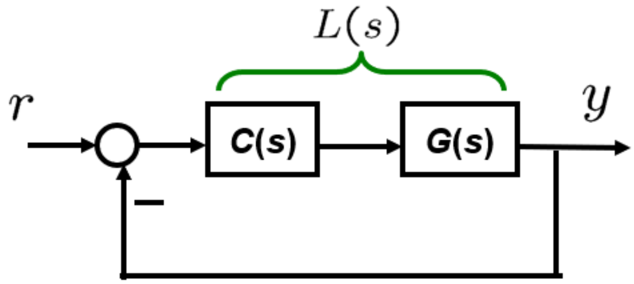
Assignment 20 (ELEC 341 L20_FreqDomSpec)

Problem 1:

Given the unity feedback system below, use frequency response methods to determine the value of gain, $C(s)$, to yield a step response with a 20% overshoot. Use Matlab to graph the step response.



$$G(s) = \frac{(s+4)}{s(s+8)(s+10)(s+15)}$$

Solution:

Assignment 20, problem 1:

$$L(s) = \frac{(s+4)}{s(s+8)(s+10)(s+15)}$$

Find $C(s)$ (gain controller compensator) for $PO = 20\%$

Method 1 (use $PM = 100\zeta$):

$$\zeta = \frac{\left|\ln\frac{PO}{100}\right|}{\sqrt{\pi^2 + (\ln\frac{PO}{100})^2}} = \frac{\left|\ln\frac{20}{100}\right|}{\sqrt{\pi^2 + (\ln\frac{20}{100})^2}} \rightarrow \underline{\zeta = 0.455} \;;\; PM = 100\zeta \rightarrow \underline{PM = 45.59^\circ}$$

We must reduce the PM to 45.59°, i.e., the Bode magnitude must be 0 dB when the Bode plot is: $-180^\circ + PM_{compen.} = -180^\circ + 45.59^\circ = \underline{-134.41^\circ}$

Find ω at which $\angle L(j\omega) = -134.41^\circ \rightarrow$

$$\angle L(j\omega) = \tan^{-1}\left(\frac{\omega}{4}\right) - \left\{\underbrace{\tan^{-1}\left(\frac{\omega}{0}\right)}_{90^\circ} + \tan^{-1}\left(\frac{\omega}{8}\right) + \tan^{-1}\left(\frac{\omega}{10}\right) + \tan^{-1}\left(\frac{\omega}{15}\right)\right\} = -134.41$$

$\rightarrow \underline{\omega = 7.5}$

At $\omega = 7.5 \rightarrow$ gain $= ? \rightarrow |L(j \times 7.5)| = \dfrac{\sqrt{7.5^2 + 4^2}}{7.5\sqrt{7.5^2+8^2}\sqrt{7.5^2+10^2}\sqrt{7.5^2+15^2}}$

$\rightarrow |L(j7.5)| = 4.930 \times 10^{-4}$;

$$C(s) = \frac{1}{|L(j \times 7.5)|} \rightarrow \boxed{C_{compen.} = C(s) \cong 2028}$$

Method 2 ($PM = \tan^{-1}\left\{\dfrac{2\zeta}{\sqrt{-2\zeta^2 + \sqrt{1+4\zeta^4}}}\right\}$):

$\underline{PM = 48.06^\circ}$; $-180^\circ + PM_{compen.} = -180^\circ + 48.06^\circ = \underline{-131.94^\circ} \rightarrow \underline{\omega = 7.14}$

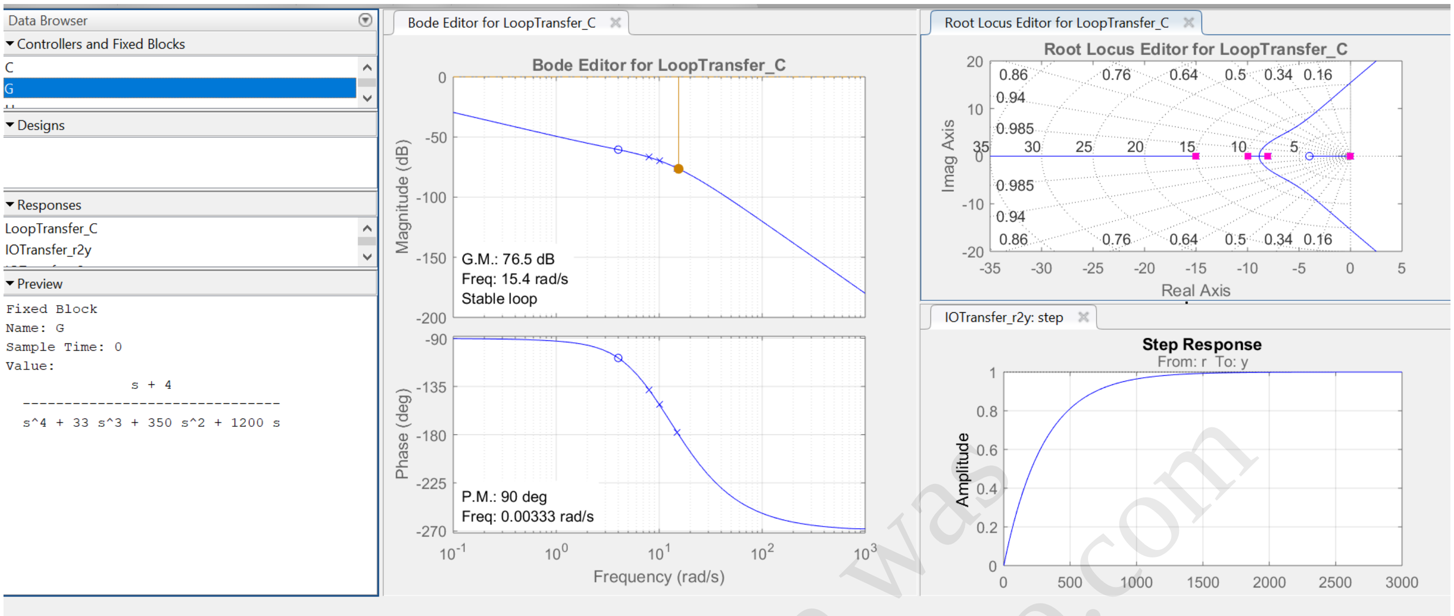
$\rightarrow |L(j7.14)| = 5.236 \times 10^{-4} \rightarrow \boxed{C_{compen.} = C(s) = 1908}$

