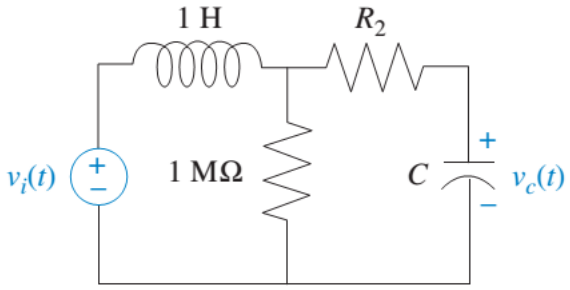
Problem 1:

For the circuit shown in Figure , find the values of R_2 and C to yield 8% overshoot with a settling time of 1 ms for the voltage across the capacitor, with $v_i(t)$ as a step input.

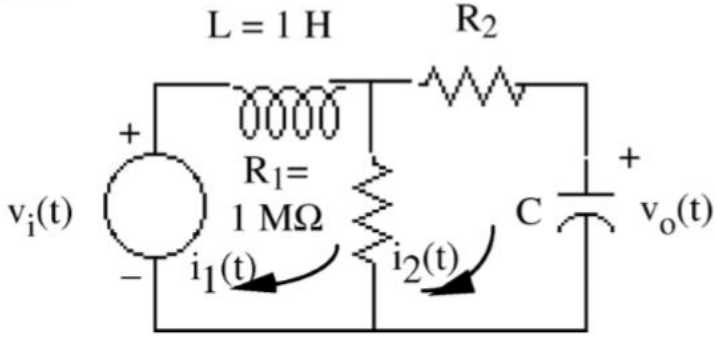


FIGURE

The given settling time is a 2% settling time.

Solution:

For the circuit shown below



write the loop equations as

$$(R_1 + Ls)I_1(s) - R_1 I_2(s) = V_i(s)$$

$$-R_1 I_1(s) + \left(R_1 + R_2 + \frac{1}{Cs}\right) I_2(s) = 0$$

Solving for $I_2(s)$

$$I_2(s) = \frac{\begin{vmatrix} R_1 + Ls & Vi(s) \\ -R_1 & 0 \end{vmatrix}}{\begin{vmatrix} R_1 + Ls & -R_1 \\ -R_1 & R_1 + R_2 + \dfrac{1}{Cs} \end{vmatrix}}$$

But, $V_o(s) = \dfrac{1}{Cs} I_2(s)$. Thus,

$$\frac{V_o(s)}{V_i(s)} = \frac{R_1}{(R_2 + R_1)CLs^2 + (CR_2R_1 + L)s + R_1}$$

Substituting component values,

$$\frac{V_o(s)}{V_i(s)} = 1000000 \frac{\dfrac{1}{(R_2 + 1000000)C}}{s^2 + \dfrac{(1000000CR_2 + 1)}{(R_2 + 1000000)C} s + 1000000 \dfrac{1}{(R_2 + 1000000)C}}$$

For 8% overshoot, $\zeta = 0.6266$. For $T_S = 0.001$, $\zeta\omega_n = \dfrac{4}{0.001} = 4000$. Hence, $\omega_n = 6383.66$. Thus,

$$1000000 \frac{1}{(R_2 + 1000000)C} = 6383.66^2$$

or,

$$C = 0.0245 \frac{1}{R_2 + 1000000} \tag{1}$$

Also,

$$\frac{1000000CR_2 + 1}{(R_2 + 1000000)C} = 8000 \tag{2}$$

Solving (1) and (2) simultaneously, $R_2 = 8023\ \Omega$, and $C = 2.4305 \times 10^{-2}\ \mu F$.