SPRING 2023 FINC B 9325 Financial Econometrics – Time Series Homework 1

I. In this assignment you will explore the variance ratio statistic defined as

$$VR(K) = \frac{Var\left(\sum_{j=1}^{K} r_{t+j}\right)}{KVar(r_{t+1})}$$

where r_t denotes the continuously compounded rate of return on the CRSP index and $Var(\cdot)$ denotes the sample variance. Monthly data on r_t are available on Canvas from 1926 to the present from which you will calculate point estimates of your $VR(K)$ statistic for $K = 12, 24, 36, 48, 60, 90, 120$. I want you to compare these values to the empirical distributions of the statistics generated from the model in Poterba and Summers (1988). In all cases, $r_t = p_t - p_{t-1}$, where p_t represents the natural logarithm of the price.

Under the null hypothesis, the model is

$$p_t = p_{t-1} + \epsilon_t$$

where ϵ_t is *i.i.d.* $N(0, 1)$.

[Note that I normalized the standard deviation to 1--which is not realistic-as the level of volatility does not matter in variance ratio tests.]

Under the alternative hypothesis, the model is

$$\begin{aligned} p_t &= p_t^* + u_t \\ p_t^* &= p_{t-1}^* + \epsilon_t \\ u_t &= \rho u_{t-1} + \nu_t \end{aligned}$$

where ϵ_t is *i.i.d.* $N(0,1)$, $\rho = 0.98$, ν_t is *i.i.d* $N(0, \sigma_\nu^2)$, the two innovations are uncorrelated, and σ_ν^2 is calibrated such that 25% of the variance of returns is due to the transitory component of prices.

To get the empirical distributions, you need to simulate the test statistic 1000 times for the appropriate sample size T.

What percent of the empirical distribution under the null hypothesis is smaller than your point estimate?

What percent of the empirical distribution under the alternative hypothesis is smaller than your point estimate?

What explanation can you give for these findings?

For extra credit, redo the analysis from 1952 to the present and provide some interpretation of why things are different.

II. Consider the stochastic process:

$$y_t = \eta + u_t$$

Here, η is a $N(0, 1)$ random variable and u_t is i.i.d, independent of η and uniformly distributed on $[0, 1]$. Is the process stationary? If it is, compute its first and second moment and autocovariances. Is it ergodic? Why or why not?