a place of mind

ELEC 341: Systems and Control

Lecture 20

Stability margin: Examples on frequency domain specifications

a place of mind

UBC

Course roadmap

Modeling

- Laplace transform
- Transfer function
- Models for systems
  - mechanical
  - electrical
  - electromechanical
- Linearization, delay

Analysis

- Stability
  - Routh-Hurwitz
  - Nyquist
- Time response
  - Transient
  - Steady state
- Frequency response
  - Bode plot

Design

- Design specs
- Root locus
- Frequency domain
- PID & Lead-lag
- Design examples

Matlab simulations

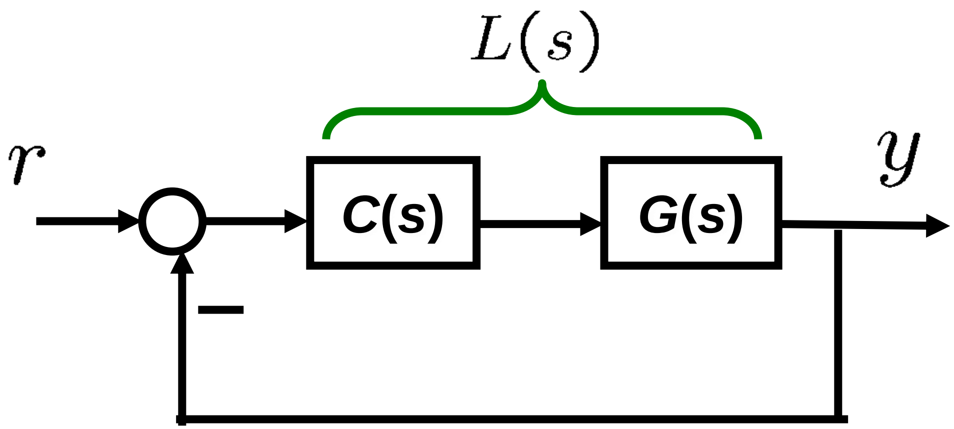
Closed-loop stability criterion (review)

- Closed-loop stability can be determined by the roots of the characteristic equation

$$1 + L(s) = 0,\ \ L(s) \ = G(s)C(s)$$

- Closed-loop system is stable if the Ch. Eq. has all roots in the open left half plane.
- How to check the closed-loop stability?
  - Computation of all the roots
  - Routh-Hurwitz stability criterion
  - *Nyquist stability criterion: Open-loop FRF L(jω) contains information of closed-loop stability.*

Nyquist stability criterion (review)



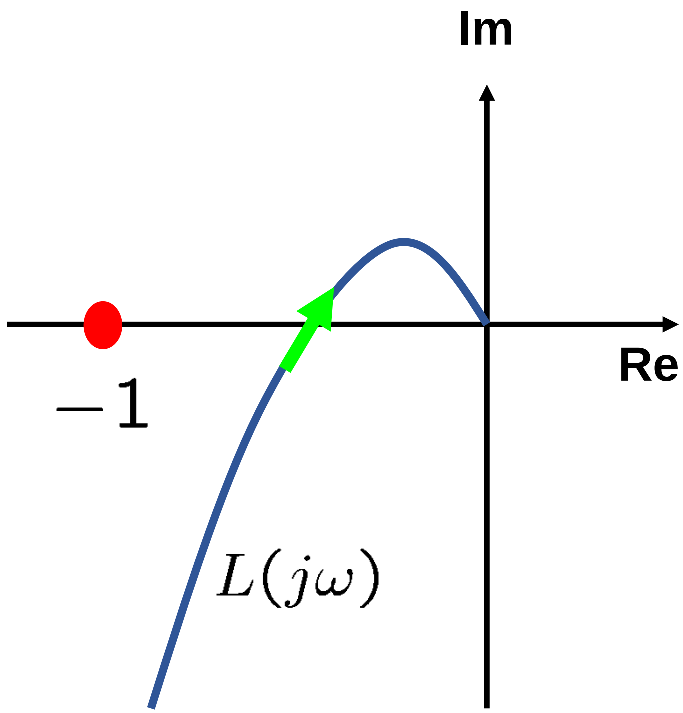
$$\boxed{\text{CL system is stable} \Leftrightarrow Z = P + N = 0}$$

Z: # of CL poles in open RHP

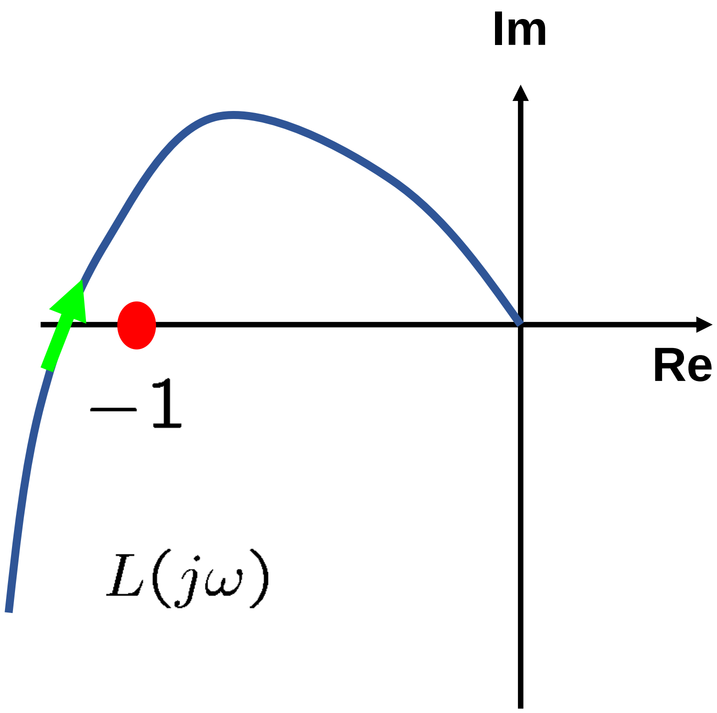
P: # of OL poles in open RHP (given)

N: # of clockwise/counterclockwise encirclement of -1 by Nyquist plot of *L*(*s*)

Examples when $P = 0$ (review)



CL stable



CL unstable

Gain margin (GM) (review)

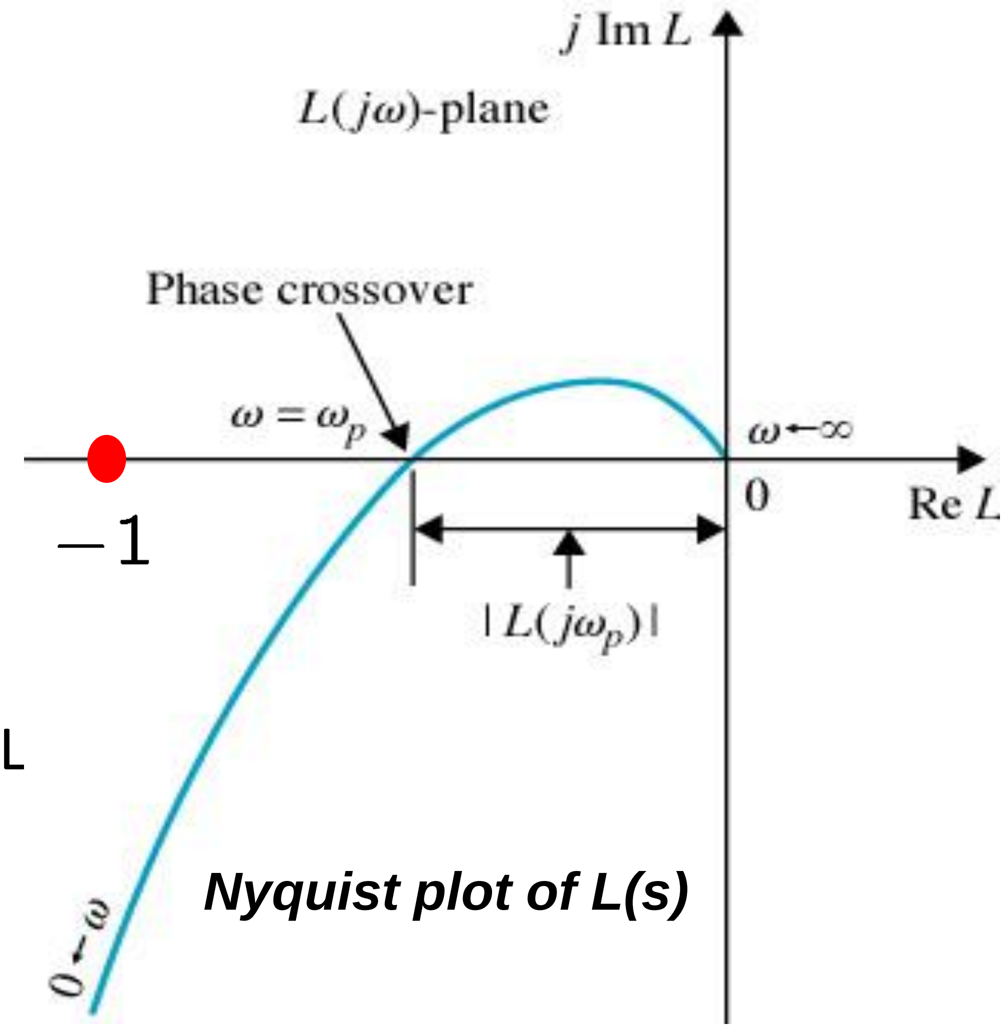
- **Phase** crossover frequency ω_p:

