# Mitali Pset 1

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R04
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1.3) $\vec{A}\cdot\vec{B} = |\vec{A}||\vec{B}|\cos\theta$

$$\cos\theta = \frac{\vec{A}\cdot\vec{B}}{|\vec{A}||\vec{B}|}$$

$$\boxed{= \frac{-4}{\sqrt{66}}}$$

$$\vec{A}\cdot\vec{B} = A_x B_x + A_y B_y + A_z B_z$$
$$= -6 + 1 + 1$$
$$= -4$$

$$|\vec{A}\times\vec{B}| = |\vec{A}|\,|\vec{B}|\sin\theta$$

$$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \vec{A}_x & \vec{A}_y & \vec{A}_z \\ \vec{B}_x & \vec{B}_y & \vec{B}_z \end{vmatrix}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & 1 \\ -2 & 1 & 1 \end{vmatrix}$$

$$= 0\hat{i} - 5\hat{j} + 5\hat{k}$$

$$|\vec{A}\times\vec{B}| = \sqrt{0+25+25}$$
$$= 5\sqrt{2}$$

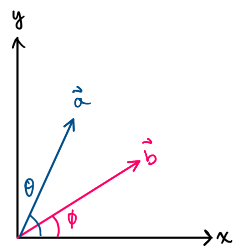
$$|\vec{A}| = \sqrt{9+1+1}$$
$$= \sqrt{11}$$
$$|\vec{B}| = \sqrt{4+1+1}$$
$$= \sqrt{6}$$

$$\sin\theta = \frac{|\vec{A}\times\vec{B}|}{|\vec{A}|\,|\vec{B}|}$$

$$\sin\theta = \frac{5\sqrt{2}}{\sqrt{11}\cdot\sqrt{6}}$$

$$\boxed{= \frac{5}{\sqrt{33}}}$$

1.8)




$$\vec{a} \stackrel{?}{=} \cos\theta\,\hat{i} + \sin\theta\,\hat{j}$$
$$|\vec{a}| = 1 = \sqrt{\sin^2\theta + \cos^2\theta}$$
$$\left.\begin{aligned} \vec{a}\cdot\hat{i} &= \cos\theta \\ \vec{a}\cdot\hat{j} &= \sin\theta \end{aligned}\right\} \vec{a} = \cos\theta\,\hat{i} + \sin\theta\,\hat{j}$$

$$\vec{b} \stackrel{?}{=} \cos\phi\,\hat{i} + \sin\phi\,\hat{j}$$
$$|\vec{b}| = 1 = \sqrt{\sin^2\phi + \cos^2\phi}$$
$$\left.\begin{aligned} \vec{b}\cdot\hat{i} &= \cos\phi \\ \vec{b}\cdot\hat{j} &= \sin\phi \end{aligned}\right\} \vec{b} = \cos\phi\,\hat{i} + \sin\phi\,\hat{j}$$

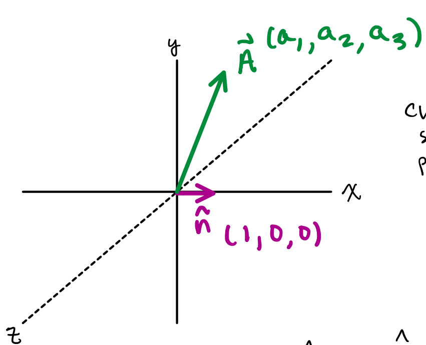
projection of $\vec{a}$ onto $\vec{b}$:

$$\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos(\theta-\phi) = a_x b_x + a_y b_y$$
$$1\cdot 1\cdot\cos(\theta-\phi) = \cos\theta\cos\phi + \sin\theta\sin\phi$$
$$\boxed{\cos(\theta-\phi) = \cos\theta\cos\phi + \sin\theta\sin\phi}$$

