EECE 360: Systems and Control
Midterm #1

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This midterm is closed-note, closed-book. A one-sided cheat-sheet is allowed. To get full credit, show all of your work.

1 Mass on a cart (50 points)

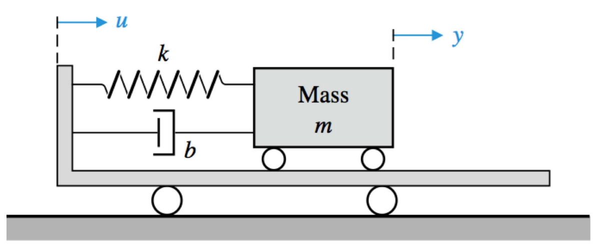

Figure 1: Mass m attached to a massless cart.

A mass as shown in Figure 1 sits on a massless cart. There is no friction between the cart and the ground, or between the mass and the cart. The output is the position of the mass, labeled $y(t)$, and the input is the position of the cart, labeled $u(t)$. The equations of motion for the mass m are:

$$m\ddot{y}(t) = -k(y(t) - u(t)) - b(\dot{y}(t) - \dot{u}(t)) \tag{1}$$

1. Find the transfer function $G(s)$ with input $u(t)$ and output $y(t)$.
2. Write the system in control canonical form. What are the matrices A, B, C, D?
3. Find the state transition matrix $\Phi(s)$ for general k, b, M. Now assume for the rest of the problem that $k = 4$, $b = 0$, and $M = 1$. Rewrite $\Phi(s)$ with these particular values.
4. What is the forced response in the s-domain for the input $u(t) = \mathbf{1}(t)$? What is the forced response in the s-domain for the input $u(t) = e^{-3t}$?
5. Given the initial condition $x(0) = \begin{bmatrix} 1 & 1 \end{bmatrix}^T$, what is the natural response in the s-domain? What is the natural response in the time-domain?

2 RLC Op-Amp Circuit (50 points)

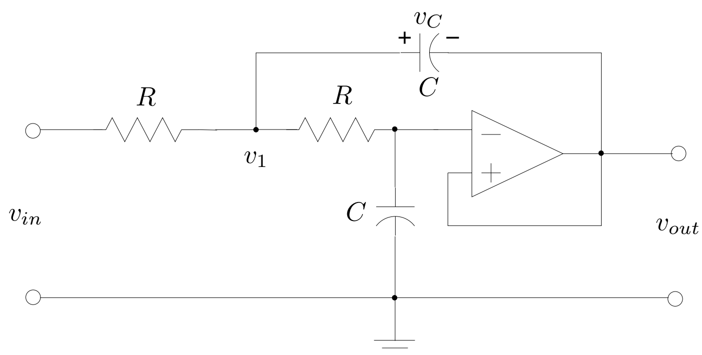

Figure 2: RLC op-amp circuit. Note that $v_c = v_1 - v_{out}$.

Consider the circuit shown in Figure 2.

1. Find the integro-differential equations which describe the circuit behavior. Write enough equations to be able to relate the output v_{out} to the input v_{in}.
2. Find the state-space representation using the state vector $x = [v_{out}, v_C]$, with input v_{in} and output v_0. Show that the state matrix is $\begin{bmatrix} 0 & 1/(RC) \\ -1/(RC) & -2/(RC) \end{bmatrix}$. What are B, C, D?
3. Use these matrices to find the transfer function $G(s)$ with input v_{in} and output v_C. (Only partial credit will be given if you find the transfer function in any other way, e.g. directly from Step 1).)
4. Show that the eigenvalues of A are equal to the poles of $G(s)$.
5. Multiple state-space representations exist for any given SISO system. Do multiple characteristic equations exist? Why or why not?