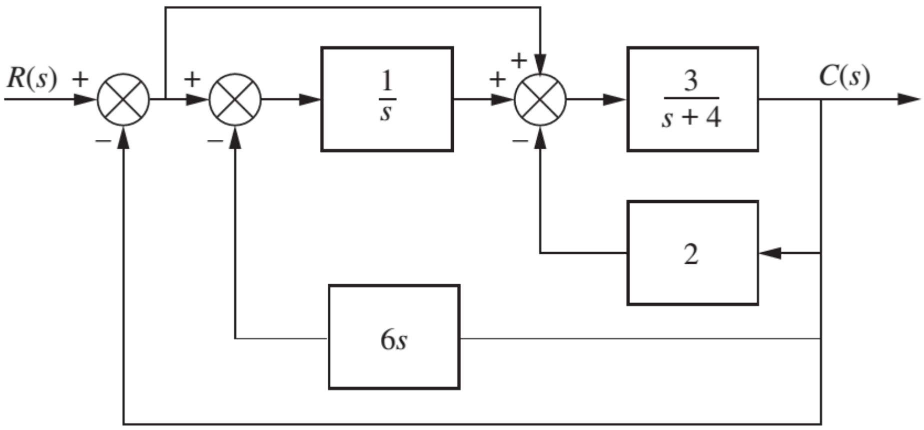
Assignment 8 (ELEC 341 L8_SSError)

Problem 1:

For the system shown in Figure， what steady-state error can be expected for the following test inputs: $10u(t)$, $10tu(t)$, $10t^2u(t)$.



FIGURE

Solution:

Reduce the system to an equivalent unit feedback system by first moving 1/s to the left past the summing junction. This move creates a forward path consisting of a parallel pair $\left(\frac{1}{s}+1\right)$ in cascade with a feedback loop consisting of $G(s)=\frac{3}{s+4}$ and $H(s)=2$. Thus,

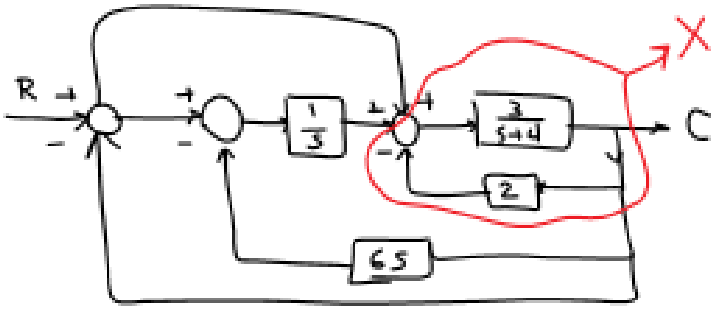
$$G_e(s)=\left(\frac{s+1}{s}\right)\left(\frac{3/(s+4)}{1+24/(s+4)}\right)$$

Hence the system is Type 1, and the steady-state errors are as follows:

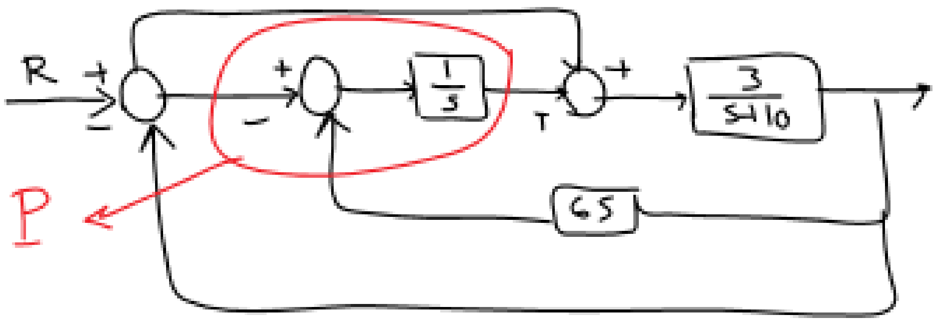
Steady state error for $10u(t)=0$

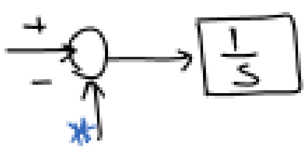
Steady state error for $10tu(t)=\frac{10}{K_v}=\frac{10}{3/28}=93.33$

Steady state error for $10t^2u(t)=\infty$

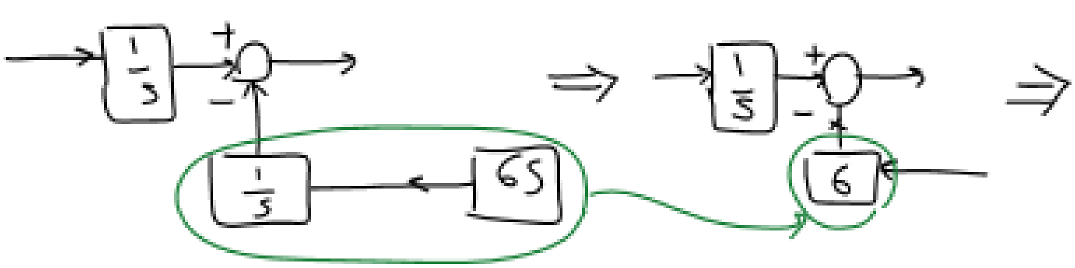


$$X = \frac{\frac{3}{s+4}}{1+2\times\frac{3}{s+4}} = \frac{\frac{3}{\cancel{s+4}}}{\frac{s+10}{\cancel{s+4}}} \rightarrow X = \frac{3}{s+10}$$

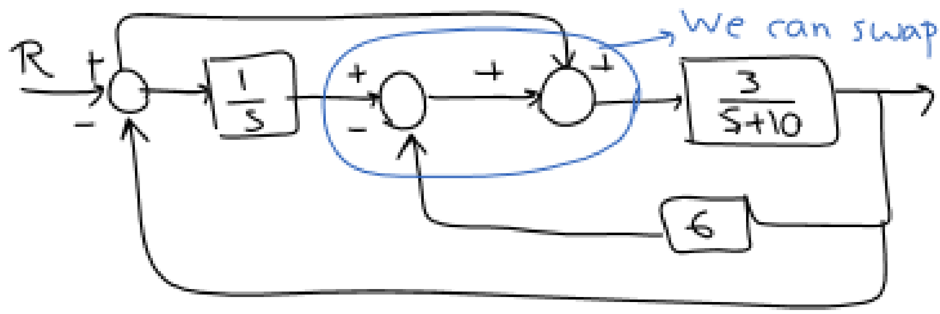




Let us move $\boxed{\frac{1}{s}}$ to the left of summing junction. If we do this, we need to add a block of negative feedback with the same gain on branck *

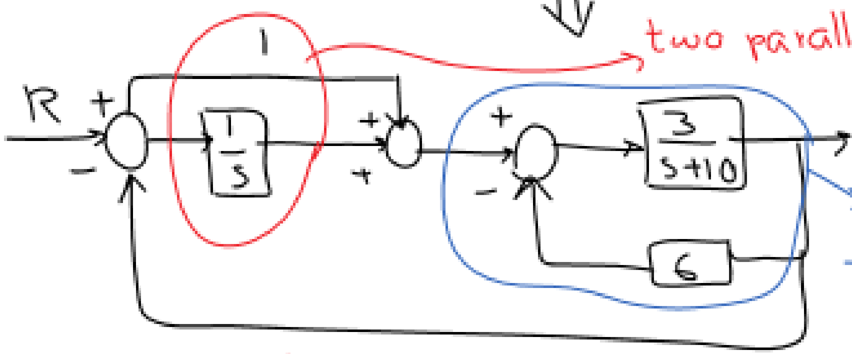


We can swap them



$\Downarrow$

two parallel path (add the gain up)

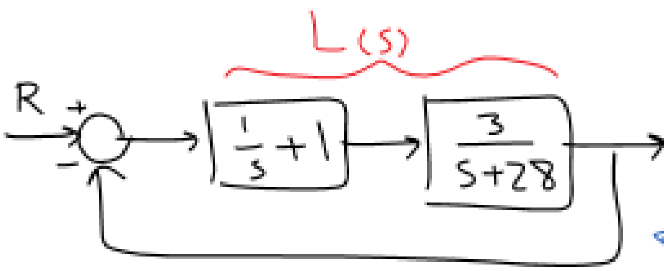


a negative feedback loop (Y)

$$Y = \frac{\text{Forward path}}{1 + \text{OLTF}}$$

$$Y = \frac{\frac{3}{s+10}}{1+6\left(\frac{3}{s+10}\right)}$$

$$Y = \frac{3}{s+28}$$



$$L(s) = G_{OLTF} = \left(\frac{1}{s}+1\right)\left(\frac{3}{s+28}\right)$$

$$\boxed{L(s) = \frac{3(s+1)}{s(s+28)}}$$

Note $L(s) = \frac{3(s+1)}{s^{1}(s+28)}$

Type 1 system

a) For step input of $r(t) = 10\,u(t)$
$\hookrightarrow R = 10$

$$e_{ss} = \frac{R}{1+K_p} \; ; K_p = \lim_{s\to 0} L(s) = \frac{3(0+1)}{0(0+28)} = \infty \rightarrow$$

$$e_{ss} = \frac{10}{1+\infty} \rightarrow \boxed{e_{ss} = 0}$$

b) $r(t) = 10t \cdot u(t)$; $e_{ss} = \dfrac{R}{K_v}$; $K_v = \lim_{s\to 0} s\,L(s) =$

$$= \frac{3(0+1)}{0+28} = \frac{3}{28} \rightarrow e_{ss} = \frac{10}{3/28} = \frac{280}{3} \rightarrow \boxed{e_{ss} \approx 93.33}$$

c) $r(t) = 10t^2 \cdot u(t)$; $e_{ss} = \dfrac{R}{K_a}$; $K_a = \lim_{s\to 0} s^2 L(s) =$

$$= \frac{0(3)(0+1)}{(0+28)} = 0 \rightarrow e_{ss} = \frac{10}{0} \rightarrow \boxed{e_{ss} = \infty}$$