Assignment 17 (ELEC 341 L17_BodeDiagram)

Problem 1:

Find analytical expressions for the magnitude and phase response for each $G(s)$ below.

a. $G(s) = \dfrac{1}{s(s+2)(s+4)}$

b. $G(s) = \dfrac{(s+5)}{(s+2)(s+4)}$

For each function in Problem 1, make a plot of the log-magnitude and the phase, using log-frequency in rad/s as the ordinate. Do not use asymptotic approximations.

The graphs need only be plotted approximately using only a few points.

For each function in Problem 1, sketch the Bode asymptotic magnitude and asymptotic phase plots. Compare your results with your answers to Problem 1.

In both (a) and (b), find $20\log M(\omega)$ and $\phi(\omega)$ at $\omega = 20$.

Solution:

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a) $G(j\omega) = \dfrac{1}{(j\omega)(j\omega+2)(j\omega+4)} \rightarrow |G(j\omega)| = \dfrac{1}{\omega\sqrt{\omega^2+4}\sqrt{\omega^2+16}}$

$\angle G(j\omega) = \angle 1 - \{\angle(j\omega) + \angle(j\omega+2) + \angle(j\omega+4)\}$

$= 0^\circ - \{90^\circ + \tan^{-1}(\frac{\omega}{2}) + \tan^{-1}(\frac{\omega}{4})\} \rightarrow$

$\angle G(j\omega) = -90^\circ - \tan^{-1}(\frac{\omega}{2}) - \tan^{-1}(\frac{\omega}{4})$

e.g.: @ $\omega = 20 \rightarrow |G(j\times 20)| = 1.2241\times 10^{-4} \rightarrow$

$20\log_{10}|G(j\times 20)| = -78.24$

$\angle G(j\times 20) = -90^\circ - \tan^{-1}(\frac{20}{2}) - \tan^{-1}(\frac{20}{4}) \rightarrow$

$\angle G(j\times 20) = -252.97^\circ$

b) $G(j\omega) = \dfrac{(j\omega+5)}{(j\omega+2)(j\omega+4)} \rightarrow |G(j\omega)| = \dfrac{\sqrt{\omega^2+25}}{\sqrt{\omega^2+4}\sqrt{\omega^2+16}}$

$\angle G(j\omega) = \angle(j\omega+5) - \{\angle(j\omega+2) + \angle(j\omega+4)\}$

$= \tan^{-1}(\frac{\omega}{5}) - \tan^{-1}(\frac{\omega}{2}) - \tan^{-1}(\frac{\omega}{4})\} \rightarrow$

$\angle G(j\omega) = \tan^{-1}(\frac{\omega}{5}) - \tan^{-1}(\frac{\omega}{2}) - \tan^{-1}(\frac{\omega}{4})$

e.g.: @ $\omega = 20 \rightarrow |G(j\times 20)| = 0.05028 \rightarrow$

$20\log_{10}|G(j\times 20)| = -25.97$

$\angle G(j\times 20) = \tan^{-1}(\frac{20}{5}) - \tan^{-1}(\frac{20}{2}) - \tan^{-1}(\frac{20}{4}) \rightarrow$

