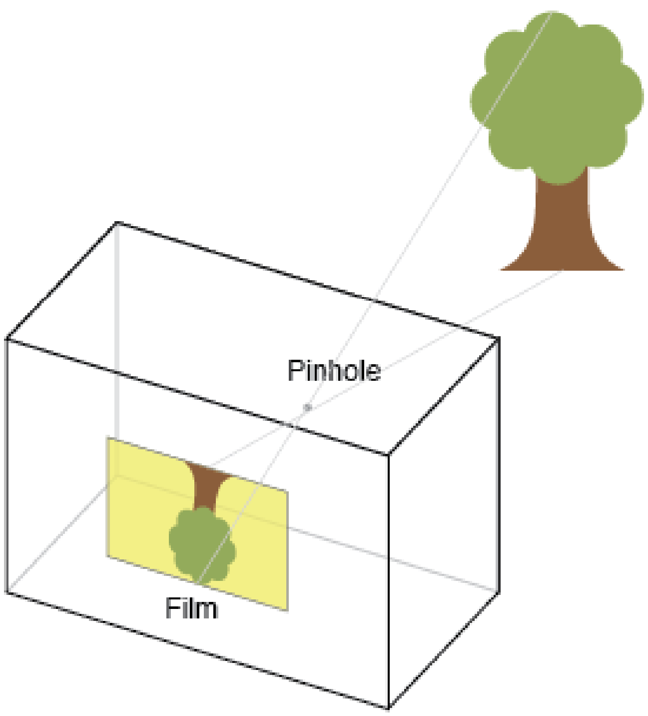
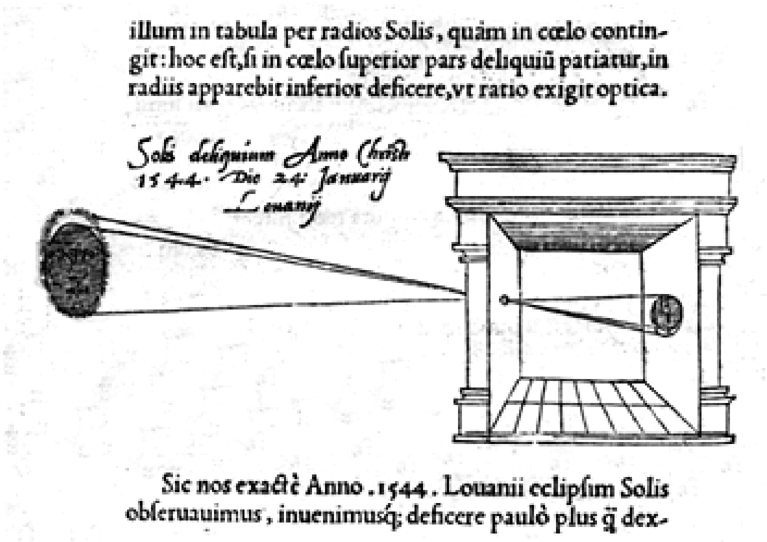
# Advanced camera calibration methods

# Pinhole camera model

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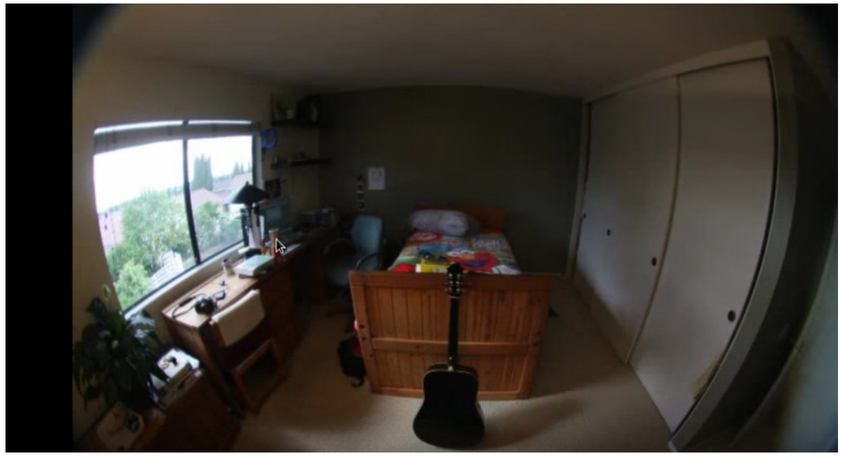
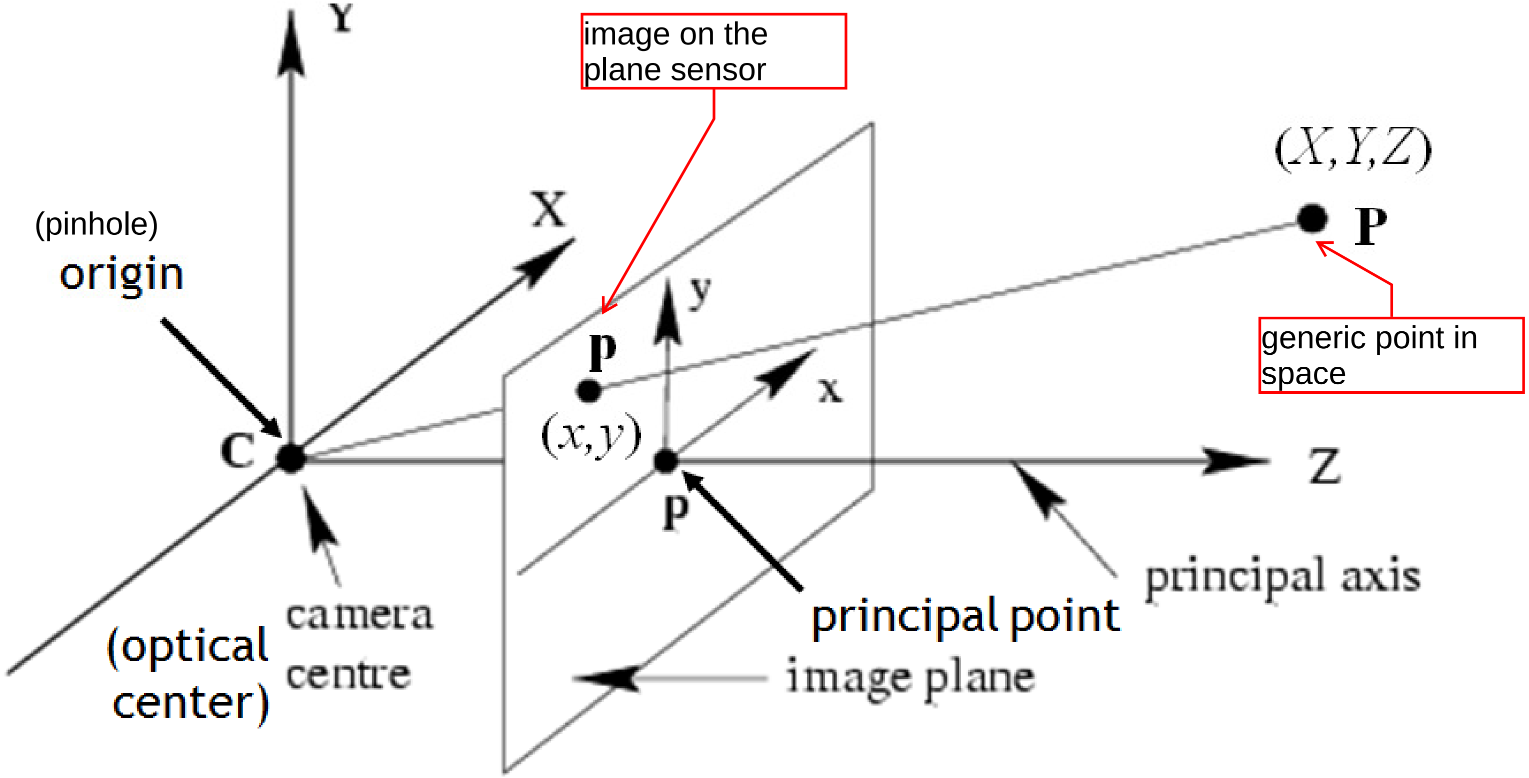
# Pinhole camera model




# Pinhole camera model

# Pinhole camera model (reference systems)

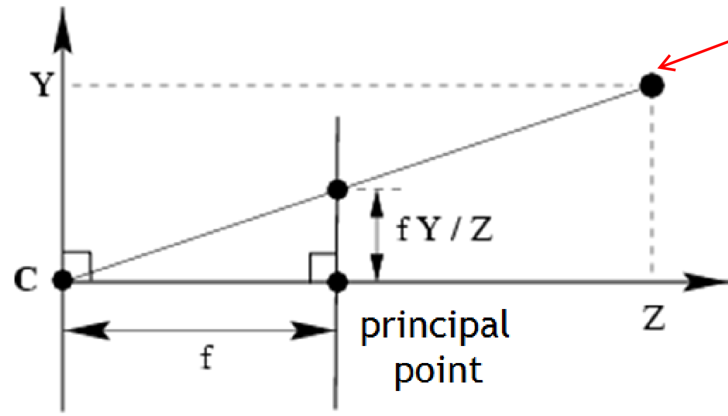



- The coordinate system
  - We will use the pin-hole model as an approximation
  - Put the optical center (**C**enter **O**f **P**rojection) at the origin
  - Put the image plane (**P**rojection **P**lane) *in front* of the COP

# Pinhole camera model

- ✓ *f* is the focal length
- ✓ triangles similarity properties allow establishing relationships between **P** camera co-ordinates (*X,Y,Z*) and its projection co-ordinates on the image plane (*x,y*)
- ✓ We can express relationships in matrix form:

respect to camera mount system




$$x = \frac{fX}{Z}$$

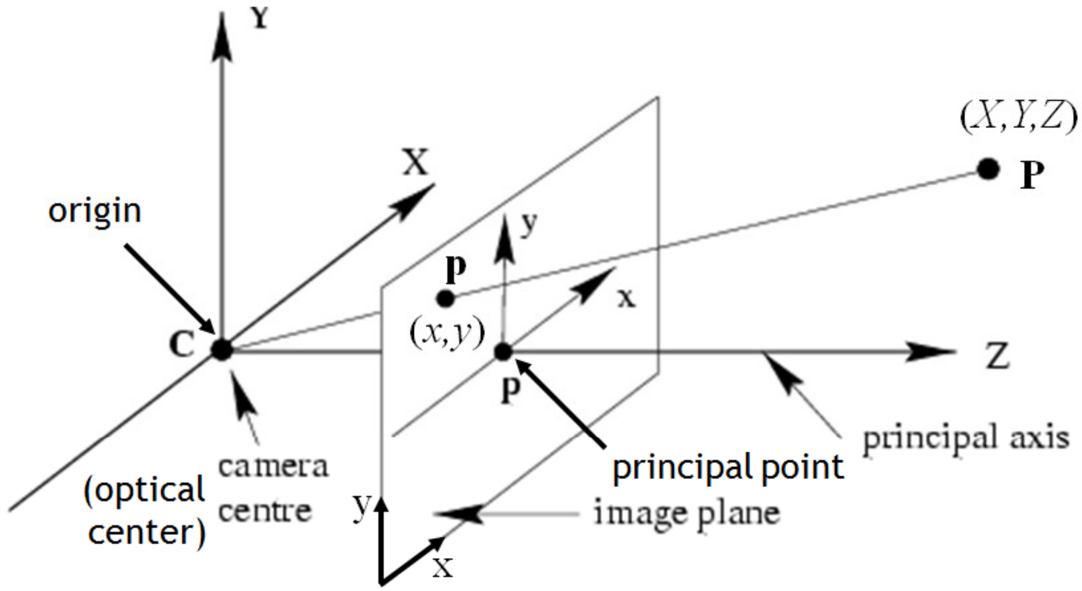
$$y = \frac{fY}{Z}$$

$$\begin{pmatrix} x \\ y \\ 1 \end{pmatrix} \sim \begin{pmatrix} fX \\ fY \\ Z \end{pmatrix} = \begin{bmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$

I get the position of the projection of the point on the image (plane) sensor.

# Pinhole camera model

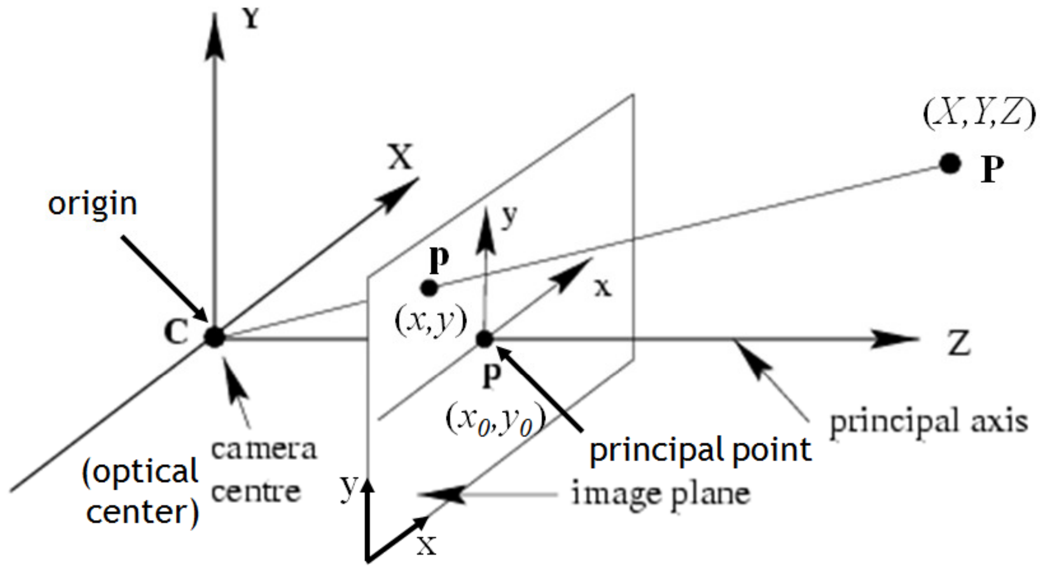
- ✓ Relevant reference systems highlighted




$$\begin{pmatrix} x \\ y \\ 1 \end{pmatrix} \sim \begin{pmatrix} fX \\ fY \\ Z \end{pmatrix} = \begin{bmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$

# Pinhole camera model

- ✓ Relevant reference systems highlighted
- ✓ Principal point co-ordinates (projection of COP onto the image plane)
- ✓ <u>This is the ideal case</u>




$$\begin{pmatrix} x \\ y \\ 1 \end{pmatrix} \sim \begin{pmatrix} fX \\ fY \\ Z \end{pmatrix} = \begin{bmatrix} f & 0 & x_0 \\ 0 & f & y_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$

# Pinhole camera model

Model refinement:

- ✓ Non square pixels (digital): apply scale factors (from pixel to millimeters)

$f_x = l\,f$

$f_x = kf$

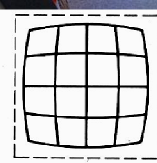
$$\mathbf{K} = \begin{bmatrix} f_x & 0 & x_0 \\ 0 & f_y & y_0 \\ 0 & 0 & 1 \end{bmatrix}$$

- ✓ Skew factor *s* (image plane not perfectly rectangular)

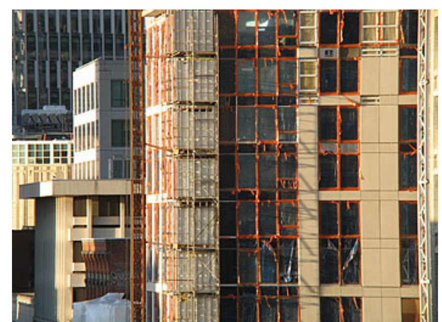
$$\mathbf{K} = \begin{bmatrix} f_x & s & x_0 \\ 0 & f_y & y_0 \\ 0 & 0 & 1 \end{bmatrix}$$

