ELEC 341 Assignment #1

Summer 2019

Group 9

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Assignment 1 (ELEC 341 L1_Introduction)

Problem 1:

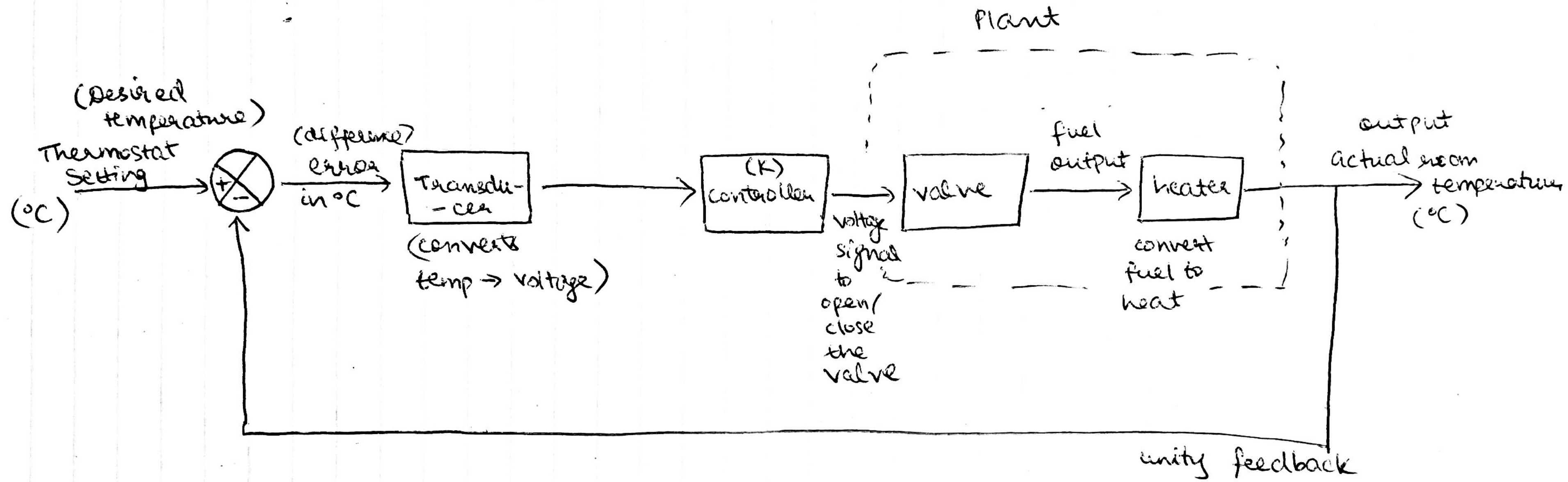
A temperature control system operates by sensing the difference between the thermostat setting and the actual temperature and then opening a fuel valve an amount proportional to this difference. Draw a functional closed-loop block diagram identifying the input and output transducers, the controller, and the plant. Further, identify the input and output signals of all subsystems.

Problem 2:

During a medical operation an anesthesiologist controls the depth of unconsciousness by controlling the concentration of isoflurane in a vaporized mixture with oxygen and nitrous oxide. The depth of anesthesia is measured by the patient's blood pressure. The anesthesiologist also regulates ventilation, fluid balance, and the administration of other drugs. In order to free the anesthesiologist to devote more time to the latter tasks, and in the interest of the patient's safety, we wish to automate the depth of anesthesia by automating the control of isoflurane concentration. Draw a functional block diagram of the system showing pertinent signals and subsystems.

Assignment 1

1)

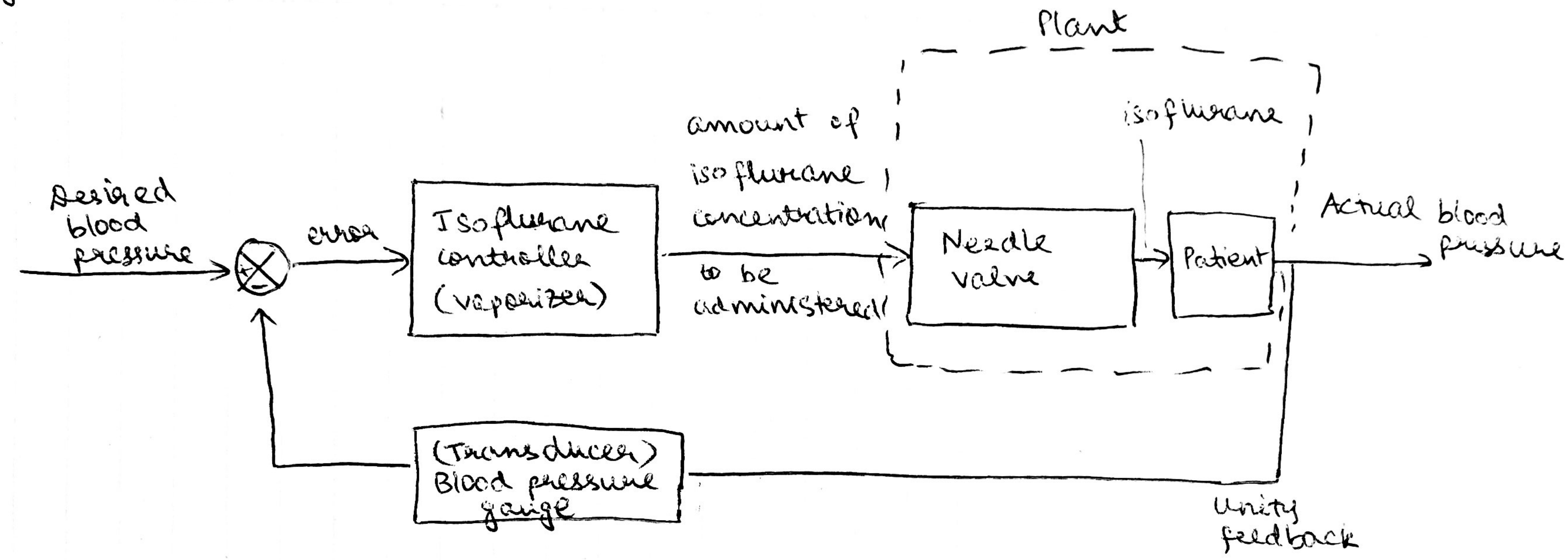


Input: Desired temperature (Thermostat Setting)

Output: Actual room temperature

Assignment 1

2)



Input: Desired blood pressure

Output: Actual blood pressure

The controller is used to automate the concentration of isoflurane which in return automates the depth of anesthesia (or unconsciousness)

Assignment 2 (ELEC 341 L2_LaplaceTransform)

Problem 1:

Find the inverse Laplace transform of the following function using tabulated Laplace transform pairs:

$F_1(s) = 1/(s+3)^2$

TABLE 1 Laplace transform table

Item no.	$f(t)$	$F(s)$
1.	$\delta(t)$	1
2.	$u(t)$	$\frac{1}{s}$
3.	$tu(t)$	$\frac{1}{s^2}$
4.	$t^n u(t)$	$\frac{n!}{s^{n+1}}$
5.	$e^{-at}u(t)$	$\frac{1}{s+a}$
6.	$\sin \omega t u(t)$	$\frac{\omega}{s^2+\omega^2}$
7.	$\cos \omega t u(t)$	$\frac{s}{s^2+\omega^2}$

TABLE 2 Laplace transform theorems

Item no.	Theorem		Name
1.	$\mathscr{L}[f(t)] = F(s)$	$= \int_{0-}^{\infty} f(t)e^{-st}dt$	Definition
2.	$\mathscr{L}[kf(t)]$	$= kF(s)$	Linearity theorem
3.	$\mathscr{L}[f_1(t) + f_2(t)]$	$= F_1(s) + F_2(s)$	Linearity theorem
4.	$\mathscr{L}[e^{-at}f(t)]$	$= F(s+a)$	Frequency shift theorem
5.	$\mathscr{L}[f(t-T)]$	$= e^{-sT}F(s)$	Time shift theorem
6.	$\mathscr{L}[f(at)]$	$= \frac{1}{a}F\left(\frac{s}{a}\right)$	Scaling theorem
7.	$\mathscr{L}\left[\frac{df}{dt}\right]$	$= sF(s) - f(0-)$	Differentiation theorem
8.	$\mathscr{L}\left[\frac{d^2f}{dt^2}\right]$	$= s^2F(s) - sf(0-) - f'(0-)$	Differentiation theorem
9.	$\mathscr{L}\left[\frac{d^nf}{dt^n}\right]$	$= s^nF(s) - \sum_{k=1}^{n} s^{n-k}f^{k-1}(0-)$	Differentiation theorem
10.	$\mathscr{L}\left[\int_{0-}^{t} f(\tau)d\tau\right]$	$= \frac{F(s)}{s}$	Integration theorem
11.	$f(\infty)$	$= \lim_{s\to 0} sF(s)$	Final value theorem[1]
12.	$f(0+)$	$= \lim_{s\to\infty} sF(s)$	Initial value theorem[2]

[1]For this theorem to yield correct finite results, all roots of the denominator of $F(s)$ must have negative real parts, and no more than one can be at the origin.
[2]For this theorem to be valid, $f(t)$ must be continuous or have a step discontinuity at $t = 0$ (that is, no impulses or their derivatives at $t = 0$).

