Assignment 13 (ELEC 341 L13_RLDesign)

Problem 1:

Given a unity feedback system that has the forward transfer function

$$G(s) = \frac{K(s-2)(s-4)}{(s^2+6s+25)}$$

do the following:

a. Sketch the root locus.

b. Find the imaginary-axis crossing.

c. Find the gain, K, at the $j\omega$-axis crossing.

d. Find the break-in point.

e. Find the point where the locus crosses the 0.5 damping ratio line.

f. Find the gain at the point where the locus crosses the 0.5 damping ratio line.

g. Find the range of gain, K, for which the system is stable.

Please note that we do not expect an accurately drawn diagram, i.e., a rough sketch will suffice. You will only need to find and show the requested key points on the diagram and the rest can be drawn approximately.

Solution:

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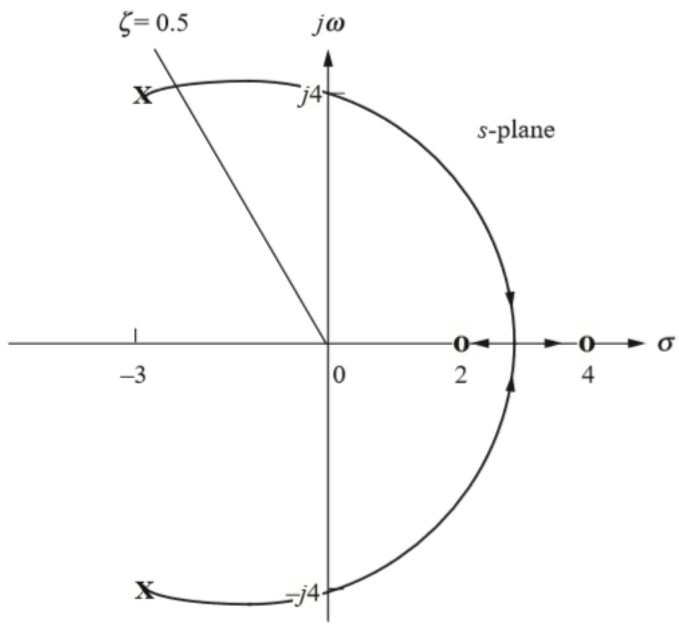
a.



b. Search along the imaginary axis and find the 180° point at $s = \pm j4.06$.
c. For the result in part (b), $K = 1$.
d. Searching between 2 and 4 on the real axis for the minimum gain yields the break-in at $s = 2.89$.
e. Searching along $\zeta = 0.5$ for the 180° point we find $s = -2.42 + j4.18$.
f. For the result in part (e), $K = 0.108$.
g. Using the result from part (c) and the root locus, $K < 1$.

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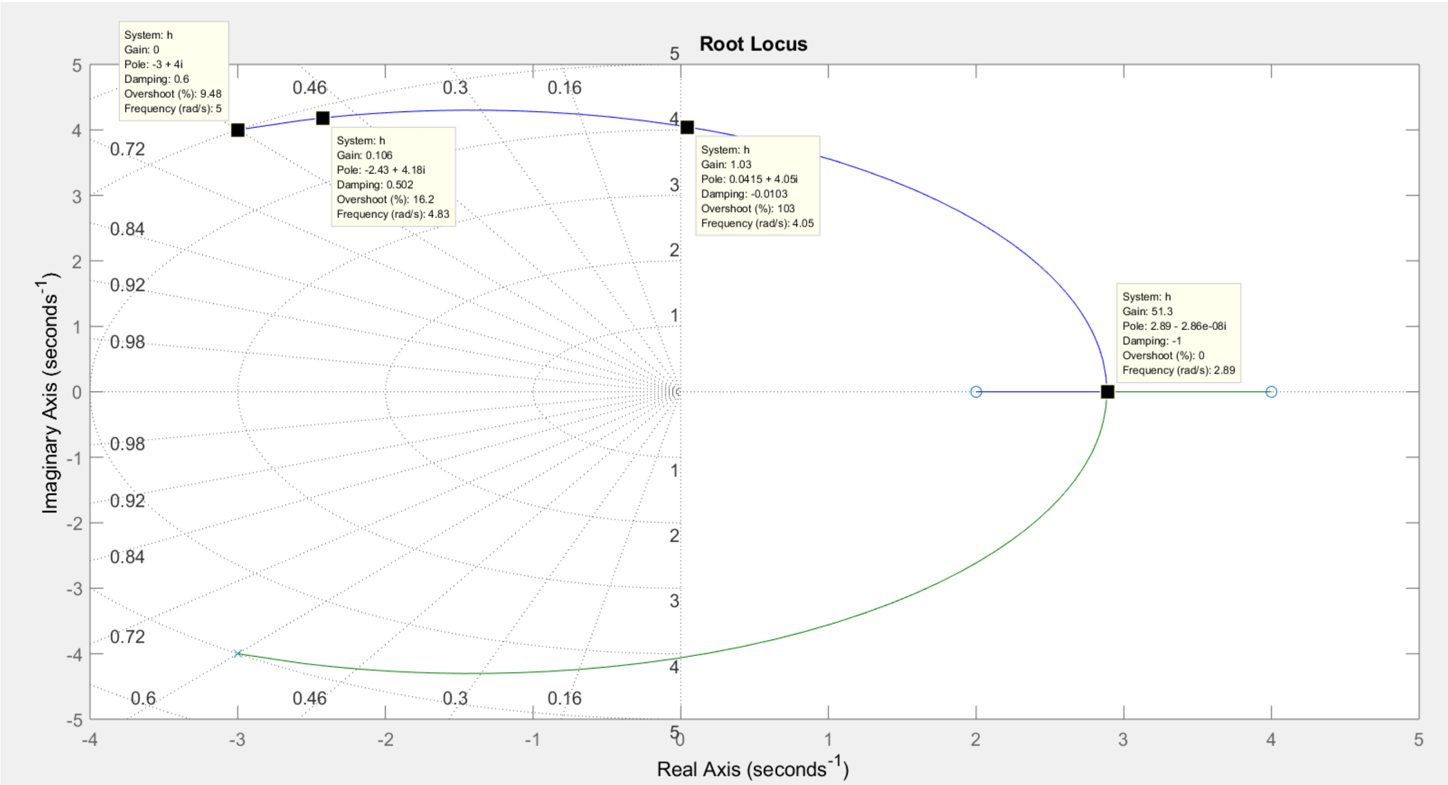