$\angle G(j\times 20) = -87.01^\circ$

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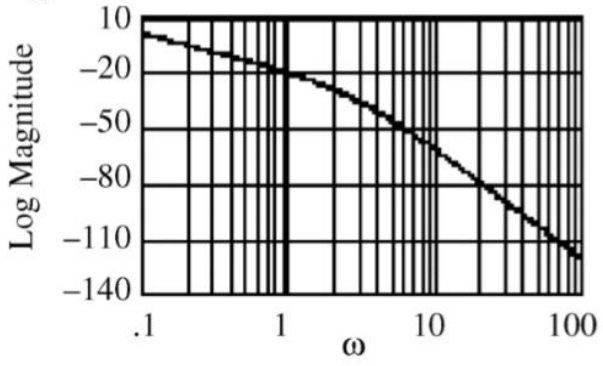
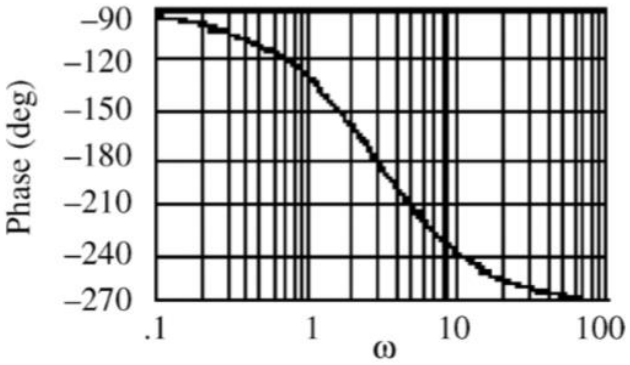
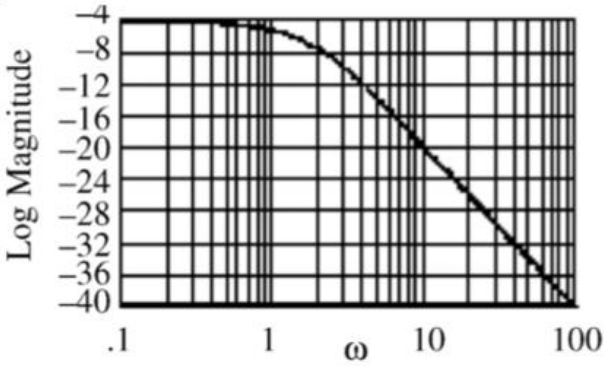
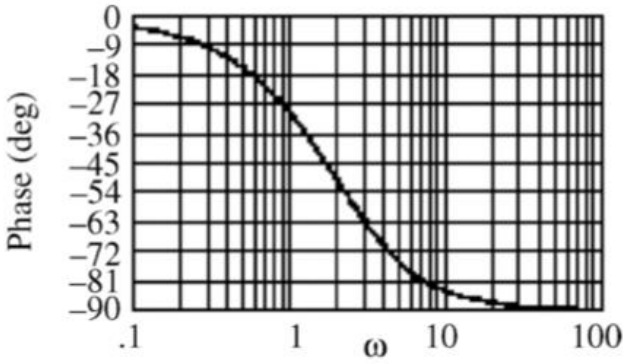
a.





b.





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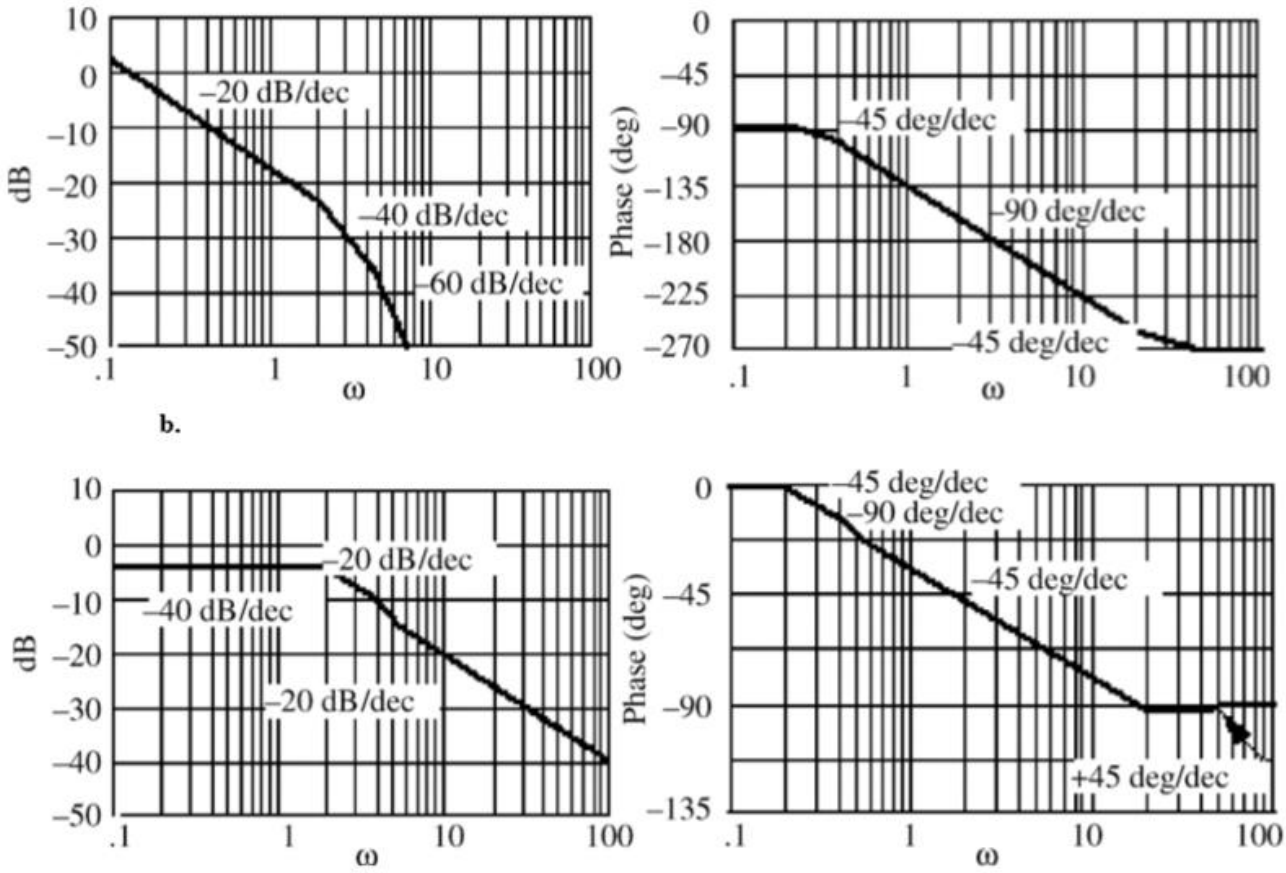

