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UBC

ELEC 341: Systems and Control

Lecture 19

Nyquist stability criterion: Examples on relative stability

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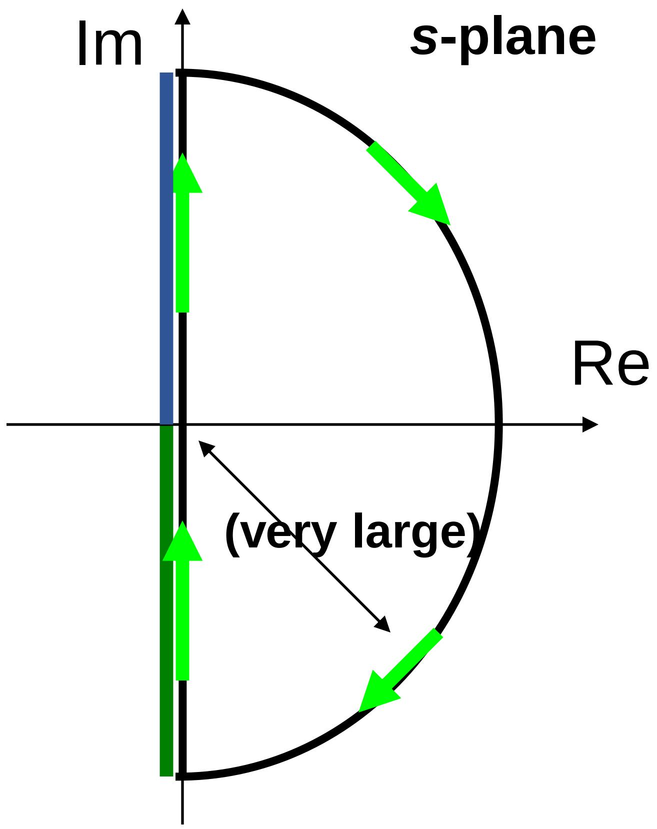
Course roadmap

Modeling	*Analysis*	*Design*
✓ Laplace transform	✓ Stability	Design specs
✓ Transfer function	✓ • Routh-Hurwitz	✓ Root locus
Models for systems	• Nyquist	Frequency domain
✓ • Electrical	✓ Time response	✓ PID & Lead-lag
✓ • Electromechanical	✓ • Transient	Design examples
✓ • Mechanical	✓ • Steady state	
✓ Linearization, delay	✓ Frequency response	
	✓ • Bode plot	

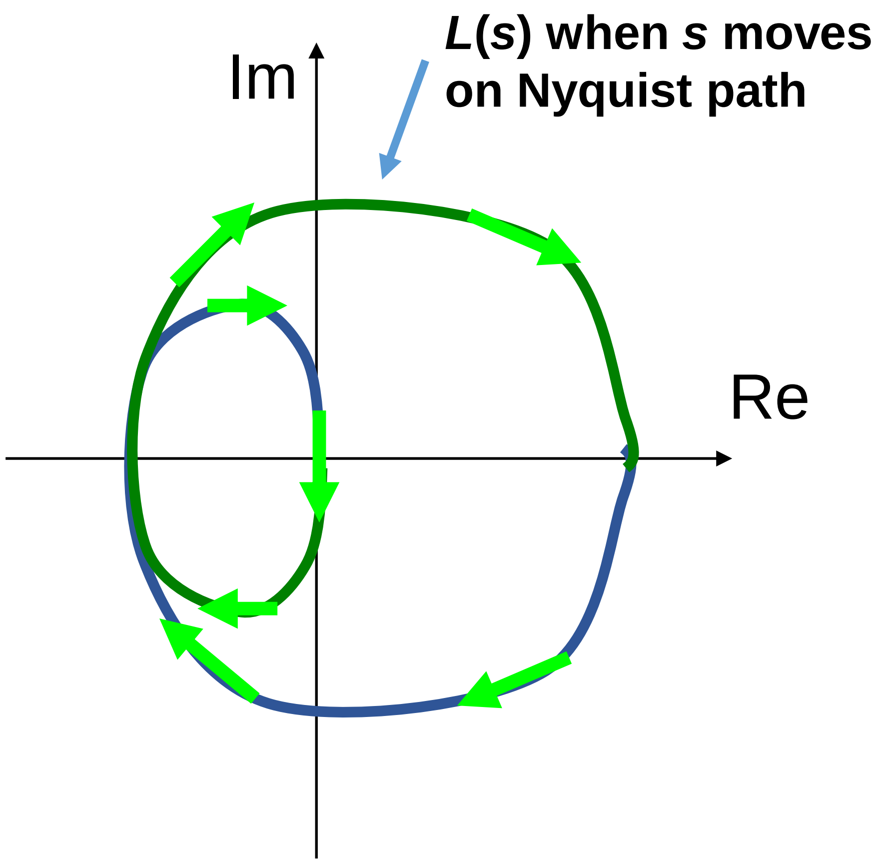
Matlab simulations

Nyquist plot (review)

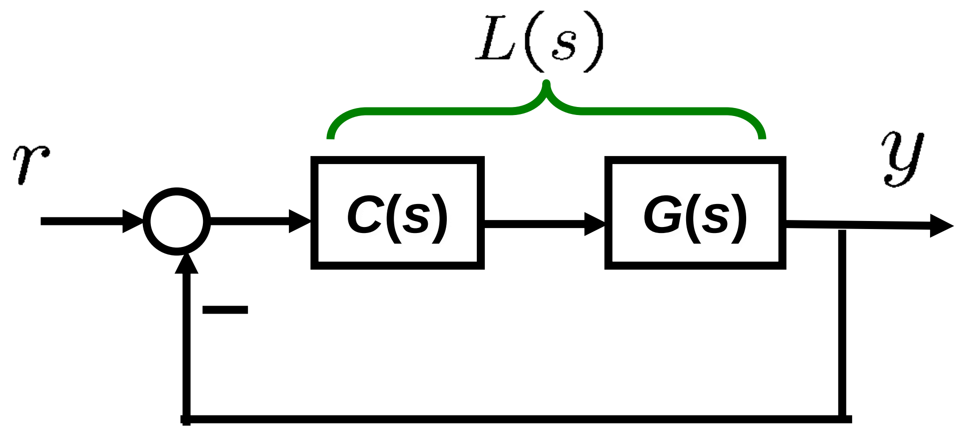
- Nyquist path



- Nyquist plot



Nyquist stability criterion (review)



$$\boxed{\text{CL system is stable} \Leftrightarrow Z = P + N = 0}$$

Z: # of CL poles in open RHP

P: # of OL poles in open RHP (given)

N: # of clockwise/counterclockwise encirclement of -1 by Nyquist plot of OL transfer function *L*(*s*) (counted by using Nyquist plot of *L*(*s*))

Advantages of Nyquist stability criterion

- It can deal with closed-loop systems with a time-delay system in the open-loop, while Routh-Hurwitz criterion cannot.
- It does not require transfer functions, just experimental frequency response data of the open-loop system are necessary to judge the closed-loop stability. On the other hand, Routh-Hurwitz criterion needs transfer functions.
- Nyquist criterion leads to the concept of "stability margin", i.e., gain-margin and phase-margin. From Routh-Hurwitz criterion, we can only judge "stable or not".

Nyquist criterion: A special case

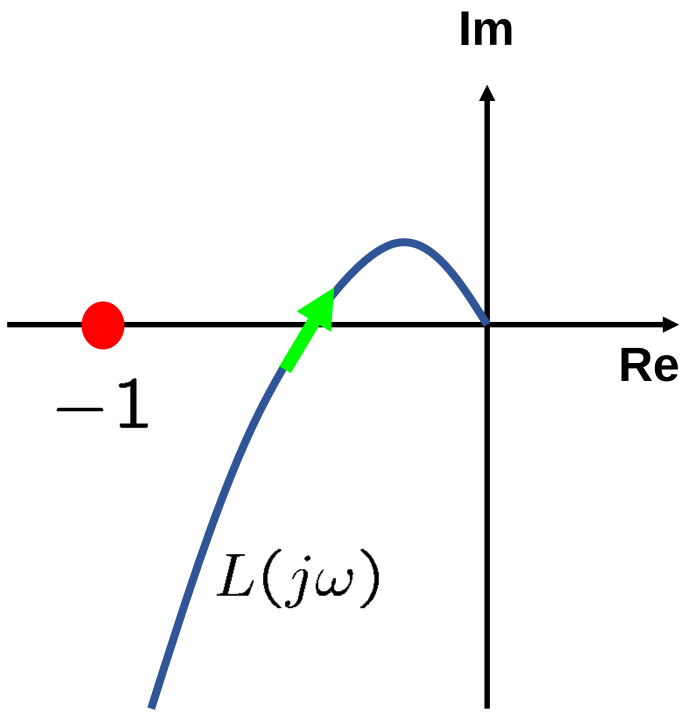
$$\boxed{\text{CL system is stable} \Leftrightarrow Z = P + N = 0}$$

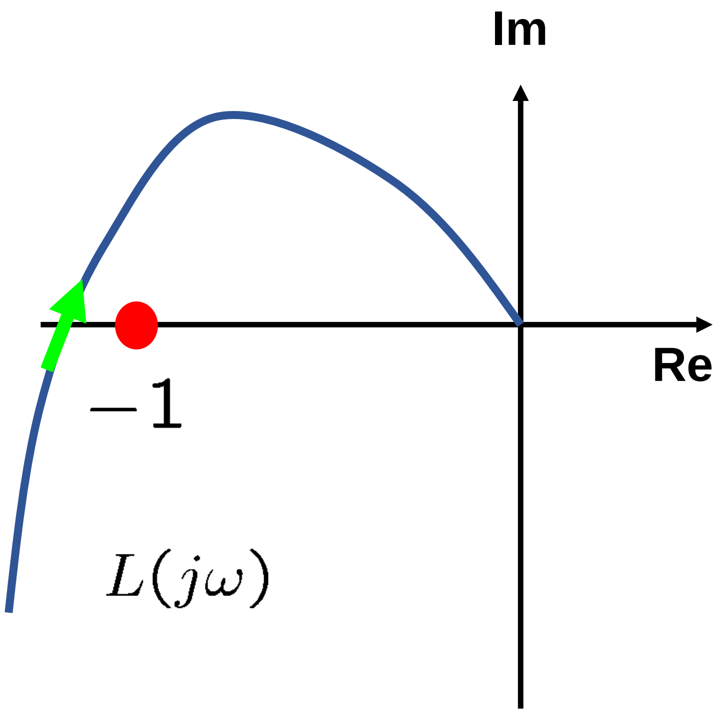
- If P = 0 (i.e., if $L(s)$ has no pole in open RHP)

$$\boxed{\text{CL system is stable} \Leftrightarrow N = 0}$$

We assume P = 0 from now on!

