ELEC 341 Assignment 2

Summer 2019 Siamak Najarian

Group 9

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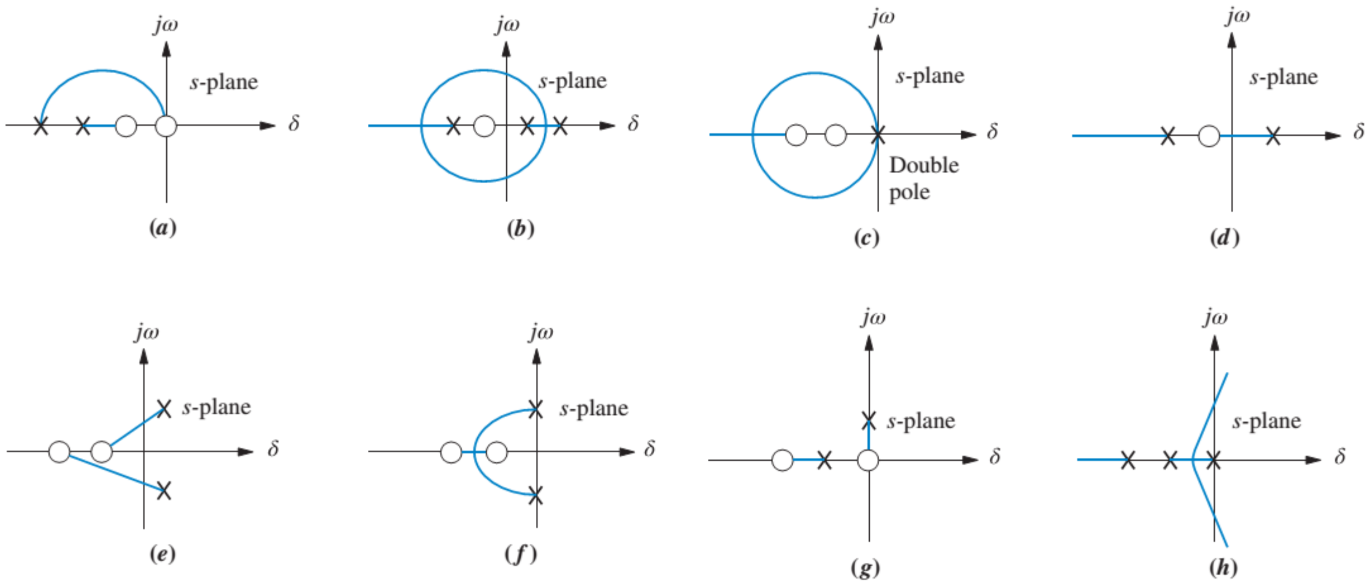
Problem 1:

For each of the root loci shown in Figure, tell whether or not the sketch can be a root locus. If the sketch cannot be a root locus, explain why. Give *all* reasons.

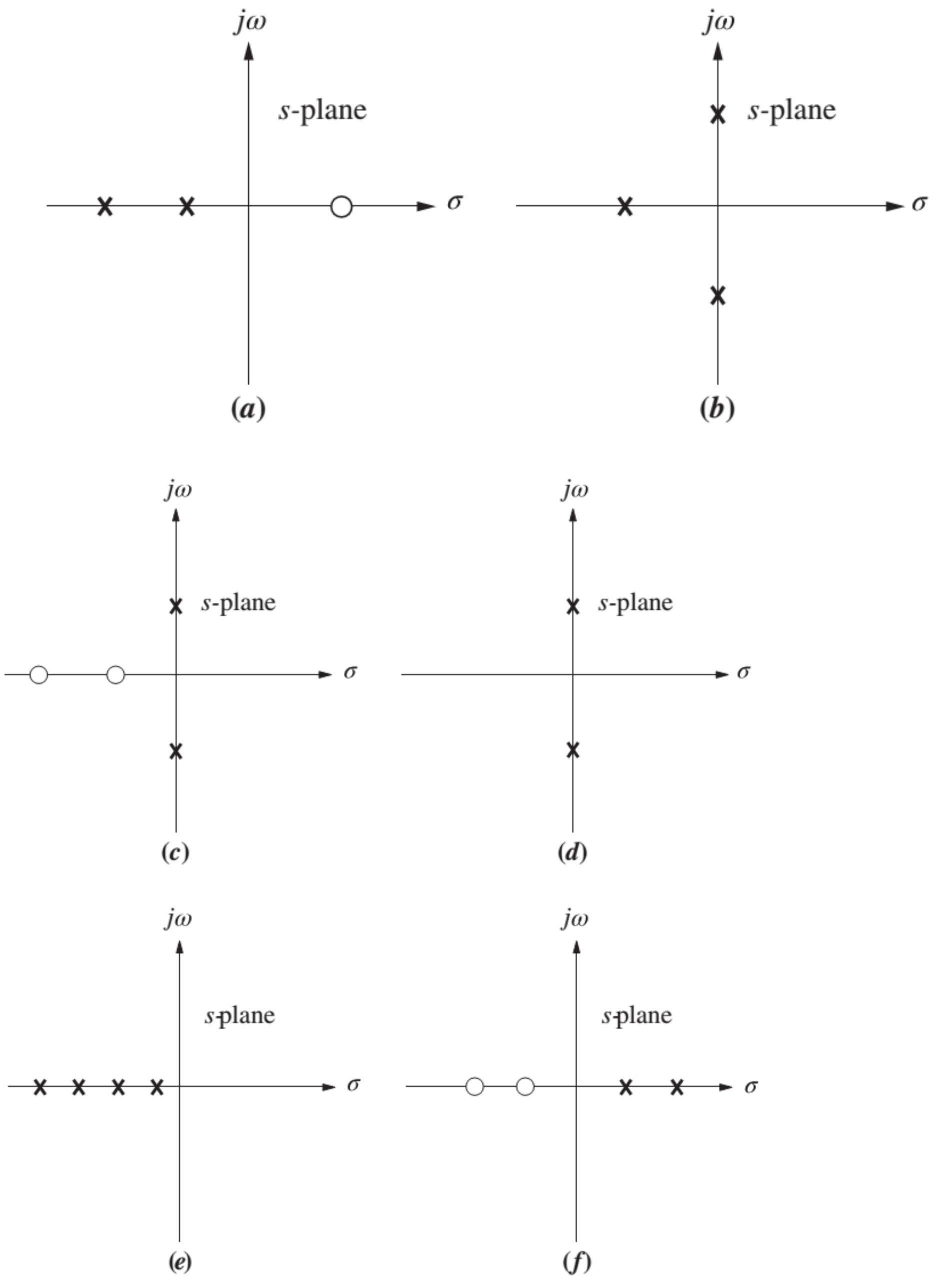


FIGURE

Assignment 11 (ELEC 341 L11_RootLocus)

Problem 2:

Sketch the general shape of the root locus for each of the open-loop pole-zero plots shown in Figure .



FIGURE

Asn 11

Q.) A) No, it cannot be a RL.

- Not symmetric WRT Re axis
- Left of even # of poles/zeros

B) No, it cannot be a RL

- Left of even # of poles/Zeros

C) No, it cannot be a RL

- left of even # of poles/Zeros.
- Doesn't have poles to zeros

D) Yes!

E) No, it cannot be a RL

- Not symmetric WRT Re axis
- Not on Re axis to the left of odd # of poles/zeros

F) Yes!

G) No, it cannot be a RL

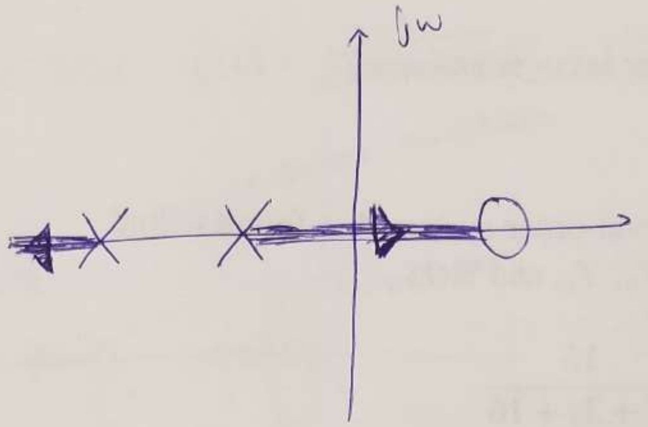
- Not symmetric WRT Re axis
- Re axis segment not to left of odd # poles/zeros

H) Yes!