Problem 2:

Find the final value of $f(t)$ for the given $F(s)$ without calculating explicitly $f(t)$

$$F(s) = \frac{2s + 51}{47s^2 + 67s}$$

Assignment 2

1. $F_1(S) = \frac{1}{(S+3)^2}$ $\quad \mathcal{L}^{-1}\{F_1(S)\} = te^{-3t}u(t)$ Frequency shift theorem

2. $F(S) = \frac{2S+51}{47S^2+67S}$

Final Value Thm

$SF(S) = \frac{2S^2+51S}{47S^2+67S}$

$= \frac{S}{S}\left(\frac{2S+51}{47S+67}\right)$

$\hookrightarrow S = -\frac{67}{47}$

open left hand plane

$\therefore$ use FVT $\Rightarrow \lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s)$

$= \lim_{s\to 0} \frac{2S+51}{47S+67}$

$= \frac{51}{67}$

Assignment 3 (ELEC 341 L3_ODESolution)

Problem 1:

Using Laplace transforms, solve the following differential equations assuming zero initial conditions.

a. $\dfrac{dx}{dt} + 7x = 5\cos 2t$

b. $\dfrac{d^2x}{dt^2} + 6\dfrac{dx}{dt} + 8x = 5\sin 3t$

c. $\dfrac{d^2x}{dt^2} + 8\dfrac{dx}{dt} + 25x = 10u(t)$

Assume that the forcing functions are zero prior to *t* = 0-.

Assignment 3 (ELEC 341 L3_ODESolution)

Problem 2:

For each of the following transfer functions, write the corresponding differential equation:

a. $\dfrac{X(s)}{F(s)} = \dfrac{7}{s^2 + 5s + 10}$

b. $\dfrac{X(s)}{F(s)} = \dfrac{15}{(s + 10)(s + 11)}$

c. $\dfrac{X(s)}{F(s)} = \dfrac{s + 3}{s^3 + 11s^2 + 12s + 18}$

Assignment 3

1. a) $x' + 7x = 5\cos(2t)$

$$sX(s) + 7X(s) = \frac{5s}{s^2+2^2}$$

$$X(s) = \frac{5s}{(s^2+4)(s+7)}$$

Partial Fraction $\Rightarrow$ $\frac{5s}{(s+7)(s^2+4)} = \frac{A}{s+7} + \frac{Bs+C}{s^2+4}$

$$5s = A(s^2+4) + (Bs+C)(s+7)$$

$$5s = As^2 + 4A + Bs^2 + Cs + 7Bs + 7C$$

$5 = C + 7B$ (1) $\quad 5 = C - 7A$ (4)

$0 = A + B$ (2) $\quad A = -B$

$0 = 4A + 7C$ (3)

$$5 = -7A + C$$
$$0 = 4A + 7C$$

$7(4) - (3) \Rightarrow 35 = -53A$

$A = -35/53$

$B = 35/53$

$C = 5 - 7B = 20/53$

$$X(s) = \left[\frac{-35/53}{s+7} + \frac{35/53\ s}{s^2+2^2} + \frac{20/53}{s^2+2^2}\right]$$

$$x(t) = \left[-\frac{35}{53}e^{-7t} + \frac{35}{53}\cos(2t) + \frac{10}{53}\sin(2t)\right]u(t)$$

1.(b) $x'' + 6x' + 8x = 5\sin(3t)$

$$s^2X(s) + 6sX(s) + 8X(s) = 5\,\frac{3}{s^2+9}$$

$$X(s) = \frac{15}{(s^2+9)(s^2+6s+8)} = \frac{15}{(s^2+9)(s+4)(s+2)}$$

$$\frac{15}{(s^2+9)(s+4)(s+2)} = \frac{A}{s+4} + \frac{B}{s+2} + \frac{Cs}{s^2+9} + \frac{D}{s^2+9}$$

$$15 = A(s+2)(s^2+9) + B(s+4)(s^2+9) + (Cs+D)(s^2+6s+8)$$

$$15 = A(s^3+9s+2s^2+18) + B(s^3+9s+4s^2+36) + Cs^3 + 6Cs^2 + 8Cs + Ds^2 + 6Ds + 8D$$

$$15 = As^3 + 2As^2 + 9As + 18A + Bs^3 + 4Bs^2 + 9Bs + 36B + Cs^3 + (6C+D)s^2 + (8C+6D) + 8D$$

$$\begin{cases} A+B+C = 0 \\ 2A+4B+6C+D = 0 \\ 9A+9B+8C+6D = 0 \\ 18A+36B+8D = 15 \end{cases}$$

$\therefore$
$$\begin{cases} A = -\frac{3}{10} \\ B = \frac{15}{26} \\ C = -\frac{18}{65} \\ D = -\frac{3}{65} \end{cases}$$

$$\therefore X(s) = -\frac{3}{10}\left(\frac{1}{s+4}\right) + \frac{15}{26}\left(\frac{1}{s+2}\right) - \frac{18}{65}\left(\frac{s}{s^2+9}\right) - \frac{3}{65}\left(\frac{1}{s^2+9}\right)$$

$$\therefore x(t) = \mathcal{L}^{-1}(X(s)) = -\frac{3}{10}e^{-4t} + \frac{15}{26}e^{-2t} - \frac{18}{65}\cos(3t) - \frac{1}{65}\sin(3t)$$

1. (c) $\frac{d^2x}{dt^2} + 8\frac{dx}{dt} + 25x = 10\,u(t)$

$$\mathcal{L}\left(\frac{d^2x}{dt^2} + 8\frac{dx}{dt} + 25x\right) = s^2X(s) + 8sX(s) + 25X(s)$$
$$= (s^2+8s+25)X(s)$$

$$\mathcal{L}(10\,u(t)) = \frac{10}{s}$$

$\therefore (s^2+8s+25)X(s) = \frac{10}{s}$

$$X(s) = \frac{10}{s(s^2+8s+25)} = \frac{A}{s} + \frac{Bs+C}{s^2+8s+25}$$

$\therefore As^2 + 8As + 25A + Bs^2 + Cs = 10$

$\therefore \begin{cases} A + B = 0 \\ 8A + C = 0 \\ 25A = 10 \end{cases}$ $\therefore$ $A = \frac{2}{5}$, $B = -\frac{2}{5}$, $C = -\frac{16}{5}$

$$\therefore X(s) = \frac{2}{5}\cdot\frac{1}{s} - \frac{2}{5}\,\frac{s+8}{s^2+8s+25}$$

$$= \frac{2}{5}\cdot\frac{1}{s} - \frac{2}{5}\,\frac{s+8}{s^2+8s+16+9} = \frac{2}{5}\cdot\frac{1}{s} - \frac{2}{5}\cdot\frac{s+8}{(s+4)^2+9}$$

$$= \frac{2}{5}\cdot\frac{1}{s} - \frac{2}{5}\cdot\left(\frac{s}{(s+4)^2+9} + \frac{8}{3}\cdot\frac{3}{(s+4)^2+9}\right)$$

$\therefore x(t) = \mathcal{L}^{-1}(X(s))$

$$= \frac{2}{5} - \frac{2}{5}e^{-4t}\left(\cos(3t) + \frac{8}{3}\sin(3t)\right)$$

Assignment 3

2 a.) $$\frac{X(s)}{F(s)} = \frac{7}{s^2+5s+10}$$

$$\Rightarrow (s^2+5s+10)X(s) = 7F(s)$$

$$= s^2X(s) + 5sX(s) + 10X(s) = 7F(s)$$

$$= x'' + 5x' + 10x = 7f$$

$$= \frac{d^2x}{dt} + \frac{5dx}{dt} + 10x = 7f$$

b.) $$\frac{X(s)}{F(s)} = \frac{15}{(s+10)(s+11)}$$

$$\Rightarrow (s^2+21s+110)X(s) = 15 \cdot F(s)$$

$$= \frac{d^2x}{dt} + 21\frac{dx}{dt} + 110x = 15f$$

c.) $$\frac{X(s)}{F(s)} = \frac{s+3}{s^3+11s^2+12s+18}$$

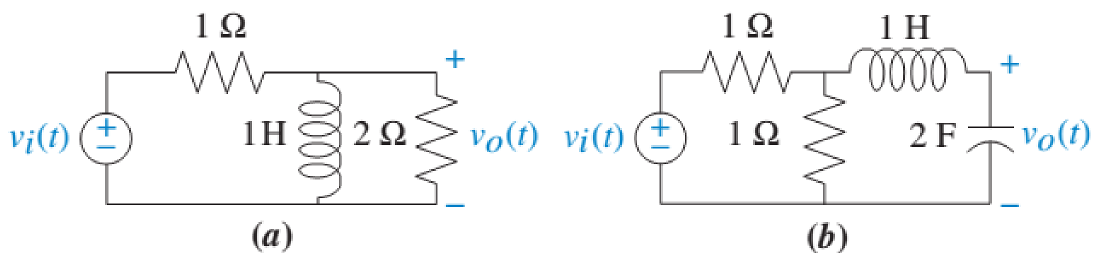
$$= (s^3+11s^2+12s+18)X(s) = (s+3)F(s)$$

$$= \frac{d^3x}{dt} + 11\frac{d^2x}{dt} + 12\frac{dx}{dt} + 18x = \frac{df}{dt} + 3f$$

