Assignment 14 (ELEC 341 L14_RLLeadLag)

Problem 1:

For a control system with open loop transfer function of $C(s).G(s) = C(s)/[s(s+4)]$, design a lead compensator by selecting $C_{lead}(s)$ so that the damping frequency is 4 rad/s and $\zeta = 0.707$.

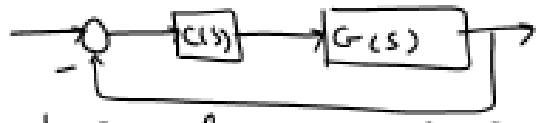
Solution:

$C(s)\,G(s) = C(s)\cdot\dfrac{1}{s(s+4)}$

$C_{lead} = ?$

$\omega_d = 4$ rad/s

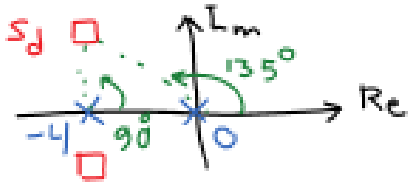
$\zeta = 0.707$



Step 1: Find $G(s_d)$ & φ (angle of deficiency)

$\omega_d = \omega_n\sqrt{1-\zeta^2} \rightarrow 4 = \omega_n\sqrt{1-(0.707)^2} \rightarrow \omega_n = 5.656$

$s_d = -\zeta\omega_n \pm \omega_d j \rightarrow s_d = -0.707 \times 5.656 \pm 4j \rightarrow s_d = -4 \pm 4j$



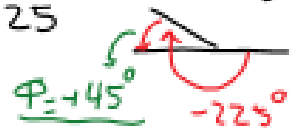
Find $G(s_d)$: $\quad G(s) = \dfrac{1}{s(s+4)} \rightarrow G(s_d) = \dfrac{1}{s_d(s_d+4)} =$

$= \dfrac{1}{(-4+4j)(-4+4j+4)} \rightarrow \boxed{G(s_d) = \dfrac{1}{(4j)(-4+4j)}}$

Check if the angle condition is satisfied (i.e., $\angle G(s_d) = 180^\circ(2k+1);\ k = 0, \pm 1, \pm 2$)

$\rightarrow \angle G(s_d) = \angle 1 - \angle\{\angle(-4+4j) + \angle(4j)\} =$

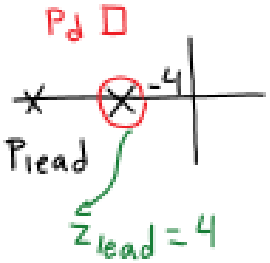
$= 0^\circ - \left\{\tan^{-1}\left(\dfrac{4}{-4}\right) + \tan^{-1}\left(\dfrac{4}{0}\right)\right\} = 0^\circ - \{135^\circ + 90^\circ\} = -225^\circ$



$\rightarrow \boxed{\varphi = +45^\circ}$

Step 2: Find $C_{lead}(s)$

Select $z_{lead} = 4$ (The real part of s_d)



$\Rightarrow$ Find $P_{lead} = ?$; $\begin{cases} C_{lead}(s) = K\dfrac{s+4}{s+P_{lead}} \text{ should} \\ \text{satisfy } \angle C(s_d) = \varphi = +45^\circ \end{cases}$

ignore K: $\quad \angle\dfrac{s_d+4}{s_d+P_{lead}} = 45^\circ \rightarrow \angle(s_d+4) - \angle(s_d+P_{lead}) = 45^\circ \rightarrow$

$\angle(-4+4j+4) - \angle(-4+4j+P_{lead}) = 45^\circ \rightarrow \tan^{-1}(4j) - \tan^{-1}\left(\dfrac{4}{-4+P_{lead}}\right) = 45^\circ$

$\rightarrow 90^\circ - 45^\circ = \tan^{-1}\left(\dfrac{4}{-4+P_{lead}}\right) \rightarrow \tan 45^\circ = \dfrac{4}{-4+P_{lead}} \rightarrow 1 = \dfrac{4}{-4+P_{lead}} \rightarrow$

$\rightarrow \underline{C_{lead}(s) = \dfrac{K(s+4)}{(s+8)}}$

Design K: Find K at s_d:

We know that $\left|C_{lead}(s) \cdot G(s)\right|_{@\, s=s_d} = \left|\frac{K}{s(s+8)}\right|_{s_d} = 1$

$\rightarrow K \left|\frac{1}{s(s+8)}\right|_{s_d=-4+4j} = 1 \rightarrow \frac{K}{\sqrt{16+16}\sqrt{16+16}} = 1 \rightarrow$

$K = 32 \Rightarrow$ $C_{lead}(s) = \frac{32(s+4)}{(s+8)}$