20% overshoot => $\zeta = 0.456$ => $\phi_M = 48.15°$.

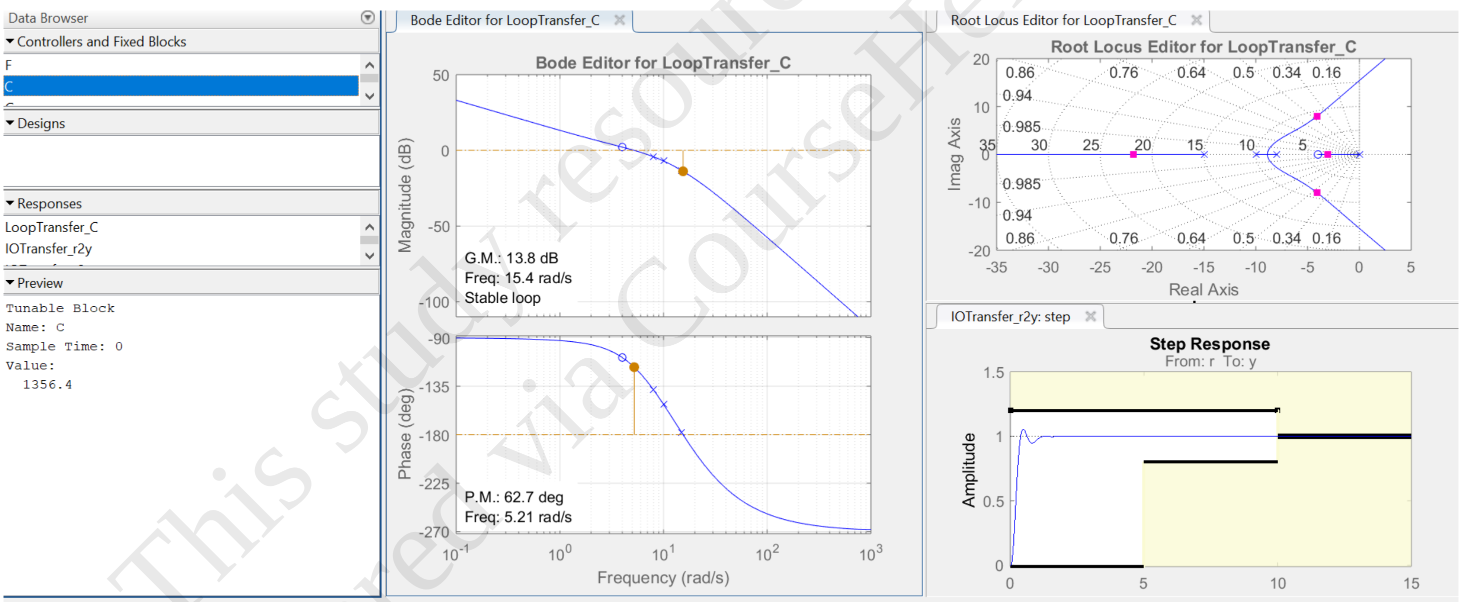
Looking at the phase diagram, where $\phi_M = 48.15°$ (i.e. $\phi = -131.85°$), the phase margin frequency = 7.14 rad/s. At this frequency, the magnitude curve is – 65.6 dB. Thus the magnitude curve has to be raised by 65.6 dB (K = 1905).

Assignment 20 (ELEC 341 L20_FreqDomSpec)

The following graph is for C(s) =1:



If we use root locus design for ζ = 0.455. we obtain the following:



Assignment 20 (ELEC 341 L20_FreqDomSpec)

If I base my design on Bode plot instead of root locus and using Matlab, I will try to put PM at around the requested value of 45.51 degrees (I managed 47.2 deg), then I will get the following graph (which corresponds to C(s) = 1944.1):