Assignment 4 (ELEC 341 L4_Model_ElecMech)

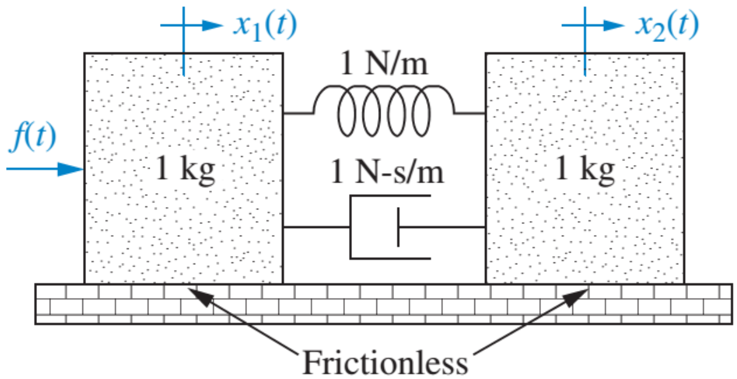
Problem 1:

Find the transfer function, $G(s) = V_o(s)/V_i(s)$ for each network shown in the following figure:



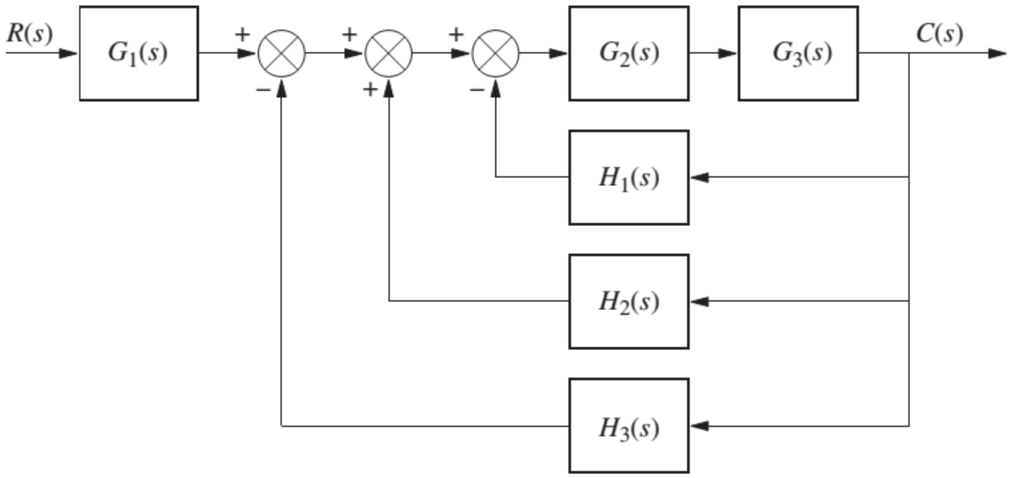
Problem 2:

Find the transfer function, $G(s) = X_2(s)/F(s)$, for the translational mechanical network shown in figure:



Problem 3:

Reduce the block diagram shown in figure to a single transfer function:



Problem 4:

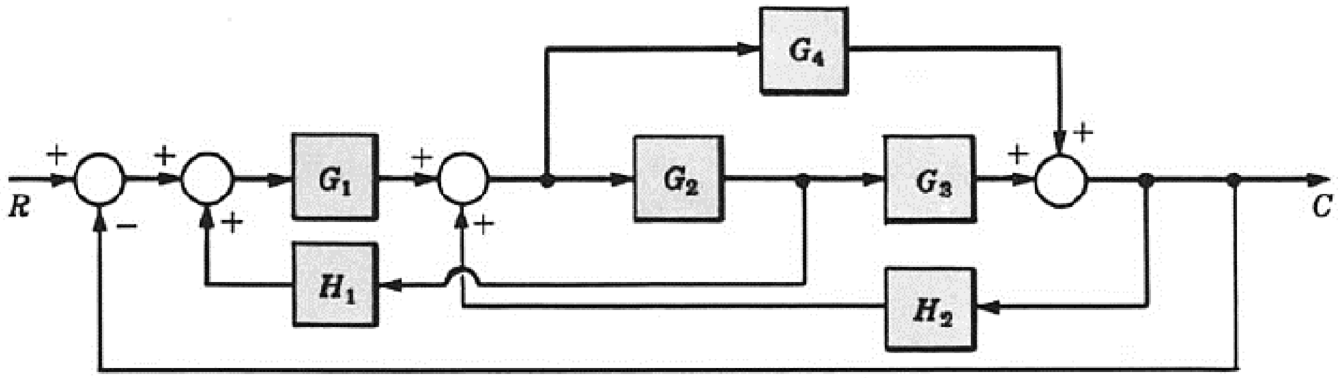
Calculate the transfer function of the system represented in the state space model as

$$\dot{X} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -1 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} [u]$$

$$Y = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Problem 5:

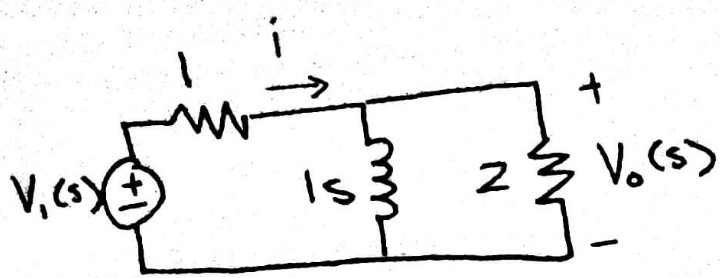
Find the signal flow graph for the following block diagram:



Assignment 4

1.

a.)



$$G(s) = \frac{V_o(s)}{V_i(s)} = \frac{2s}{3s+2}$$

$$V_i(s) = [1 + (s \| 2)] \cdot i$$

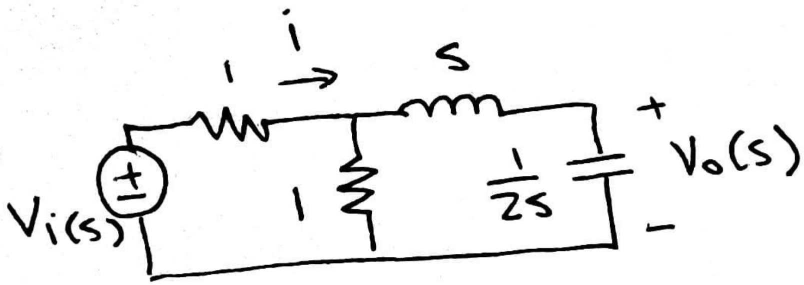
$$= \left[1 + \frac{2s}{s+2}\right] i$$

$$V_o(s) = \left[\frac{s}{s+2} \cdot i\right] \cdot 2$$

$$\frac{V_o(s)}{V_i(s)} = \frac{\frac{s}{s+2} \cdot 2 \cdot \cancel{i}}{\left[1 + \frac{2s}{s+2}\right] \cdot \cancel{i}}$$

$$= \frac{2s}{(s+2)+2s} = \frac{2s}{3s+2}$$

b.)



$$V_i(s) = \left[1 + \left(1 \| \left(s + \frac{1}{2s}\right)\right)\right] \cdot i$$

$$V_o(s) = \left[\frac{1}{1+s+\frac{1}{2s}} \cdot i\right] \cdot \frac{1}{2s}$$

$$G(s) = \frac{V_o(s)}{V_i(s)} = \frac{1}{4s^2+2s+2}$$

$$\frac{V_o}{V_i} = \frac{\frac{1}{2s\left(1+s+\frac{1}{2s}\right)}}{1 + \frac{s+\frac{1}{2s}}{1+s+\frac{1}{2s}}}$$