  $$\angle L(j\omega_p) = -180^\circ$$

- **Gain margin** (in dB)

$$\boxed{GM = 20\log_{10}\frac{1}{|L(j\omega_p)|}}$$

  - Indicates how much OL gain can be multiplied without violating CL stability.



Nyquist plot of L(s)

Phase margin (PM) (review)

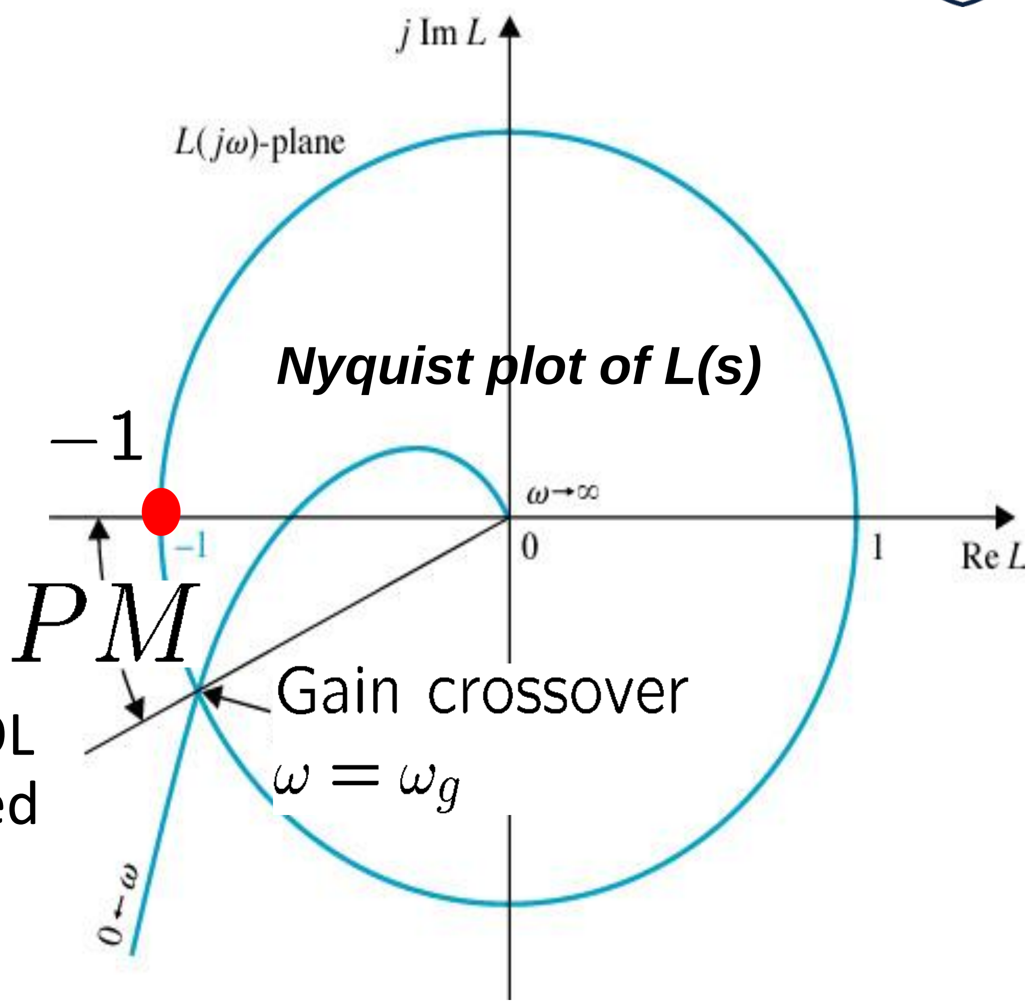
- **Gain** crossover frequency ω_g:

