Assignment 19 (ELEC 341 L19_RelativeStability)

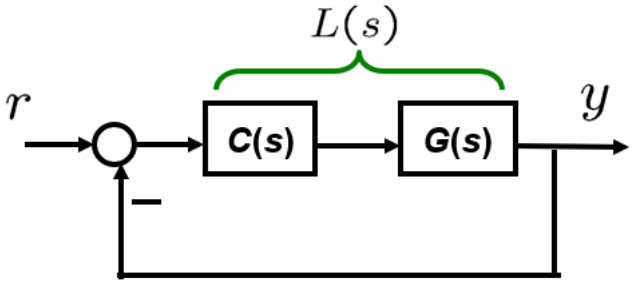
Problem 1:

For the system shown in the following figure, where

$$G(s) = \frac{1}{(s+5)(s+20)(s+50)}$$

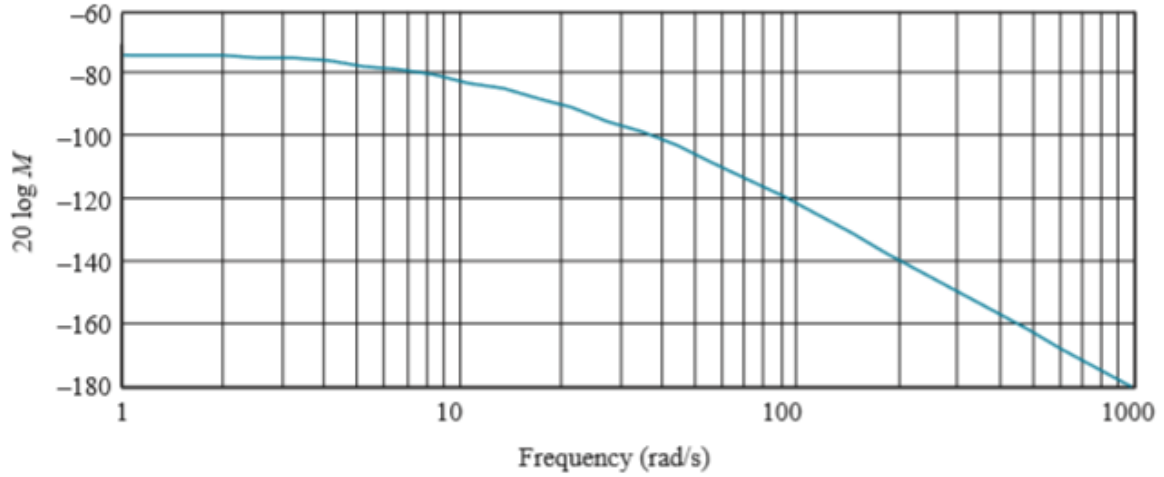
Do the following:

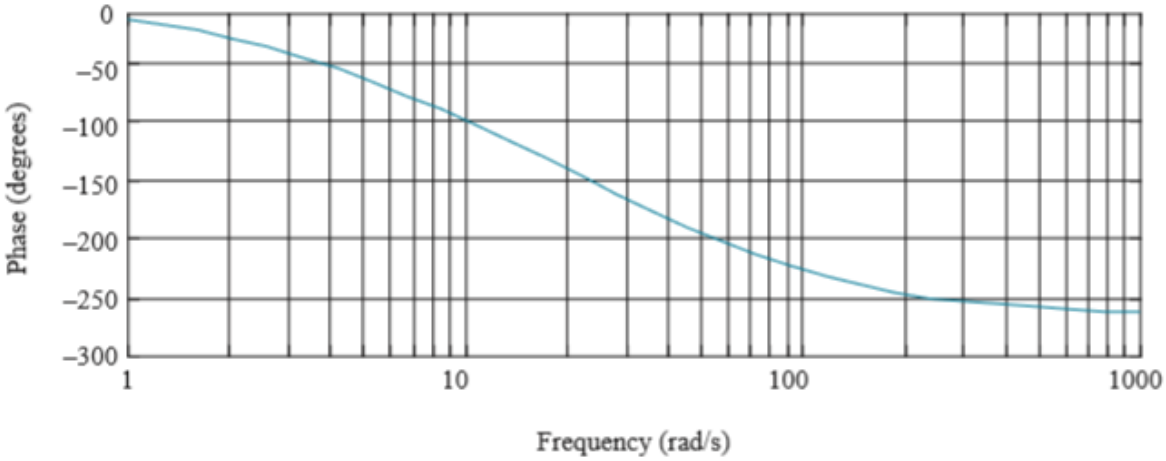
a) Draw the Bode log-magnitude and phase plots.
b) Evaluate gain margin, phase margin, gain cross-over frequency, and phase cross-over frequency analytically and show them in your Bode plots for *C*(*s*) = 10,000.



Solution:

a.


−60
−80
−100
−120
−140
−160
−180
20 log M
1
10
100
1000
Frequency (rad/s)


0
−50
−100
−150
−200
−250
−300
Phase (degrees)
1
10
100
1000
Frequency (rad/s)

$$L(s) = \frac{10^4}{(s+5)(s+20)(s+50)}$$

Find ω_g: $L(j\omega) = \frac{10^4}{(j\omega+5)(j\omega+20)(j\omega+50)} \rightarrow |L(j\omega)| = \frac{10^4}{\sqrt{\omega^2+25}\sqrt{\omega^2+400}\sqrt{\omega^2+2500}}$

$|L(j\omega_g)| = 1 \rightarrow \frac{10^4}{\sqrt{\omega_g^2+25}\sqrt{\omega_g^2+400}\sqrt{\omega_g^2+2500}} = 1 \rightarrow \boxed{\omega_g = 7.74}$

$\angle L(j\omega_g) = \angle \frac{10^4}{(7.74j+5)(7.74j+20)(7.74j+50)} = 0 - \{\tan^{-1}(\frac{7.74}{5}) + \tan^{-1}(\frac{7.74}{20}) + \tan^{-1}(\frac{7.74}{50})\} \rightarrow$

$\rightarrow \underline{\angle L(j\omega_g) = -87.09^\circ}$; $PM = \angle L(j\omega_g) + 180^\circ = -87.09^\circ + 180^\circ \rightarrow$

$\rightarrow \boxed{PM = 92.91^\circ}$; $\angle L(j\omega_p) = -180^\circ \rightarrow \tan^{-1}(\frac{\omega_p}{5}) + \tan^{-1}(\frac{\omega_p}{20}) + \tan^{-1}(\frac{\omega_p}{50}) = 180^\circ$

$\rightarrow \boxed{\omega_p = 36.7 \text{ rad/s}}$; $|L(j36.7)| = \frac{10^4}{\sqrt{36.7^2+25}\sqrt{36.7^2+400}\sqrt{36.7^2+2500}} \rightarrow$

$\rightarrow \underline{|L(j36.7)| = 0.1041}$; $GM = 20\log_{10}\frac{1}{|L(j\omega_p)|} = 20\log_{10}\frac{1}{0.1041}$

$\rightarrow \boxed{GM = 19.64 \text{ dB}}$

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The following graph is for C(s) = 10,000: