

# Kern- und Teilchenphysik II Spring Term 2015

## Exercise Sheet 2

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### 1. Eigenspinors of the $S_z$ operator

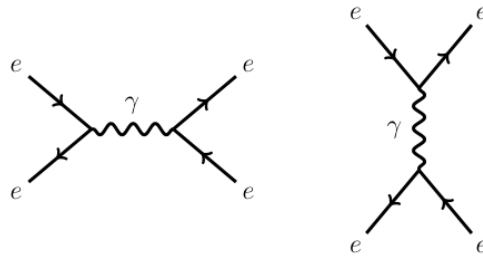
- a) If the  $z$  axis is orientated so that it points along the direction of motion, show that the canonical solution  $u^{(1)}$  reduces to:

$$u^{(1)} = \begin{pmatrix} \sqrt{(E + mc^2)/c} \\ 0 \\ \sqrt{(E - mc^2)/c} \\ 0 \end{pmatrix}$$

- b) Construct the equivalent expressions for  $u^{(2)}$ ,  $v^{(1)}$  and  $v^{(2)}$   
c) Show that they are all eigenspinors of  $S_z$ , and find their eigenvalues

### 2. Electron-muon scattering

Let's consider electron-positron scattering happens by the two diagrams shown below:



- a) Usin Feyman rules please write down the amplitude for the sum od the two processes.  
b) Evaluate the amplitude for electron-positron scattering in the CM system, assuming the  $e^-$  and  $e^+$  approach each other along the  $z$  axis, repel, and return along the  $z$  axis. Assume the initial and final particles all have helicity  $+1$ . [Hint: Use the spinors you calculated in the previous question]

### 3. Casimir's trick

Casimir's trick states that:

$$\sum_{\text{all spins}} [\bar{u}(a)\Gamma_1 u(b)][\bar{u}(a)\Gamma_2 u(b)]^* = \text{Tr}[\Gamma_1(\not{p}_b + m_b c)\bar{\Gamma}_2(\not{p}_a + m_a c)]$$

- a) Find the analogue expression for antiparticles:

$$\sum_{\text{all spins}} [\bar{v}(a)\Gamma_1 v(b)][\bar{v}(a)\Gamma_2 v(b)]^*$$