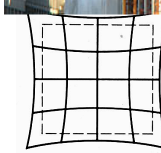
- ✓ Lens distortions:
  - ✓ Polynomial models (for radial particularly)

Radial



Barrel





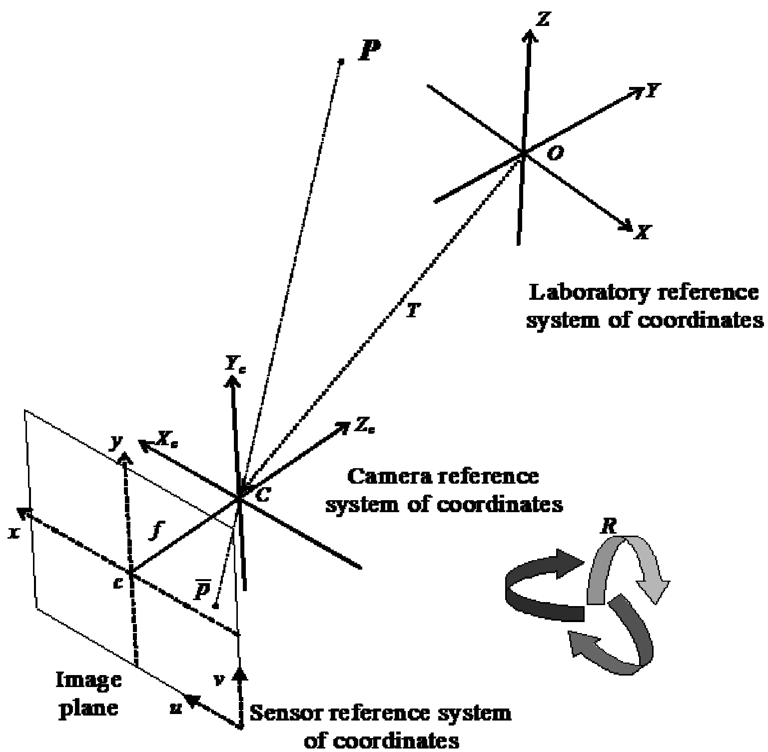
Pin-cushion

# Pinhole camera model

- Extrinsic parameters:

  if **R** and **t** express position and orientation of the camera w.r.t. a laboratory reference frame, it holds:

$$P^{TVC} = \begin{bmatrix} R & -Rt \\ 0 & 1 \end{bmatrix} P^{LAB}$$




Composing:

$$p = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} f & 0 & x_0 \\ 0 & f & y_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} R & -Rt \\ 0 & 1 \end{bmatrix} P^{LAB}$$

In matrix notation: **p=KDP**

# Camera calibration (with known control points)

- Estimate directly the 11 parameters of the matrix **M**

$$\begin{pmatrix} x \\ y \\ 1 \end{pmatrix} \sim \begin{bmatrix} f & 0 & x_0 \\ 0 & f & y_0 \\ 0 & 0 & 1 \end{bmatrix} [\mathbf{R} | -\mathbf{Rt}] \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix} = \begin{bmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \end{bmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

we want to find this? you need to know the other 2

$$\mathbf{x} \sim \mathbf{K}[\mathbf{R} | -\mathbf{Rt}]\mathbf{X} = \mathbf{MX}$$

$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} \sim \begin{bmatrix} m_{00} & m_{01} & m_{02} & m_{03} \\ m_{10} & m_{11} & m_{12} & m_{13} \\ m_{20} & m_{21} & m_{22} & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

For each point known in3D and known in 2D, it holds:

$$u_i = \frac{m_{00}X_i + m_{01}Y_i + m_{02}Z_i + m_{03}}{m_{20}X_i + m_{21}Y_i + m_{22}Z_i + 1}$$

$$v_i = \frac{m_{10}X_i + m_{11}Y_i + m_{12}Z_i + m_{13}}{m_{20}X_i + m_{21}Y_i + m_{22}Z_i + 1}$$

# Camera calibration (with known control points)

- Linear approach (DLT)

$$u_i(m_{20}X_i + m_{21}Y_i + m_{22}Z_i + 1) = m_{00}X_i + m_{01}Y_i + m_{02}Z_i + m_{03}$$

$$v_i(m_{20}X_i + m_{21}Y_i + m_{22}Z_i + 1) = m_{10}X_i + m_{11}Y_i + m_{12}Z_i + m_{13}$$

