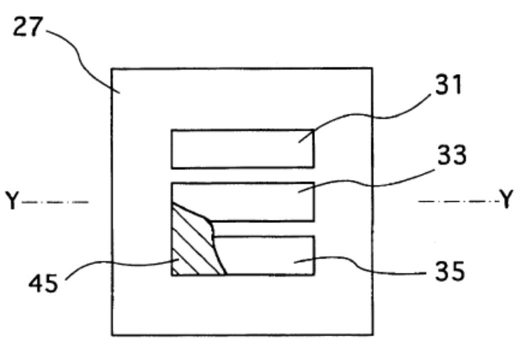
[54] **CAPACITIVE ACCELERATION SENSOR**



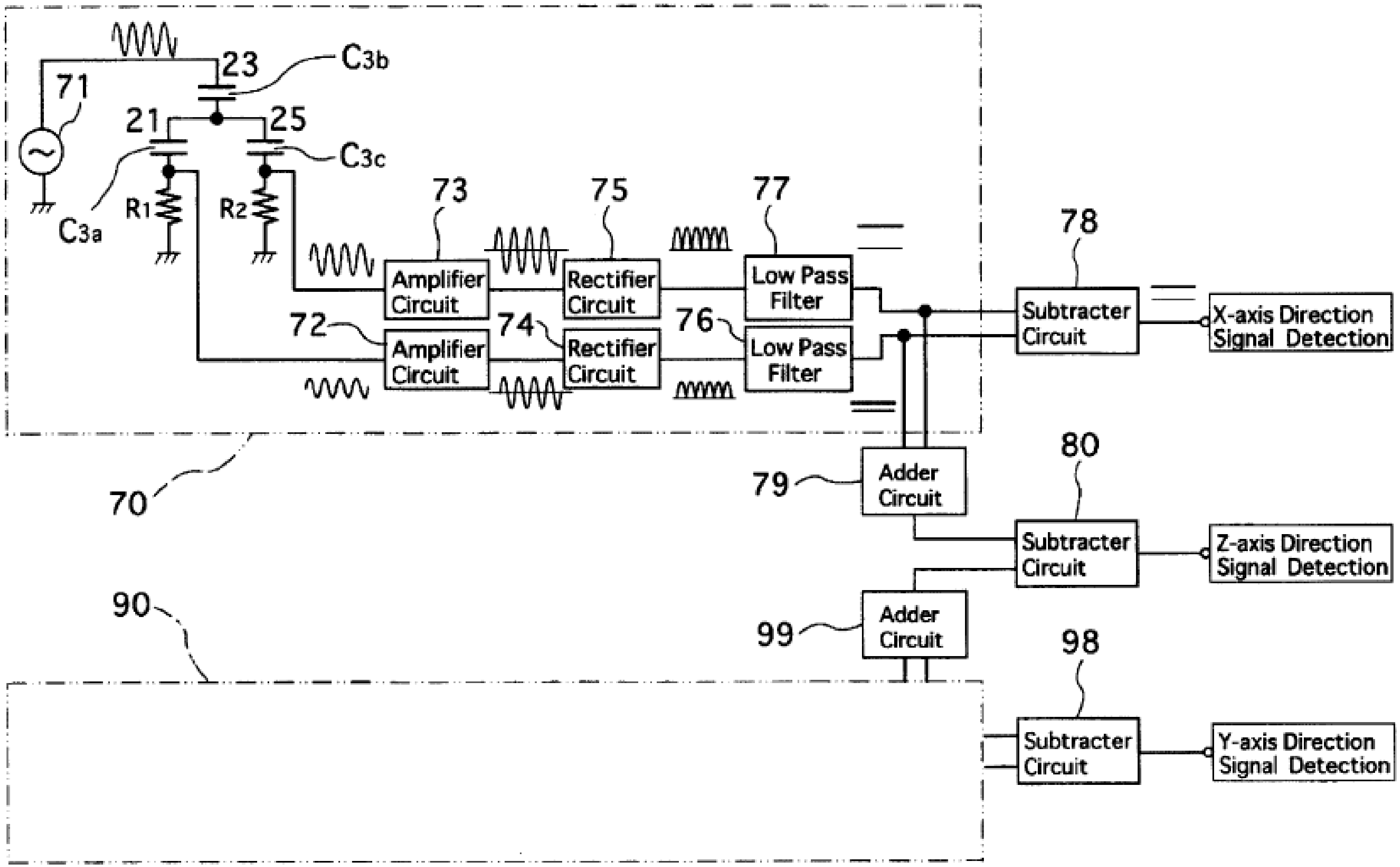
A seismic mass **41** is supported by a beam **37** which has opposite ends secured to a housing **11** so that its gravity center G is positioned away from a straight line connecting the opposite ends the beam **37**. Thus, a rotating force is given to the seismic mass **41** in response to an acceleration. The rotating force given to the seismic mass **41** causes displacement of the beam **37**. Then, a capacitance between a movable electrode **43**, **45** and a fixed electrode **21**, **23**, **25**, **31**, **33**, **35** changes, and such a change defines a magnitude of the acceleration.


21
23
25
43
41
45
G
31


C3a
C3b
C3c
23
43
21
25
43
43
45
45
31
45
35
33
C5a
C5b
C5c

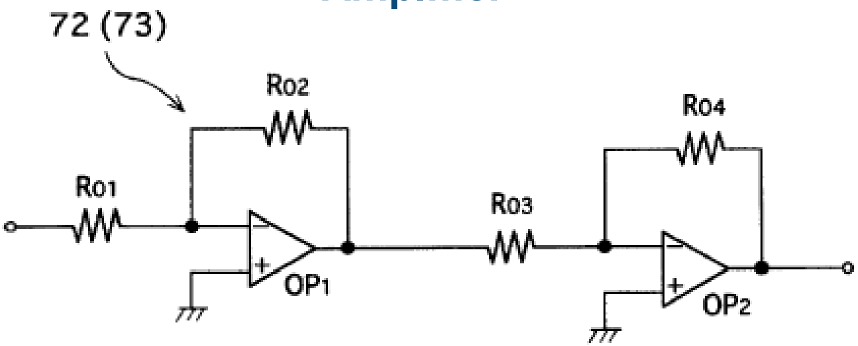

27
31
33
Y
Y
45
35

Capacitive acceleration sensor (3)

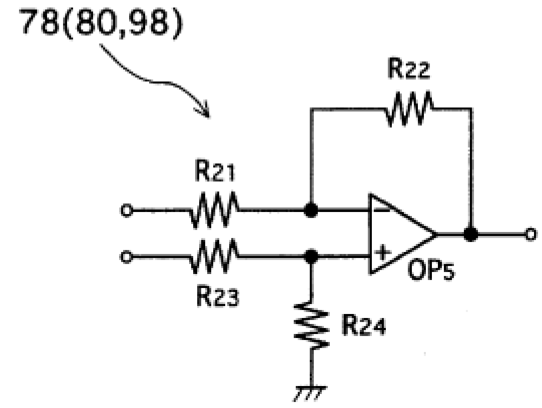


Capacitive acceleration sensor (4)

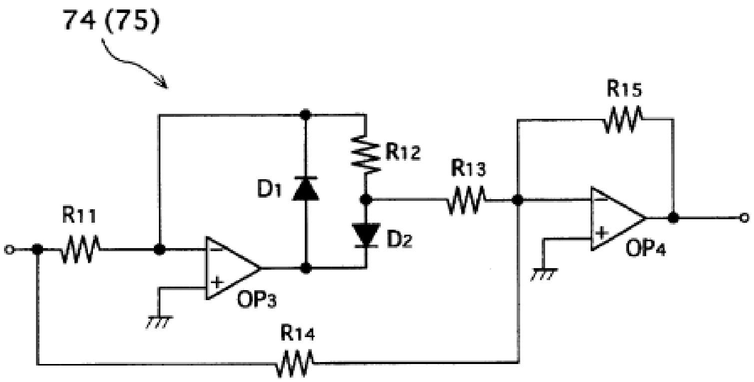
Amplifier



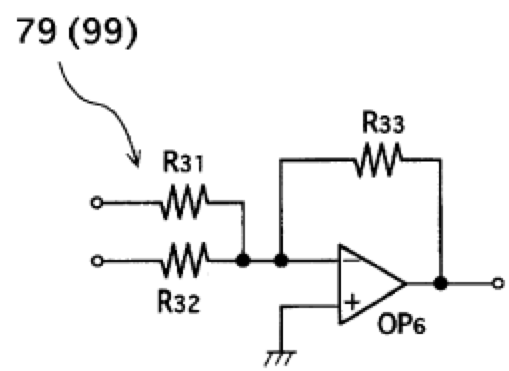
Differential amplifier (difference amplifier)



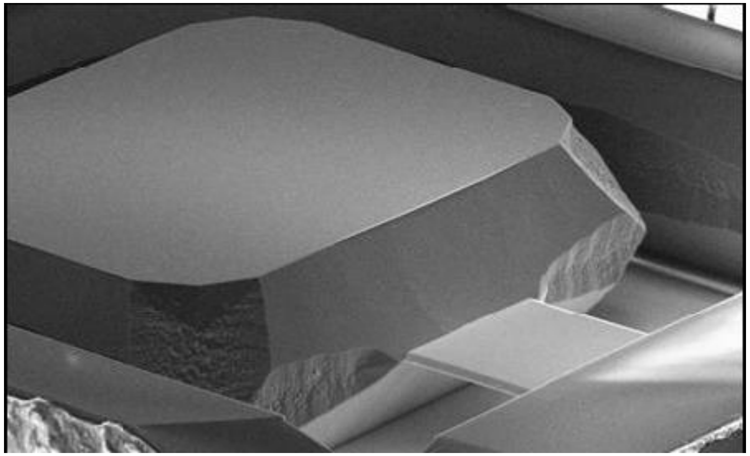
Rectifier

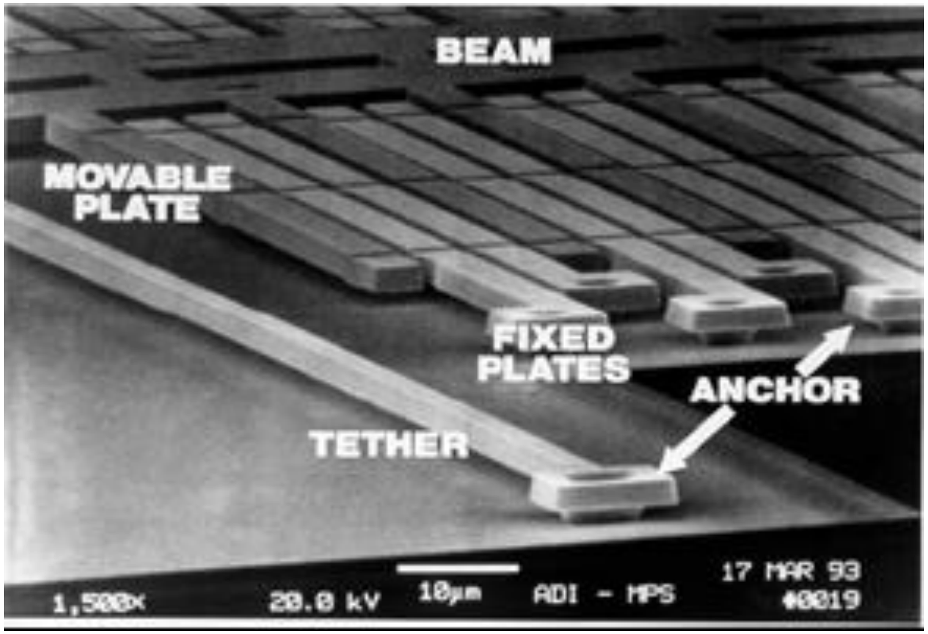


Summing amplifier



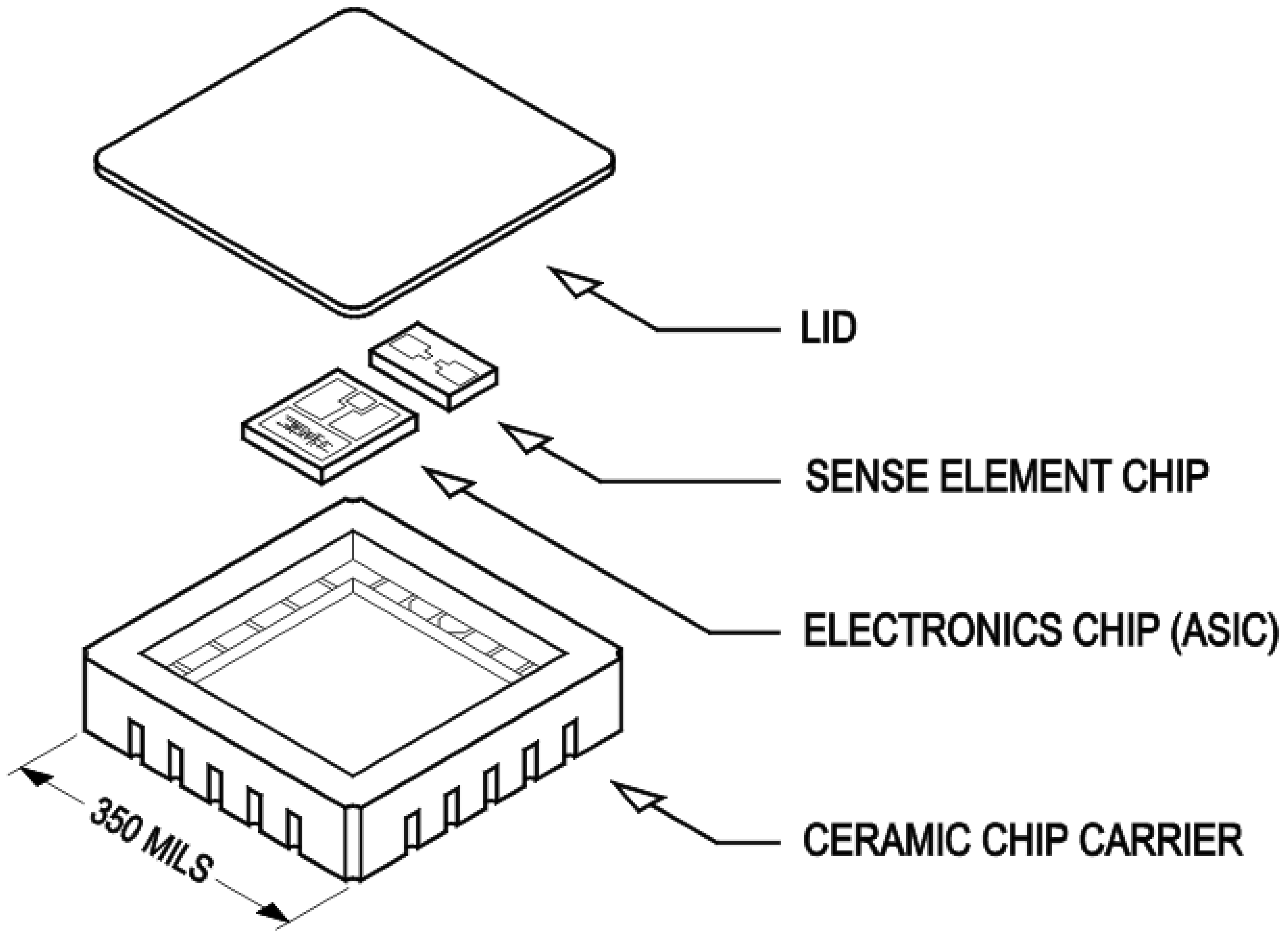
MEMS (Micro-ElectroMechanical Systems) technology