$$|L(j\omega_g)| = 1$$

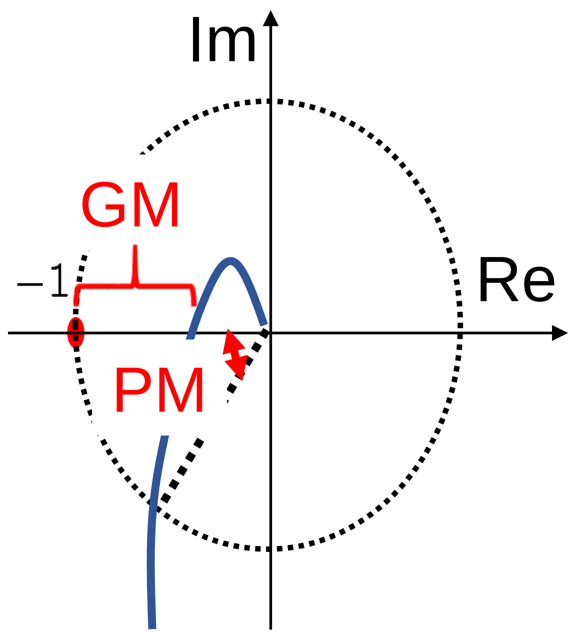
- **Phase margin**

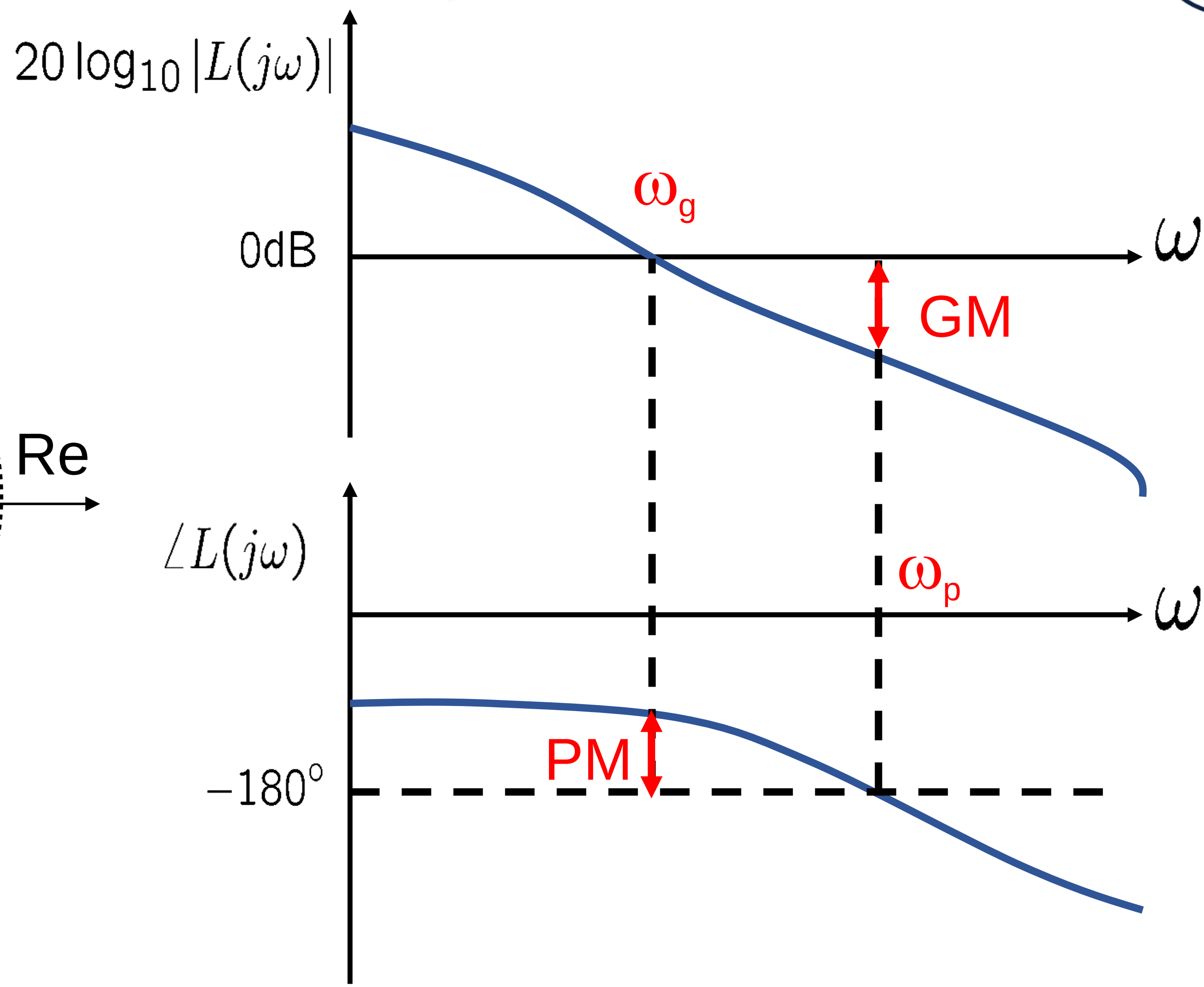
$$\boxed{PM = \angle L(j\omega_g) + 180^\circ}$$

  - Indicates how much OL phase lag can be added without violating CL stability.



Relative stability on Nyquist plot and Bode plot





Example 1: (GM & PM)

$$G_1(s) = \frac{1}{s+1}$$

$$\times$$

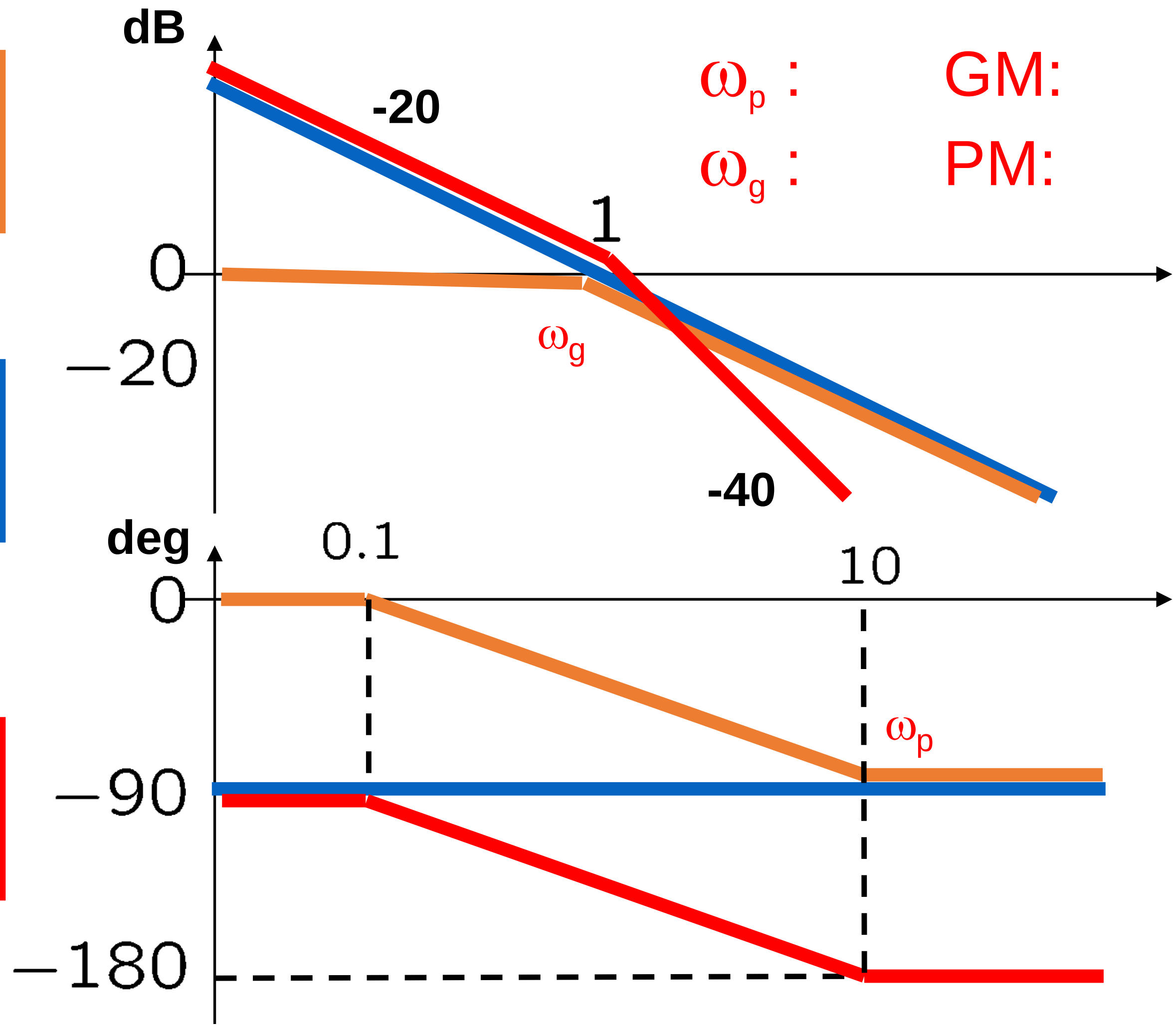
$$G_2(s) = \frac{1}{s}$$

$$\Downarrow$$

$$L(s) = \frac{1}{s(s+1)}$$

ω_p : GM:

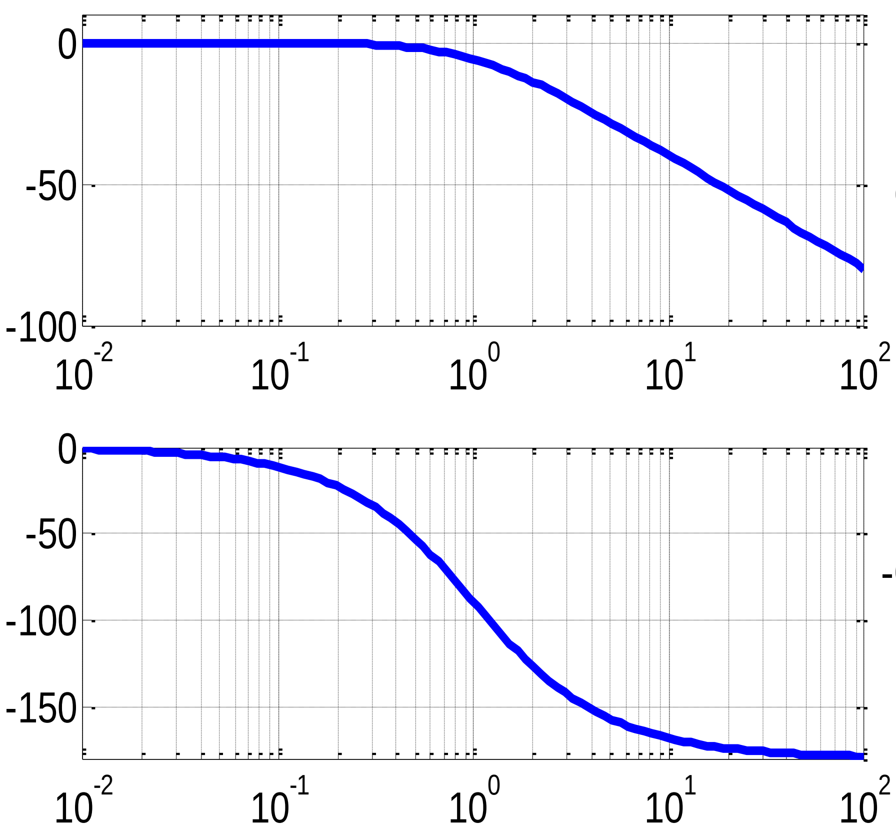
ω_g : PM:



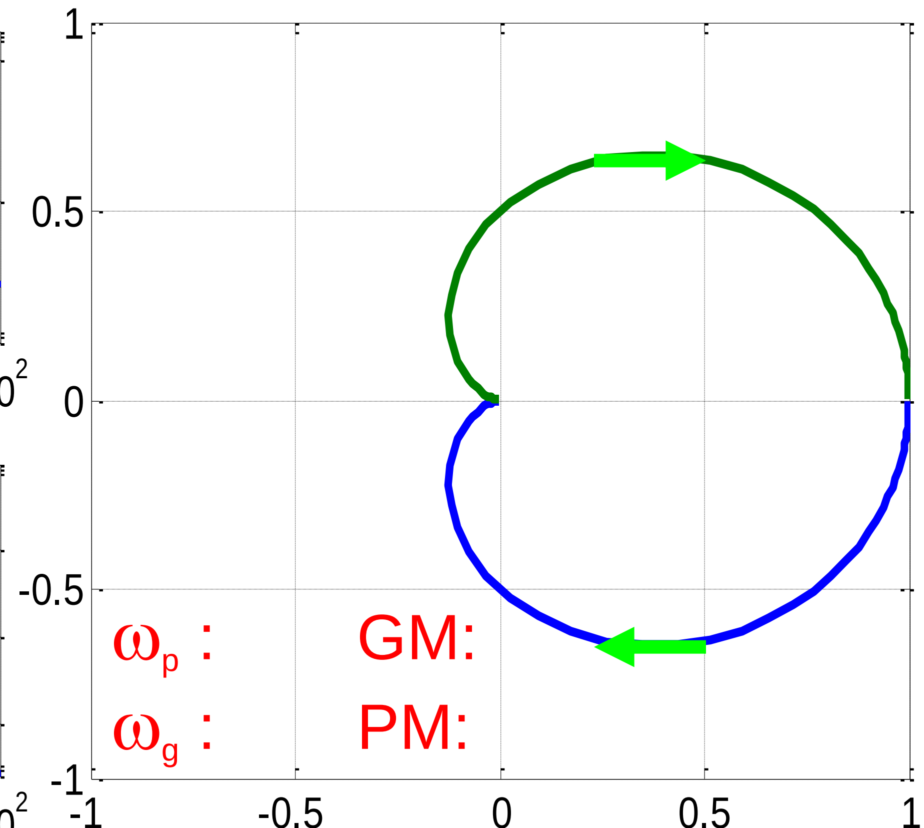
Example 1 (cont'd): (GM & PM)

$$L(s) = \frac{1}{(s+1)^2}$$

Bode plot

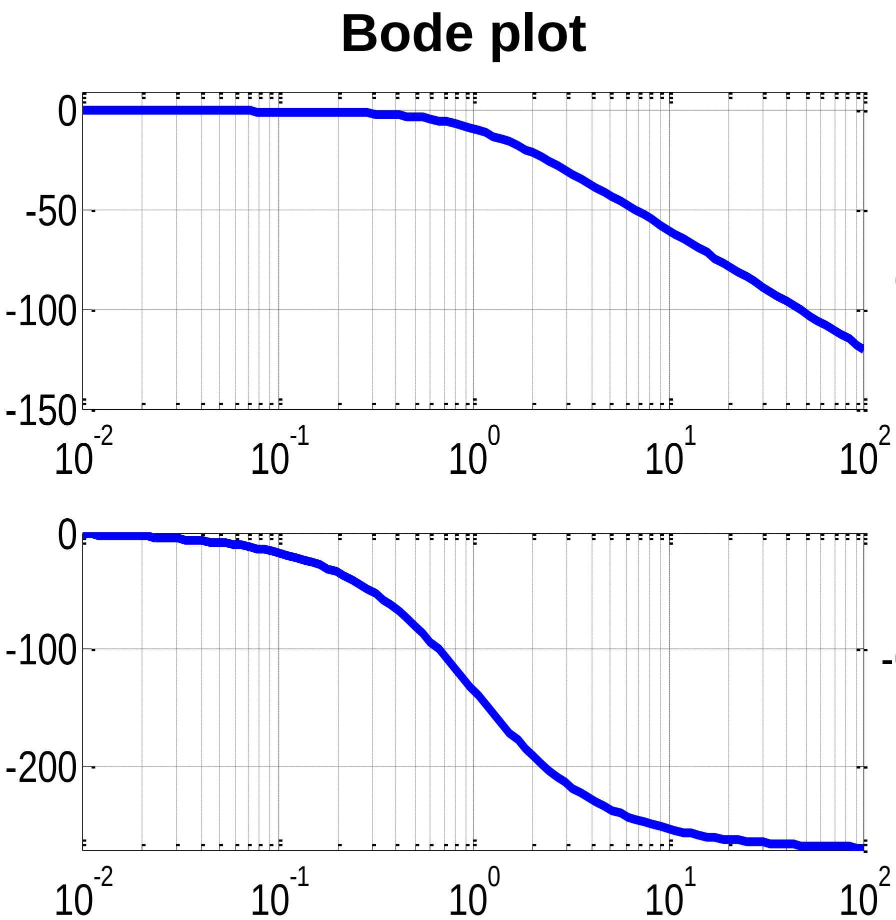


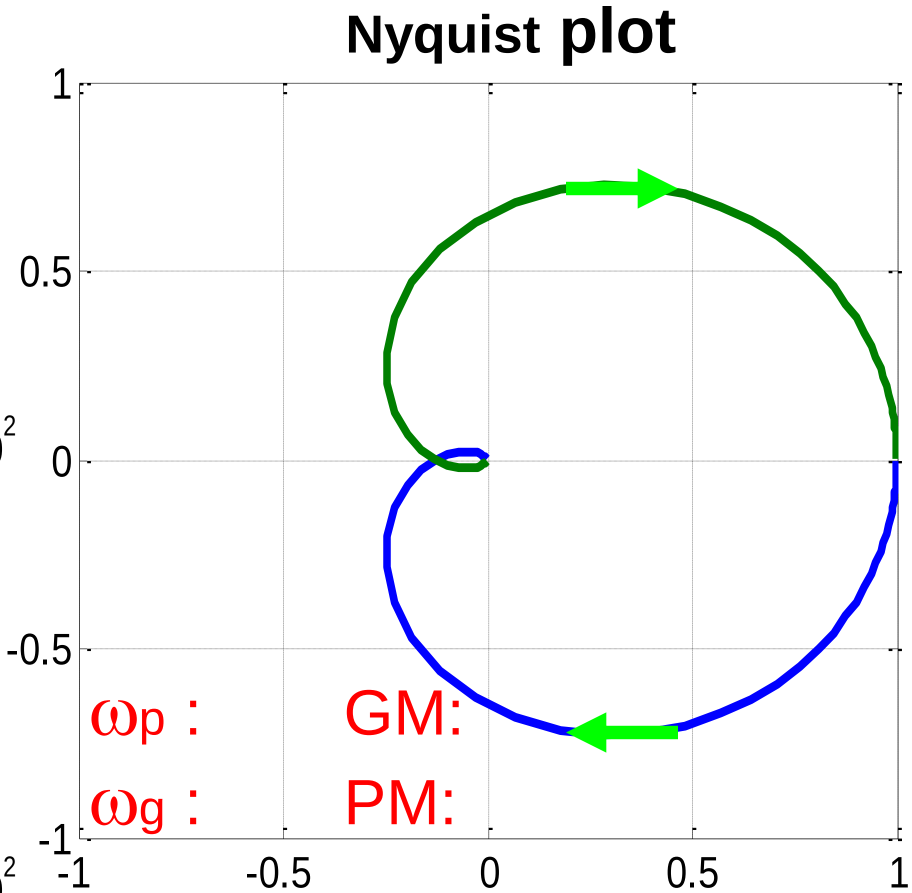
Nyquist plot



Example 2: (GM & PM)

$$L(s) = \frac{1}{(s+1)^3}$$





Example 2 (cont'd): How to compute GM?