Examples when $P = 0$



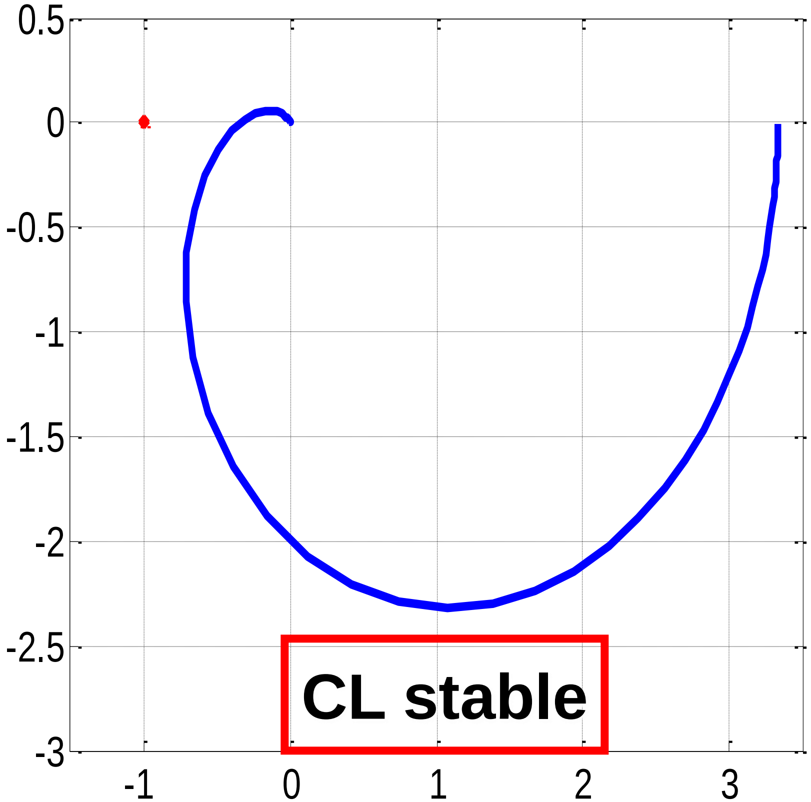


CL stable

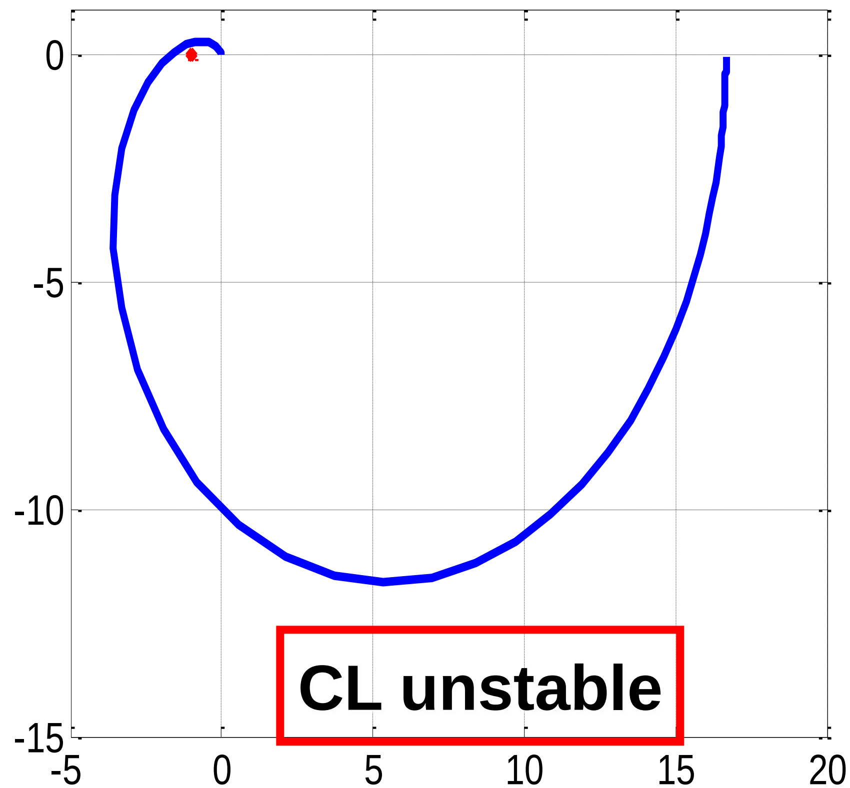
CL unstable

Example 1: Stable $L(s)$

$$L(s) = \frac{20}{(s+1)(s+2)(s+3)}$$



$$L(s) = \frac{100}{(s+1)(s+2)(s+3)}$$

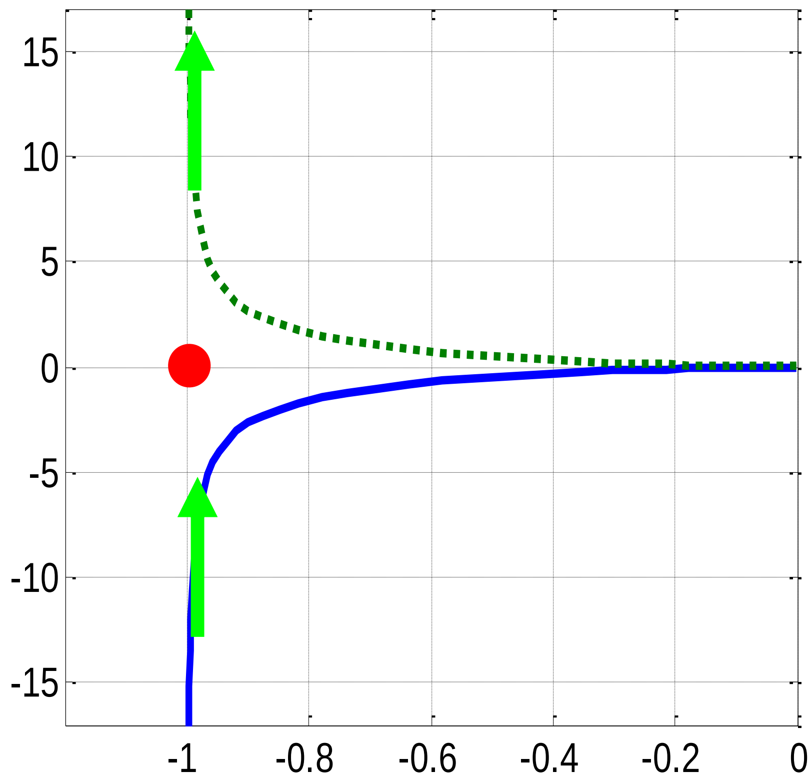


Example 2: *L*(*s*) with an integrator

$$L(s) = \frac{1}{s(s+1)}$$

```
sys = tf(1,[1 1 0]);
nyquist(sys)
```

How to use the Nyquist stability criterion?

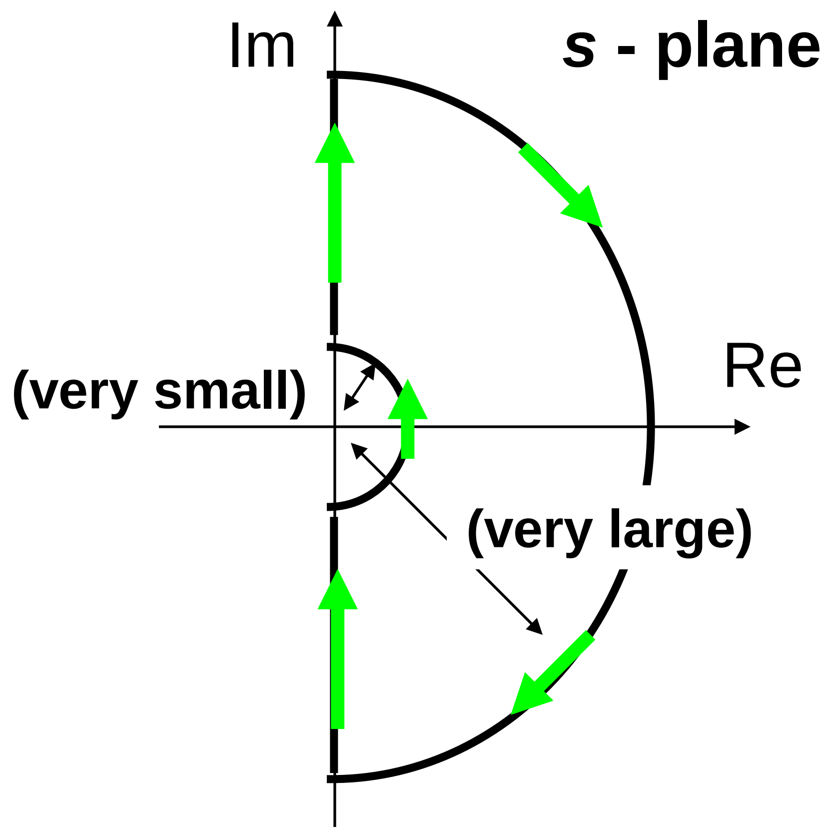


Example 2 (cont'd): *L*(*s*) with an integrator

- When open loop poles lie on the imaginary axis, we modify Nyquist path, by detouring to the right of the poles.

Ex:

$$L(s) = \frac{1}{s(s+1)}$$

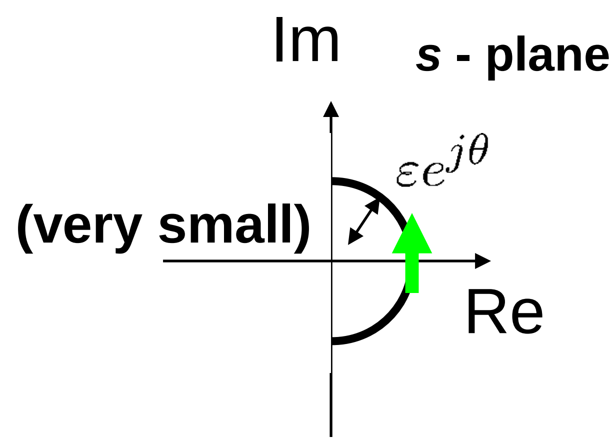


Example 2 (cont'd): $L(s)$ for modified Nyquist path

- For small $|s|$,

$$L(s) = \frac{1}{s(s+1)} = \frac{1}{s^2+s} \approx \frac{1}{s}$$

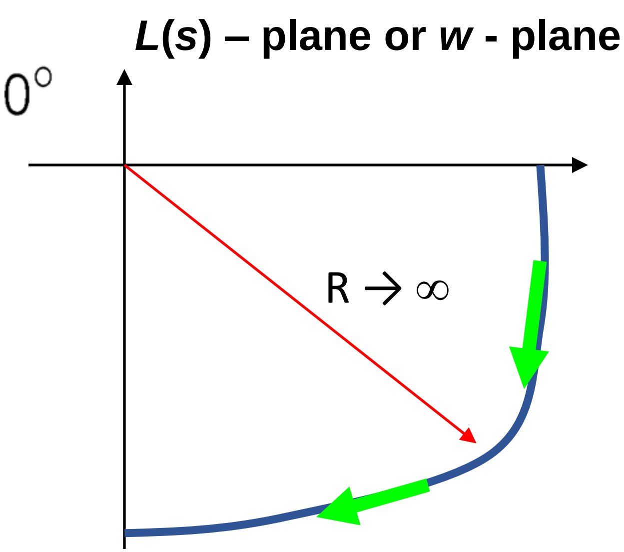
$$s = \varepsilon e^{j\theta} \to L(s) \approx \frac{1}{\varepsilon} e^{-j\theta}$$