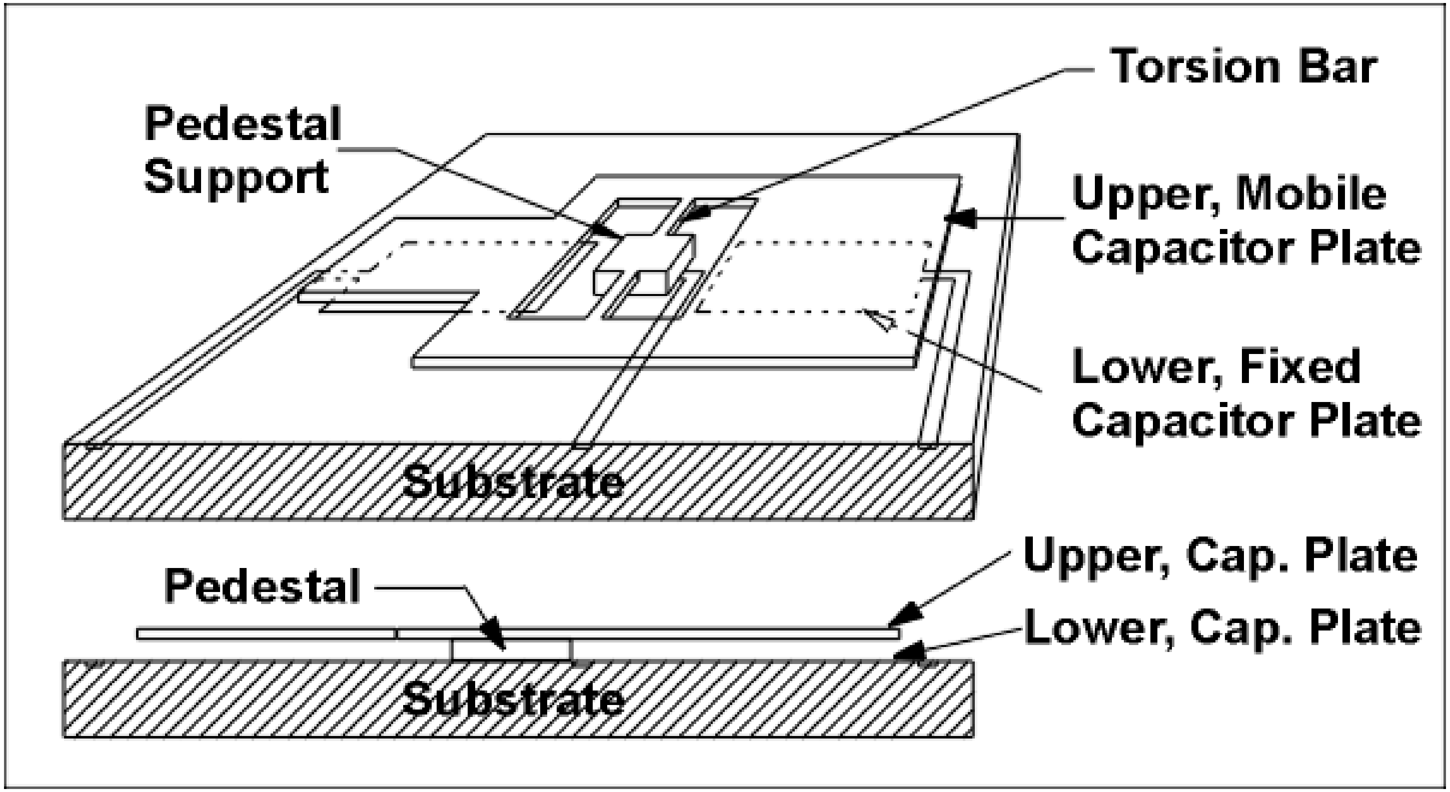


Spring, proof mass and capacitor of a bulk MEMS accelerometer (VTI) and the same parts of a surface MEMS accelerometer (Analog Devices) at 10x magnification

MEMS accelerometers



MEMS accelerometers



MEMS accelerometers: ADXL150

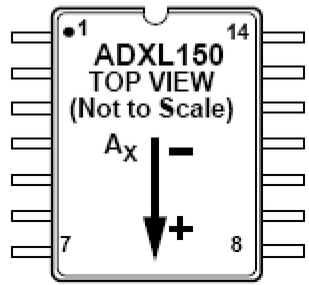
ANALOG DEVICES

±5 *g* to ±50 *g*, Low Noise, Low Power, Single/Dual Axis *i*MEMS® Accelerometers

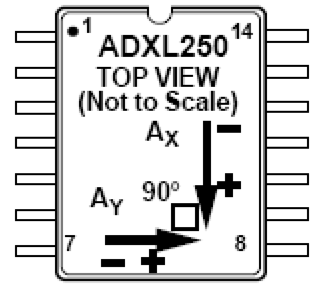
ADXL150/ADXL250

FEATURES

Complete Acceleration Measurement System on a Single Monolithic IC
80 dB Dynamic Range
Pin Programmable ±50 *g* or ±25 *g* Full Scale
Low Noise: 1 m*g*/$\sqrt{Hz}$ Typical
Low Power: <2 mA per Axis
Supply Voltages as Low as 4 V
2-Pole Filter On-Chip
Ratiometric Operation
Complete Mechanical & Electrical Self-Test
Dual & Single Axis Versions Available
Surface Mount Package

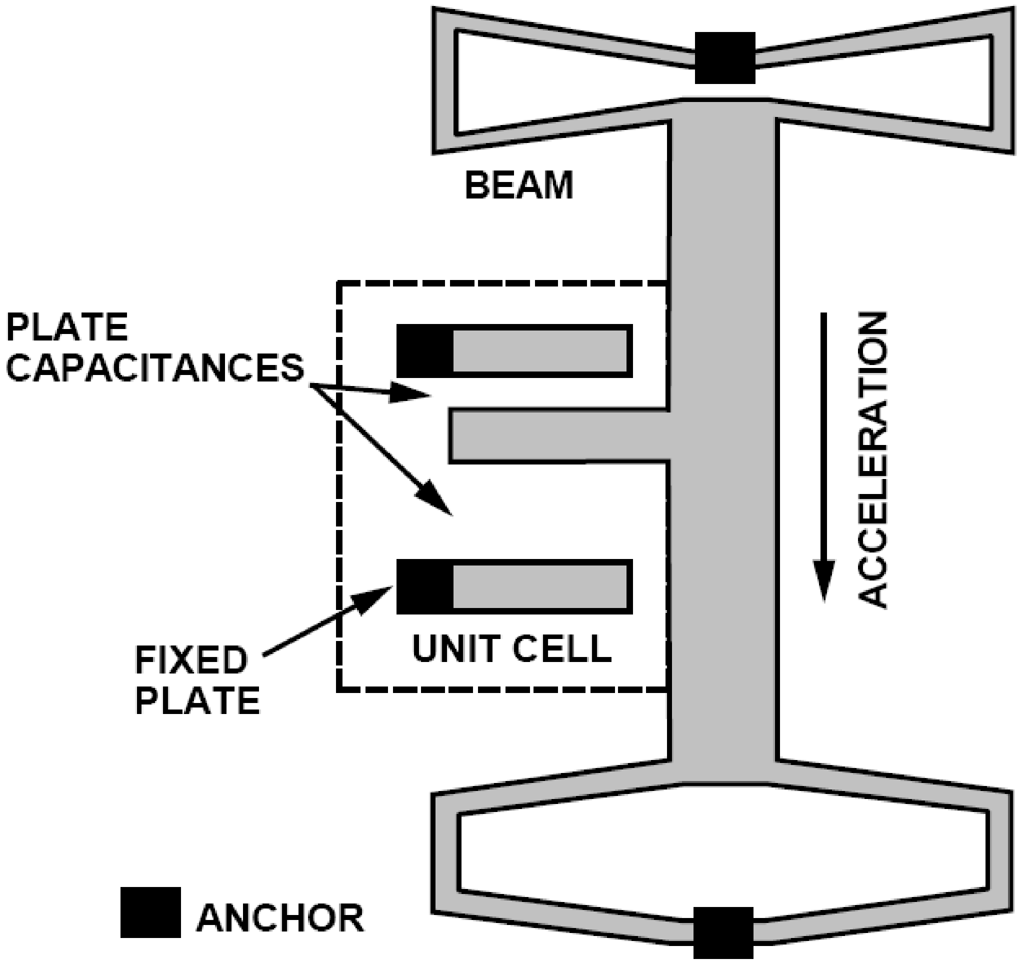


POSITIVE A = POSITIVE V_{OUT}

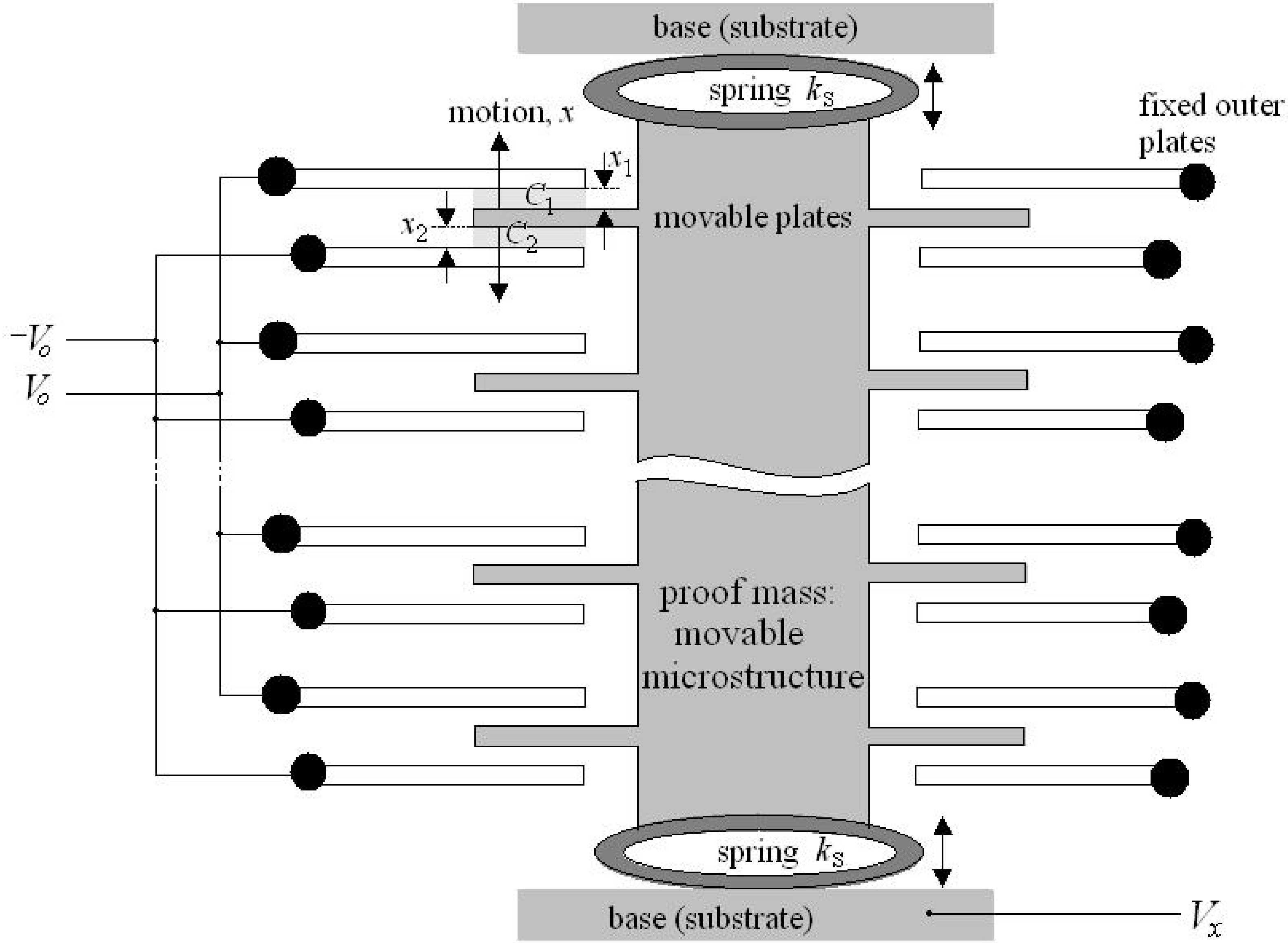


POSITIVE A = POSITIVE V_{OUT}

MEMS accelerometers: ADXL150



Simplified View of Sensor Under Acceleration


base (substrate)
spring k_S
motion, x
fixed outer plates
x_1
C_1
x_2
C_2
movable plates
$-V_o$
V_o
proof mass: movable microstructure
spring k_S
base (substrate)
V_x

Neglecting the fringing effect near the edges, the parallel-plate capacitance is

$$C_0 = \varepsilon_0 \varepsilon \frac{A}{d} = \varepsilon_A \frac{1}{d}$$

The deflection of proof mass is measured using the capacitance difference. The free-space (air) capacitances between the movable plate and two stationary outer plates C_1 and C_2 are functions of the corresponding displacements x_1 and x_2:

$$C_1 = \varepsilon_A \frac{1}{x_1} = \varepsilon_A \frac{1}{d+x} = C_0 - \Delta C$$

(equations 1)

$$C_2 = \varepsilon_A \frac{1}{x_2} = \varepsilon_A \frac{1}{d-x} = C_0 + \Delta C$$

If the acceleration is zero, the capacitances C_1 and C_2 are equal because $x_1 = x_2$. The proof mass displacement x results due to acceleration.
If $x \neq 0$, the capacitance difference is found to be

$$C_2 - C_1 = 2\Delta C = 2\varepsilon_A \frac{x}{d^2 - x^2}$$

Measuring ΔC, one finds the displacement x by solving the nonlinear algebraic equation

$$\Delta C x^2 + \varepsilon_A x - \Delta C d^2 = 0$$

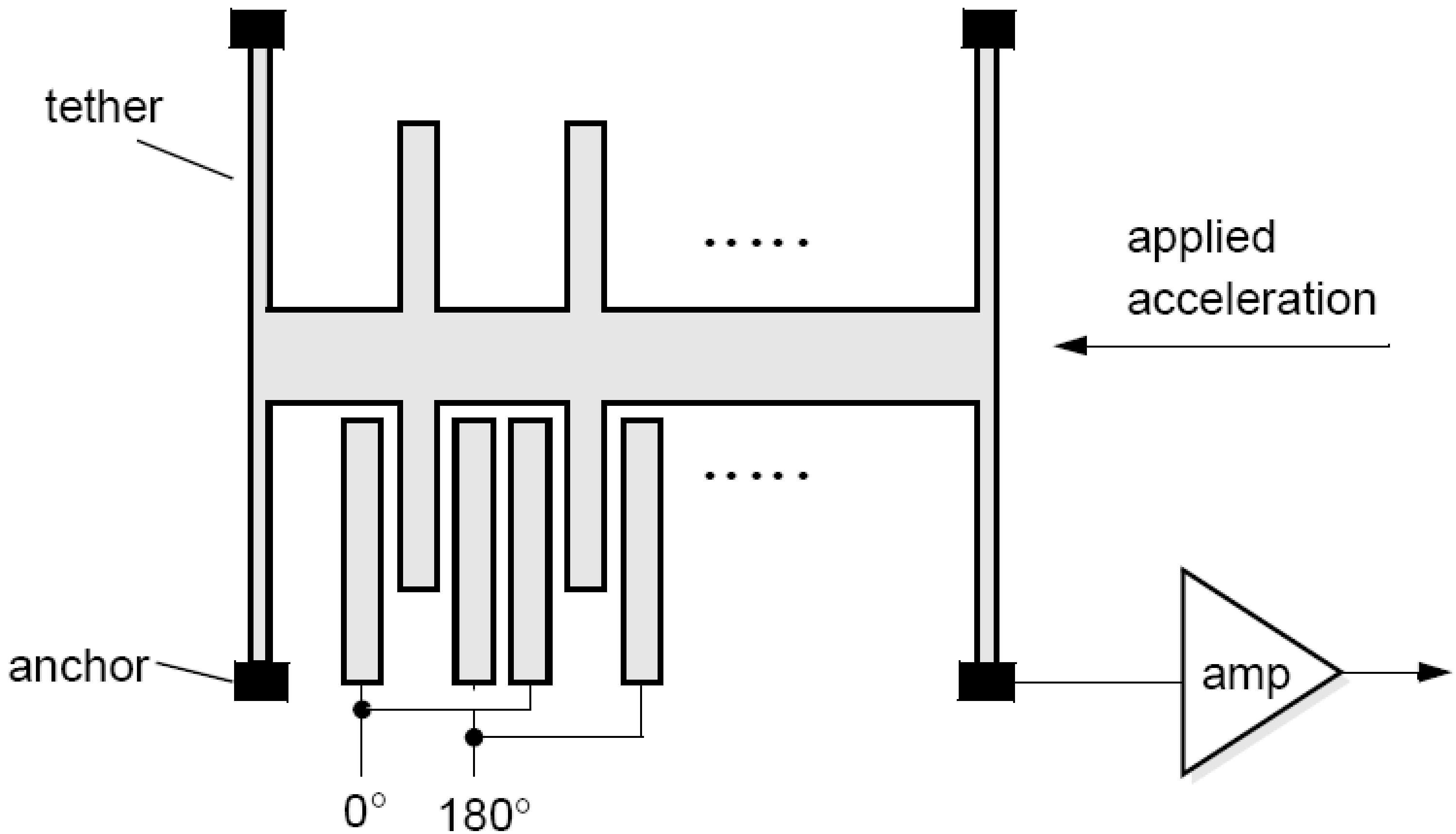
This equation can be simplified. For small displacements, the term ΔCx^2 is negligible. Thus, ΔCx^2 can be omitted. Then, from