- Frequency response function

$$\left[(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3\right]$$

$$\begin{aligned} L(j\omega) &= \frac{1}{(j\omega+1)^3} \\ &= \frac{1}{(j\omega)^3 + 3(j\omega)^2 + 3j\omega + 1} \\ &= \frac{1}{1 - 3\omega^2 + j\omega(3-\omega^2)} \\ &= \frac{1 - 3\omega^2 - j\omega(3-\omega^2)}{(1-3\omega^2)^2 + \omega^2(3-\omega^2)^2} \end{aligned}$$

$$\mathrm{Im}\left\{L(j\omega)\right\} = 0 \Rightarrow \omega_p = \sqrt{3} \Rightarrow \boxed{L(j\sqrt{3}) = -\frac{1}{8}}$$

(PM computation often requires computational tools.)

Course roadmap

Modeling

- Laplace transform
- Transfer function
- Models for systems
  - mechanical
  - electrical
  - electromechanical
- Linearization, delay

Analysis

- Stability
  - Routh-Hurwitz
  - Nyquist
- Time response
  - Transient
  - Steady state
- Frequency response
  - Bode plot

Design

- Design specs
- Root locus
- Frequency domain
- PID & Lead-lag
- Design examples

Matlab simulations

Controller design comparison

Design specifications in time domain
(rise time, settling time, overshoot, steady state error, etc.)

Approximate translation

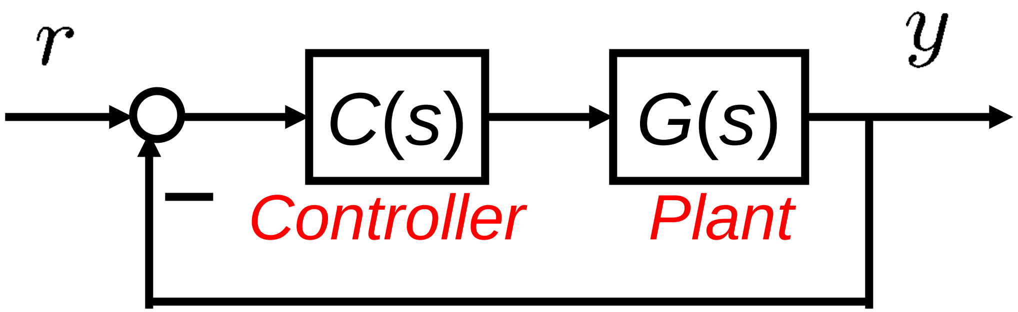
Desired closed-loop
pole location
in *s*-domain

Desired open-loop
frequency response
in *s*-domain

Root locus shaping

Frequency response shaping (loop shaping)

Feedback control system design



OL: $L(s) = G(s)C(s)$

CL: $T(s) = \dfrac{L(s)}{1+L(s)}$

- Given $G(s)$, design $C(s)$ that satisfies CL stability and time-domain specs, i.e., transient and steady-state responses.
- We will learn about typical modifications of OL Bode plot and their effect on closed-loop properties such as stability and time-domain responses.

How to design C (or K) to yield a specific PO in step response using frequency response?

Here is a summary of steps to take:

- **Step 1:** Use the given PO, find ζ.
- **Step 2:** Use ζ. Find PM (see note below). Call this $\mathrm{PM}_{\text{compensator}}$.
- **Step 3:** Find ω^* at which the following equation is satisfied:

$$\angle L(j\omega^*) = -180^0 + \mathrm{PM}_{\text{compensator}}$$

- **Step 4:** Find the requested gain, i.e., C,

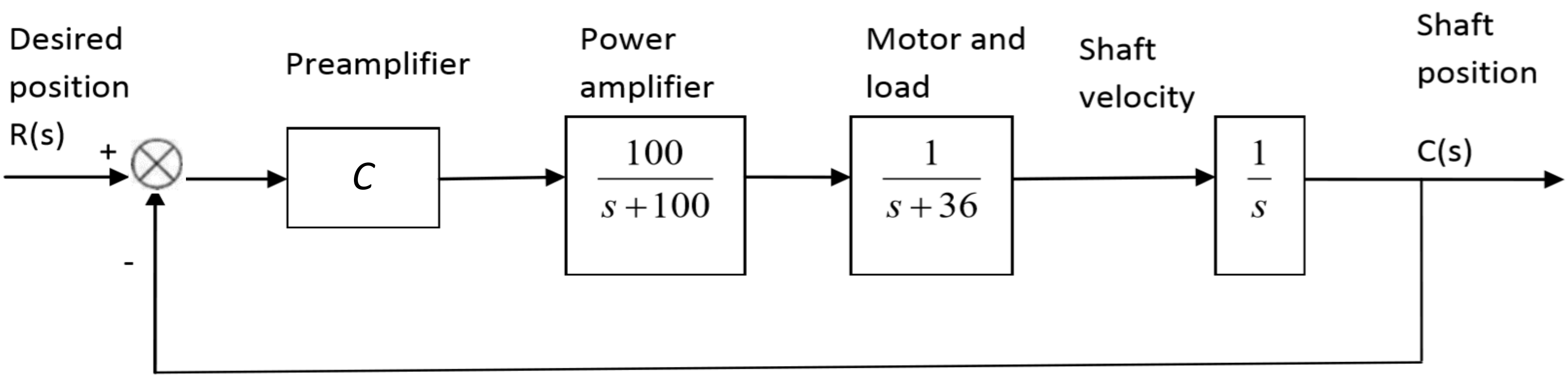
$$C = \frac{1}{|L(j\omega^*)|}$$

Note: There are two methods to calculate PM (the simplified one is preferred):

$$\mathrm{PM}_{\text{compensator}} = 100\zeta \qquad \text{or} \qquad \mathrm{PM}_{\text{compensator}} = \tan^{-1}\left(\frac{2\zeta}{\sqrt{-2\zeta^2 + \sqrt{1 + 4\zeta^4}}}\right)$$

Example 3

For the position control system shown below, find the value of preamplifier gain, *C*, to yield a 9.5% overshoot in the transient response for a step input. Use frequency response methods.