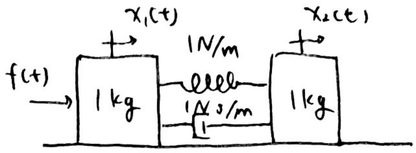
$$= \frac{1}{2s\left(1+s+\frac{1}{2s}\right) + 2s\left(s+\frac{1}{2s}\right)}$$

$$= \frac{1}{4s^2+2s+2}$$

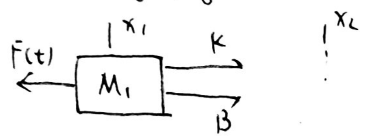
Assignment 4

Problem 2:

$$G(s) = \frac{X_2(s)}{F(s)}$$



① Free Body Diagram 1:



$$M_1 \cdot x_1'' = -F(t) - k(x_1 - x_2) - B(x_1' - x_2')$$

$$M_1 \cdot x_1''(t) = f(t) - k(x_1(t) - x_2(t)) - B(x_1'(t) - x_2'(t))$$

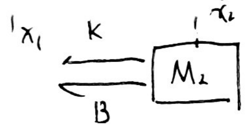
let $X(s) = \mathcal{L}(x(t))$, $\mathcal{L}(f(t)) = F(s)$

$$M_1 s^2 X_1 = F(s) - k(X_1 - X_2) - B(sX_1 - sX_2)$$

$\because M_1 = K = B = 1$

$$s^2 X_1 = F(s) - X_1 + X_2 - sX_1 + sX_2 \quad ①$$

② Free Body Diagram 2:



$$M_2 x_2'' = +k(x_1 - x_2) + B(x_1' - x_2')$$

let $X(s) = \mathcal{L}(x(t))$

$$M_2 s^2 X_2 = +k(X_1 - X_2) + B(sX_1 - sX_2)$$

$\because M_2 = k = B = 1$

$$s^2 X_2 = X_1 - X_2 + sX_1 - sX_2 \quad ②$$

$\therefore$ To solve for $G(s) = \dfrac{X_2(s)}{F(s)}$

$\because$ ① $\rightarrow (s^2 + s + 1)X_1 = F(s) + (s+1)X_2$

② $\rightarrow (1+s)X_1 = (s^2 + s + 1)X_2$

$\therefore$ replace X_1 in ① with ②

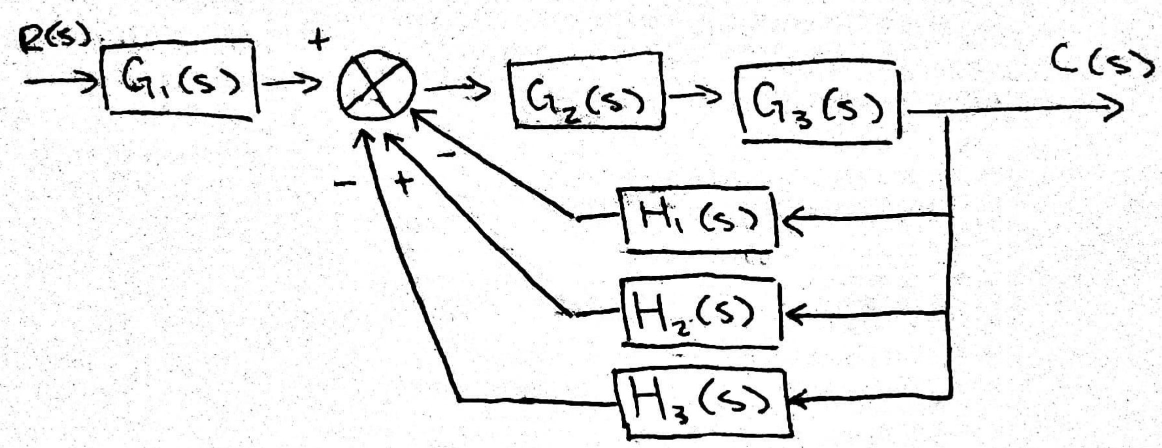
$$F(s) = -(s+1)X_2 + (s^2+s+1)\frac{s^2+s+1}{s+1}X_2$$

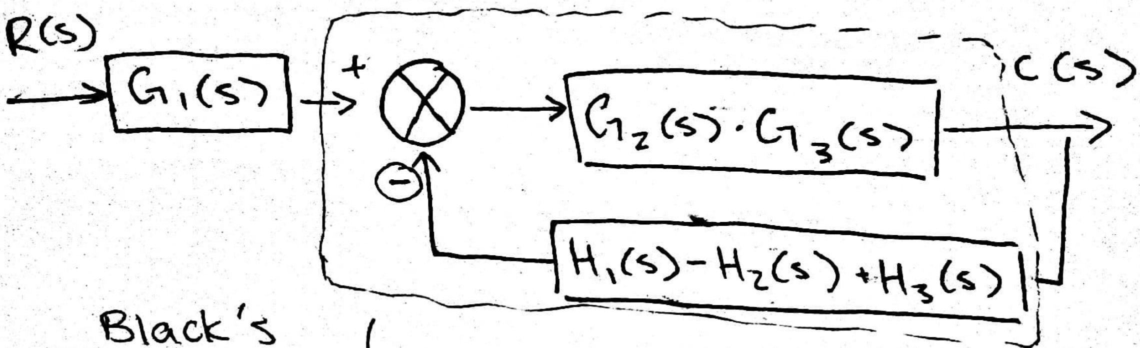
$$= \left(\frac{(s^2+s+1)^2}{s+1} - \frac{(s+1)^2}{s+1}\right)X_2(s)$$

$$\therefore G(s) = \frac{X_2(s)}{F(s)} = \frac{s+1}{(s^2+s+1)^2 - (s+1)^2} = \frac{s+1}{s^4+s^3+s^2+s^3+s^2+\cancel{s}+\cancel{s^2}+\cancel{s}+\cancel{1}-\cancel{s^2}-\cancel{2s}-\cancel{1}}$$

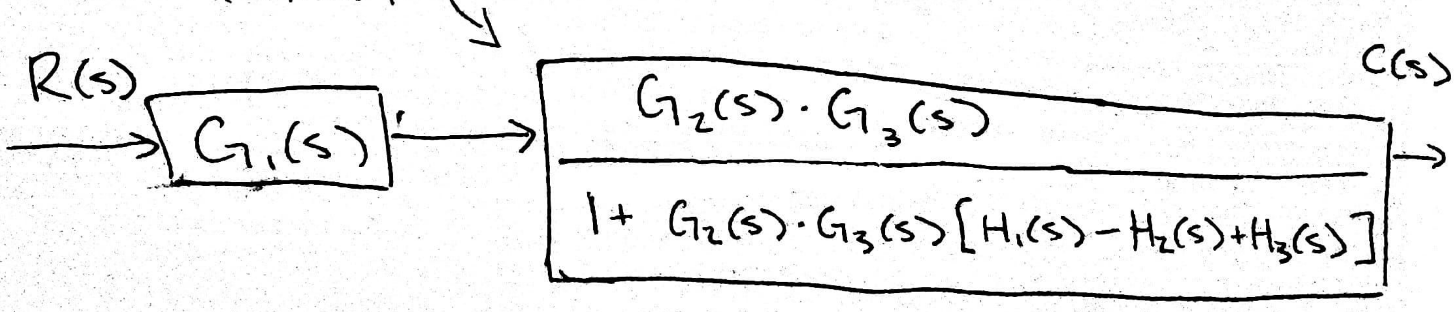
$$= \frac{s+1}{s^2(s^2+2s+2)}$$

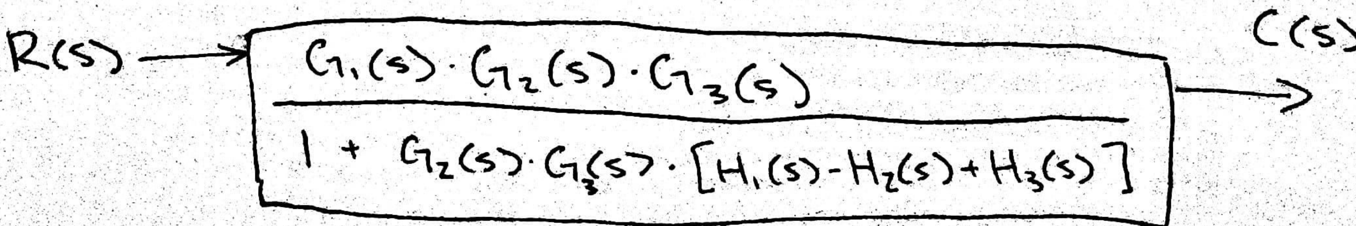
3





Black's formula





Assignment 4 (ELEC 341 L4_Model_ElecMech)

Problem 4:

Calculate the transfer function of the system represented in the state space model as

$$\dot{X} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -1 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} [u]$$

$$Y = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Here ; $A = \begin{bmatrix} -1 & -1 \\ 1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $C = \begin{bmatrix} 0 & 1 \end{bmatrix}$, $D = 0$

We know: T.F $\frac{Y(s)}{U(s)} = C(sI - A)^{-1}B + D$

$$\therefore (sI - A) = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} -1 & -1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} s+1 & 1 \\ -1 & s \end{bmatrix}$$

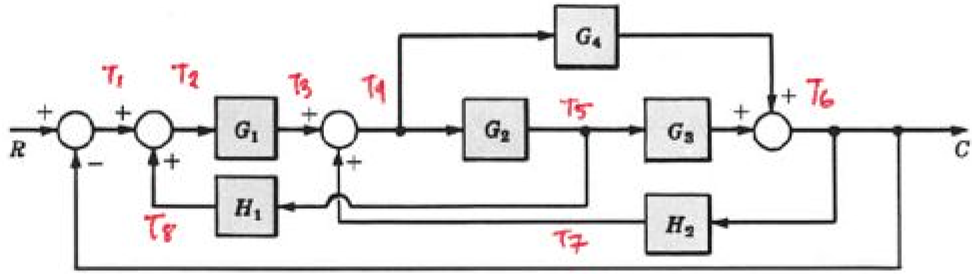
$$(sI - A)^{-1} = \frac{1}{s(s+1) + 1} \begin{bmatrix} s & -1 \\ 1 & s+1 \end{bmatrix} = \frac{1}{s^2 + s + 1} \begin{bmatrix} s & -1 \\ 1 & s+1 \end{bmatrix}$$