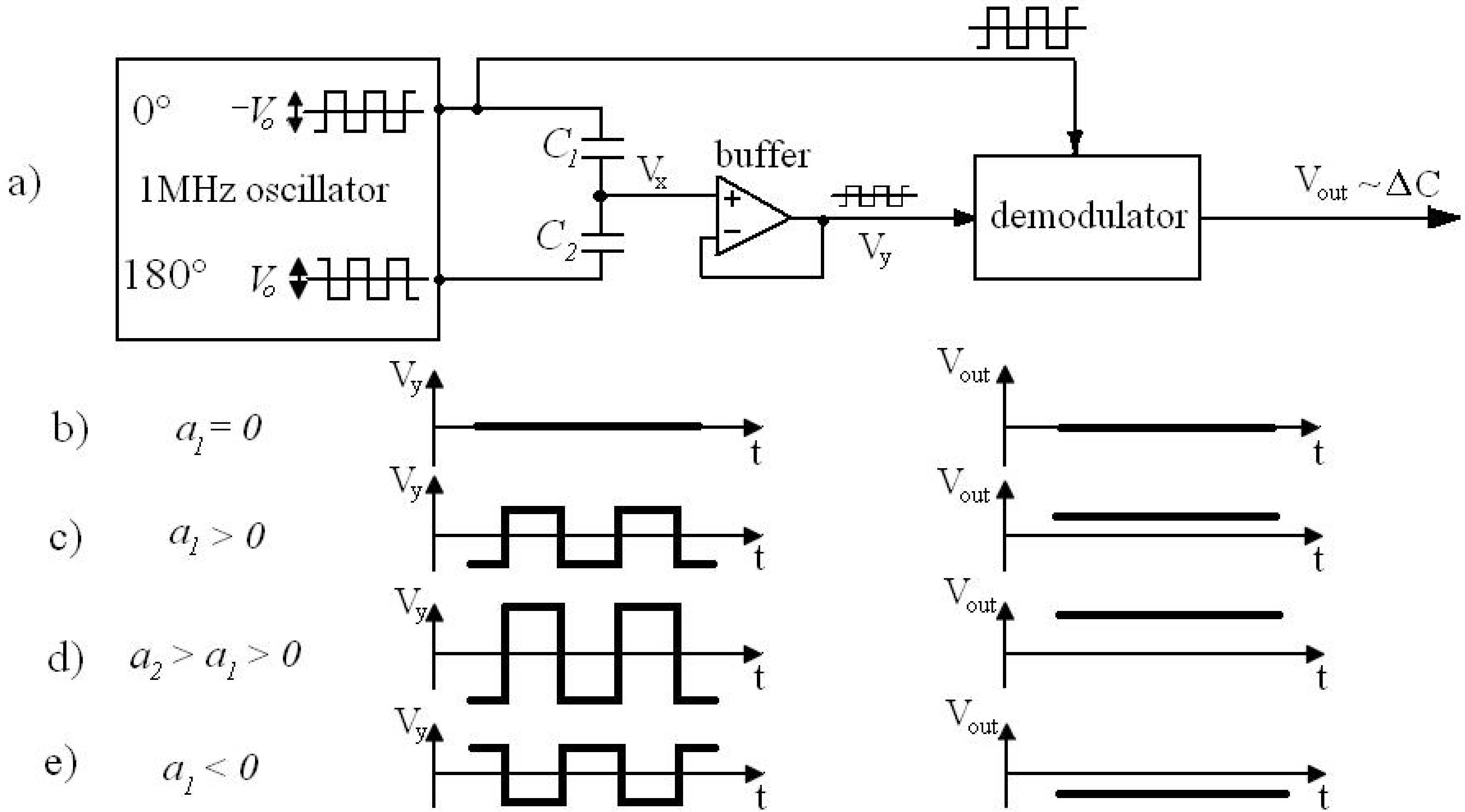
$$x \approx \frac{d^2}{\varepsilon_A} \Delta C = d\frac{\Delta C}{C_0} \qquad \textit{(equation 2)}$$

one concludes that the displacement is approximately proportional to the capacitance difference ΔC.

Every sensor has a lot of capacitor sets. All upper capacitors are wired parallel for an overall capacitance C_1 and like wise all lower ones for overall capacitance C_2, otherwise capacitance difference would be negligible to detect.
Equation 2 now doesn't hold true just for one pair of capacitors, but for all system.

As an example Analog Devices accelerometer ADXL05, has 46 pairs of capacitors.
Sensor's fixed plates are driven by 1MHz square waves with voltage amplitude V_0 coming out of oscillator. Phases of the square waves that drives upper and lower fixed plates differs for 180°.


tether
applied
acceleration
anchor
amp
0°
180°


a)
0°
$-V_o$
1MHz oscillator
180°
V_o
C_1
C_2
V_x
buffer
V_y
demodulator
$V_{out} \sim \Delta C$
b) $a_1 = 0$
c) $a_1 > 0$
d) $a_2 > a_1 > 0$
e) $a_1 < 0$
V_y
V_{out}
t

This system can be seen as a simple voltage divider whose output goes forward through buffer and demodulator.
First of all we are interested in voltage output V_x, that is actually the voltage of the proof mass. It holds true that

$$(V_x + V_0)C_1 + (V_x - V_0)C_2 = 0$$

and if we use Equations 1 and 2 we get for voltage output

$$V_x = V_0 \frac{C_2 - C_1}{C_2 + C_1} = \frac{x}{d} V_0$$

(equation 3)

V_x is square wave with the right amplitude proportional to acceleration. We also can't just simply use this output signal, because it is weak and noisy.

When there is no acceleration (**a_1=0**), the proof mass doesn't move, and therefore, the voltage output is zero.
If we accelerate the sensor (**a_1>0**), the voltage output V_x changes proportional to alternating voltage input V_0 (Equation 3). To avoid signal attenuation, we read V_x with voltage follower (buffer), therefore signal V_y is actually V_x multiplied by 1.
If we inverse the acceleration (**a_1<0**), signals V_x and V_y get negative sign.

Demodulator then gives us the sign of the acceleration, because it multiplies the input signal V_y with the square waves V_0 coming from oscillator. Now we finally have voltage output V_{out} with the right sign of acceleration and the right amplitude.

For an ideal spring, according to Hooke's law, the spring exhibit a restoring force F_S which is proportional to the displacement x. Thus, $\boldsymbol{F_S = k_S x}$, where k_S is the spring constant.

From Newton's second law of motion, neglecting the air friction (which is negligibly small), $\boldsymbol{ma = md^2x/dt^2 = k_S x}$. Thus, the acceleration, as a function of the displacement, is

$$a = \frac{k_S}{m} x$$

Then, making use of Equation 3 (*), the acceleration is found to be proportional to voltage output:

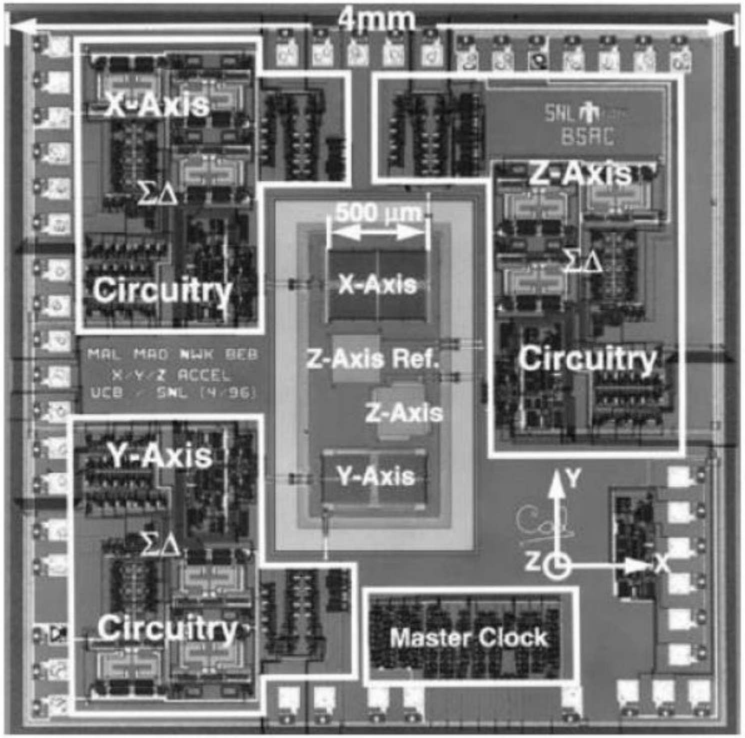
$$\boldsymbol{a = \frac{k_S d}{m V_0} V_x}$$

$$(*)\ \ V_x = \frac{x}{d} V_0 \rightarrow x = \frac{d}{V_0} V_x$$

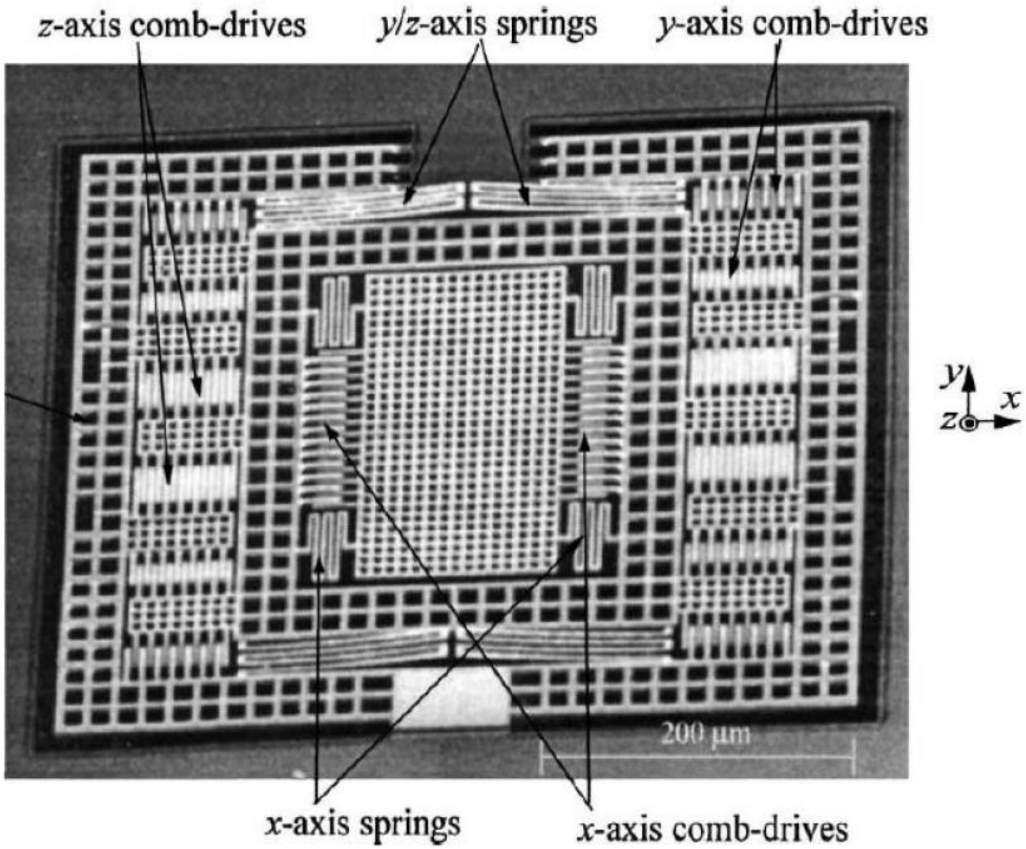
The mass of the proof mass is approximately 0.1 μg, the smallest detectable capacitance change is 20 aF and gaps between capacitor plates are approximately 1.3 μm.

This is the simplest example of one axis accelerometer.
Its capacitance changes due to changes of distance *d* between capacitor plates.

Considering sets of capacitors turned in perpendicular directions, it is possible to obtain 2- or even 3-axis accelerometers



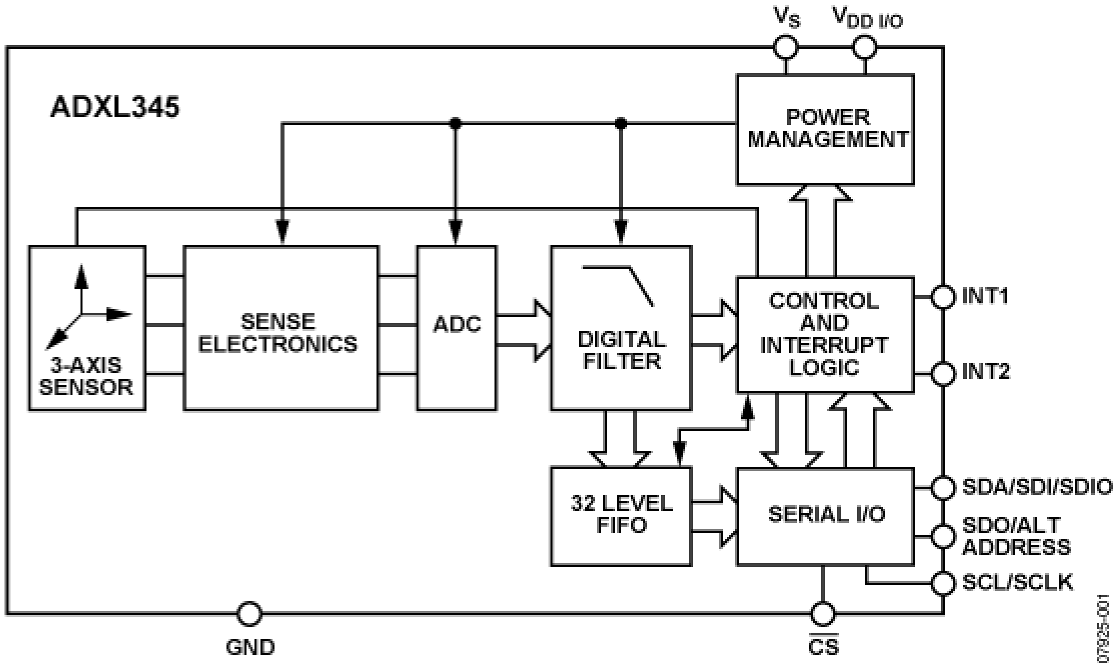
3D accelerometer structure.
It has three different sensors
for x-/y-/z-axis acceleration and
three different electronic circuitry
for each axis



3D accelerometer structure without
electronics. All three sensors
are linked with the same proof mass.

The ADXL345 is a small, thin, ultralow power, **3-axis accelerometer** with high resolution (13-bit) measurement at up to ±16 *g*. Digital output data is formatted as 16-bit twos complement and is acces-sible through either a SPI (3- or 4-wire) or I^2C digital interface.

The ADXL345 is well suited for mobile device applications. It measures the static acceleration of gravity in tilt-sensing applications, as well as dynamic acceleration resulting from motion or shock. Its high resolution (3.9 m*g*/LSB) enables measurement of inclination changes less than 1.0°.



