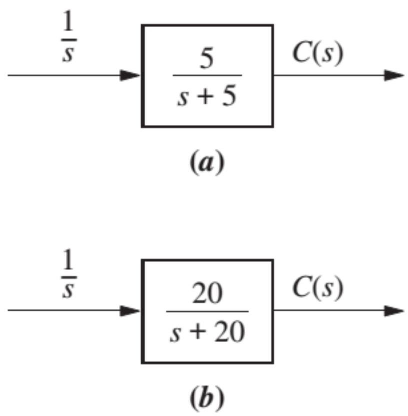
Problem 1:

Find the output response, $c(t)$, for each of the systems shown in Figure. Also find the time constant, rise time, and settling time for each case.



FIGURE

Use the difference between 10% to 90% of y_{ss} as the rise time and also use a 2% settling time.

Solution:

a. $C(s) = \frac{5}{s(s+5)} = \frac{1}{s} - \frac{1}{s+5}$. Therefore, $c(t) = 1 - e^{-5t}$.

Also, $T = \frac{1}{5}$, $T_r = \frac{2.2}{a} = \frac{2.2}{5} = 0.44$, $T_s = \frac{4}{a} = \frac{4}{5} = 0.8$.

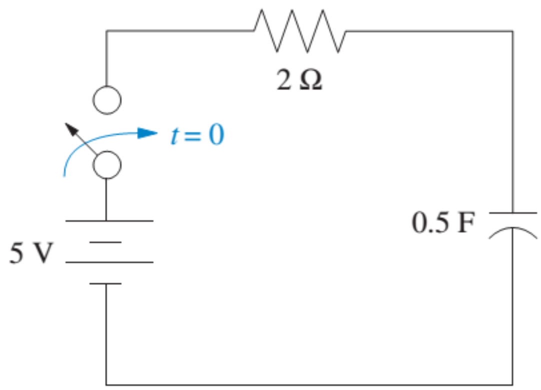
b. $C(s) = \frac{20}{s(s+20)} = \frac{1}{s} - \frac{1}{s+20}$. Therefore, $c(t) = 1 - e^{-20t}$. Also, $T = \frac{1}{20}$,

$T_r = \frac{2.2}{a} = \frac{2.2}{20} = 0.11$, $T_s = \frac{4}{a} = \frac{4}{20} = 0.2$.

In the above calculations, a = 1/T.

Problem 2:

Find the capacitor voltage in the network shown in Figure if the switch closes at $t = 0$. Assume zero initial conditions. Also find the time constant, rise time, and settling time for the capacitor voltage.



FIGURE

Use the difference between 10% to 90% of y_{ss} as the rise time and also use a 2% settling time. Use Matlab to verify and plot your output response.

Solution:

Using voltage division, $\dfrac{V_C(s)}{V_i(s)} = \dfrac{1/RC}{s + \dfrac{1}{RC}} = \dfrac{1}{s+1}$. Since $V_i(s) = \dfrac{5}{s}$

$$V_C(s) = \frac{5}{s}\left(\frac{1}{s+1}\right) = \frac{5}{s} - \frac{5}{s+1}.$$

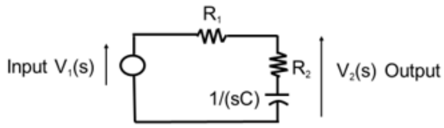
Therefore: $v_c(t) = 5 - 5e^{-t}$.

Also, $T = \dfrac{1}{1} = 1\,\text{sec}; T_r = \dfrac{2.2}{1} = 2.2\,\text{sec}; T_s = \dfrac{4}{1} = 4\,\text{sec}$.

Assignment 9 (ELEC 341 L9_StepResponse)

Modeling example (cont'd)

- **Impedance method**
  - **Step 1:** Replace electrical elements with impedances.
  - **Step 2:** Deal with impedances as if they were resistances.



$$G(s) = \frac{V_2(s)}{V_1(s)} = \frac{\text{(Impedance for output)}}{\text{(Total impedance)}} = \frac{R_2 + \frac{1}{sC}}{R_1 + R_2 + \frac{1}{sC}}$$

$$= \frac{R_2Cs+1}{(R_1+R_2)Cs+1}$$

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For the problem at hand: $R_2 = 0 \rightarrow$

$$\frac{V_2}{V_1} = \frac{V_c}{V_i} = \frac{\frac{1}{sC}}{R+\frac{1}{sC}} = \frac{1}{RCs+1} \xrightarrow[C=0.5]{R=2}$$

$$\boxed{\frac{V_c}{V_i} = \frac{1}{s+1}}$$

Problem 3:

For the second-order system that follows, find ζ, ω_n, T_s, T_p, T_r, and $\%OS$.

$$T(s) = \frac{16}{s^2 + 3s + 16}$$

Solution:

$\omega_n^2 = 16$ r/s, $2\zeta\omega_n = 3$. Therefore $\zeta = 0.375$, $\omega_n = 4$. $T_s = \dfrac{4}{\zeta\omega_n} = 2.667$ s; $T_P = \dfrac{\pi}{\omega_n\sqrt{1-\zeta^2}} =$ 0.8472 s; $\%OS = e^{-\zeta\pi / \sqrt{1-\zeta^2}} \times 100 = 28.06$ %; $\omega_n T_r = (1.76\zeta^3 - 0.417\zeta^2 + 1.039\zeta + 1) = 1.4238$; therefore, $T_r = 0.356$ s.