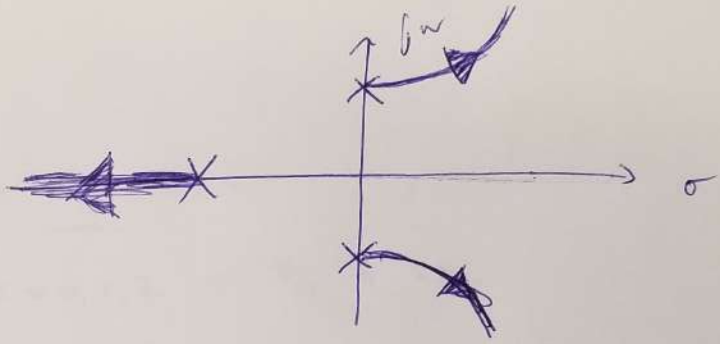
Asn 11

Q_2)

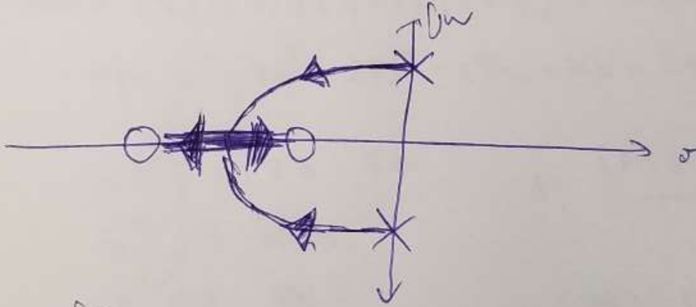
A)



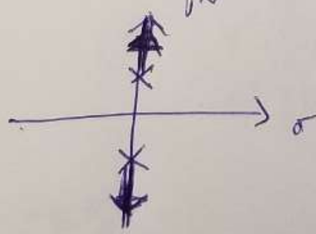
B)



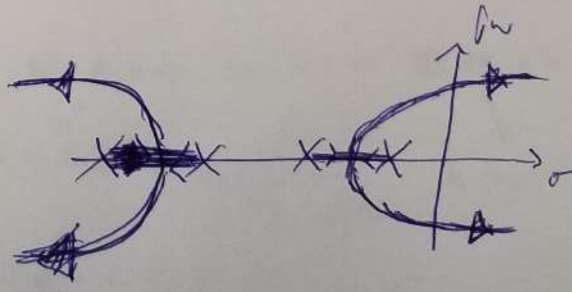
C)



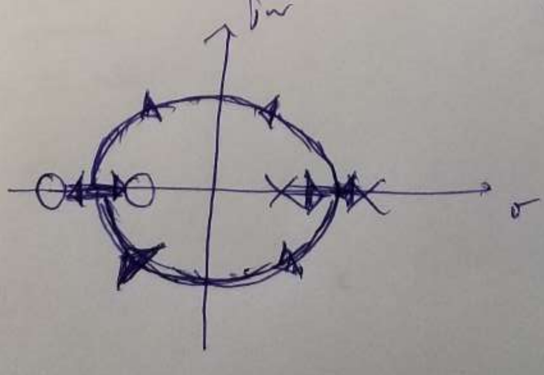
D)



E)



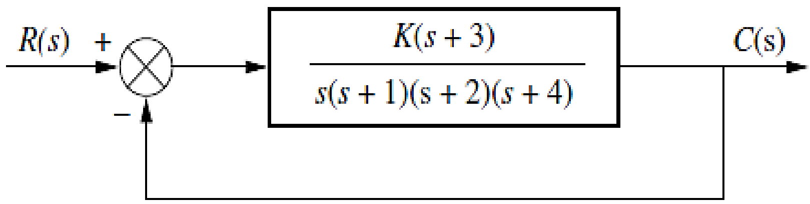
F)



Assignment 12 (ELEC 341 L12_RLExamples)

Problem 1:

Sketch the root locus for the following system:



Assignment 12 (ELEC 341 L12_RLExamples)

Problem 2:

Given the unity feedback system of Figure , where

$$G(s) = \frac{K(s+2)}{s(s+1)(s+3)(s+5)}$$

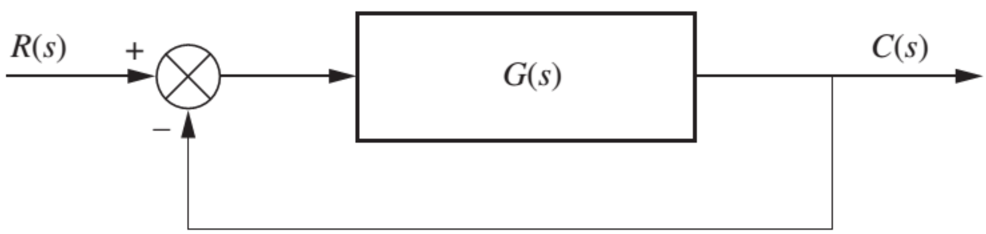
do the following:

a. Sketch the root locus.

b. Find the asymptotes.

c. Find the value of gain that will make the system marginally stable.

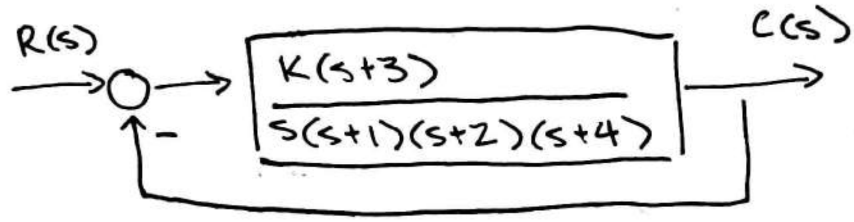
d. Find the value of gain for which the closed-loop transfer function will have a pole on the real axis at -0.5.



FIGURE

Assignment 12

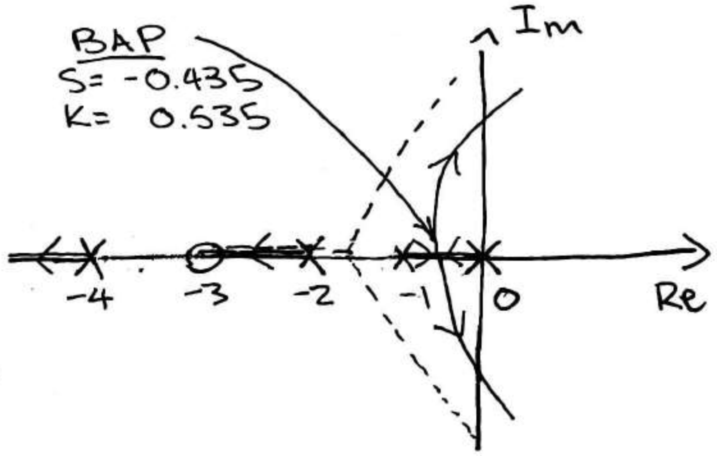
(1.)



$r = n - m = 4 - 1 = 3$

$\frac{\pi}{r} \times (2k+1)$, $K = 0, 1, \ldots (r-1)$

$= \frac{\pi}{3}, \pi, \frac{5\pi}{3}$, angles of asymptotes



$\alpha = \dfrac{\Sigma \text{pole} - \Sigma \text{zero}}{r}$ intersection of asymptotes

$= \dfrac{((0) + (-1) + (-2) + (-4)) - (-3)}{3}$

$= -\frac{4}{3}$

Breakaway points:

$\frac{dL(s)}{ds} = 0$ $\quad L(s) = (s+3)(s(s+1)(s+2)(s+4))^{-1}$

$L'(s) = \dfrac{1}{s(s+1)(s+2)(s+4)} + \dfrac{-(s+3)[(s^2+s)(s^2+6s+8)]'}{[s(s+1)(s+2)(s+4)]^2}$

$= \dfrac{s^4 + 6s^3 + 8s^2 + s^3 + 6s^2 + 8s - [(s+3)(4s^3 + 21s^2 + 28s + 8)]}{[s(s+1)(s+2)(s+4)]^2}$

$= \dfrac{s^4 + 7s^3 + 14s^2 + 8s - [4s^4 + 21s^3 + 28s^2 + 8s + 12s^3 + 63s^2 + 84s + 24]}{(*)}$

$= \dfrac{-3s^4 - 26s^3 - 77s^2 - 84s - 24}{(*)}$ $\quad s = -0.435, K = 0.535$
$\quad\quad = -1.6097, \quad = -0.659$

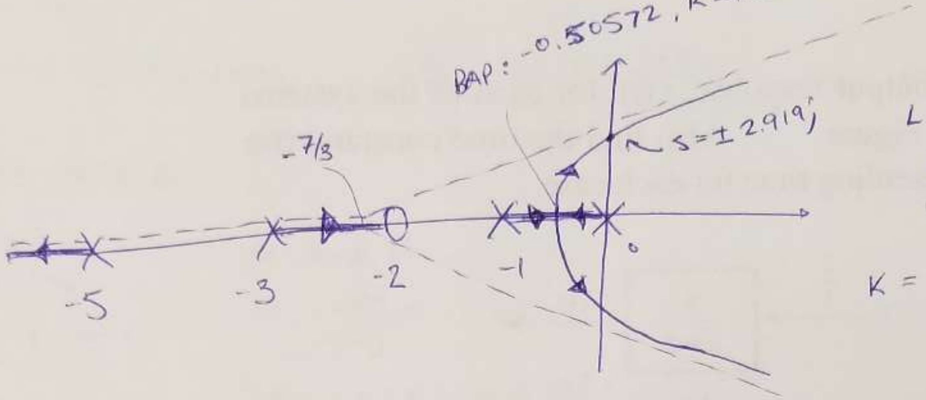
positivity testing:

$K = -\frac{1}{L(s)}$ $\quad \therefore$ BAP @ $s = -0.435$
$K = 0.535$

Asn 12

Q2) $G(s) = K \frac{s+2}{s(s+1)(s+3)(s+5)}$, $L(s) = \frac{s+2}{s(s+1)(s+3)(s+5)}$, $L'(s) = \frac{-3s^4 - 26s^3 - 77s^2 - 92s - 30}{(*)}$

A)



$L'(s) = 0$

↳ $s_1 = -4.04317$

↳ $s_2 = -0.50572$

$K = \frac{-1}{L(s)} \Rightarrow$ × $K_1 = \cancel{0.166367}$ -6.01082

✓ $K_2 = \underline{1.87524}$

B) Degree ⇒ $4 - 1 = \underline{3}$

Asymptote on Re ⇒ $\frac{2-1-3-5}{3} = \underline{-7/3} = -2.\bar{3}$

Angle ⇒ $\frac{\pi}{3}(2K+1)$, $K = 0, 1, 2$ $= \pi/3, \pi, 5\pi/3$

c) $1 + KL(s) = 0 \iff (s)(s+1)(s+3)(s+5) + K(s+2) = 0$

$\Rightarrow s^4 + 9s^3 + 23s^2 + 15s + Ks + 2K = 0$

s^4	1	23	2K
s^3	9	15+K	
s^2	A	2K	
s^1	B		
s^0	2K		

A: $\frac{(23)(9) - (15+K)}{9}$

B: $\frac{(A)(15+K) - (9)(2K)}{A}$

① $A > 0$ ② $B > 0$ ③ $2K > 0$

$K < 192$ $-46.687 < K$, $K < 61.6872$ $K > 0$

$\underline{0 < K < 61.6872}$

↳ $A' = 14.4792...$

↳ $B' = 0$ (ROZ)

↳ ~~B...(A')~~

$A's^2 + 2K = 0$ ~~≈28.9584~~

$\underline{s = \pm 2.919j}$

D) Pole @ $Re = -0.5$

$\underline{K = 1.87524}$

Work is beside RL sketch!

Assignment 13 (ELEC 341 L13_RLDesign)

Problem 1:

Given a unity feedback system that has the forward transfer function

$$G(s) = \frac{K(s-2)(s-4)}{(s^2+6s+25)}$$

do the following:

a. Sketch the root locus.

b. Find the imaginary-axis crossing.

c. Find the gain, K, at the $j\omega$-axis crossing.

d. Find the break-in point.

e. Find the point where the locus crosses the 0.5 damping ratio line.

f. Find the gain at the point where the locus crosses the 0.5 damping ratio line.

g. Find the range of gain, K, for which the system is stable.

Please note that we do not expect an accurately drawn diagram, i.e., a rough sketch will suffice. You will only need to find and show the requested key points on the diagram and the rest can be drawn approximately.

Assignment 13

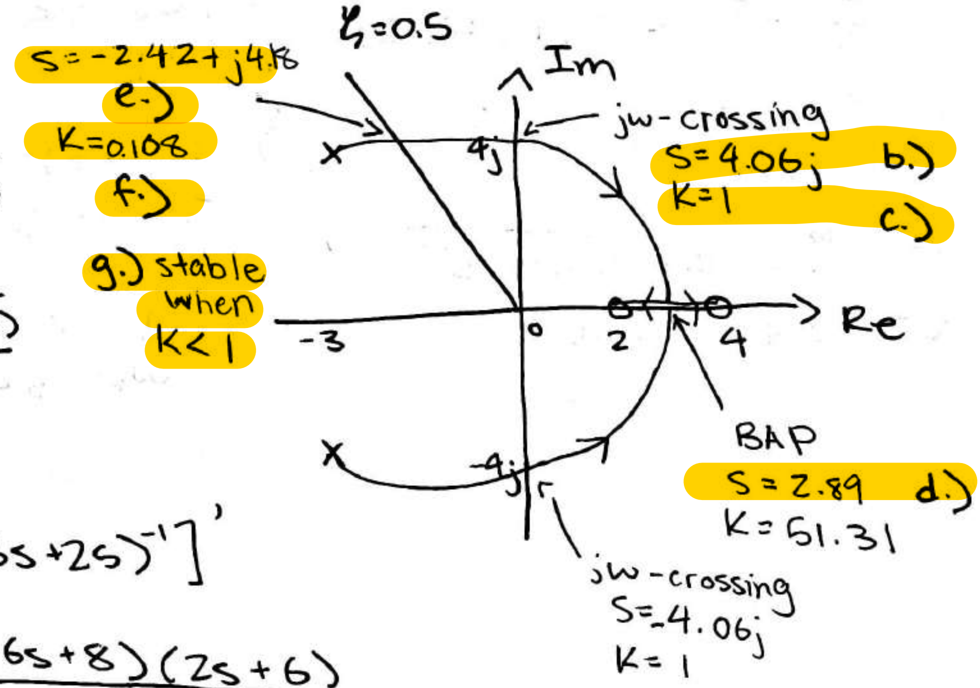
$$G(s) = \frac{K(s-2)(s-4)}{(s^2+6s+25)}$$

zeros = $+2, +4$

poles = $\dfrac{-6 \pm \sqrt{6^2 - 4(25)}}{2}$

$= -3 \pm 4j$

a.)



$$\frac{dL(s)}{ds} = \left[(s^2-6s+8)(s^2+6s+25)^{-1}\right]'$$

$$= \frac{2s-6}{s^2+6s+25} + \frac{-(s^2-6s+8)(2s+6)}{(s^2+6s+25)^2}$$

$$= \frac{[2s^3+12s^2+50s-6s^2-36s-150]-[2s^3+6s^2-12s^2-36s+16s+48]}{(*)}$$

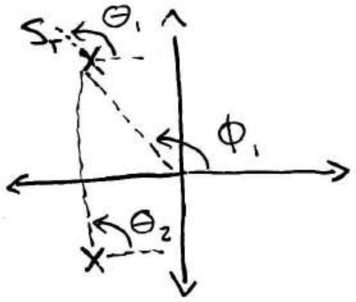
$$= \frac{12s^2+34s-198}{(*)}$$

$$s = \frac{-34 \pm \sqrt{34^2 - 4(198)(12)}}{2(12)}$$

$$K = -\frac{1}{L(s)}$$

$s = 2.88, -5.7187$

$K = 51.31, -0.312$



$\phi_1 = 180° - \tan^{-1}\left[\frac{4}{3}\right] = 126.9°$

$\theta_2 = 90°$

$\theta_1 = 180 - 126.9 - 90 = -36.9°$

CE: $1 + \dfrac{K(s-2)(s-4)}{(s^2+6s+25)} = 0$

$K[s^2-4s-2s+8]+s^2+6s+25=0$

$(K+1)s^2 + (-6K+6)s + (8K+25) = 0$

$s^2 + \left[\dfrac{-6K+6}{K+1}\right]s + \left[\dfrac{8K+25}{K+1}\right] = 0$

$s^2 + [2\zeta\omega_n]s + \omega_n^2 = 0$

s^2	$(K+1)$	$(8K+25)$
s^1	$(-6K+6)$	
s^0	$8K+25$	

ROZ → $8K+25=0$

or → $-6K+6=0$

$K = 1$

$P(s) = 2s^2 + 33$

$s = \pm j4.06$

① $\dfrac{-6K+6}{K+1} = 2(0.5)\omega_n$

② $\dfrac{8K+25}{K+1} = \omega_n^2$

$s = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$

$s = -2.42 + j4.18$

Assignment 13 cont.

$$\frac{8K+25}{K+1} = \frac{(-6K+6)^2}{(K+1)^2}$$

$(8K+25)(K+1) = (-6K+6)(-6K+6)$

$8K^2 + 33K + 25 = 36K^2 - 72K + 36$

$0 = 28K^2 - 105K + 11$

$K = 3.6421,\ 0.10786$

$\omega_n = -3.41496,\ 4.8317$

Assignment 14 (ELEC 341 L14_RLLeadLag)

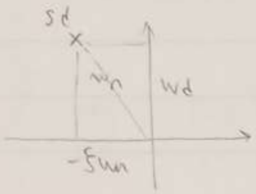
Problem 1:

For a control system with open loop transfer function of $C(s).G(s) = C(s)/[s(s+4)]$, design a lead compensator by selecting $C_{lead}(s)$ so that the damping frequency is 4 rad/s and $\zeta = 0.707$.

Assignment 14

$C(s)\,G(s) = C(s)/s(s+4)$, Let $C_{lead}(s) = k\,\dfrac{s+Z_{lead}}{s+P_{lead}}$, $w_d = 4$ rad/s , $\zeta = 0.707$

1. find the desired pole, s_d.