- When s moves as $s = \varepsilon e^{j\theta}$ and $\theta = 0 \to 90^\circ$

$L(s)$ moves as shown on the right:

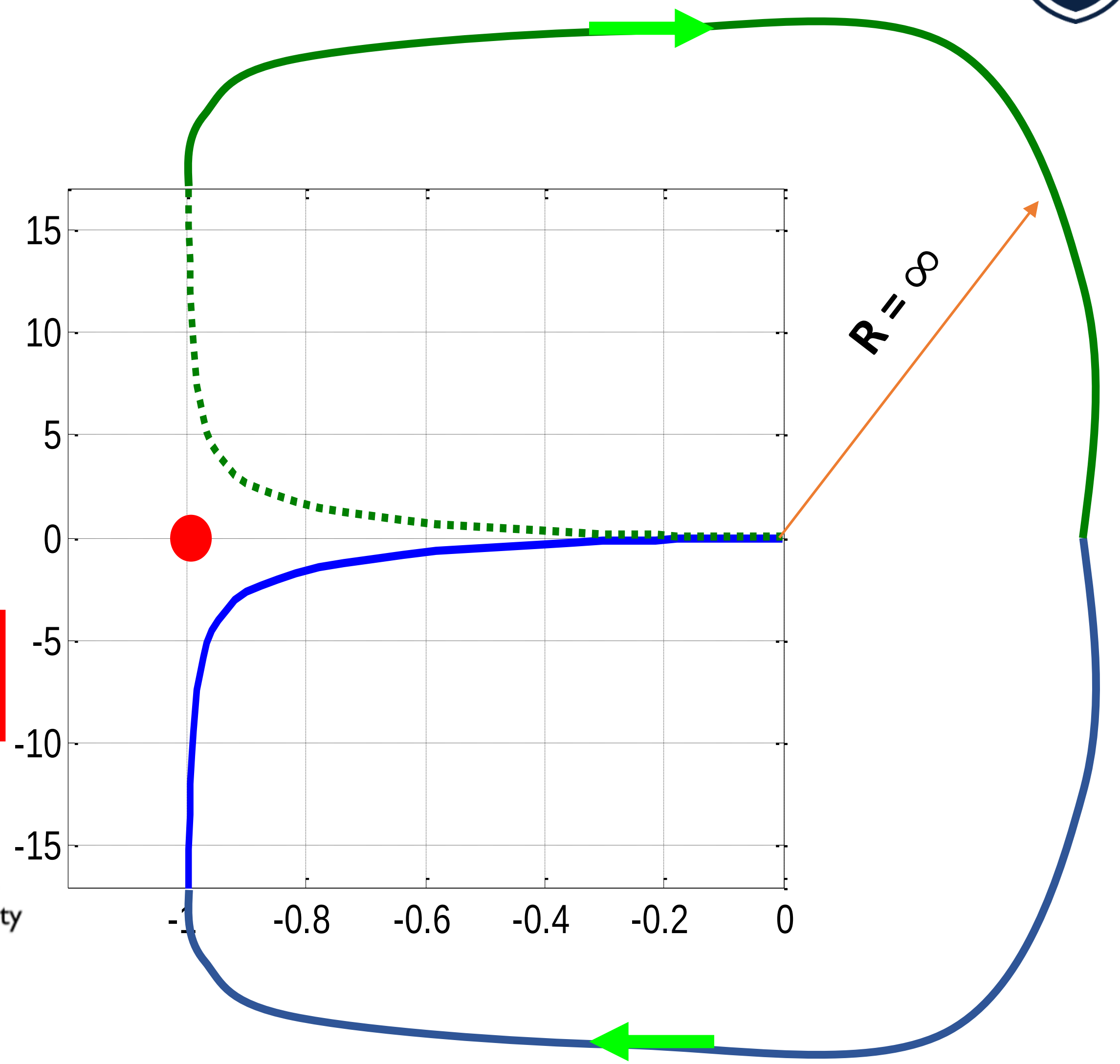
$$L(s) \approx \frac{1}{\varepsilon} e^{-j\theta}, \ \theta = 0 \to 90^\circ$$

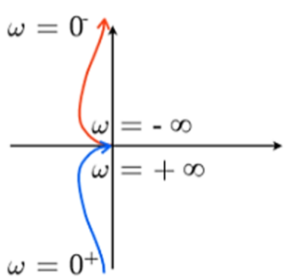


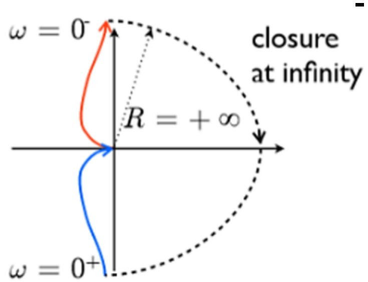
Example 2 (cont'd): $L(s)$ with an integrator