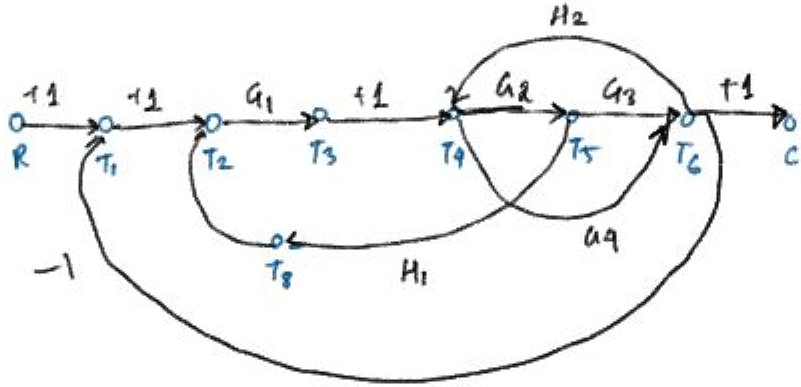
$$C(sI - A)^{-1} = \frac{1}{s^2 + s + 1} \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} s & -1 \\ 1 & s+1 \end{bmatrix} = \frac{1}{s^2 + s + 1} \begin{bmatrix} 1 & s+1 \end{bmatrix}$$

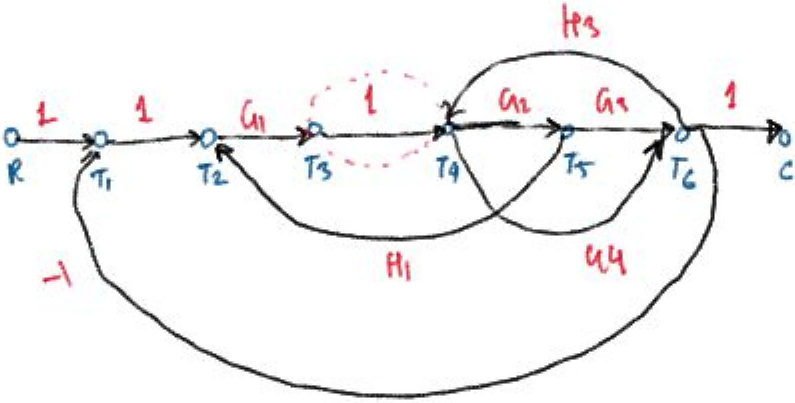
$$C(sI - A)^{-1}B = \frac{1}{s^2 + s + 1} \begin{bmatrix} 1 & s+1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \frac{1}{s^2 + s + 1}$$ Ans

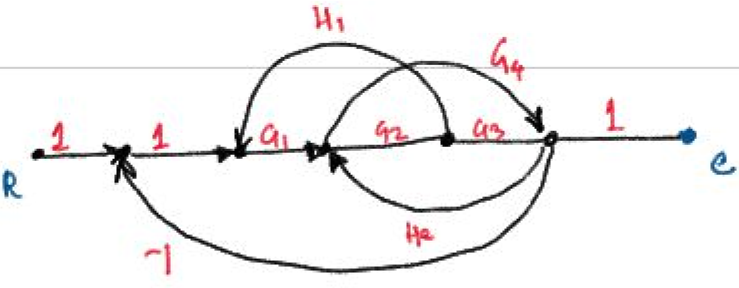
Problem 5:

Find the signal flow graph for the following block diagram:









Assignment 5 (ELEC 341 L5_Model_DCMotor)

Problem 1:

Find the linearize form of the following system using the equilibrium point of $v_0 = 30$. That is, re-write the ODE in terms of perturbation variables:

$$m\dot{v}(t) + K_f \cdot v(t) + K_a . v^2(t) = F_m(t)$$

Use the following numerical values for the constant parameters in your calculations:
$m = 1000\ ;\ K_f = 0.1\ ;\ K_a = 1$

Assignment 5 (ELEC 341 L5_Model_DCMotor)

Problem 1:

Find the linearize form of the following system using the equilibrium point of $v_0 = 30$. That is, re-write the ODE in terms of perturbation variables:

$$m\dot{v}(t) + K_f \cdot v(t) + K_a \cdot v^2(t) = F_m(t)$$

Use the following numerical values for the constant parameters in your calculations:
$m = 1000$; $K_f = 0.1$; $K_a = 1$

1) Input: $F_m(t)$, Output: $v(t)$

2) $f(\dot{v}, v, F_m) = 0 \Rightarrow m\dot{v}(t) + K_f v(t) + K_a v^2(t) - F_m(t) = 0$

3) $v(t) = V_0$, $\dot{v}(t) = 0 \Rightarrow 0 + K_f V_0 + K_a V_0^2 - \cancel{F_m(t)} = 0$

$\Rightarrow F_{m_0} = K_f V_0 + K_a V_0^2$

4) $f(\dot{v}, v, F_m) = f(\dot{v}_0, v_0, F_{m_0}) + \frac{\partial f}{\partial \dot{v}}\Big|_{op} (\dot{v} - \dot{v}_0) + \frac{\partial f}{\partial v}\Big|_{op} (v - v_0)$

$+ \frac{\partial f}{\partial F_m}\Big|_{op} (F_m - F_{m_0})$

5) $\begin{cases} \delta v = v - v_0 \\ \delta F_m = F_m - F_{m_0} \\ \delta \dot{v} = \dot{v} - \dot{v}_0 \end{cases} \Rightarrow f(\dot{v}, v, F_m) = f(\dot{v}_0, v_0, F_{m_0}) + \frac{\partial f}{\partial \dot{v}}\Big|_{op} \delta \dot{v} + \frac{\partial f}{\partial v}\Big|_{op} \delta v$

$+ \frac{\partial f}{\partial F_m}\Big|_{op} \delta F_m$

6) $\frac{\partial f}{\partial \dot{v}}\Big|_{op} = m = 1000$

$\frac{\partial f}{\partial v}\Big|_{op} = K_f + 2K_a V_0 = 0.1 + (2)(1)(30) = 60.1$

$\frac{\partial f}{\partial F}\Big|_{op} = -1$

$\Rightarrow f(\dot{v}, v, F) = 0 + \overset{1000}{m}\, \delta \dot{v} + (K_f + 2K_a V_0)\, \delta v + (-1)\, \delta F_m$

$= 1000\, \delta \dot{v} + 60.1\, \delta v - \delta F_m$

$\Rightarrow \delta F_m = 1000\, \delta \dot{v} + 60.1\, \delta v$

$\mathcal{L} \rightarrow \frac{\tilde{V}(s)}{F(s)} = \frac{1}{1000s + 60.1}$

Assignment 6 (ELEC 341 L6_Stability)

Problem 1:

Tell how many roots of the following polynomial are in the right half-plane, in the left half-plane, and on the $j\omega$-axis:

$$P(s) = s^5 + 3s^4 + 5s^3 + 4s^2 + s + 3$$

Use Routh-Hurwitz criterion.

Assignment 6 28 th May

1. $P(s) = s^5 + 3s^4 + 5s^3 + 4s^2 + s + 3$

- set up Routh Array :

$$\begin{array}{llll}
s^5 & 1 & 5 & 1 \\
s^4 & 3 & 4 & 3 \\
s^3 & \frac{11}{3} & \cancel{0}\ \varepsilon & \\
s^2 & A & 3 & \\
s^1 & B & & \\
s^0 & 3 & &
\end{array}$$

$$A = \frac{4 \times \frac{11}{3} - 3\varepsilon}{\frac{11}{3}} = \frac{44 - 9\varepsilon}{11} \approx 4$$

$$B = \frac{A\varepsilon - 11}{A} = \frac{4\varepsilon - 11}{4} = -\frac{11}{4}$$

$\therefore$ the first column progress from

$1 \to 3 \to \frac{11}{3} \to 4 \to -\frac{11}{4} \to 3$

$\therefore$ there are 2 roots in the left hand plane

$\because$ $P(s)$ is a polynomial of 5,

$\therefore$ there are 5 roots in total

$\therefore$ there are 3 roots left

By using matlab's roots function, ~~we~~ we get the result

$$\begin{cases}
s_1 = -1.6313 \\
s_2 = -0.9407 + 1.5042i \\
s_3 = -0.9407 - 1.5042i \\
s_4 = 0.2563 + 0.7201i \\
s_5 = 0.2563 + 0.7201i
\end{cases}$$

$\therefore$ There are 2 in the left hand plane, 3 in the right hand plane.

Assignment 7 (ELEC 341 L7_RouthHurwitzEx)

Problem 1:

Tell how many roots of the following polynomial are in the right half-plane, in the left half-plane, and on the $j\omega$-axis:

$$P(s) = s^5 + 6s^3 + 5s^2 + 8s + 20$$

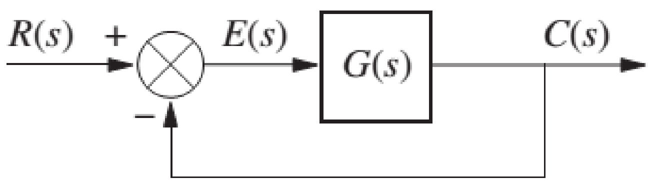
Use Routh-Hurwitz criterion.