PHYSICAL MOUNTING

The ADXL345 is 3-axis accelerometer. The sensing axes are shown in Figure 1.

The ADXL345 senses positive acceleration when it is accelerated in the positive direction of the sensing axes. The user must be careful when sensing gravity because positive acceleration is sensed when the direction of the sensing axis is opposite to gravity. Figure 2 shows the output response to gravity.

The ADXL345 is supplied in a small, thin, 3 mm × 5 mm × 1 mm, 14-lead, plastic package. Refer to the ADXL345 data sheet for recommended printed circuit board land pattern.

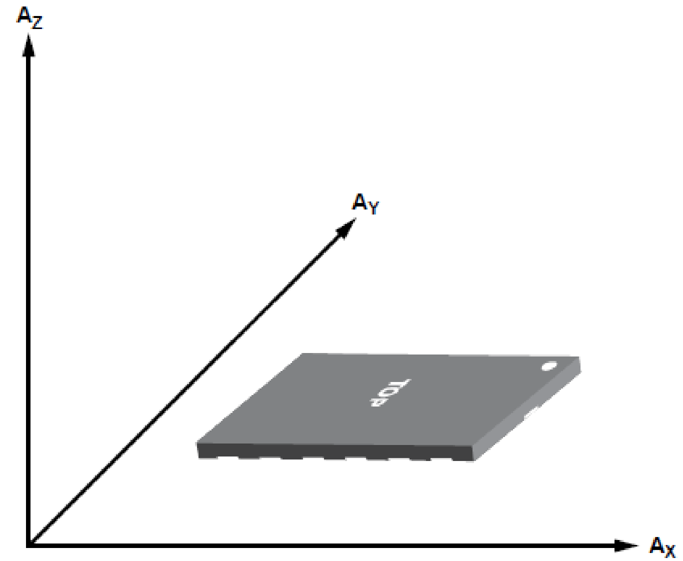


Figure 57. Axes of Acceleration Sensitivity (Corresponding Output Voltage Increases When Accelerated Along the Sensitive Axis)

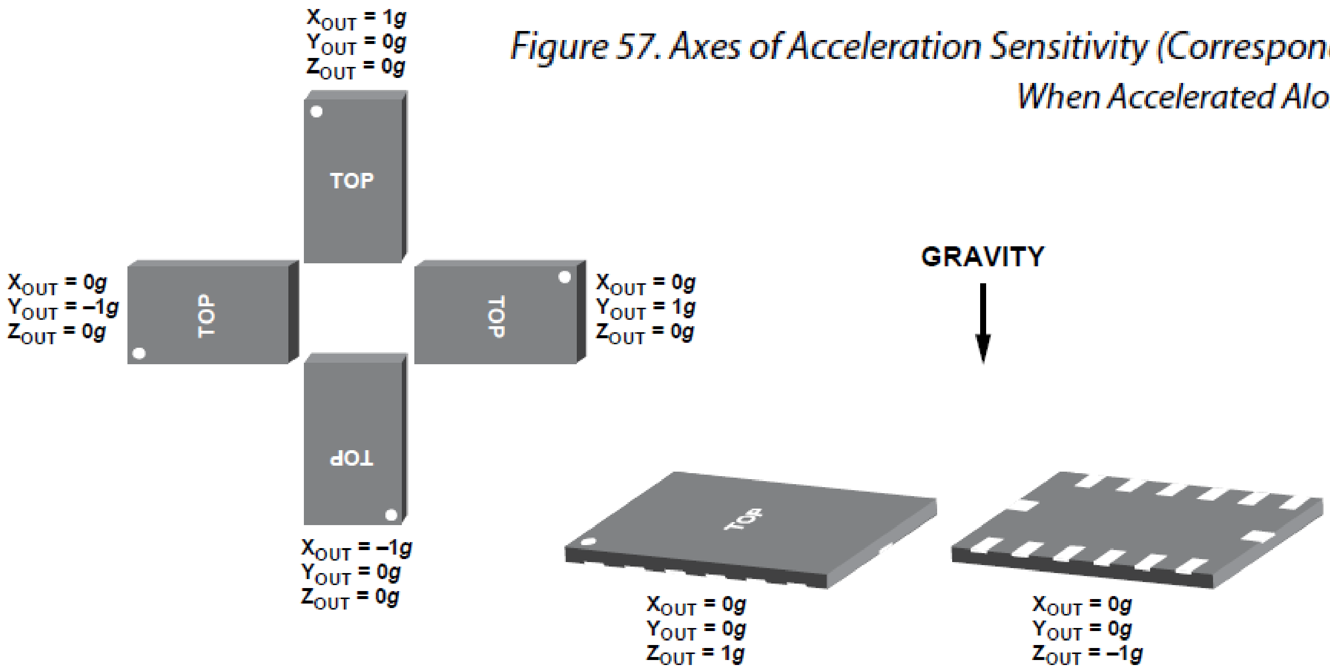


Figure 58. Output Response vs. Orientation to Gravity

Accelerometers: characteristics (1)

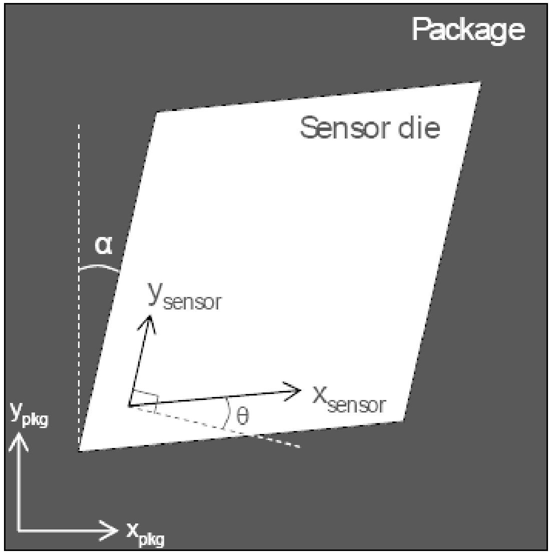
- **Measurement range** is the level of acceleration supported by the sensor's output signal specifications, typically specified in ±g. This is the greatest amount of acceleration the part can measure and accurately represent as an output. For example, the output of a ±3g accelerometer is linear with acceleration up to ±3g. If it is accelerated at 4g, the output may rail. Note that the breaking point is specified by the Absolute Maximum Acceleration, NOT by the measurement range. A 4g acceleration will not break a ±3g accelerometer.

- **Sensitivity** is the ratio of change in acceleration (input) to change in the output signal. This defines the ideal, straight-line relationship between acceleration and output. Sensitivity is specified at a particular supply voltage and is typically expressed in units of mV/g for analog-output accelerometers (*g =9.80665 m/s*^{2} *at sea level, 45*$^\circ$ *latitude*, LSB/g, or mg/LSB for digital-output accelerometers. It is usually specified in a range (min, typ, max) or as a typical figure and % deviation. For analog-output sensors, sensitivity is ratiometric to supply voltage; doubling the supply, for example, doubles the sensitivity.

Accelerometers: characteristics (2)

- **Nonlinearity**: Ideally, the relationship between voltage and acceleration is linear and described by the sensitivity of the device. Nonlinearity is a measurement of deviation from a perfectly constant sensitivity, specified as a percentage with respect to either full-scale range (%FSR) or ± full scale (%FS). Typically, FSR = FS+FS. Nonlinearity of MEMS accelerometers is low enough that it can most often be ignored.

- **Package Alignment Error** is the angle between the accelerometer-sensing axes and the referenced package feature (see Figure). "Input Axis Alignment" is another term used for this error. The units for package alignment error are "degrees." Packaging technology typically aligns the die to within about 1° of the package.



Accelerometers: characteristics (3)

- **(Orthogonal) Alignment Error** is the deviation from the ideal angular displacement (typically 90°) between multi-axis devices. MEMS accelerometers are manufactured using photolithography on a single piece of silicon, so axis-to-axis alignment error is not generally a problem.

- **Cross-axis sensitivity** is a measure of how much output is seen on one axis when acceleration is imposed on a different axis, typically specified as a percentage. The coupling between two axes results from a combination of alignment errors, etching inaccuracies, and circuit crosstalk.

Accelerometers: characteristics (4)

- **Zero-*g* Bias Level** specifies the output level when there is no acceleration (zero input). Analog sensors typically express this in volts (or mV) and digital sensors in codes (LSB). Zero-*g* Bias is specified at a particular supply voltage and is typically ratiometric with supply voltage (most often, zero-*g* bias is nominally half the supply voltage).
- Several aspects of zero-g bias are often specified:
- **Zero-*g* Voltage**, in V, specifies the range of voltages that may be expected at the output under 0*g* of acceleration.
- **Output Deviation from Ideal**, also called **Initial Bias Error**, is specified at 25°C, either in terms of acceleration error (*g*) or output signal: mV for analog sensors and LSB for digital sensors.
- **Zero-*g* Offset vs. Temperature**, or **Bias Temperature Coefficient**, in m*g*/°C, describes how much the output shifts for each °C temperature change; and
- **Bias Voltage Sensitivity** is the change in "Zero-Bias Level" with respect to change in power supply. The units for this parameter are typically, mv/V, mg/V, or LSB/V.
- **Zero-g Total Error** includes all errors.

Frequency response of an accelerometer

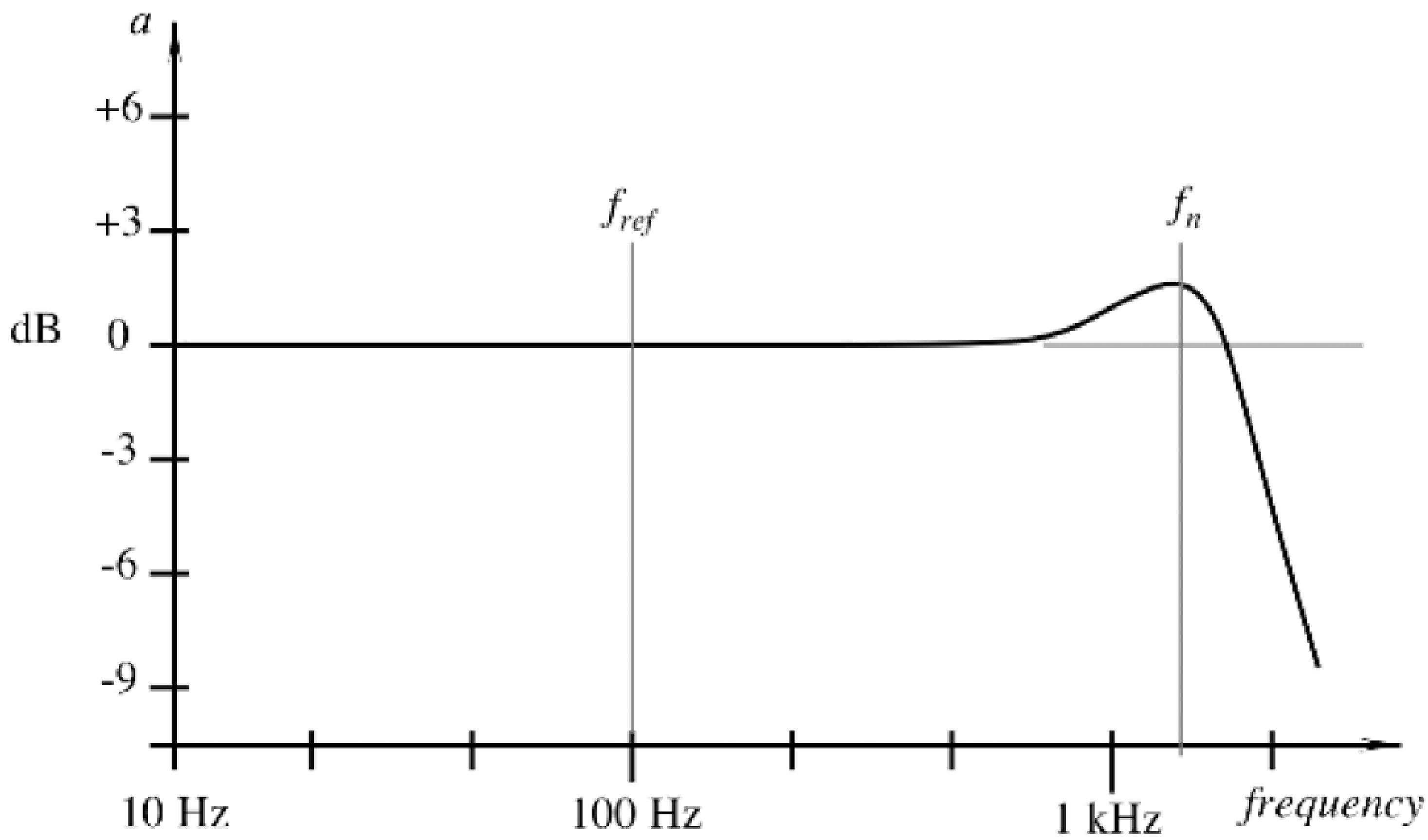


Fig. 8.2. A frequency response of an accelerometer. f_n is a natural frequency and f_{ref} is the reference frequency.

ADXL150/ADXL250: frequency response

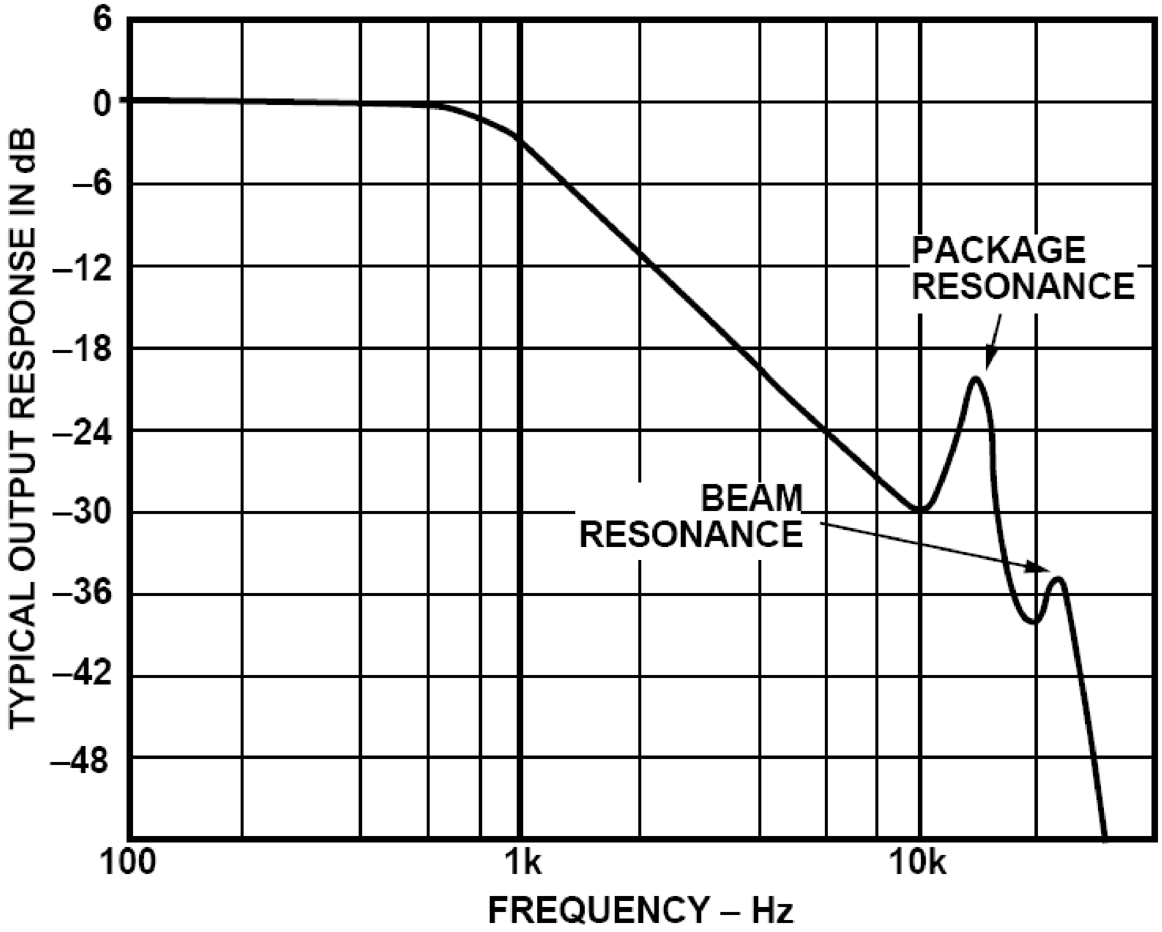


Figure 6. Typical Output Response vs. Frequency of ADXL150/ADXL250 on a PC Board that Has Been Conformally Coated

Free-fall Sensing

When a tri-axis accelerometer is stationary, its total acceleration it measures is 1g (9.8m/s^2), regardless of orientation. This total acceleration can be calculated from the X, Y, and Z outputs of the accelerometer:

$$a_{total} = \sqrt{a_x^2 + a_y^2 + a_z^2}$$

When a tri-axis accelerometer is dropped in any orientation, it is in free-fall and the measured acceleration on all three axes is 0g. Therefore, the total acceleration is zero as well. Total acceleration can be monitored to sense that the accelerometer has been dropped, and to measure its free-fall time.