$$L(s) = \frac{1}{s(s+1)}$$

$$\boxed{P = 0,\ N = 0 \Rightarrow \text{CL stable}}$$







Example 2 (cont'd): Bode plot of $L(s)$

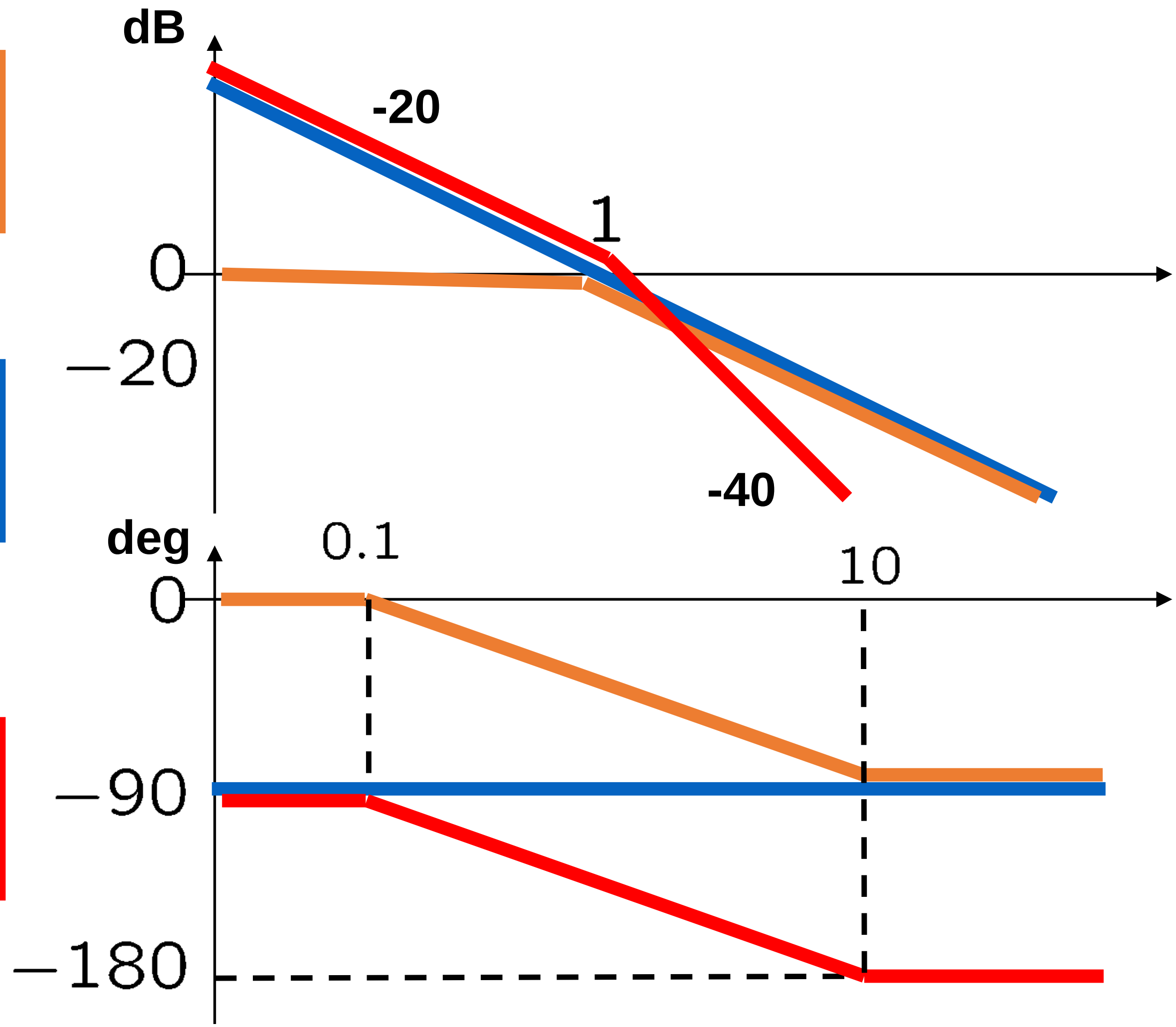
$$G_1(s) = \frac{1}{s+1}$$

$$\times$$

$$G_2(s) = \frac{1}{s}$$

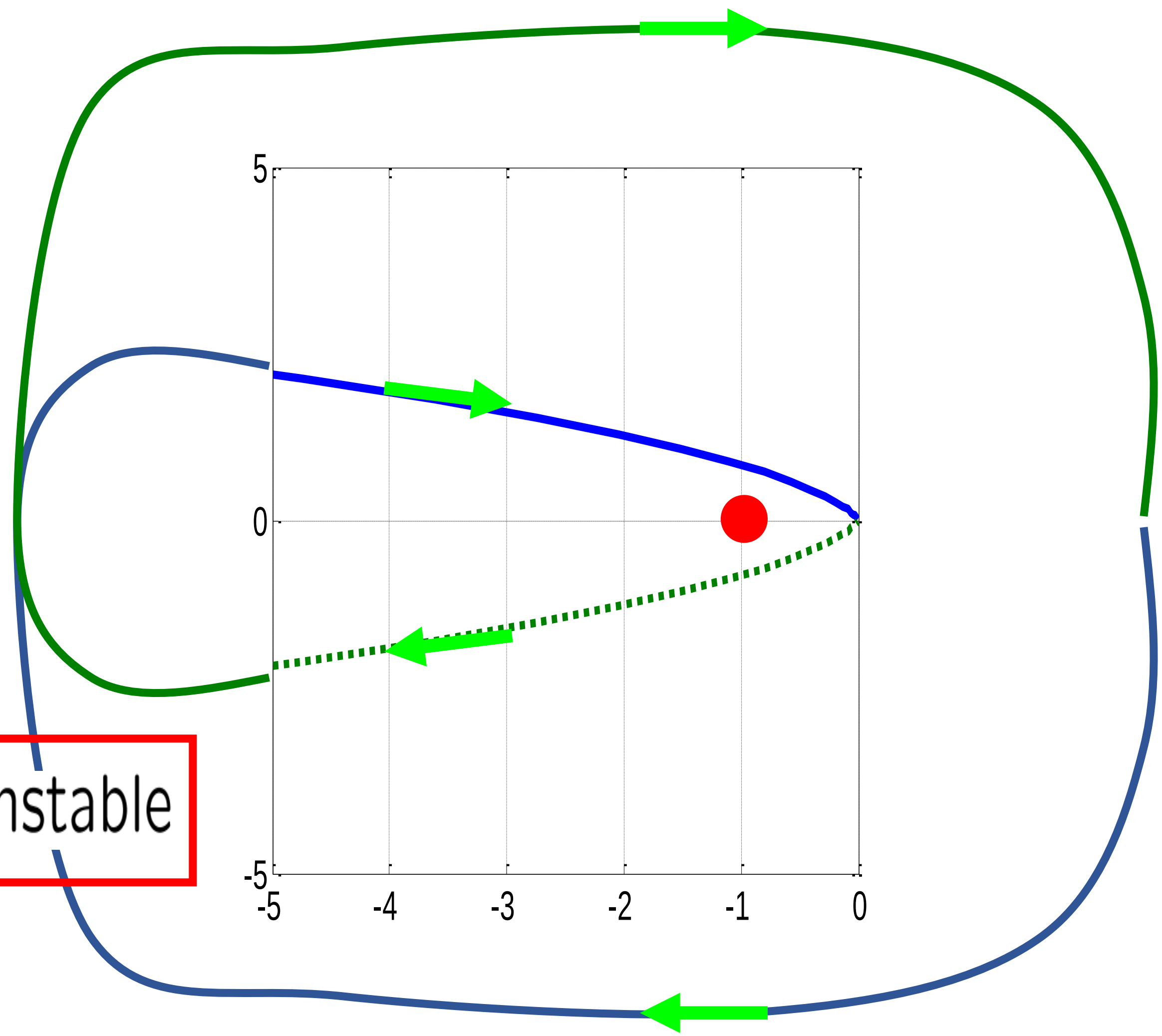
$$\Downarrow$$

$$L(s) = \frac{1}{s(s+1)}$$



Example 3: $L(s)$ with double integrator

$$L(s) = \frac{1}{s^2(s+1)}$$



$$\boxed{P = 0,\ N = 2 \Rightarrow \text{CL unstable}}$$

Example 3 (cont'd): Bode plot of *L*(*s*)