1.13) $\vec{A}$ = an arbitrary vector

$\hat{n}$ = unit vector in fixed direction

$$\vec{A} = (\vec{A}\cdot\hat{n})\hat{n} + (\hat{n}\times\vec{A})\times\hat{n}$$




Choose coordinates so that $\hat{n}$ is parallel to $x$-axis

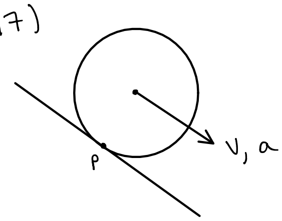
$$\vec{A}\cdot\hat{n} = \langle a_1, a_2, a_3\rangle \cdot \langle 1, 0, 0\rangle$$
$$= a_1$$

$$\hat{n}\times\vec{A} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 1 & 0 & 0 \\ a_1 & a_2 & a_3 \end{vmatrix}$$
$$= -a_3\hat{y} + a_2\hat{z}$$

$$(\hat{n}\times\vec{A})\times\hat{n} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0 & -a_3 & a_2 \\ 1 & 0 & 0 \end{vmatrix}$$
$$= a_2\hat{y} + a_3\hat{z}$$

$$\vec{A} = (a_1)\hat{n} + (a_2\hat{y} + a_3\hat{z})$$
$$= a_1\hat{x} + a_2\hat{y} + a_3\hat{z}$$
$$= \vec{A} \checkmark$$

1.17)

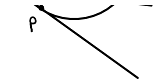



at point P, tangential velocity must be 0
– no slipping

therefore:

$v = R\omega$ where $\omega$ = angular velocity

$$\frac{d}{dt}(v) = \frac{d}{dt}(R\omega)$$



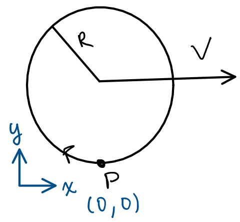
$V = R\omega$ where $\omega$ = angular velocity

$\frac{d}{dt}(v) = \frac{d}{dt}(R\omega)$

$a = R\alpha$

$\Rightarrow \boxed{\alpha = \frac{a}{R}}$

1.24)




position:
relative to tire center,

$x_p(t) = -R\sin\omega t$

$y_p(t) = -R\cos\omega t$

center:

$x_c(t) = vt = R\omega t$

$y_c(t) = R$

where $\omega = V/R$

motion of pebble w.r.t. road:

$x(t) = x_p + x_c = R\omega t - R\sin\omega t$

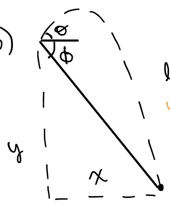
$y(t) = y_p + y_c = R - R\cos\omega t$

$$\boxed{\vec{r}(t) = (R\omega t - R\sin\omega t)\hat{i} + (R - R\cos\omega t)\hat{j}}$$

$$\boxed{\vec{V}(t) = \dot{\vec{r}}(t) = (R\omega - R\omega\cos\omega t)\hat{i} + (R\omega\sin\omega t)\hat{j}}$$

$$\boxed{\vec{a}(t) = \ddot{\vec{r}}(t) = (R\omega^2\sin\omega t)\hat{i} + (R\omega^2\cos\omega t)\hat{j}}$$

1.26)




line equation: $y = mx + b$

$y = -x\tan\phi$

$m = \frac{\sin\phi}{\cos\phi} = \tan\phi$

assume $b = 0$

parabola:

if initial speed $= v_0$,

$V_x = v_0\cos\theta$, $V_y = v_0\sin\theta - gt$

$x = v_0 t\cos\theta$, $y = v_0 t\sin\theta - \frac{1}{2}gt^2$

$\Rightarrow t = \frac{x}{v_0\cos\theta}$

$\Rightarrow y = \cancel{v_0}\sin\theta \cdot \frac{x}{\cancel{v_0}\cos\theta} - \frac{1}{2}g\left(\frac{x}{v_0\cos\theta}\right)^2$

$y = x\tan\theta - \frac{gx^2}{2v_0^2\cos^2\theta}$

Intersection: $(x \neq 0)$

$-x\tan\phi = x\tan\theta - \frac{gx^2}{2v_0^2\cos^2\theta}$

$\frac{gx^{\cancel{2}}}{2v_0^2\cos^2\theta} = \cancel{x}\tan\theta + \cancel{x}\tan\phi$

$\frac{gx}{2v_0^2} = \tan\theta\cos^2\theta + \tan\phi\cos^2\theta$

$x = (\sin\theta\cos\theta + \tan\phi\cos^2\theta)\frac{2v_0^2}{g}$

$\frac{dx}{d\theta} = \frac{v_0^2}{g}\frac{d}{d\theta}(\sin 2\theta + 2\tan\phi\cos^2\theta)$

