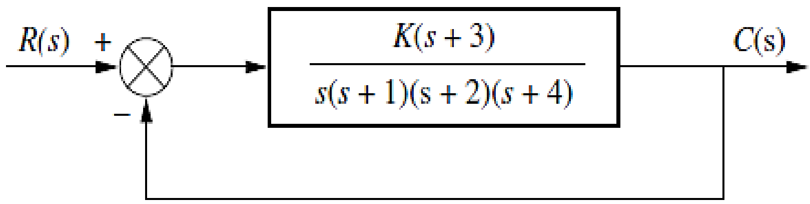
Assignment 12 (ELEC 341 L12_RLExamples)

Problem 1:

Sketch the root locus for the following system:



Solution:

Number of finite poles = n = 4
Number of finite zeros = m = 1
Number of asymptotes = n - m = 3
Number of branches or loci equals to the number of finite poles (n) = 4
Let us begin by calculating the asymptotes. The real-axis intercept is evaluated as:

$$\boxed{\sigma_a = \frac{\sum \text{finite poles} - \sum \text{finite zeros}}{\#\text{finite poles} - \#\text{finite zeros}}}$$

$$\sigma_a = \frac{(-1-2-4)-(-3)}{4-1} = -\frac{4}{3}$$

The angles of the lines that intersect at - 4/3 are:

$$= \frac{(2k+1)\pi}{4 \;-1}$$

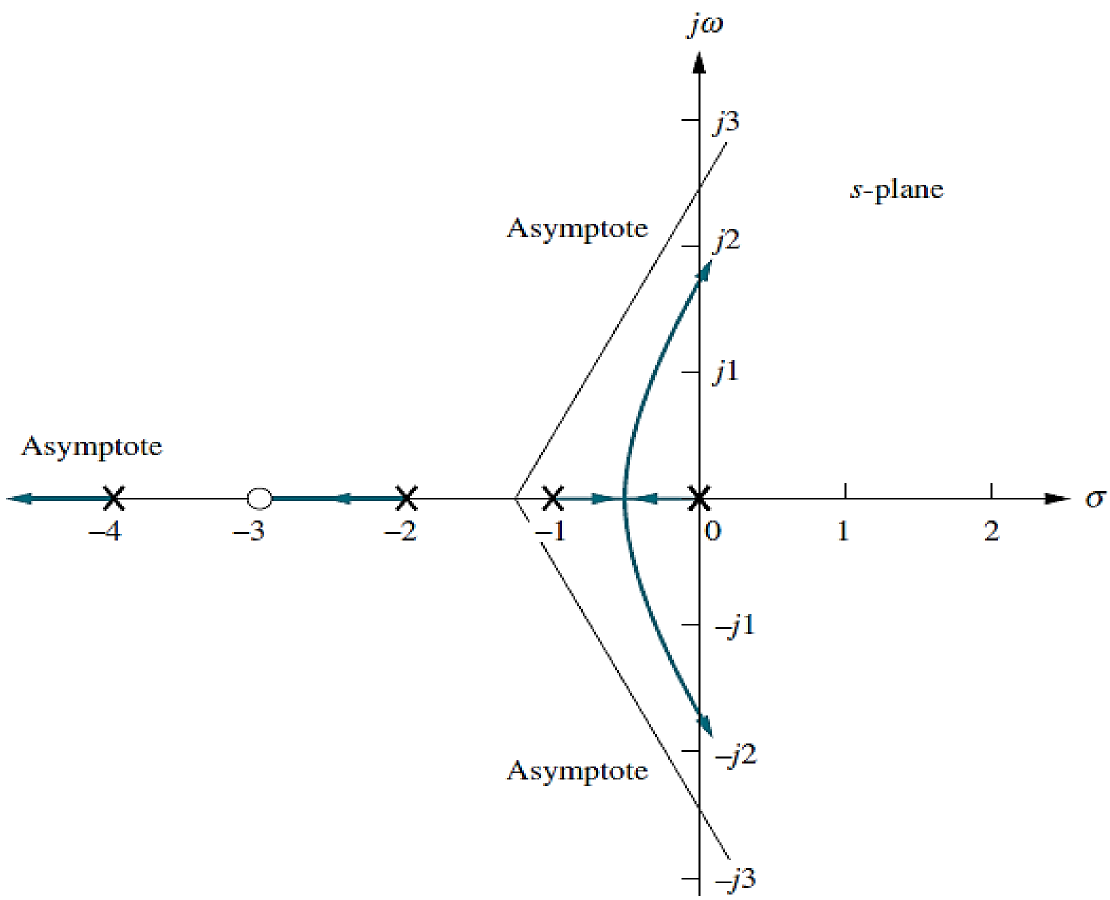
$$\text{where } k = 0, \pm 1, \pm 2, \pm 3$$

$$\theta_a = \frac{(2k+1)\pi}{\#\text{finite poles} - \#\text{finite zeros}} = \frac{(2k+1)}{4\ -1}$$

$$= \pi/3 \qquad \text{for } k = 0$$

$$= \pi \qquad \text{for } k = 1$$

$$= 5\pi/3 \qquad \text{for } k = 2$$



In this problem there are 4 open-loop poles and 1 open-loop zeros. Hence only one pole has made the pair with the zeros. Thus, there must be 3 zeros at infinity.
The asymptotes tell us how we get to these zeros at infinity.
The real-axis segments lie to the left of an odd number of poles and/or zeros.
The locus starts at the open-loop poles and ends at the open-loop zeros.
There is only one open-loop finite zero and three infinite zeros.
The three zeros at infinity are at the ends of the asymptotes.

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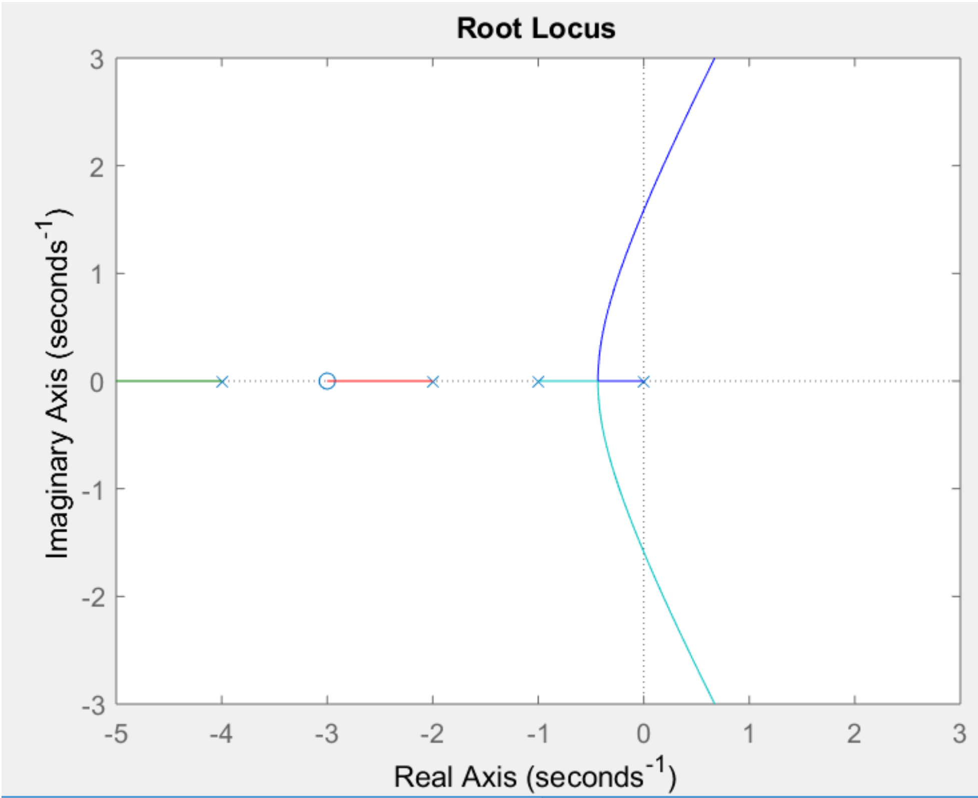

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Problem 2:

Given the unity feedback system of Figure , where

$$G(s) = \frac{K(s+2)}{s(s+1)(s+3)(s+5)}$$

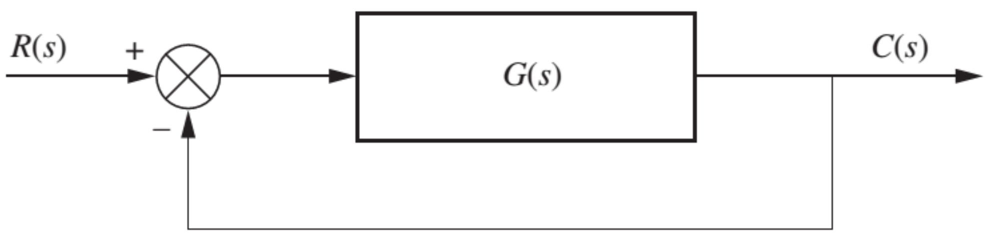
do the following:

a. Sketch the root locus.

b. Find the asymptotes.

c. Find the value of gain that will make the system marginally stable.