$$G(s) = \frac{K(s-2)(s-4)}{(s^2+6s+25)}$$

a) RL

b) jw-axis crossing

c) K at jw-axis crossing

d) Break-in point

e) s at which $\zeta = 0.5$

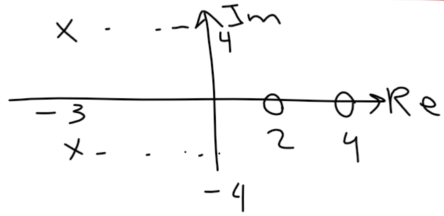
f) K at which $\zeta = 0.5$

g) range of K for which the system is stable

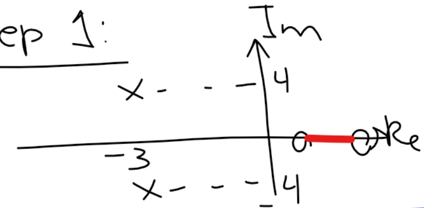
$$s^2+6s+25=0 \rightarrow s_{1,2}=-3\pm4j \Rightarrow s^2+6s+25=(s+3-4j)(s+3+4j)$$

Note that $s^2+a\cdot s+b=(s-s_1)(s-s_2)$

Step 0:



Step 1:



Step 2: $n=2,\ m=2 \rightarrow r=n-m=2-2=0 \rightarrow \boxed{r=0} \rightarrow$ no asymptote

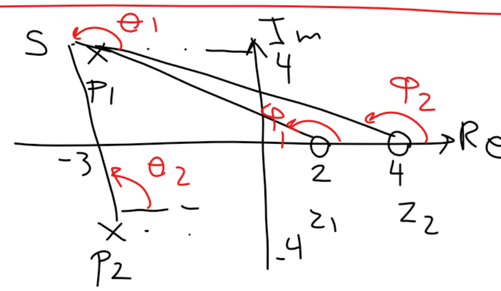
Step 3: $L=G=\dfrac{K(s-2)(s-4)}{D}=\dfrac{K(s^2-6s+8)}{D} \rightarrow$

$$G'=\frac{K(12s^2+34s-198)}{(s^2+6s+25)^2}=0 \rightarrow 12s^2+34s-198=0$$

$\rightarrow$
$$\begin{cases} s_1=2.8853 \xrightarrow{K=-\frac{1}{L(s)}} K=-\dfrac{1}{\dfrac{(2.8853^2-6\times2.8853+8)}{(2.8853^2+6\times2.8853+25)}} \rightarrow K=51.31 \\ s_2=-5.71864 \xrightarrow{K=-\frac{1}{L(s)}} k=-0.311 \text{ (not acceptable)} \end{cases} \Rightarrow$$

$\boxed{K=51.31} \Rightarrow$ $\boxed{S=2.8853 \text{ is break-in point and } K=51.31}$