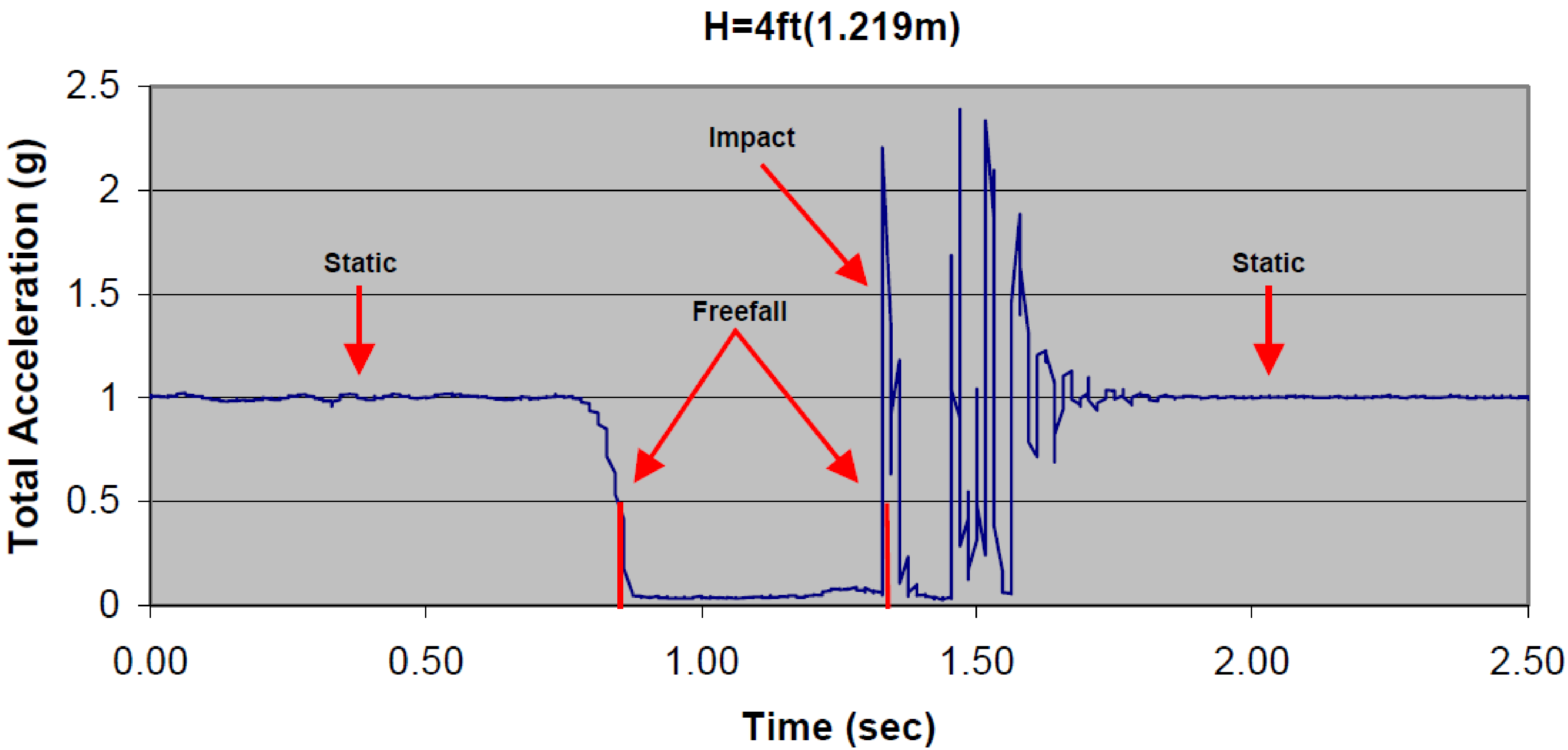
Note that this signature cannot accurately be observed when using a dual axis accelerometer because, when horizontally oriented, the X and Y-axis outputs are the same (0g), whether the accelerometer is stationary or in free-fall.

The purpose of a drop test is to address a product warranty situation in which a manufacturer guarantees that a product will survive at a given drop to concrete, e.g. 4ft (1.219 meters).

The accelerometer (Kionix KXM52) is mounted in a metal project box such that the accelerometer is located near the center of mass.
Mounting near the center of mass will ensure that centripetal accelerations remain negligible throughout the experiment.

With the PC logging data at a rate of 250 samples per second, the enclosure was dropped from a height of approximately 4ft.

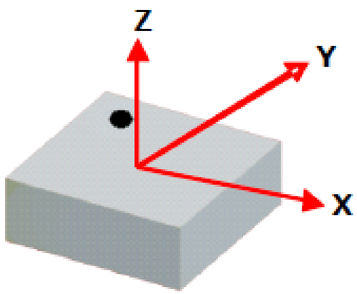
Drop test



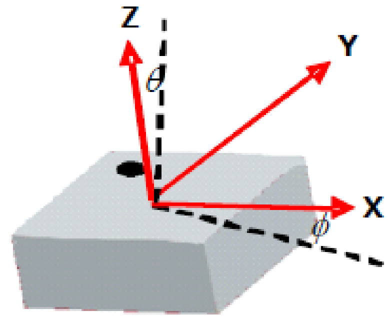
The total time in free-fall was considered to be the interval of time between the point at which the object crossed .0.5g on the way to free-fall to the point at which the object again crossed .5g upon impact. The object did fall from a height of approximately 1.219m (4ft), in fact

$$h = g \cdot t^2 / 2$$

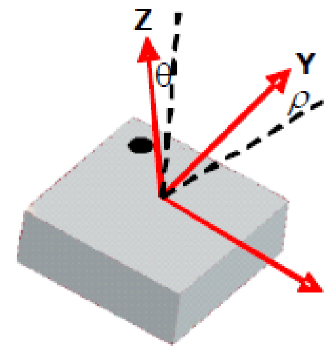
(h = drop height (m), t = time (s), g = 9.8m/s²)



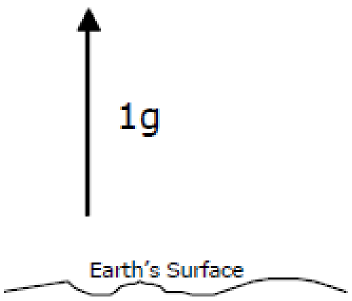
No X-tilt or Y-tilt; level with the ground.



X-tilt (ϕ) is the angle of the x axis relative to the ground. θ is the angle of the z axis relative to gravity.



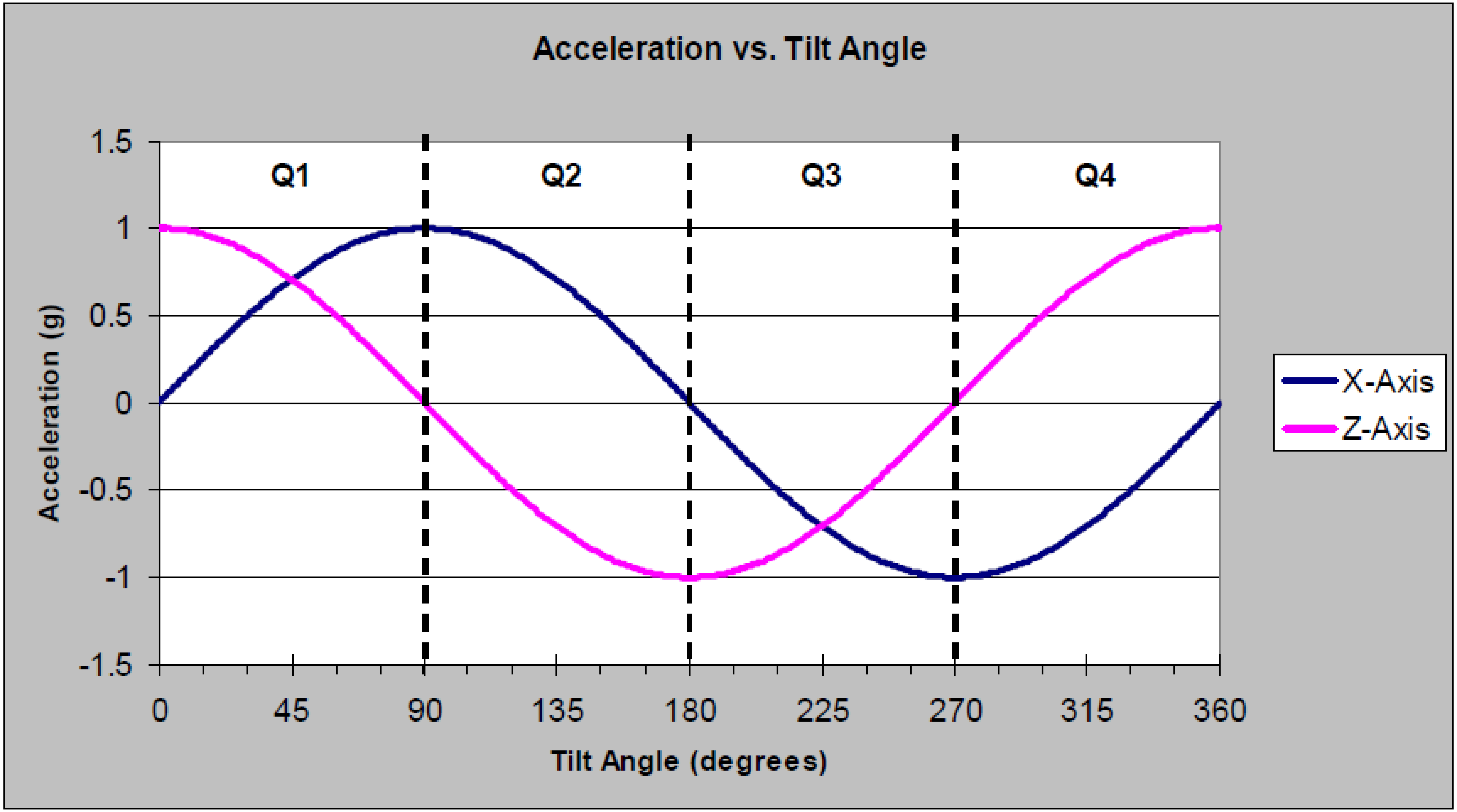
Y-tilt (ρ) is the angle of the Y axis relative to the ground. θ is the angle of the z axis relative to gravity.



$$\phi = \arcsin(a_x)$$
$$\rho = \arcsin(a_y)$$
$$\theta = \arccos(a_z)$$

Tilt table

	X-axis		Z-axis	
X-tilt (ϕ) in (°)	Acceleration (g)	Change per ° of Tilt (mg)	Acceleration (g)	Change per ° of Tilt (mg)
0	0	17.452	1	-0.152
15	0.259	16.818	0.966	-4.664
30	0.5	15.038	0.866	-8.858
45	0.707	12.233	0.707	-12.448
60	0.866	8.594	0.5	-15.19
75	0.966	4.37	0.259	-16.897
90	1	-0.152	0	-17.452
105	0.966	-4.664	-0.259	-16.897
120	0.866	-8.858	-0.5	-15.19
135	0.707	-12.448	-0.707	-12.448
150	0.5	-15.19	-0.866	-8.858
165	0.259	-16.897	-0.966	-4.664
180	0	-17.452	-1	0.152
195	-0.259	-16.897	-0.966	4.37
210	-0.5	-15.19	-0.866	8.594
225	-0.707	-12.448	-0.707	12.233
240	-0.866	-8.858	-0.5	15.038
255	-0.966	-4.664	-0.259	16.818
270	-1	0.152	0	17.452
285	-0.966	4.37	0.259	16.818
300	-0.866	8.594	0.5	15.038
315	-0.707	12.233	0.707	12.233
330	-0.5	15.038	0.866	8.594
345	-0.259	16.818	0.966	4.37
360	0	17.452	1	-0.152


Acceleration vs. Tilt Angle
Q1
Q2
Q3
Q4
1.5
1
0.5
0
-0.5
-1
-1.5
Acceleration (g)
0
45
90
135
180
225
270
315
360
Tilt Angle (degrees)
X-Axis
Z-Axis

Sign recognition

Sign recognition is imperative because, for example, tilts of 45° and 135° are represented by the same sensor output, .707g. The methods presented up to now do not explain how to tell the difference between two tilt angles that result in the same sensor output. This ambiguity can be resolved by monitoring the sign of the Z-axis output. In this case, when the accelerometer is tilted at 45°, the Z-axis output is a positive value. When the accelerometer is tilted to 135°, the Z-axis output is a negative value. The sign of the X-axis and the Z-axis will indicate in the device which quadrant the accelerometer is being tilted, consequently indicating inversion.

The following table shows the expected signs for each output in each quadrant.

Acceleration Signs		
Quadrant	Xaccel	Zaccel
1	+	+
2	+	-
3	-	-
4	-	+

Conditioning of capacitive sensors (1)

Signal conditioning for sensors based on variation of reactance must include a **supply of exciting alternating current**.

Capacitive sensors usually have capacitance smaller than 100 pF. The supply frequency must then be from 10 kHz to 100 MHz in order to yield reasonable values for their impedances. In order to avoid capacitive interference because of their high output impedance, we frequently connect capacitive sensors with shielded **cables**. But this adds a capacitance in parallel with that of the sensor, which reduces sensitivity and decreases linearity. Furthermore, any relative movement between cable conductors and the insulating dielectric can increase errors. The usual solution is to **place the electronic circuits as close as possible to the sensor**, thus using short cables and even rigid cables, and to apply driven shield techniques or impedance transformers.

Conditioning of capacitive sensors (2)

When the measurement system requires all the signals to be converted to dc voltages, some available options for the sensors working at alternating frequencies are

- peak detection
- rms measurement
- mean value calculation after rectification(most common).

A common solution to obtain an electric signal from a variable reactance sensor is just to apply Ohm's law.

A change in impedance can be detected by measuring the change in current when a constant ac voltage is supplied to it, or by the change in the drop in voltage across it when driven by a constant alternating current.

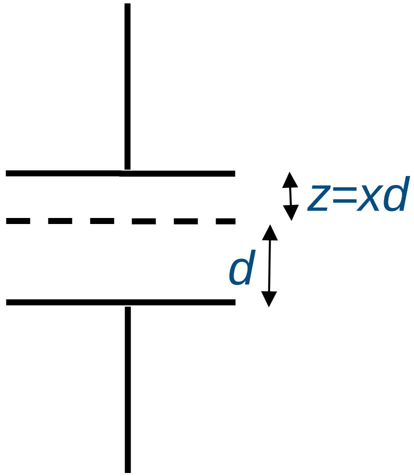
Conditioning of capacitive sensors (3)

For any impedance, we define the quality factor Q as its reactance divided by its resistance at a given frequency.