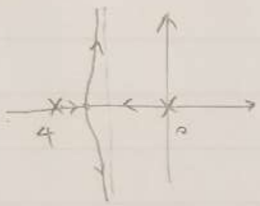
$\because w_d = w_n\sqrt{1-\zeta^2}$

$\therefore w_n = \dfrac{w_d}{\sqrt{1-\zeta^2}} = 5.656$

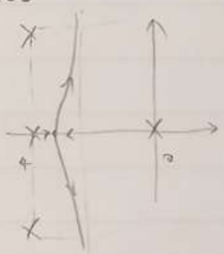
$\therefore \zeta w_n = 0.707 \times 5.656 \approx 3.998 \approx 4$

$\therefore s_d = 4 \pm 4i$

2. plot the current RL, for $G(s)$.



$\Rightarrow$



3. Find Z_{lead} and P_{lead} (use $s_d = -4+4i$)

① find ϕ, which is $\angle(s_d + Z_{lead}) - \angle(s_d + P_{lead}) = \phi$

$\because \phi = 180^\circ - \angle G(s_d)$

$\angle G(s_d) = \angle \dfrac{1}{s_d\,(s_d+4)} = -(\angle(4+4i) + \angle 4i)$

$= -(135^\circ + 90^\circ) = -225^\circ$

$\Rightarrow$ flip $\angle G(s_d)$ to the real axis

$\therefore \angle G(s_d) = 360^\circ - 225^\circ$
$= 135^\circ$

$\therefore \phi = 180^\circ - 135^\circ = 45^\circ$

$\because$ We always take $Z_{lead} = |Re(s_d)|$ for analytic method

$\therefore Z_{lead} = |(-4)| = 4$

$\therefore \phi = \angle(s_d + Z_{lead}) - \angle(s_d + P_{lead})$

$\angle(s_d + P_{lead}) = \angle(s_d + Z_{lead}) - \phi$

$= \angle(4i) - 45^\circ = 90^\circ - 45^\circ = 45^\circ$

$\angle((-4+P_{lead}) + 4i) = 45^\circ$

$\tan\dfrac{4}{P_{lead}-4} = 45^\circ \Rightarrow \dfrac{4}{P_{lead}-4} = \operatorname{atan} 45^\circ$

$\dfrac{4}{\underbrace{\operatorname{atan} 45^\circ}_{1}} + 4 = P_{lead}$

$\therefore P_{lead} = 8$

$C_{lead}(s) = k\,\dfrac{s+Z_{lead}}{s+P_{lead}}$

$= k\left(\dfrac{s+4}{s+8}\right)$

4. find k in $C_{lead}(s)$ $(s_d = -4+4i)$

$\because$ By magnitude condition

$$|L(s_d)| = 1$$

$$= |C(s)G(s)| = \left| k\left(\frac{s+4}{s+8}\right)\left(\frac{1}{s(s+4)}\right) \right|$$

$$= k \left| \frac{1}{s_d(s_d+8)} \right| = 1$$

$$\therefore \; k = |s_d(s_d+8)|$$

$$k = |(-4+4i)(4+4i)|$$

$$k = 32$$

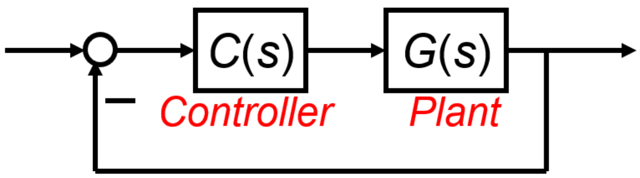
$$\therefore \; C_{lead}(s) = 32\left(\frac{s+4}{s+8}\right)$$

Assignment 15 (ELEC 341 L15_RLPID)

Problem 1:

Determine the Ziegler-Nichols tuning parameters for a PID controller, with the given plant transfer function:

$$G(s) = \left(\frac{5}{2s+1}\right)\left(\frac{0.4}{5s+1}\right)\left(\frac{2}{s+1}\right)$$



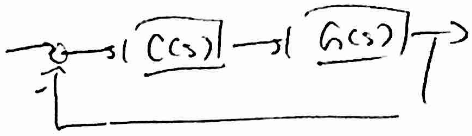
Assume that the time constants have units of minutes and the controller transfer function *C*(*s*) is as follows:

$$C(s) = K_p\left(1 + \frac{1}{T_I s} + T_D s\right)$$

Use Matlab to graph the step response.

Asn 15

Q1) $G(s) = \left(\frac{5}{2s+1}\right)\left(\frac{0.4}{5s+1}\right)\left(\frac{2}{s+1}\right)$



$C(s) = K_p\left(1 + \frac{1}{T_I s} + T_D s\right)$

Let $c(s) = K_p = K_c$, $s = jw$

↳ $1 + K_c G(jw) = 0$ [Characteristic Eqn]

$$\Rightarrow 1 + K_c\left[\left(\frac{5}{2\underset{jw}{s}+1}\right)\left(\frac{0.4}{5\underset{jw}{s}+1}\right)\left(\frac{2}{\underset{jw}{s}+1}\right)\right] = 0$$

$$\Rightarrow (2\underset{jw}{s}+1)(5\underset{jw}{s}+1)(\underset{jw}{s}+1) + K_c(5)(0.4)(2) = 0$$

$$\Rightarrow 10\underset{jw}{s}^3 + 17\underset{jw}{s}^2 + 8\underset{jw}{s} + 1 + K_c(4) = 0$$

$$\Rightarrow -10jw^3 - 17w^2 + 8jw + [4K_c + 1] = 0 \qquad \underset{①}{a} + j\underset{②}{b} = 0$$

$$\Rightarrow \begin{cases} \sigma = 0 \Rightarrow ① \; -17w^2 + 4K_c + 1 = 0 \\ jw = 0 \Rightarrow ② \; -10jw^3 + 8jw = 0 \end{cases}$$

↳ $-10w^3 + 8w = 0$

$\rightarrow w^2 = \cancel{8/10}\ 4/5$

$w = \cancel{\sqrt{8/10}}\ \frac{2}{\sqrt{5}}$

Plug in to ①: $K_c = \frac{-1 + 17\left(\frac{2}{\sqrt{5}}\right)^2}{4}$

$= \frac{-1 + 17 \cdot \cancel{8/10}\ 4/5}{4}$

$= \cancel{\frac{26/5}{4}} = \frac{63}{20}$

$K_c = \cancel{84/...}\ 63/20 = 3.15$

$w = 2\pi / T_c \Rightarrow \sqrt{5}\,\pi = T_c$

$T_c = \cancel{4\pi/\sqrt{5}} = \cancel{5.61985...}\ 7.0248$

$K_p = 0.6K_c = \cancel{3.0375}\ 1.89$

$T_I = 0.5\,T_c = \cancel{2.8099...}\ 3.51241$ min

$T_D = 0.125\,T_c = \cancel{0.7024815}\ 0.878102$ min

Matlab? Nope :)

Step Response: OLTF $= C(s)G(s)$

$= (1.89)\left(1 + \frac{1}{3.51s} + 0.88s\right)\left(\frac{4}{(2s+1)(5s+1)(s+1)}\right)$

Assignment 16 (ELEC 341 L16_FrequencyResponse)

Problem 1:

Convert the given transfer function into frequency response function, i.e., into $G(j\omega)$ form. Also, compute $M(\omega) = |G(j\omega)|$ and $\phi(\omega) = \angle G(j\omega)$. Find 20log $M(\omega)$ and $\phi(\omega)$ at $\omega = 20$.

$$G(s) = \frac{(s+3)(s+5)}{s(s+2)(s+4)}$$

Assignment 16

$$G(s) = \frac{(s+3)(s+5)}{s(s+2)(s+4)}$$

$$|G(j\omega)| = \frac{\sqrt{(\omega)^2 + (3)^2} \cdot \sqrt{(\omega)^2 + (5)^2}}{\sqrt{(\omega)^2} \cdot \sqrt{(\omega)^2 + (2)^2} \cdot \sqrt{(\omega)^2 + (4)^2}}$$

$$= \frac{\sqrt{\omega^2 + 9} \cdot \sqrt{\omega^2 + 25}}{\omega \cdot \sqrt{\omega^2 + 4} \cdot \sqrt{\omega^2 + 16}}$$

$$\angle G(j\omega) = \left[\tan^{-1}\left(\frac{\omega}{3}\right) + \tan^{-1}\left(\frac{\omega}{5}\right)\right] - \left[90° + \tan^{-1}\left(\frac{\omega}{2}\right) + \tan^{-1}\left(\frac{\omega}{4}\right)\right]$$

$$M(\omega = 20) = |G(j20)| = 0.0508$$

$$20\log M(\omega = 20) = 20\log_{10} 0.0508 = -25.8$$

$$\phi(\omega = 20) = \angle G(j20)$$

$$= \left[\tan^{-1}\left(\frac{20}{3}\right) + \tan^{-1}\left(\frac{20}{5}\right)\right] - \left[90° + \tan^{-1}\left(\frac{20}{2}\right) + \tan^{-1}\left(\frac{20}{4}\right)\right]$$

$$= -95.54°$$

Assignment 17 (ELEC 341 L17_BodeDiagram)

Problem 1:

Find analytical expressions for the magnitude and phase response for each $G(s)$ below.

a. $G(s) = \dfrac{1}{s(s+2)(s+4)}$

b. $G(s) = \dfrac{(s+5)}{(s+2)(s+4)}$

For each function in Problem 1, make a plot of the log-magnitude and the phase, using log-frequency in rad/s as the ordinate. Do not use asymptotic approximations.