$= \frac{v_0^2}{g}(2\cos 2\theta - 4\tan\phi\sin\theta\cos\theta)$

$= \frac{2v_0^2}{g}(\cos 2\theta - \tan\phi\sin 2\theta)$

maximize $x$:

$\frac{dx}{d\theta} = 0 = \frac{2v_0^2}{g}(\cos 2\theta - \tan\phi\sin 2\theta)$

$\cos 2\theta = \tan\phi\sin 2\theta$

$\cot 2\theta = \tan\phi$

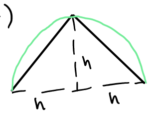
$\tan(\frac{\pi}{2} - 2\theta) = \tan\phi$

$\frac{\pi}{2} - 2\theta = \phi$

$2\theta = \frac{\pi}{2} - \phi$

$\boxed{\theta = \frac{\pi}{4} - \frac{\phi}{2}}$

1.27)




find speed such that:

$z_{max} = h$

coordinate system such that:

$x_0 = 0$

$z_0 = 0$

equations:

$x(t) = x_0 + v_x t$

$z(t) = z_0 + v_z t - \frac{1}{2}gt^2$

$V_z(t) = V_z - gt$

at highest point:

$V_z(t) = 0 = V_z - gt$

$t = \frac{V_z}{g}$

$z\left(\frac{V_z}{g}\right) = h = V_z \cdot \frac{V_z}{g} - \frac{1}{2}\cancel{g} \cdot \frac{V_z^2}{g^{\cancel{2}}}$

$h = \frac{V_z^2}{2g}$

$V_z^2 = 2gh$

$V_z = \sqrt{2gh}$

$x\left(\frac{V_z}{g}\right) = h = V_x \cdot \frac{V_z}{g}$

$V_x = \frac{gh}{V_z} = \frac{gh}{\sqrt{2gh}} = \sqrt{\frac{gh}{2}}$

$s^2 = V_x^2 + V_z^2$

$= \frac{gh}{2} + 2gh$

$\boxed{s = \sqrt{\frac{5}{2}gh}}$

8) a. Bee, first leg:

$u = \frac{d_0}{t_0}$ $d_0 = d - vt_0$

relative to train,

$u + v = \frac{d}{t_0} \Rightarrow t_0 = \frac{d}{u+v}$

$d_0 = d - \frac{vd}{u+v} = d\left(1 - \frac{v}{u+v}\right)$

$= \frac{du}{u+v}$

second leg:

$u = \frac{d_1}{t_1}$ $d_1 = (d_0 - vt_0) - vt_1$

$t_1 = \frac{d_0 - vt_0}{u+v}$

$d_1 = (d_0 - vt_0) - v\left(\frac{d_0 - vt_0}{u+v}\right)$

$= (d_0 - vt_0)\left(1 - \frac{v}{u+v}\right)$

$= (d_0 - vt_0)\left(\frac{u}{u+v}\right)$

$= \left(\frac{du}{u+v} - \frac{dv}{u+v}\right)\left(\frac{u}{u+v}\right)$

$= \frac{du}{u+v}\left(\frac{u-v}{u+v}\right)$

third leg:

$d_3 = \frac{du}{u+v}\left(\frac{u-v}{u+v}\right)^2$

n-th leg:

$d_n = \frac{du}{u+v}\left(\frac{u-v}{u+v}\right)^n$

sum:

$\sum_{n=0}^{\infty} d_n = \sum_{n=0}^{\infty} \frac{du}{u+v}\left(\frac{u-v}{u+v}\right)^n$

$= \frac{du}{u+v} \cdot \left(1 - \frac{u-v}{u+v}\right)^{-1}$

$= \frac{du}{\cancel{u+v}} \cdot \frac{\cancel{u+v}}{2v}$

$\boxed{= \frac{du}{2v}}$

b. $\Delta x = vt$

Trains: Bee's speed: $u$

$v = \frac{d/2}{t}$

$\Rightarrow t = \frac{d}{2v}$

$v = \frac{\cdot\,\cdot}{t}$

$\Rightarrow t = \frac{d}{2v}$

$$\boxed{d_n = \frac{du}{2v}}$$

9) Code at: https://github.com/mchow101/physics-8.012/blob/master/PSet%201.ipynb