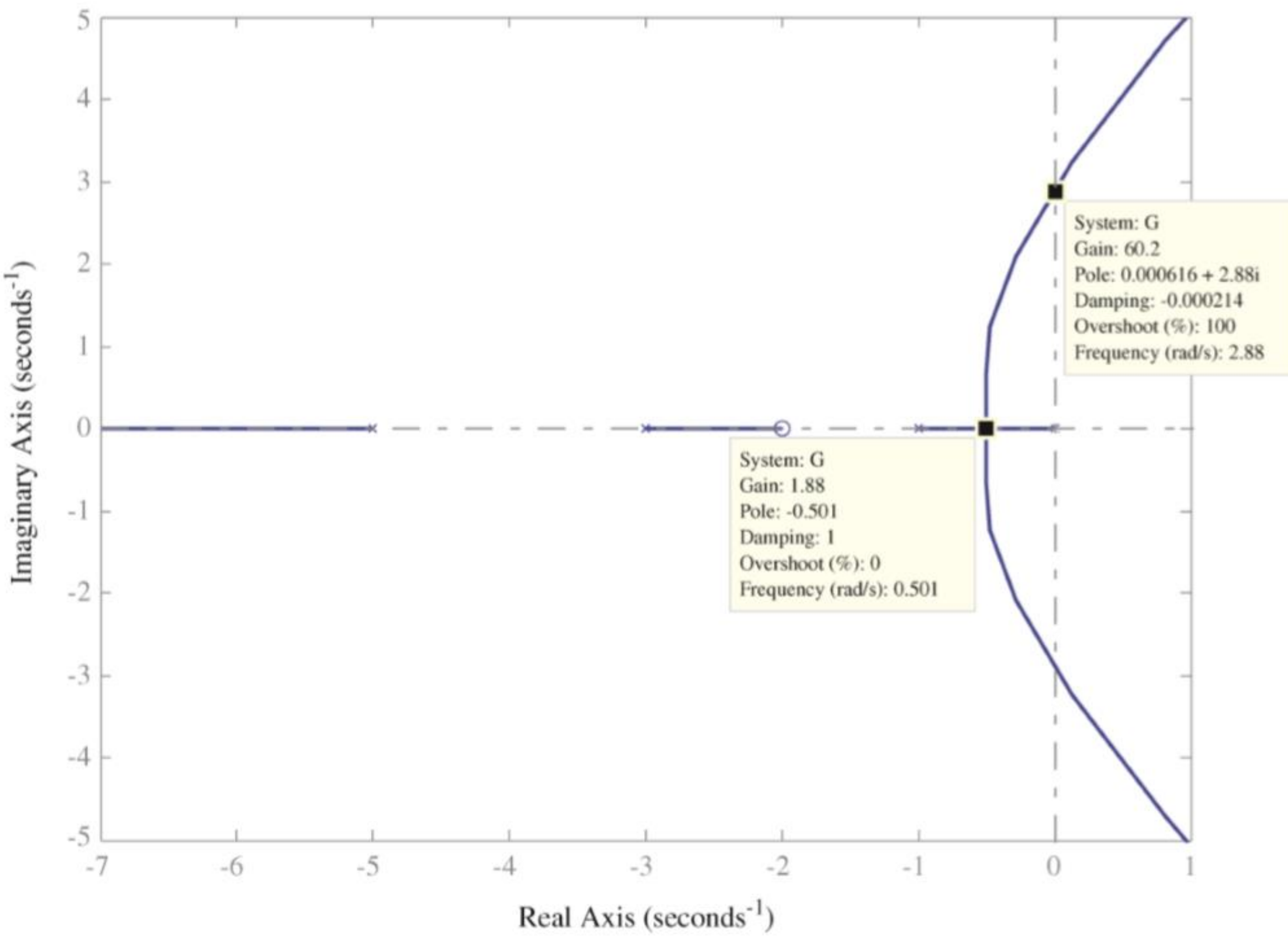
d. Find the value of gain for which the closed-loop transfer function will have a pole on the real axis at −0.5.



FIGURE

Solution:

a.


System: G
Gain: 60.2
Pole: 0.000616 + 2.88i
Damping: -0.000214
Overshoot (%): 100
Frequency (rad/s): 2.88
System: G
Gain: 1.88
Pole: -0.501
Damping: 1
Overshoot (%): 0
Frequency (rad/s): 0.501
Imaginary Axis (seconds^-1)
Real Axis (seconds^-1)
5
4
3
2
1
0
-1
-2
-3
-4
-5
-7
-6
-5
-4
-3
-2
-1
0
1

$$G(s) = \frac{k(s+2)}{s(s+1)(s+3)(s+5)}$$

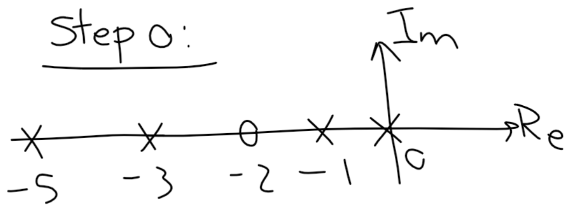
a) RL = ?

b) asymptotes = ?

