

# 1 Gbit/s Ethernet Transceiver - Fully Differential OTA

Due Date 2018, Jan 8, 4 PM

## Motivation

In the previous homework, the top level specification of the amplifier were derived which can be used in a 1000Base-T Ethernet transceiver shown Figure 1. The specification are summarized as in Table 1.

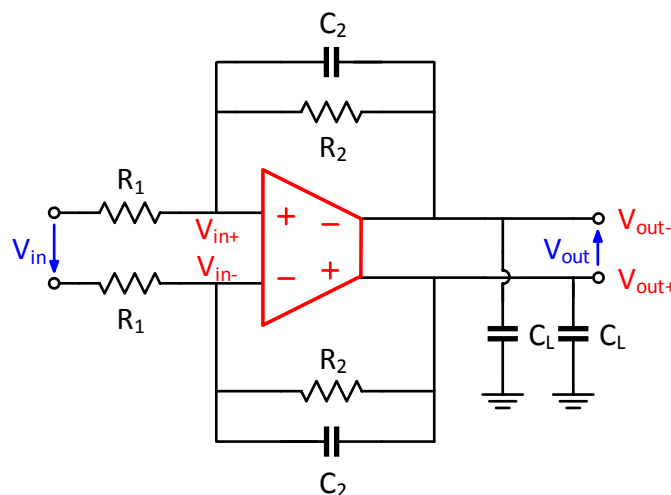


Figure 1: Fully differential amplifier deployed in an Anti-aliasing filter and inverting amplifier.

In this homework, the goal is to study the architecture of the amplifier that meets the specifications. First, the small signal model of the whole architecture is studied to analyze the frequency behavior of the amplifier.

In practicum 2, the dimensions of the transistors are calculated and then simulated to verify the specifications.

Table 1: Amplifier specifications.

$V_{DD}$	1.5 V
$A_{v0}$	$\geq 1000 \frac{V}{V}$ or 60 dB
$C_L$	1 pF
$\omega_0$	$2\pi 250 \frac{\text{krad}}{\text{s}}$
$\omega_u$	$\geq 2\pi 250 \frac{\text{Mrad}}{\text{s}}$
$V_{\text{out-diff,swing}}$	$\geq 1 \text{ V}$
$R_1$	7 k $\Omega$
$R_2$	14 k $\Omega$
$C_2$	175 fF
SR	$\geq 250 \frac{\text{MV}}{\text{s}}$
PM	$\geq 60^\circ$
$I_b$	100 $\mu\text{A}$

### amplifier Architecture

The architecture of the amplifier without the CMFB and biasing circuit is shown in Figure 2. The amplifier is a fully differential two stage telescopic RC compensated amplifier. Since the circuit is fully symmetric, the MOSFET pairs are subscripted with  $M_{iL}$  and  $M_{iR}$  depending on their location in the circuit. A parameter, for example for  $g_{m1}$ , means that it's equal to the properties within  $g_{m1L}$  and  $g_{m1R}$ .

The circuit for the biasing and the CMFB is not included here. Noted that, if the amplifier is simulated in an open loop configuration, the feedback network might be placed at the output. For simplicity,  $Z_L$  is modeled with  $C_L$  and  $R_2$ ,  $R_1$  and  $C_2$  are ignored.

Noted that, from now on  $Z_L = \frac{1}{sC_L}$  and  $C_L = 1 \text{ pF}$ .

### Exercise 1: Small Signal Model

(15P)

- a) The small signal model of the two-stage amplifier is shown in Figure 3. Noted that for the simplicity, you may ignore the effect of  $C_{DS5}$  since it's much smaller than  $C_C$ .  $R_A$  and  $R_B$  represent the output resistance of the first stage and the second stage respectively.  $C_A$  and  $C_B$  represent the sum of all capacitors at the node of A and B respectively. (5P)

Determine the values of  $R_A$ ,  $R_B$ ,  $C_A$  and  $C_B$  with respect to  $M_1$  to  $M_6$ .

**Note:** Do not include  $C_C$  with miller effect in  $C_A$  and  $C_B$ . However, include all parasitic capacitors at node A and B, as well as the load capacitor.