The direct application of Ohm's law to sensors whose Q is not very high implies the measurement of two components of the output signal: the one in phase and the one 90° out of phase - in quadrature- with respect to the supply signal. However, only the in-phase signal carries information about the measurand.

Moreover, the actual impedance variations are often very small, and stray capacitances usually interfere with the changes to be measured. Thus reactance measurement techniques must consider those problems.

Variable capacitance (distance)

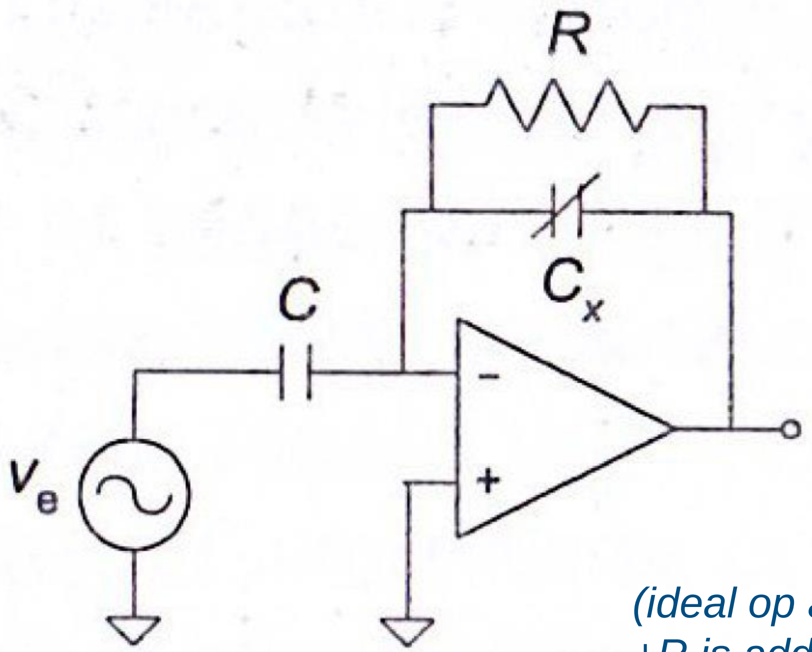


$$C_x = \varepsilon \frac{A}{d+z} = \varepsilon \frac{A}{d+xd} = \varepsilon \frac{A}{d(1+x)}$$

$$\left(C_0 = \varepsilon \frac{A}{d}\right) \Rightarrow C_x = \frac{C_0}{(1+x)}$$

Variable capacitance (distance): signal conditioning with linear output

$$C_x = \epsilon \frac{A}{d(1+x)} = \frac{C_0}{1+x}$$



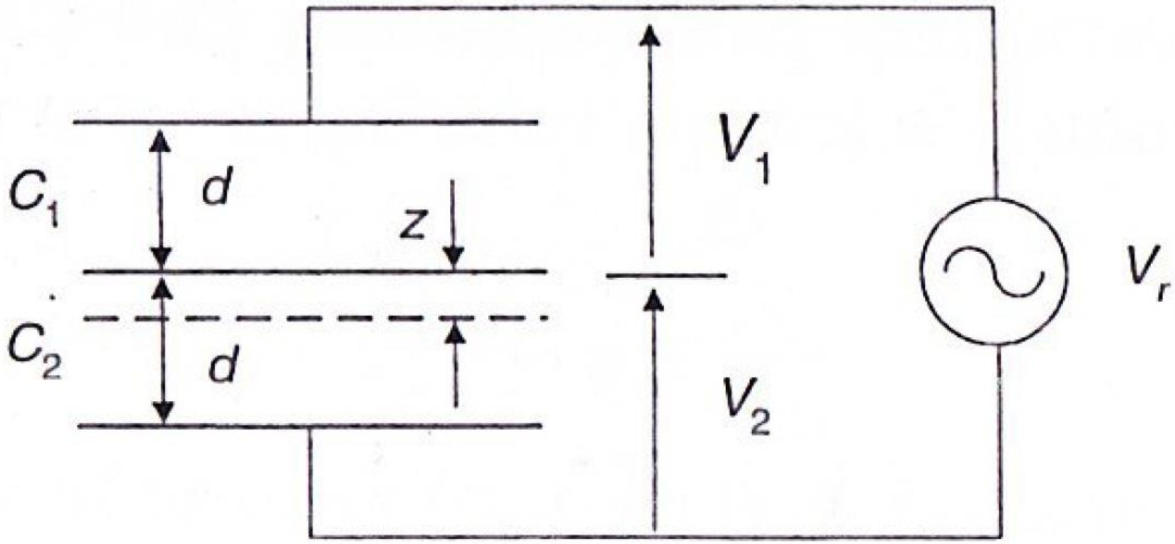
$$v_o = -v_e \frac{Z_x}{Z} = -v_e \frac{C}{C_0}(1+x)$$

(ideal op amp
+R is added to bias the op amp and is much higher than the sensor impedance at the excitation frequency)

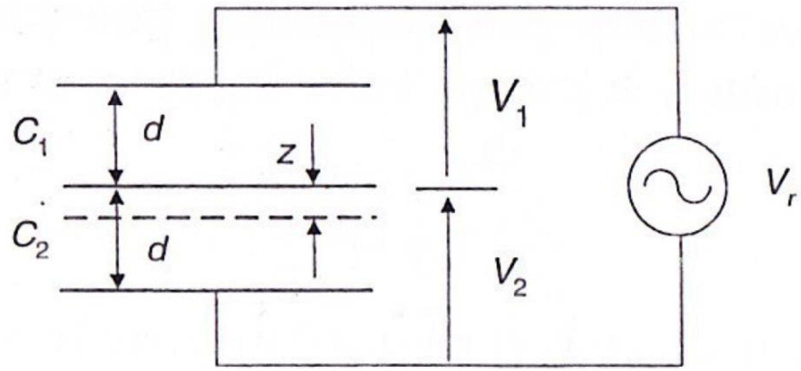
Differential capacitance (distance)

$$C_1 = \frac{\epsilon A}{d + z}$$

$$C_2 = \frac{\epsilon A}{d - z}$$



Differential capacitance (distance)



$$V_1 = \frac{V_r}{\frac{1}{j\omega C_1} + \frac{1}{j\omega C_2}} \frac{1}{j\omega C_1} = V_r \frac{C_2}{C_1 + C_2}$$

$$V_2 = \frac{V_r}{\frac{1}{j\omega C_1} + \frac{1}{j\omega C_2}} \frac{1}{j\omega C_2} = V_r \frac{C_1}{C_1 + C_2}$$

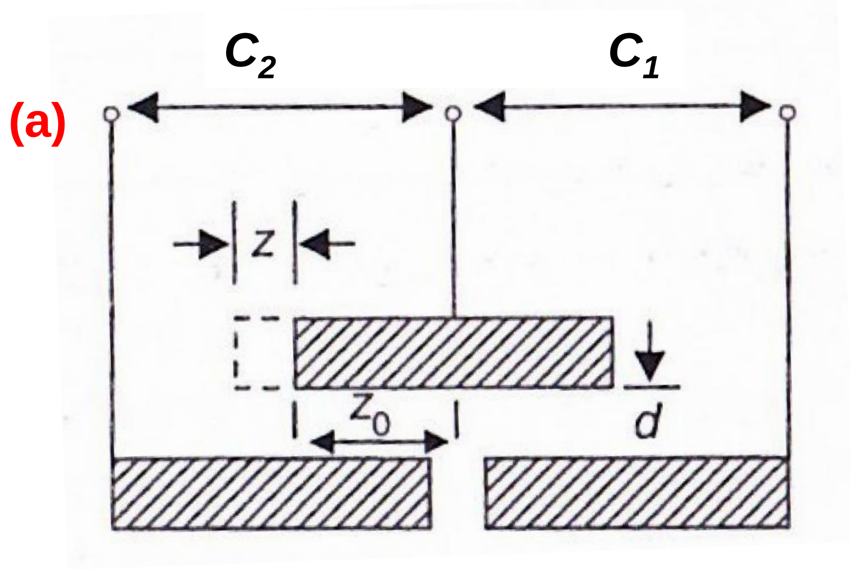
Differential capacitance (distance)

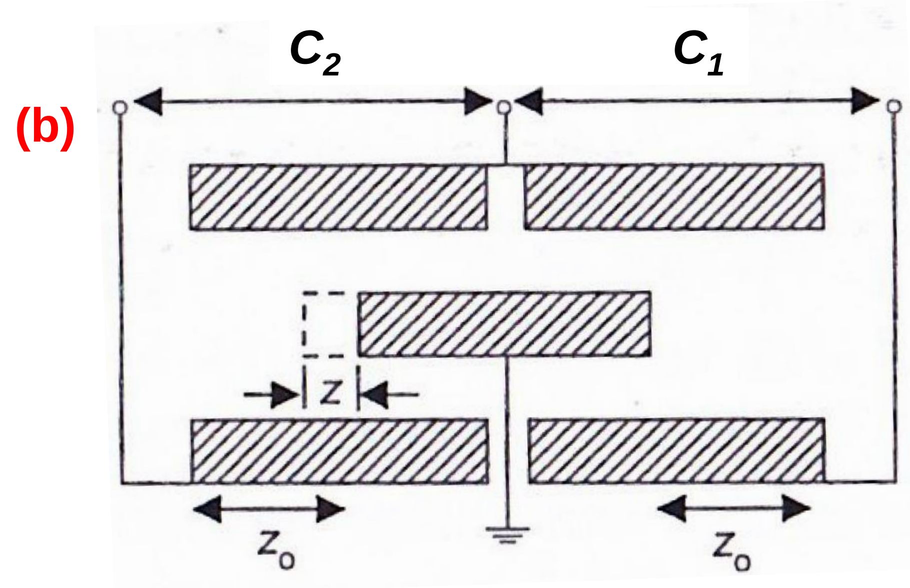
$$V_1 = V_\mathrm{r} \frac{1/(d-z)}{1/(d+z) + 1/(d-z)} = V_\mathrm{r} \frac{d+z}{2d}$$

$$V_2 = V_\mathrm{r} \frac{1/(d+z)}{1/(d+z) + 1/(d-z)} = V_\mathrm{r} \frac{d-z}{2d}$$

$$\boxed{V_1 - V_2 = V_\mathrm{r} \left(\frac{d+z}{2d} - \frac{d-z}{2d} \right) = V_\mathrm{r} \frac{z}{d}}$$

Differential capacitance (area)





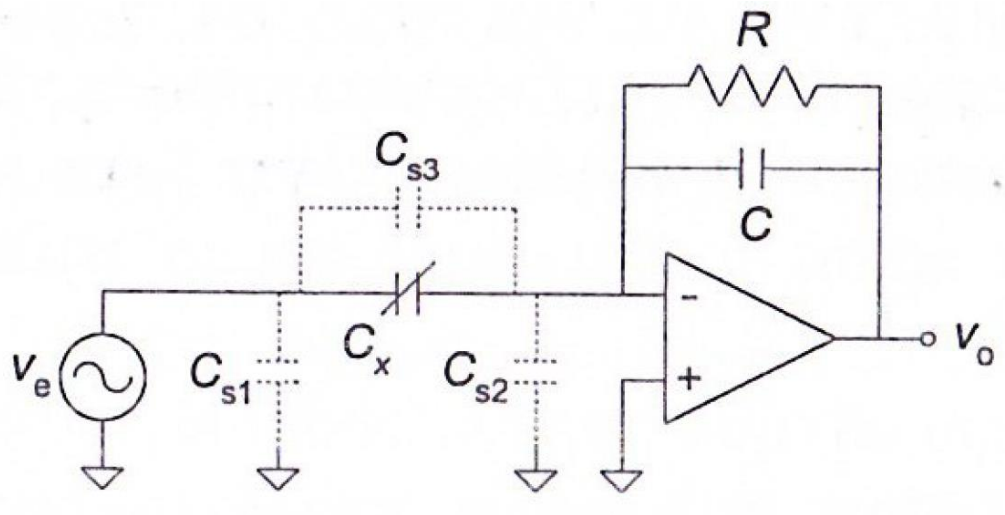
$$C_1 = \epsilon \frac{w(z_0 - z)}{d} = \epsilon \frac{w}{d} z_0 \frac{z_0 - z}{z_0} = C_0 \frac{z_0 - z}{z_0}$$

(a)

$$C_2 = \epsilon \frac{w(z_0 + z)}{d} = \epsilon \frac{w}{d} z_0 \frac{z_0 + z}{z_0} = C_0 \frac{z_0 + z}{z_0}$$

$$C_2 - C_1 \propto z$$

Charge amplifier



The charge amplifier applies a constant voltage to the sensor and measures the resulting current by converting it to a voltage through C. Disregarding R (op amp bias) and stray capacitance C_{s3}, the output voltage is

$$v_o = -v_e \frac{C_x}{C}$$

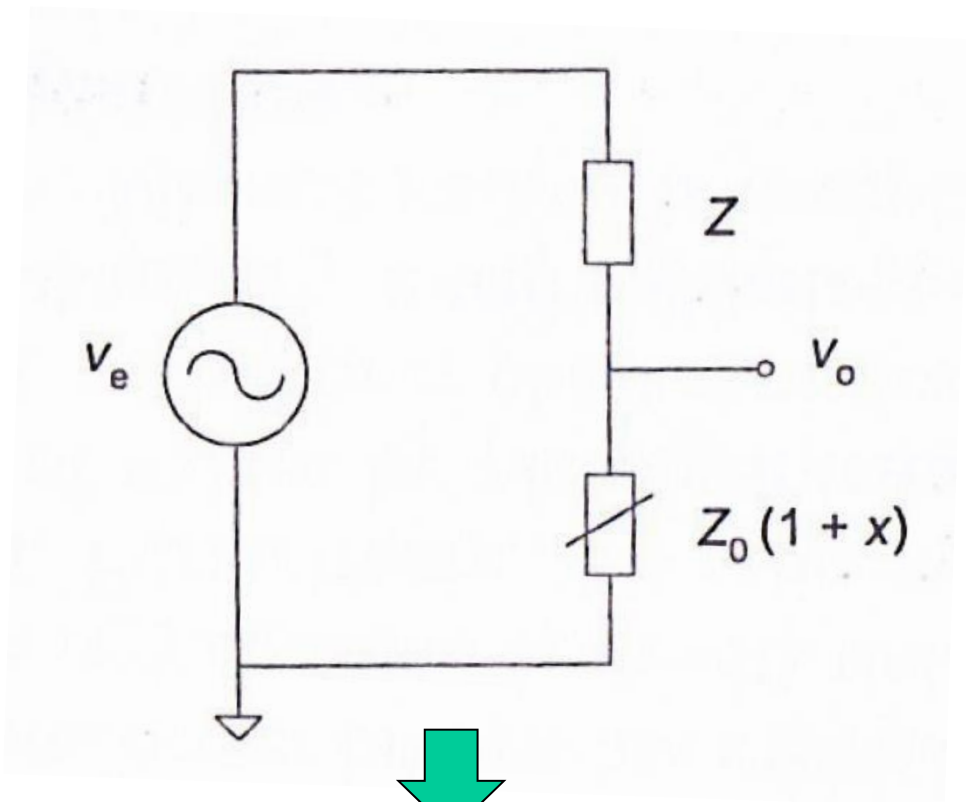
Thus, it is proportional to the sensor capacitance.

Note that stray capacitances C_{s1} and C_{s2} do not contribute to the output. C_{s1} is in parallel with a voltage source and C_{s2} has both ends at the same voltage because of the opamp. Nevertheless, a large C_{s2} may cause oscillation. Shielding sensor leads reduces C_{s3}.