Example 3 (cont'd)

Solution:

$$L(s) = \frac{100}{s(s+36)(s+100)} \rightarrow \text{ Find } C \text{ so that PO} = 9.5\%$$

$$\zeta = \frac{\left|\ln\frac{PO}{100}\right|}{\sqrt{\pi^2+\left(\ln\frac{PO}{100}\right)^2}} = \frac{\left|\ln\frac{9.5}{100}\right|}{\sqrt{\pi^2+\left(\ln\frac{9.5}{100}\right)^2}} \rightarrow \zeta \approx 0.6$$

$$\text{PM}_{\text{compensator}} = 100\zeta \rightarrow \text{PM}_{\text{compensator}} = 60°$$

$$\angle L(j\omega^*) = -180^0 + \text{PM}_{\text{compensator}} = -180° + 60° = -120° \quad \rightarrow \angle L(j\omega^*) = -120°$$

Find ω^* at which $\angle L(j\omega^*) = -120^0$

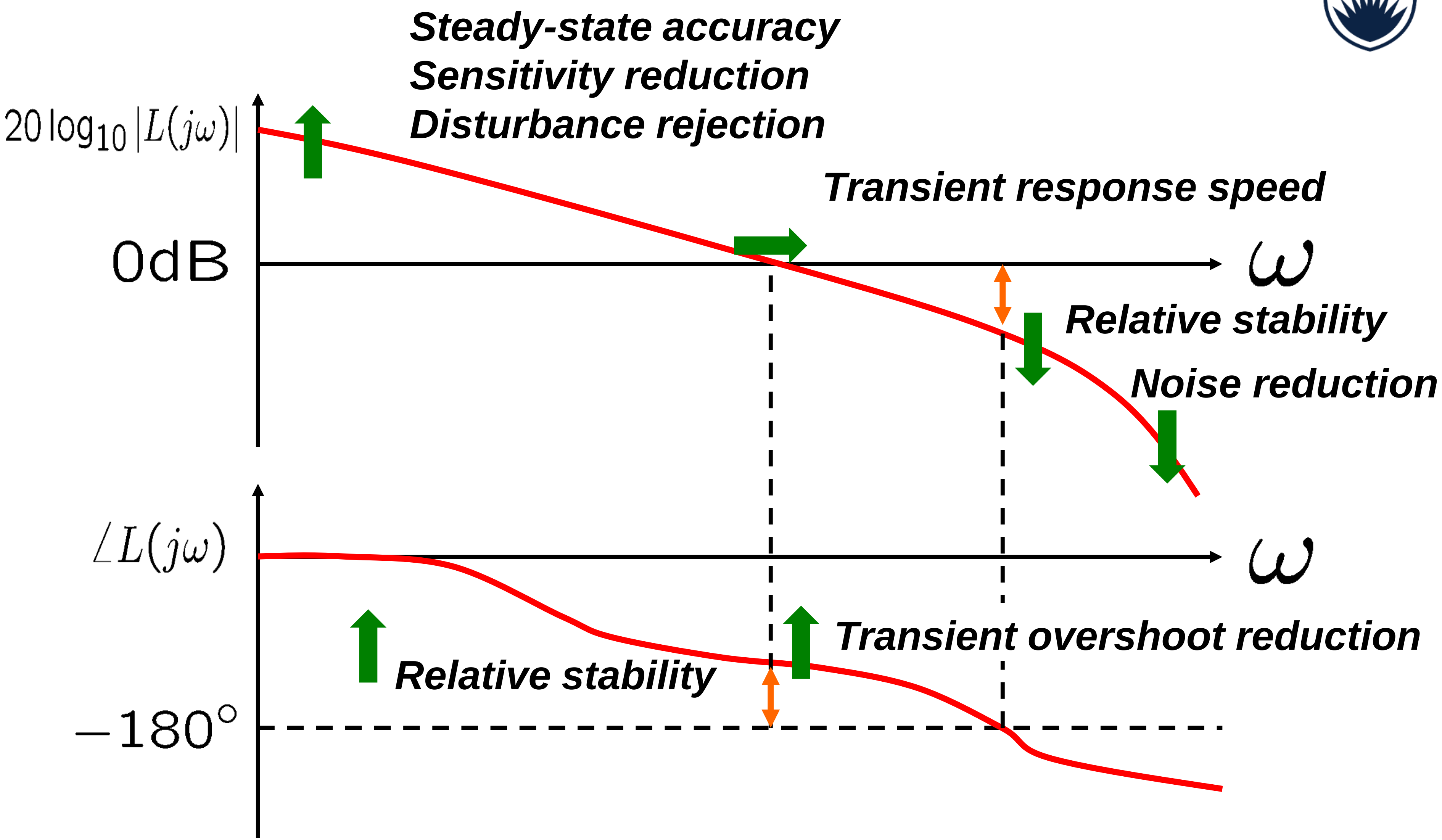
$$\angle L(j\omega^*) = 0^0 - \{90^0 + \tan^{-1}\left(\frac{\omega^*}{36}\right) + \tan^{-1}\left(\frac{\omega^*}{100}\right)\} = -120^0 \quad \rightarrow \omega^* = 14.45$$

At $\omega^* = 14.45 \rightarrow C = ?$

$$|L(j \times 14.45)| = \frac{100}{14.45\sqrt{14.45^2+36^2}\sqrt{14.45^2+100^2}} \rightarrow |L(j \times 14.45)| = 1.7656 \times 10^{-3}$$

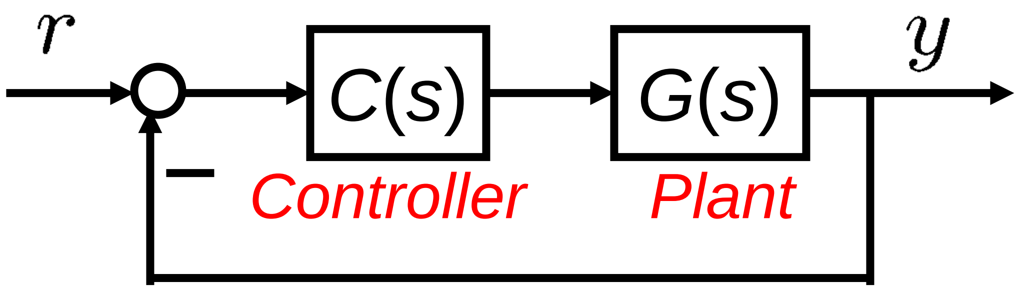
$$C = \frac{1}{|L(j\omega^*)|} = \frac{1}{1.7656\times10^{-3}} \rightarrow \boxed{C = 566}$$

Typical modification of OL Bode plot



- Using the above is called **frequency shaping (loop shaping) design.**

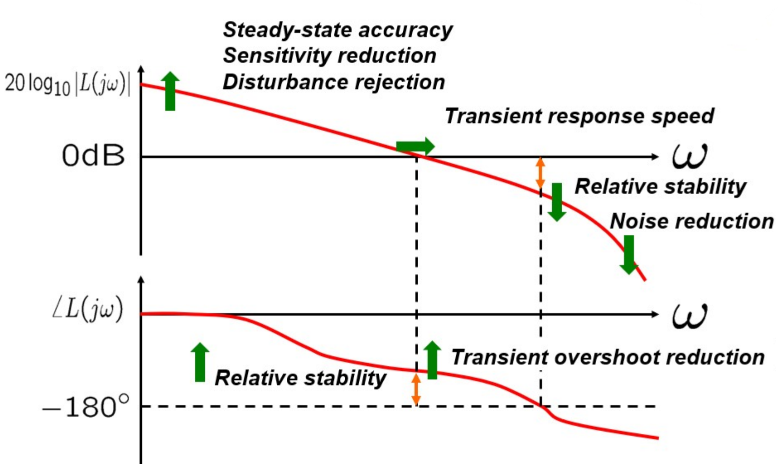
Steady-state accuracy



$$L(s) = G(s)C(s)$$

$$T(s) = \frac{L(s)}{1+L(s)}$$

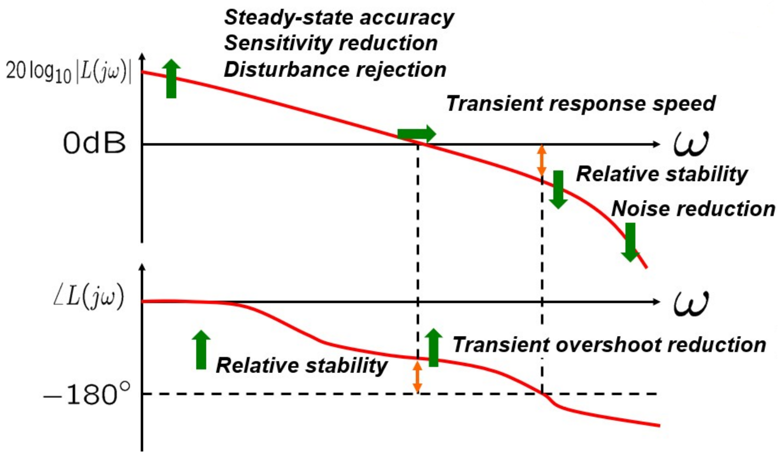
y(*t*) tracks *r*(*t*) very well.



Sensitivity reduction

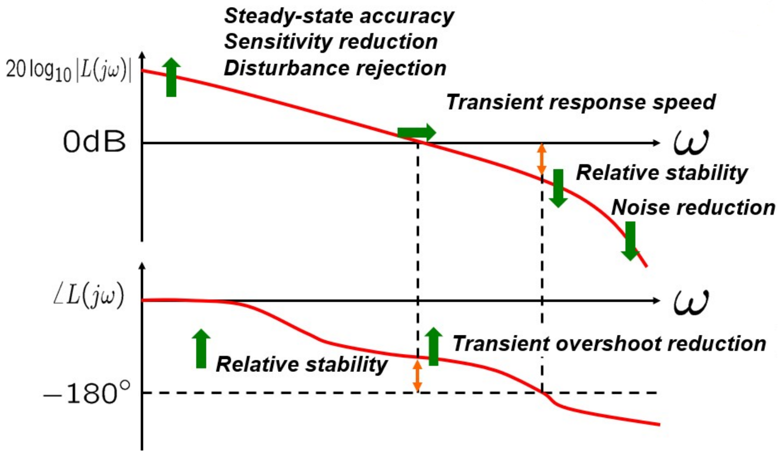
- Sensitivity indicates the influence of plant variations (due to temperature, humidity, age, etc.) on closed-loop performance.
- Sensitivity function

$$S(s) = \frac{\partial T(s)/T(s)}{\partial G(s)/G(s)} = \frac{1}{1+G(s)C(s)} = \frac{1}{1+L(s)}$$

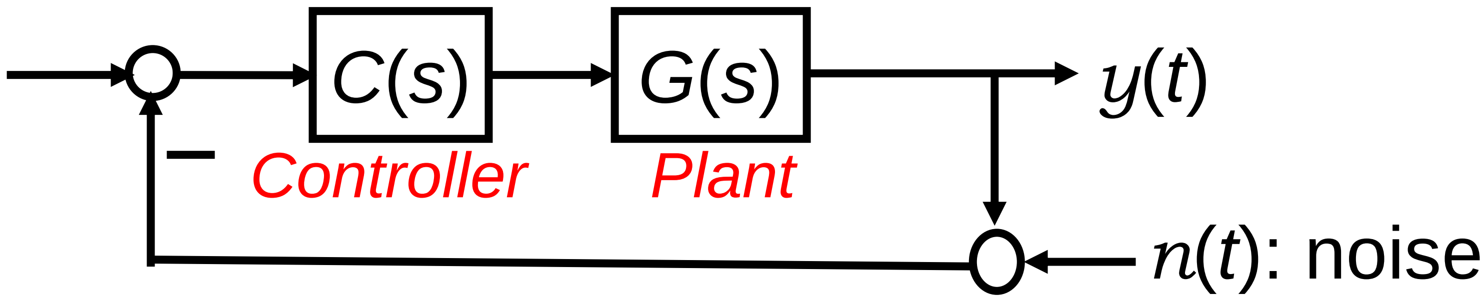


Disturbance

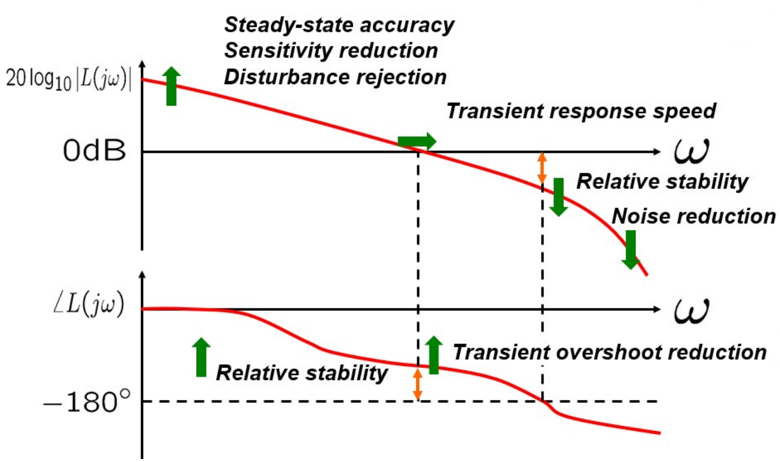
- Unwanted signal
- Examples
  - Wind turbulence in airplane altitude control
  - Wave in ship direction control
  - Sudden temperature change outside the temperature-controlled room
  - Bumpy road in cruise control
- Often, disturbance is neither measurable nor predictable. Use feedback to compensate for it!



Noise reduction



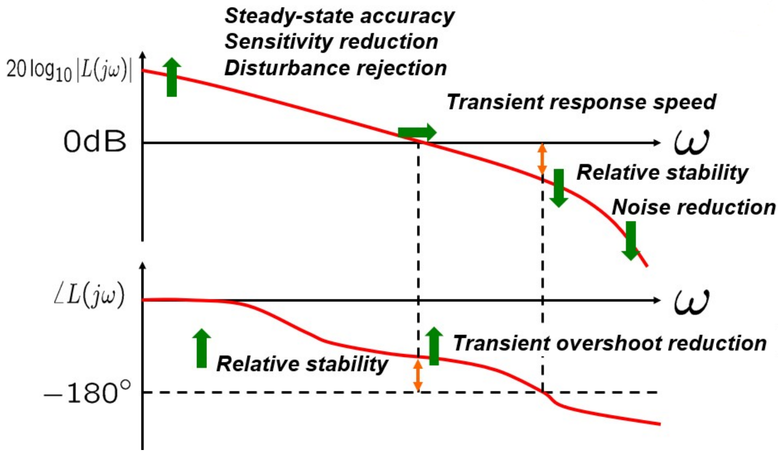
$y(t)$ is not affected by $n(t)$.



Relative stability

- We require adequate GM and PM for:
  - safety against inaccuracies in modeling
  - reasonable transient response (overshoot)
- It is difficult to give reasonable numbers of GM and PM for general cases, but usually,
  - **GM** should be at least **6 dB**
  - **PM** should be at least **45 deg**

  (These values are not absolute but approximate!)
- In controller design, we are especially interested in PM (which typically leads to good GM).



a place of mind

Control in automobiles

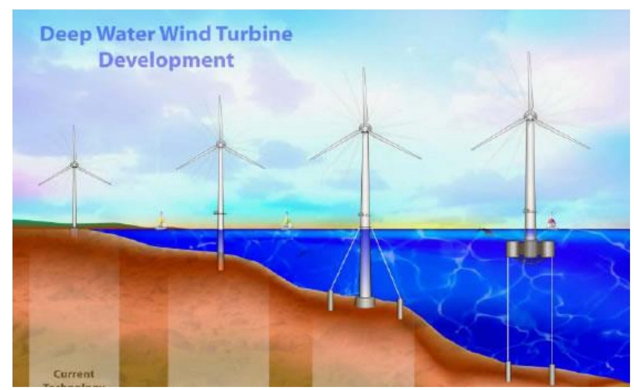
Automobile is a collection of control technologies.