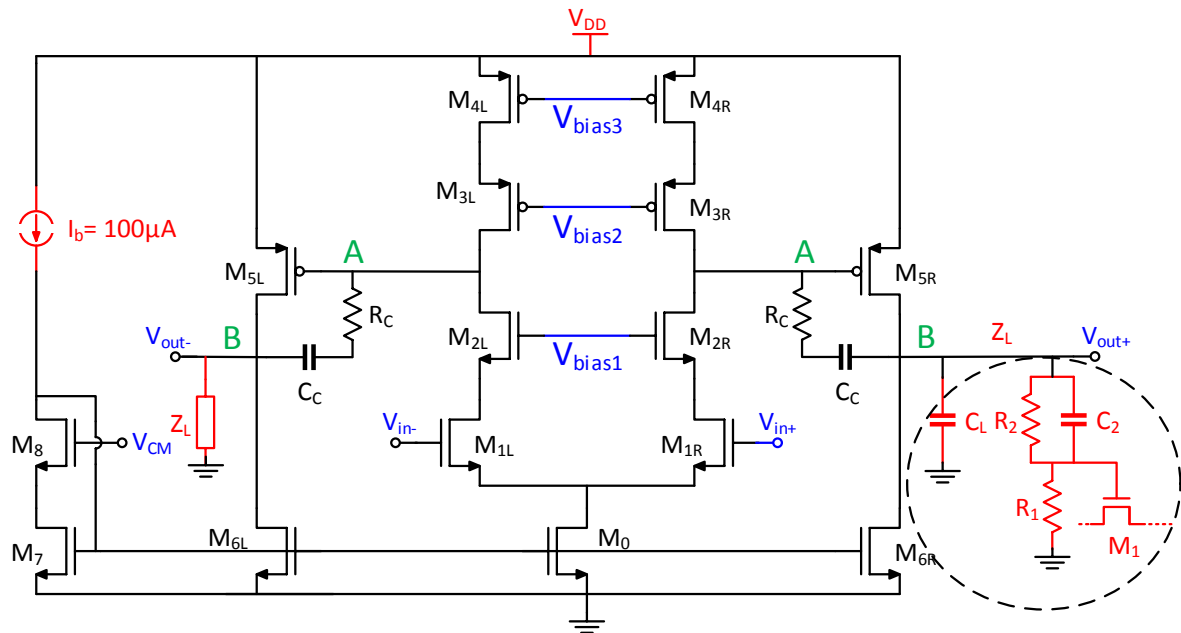


Figure 2: amplifier circuit schematic. It is a fully differential telescopic two stage amplifier with RC compensation. The load of the amplifier is an impedance defined as  $Z_L$ . In the schematic, components colored *red* are fixed by the design and nodes colored *blue* will be connected to the biasing circuitry or CMFB.

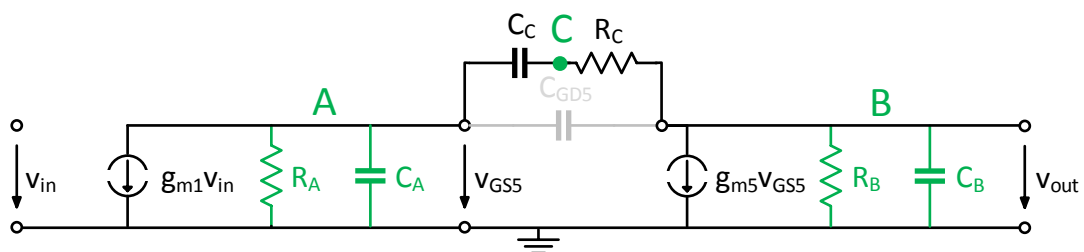


Figure 3: Small signal model of the two-stage amplifier shown in Figure 2.

- b) Determine  $\frac{V_{out}(s)}{V_{in}(s)}$  by applying KCL at node A, B and C. Bring it to the general form

$$\frac{V_{out}}{V_{in}} = \frac{b_1 s + b_0}{a_3 s^3 + a_2 s^2 + a_1 s + 1}.$$

Determine  $a_1$ ,  $a_2$ ,  $a_3$ ,  $b_0$  and  $b_1$  in terms of  $g_{m1}$ ,  $R_A$ ,  $C_A$ ,  $g_{m5}$ ,  $R_B$ ,  $C_B$ ,  $C_C$  and  $R_C$ .

(10P)

## Exercise 2: Determination of Poles and Zeros (15P)

- a) How many poles and zeros does the transfer function exhibit? (2.5P)
- b) Determine the zero of the transfer function. (2.5P)
- c) Determine the poles of the transfer function using the following approximation for widely spaced poles: (5P)

$$\begin{aligned} & \left(1 + \frac{s}{\omega_{P1}}\right) \cdot \left(1 + \frac{s}{\omega_{P2}}\right) \cdot \left(1 + \frac{s}{\omega_{P3}}\right) \\ &= 1 + \left(\frac{1}{\omega_{P1}} + \frac{1}{\omega_{P2}} + \frac{1}{\omega_{P3}}\right) \cdot s + \left(\frac{1}{\omega_{P1}\omega_{P2}} + \frac{1}{\omega_{P1}\omega_{P3}} + \frac{1}{\omega_{P2}\omega_{P3}}\right) s^2 + \frac{1}{\omega_{P1}\omega_{P2}\omega_{P3}} s^3 \\ & \omega_{P3} \gg \omega_{P2} \gg \omega_{P1} : \\ & \approx 1 + \frac{1}{\omega_{P1}} s + \frac{1}{\omega_{P1}\omega_{P2}} s^2 + \frac{1}{\omega_{P1}\omega_{P2}\omega_{P3}} s^3 \end{aligned}$$

- d) Simplify  $\omega_{P1}$ ,  $\omega_{P2}$  and  $\omega_{P3}$ . (5P)

**Note 1:** You may approximate  $C_B$  with  $C_L$ , since the parasitic capacitors of the transistors are much smaller than the load capacitor.