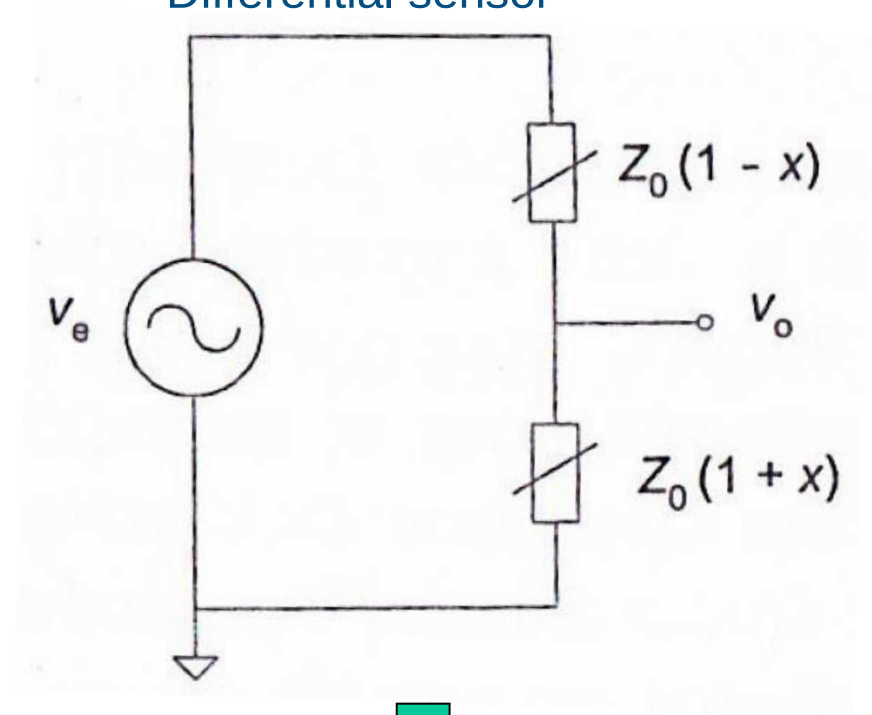
Charge amplifier

- If C is small, a small change in charge at the input can generate a large voltage swing in the output.
- The main method of connecting capacitive sensors such as pyroelectric sensors whose output is low (piezoelectric sensors, on the other hand produce a higher voltage).
- It is necessary for the input impedance to be very high and care must be taken in connections (such as the use of very good capacitors).
- Commercial charge amplifiers use FET transistors to ensure the necessary high input impedance.

Voltage dividers



Differential sensor



$$v_o = v_e \frac{Z_0(1+x)}{Z + Z_0(1+x)} \overset{(Z=Z_0)}{=} v_e \frac{1+x}{2+x}$$

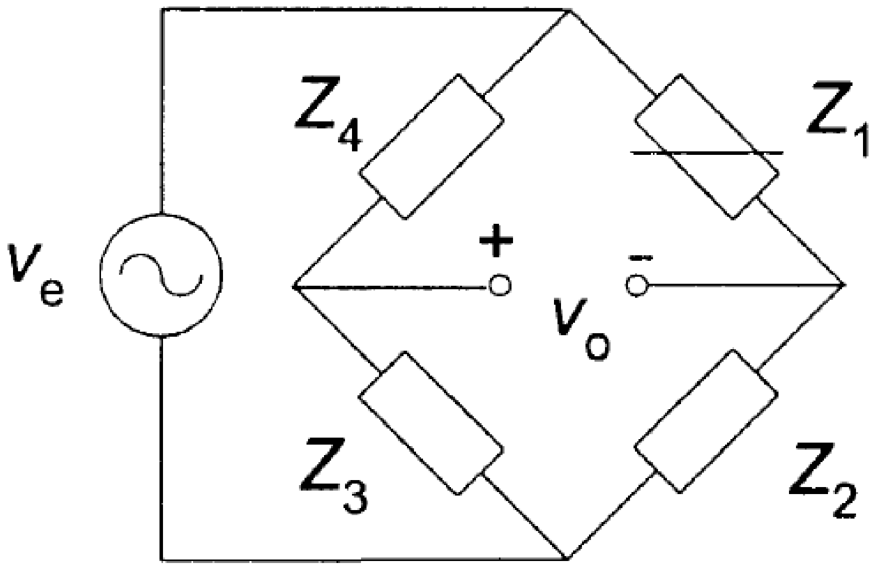
(non linear output)

$$v_o = v_e \frac{Z_0(1+x)}{Z(1-x) + Z_0(1+x)} \overset{(Z=Z_0)}{=} v_e \frac{1+x}{2}$$

(linear output)

The classical solution to cancel the constant term appearing at the output of a voltage divider (see previous slide) is a **bridge circuit**.

Because the bridge involves reactive impedances, it must be supplied by an alternating current or voltage.



When the bridge includes only one sensor whose impedance changes linearly with the measurand (e.g. $Z_1=Z_0(1+x)$ and and $Z_0=Z_2=Z_3=Z_4$), the output voltage shows a nonlinear relationship with x:

$$v_o = v_e \frac{x}{2(2+x)}$$

But for a differential sensor placed in adjacent arms (e.g. $Z_1=Z_0(1+x)$; $Z_2=Z_0(1-x)$ and $Z_0=Z_3=Z_4$), the output is

$$v_o = v_e \frac{x}{2}$$

and therefore v_o is proportional to x.

Furthermore, the same as in voltage dividers (and dc bridges), **this circuit cancels changes that are simultaneous for both sensor elements (such as the changes due to temperature)**.

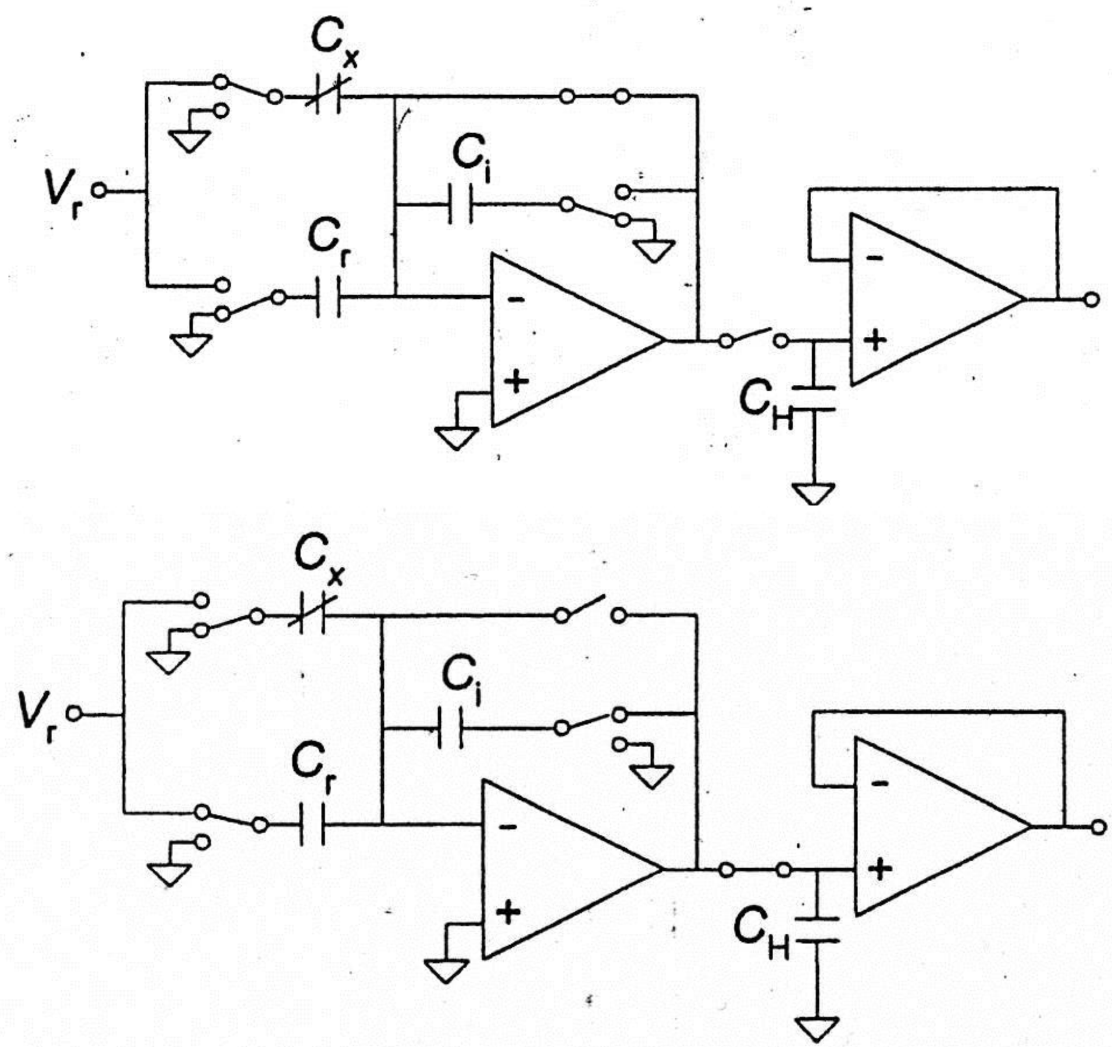
→ This makes ac bridges the most attractive solution for differential sensors.

The impedances for the two remaining bridge arms not occupied by the sensor are chosen depending on the type of sensor.

For a differential <u>inductive</u> sensor, resistors can be used because they have low to medium impedance.

Because single and differential <u>capacitive</u> sensors have large impedance, using resistors for the remaining bridge arms may produce large errors due to parasitic impedances to earth (which may have similar or lower impedance than the resistors).

Conditioning of capactive sensors: charge redistribution method



a) **"Autozero phase":** a dc reference voltage charges Cx to Vr and discharges Cr and Ci to ground. The output is zero.

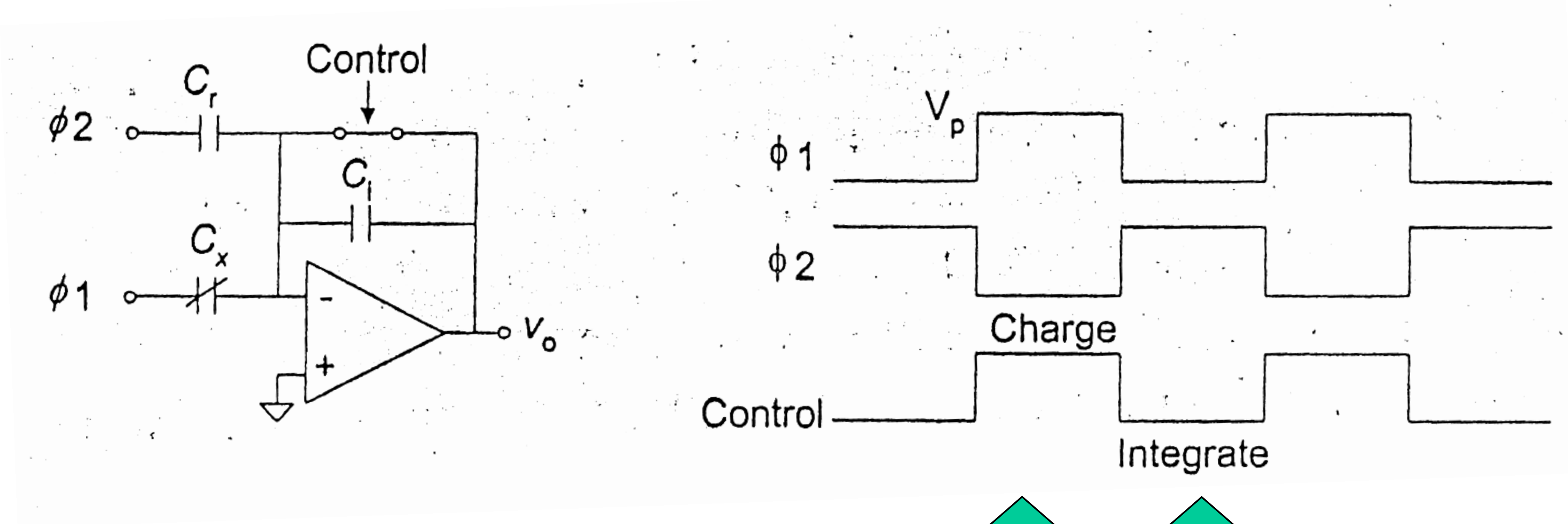
b) **"Measurement phase"**: Cx is connected to ground, Cr to Vr and Ci makes a feedback for the op-amp (integrator).

If Cx = Cr, the charge stored in Cx is redistributed between them and the output is zero.

If Cx ≠ Cr, there is a net charge flow through Ci and op-amp output voltage signal is proportional to Cx-Cr

Output stage : sample-and-hold which maintains the last output voltage during the following autozero phase.

Conditioning of capacitive sensors: charge integration method



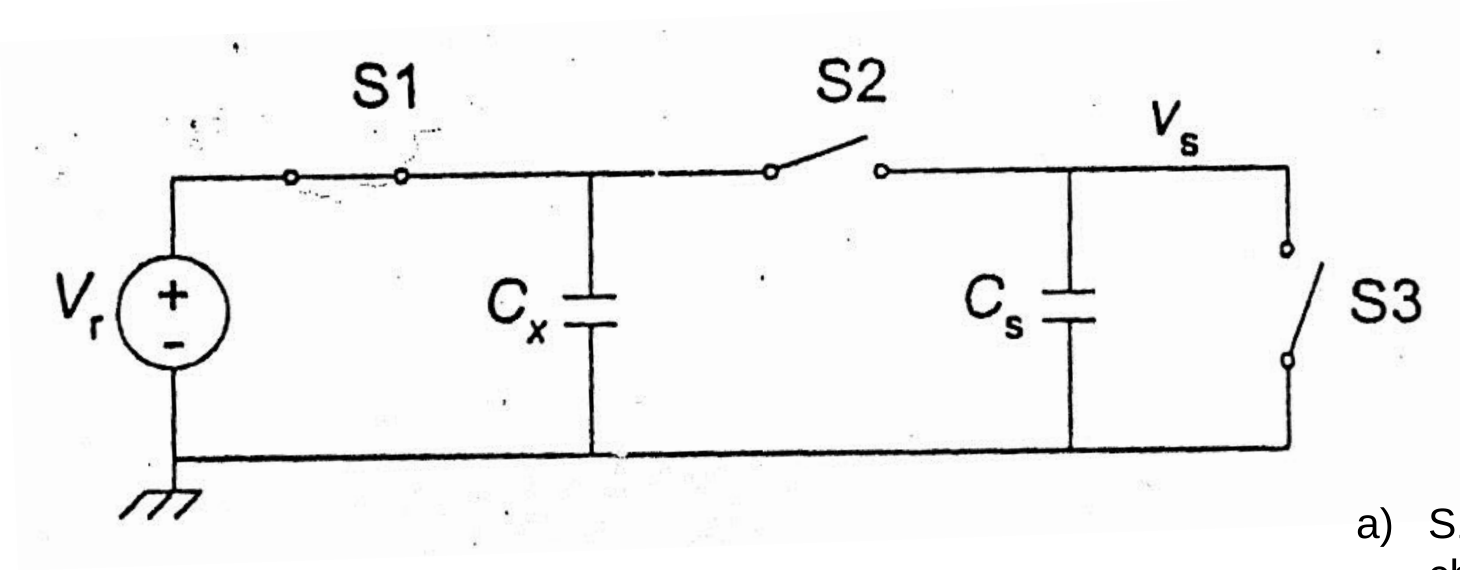
The two capacitances are connected to out-of-phase clock signals