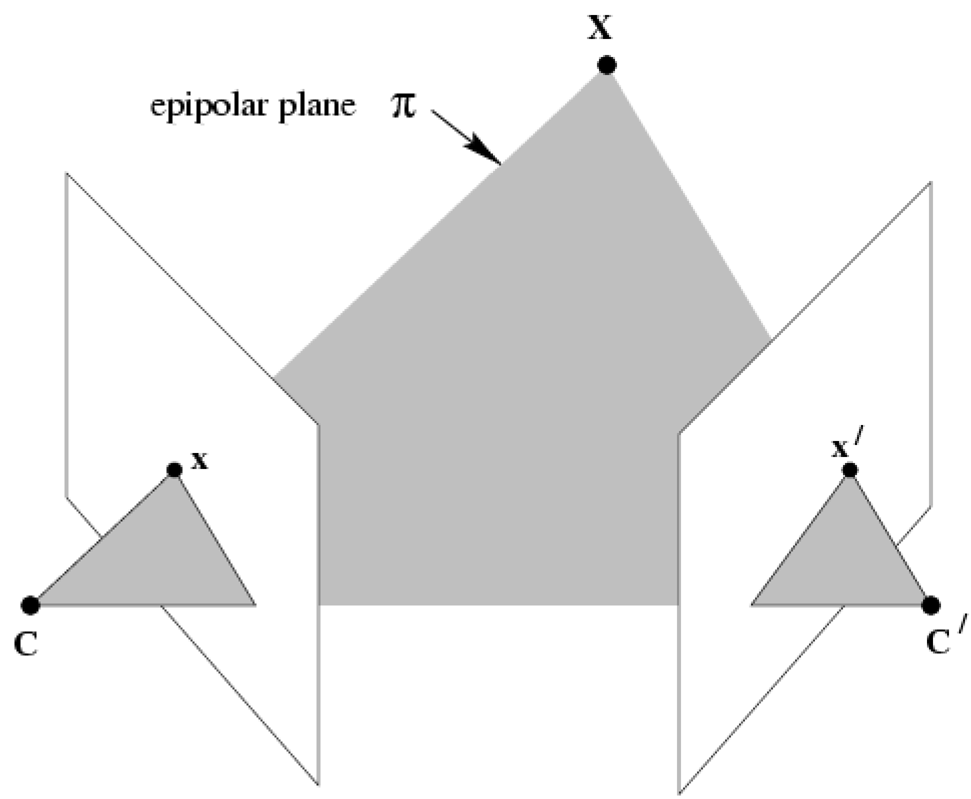
- Non-linear approach

$$\sum_{i=1}^{N}\left(u_i - \frac{m_{00}X_i + m_{01}Y_i + m_{02}Z_i + m_{03}}{m_{20}X_i + m_{21}Y_i + m_{22}Z_i + 1}\right)^2 + \left(v_i - \frac{m_{10}X_i + m_{11}Y_i + m_{12}Z_i + m_{13}}{m_{20}X_i + m_{21}Y_i + m_{22}Z_i + 1}\right)^2$$

# Camera calibration (with unknown control points)

- ✓ Need to introduce geometric constraints on the perspective projection model involving at least two cameras
- ✓ These constraints are given by the epipolar geometry