Problem 2:

In the system of Figure , let

$$G(s) = \frac{K(s+2)}{s(s-1)(s+3)}$$

Find the range of K for closed-loop stability.



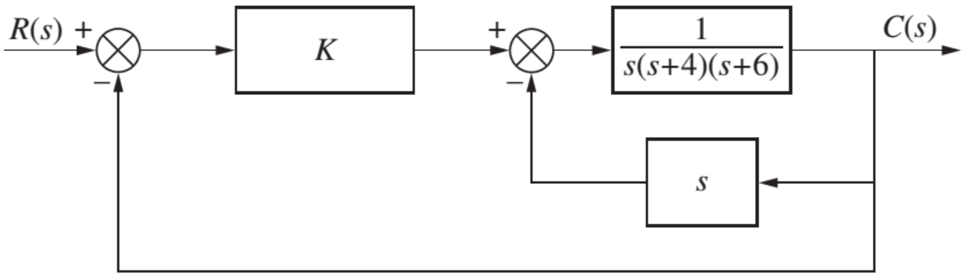
FIGURE

Use Routh-Hurwitz criterion.

Assignment 7 (ELEC 341 L7_RouthHurwitzEx)

Problem 3:

For the system shown in Figure, find the value of gain, K, that will make the system oscillate. Also, find the frequency of oscillation.



FIGURE

Use Routh-Hurwitz criterion.

Assignment 7

1. $P(s) = s^5 + 6s^3 + 5s^2 + 8s + 20$

s^5	1	6	8
s^4	$\cancel{0}\ \varepsilon$	5	20
s^3	a	b	
s^2	c	d	
ROZ → s^1	e		
s^0	f		

$a = \dfrac{6\varepsilon - 5}{\varepsilon}$ $\qquad \lim_{\varepsilon \to 0} a = \dfrac{-5}{\varepsilon}$

$b = \dfrac{8\varepsilon - 20}{\varepsilon}$ $\qquad \lim_{\varepsilon \to 0} b = \dfrac{-20}{\varepsilon}$

$c = \dfrac{5a - b\varepsilon}{a} = \dfrac{-\frac{25}{\varepsilon} + \frac{20\varepsilon}{\varepsilon}}{-\frac{5}{\varepsilon}}$

$= \dfrac{-25 + 20\varepsilon}{-5}$

$\lim_{\varepsilon \to 0} c = 5$

$d = \dfrac{20a - \varepsilon 0}{a} = 20$

$e = \dfrac{cb - ad}{c} = \dfrac{5\left(\frac{-20}{\varepsilon}\right) - \left(\frac{-5}{\varepsilon}\right)(20)}{5} = 0$

ROZ ⇒ Aux. polynomial $\Rightarrow \dfrac{d}{ds}(5s^2 + 20) = 10$

Replace $e = 10$

$f = \dfrac{de - 0c}{e} = d = 20$

s^5	1	6	8
s^4	ε	5	20
s^3	$-5/\varepsilon$	$-20/\varepsilon$	
s^2	5	20	
s^1	10		
s^0	20		

2 sign changes ⇒ 2 poles in ORHP

Aux. pol ⇒ $5s^2 + 20 = 0$

$s = \pm 2i$ ⇒ 2 poles on $j\omega$-axis

5 poles − 4 poles ⇒ 1 pole in OLHP

Summary of Poles: Open Right Hand Plane = 2
Open Left Hand Plane = 1
$j\omega$-axis = 2

2. $$G(s) = \frac{K(s+2)}{s(s-1)(s+3)}$$

Let Closed loop transfer function = $F(s)$

$$F(s) = \frac{G(s)}{1 + G(s)}$$

$$= \frac{\dfrac{K(s+2)}{s(s-1)(s+3)}}{1 + \dfrac{K(s+2)}{s(s-1)(s+3)}}$$

$$= \frac{K(s+2)}{s(s-1)(s+3) + K(s+2)}$$

$$= \frac{K(s+2)}{s^3 + 2s^2 - 3s + Ks + 2K}$$

Characteristic eqn : $s^3 + 2s^2 + (-3+K)s + 2K$

s^3	1	$-3+K$
s^2	2	$2K$
s^1	$a = -3$	
s^0	$b = 2K$	

$$a = -\frac{(1)(2K) + (-3+K)(2)}{2}$$

$$= K - 3 - K$$

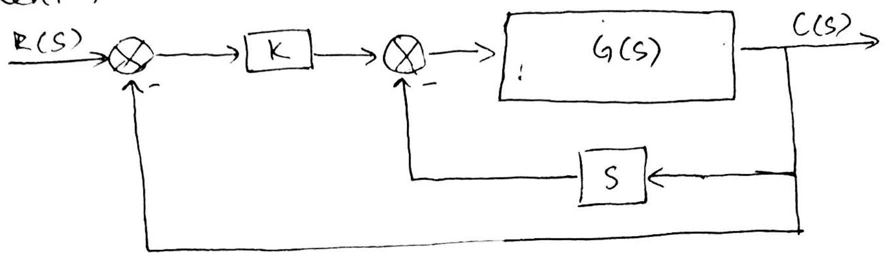
$$= -3$$

$$b = 2K$$

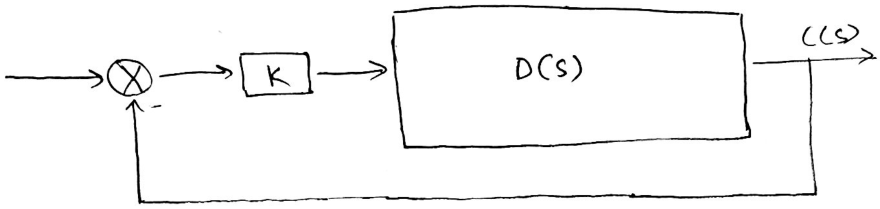
Since $a = -3$, the first column of the Routh-Hurwitz array will always have a sign change, for any value of K. Thus, no range of K will offer the system closed-loop stability.

Assignment 7

3.



$G(S) = \frac{1}{S(S+4)(S+6)}$



$D(S) = \frac{G(S)}{1 + S\,G(S)}$

$$D(S) = \frac{\frac{1}{S(S+4)(S+6)}}{1 + \frac{S}{S(S+4)(S+6)}} = \frac{1}{S(S+4)(S+6) + S}$$

$$= \frac{1}{S^3 + 10S^2 + 25S}$$

$$T(S) = \frac{K\,D(S)}{1 + K\,D(S)} = \frac{K}{S^3 + 10S^2 + 25S + K}$$

S^3	1	25
S^2	10	K
S^1	a	
S^0	b	

$a = \frac{(25)(10) - (K)(1)}{10} = \frac{250 - K}{10}$

$b = K$

Oscillation $\Rightarrow$ S^1 should be ROZ $\Rightarrow K = 250$

Aux. pol. $P(S) = 10S^2 + 250 = 0$

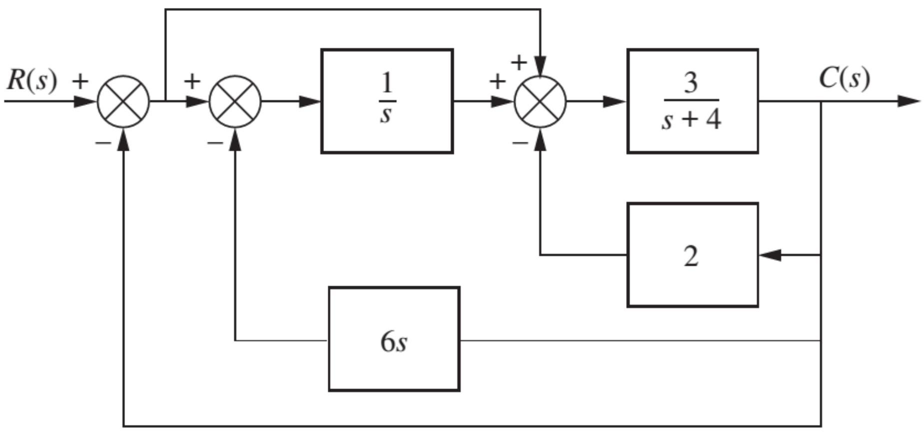
$S = \pm j5$

$\therefore \omega_{oscillation} = 5$ rad/s

Assignment 8 (ELEC 341 L8_SSError)

Problem 1:

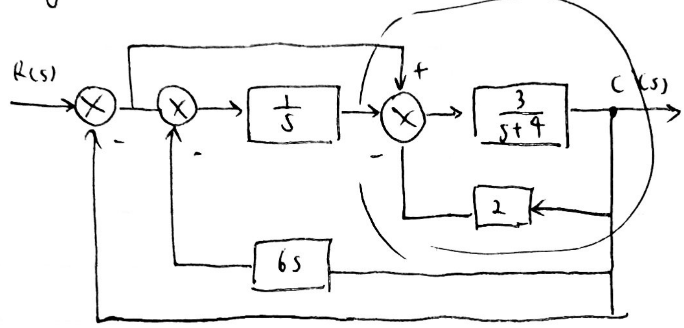
For the system shown in Figure, what steady-state error can be expected for the following test inputs: $10u(t)$, $10tu(t)$, $10t^2u(t)$.



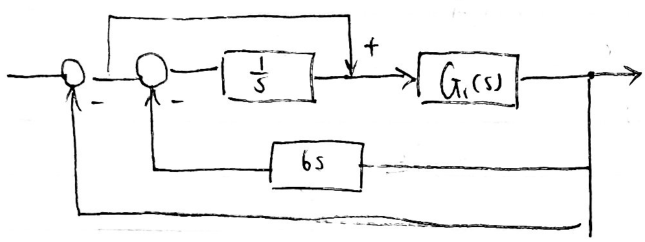
FIGURE

Assignment 8

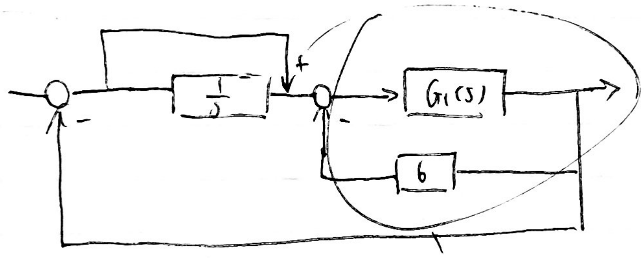
1)



Simplify to one

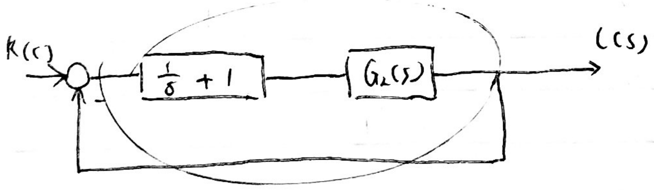


$$G_1(s) = \frac{\frac{3}{s+4}}{1 + 2 \times \frac{3}{s+4}} = \frac{3}{s+4+6} = \frac{3}{s+10}$$



$$G_2(s) = \frac{G_1(s)}{1 + 6\,G_1(s)} = \frac{\frac{3}{s+10}}{1 + 6 \times \frac{3}{s+10}}$$

$$= \frac{3}{s+10+18} = \frac{3}{s+28}$$

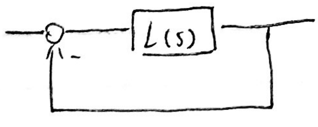


$$G_3(s) = \frac{(\frac{1}{s}+1)\,G_2(s)}{1 + (\frac{s+1}{s})(\frac{3}{s+28})}$$

$$= \frac{\frac{s+1}{s} \times \frac{3}{s+28}}{1 + \frac{3(s+1)}{s(s+28)}} = \frac{3(s+1)}{s(s+28) + 3(s+1)}$$

$$= \frac{3s+3}{s^2+28s+3s+3}$$

$$= \frac{3s+3}{s^2+31s+3}$$



$$L(s) = (\frac{1}{s}+1)(\frac{3}{s+28})$$

$$= \frac{3(s+1)}{s(s+28)}$$

∴ ~~step~~ for $10u(t)$, step error:

$$K_p = \lim_{s\to 0} L(s) \to \infty$$

∴ ess (steady state error)

$$= \frac{R}{1+K_p} \to 0$$

for $R(s) = \mathcal{L}(10t\,u(t)) = \frac{10}{s^2}$

$$K_v = \lim_{s\to 0} s\,L(s) = \frac{3s+3}{s+28} = \frac{3}{28}$$

$$\therefore \; ess = \frac{R}{K_v} = \frac{10}{\frac{3}{28}} = \frac{280}{3}$$

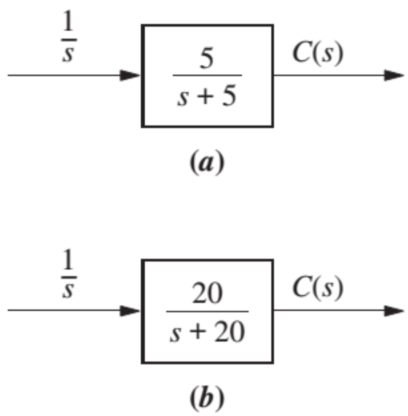
for $R(s) = \mathcal{L}(10t^2u(t)) = \frac{10}{s^3}$

$$K_a = \lim_{s\to 0} s^2 L(s) \to 0$$

$$\therefore \; ess = \frac{R}{K_a} \to \infty$$

Problem 1:

Find the output response, $c(t)$, for each of the systems shown in Figure. Also find the time constant, rise time, and settling time for each case.

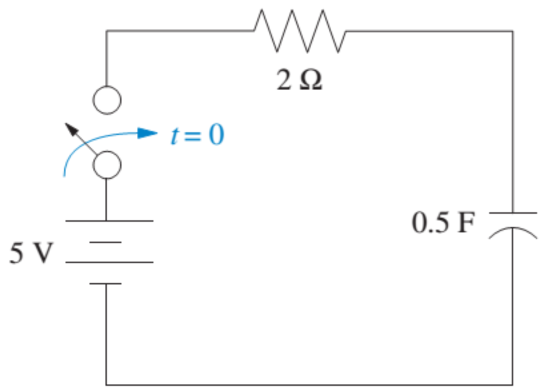


FIGURE

Use the difference between 10% to 90% of y_{ss} as the rise time and also use a 2% settling time.

Problem 2:

Find the capacitor voltage in the network shown in Figure if the switch closes at $t = 0$. Assume zero initial conditions. Also find the time constant, rise time, and settling time for the capacitor voltage.



FIGURE

Use the difference between 10% to 90% of y_{ss} as the rise time and also use a 2% settling time. Use Matlab to verify and plot your output response.

Problem 3:

For the second-order system that follows, find ζ, ω_n, T_s, T_p, T_r, and $\%OS$.

$$T(s) = \frac{16}{s^2 + 3s + 16}$$

Ass 9

1.

a) $\xrightarrow{\frac{1}{s}} \boxed{\frac{5}{s+5}} \xrightarrow{C(s)}$

$$C(s) = \frac{1}{s}\cdot\frac{5}{s+5} = \frac{5}{s^2+5s} = \frac{A}{s} + \frac{B}{s+5}$$

$$\frac{5}{s(s+5)} = \frac{A}{s} + \frac{B}{s+5}$$

$$\Rightarrow \frac{5}{s+5} = A + \frac{Bs}{s+5} \Rightarrow s=0, \Rightarrow 1 = A$$

$$\Rightarrow \frac{5}{s} = \frac{A(s+5)}{s} + B \Rightarrow s=-5 \Rightarrow -1 = B$$

$$C(s) = \frac{1}{s} - \frac{1}{s+5} \Rightarrow c(t) = u(t) - e^{-5t}u(t)$$

$$c(t) = u(t)\left[1-e^{-5t}\right]$$

1st order

$$G(s) = \frac{K}{Ts+1} = \frac{5}{s+5} = \frac{1}{\frac{1}{5}s+1} \qquad K=1 \quad T=\frac{1}{5}$$

rise time = $10\% \, y_{ss} \xrightarrow{\text{to}} 90\% \, y_{ss}$

$$y(t) = K(1-e^{-t/T})$$

$$= 1(1-e^{-\frac{t}{1/5}})$$

$$y(t) = 1-e^{-5t}$$

$$0.9y_{ss} = y_{ss}(1-e^{-5t})$$

$$e^{-5t} = 0.1$$

$$-5t = \ln(0.1)$$

$$t_1 = 0.460517s$$

$$e^{-5t} = 0.9$$

$$t_2 = 0.02107$$

$$t_2 - t_1 = 0.4395s$$

$$T = 0.4395s$$

$\Updownarrow$

$$\text{rise time} = 2.2T$$

$$= 2.2\left(\tfrac{1}{5}\right)$$

$$= 0.44s$$

$$T_s = 0.98 = 1-e^{-5t}$$

$$t = 0.7824$$

$\Updownarrow$

$$T_s = 4T = 0.8s$$

b.) $C(s) = \frac{1}{s} \cdot \frac{20}{s+20} = \frac{A}{s} + \frac{B}{s+20}$

$\Rightarrow \frac{20}{s+20} = A + \frac{Bs}{s+20}$, $s=0$, $\Rightarrow 1 = A$

$\frac{20}{s} = \frac{A(s+20)}{s} + B$, $s=-20$, $\Rightarrow -1 = B$

$C(s) = \frac{1}{s} - \frac{1}{s+20} \Rightarrow c(t) = u(t)\left[1 - e^{-20t}\right]$

1st order

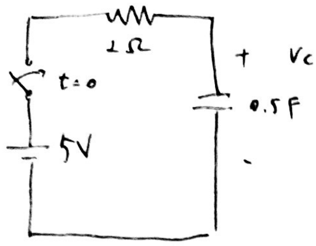
$G(s) = \frac{K}{Ts+1} = \frac{20}{\frac{1}{20}s + 1}$ $\quad K = 20$, $T = \frac{1}{20}$

rise time $= 2.2T$
$= 2.2\left(\frac{1}{20}\right)$
$= 0.11s$

$T_s = 4T = 4 \cdot \left(\frac{1}{20}\right)$
$= \frac{1}{5}s$

Assignment 9

2.



$T = ?$

$T_r = ?$

$T_s = ?$

$$G(s) = \frac{V_c(s)}{V_i(s)}$$

$$\because V_c(s) = \frac{\frac{1}{0.5s}}{2 + \frac{1}{0.5s}} \cdot V_i(s)$$

$$= \frac{1}{s+1} \cdot V_i(s)$$

$$\therefore G(s) = \frac{1}{s+1}$$

$$\because V_i(t) = \underset{u(t)}{5V} \rightarrow V_i(s) = \frac{5}{s}$$

$$\therefore \cancel{G(s)} V_c(s) = \frac{1}{s+1} \cdot \frac{5}{s}$$

$$= \frac{A}{s+1} + \frac{B}{s}$$

where $B(s+1) + As = 5$

$$\begin{cases} B + A = 0 \\ B = 5 \end{cases} \rightarrow A = -5$$

$$V_c(s) = \frac{-5}{s+1} + \frac{5}{s} = 5\left(\frac{1}{s} - \frac{1}{s+1}\right)$$

$$\therefore V_c(t) = \mathcal{L}^{-1}(V_c(s)) = 5(u(t) - e^{-t}) = 5(1 - e^{-t})$$

$$\because G(s) = \frac{1}{s+1} = \frac{K}{Ts+1}$$

$$\therefore \begin{cases} T = 1\,s \\ T_r = 2.2T = 2.2\,s \\ T_s = 4T = 4\,s \end{cases}$$

Assignment 9

3. $T(s) = \dfrac{16}{s^2+3s+16}$

$\therefore \begin{cases} 2\zeta w_n = 3 \\ w_n^2 = 16 \end{cases} \rightarrow \begin{cases} w_n = 4 \\ \zeta = \dfrac{3}{8} = 0.375 \end{cases} \rightarrow T = \dfrac{1}{\zeta w_n} = \dfrac{2}{3} = 0.666..7$

$\%OS\ \zeta = 100 e^{-\zeta\pi/\sqrt{1-\zeta^2}} = \cancel{0.6199112458461} \ 28.0596719899\%$

$T_p = \dfrac{\pi}{w_d} = \dfrac{\pi}{w_n\sqrt{1-\zeta^2}} = 0.847224533986\ s$

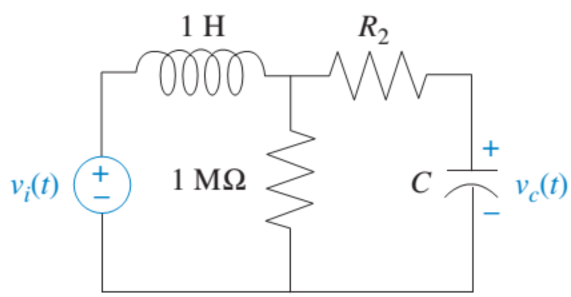
$T_r = \dfrac{1.76\zeta^3 - 0.417\zeta^2 + 1.039\zeta + 1}{w_n} = 0.3559492815$

$T_s = \begin{cases} 2\%: & 4T = \dfrac{4}{\zeta w_n} = \dfrac{8}{3} = 2.66667 \\ 5\%: & 3T = \dfrac{3}{\zeta w_n} = 2 \end{cases}$

Assignment 10 (ELEC 341 L10_TimeResponseEx)

Problem 1:

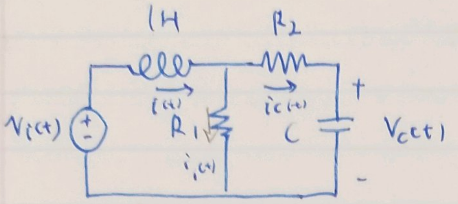
For the circuit shown in Figure , find the values of R_2 and C to yield 8% overshoot with a settling time of 1 ms for the voltage across the capacitor, with $v_i(t)$ as a step input.



FIGURE

The given settling time is a 2% settling time.

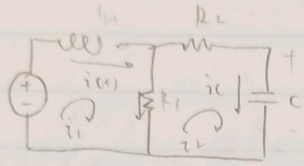
Assignment 10



$R_1 = 1M\Omega$

$PO = 8\%$

$T_s = 0.001\,s = 4T$



$$\begin{cases} -V_i(s) + I_1(s)\cdot s(1) + (I_1(s) - I_2(s))R_1 = 0 \\ (I_2(s) - I_1(s))R_1 + I_2(s)R_2 + I_2(s)\dfrac{1}{sC} = 0 \end{cases}$$

$$\begin{cases} V_i(s) = I_1(s)\cdot(s+R_1) - I_2(s)R_1 \\ (R_1 + R_2 + \dfrac{1}{sC})I_2(s) - R_1 I_1(s) = 0 \end{cases}$$

$V_C(s) = I_2(s)\cdot\dfrac{1}{sC}$, $G(s) = \dfrac{V_C(s)}{V_i(s)}$

$\therefore I_2(s) = \dfrac{R_1}{R_1 + R_2 + \frac{1}{sC}} I_1(s)$

$V_i(s) = \dfrac{R_1 + R_2 + \frac{1}{sC}}{R_1} I_2(s)\,(s+R_1) - R_1 I_2(s)$

$= \dfrac{(R_1 + R_2 + \frac{1}{sC})(s+R_1) - R_1^2}{R_1}$

$V_C(s) = \dfrac{1}{sC} I_2(s)$

$\therefore G(s) = \dfrac{V_C(s)}{V_i(s)} = \dfrac{\frac{1}{sC}\cdot R_1}{(R_1+R_2+\frac{1}{sC})\cdot(s+R_1) - R_1^2} = \dfrac{R_1}{(sCR_1 + sCR_2 + 1)(s+R_1) - sCR_1^2}$

$= \dfrac{R_1}{s^2CR_1 + s^2CR_2 + s + \cancel{sCR_1^2} + sCR_1R_2 + R_1 - \cancel{sCR_1^2}}$

$= \dfrac{R_1}{s^2C(R_1+R_2) + s(CR_1R_2+1) + R_1}$

$= \dfrac{R_1/C(R_1+R_2)}{s^2 + \dfrac{CR_1R_2+1}{C(R_1+R_2)}s + \dfrac{R_1}{C(R_1+R_2)}} =$

$\because PO = 8\% = 100e^{-\zeta\pi/\sqrt{1-\zeta^2}}$

$T_s = \dfrac{4}{\zeta\omega_n}$, when $2\zeta\omega_n = \dfrac{CR_1R_2+1}{C(R_1+R_2)}$

$= 0.001$

$\therefore$ Use calculating device, we get $\zeta = 0.626577186806$

$\because \dfrac{4}{\zeta\omega_n} = 0.001 \rightarrow \omega_n = \dfrac{4}{\zeta \times 0.001}$

$\therefore \omega_n = 6383.89025951$

$$G(s) = \frac{A}{s^2 + Bs + D}, \quad \text{where } A = \frac{R_1}{C(R_1+R_2)}$$

$$B = \frac{CR_1R_2 + 1}{C(R_1+R_2)}$$

$$D = \frac{R_1}{C(R_1+R_2)}$$

$\because \omega_n = 6383.89025951$

$\therefore D = \omega_n^2 \Rightarrow \frac{R_1}{C(R_1+R_2)} = \omega_n^2$ ① $B = 2\xi\omega_n \Rightarrow \frac{CR_1R_2+1}{C(R_1+R_2)} = 2\xi\omega_n$ ②

$$\begin{cases} ① \\ ② \end{cases} \Rightarrow \begin{cases} C(R_1+R_2) = \frac{R_1}{\omega_n^2} \\ C(R_1+R_2) = \frac{CR_1R_2+1}{2\xi\omega_n} \end{cases} \Rightarrow \frac{\frac{R_1 \cdot 2\xi\omega_n}{\omega_n^2} - 1}{R_1} = CR_2$$

$$CR_2 = \frac{2\xi}{\omega_n} - \frac{1}{R_1} = G$$

$$\therefore C = \frac{G}{R_2}$$

$$\therefore C\left(R_1 + \frac{G}{C}\right) = \frac{R_1}{\omega_n^2}$$

$$CR_1 + G = \frac{R_1}{\omega_n^2}$$

$$\begin{cases} C = \left(\frac{1}{\omega_n^2} - \frac{G}{R_1}\right) \\ R_2 = \frac{G}{\frac{1}{\omega_n^2} - \frac{G}{R_1}} = \frac{\omega_n^2 \cdot R_1 \cdot G}{R_1 - \omega_n^2 G} \end{cases}$$

Use Calculator:

$$\begin{cases} \xi = 0.62657718 \quad (B) \\ \omega_n = 6383.89025 \quad (C) \\ R_1 = 1\times10^6\,\Omega \quad (A) \end{cases} \qquad \begin{cases} G = 1.95299485544\times10^{-4} \\ C = 2.43421362075\times10^{-8}\,F \\ R_2 = 8023.10380155\,\Omega \end{cases}$$