The graphs need only be plotted approximately using only a few points.

For each function in Problem 1, sketch the Bode asymptotic magnitude and asymptotic phase plots. Compare your results with your answers to Problem 1.

In both (a) and (b), find 20log $M(\omega)$ and $\phi(\omega)$ at ω = 20.

Asn 17 ①

Q$_1$) A) $G(s) = \dfrac{1}{s(s+2)(s+4)}$, $s \Rightarrow j\omega$

$\hookrightarrow \dfrac{1}{s^3+6s^2+8s} \Rightarrow \dfrac{1}{-j\omega^3 - 6\omega^2 + 8j\omega}$ $\quad a + bj \Rightarrow \begin{cases} a = -6\omega^2 \\ b = -\omega^3 + 8\omega \end{cases}$

$$|G(\omega)| = \frac{1}{\sqrt{(a)^2+(b)^2}} = \boxed{\frac{1}{\sqrt{(+6\omega^2)^2 + (-\omega^3+8\omega)^2}} = M(\omega)}$$

$\angle G(\omega) = \arctan(b/a)$ [a is negative, b is negative, so add π]

$$= \boxed{-\left[\pi + \tan^{-1}\left(\frac{8-\omega^2}{-6\omega}\right)\right] = \phi(\omega)}$$

B) $G(s) = \dfrac{(s+5)}{(s+2)(s+4)}$, $s \Rightarrow j\omega$

conjugate

$\hookrightarrow \dfrac{s+5}{s^2+6s+8} \rightarrow \dfrac{j\omega+5}{-\omega^2+6j\omega+8} \Rightarrow \dfrac{j\omega+5}{(-\omega^2+8)+j(6\omega)} \cdot \dfrac{(-\omega^2+8)-j(6\omega)}{(-\omega^2+8)-j(6\omega)}$

$\Rightarrow \dfrac{j(-\omega^3+8\omega) + 6\omega^2 - 5\omega^2 + 40 - 5j6\omega}{(-\omega^2+8)^2 + (6\omega)^2} \Rightarrow \dfrac{(\omega^2+40) - j(\omega^3+22\omega)}{\omega^4+20\omega^2+64}$

$$|G(\omega)| = \boxed{\frac{\sqrt{(\omega^2+40)^2 + \omega^2(\omega^2+22)^2}}{\omega^4+20\omega^2+64} = M(\omega)}$$

$\angle G(\omega) = \arctan(Im/Re)$

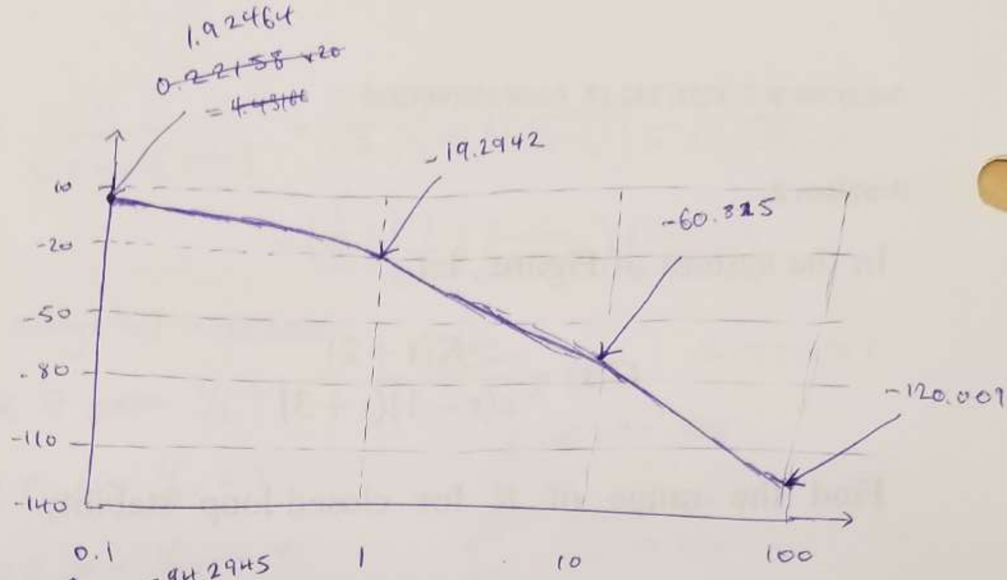
$$= \boxed{\tan^{-1}\left[\frac{-\omega(\omega^2+22)}{\omega^2+40}\right] = \phi(\omega)}$$

Asn 17 ②

plots:

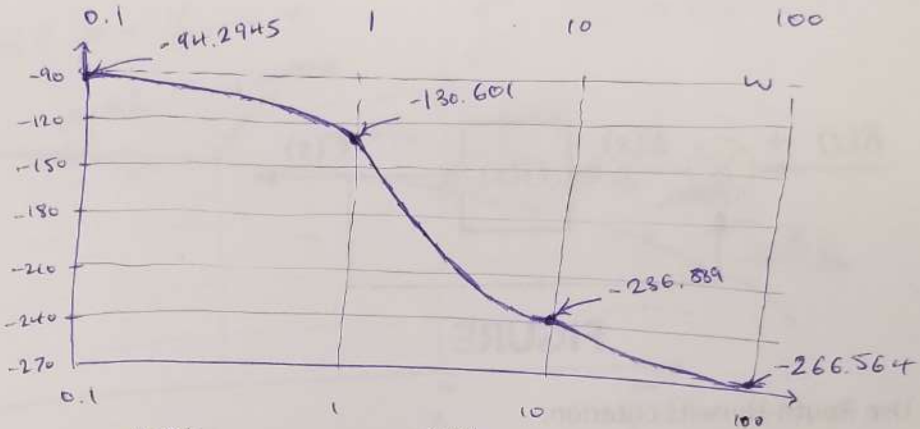
A)

20 log magnitude



W=20 => -78.2753

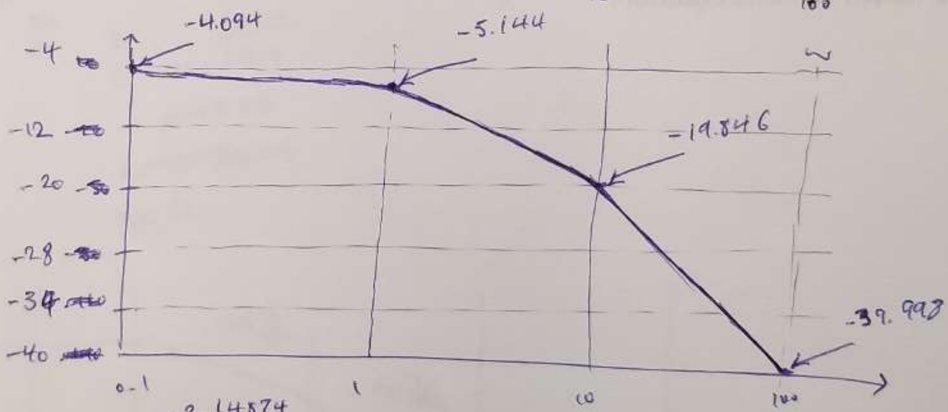
phase (deg)



W=20 => -252.98

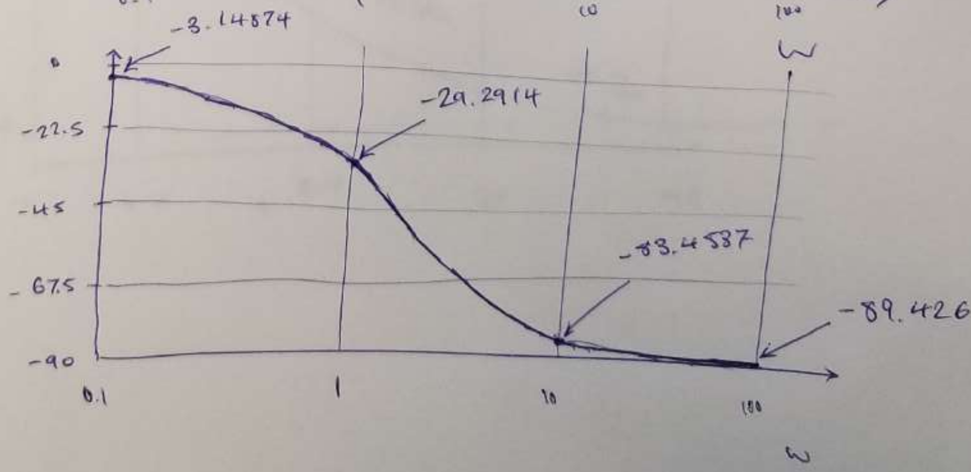
B)

