Assignment 3 (ELEC 341 L3_ODESolution)

Problem 1:

Using Laplace transforms, solve the following differential equations assuming zero initial conditions.

a. $\dfrac{dx}{dt} + 7x = 5\cos 2t$

b. $\dfrac{d^2x}{dt^2} + 6\dfrac{dx}{dt} + 8x = 5\sin 3t$

c. $\dfrac{d^2x}{dt^2} + 8\dfrac{dx}{dt} + 25x = 10u(t)$

Assume that the forcing functions are zero prior to t = 0-.

Solution:

a. The Laplace transform of the differential equation, assuming zero initial conditions,

is, $(s+7)X(s) = \dfrac{5s}{s^2+2^2}$. Solving for X(s) and expanding by partial fractions,

$$\frac{5s}{(s+7)(s^2+4)} = -\frac{35}{53}\frac{1}{s+7} + \frac{5}{53}\frac{7s+4}{s^2+4}$$

Or,

$$\frac{5s}{(s+7)(s^2+4)} = -\frac{35}{53}\frac{1}{s+7} + \frac{5}{53}\frac{7s+2\sqrt{4}}{s^2+4}$$

Taking the inverse Laplace transform, $x(t) = -\dfrac{35}{53}\ e^{-7t} + (\dfrac{35}{53}\ \cos 2t + \dfrac{10}{53}\ \sin 2t)$.

b. The Laplace transform of the differential equation, assuming zero initial conditions, is,

$(s^2+6s+8)X(s) = \dfrac{15}{s^2+9}$.

Solving for X(s)

$$X(s) = \frac{15}{(s^2+9)(s^2+6s+8)}$$

and expanding by partial fractions,

$$X(s) = -\frac{3}{65}\frac{6s+\frac{1}{\sqrt{9}}\sqrt{9}}{s^2+9} - \frac{3}{10}\frac{1}{s+4} + \frac{15}{26}\frac{1}{s+2}$$

Taking the inverse Laplace transform,

$$x(t) = -\frac{18}{65}\cos(3t) - \frac{1}{65}\sin(3t) - \frac{3}{10}e^{-4t} + \frac{15}{26}e^{-2t}$$

c. The Laplace transform of the differential equation is, assuming zero initial conditions,
$(s^2+8s+25)x(s) = \dfrac{10}{s}$. Solving for X(s)

$$X(s) = \frac{10}{s(s^2+8s+25)}$$

and expanding by partial fractions,

$$X(s) = \frac{2}{5}\frac{1}{s} - \frac{2}{5}\frac{1(s+4)+\frac{4}{\sqrt{9}}\sqrt{9}}{(s^2+8s+25)}$$

Taking the inverse Laplace transform,

$$x(t) = \frac{2}{5} - e^{-4t}\left(\frac{8}{15}\sin(3t) + \frac{2}{5}\cos(3t)\right)$$

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Problem 2:

For each of the following transfer functions, write the corresponding differential equation:

a. $\dfrac{X(s)}{F(s)} = \dfrac{7}{s^2 + 5s + 10}$

b. $\dfrac{X(s)}{F(s)} = \dfrac{15}{(s+10)(s+11)}$

c. $\dfrac{X(s)}{F(s)} = \dfrac{s+3}{s^3 + 11s^2 + 12s + 18}$

Solution:

a. Cross multiplying, $(s^2+5s+10)X(s) = 7F(s)$.

Taking the inverse Laplace transform, $\dfrac{d^2x}{dt^2} + 5\dfrac{dx}{dt} + 10x = 7f$.

b. Cross multiplying after expanding the denominator, $(s^2+21s+110)X(s) = 15F(s)$.

Taking the inverse Laplace transform, $\dfrac{d^2x}{dt^2} + 21\dfrac{dx}{dt} + 110x = 15f$.

c. Cross multiplying, $(s^3+11s^2+12s+18)X(s) = (s+3)F(s)$.

Taking the inverse Laplace transform, $\dfrac{d^3x}{dt^3} + 11\dfrac{d^2x}{dt^2} + 12\dfrac{dx}{dt} + 18x = df/dt + 3f$.