**Problem 1:**

Tell how many roots of the following polynomial are in the right half-plane, in the left half-plane, and on the $j\omega$-axis:

$$P(s) = s^5 + 6s^3 + 5s^2 + 8s + 20$$

Use Routh-Hurwitz criterion.

**Solution:**

$$s^5 + 6s^3 + 5s^2 + 8s + 20 = Q(s)$$

$$\rightarrow Q(s) = (1)s^5 + (0)s^4 + (6)s^3 + (5)s^2 + (8)s + 20$$

$$\begin{array}{c|ccc}
s^5 & 1 & 6 & 8 \\
s^4 & \cancel{0} \rightarrow \varepsilon & 5 & 20 \\
s^3 & a & b & \\
s^2 & c & d & \\
s^1 & e & & \\
s^0 & f & &
\end{array}$$

$a = \dfrac{6\varepsilon - 5}{\varepsilon}$; $\lim\limits_{\varepsilon \to 0} a = -\dfrac{5}{\varepsilon}$

$b = \dfrac{8\varepsilon - 20}{\varepsilon}$; $\lim\limits_{\varepsilon \to 0} b = \dfrac{-20}{\varepsilon}$

$c = \dfrac{5a - \varepsilon b}{a} = \dfrac{5(-\frac{5}{\varepsilon}) - \varepsilon(-\frac{20}{\varepsilon})}{-\frac{5}{\varepsilon}}$

large small

$= \dfrac{\frac{25}{\varepsilon} + 20}{-\frac{5}{\varepsilon}} = \dfrac{-\frac{25}{\varepsilon}}{-\frac{5}{\varepsilon}} \rightarrow \lim\limits_{\varepsilon \to 0} c = 5$

$d = \dfrac{20a - \varepsilon \times 0}{a} = 20 \rightarrow$ $d = 20$

summary of what we found so far

$$\begin{array}{c|ccc}
s^5 & 1 & 6 & 8 \\
s^4 & \varepsilon & 5 & 20 \\
s^3 & -\frac{5}{\varepsilon} & -\frac{20}{\varepsilon} & \\
s^2 & 5 & 20 & \\
\text{ROZ} \rightarrow s^1 & \cancel{0} \rightarrow 10 & & \\
s^0 & 20 & &
\end{array}$$

$e = \dfrac{(-\frac{20}{\varepsilon})(5) - 20(-\frac{5}{\varepsilon})}{5}$

$= -\dfrac{20}{\varepsilon} + \dfrac{20}{\varepsilon} = 0 \rightarrow$ $e = 0$

$P(s)$ = auxillary polynomial

$= (5)s^2 + (20)s^0 \rightarrow P(s) = 5s^2 + 20 \rightarrow$

$\hookrightarrow P'(s) = 10 \rightarrow$ replace '0' in ROZ with 10.

Summay:

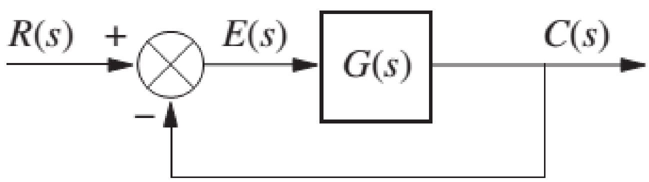
- We have one auxillary polynomial ($p(s)$), where $p(s) = 5s^2 + 20$; $P(s) = 0 \rightarrow$
  $\rightarrow 5s^2 + 20 = 0 \rightarrow s = \pm 2i \Rightarrow$ $\boxed{\text{Two poles on } j\omega\text{-axis.}}$
- We have two sign changes $\left(\begin{array}{l} s^4 \rightarrow s^3 \\ s^3 \rightarrow s^2 \end{array}\right) \rightarrow$ We have two poles in RHP
  $\Rightarrow$ $\boxed{\text{Two poles in RHP.}}$
- Order of chara. eqn. is '5'. $\hookrightarrow$ We have 5 total poles. Therefore, 5-4=1 and we have 1 pole in LHP $\Rightarrow$
  $\Rightarrow$ $\boxed{\text{One pole in LHP.}}$

**Problem 2:**

In the system of Figure , let

$$G(s) = \frac{K(s+2)}{s(s-1)(s+3)}$$

Find the range of $K$ for closed-loop stability.




FIGURE

Use Routh-Hurwitz criterion.

**Solution:**

**Assignment 7 (ELEC 341 L7_RouthHurwitzEx)**

The characteristic equation is:

$$1+K\frac{(s+2)}{s(s-1)(s+3)}=0 \text{ or}$$

$$s(s-1)(s+3)+K(s+2)=0 \text{ or}$$

$$s^3+2s^2+(K-3)s+2K=0$$

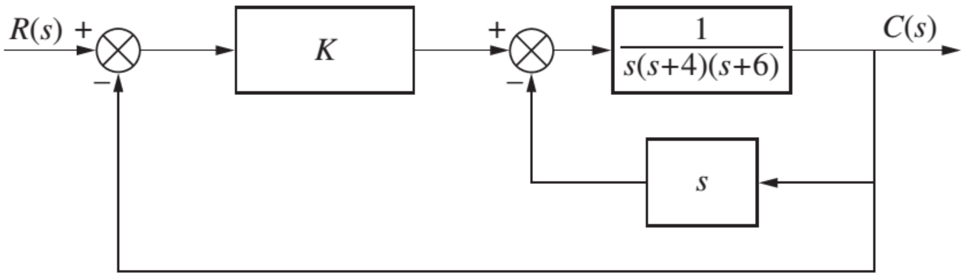
The Routh array is:

| | | |
|---|---|---|
| $s^3$ | 1 | $K-3$ |
| $s^2$ | 2 | $2K$ |
| s | $-3$ | |
| 1 | $2K$ | |

The first column will always have a sign change regardless of the value of $K$. There is no value of $K$ that will stabilize this system.

**Problem 3:**

For the system shown in Figure, find the value of gain, $K$, that will make the system oscillate. Also, find the frequency of oscillation.




**FIGURE**

Use Routh-Hurwitz criterion.

**Solution:**

Applying the feedback formula on the inner loop and multiplying by K yields

$$G_e = \frac{K}{s(s^2+10s+25)}$$

Thus,

$$T = \frac{K}{s^3+10s^2+25s+K}$$

Making a Routh table:

| | | |
|---|---|---|
| $s^3$ | 1 | 25 |
| $s^2$ | 10 | K |
| $s^1$ | $\frac{250-K}{10}$ | 0 |
| $s^0$ | K | 0 |

For oscillation, the $s^1$ row must be a row of zeros. Thus, $K = 250$ will make the system oscillate. The previous row now becomes: $10s^2 + 250$. Thus, $s^2 + 25 = 0$, or $s = \pm\sqrt{-25} = \pm j5$. Hence, the frequency of oscillation is 5 rad/s.