Kern- und Teilchenphysik II
Spring Term 2015

Exercise Sheet 2

Lecturers: Prof. F. Canelli, Prof. N. Serra
Assistants: Dr. M. Chrzaszcz, Dr. A.de Cosa

1. Eigenspinors of the S_z operator

a) If the z axis is orientated so that it points along the direction of motion, show that the canonical solution $u^{(1)}$ reduces to:

$$u^{(1)} = \begin{pmatrix} \sqrt{(E+mc^2)/c} \\ 0 \\ \sqrt{(E-mc^2)/c} \\ 0 \end{pmatrix}$$

b) Construct the equivalent expressions for $u^{(2)}$, $v^{(1)}$ and $v^{(2)}$

c) Show that they are all eigenspinors of S_z, and find their eigenvalues

2. Electron-muon scattering

The amplitude for electron-muon scattering is given by:

$$\mathcal{M} = -\frac{g_e^2}{(p_1-p_3)^2}[\bar{u}^{(s_3)}(p_3)\gamma^\mu u^{(s_1)}(p_1)][\bar{u}^{(s_4)}(p_4)\gamma_\mu u^{(s_2)}(p_2)]$$

a) Evaluate the amplitude for electron-muon scattering in the CM system, assuming the e and μ approach each other along the z axis, repel, and return along the z axis. Assume the initial and final particles all have helicity $+1$. [Hint: Use the spinors you calculated in the previous question, Answer: $\mathcal{M} = -2g_e^2$]

3. Casimir's trick

Casimir's trick states that:

$$\sum_{\text{all spins}} [\bar{u}(a)\Gamma_1 u(b)][\bar{u}(a)\Gamma_2 u(b)]^* = \mathrm{Tr}[\Gamma_1(\not{p}_b + m_b c)\overline{\Gamma}_2(\not{p}_a + m_a c)]$$

a) Find the analogue expression for antiparticles:

$$\sum_{\text{all spins}} [\bar{v}(a)\Gamma_1 v(b)][\bar{v}(a)\Gamma_2 v(b)]^*$$