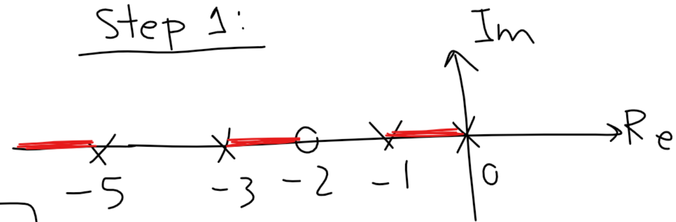
c) $K\big|_{\text{marginally stable}} = ?$

d) $K\big|_{s=-0.5} = ?$

Step 0:



Step 1:



step 2: $n = 4, m = 1$

$r = n - m = 4 - 1 = 3 \longrightarrow \boxed{r = 3}$; $\alpha = \dfrac{\sum \text{Pole} - \sum \text{zero}}{r} =$

$$= \frac{\{(-5)+(-3)+(-1)+(0)\} - \{-2\}}{3} \longrightarrow \boxed{\alpha = -2.33 = -\frac{7}{3}}$$

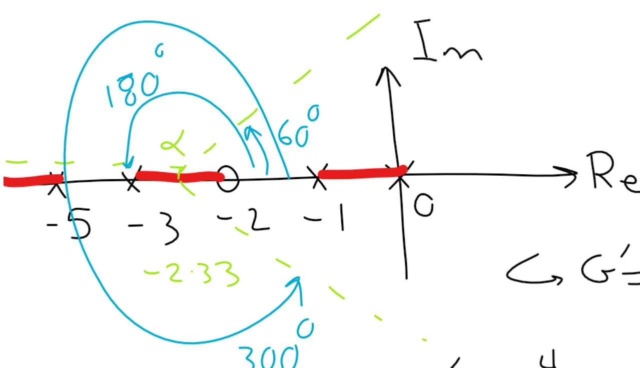
$$\text{angle} = \frac{(2K+1)\pi}{r} = \frac{(2k+1)\pi}{3}$$

$K=0 \rightarrow \pi/3 = 60^\circ$

$K=1 \rightarrow \pi = 180^\circ$

$K=2 \rightarrow \dfrac{5\pi}{3} = 300^\circ$

$r - 1 = 3 - 1 = 2$



step 3: $L = G = \dfrac{K(s+2)}{S(s+1)(s+3)(s+5)}$

$\hookrightarrow G' = L' = 0 \rightarrow \dfrac{-3s^4 - 26s^3 - 77s^2 - 92s - 30}{s^2(s+1)^2(s+3)^2(s+5)^2} = 0 \rightarrow$

$\hookrightarrow 3s^4 + 26s^3 + 77s^2 + 92s + 30 = 0$

(it has only 2 roots)

$\hookrightarrow s_1 = -0.50572$

$s_2 = -4.04317$

$s_1 = -0.50572$; $K = -\frac{1}{L(s)} = -\frac{1}{\left\{ \frac{(-0.50572+2)}{(-0.50572)(-0.50572+1)\ldots} \right\}}$

$\rightarrow \boxed{K = 1.875} \rightarrow$ $\boxed{\text{s = -0.5057 is a break-away point \& K is 1.875 at this point}}$

Just by accident, this answers part 'd' as well. That is,
at $s \approx -0.5 \longrightarrow \boxed{K \approx 1.875}$

$s_2 = -4.04317 \longrightarrow K = -6.01$ (not acceptable)

Find $K\Big|_{\text{marginally stable}} = ?$ $\quad CLTF = \dfrac{\frac{K(s+2)}{s(s+1)(s+3)(s+5)}}{1 + \frac{K(s+2)}{s(s+1)(s+3)(s+5)}} \longrightarrow$

$\longrightarrow CLTF = \dfrac{\frac{K(s+2)}{\varnothing}}{\frac{\varnothing + K(s+2)}{\varnothing}} \longrightarrow CLTF = \dfrac{K(s+2)}{s(s+1)(s+3)(s+5) + K(s+2)}$

$\longrightarrow$ Ch. eq: $\underline{s^4 + 9s^3 + 23s^2 + (15+K)s + 2K = 0}$

s^4	1	23	2K
s^3	9	15+K	0
s^2	21.3334−0.1111K	2K	
s^1	a	0	
s^0	2K		

$\dfrac{23\times 9 - (15+K)}{9} = \underline{21.3334 - 0.1111K}$

$\dfrac{(15+K)(21.3334-0.1111K) - 9\times 2K}{21.3334 - 0.1111K} =$

$\underline{a = 15 + K - \dfrac{18K}{21.3334 - 0.1111K}}$