**Note 2:**  $C_L > C_C \gg C_A$ ,  $R_A > R_B \gg R_C$

## Exercise 3: Frequency Analysis: Single Pole (30P)

- a) Calculate the magnitude and phase of the transfer function: (10P)

$$A_v(s) = \frac{A_{v0}}{1 + \frac{s}{\omega_0}}$$

- b) Sketch the magnitude and phase for  $A_{v0} = 100 \frac{V}{V}$  and  $\omega_0 = 10 \frac{\text{Mrad}}{s}$ . (Matlab, Mathematica, ...) (5P)
- c) Consider your sketch. At which frequencies does the pole approximately start and stop to modify the phase, phase having approx.  $5^\circ$  shift? (5P)
- d) What is the value of the magnitude and the phase for  $s = -j\omega_0$ ? (5P)
- e) Calculate the unity-gain frequency  $\omega_u$ , i.e.,  $|A_v(j\omega)| = 1$ . Simplify your result. Assume that  $A_{v0}$  is very large. (5P)

## Exercise 4: Frequency Analysis: Positive Zero (15P)

- a) Calculate the magnitude and phase of the transfer function : (5P)

$$A_v(s) = A_{v0} \left(1 - \frac{s}{\omega_Z}\right)$$

- b) Sketch the magnitude and phase for  $A_{v0} = 100 \frac{V}{V}$  and  $\omega_Z = 10 \frac{\text{Mrad}}{s}$ . (Matlab, Mathematica,...) (5P)
- c) Consider your sketch. At which frequencies does the zero approximately start and stop to modify the phase, having approx.  $5^\circ$  shift? (5P)

## Exercise 5: Two Stage Amplifier (15P)

Assume the small signal gain of the two-stage amplifier given by (ignoring the third pole):

$$A_v(s) = \frac{A_{v0}}{\left(1 + \frac{s}{\omega_{P1}}\right)} \cdot \frac{\left(1 - \frac{s}{\omega_Z}\right)}{\left(1 + \frac{s}{\omega_{P2}}\right)}$$

- a) Apply your results from Exercise 3 & 4 in order to explain in a few words why the unity gain frequency  $\omega_u$  is determined by  $\omega_{P1}$ , assuming that  $\omega_{P2} \geq 3\omega_u$  and  $\omega_Z \geq 10\omega_u$ . Determine  $\omega_u$  in terms of  $A_{v0}$  and  $\omega_{P1}$ . (5P)
- b) Determine the location of  $\omega_{P2}$  in multiples of  $\omega_u$  so that the phase margin of  $A_v(s)$  equals  $45^\circ$ ,  $60^\circ$  and  $75^\circ$  respectively. Assume that  $\omega_Z = 10\omega_u$  and  $A_{v0}$  is really large. (5P)
- c) Explain why the phase margin will be increased or decreased if the zero ( $\omega_Z$ ) moves from  $10\omega_u$  to  $5\omega_u$ . (5P)

**Exercise 6: Phase Margin and Feedback****(10P)**

- a) Calculate the small signal transfer function of  $H(s) = \frac{V_{out}(s)}{V_{in}(s)}$  of the circuit shown in Figure 4. Assuming a two-pole model for the amplifier: (5P)

$$A_v(s) = \frac{A_{v0}}{\left(1 + \frac{s}{\omega_{P1}}\right) \left(1 + \frac{s}{\omega_{P2}}\right)}$$

- b) How the step response of the circuit based on the calculated transfer function for  $A_{v0} = 10\,000$  and  $\omega_{P2} = 2\pi\,100\,\frac{\text{rad}}{\text{s}}$  and

$$\omega_{P1} = 2\pi\,0.014\,14\,\frac{\text{rad}}{\text{s}}, \quad \text{PM} = 45^\circ$$

$$\omega_{P1} = 2\pi\,0.006\,67\,\frac{\text{rad}}{\text{s}}, \quad \text{PM} = 60^\circ$$

$$\omega_{P1} = 2\pi\,0.001\,79\,\frac{\text{rad}}{\text{s}}, \quad \text{PM} = 75^\circ$$

Explain, what can be observed.

(5P)

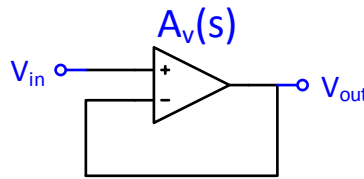


Figure 4: Unity gain buffer.