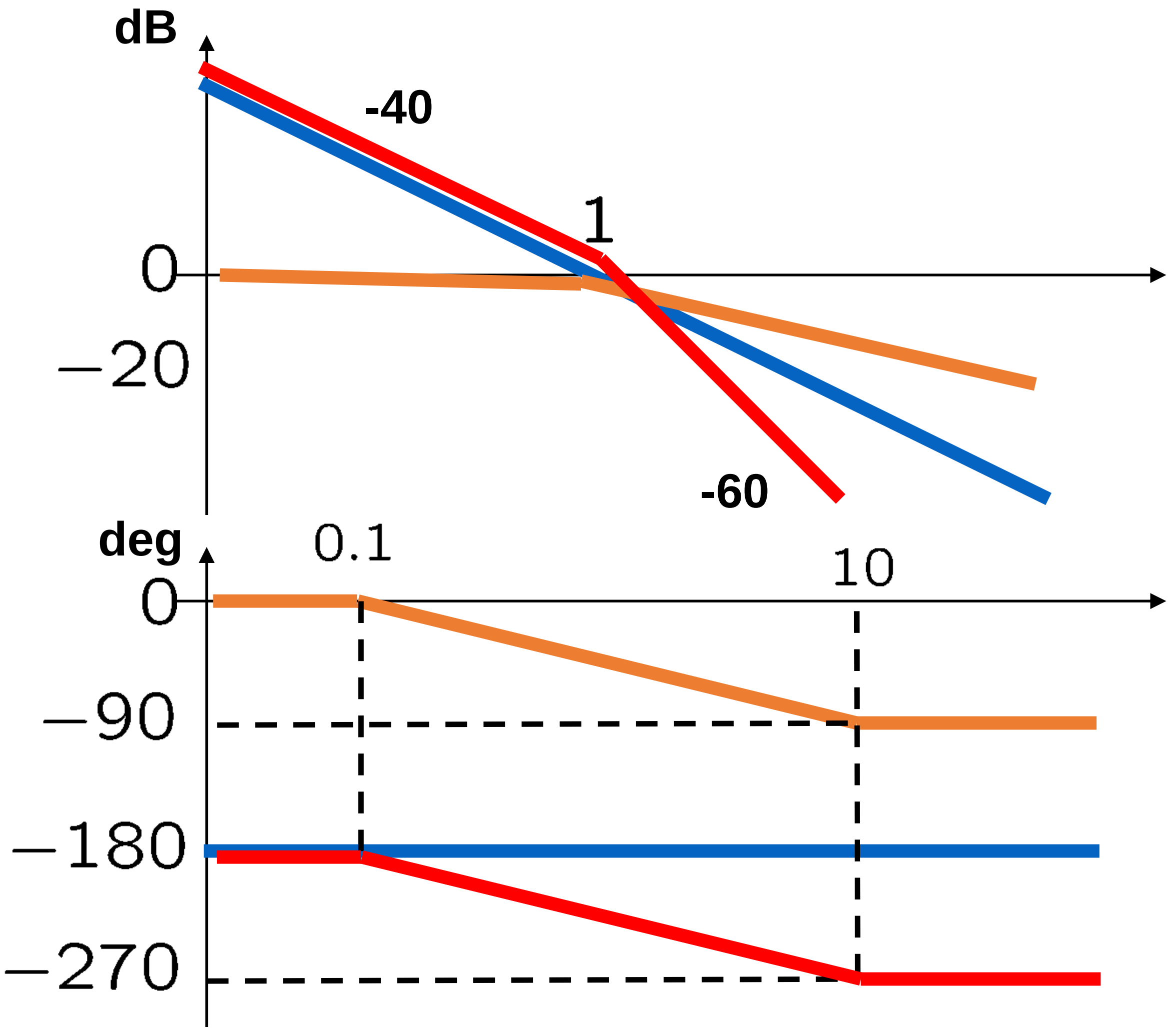
$$G_1(s) = \frac{1}{s+1}$$

$$\times$$

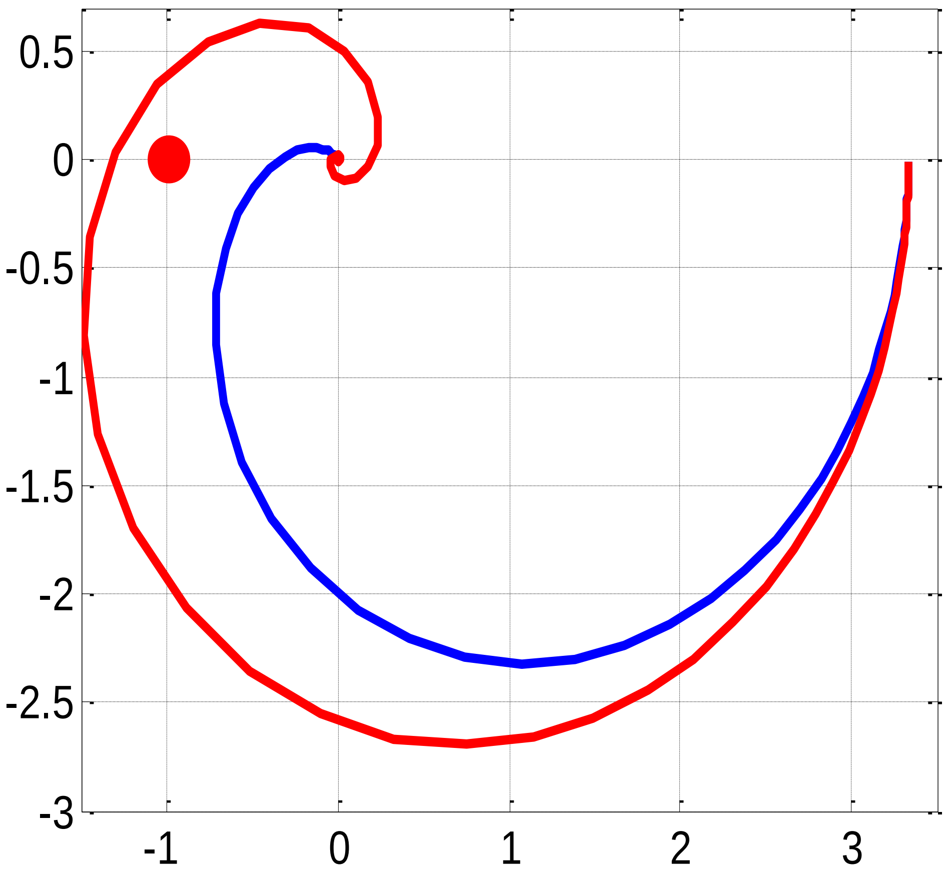
$$G_2(s) = \frac{1}{s^2}$$

$$\Downarrow$$

$$L(s) = \frac{1}{s^2(s+1)}$$



Example 4: $L(s)$ with a time-delay



$$L(s) = \frac{20}{(s+1)(s+2)(s+3)}$$

$$\boxed{P = 0,\ \ N = 0 \Rightarrow \text{CL stable}}$$

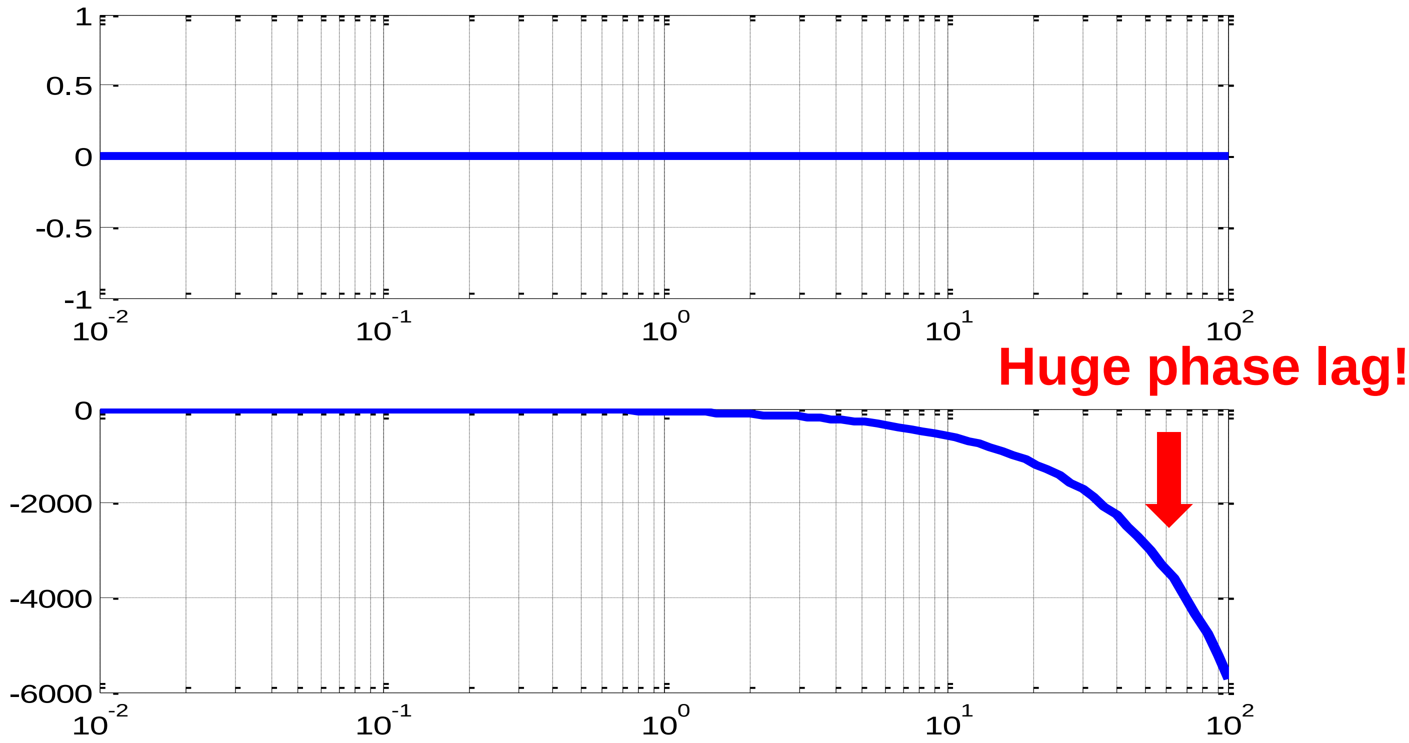
$$L(s) = \frac{20e^{-0.7s}}{(s+1)(s+2)(s+3)}$$

$$\boxed{P = 0,\ \ N = 2 \Rightarrow \text{CL unstable}}$$

Routh-Hurwitz is NOT applicable!

Bode plot of a time delay (review)

$$G(s) = e^{-Ts} \Rightarrow |G(j\omega)| = 1, \forall\omega, \ \angle G(j\omega) = -\omega T(\text{rad})$$



The phase lag causes instability of the closed-loop system, and thus, the difficulty in control.

Remarks on Nyquist stability criterion

- Nyquist stability criterion gives not only *absolute* but also *relative stability*.
  - Absolute stability: Is the closed-loop system stable or not? (Answer is yes or no.)
  - Relative stability: How "much" is the closed-loop system stable? (Margin of safety or margin of stability)
- Relative stability (stability margin) is important because a math model is never accurate.
- How to measure relative stability?
  - Use a "distance" from the critical point -1.
  - Gain margin (GM) & Phase margin (PM)

Gain margin (GM)

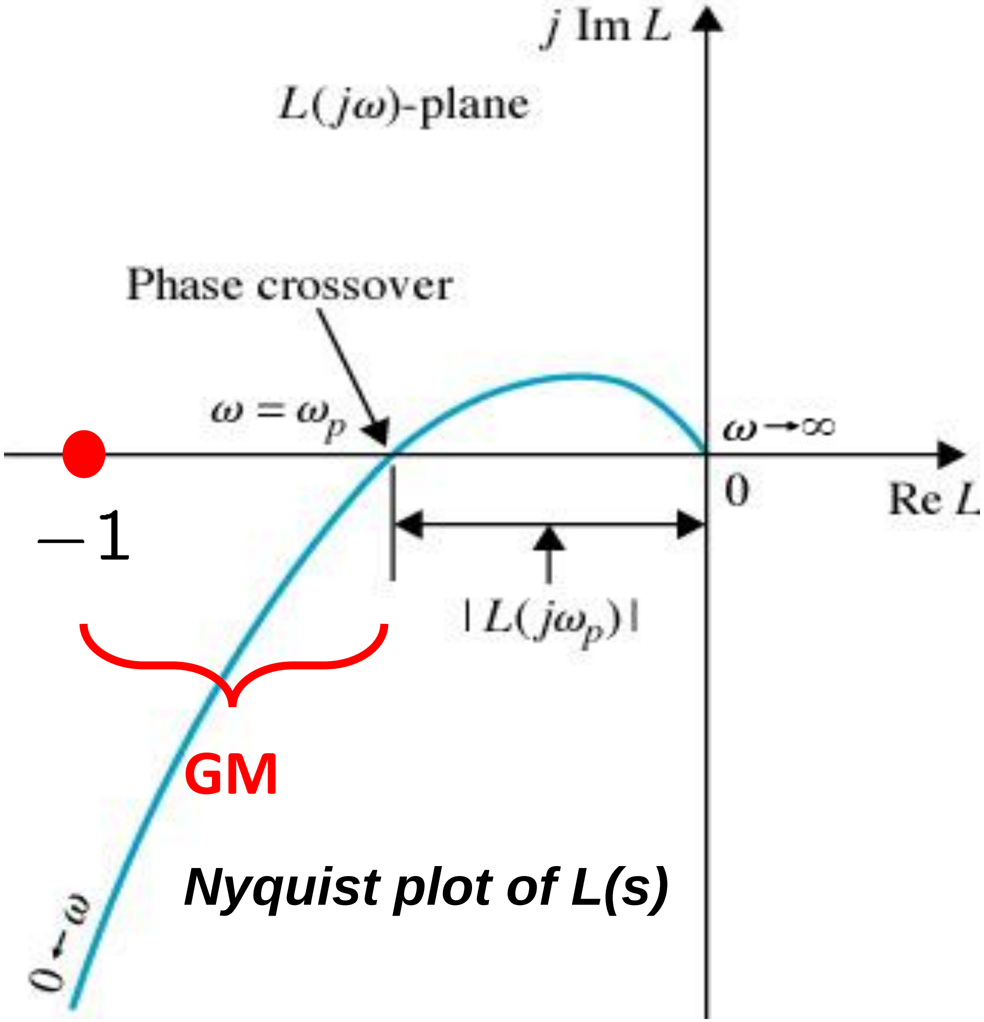
- **Phase** crossover frequency ω_p:

$$\angle L(j\omega_p) = -180^\circ$$

- **Gain margin** (in dB)

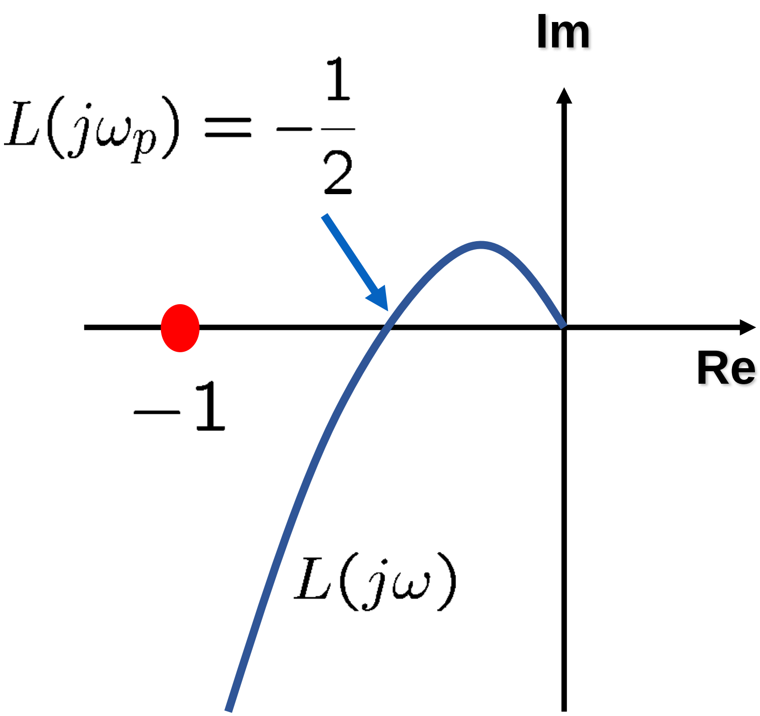
$$\boxed{GM = 20\log_{10}\frac{1}{|L(j\omega_p)|}}$$

  - GM indicates how much OL gain can be multiplied without violating CL stability.

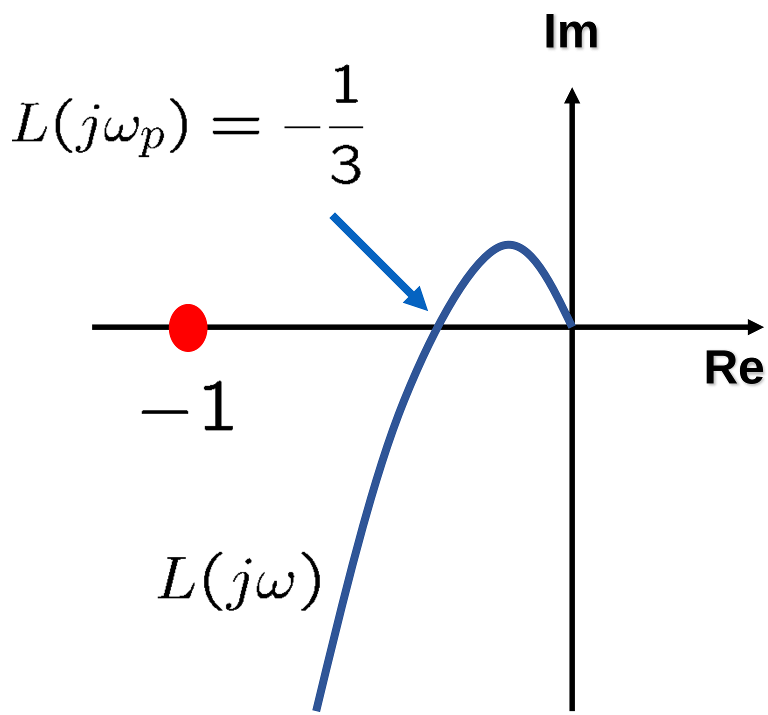


Nyquist plot of L(s)