- Vehicle stability control
  - Antilock Breaking System (ABS)
  - Traction control
  - Active suspension
- Energy efficiency and emission reduction
  - Engine control (fuel injection amount and timing, spark timing)
  - Transmission control
  - Energy management of hybrid vehicles
- Driver-assist system and autonomous car
  - Adaptive cruise control
  - Automated parallel parking
  - Automatic lane following
  - Collision prevention
  - Driverless car (Tesla, Google, Mercedes-Benz, Toyota, etc.)
  - Flying vehicle (AeroMobil)

Wind turbines

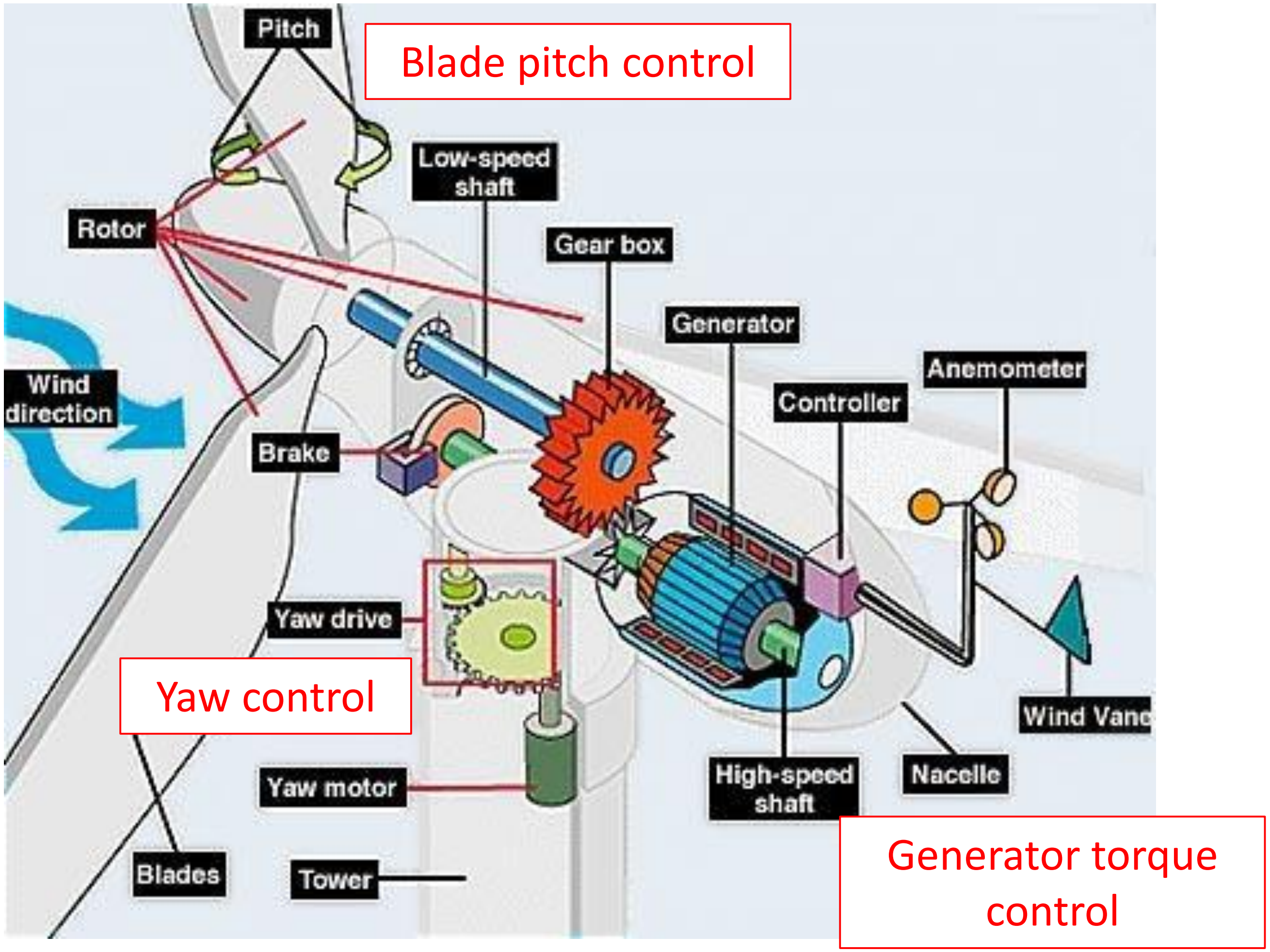
- Multi-disciplinary
  - Electrical, civil, mechanical, materials, integrated, and environmental engineering
  - Dynamics (modeling)
  - Aerodynamics (blade design)
  - Fluid dynamics (wind flow)
  - Hydro dynamics (offshore)
  - Vibration (fatigue)
  - Control
    - Yaw control
    - Blade pitch control
    - Generator torque control

The Eye of the Wind
(60m hub height, 1.5MW)
Grouse Mountain



Onshore and offshore wind turbines
(Taken from NREL report)

Inside the nacelle of a wind turbine



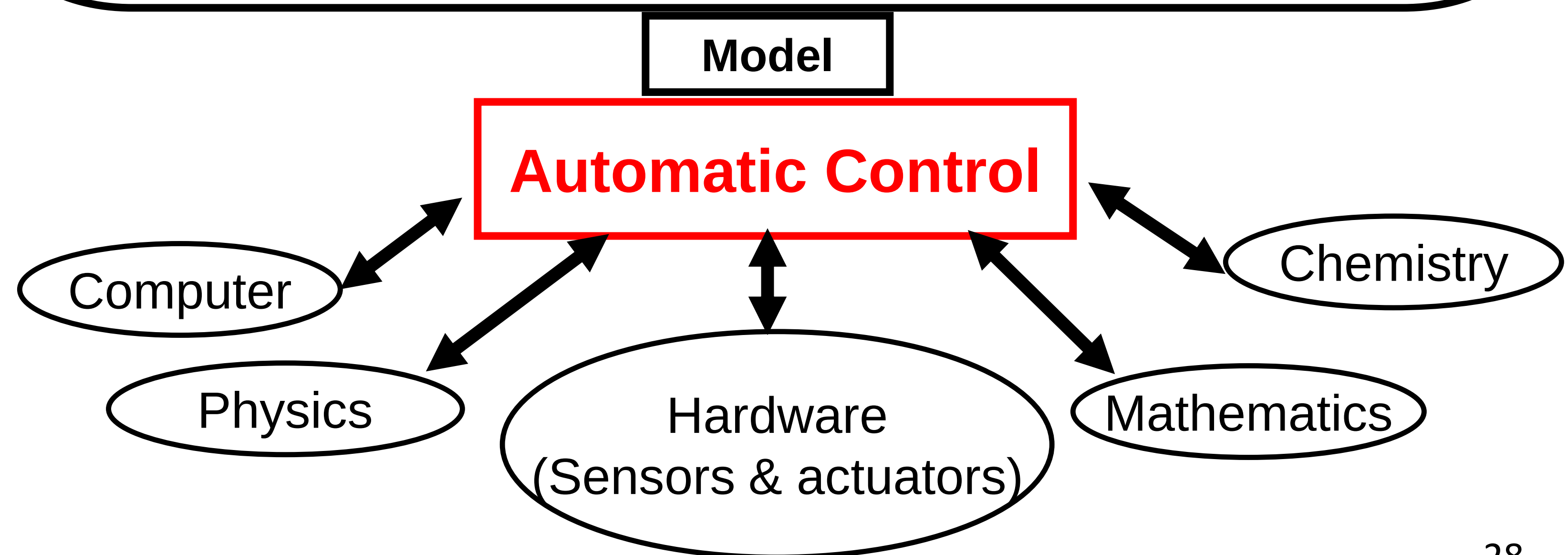
(Taken from NREL report)

Application of systems and control in various disciplines

Automatic control supports various disciplines!
(often called "hidden technology")

Electrical engineering
Biomedical engineering
Mechanical engineering
Chemical engineering
Civil engineering
Aerospace engineering
Environmental engineering

Integrated engineering
Engineering physics
Computer engineering
Mechatronics engineering
MEMS, Nanotechnology
Medicine, Economics, Biology



Control in biomedical engineering

- **Rehabilitation engineering**
  - Powered prostheses (use body signals to control external assistive devices)
  - Functional electrical stimulation (use external signal to control body)
    - Stroke
    - Spinal cord injuries
    - Parkinson's disease
- **Drug delivery and administration**
  - Blood pressure regulation
  - Blood glucose control
  - Anesthesia control
- **Medical instruments**
  - Smart surgical tools
  - Artificial tactile sensing and haptics feedback
  - Temperature and humidity regulation
  - Robotically-assisted surgery
  - Artificial hearts

Summary

- Examples of gain margin and phase margin
- Frequency domain specifications
- Next
  - Frequency response shaping (loop shaping) on Bode plot using Matlab
    - Gain
    - Lead, lag, and lead-lag compensators