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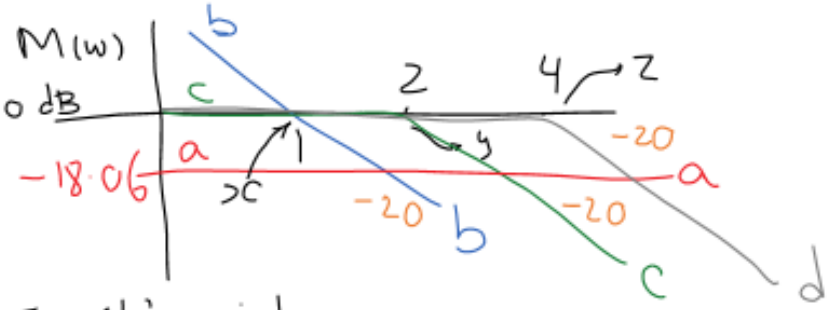
a) $G(s) = \dfrac{1}{s(s+2)(s+4)} = \left(\frac{1}{8}\right)\left(\frac{1}{s}\right)\left(\frac{1}{0.5s+1}\right)\left(\frac{1}{0.25s+1}\right)$

$\rightarrow G(s) = \underbrace{\left(\frac{1}{8}\right)}_{a}\underbrace{\left(\frac{1}{s}\right)}_{b}\underbrace{\left(\frac{1}{0.5s+1}\right)}_{c}\underbrace{\left(\frac{1}{0.25s+1}\right)}_{d}$



For 'b', point x:

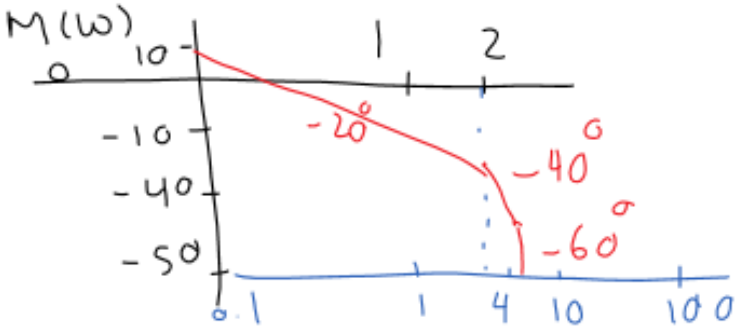
$\frac{1}{s} = \frac{1}{(\frac{1}{1}s+0)} \rightarrow x = \frac{1}{1} = 1$

For 'c', point y:

$\frac{1}{0.5s+1} = \frac{1}{Ts+1} \rightarrow \frac{1}{T} = \frac{1}{0.5} = 2 \rightarrow y = 2$

For 'd', point z:

$\frac{1}{0.25s+1} = \frac{1}{Ts+1} \rightarrow \frac{1}{T} = \frac{1}{0.25} = 4 \rightarrow z = 4$



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