Pset 2 Mitali Chowdhury, R03

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2.1) a) $m = 5$ kg

$\vec{F} = (4t^2 \hat{i} - 3t\hat{j})$ N

a) $\vec{F} = ma$

$a = \dfrac{(4t^2\hat{i} - 3t\hat{j})\text{ N}}{5 \text{ kg}}$

$= (\frac{4}{5}t^2\hat{i} - \frac{3}{5}t\hat{j})$ m/s²

$\frac{dv}{dt} = a \Rightarrow \int dv = \int a\,dt$

$v = \int (\frac{4}{5}t^2\hat{i} - \frac{3}{5}t\hat{j})dt$

$= (\frac{4}{15}t^3\hat{i} - \frac{3}{10}t^2\hat{j}) + \cancel{v_0}^{\,0}$

$\boxed{v = (\frac{4}{15}t^3\hat{i} - \frac{3}{10}t^2\hat{j})\text{ m/s}}$

b) $\frac{dr}{dt} = v \Rightarrow \int dr = \int v\,dt$

$r = \int (\frac{4}{15}t^3\hat{i} - \frac{3}{10}t^2\hat{j})dt$

$= \frac{1}{15}t^4\hat{i} - \frac{1}{10}t^3\hat{j} + \cancel{x_0}^{\,0}$

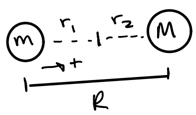
$\boxed{= (\frac{1}{15}t^4\hat{i} - \frac{1}{10}t^3\hat{j})\text{ m}}$

c) $\vec{r} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ t^4/15 & -t^3/10 & 0 \\ 4t^3/15 & -3t^2/10 & 0 \end{vmatrix}$

$= 0\hat{i} - 0\hat{j} + (-\frac{3t^3}{10}\cdot\frac{t^4}{15} + \frac{t^3}{10}\cdot\frac{4t^3}{15})\hat{k}$

$\boxed{= \frac{t^6}{150}\hat{k}\ \text{m}^2/\text{s}}$

2.4)



both particles revolve around the center of mass

$F = ma_1 = m r_1 \omega^2$

$F = -Ma_2 = -Mr_2\omega^2$

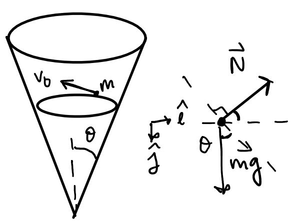
$r_1 = \frac{F}{m\omega^2}$

$r_2 = \frac{F}{M\omega^2}$

$R = r_1 + r_2 = \frac{F}{m\omega^2} + \frac{F}{M\omega^2}$

$\boxed{= \frac{F}{\omega^2}(\frac{1}{m} + \frac{1}{M})}$

2.6)



$F = ma = \vec{N}\cos\theta\,\hat{i} + (mg - \vec{N}\sin\theta)\hat{j}$

$ma = \vec{N}\cos\theta\,\hat{i} + 0\hat{j}$

$m(\cancel{r}\cdot\frac{v_0^2}{r^{\cancel{2}}}) = \frac{mg}{\sin\theta}\cdot\cos\theta$

$\frac{v_0^2}{r} = g\cdot\frac{\cos\theta}{\sin\theta}$

$\boxed{r = \frac{\tan\theta\, v_0^2}{g}}$

only horizontal motion:

$mg - \vec{N}\sin\theta = 0$

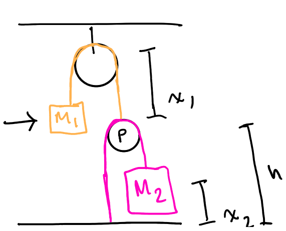
$\vec{N} = \frac{mg}{\sin\theta}$

$a = r\alpha = r\omega^2$

$\omega^2 = (\frac{v}{r})^2$

2.8)

find acc →



$l_1 = \pi r + 2x_1$

$\pi r + 2x_1 = (x_1 + \Delta x_1) + (x_1 - \Delta x_1)$

$l_2 = h + (h - x_2) + \pi r_p$

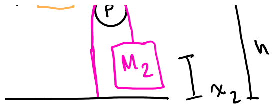
$h + x_2 + \pi r_p = h + \Delta x_1 + (h - x_2) - \Delta x_1$

Since system also rises by x_1,

$-\Delta x_2 = 2\Delta x_1$

$-\dot{x}_2 = 2\dot{x}_1$

$-\ddot{x}_2 = 2\ddot{x}_1$

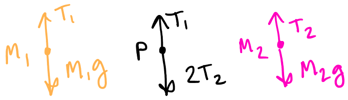
acc 

$l_2 = h + (h - x_2) + \pi r_p$

$h + x_2 + \pi r_p = h + \Delta x_1 + \underbrace{(h - x_2) - \Delta x_1}_{\text{height of } m_2}$

$-\dot{x}_2 = 2\dot{x}_1$

$-\ddot{x}_2 = 2\ddot{x}_1$



upper pulley

$F = ma$

$M_1 g - T_1 = M_1 \ddot{x}_1$

lower pulley

$M_p a_p = 2T_2 - T_1$

$0 = 2T_2 - T_1$

$T_1 = 2T_2$

lower mass

$M_2 a_2 = M_2 g - T_2$

$M_2 \ddot{x}_2 = M_2 g - T_2$

$T_2 = M_2 g - M_2 \ddot{x}_2$

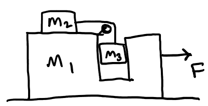
$= m_2 g + m_2 \cdot 2\ddot{x}_1$

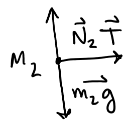
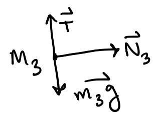
$\to M_1 g - 2T_2 = M_1 \ddot{x}_1$

$T_2 = \frac{m_1 g - m_1 \ddot{x}_1}{2} \to \frac{m_1 g - m_1 \ddot{x}_1}{2} = m_2 g + 2m_2 \ddot{x}_1$

$m_1 g - m_1 \ddot{x}_1 = 2m_2 g + 4m_2 \ddot{x}_1$

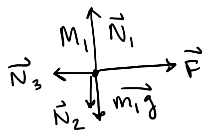
$$\boxed{\frac{m_1 g - 2m_2 g}{m_1 + 4m_2} = \ddot{x}_1}$$

2.13) 

$\Sigma F_y = 0$

$\Sigma F_x = ma$



$M_2 a = T$

$M_2 a = M_3 g \leftarrow T = m_3 g$

$0 = m_3 g - T$

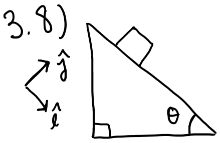
$a = \frac{M_3 g}{M_2}$

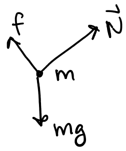
$F = (M_1 + M_2 + M_3) a$

$= M_1 a + M_2 a + M_3 a$

$= a(M_1 + M_2 + M_3)$

$$\boxed{= \frac{M_3 g}{M_2}(M_1 + M_2 + M_3)}$$

3.8) 

a) 

$F = ma = 0$

$F_{\hat{\ell}} = mg\sin\theta - f = 0$

$f = \mu N$

$= \mu \cdot mg\cos\theta$

$mg\sin\theta - f = 0$

$\mu mg\cos\theta = mg\sin\theta$

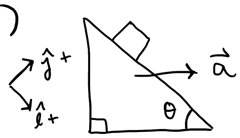
$\mu = \frac{\sin\theta}{\cos\theta}$

$\boxed{\mu = \tan\theta}$

$mg\cos\theta - N = 0 = F_{\hat{j}}$

$mg\cos\theta - N = 0$

$N = mg\cos\theta$

b) 

$F_{\hat{\ell}} = N - mg\cos\theta = ma\sin\theta$

$N = mg\cos\theta + ma\sin\theta$

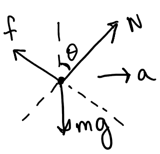
$F_{\hat{j}} = mg\sin\theta - f = ma\cos\theta$