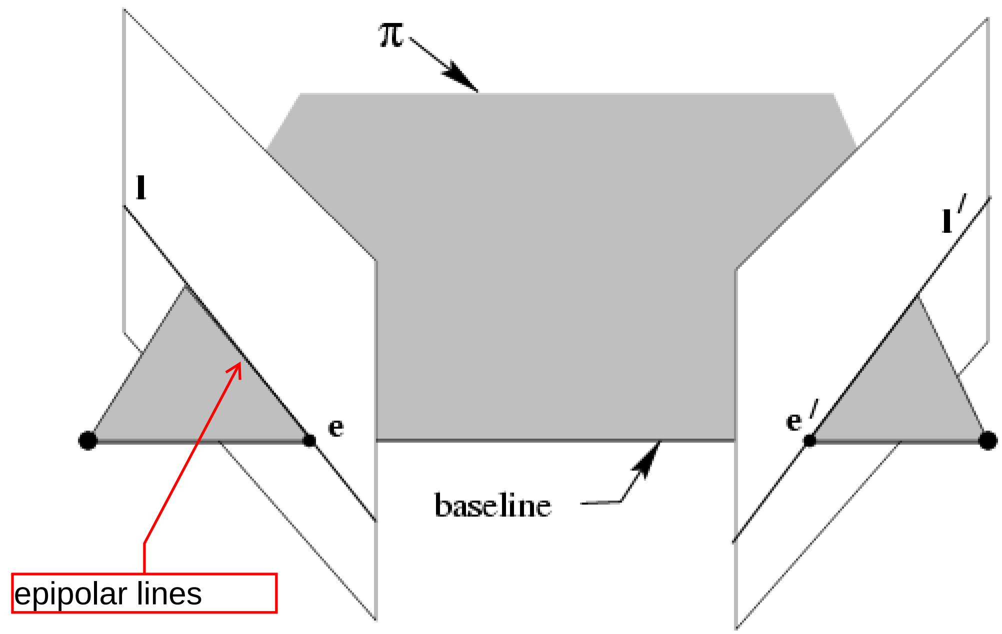
**C,C’,x,x’ and X are coplanar**



# Camera calibration (with unknown control points)

✓ Epipolar geometry

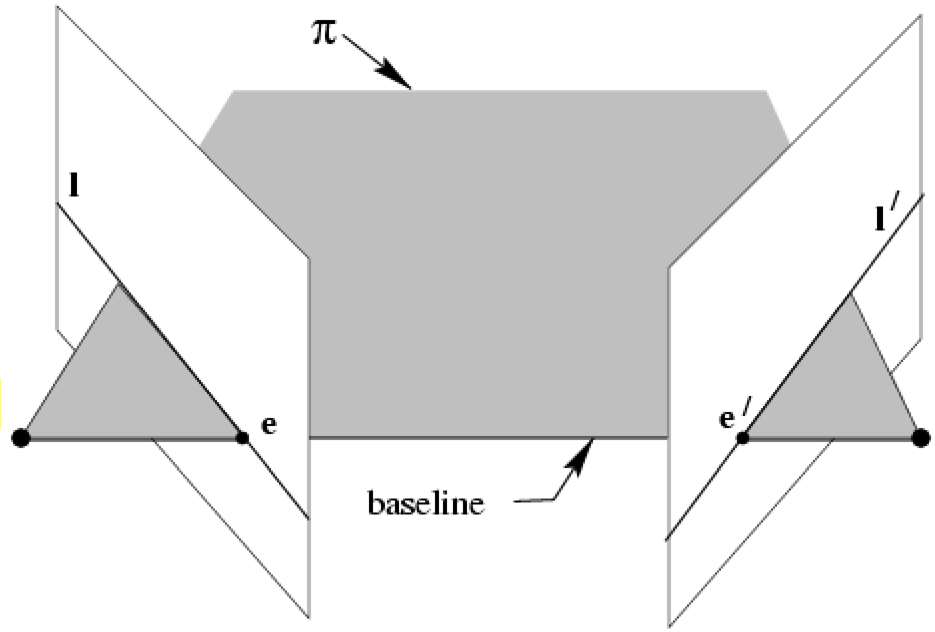
**All points on $\pi$ (epipolar plane) project on l (epipolar line on camera 1) and l' (epipolar line on camera 2)**



# Camera calibration (with unknown control points)

- ✓ Epipolar geometry

**The camera baseline intersects the image planes at the epipoles e and e'. Any plane π containing the baseline is an epipolar plane.**

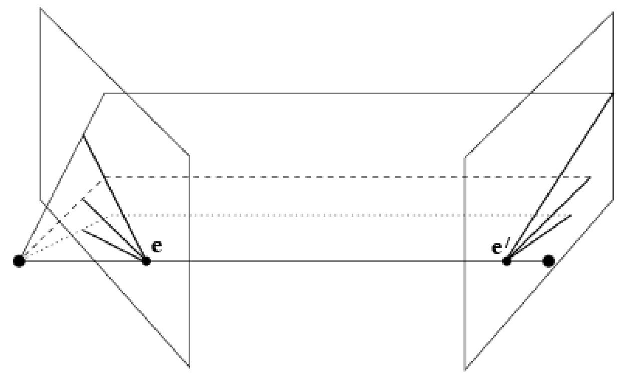


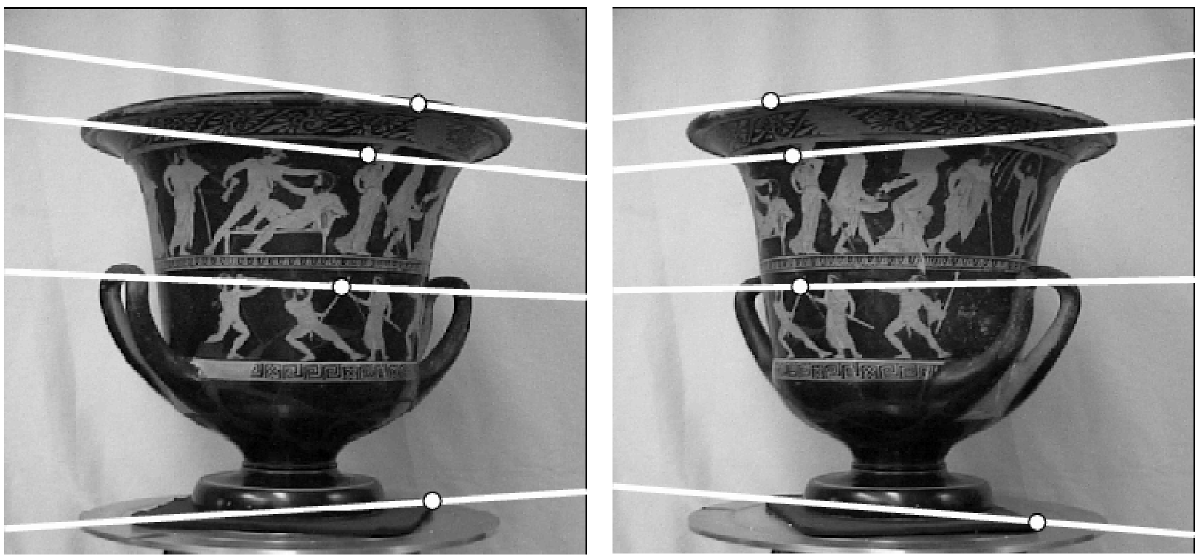

If I change the point I'll have ANOTHER
epipolar plane (and also lines etc.)! **BUT! always passing thorought the baseline.**
The base line it's like an unique line from which we can image a set of epipolar planes always passing here, and containing different points in spaces, and there corresponding images on the two plate sensors.