Angle of departure:



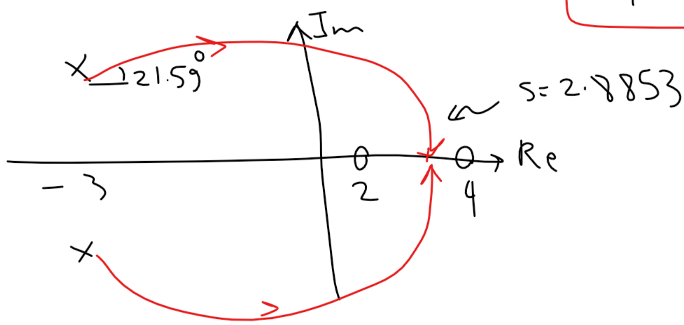
$$\angle G(s) = \angle \frac{(s-2)(s-4)}{(s-s_1)(s-s_2)} = 180^\circ \; ; \text{LHS} = \{\angle(s-2)+\angle(s-4)\} - \{\underbrace{\angle(s-s_1)}_{\theta_1} + \angle(s-s_2)\}$$

$$= \angle(-3+4j-2)+\angle(-3+4j-4)\} - \theta_1 - \{\angle(-3+4j-(-3-4j))\}$$

$$= \angle(-5+4j) + \angle(-7+4j) - \theta_1 - 90^\circ = \tan^{-1}\left(\frac{4}{-5}\right) + \tan^{-1}\left(\frac{4}{-7}\right) - \theta_1 - 90^\circ$$

$$\rightarrow 141.34^\circ + 150.25^\circ - \theta_1 - 90^\circ = 180^\circ \rightarrow \boxed{\theta_1 = +21.59^\circ} \quad \angle\theta_1 = 21.59^\circ$$

RL:



Range of K:

$$\text{CLTF} = \frac{\dfrac{K(s-2)(s-4)}{s^2+6s+25}}{1+\dfrac{K(s-2)(s-4)}{s^2+6s+25}} = \frac{K(s-2)(s-4)}{s^2+6s+25+K(s-2)(s-4)}$$

Ch. Eq: $s^2+6s+25+K(s^2-4s-2s+8)=0 \rightarrow$

$$(1+K)s^2+(6-6K)s+25+8K=0 \quad \text{or} \quad s^2 + \frac{6-6K}{1+K}s + \frac{25+8K}{1+K} = 0$$

s^2	$1+K$	$25+8K$
s^1	$6-6K$	0
s^0	$25+8K$	

$$\frac{(25+8K)(6-6K) - 0\times(1+K)}{6-6K} = \; = 25+8K$$

$$\begin{cases} 25+8K>0 \rightarrow \underline{K>-3.125} \\ 1+K>0 \rightarrow K>-1 \\ 6-6K>0 \rightarrow K<1 \end{cases}$$

given that $K>0 \Rightarrow$ $\boxed{0<K<1 \text{ or just } K<1}$

Find jw-axis crossing:

$K=1 \rightarrow$ ROZ $\quad s^1 \mid 0 \rightarrow (1+1)s^2+(25+8\times1)s^0=0$

$\hookrightarrow 2s^2+33=0 \rightarrow \boxed{s=\pm 4.06j}$

e) s at $\zeta=0.5=?$; we had: $s^2+\frac{6-6K}{1+K}\cdot s+\frac{25+8K}{1+K}=0 \rightarrow$

$$\rightarrow \begin{cases} 2\zeta\omega_n=\frac{6-6K}{1+K} \\ \omega_n^2=\frac{25+8K}{1+K} \quad (1) \end{cases}$$

; $\zeta=0.5 \rightarrow \omega_n=\frac{6-6K}{1+K}$ (3) $\xrightarrow[(1)]{\text{sub in}}$

$$\left(\frac{6-6K}{1+K}\right)^2=\frac{25+8K}{1+K} \rightarrow \underline{28K^2-105K+11=0} \rightarrow$$

$\rightarrow K=0.10786$

$K=3.64214 \longrightarrow$ sub in (3) $\rightarrow$ will give a negative ω_n

$\longrightarrow$ unacceptable $\Rightarrow \boxed{K=0.10786} \longrightarrow$

$$\rightarrow \omega_n=\frac{6-6\times(0.10786)}{1+0.10786} \rightarrow \boxed{\omega_n=4.831} \rightarrow$$

$$s=-\zeta\omega_n \pm \omega_n\sqrt{1-\zeta^2}\,j=-0.5\times4.831\pm4.831\sqrt{1-0.5^2}\,j \rightarrow$$

$$\hookrightarrow \boxed{s=-2.4155\pm4.183j}$$