20 log magnitude



W=20 => -25.9709 ~~-18.496~~

phase (deg)



w=20 => -87.0157

Asn 17 (3)

Bode plots: A)

$$G(s) = \frac{1}{s(s+2)(s+4)} = \left(\frac{1}{s}\right)\left(\frac{1}{2}\right)\left(\frac{1}{0.5s+1}\right)\left(\frac{1}{4}\right)\left(\frac{1}{0.25s+1}\right)$$

$$= \left(\frac{1}{8}\right)\left(\frac{1}{s}\right)\left(\frac{1}{0.5s+1}\right)\left(\frac{1}{0.25s+1}\right)$$

Magnitude

$1/8$: Constant $20\log(1/8) = -18.0618$

$1/s$: -20 dB @ point $1/1 = 1$

$\frac{1}{0.5s+1}$: -20 dB @ point $1/0.5 = 2$

$\frac{1}{0.25s+1}$: -20 dB @ point $1/0.25 = 4$

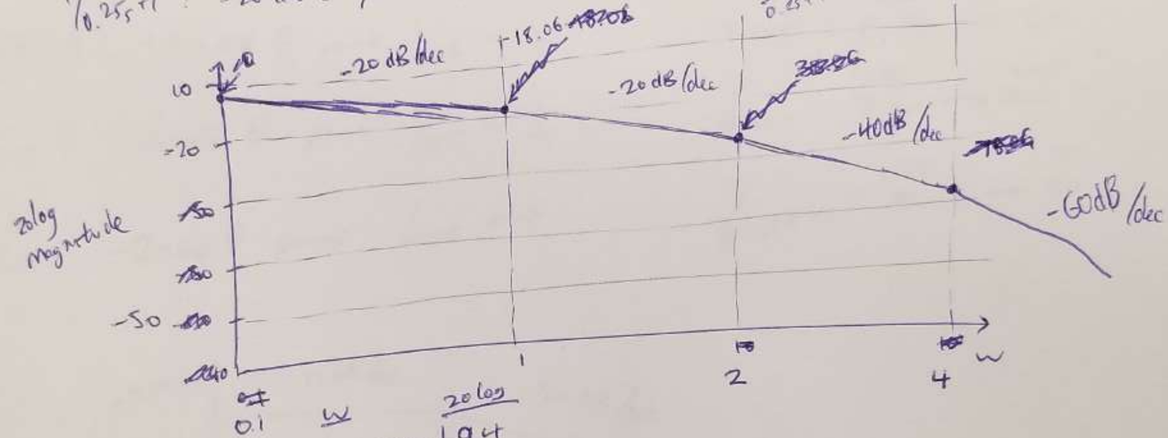
phase

$1/8$: $0°$ $\quad 0.1T \to 10T$

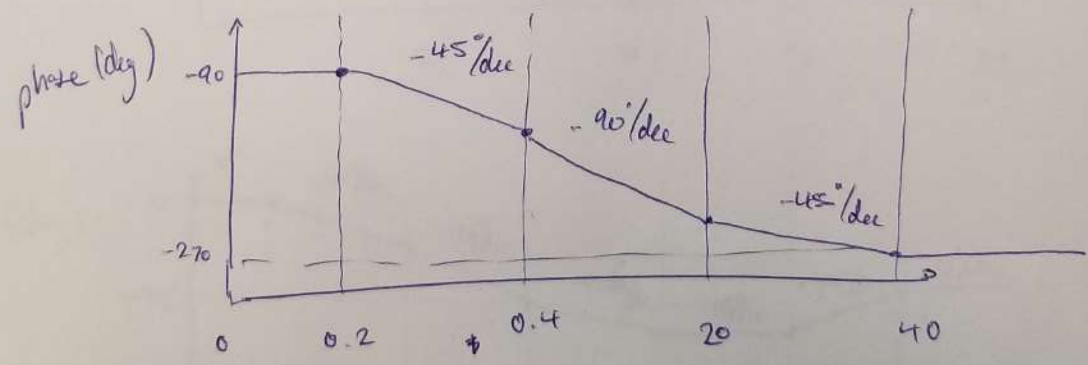
$1/s$: $-90°$ flat

$\frac{1}{0.5s+1}$: $0.2 \to 20$

$\frac{1}{0.25s+1}$: $0.4 \to 40$



w	20log
0.1	≈ +1.94
1	≈ -18.06
~~10~~	~~≈ 38.06~~
~~100~~	~~≈~~



Asn 17 (4)

Bode Plots B) $G(s) = \frac{(s+5)}{(s+2)(s+4)} \rightarrow (5)(0.2s+1)(\frac{1}{2})(\frac{1}{0.5s+1})(\frac{1}{4})(\frac{1}{0.25s+1})$

$= (\frac{5}{8})(0.2s+1)(\frac{1}{0.5s+1})(\frac{1}{0.25s+1})$

Magnitude

$\frac{5}{8}$: Const. $20\log(5/8) = -4.0824$

$0.2s+1$: $+20dB$ @ point $1/0.2 = 5$

$\frac{1}{0.5s+1}$: $-20dB$ @ point $1/0.5 = 2$

$\frac{1}{0.25s+1}$: $-20dB$ @ point $1/0.25 = 4$

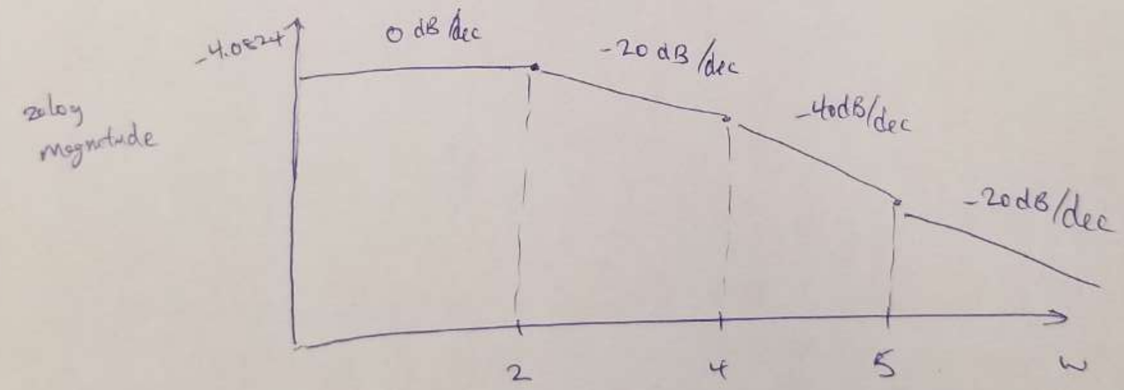
Phase

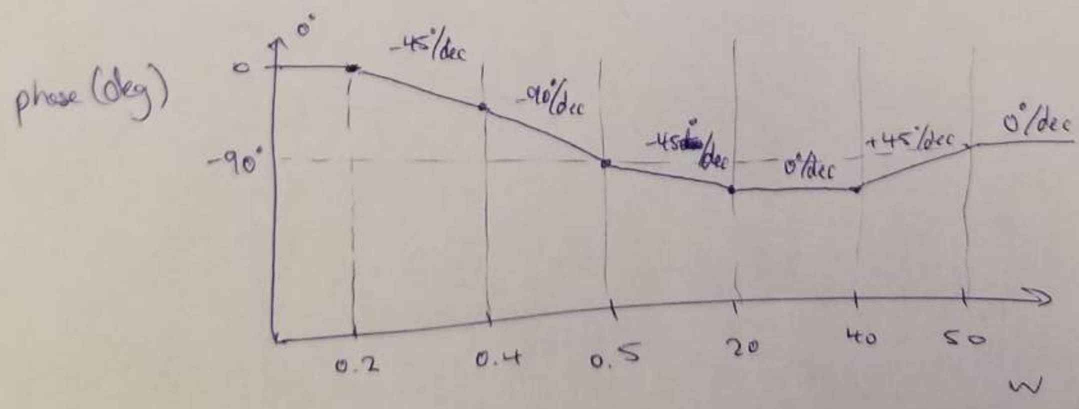
$5/8$: $0°$

$0.2s+1$: $\overset{+45}{0.5} \rightarrow \overset{-45}{50}$

$1/0.5s+1$: $\overset{-45}{0.2} \rightarrow \overset{+45}{20}$

$1/0.25s+1$: $\overset{-45}{0.4} \rightarrow \overset{+45}{40}$





Assignment 18 (ELEC 341 L18_NyquistCriterion)

Problem 1:

Draw Nyquist plot for the given *G*(*s*):

$$G(s) = \frac{20(s^2 + s + 0.5)}{s(s+1)(s+10)}$$

Assignment 18:

$$G(s) = \frac{20(s^2+s+0.5)}{s(s+1)(s+10)}$$

$$\therefore |G(iw)| = \left|\frac{20(-w^2+iw+0.5)}{iw(iw+1)(iw+10)}\right|$$

$$= \frac{20\,|-w^2+iw+0.5|}{|iw|\,|iw+1|\,|iw+10|}$$

$$= \frac{20\sqrt{(-w^2+0.5)^2+w^2}}{w\cdot\sqrt{1+w^2}\cdot\sqrt{100+w^2}}$$

$$\measuredangle G(iw) = \tan^{-1}\frac{Im}{Re}$$

$$= \underbrace{\cancel{\tan^{-1} 20}}_{0} + \tan^{-1}\left(\frac{w}{0.5-w^2}\right)$$

$$- \left(\underbrace{\cancel{\tan^{-1} iw}}_{90^\circ} + \tan^{-1}\frac{w}{1} + \tan^{-1}\frac{w}{10}\right)$$

$$= \tan\frac{w}{0.5-w^2} - 90^\circ - \tan^{-1} w - \tan^{-1}\frac{w}{10}$$

$\therefore$ at $w \to 0^+$

$$|G(i0^+)| = \frac{20\sqrt{(0+0.5)^2+0}}{0^+\cdot 1\cdot 10} \to \infty$$

$$\measuredangle G(i0^+) = \underbrace{\cancel{\tan^{-1}\frac{0^+}{0.5-0}}}_{0} - 90^\circ - \underbrace{\cancel{\tan^{-1} 0^+}}_{0} - \underbrace{\cancel{\tan^{-1}\frac{0^+}{10}}}_{0} = -90^\circ$$

at $w \to \infty$

$$|G(i\infty)| = 20\sqrt{\frac{w^2+(0.5-w^2)^2}{w^2(1+w^2)(100+w^2)}}\Bigg|_{w\to\infty}$$

$$(0.25 - w^2 + w^4)$$

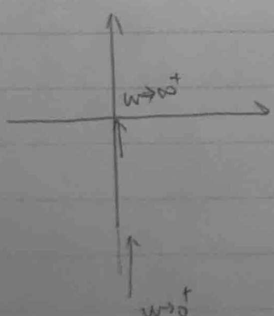
$$= 20\sqrt{\frac{1+(0.5-w^2)^2/w^2}{(1+w^2)(100+w^2)}}\Bigg|_{w\to\infty} = 20\sqrt{\frac{\cancel{\frac{1}{w^2}} + \cancel{\frac{0.25}{w^4}} - \cancel{\frac{1}{w^2}} + 1}{(\cancel{\frac{1}{w^2}}+1)(100+w^2)}}\Bigg|_{w\to\infty} = 20\sqrt{\frac{1}{100+w^2}}\Bigg|_{w\to\infty} \to 0$$

$$\measuredangle G(i\infty) = \underbrace{\cancel{\tan^{-1}\frac{\frac{1}{w}}{\frac{0.5}{w^2}-1}}}_{0}\Bigg|_{w\to\infty} - 90^\circ - \underbrace{\cancel{\tan^{-1} w}}_{0}\Big|_{w\to\infty} - \underbrace{\cancel{\tan^{-1}\frac{w}{10}}}_{0}\Big|_{w\to\infty}$$

$$= -90^\circ$$

	$\lvert G(iw)\rvert$	$\measuredangle G(iw)$
$w=0.5$	2.0	-55°
$w=0.75$	1.605	-36.5
$w=1$	1.570	-24.41
$w=1.25$	1.631	-18.1127
$w=1.5$	1.682	-15.589
$w=2$	1.766	-14.555
$w=5$	1.757	-26.71
$w=10$	1.401	-45.11

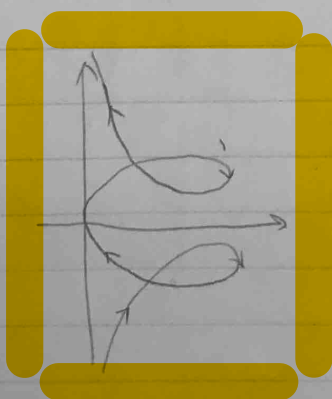
$\therefore$ Based on $w\to 0^+$, $w \to \infty$ the Nyquist plot starts with



$\Rightarrow$

use these points to plot, and mirror the points according to real axis.

$\Rightarrow$



Assignment 19 (ELEC 341 L19_RelativeStability)

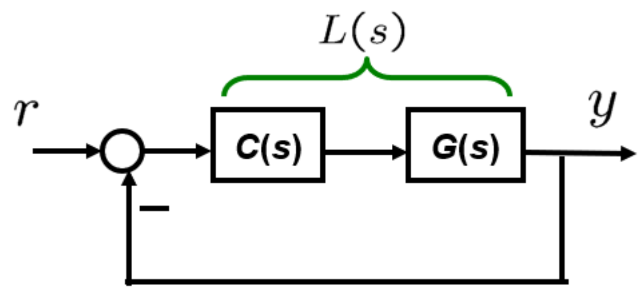
Problem 1:

For the system shown in the following figure, where

$$G(s) = \frac{1}{(s+5)(s+20)(s+50)}$$

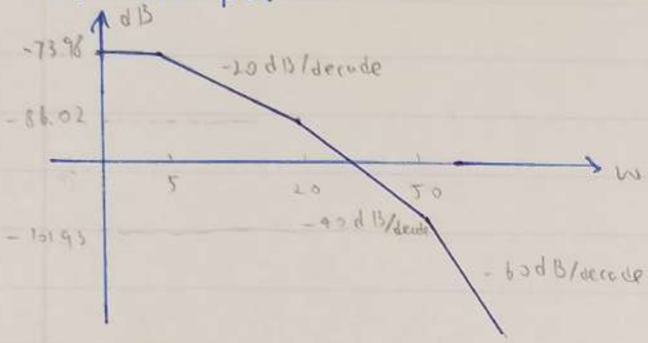
Do the following:

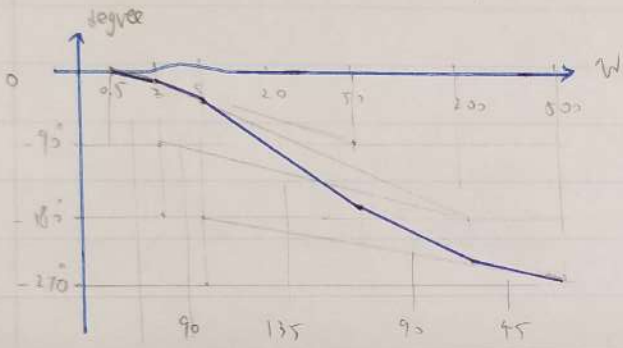
a) Draw the Bode log-magnitude and phase plots.
b) Evaluate gain margin, phase margin, gain cross-over frequency, and phase cross-over frequency analytically and show them in your Bode plots for $C(s)$ = 10,000.



Assignment 19

(a) Bode plot





45 degree / decade

for $G(s)=\frac{1}{(s+5)(s+20)(s+50)}$

$$=\frac{1}{(\frac{1}{5}s+1)(\frac{1}{20}s+1)(\frac{1}{50}s+1)\underbrace{5\times20\times50}_{5000}}$$

$\because 20\log 5000 \approx 73.98$

(b) $G(s)=\frac{1}{(s+5)(s+20)(s+50)}$ $\quad C(s)=1\times10^4$

$\because L(s)=C(s)\,G(s)=\frac{1\times10^4}{(s+5)(s+20)(s+50)}$

$\therefore \measuredangle L(i\omega_p)=-180^\circ$

$-180^\circ = -\measuredangle(5+i\omega_p)-\measuredangle(20+i\omega_p)-\measuredangle(50+i\omega_p)$

$-180^\circ = -\operatorname{atan}(\frac{\omega_p}{5})-\operatorname{atan}(\frac{\omega_p}{20})-\operatorname{atan}(\frac{\omega_p}{50})$

solve for $\omega_p = 36.7423$ rad/s

$GM = 20\log\frac{1}{|L(i\omega_p)|} = 20\log\frac{1}{\left|\frac{1\times10^4}{(i\omega_p+5)(i\omega_p+20)(i\omega_p+50)}\right|}$

$= 19.6679$ dB

$\therefore |L(i\omega_g)|=1$

$$\frac{1\times10^4}{\sqrt{\omega_g^2+5^2}\sqrt{\omega_g^2+20^2}\sqrt{\omega_g^2+50^2}}=1$$

solve for $\omega_g = 7.7416$ rad

$PM = \measuredangle L(i\omega_g)+180^\circ$

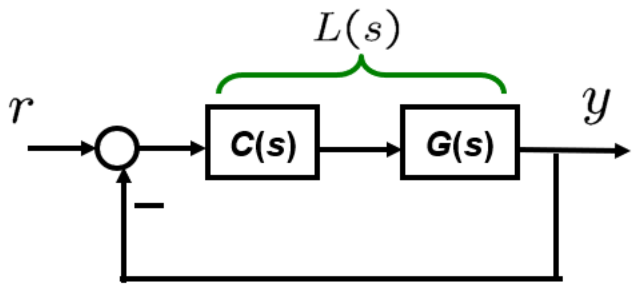
$= \measuredangle\left(\frac{1}{(i\omega_g+5)(i\omega_g+20)(i\omega_g+50)}\right)+180^\circ$