# Camera calibration (with unknown control points)

- Epipolar geometry:
  - Converging cameras:

    for each point on one image one can define the corresponding epipolar line on the other image on which the corresponding point lays



# Camera calibration (with unknown control points)

✓ Epipolar geometry:

1. Let's assume we want to calibrate one camera (TVC2) on the other (TVC1). Then the extrinsic parameters of TVC1 are **R**=$[I]_{3x3}$ and **t**=[0,0,0]

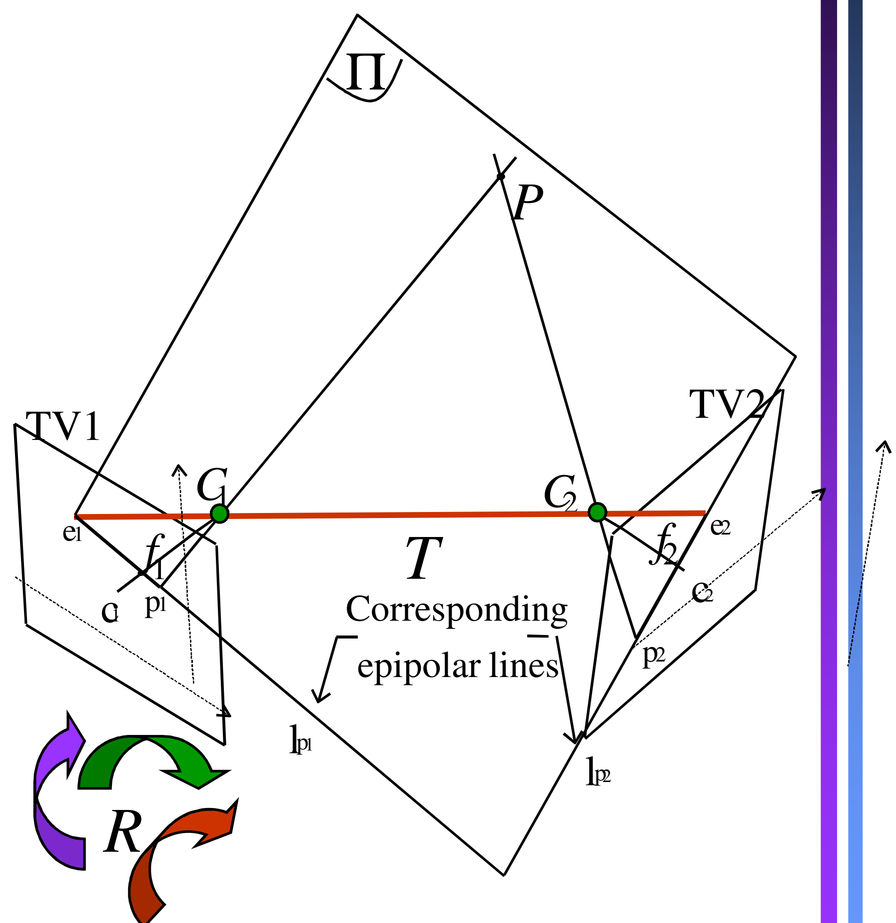
identity matrix and zero translation vector

2. It holds:

$$\mathbf{p}_1 = \mathbf{K}_1[\mathbf{I} \mid \mathbf{0}]\mathbf{P}$$

$$\mathbf{p}_2 = \mathbf{K}_2[\mathbf{R} \mid -\mathbf{R}\mathbf{t}]\mathbf{P}$$

[audio]




absolute value of baseline is my translation vector!, it connects the two center of prosp.

# Camera calibration (with unknown control points)

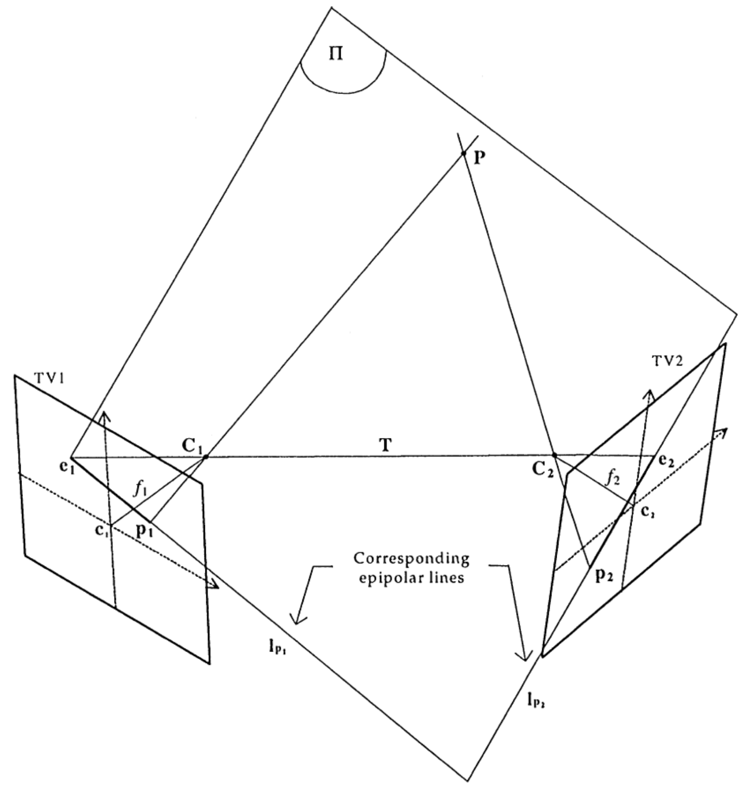
✓ Epipolar constraints:

1. Let's impose coplanarity of vectors $C_1C_2$, $PC_1$, $PC_2$
2. It holds:

$$\mathbf{PC}_2 \times \mathbf{T} \wedge \mathbf{PC}_1 = 0$$

3. By replacing with the expression of **P** in both cameras, introducing the skew symmetric matrix S for the translation vector **T**:

$$\mathbf{S} = \begin{bmatrix} 0 & -T_z & T_y \\ T_z & 0 & -T_x \\ -T_y & T_x & 0 \end{bmatrix}$$




Then I need to trasnform vector t to matrix, then a lot of calculation that are skipped.