$f = \mu N = mg\sin\theta - ma\cos\theta$

$\mu(mg\cos\theta + ma\sin\theta) = mg\sin\theta - ma\cos\theta$

$\mu mg\cos\theta - mg\sin\theta = -ma\cos\theta - \mu ma\sin\theta$

$a = \frac{\mu mg\cos\theta - mg\sin\theta}{-m\cos\theta - \mu m\sin\theta}$



$$\boxed{a = \frac{g\sin\theta - \mu g\cos\theta}{\cos\theta + \mu\sin\theta}}$$

c) $F_{\hat{\ell}} = N - mg\cos\theta = ma\sin\theta$

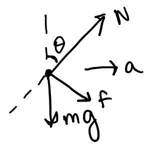
$N = mg\cos\theta + ma\sin\theta$

$F_{\hat{j}} = mg\sin\theta + f = ma\cos\theta$

$f = \mu N = ma\cos\theta - mg\sin\theta$

c) $F_{\hat{r}} = N - mg\cos\theta = ma\sin\theta$

$N = mg\cos\theta + ma\sin\theta$



$F_{\hat{j}} = mg\sin\theta + f = ma\cos\theta$

$f = \mu N = ma\cos\theta - mg\sin\theta$

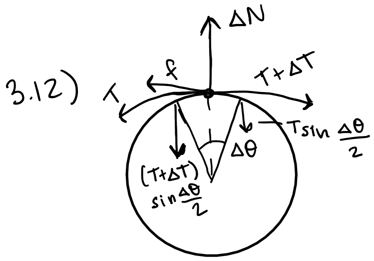
$\mu(mg\cos\theta + ma\sin\theta) = ma\cos\theta - mg\sin\theta$

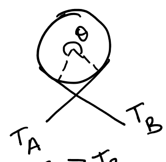
$\mu mg\cos\theta + mg\sin\theta = ma\cos\theta - \mu ma\sin\theta$

$$a = \frac{\mu mg\cos\theta + mg\sin\theta}{m\cos\theta - \mu m\sin\theta}$$

$$\boxed{a = \frac{\mu g\cos\theta + g\sin\theta}{\cos\theta - \mu\sin\theta}}$$

3.12)





$T_A > T_B$

$F_y = 0 = \Delta N - (T+\Delta T)\sin\frac{\Delta\theta}{2} + T\sin\frac{\Delta\theta}{2}$

$\Delta N = (T+\Delta T)\sin\frac{\Delta\theta}{2} + T\sin\frac{\Delta\theta}{2}$

small angle approximation:

$\Delta N = (T+\Delta T)\frac{\Delta\theta}{2} + T\frac{\Delta\theta}{2}$

$\Delta N = T\Delta\theta + \frac{\Delta T\Delta\theta}{2}$

when $\Delta T \to 0$

$\Delta N = T\Delta\theta$

$(f+T) - (T+\Delta T) = 0 = F_x$

$\mu\Delta N - \Delta T = 0$

$\mu T\Delta\theta = \Delta T$

$\frac{\Delta T}{\Delta\theta} = \mu T$

as $\Delta T \to 0$, $\Delta\theta \to 0$

$$\frac{dT}{d\theta} = \mu T$$

Solving diff eq:

$\frac{dT}{T} = \mu d\theta$

$\int_{T_B}^{T_A} \frac{dT}{T} = \int_0^\theta \mu d\theta$

$\ln|T|\Big|_{T_B}^{T_A} = \mu\theta\Big|_0^\theta$

$\ln|T_A| - \ln|T_B| = \mu\theta$

$\ln|T_B| = \ln|T_A| - \mu\theta$

$$\boxed{T_B = T_A e^{-\mu\theta}}$$

8) a) $m\ddot{x} + kx + bv = 0$

$$\begin{cases} \frac{dx}{dt} = v \\ \frac{dv}{dt} = \frac{-kx - bv}{m} \end{cases}$$

$$\frac{d^2v}{dt} = \frac{-kv - b\dot{v}}{m}$$

$$= \frac{-kv - b\left(\frac{-kx - bv}{m}\right)}{m}$$

$$= \frac{-kvm + kbx + b^2v}{m^2}$$

$x(t+\delta t) = x(\delta t) + x'(t-\delta t) + \frac{x''}{2}(t-\delta t)^2$

$= x(\delta t) + v(t-\delta t) - \frac{kx - bv}{2}(t-\delta t)^2$

$v(t+\delta t) = v(\delta t) + v'(\delta t) + \frac{v''}{2}(\delta t)^2$

$= v(\delta t) - \frac{kx - bv}{2}(\delta t) + \frac{kbx + b^2v - kvm}{m^2}(\delta t)^2$

b) https://github.com/mchow101/physics-8.012/blob/master/Pset%202.ipynb