Charge phase

Integration phase

$$v_0 = V_P \frac{C_X - C_r}{C_i}$$

Conditioning of capacitive sensors: charge transfer method

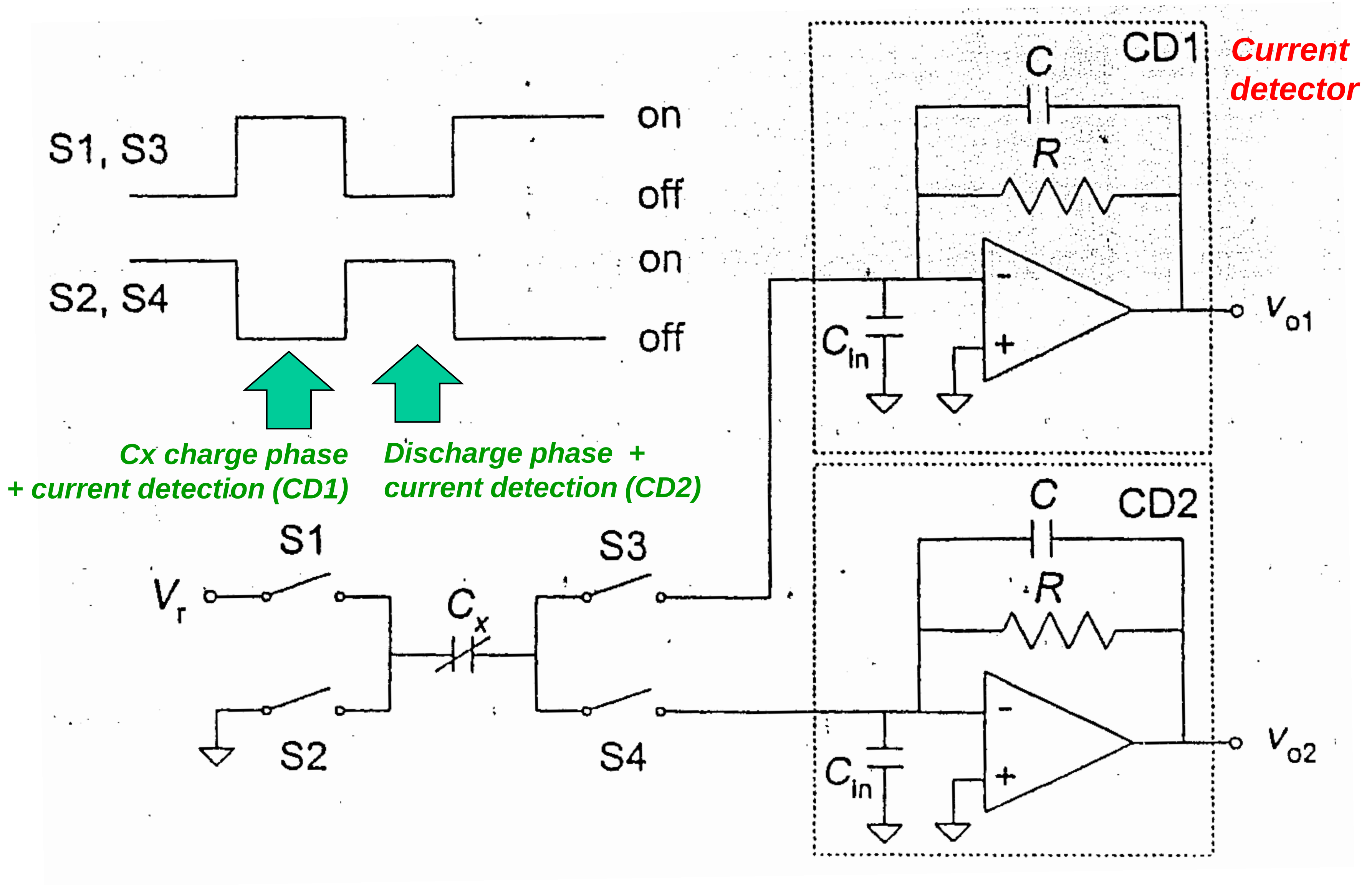


Charge transfer from a capacitor with unknown capacity (C_x) to an uncharged capacitor with known capacity (C_S).
If $C_S >> C_x$

$$v_s = V_r \frac{C_X}{C_X + C_S} \approx V_r \frac{C_X}{C_S} \Rightarrow C_X = C_S \frac{v_s}{V_r}$$

a) S1 closes and Cx charges up to Vr;
b) S1 opens and S2 closes to transfer charge from Cx to Cs;
c) S2 opens again and vs is measured;
d) S3 briefly closes to uncharge Cs
e) the cycle restarts from a)

Charge transfer method for an ungrounded capacitance



Capacitive accelerometer

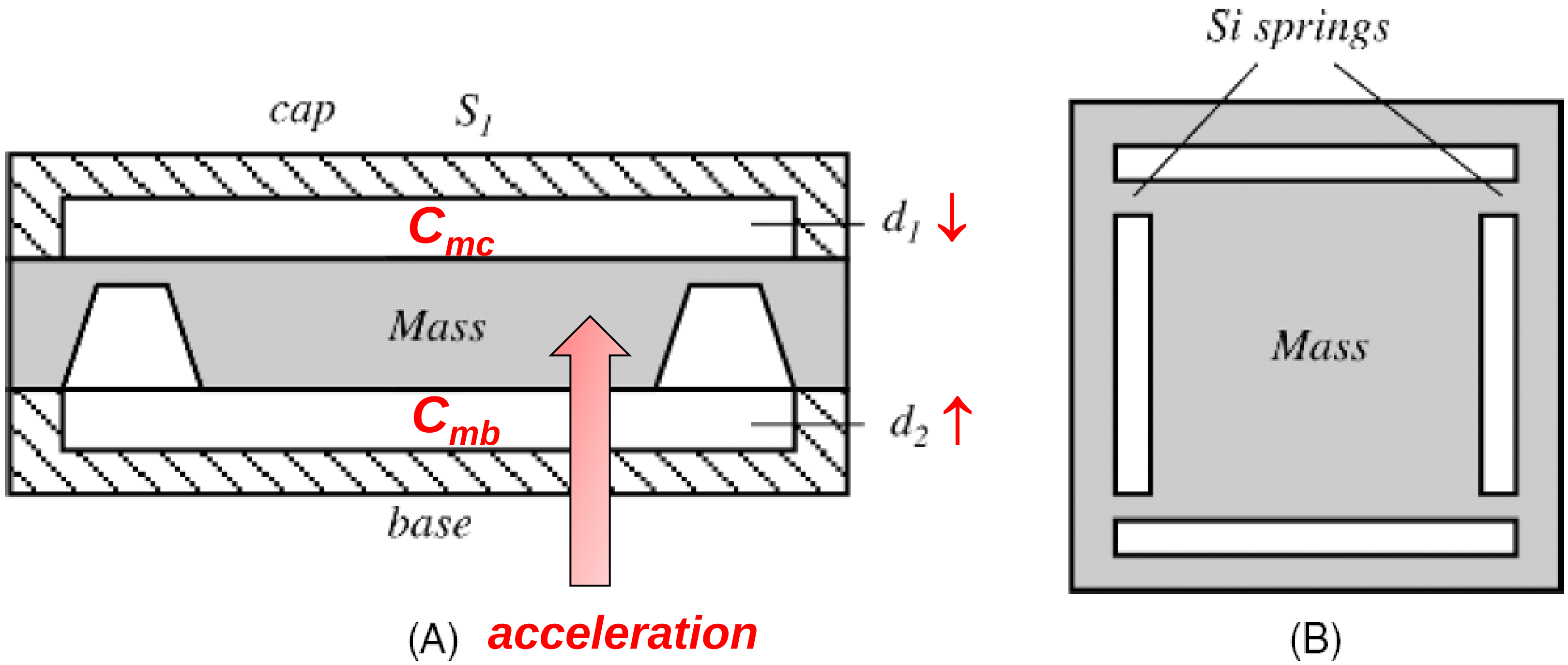


Fig. 8.3. Capacitive accelerometer with a differential capacitor: (A) side cross-sectional view; (B) top view of a seismic mass supported by four silicon springs.

Capacitive accelerometer integrated conditioning circuit

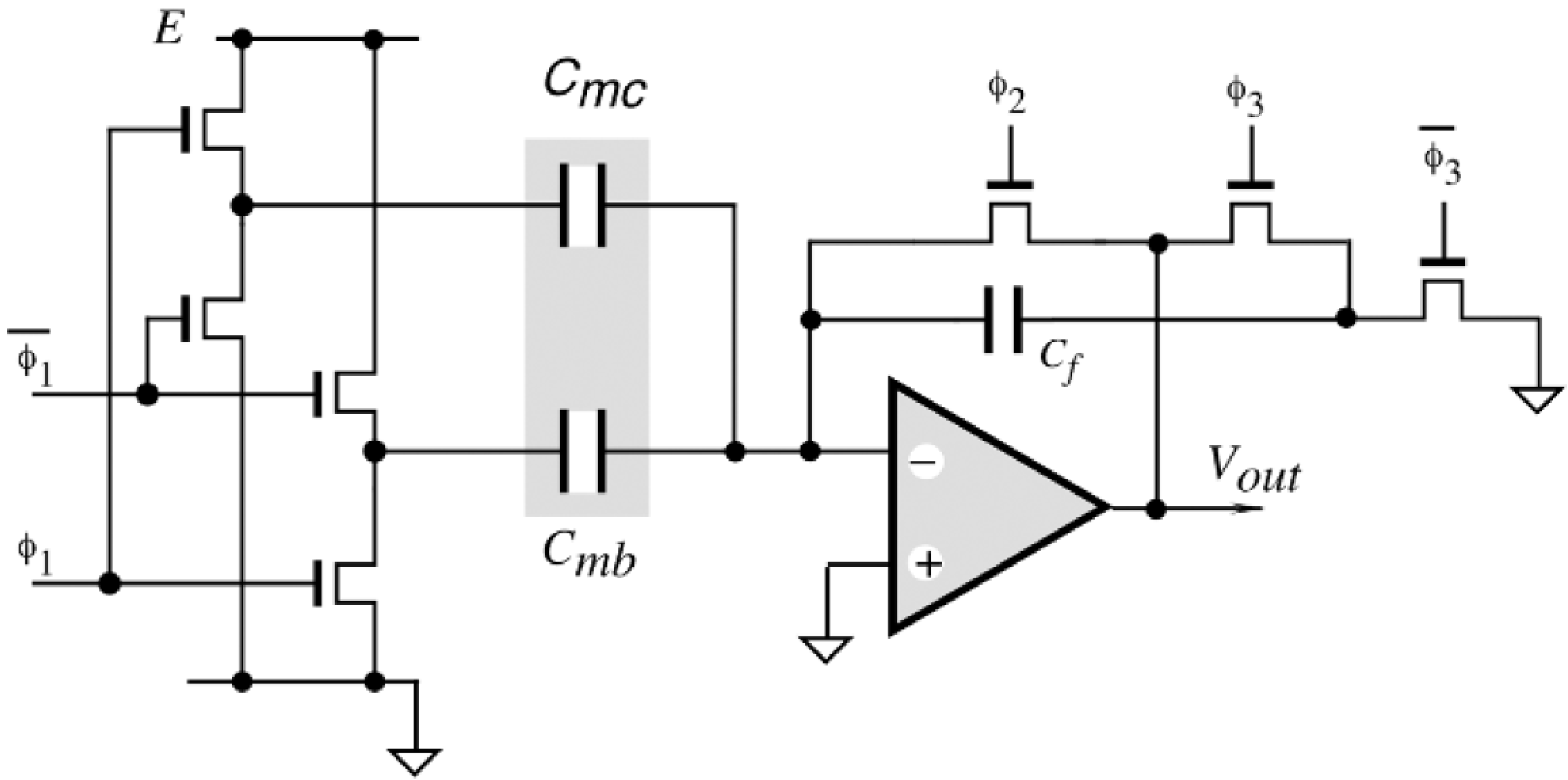


Fig. 8.4. Circuit diagram of a capacitance-to-voltage conversion suitable for an integration on silicon.

Conditioning of capacitive sensors

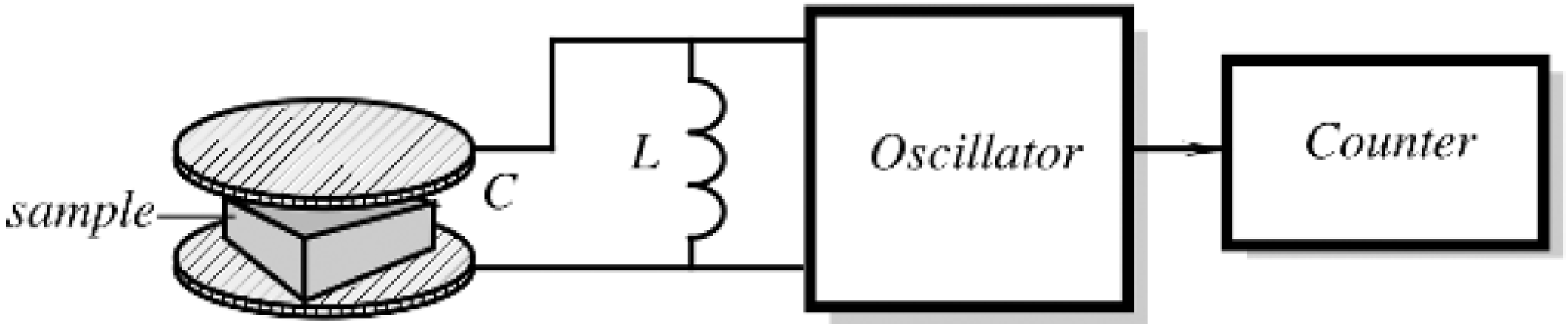
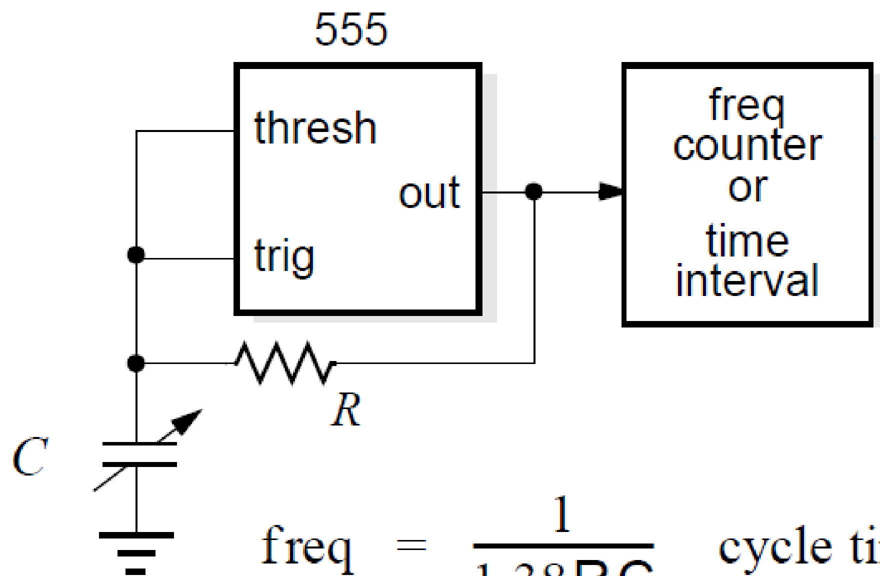


Fig. 13.3. Capacitive moisture sensing system.

RC oscillator

The RC oscillator used with a spacing-variation capacitor will produce a frequency output which is linear with spacing, while an area-variation capacitor is linearized by measuring pulse width.



$$\text{freq} = \frac{1}{1.38RC} \qquad \text{cycle time} = 1.38RC$$

The circuit as shown has no way of accommodating stray capacitance. If, for example, the capacitor is connected by a coaxial cable, the cable capacitance adds to the measured capacitance and spoils stability and sensitivity. Often a computer can be used to calibrate these errors, but the synchronous demodulator circuits (next slide) are a more accurate choice. Another drawback with the RC oscillator is that capacitance is measured relative to the fixed resistor, and the resistor stability and temperature coefficient may not track well.

Capacitive gain stages or preamplifiers
Noise reduction

Capacitive gain stages or preamplifiers are used to amplify and convert the capacitive change inputs to observable voltage outputs.
Capacitive circuits can be categorized in two main categories:

1) continuous time;
2) discrete-time or sampled-data.

There are two main techniques developed for **noise reduction in capacitive circuits**:

1) chopper stabilization

continuous-time method, the input signal is first modulated to a higher frequency away from 1/f noise corner. Then, the modulated signal is amplified, and demodulated to baseband. As the main signal is demodulated, 1/f noise of the circuit is modulated to high frequencies, so amplified input signal and 1/f noise are kept in different frequency bands. Lastly, the signal is filtered out.

2) correlated double sampling (CDS).

discrete-time method in which basically input-referred noise and offset are sampled twice while signal is sampled once in each cycle. Then, these two samples are subtracted from each other, eliminating the low-frequency noise and offsets since they do not change significantly between these samples.