Assignment 18 (ELEC 341 L18_NyquistCriterion)

Problem 1:

Draw Nyquist plot for the given *G*(*s*):

$$G(s) = \frac{20(s^2 + s + 0.5)}{s(s+1)(s+10)}$$

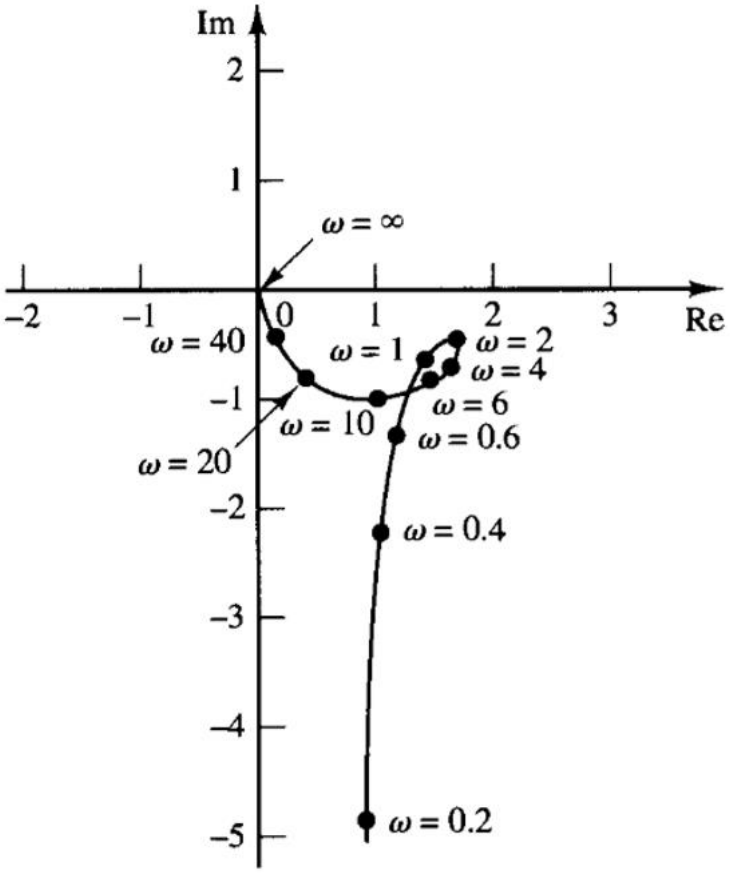
Solution:

$$|G(j\omega)| = \frac{20\sqrt{(0.5-\omega^2)^2+\omega^2}}{\omega\sqrt{(1+\omega^2)(100+\omega^2)}}$$

$$\angle G(j\omega) = tan^{-1}\frac{\omega}{0.5-\omega^2} - 90 - tan^{-1}\omega - tan^{-1}(\omega/10)$$

ω	$\lvert G(j\omega)\rvert$	$\angle G(j\omega)$
0.1	9.952	-84.5
0.2	4.91	-78.9
0.4	2.4	-64.5
0.6	1.7	-47.53
1	1.573	-24.15
2	1.768	-14.5
6	1.8	-31.09
10	1.407	-45.03
20	0.893	-63.44
40	0.485	-75.96

(a) Estimated values



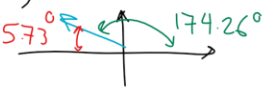
(b) Polar Plot

Method 1:

For this method, you will need to calculate the numerator and denominator in the following term, separately, and check for the quadrant in which the vector is located. After that, you should convert the angle to an angle which is measured w.r.t. the positive Re axis.

e.g.: Find $\tan^{-1}\left(\frac{\omega}{0.5-\omega^2}\right)$ at $\omega=10$ and then compute $\angle G(j\times 10)$.

for $\omega=10 \rightarrow \tan^{-1}\left(\frac{10}{0.5-10^2}\right)=\tan^{-1}\left(\frac{+10}{-99.5}\right)=5.73^\circ \rightarrow 180-5.73=174.26^\circ$



For $\omega=10$ find $\angle G(j\times 10)$:

$\angle G(j\times 10)=174.26^\circ-90^\circ-84.28^\circ-45^\circ \rightarrow$

$\angle G(j\times 10)=-45.02^\circ$

Method 2:

Decompose the quadratic equation in the numerator into two brackets, then follow the usual method of finding the angle:

$$\angle G(s) = \frac{20(s+0.5-0.5j)(s+0.5+0.5j)}{s(s+1)(s+10)}$$

$$\angle G(j\omega) = \frac{20(j\omega+0.5-0.5j)(j\omega+0.5+0.5j)}{(j\omega)(j\omega+1)(j\omega+10)}$$

$$= \frac{20\{0.5+(\omega-0.5)j\}\{0.5+(\omega+0.5)j\}}{(j\omega)\{1+j\omega\}\{10+j\omega\}}$$

$$= \angle\{0.5+(\omega-0.5)j\}+\angle\{0.5+(\omega+0.5)j\}-\{\angle j\omega+\angle(1+j\omega)+\angle(10+j\omega)\} \Rightarrow$$

$$\angle G(j\omega) = \tan^{-1}\left(\frac{\omega-0.5}{0.5}\right)+\tan^{-1}\left(\frac{\omega+0.5}{0.5}\right)-90^{\circ}-\tan^{-1}(\omega)-\tan^{-1}\left(\frac{\omega}{10}\right)$$

e.g.: at $\omega = 40 \rightarrow \angle(j\times 40) = -75.96^{\circ}$