Examples of GM

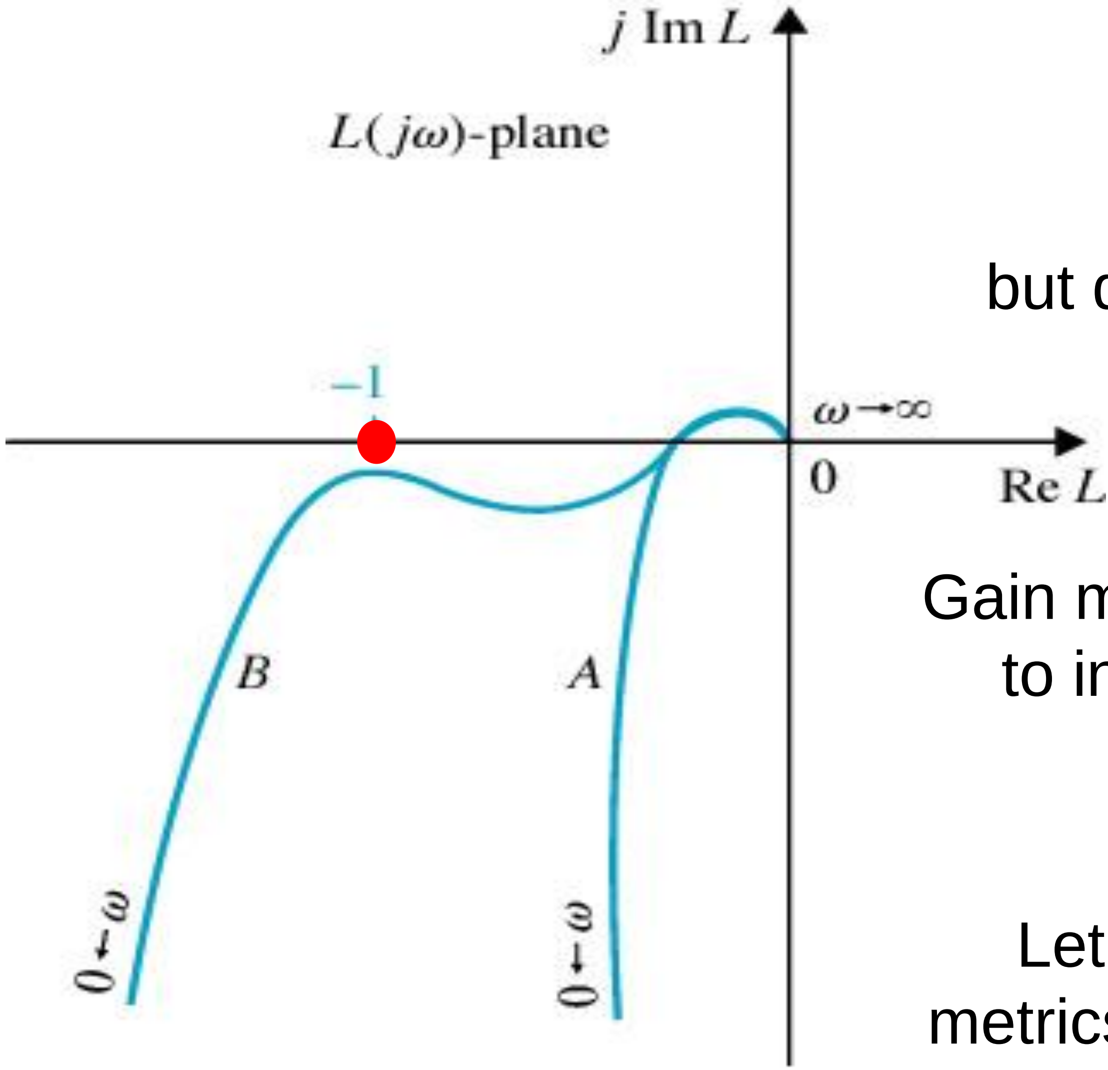


$$GM = 20\log_{10}\frac{\overbrace{1}}{\underbrace{|L(j\omega_p)|}_{2}} \approx 6\text{dB}$$



$$GM = 20\log_{10}\frac{\overbrace{1}}{\underbrace{|L(j\omega_p)|}_{3}} \approx 9.5\text{dB}$$

Reason why GM is inadequate



Same gain margin,
but different relative stability

Gain margin is often inadequate
to indicate relative stability

Let us look into another
metrics called **phase margin**!

Phase margin (PM)

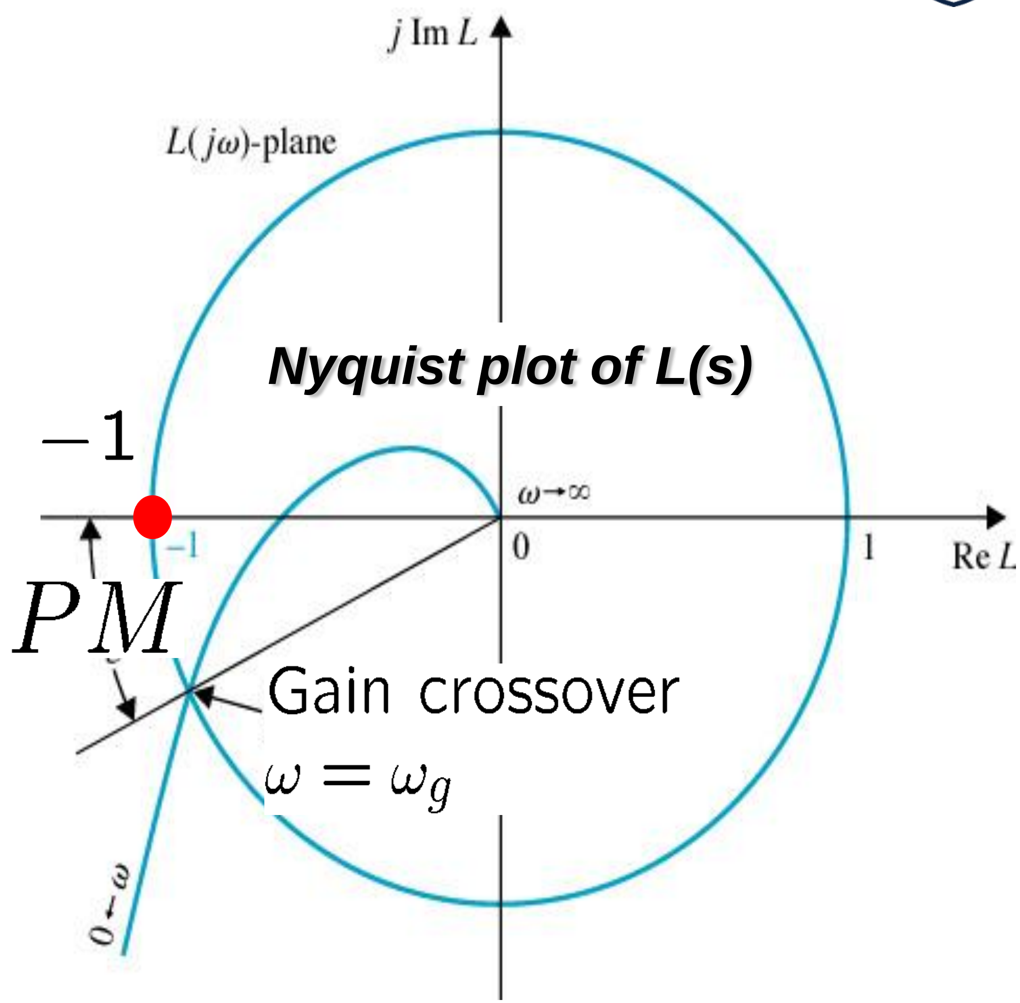
- **Gain** crossover frequency ω_g:

$$|L(j\omega_g)| = 1$$

- **Phase margin**

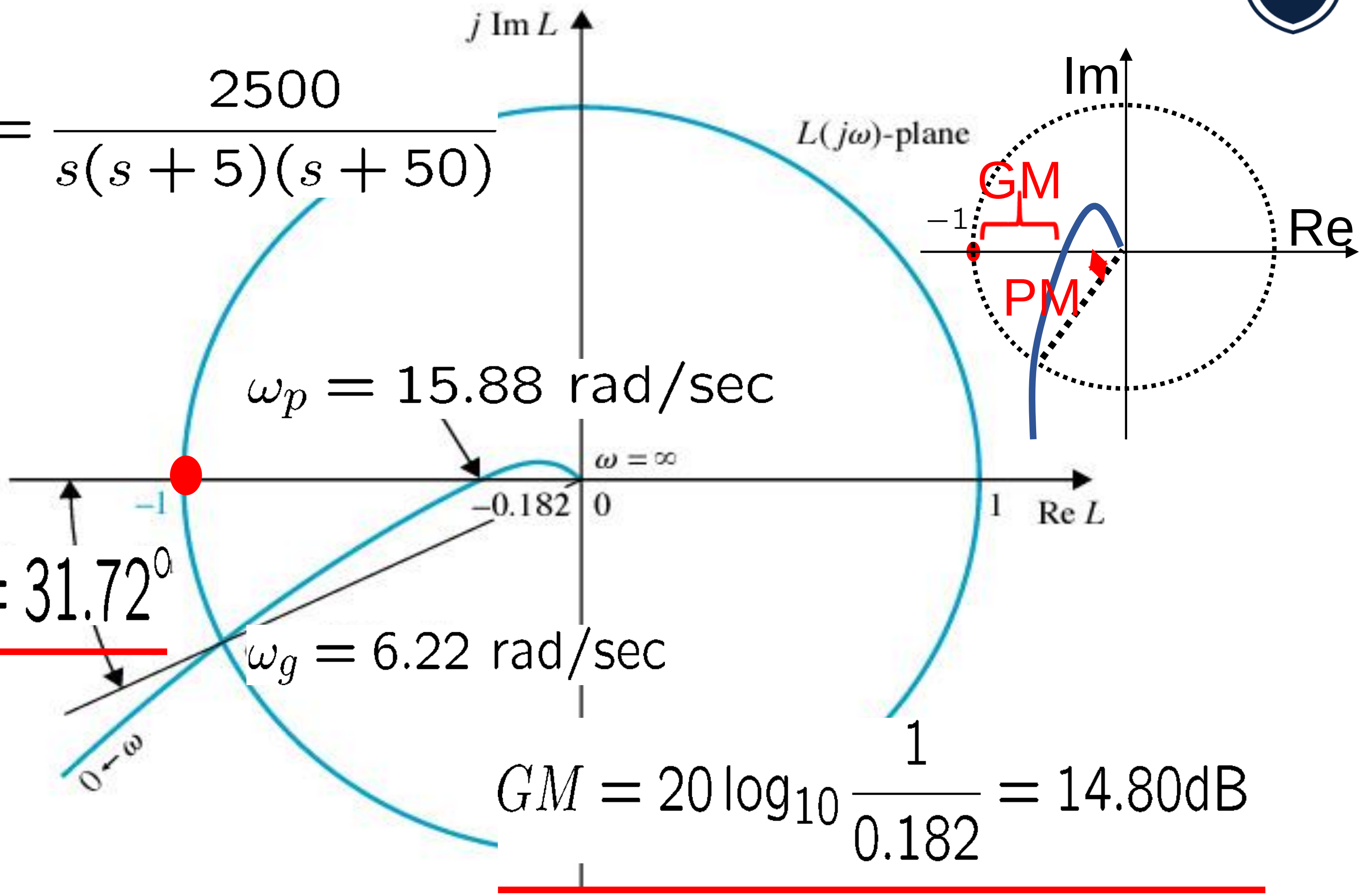
$$\boxed{PM = \angle L(j\omega_g) + 180^\circ}$$

  - PM indicates how much OL phase lag can be added without violating CL stability.