$= 92.8951^\circ$

Assignment 20 (ELEC 341 L20_FreqDomSpec)

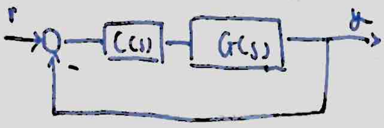
Problem 1:

Given the unity feedback system below, use frequency response methods to determine the value of gain, $C(s)$, to yield a step response with a 20% overshoot. Use Matlab to graph the step response.



$$G(s) = \frac{(s+4)}{s(s+8)(s+10)(s+15)}$$

Assignment 20



$$G(s) = \frac{s+4}{s(s+8)(s+10)(s+15)}$$

$\because$ PO = 20% , find C(s)

$\therefore$ $$\zeta = \frac{\left|\ln \frac{PO}{100}\right|}{\sqrt{\pi^2 + (\ln \frac{PO}{100})^2}} = 0.4559$$

$$PM_{compensator} = \mathrm{atan}\left(\frac{2\zeta}{-2\zeta^2 + \sqrt{1+4\zeta^4}}\right) = 53.8080^\circ \quad / \quad PM_{compensator} = 100\,\zeta = 45.59^\circ$$

$$\measuredangle L(iw^*) = -180^\circ + PM_{compensator}$$

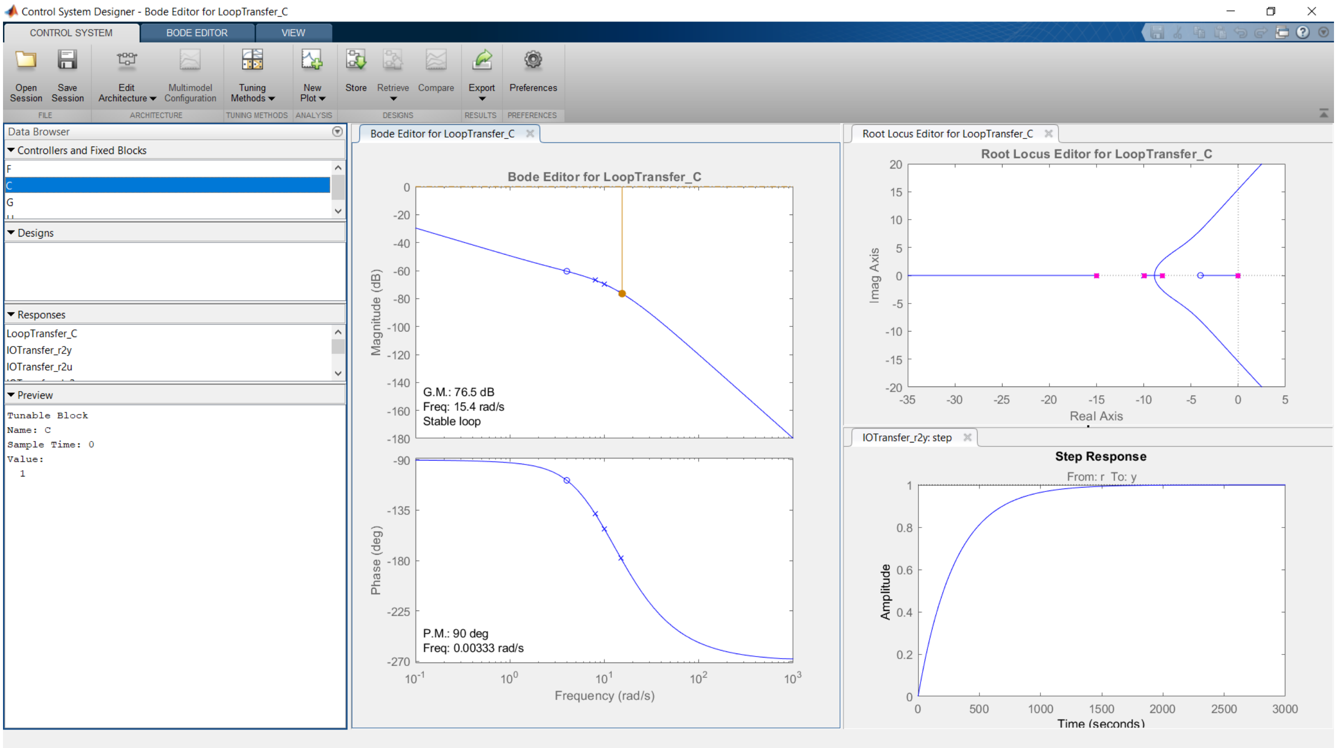
$$-134.41 = \measuredangle(iw+4) - \underbrace{\measuredangle(iw)}_{90^\circ} - \measuredangle(iw+10) - \measuredangle(iw+15) - \measuredangle(iw+8)$$

$$-44.41 = \mathrm{atan}\left(\frac{w}{4}\right) - \mathrm{atan}\left(\frac{w}{10}\right) - \mathrm{atan}\left(\frac{w}{8}\right) - \mathrm{atan}\left(\frac{w}{15}\right)$$

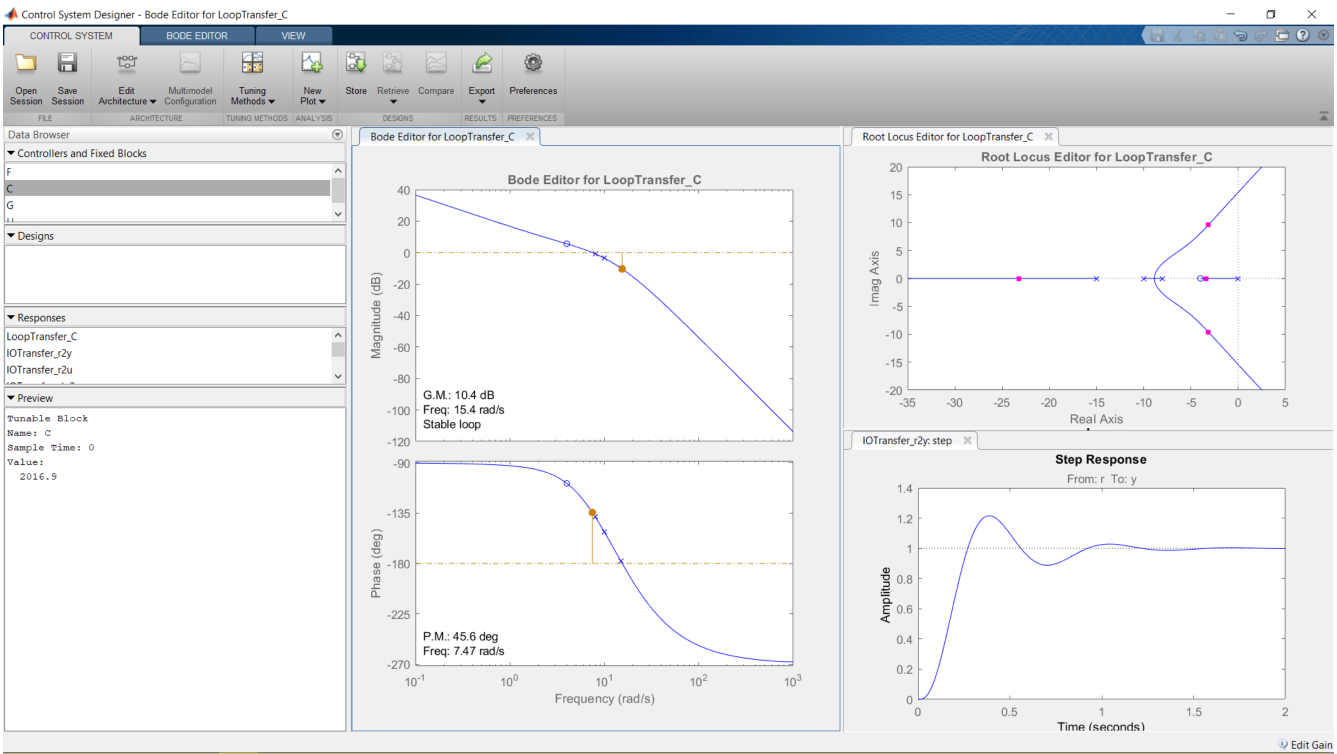
solve for $w = 7.4661$ rad/s

$$\boxed{\because\ C = \frac{1}{|L(iw)|} = 2016.9123}$$

Step respone for C(s) = 1



Step response for C(s) = 2016.9



> note that PM is 45.6 degree, close to desired 45.5 degree.