# Camera calibration (with unknown control points)

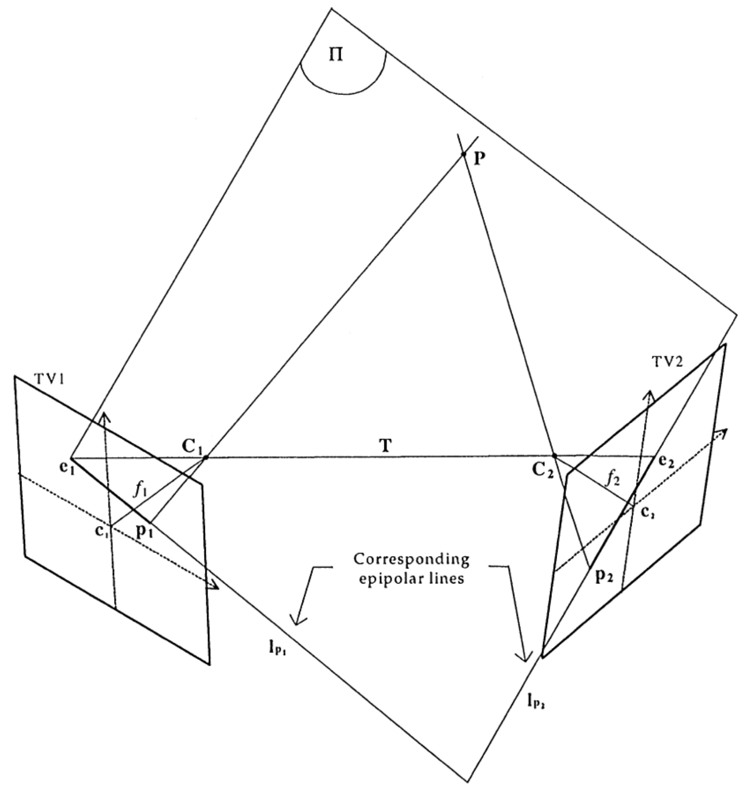
We obtain:

inverted and traposed

$$\mathbf{p}_2^T \mathbf{K}_2^{-T} \mathbf{RSK}_1^{-1} \mathbf{p}_1 = 0$$

where **E**=**RS** is the *essential matrix* containing the extrinsic parameters and **F**=**K**$^{-T}$**EK**$^{-1}$ is the *fundamental matrix* which interprets the epipolar geometric constraints (no a priori knowledge) containing intrinsic and extrinsic parameters

Matrix **F** has several interesting properties.



# Camera calibration (with unknown control points)

- Fundamental matrix properties:
  - Matrix is 3 x 3
  - Has 7 degrees of freedom. There are 9 elements, but scaling is not significant and det F = 0
  - **Transpose :** If **F** is fundamental matrix of cameras ($P_1$, $P_2$).

"DIRECTION OF F"

  - $\mathbf{F}^T$ is fundamental matrix of camera ($P_2$, $P_1$).
  - By definition, it holds: $\mathbf{p}_2^T \mathbf{F} \mathbf{p}_1 = 0$
  - **Epipolar lines:** Think of $\mathbf{p}_1$ and $\mathbf{p}_2$ as points in the projective plane then $\mathbf{F}\,\mathbf{p}_1$ is projective line in the right image,

    that is $l_2 = \mathbf{F}\,\mathbf{p}_1$ and $l_1 = \mathbf{F}^T \mathbf{p}_2$
  - **Epipole:** Since for any **p** the epipolar line $l_2 = \mathbf{F}\,\mathbf{p}_1$ contains the epipole $e_2$, thus $(e_2^T)\,\mathbf{p}_1 = 0$ for all **p** . Thus $(e_2^T)\,\mathbf{F} = 0$ and $\mathbf{F} e_1 = 0$

# Camera calibration (with unknown control points)

- Fundamental matrix:
  - Encodes information from extrinsic and intrisic parameters
  - can be estimated with at least 8 pair matches (8-point algorithm) by solving the linear equation system $\mathbf{p}_2^T \mathbf{F} \mathbf{p}_1 = 0$
  - Robust optimization needed especially because much more matches are used
- From **F**, intrinsic (from **F**) and extrinsic (from **E**, once the intrinsic ones have been estimated) parameters can be extracted by SVD
- Algebraic and geometric methods (involving the *absolute conic*) are applied to extract focal length from **F** under the assumption that principal points are known
- When the principal point is not known, large-scale optimization methods are required (for example genetic algorithms)
- A scale factor needs to be provided (known distance between markers on the *magic wand*)

# Camera calibration (with unknown control points)

## ✓ Key References


- H.C. Longuet-Higgins. A computer algorithm for reconstructing a scene from two projections. *Nature, 293:133–135, Sept 1981*
- S. J. Maybank and O. D. Faugeras. A theory of self-calibration of a moving camera. *International Journal of Computer Vision, 8:2:123 – 151, 1992.*
- Z. Zhang, Determining the epipolar geometry and is uncertainty: a review, *Int. J. Comput. Vision 27 (2) 161-195, 1998*
- Q.T. Luong, O.D. Faugeras, The fundamental matrix: theory, algorithms, and stability analysis. *Int. J. Comput. Vision 17 (1):43-76. 1996*

# Camera calibration (with unknown control points)