Example 1

$$L(s) = \frac{2500}{s(s+5)(s+50)}$$



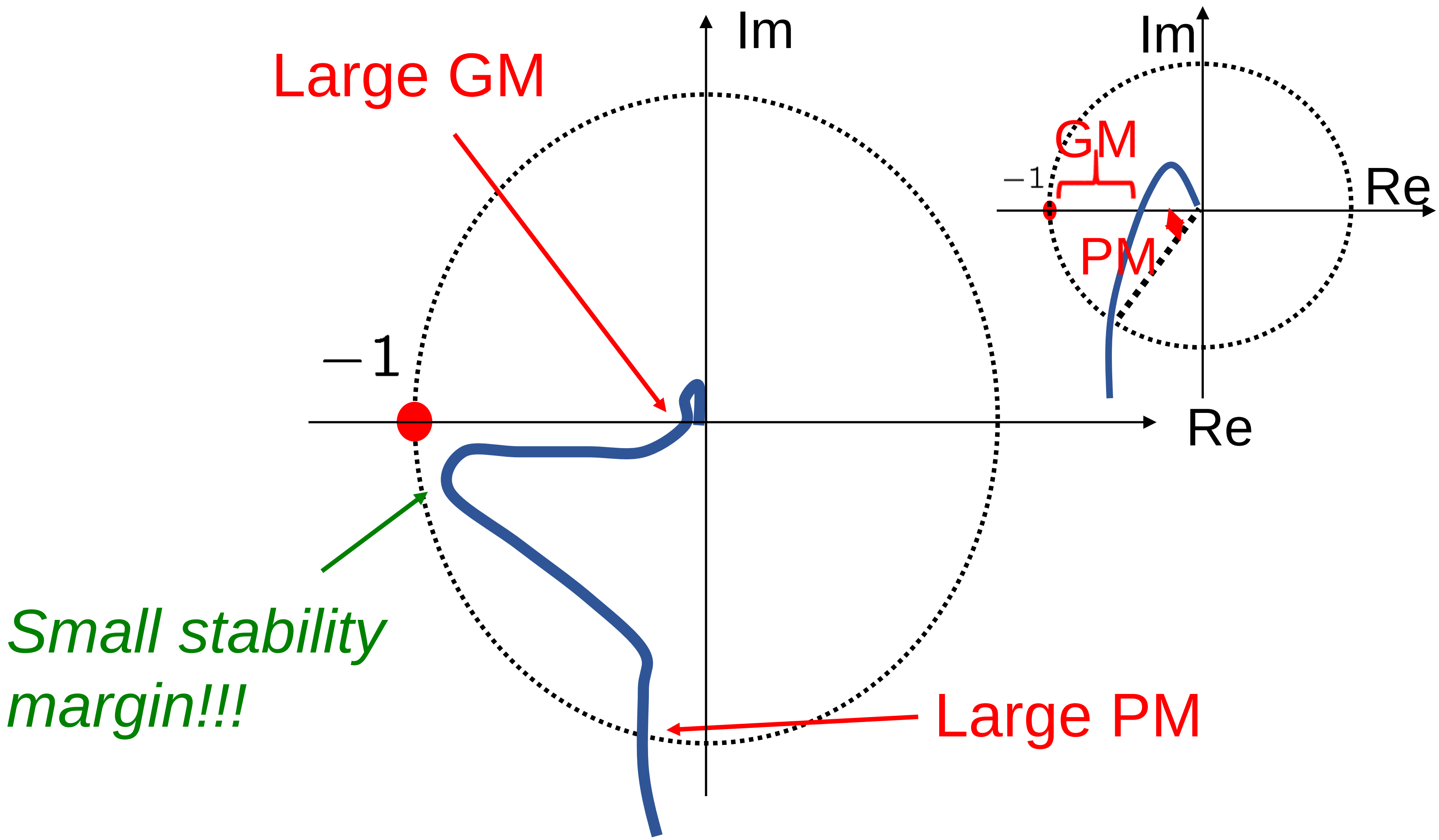
$\omega_p = 15.88$ rad/sec

$\omega_g = 6.22$ rad/sec

$$PM = 31.72^0$$

$$GM = 20\log_{10}\frac{1}{0.182} = 14.80\text{dB}$$

GM and PM: An extreme example

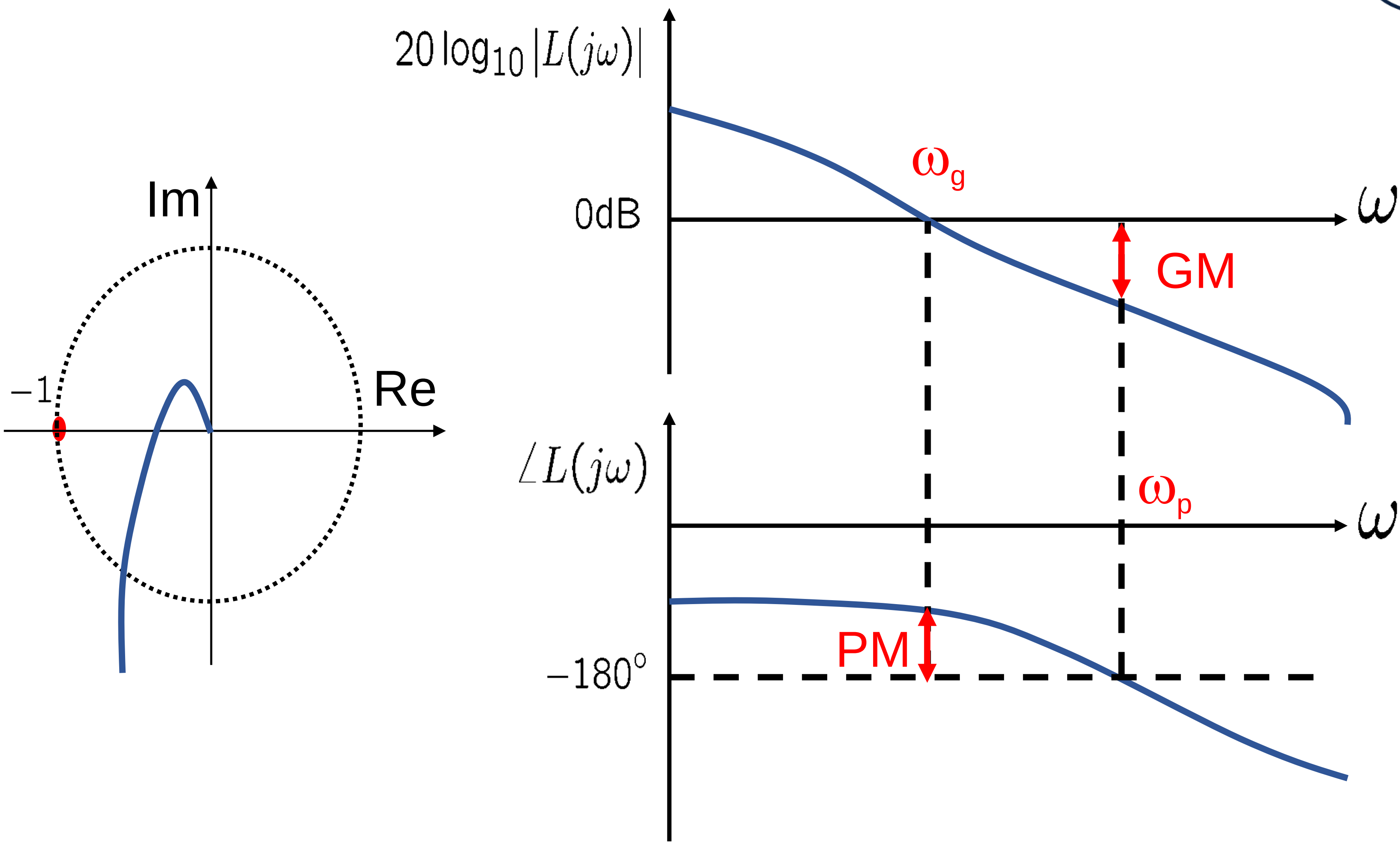


Notes on Nyquist plot

- Advantages
  - Nyquist plot can be used for study of closed-loop stability, for open-loop systems which are unstable and include time-delay.
- Disadvantage
  - Controller design on Nyquist plot is difficult. (Controller design on Bode plot is much simpler.)

We translate GM and PM defined in Nyquist plot into those in Bode plot!

