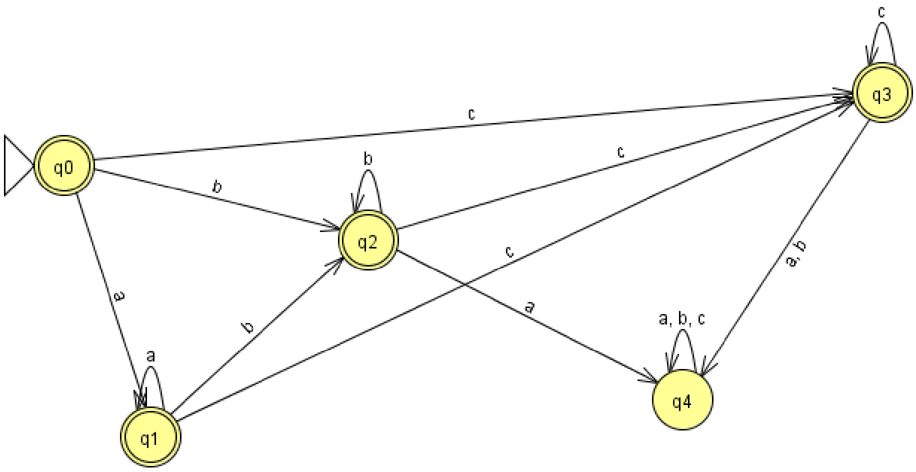
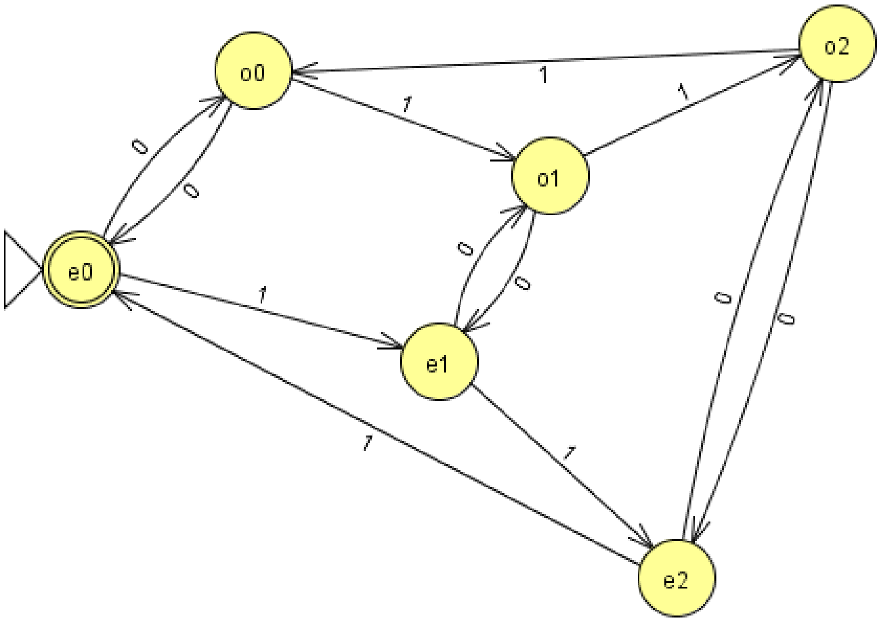
Question 1