Relative stability on Bode plot

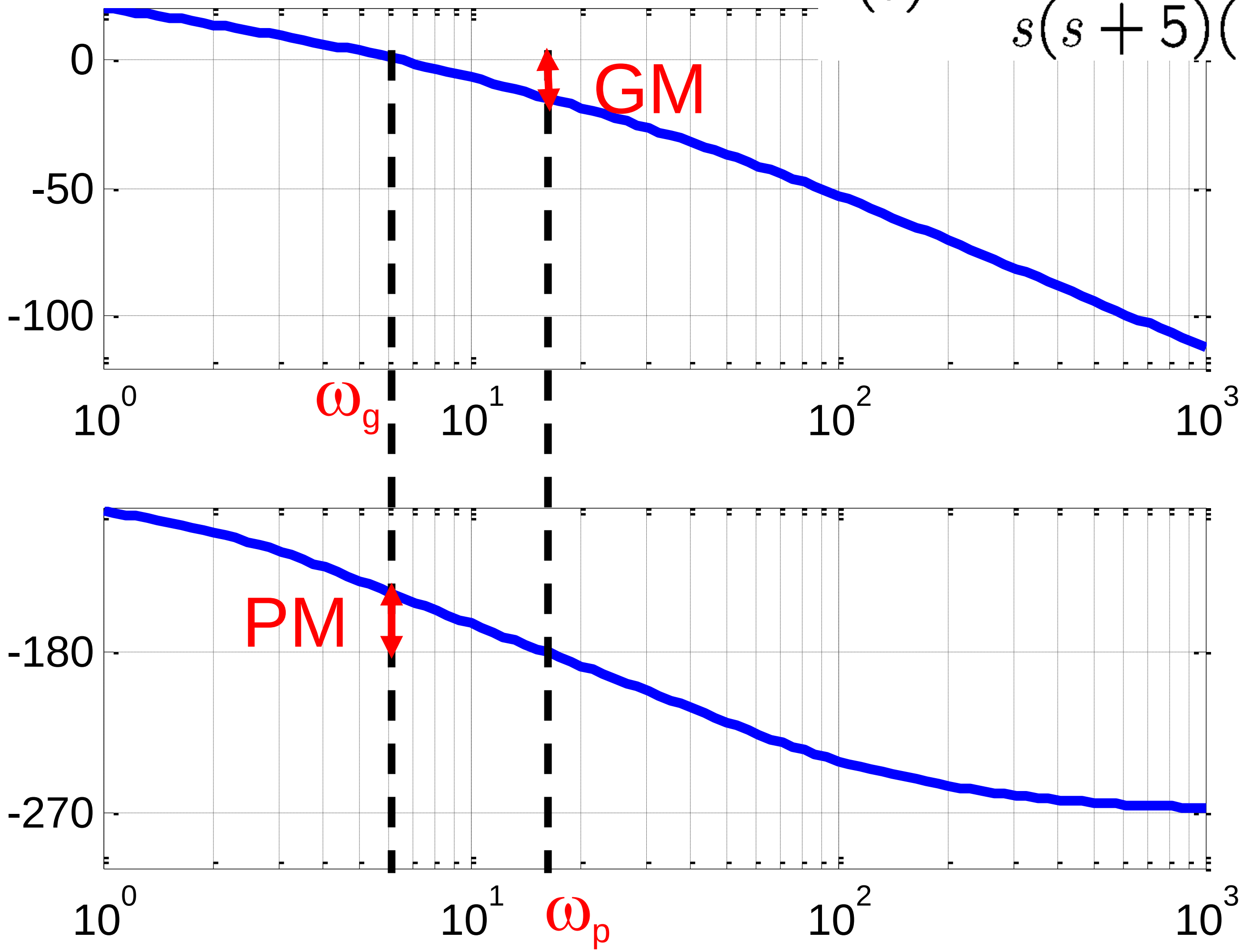


Notes on Bode plot

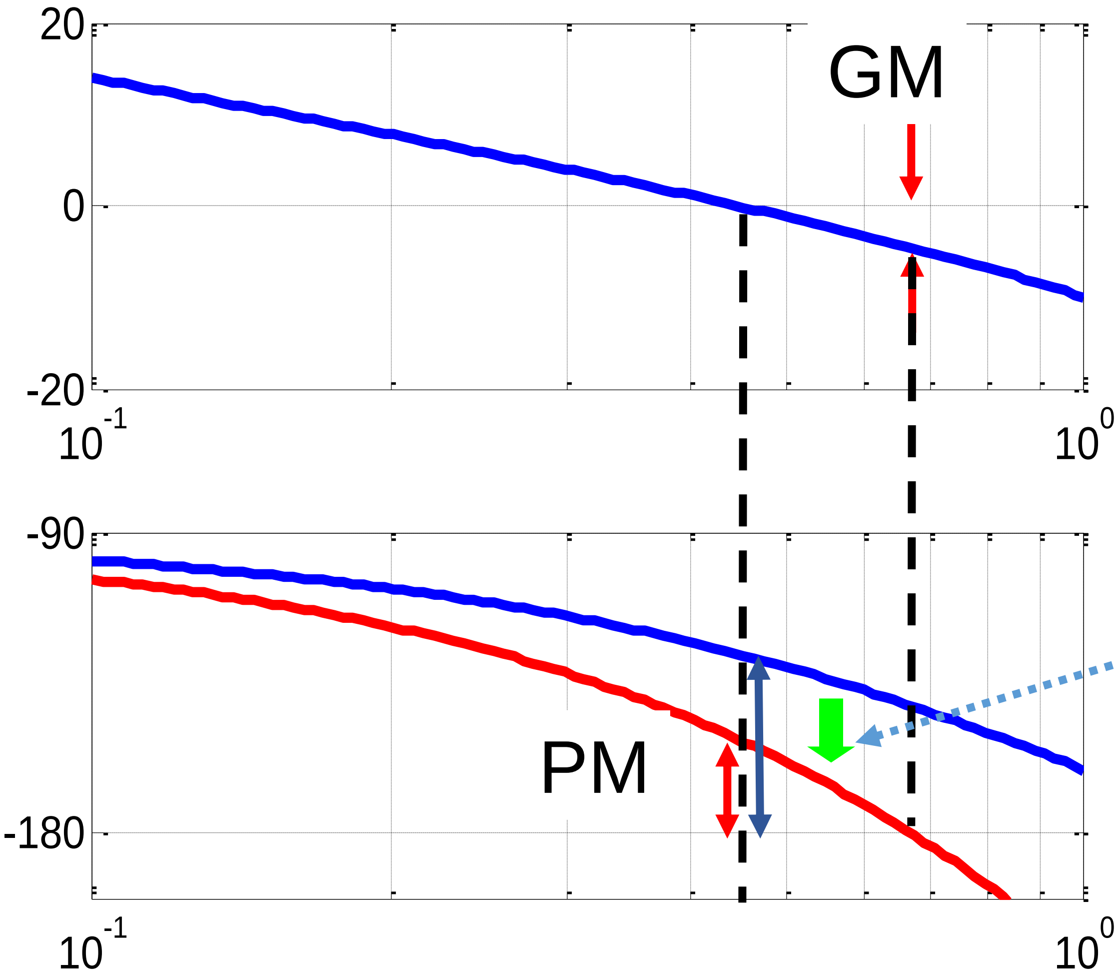
- Advantages
  - Without computer, Bode plot can be sketched easily by using straight-line approximations.
  - GM, PM, crossover frequencies are easily determined on Bode plot.
  - Controller design on Bode plot is simpler. (Next week)
- Disadvantage
  - If OL system has poles in open right half plane, it will be complicated to use Bode plot for closed-loop stability analysis.

Example 1 (revisited)

$$L(s) = \frac{2500}{s(s+5)(s+50)}$$



Relative stability with time delay



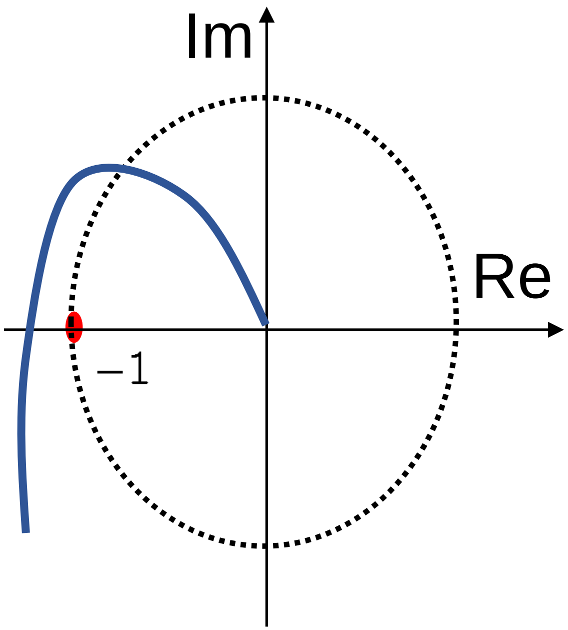
$$L(s) = \frac{1}{s(s+1)(s+2)}$$

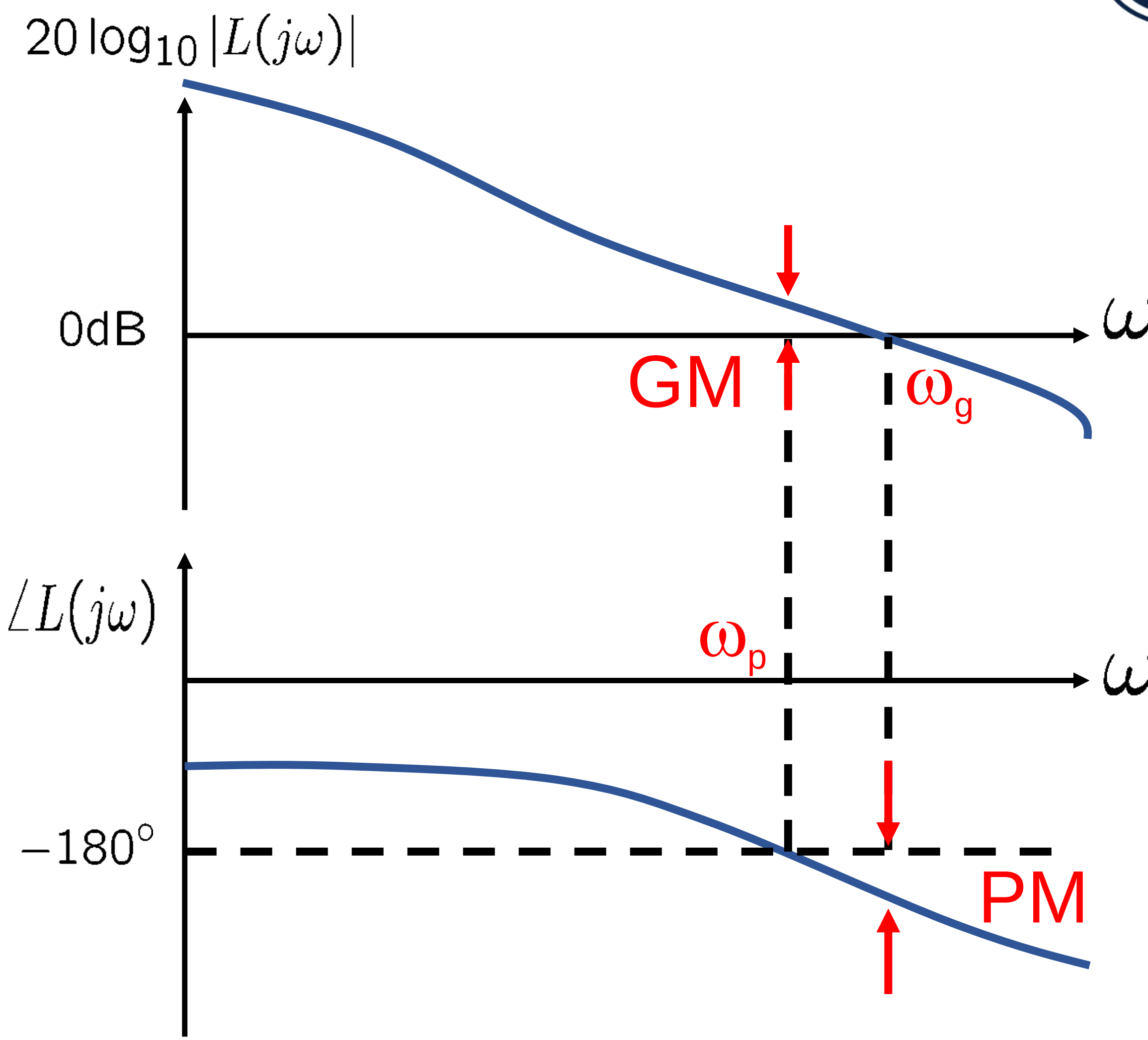
$$L(s) = \frac{e^{-s}}{s(s+1)(s+2)}$$

Time delay reduces relative stability!

Unstable closed-loop case

Assume that P = 0.





Summary

- Relative stability: Closeness of Nyquist plot to the critical point -1.
  - Gain margin, phase crossover frequency
  - Phase margin, gain crossover frequency
- Relative stability on Bode plot
- We normally emphasize PM in controller design.
- Next
  - Frequency domain specifications