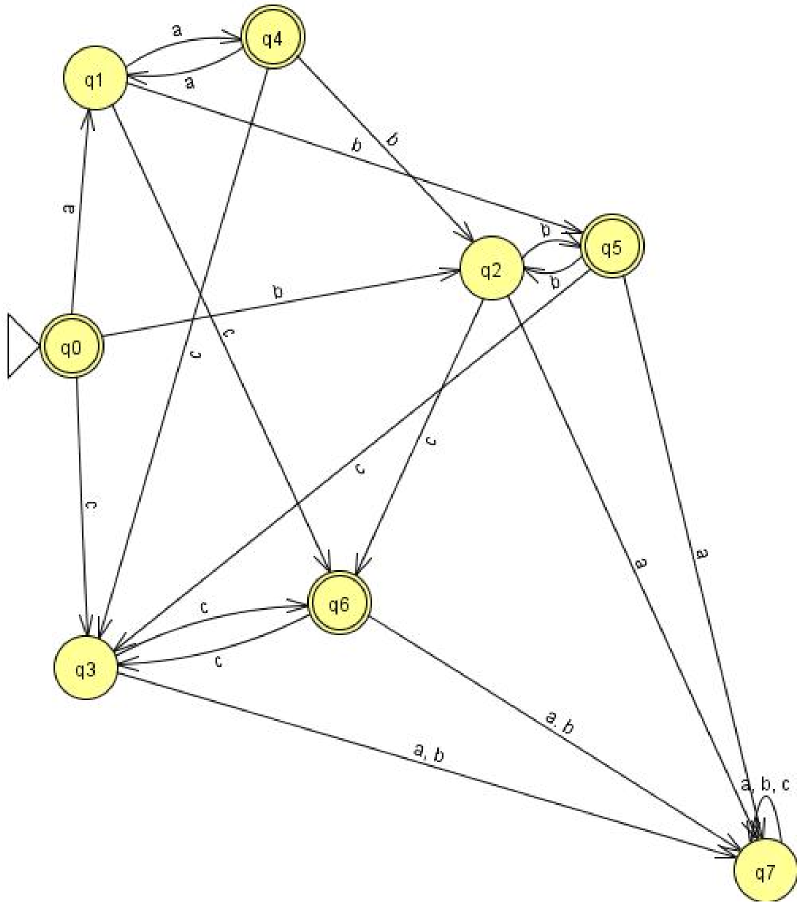


a.

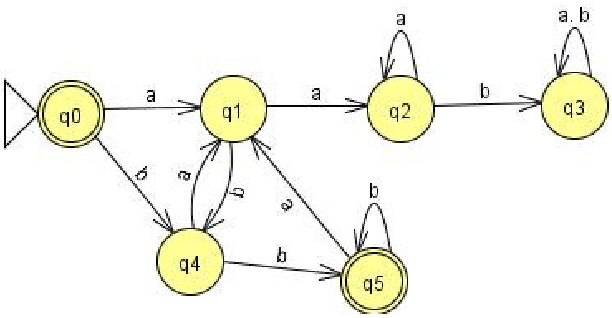
b.



c.



d.



Base: Let $y = \epsilon$ then $|y| = 0$

$$\begin{aligned}
&\hat{\delta}(\hat{\delta}(q,x),y) && \text{(assumption)}\\
&= \hat{\delta}(\hat{\delta}(q,x),\epsilon) && \text{(definition)}\\
&= \hat{\delta}(q,x) && \text{(definition)}\\
&= \hat{\delta}(q,x\epsilon) && \text{(definition)}\\
&= \hat{\delta}(q,xy) && \text{(assumption)}\\
&Q.E.D.
\end{aligned}$$

IH: if $|y| = n$ then $\hat{\delta}(q,x) = \hat{\delta}(\hat{\delta}(q,x),y)$

IS: Assume the hypothesis is true for n. We will show that it is true for $n+1$ Let $y = |za|$ where z is a string such that $|z| \geq 0$ and a is a character in Σ

$$\begin{aligned}
&\hat{\delta}(\hat{\delta}(q,x),y) && \text{(assumption } y = za\text{)}\\
&= \hat{\delta}(\hat{\delta}(q,x),za) && \text{(assumption)}\\
&= \delta(\hat{\delta}(\hat{\delta}(q,x),y),a) && \text{(definition } \hat{\delta}\text{)}\\
&= \delta(\hat{\delta}(q,xz),a) && \text{(IH)}\\
&= \hat{\delta}(q,xza) && \text{(definition } \hat{\delta}\text{)}\\
&= \hat{\delta}(q,xy) && \text{(assumption } y = za\text{)}\\
&Q.E.D
\end{aligned}$$

	0	1
$\rightarrow$ p	{q,s}	$\emptyset$
q	{r}	{s}
r	$\emptyset$	{s}
*s	$\emptyset$	$\emptyset$