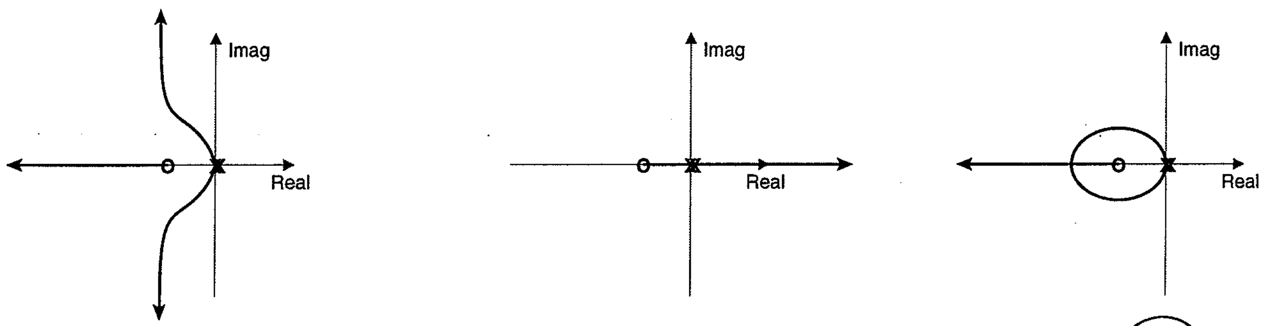
1 Root Locus (30 points)

In each of the following rows, identify the root locus plot that is correct.

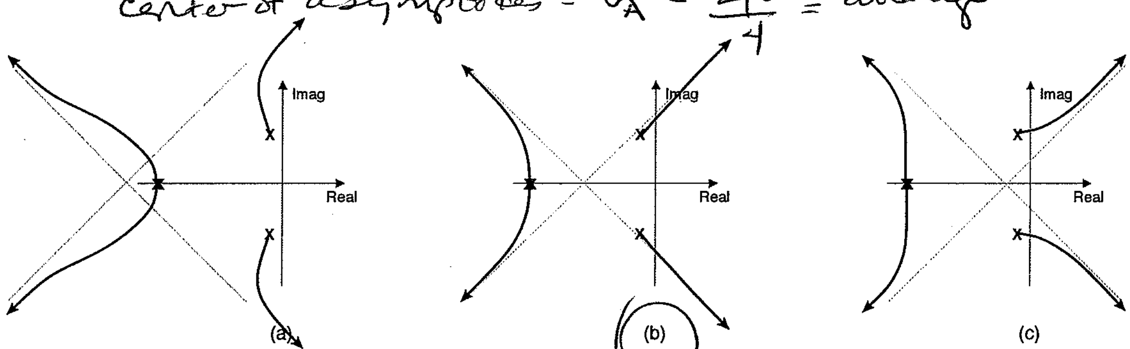
1. Co-located poles and one zero $\Longrightarrow$ 1 asymptote $(n-m = 2-1 = 1)$



(a) (b) (c)

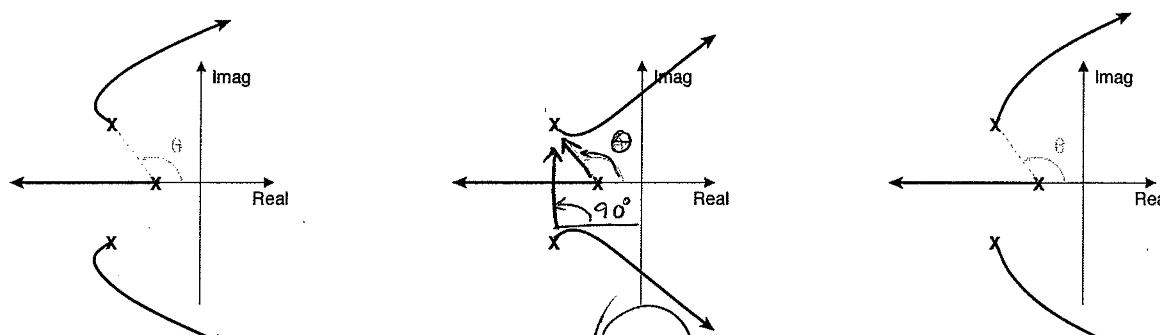
2. Co-located poles and complex conjugate poles

center of asymptotes $= \sigma_A = \frac{\sum p_i}{4}$ = average location of poles



(a) (b) (c)

3. Single pole and complex conjugate poles (assume $\theta \approx 135°$)



(a) (b) (c)

$$-(\theta_p + \theta + 90) = 180$$

$$\theta_p = -180 - 90 - \theta$$

$$\cong 90 - 135^\circ$$

$$\simeq -45^\circ$$

2 Satellite Attitude Control (70 points)

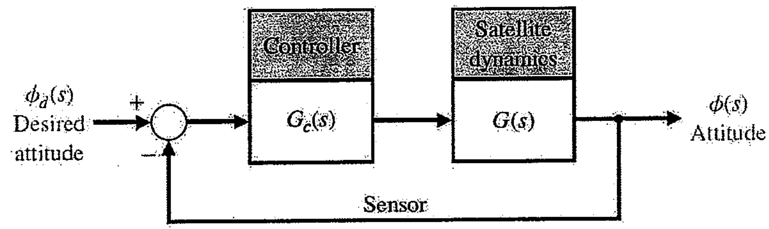


Figure 1: Block-diagram model of attitude control of an orbiting satellite.

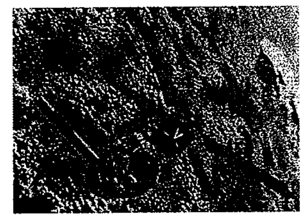


Figure 2: Satellite in space.

Consider the problem of attitude control for satellite design. The satellite must maintain a certain orientation with respect to the sun in order to keep its solar cells charged. Thrusters attached to the satellite can be used to change the satellite's orientation. Assume that the satellite dynamics are $G(s) = \frac{1}{s^2}$ and a lead controller $G_c(s) = K\frac{s+z}{s+p}$ is used.

1. What values of $K > 0$ will stabilize the closed-loop system? (Recall $p > z > 0$).

$0 = 1 + kG(s)$

$= (s+p)s^2 + k(s+z)$

$= s^3 + ps^2 + ks + kz$

$\therefore$ $k > 0$ and

$kp - kz > 0$

$k(p-z) > 0$

$k > 0$.

any $k > 0$ will work.

2. The closed-loop system should have a non-dominant pole at $s = -10$, and a dominant complex conjugate pair of poles closer to the origin. What are z, p, K such that the settling time is equal to 4 seconds and the peak time is equal to π seconds? (Hint: Start by finding ζ, ω_n to meet transient requirements.)

$$t_s = \frac{4}{\zeta \omega_n} = 4 \Rightarrow \zeta \omega_n = 1$$

$$t_p = \frac{\pi}{\omega_n \sqrt{1-\zeta^2}} = \pi \Rightarrow \omega_n = \frac{1}{\sqrt{1-\zeta^2}}$$

$$\frac{1}{\zeta} = \frac{1}{\sqrt{1-\zeta^2}}$$

$$\zeta^2 = 1-\zeta^2$$

$$\zeta = 1/\sqrt{2}, \quad \omega_n = \sqrt{2}$$

$$(s+10)(s^2 + 2\zeta\omega_n s + \omega_n^2)$$

$$= (s+10)(s^2 + 2 \cdot 1 \cdot s + 2)$$

$$= s^3 + 2s^2 + 2s + 10s^2 + 20s + 20$$

$$= s^3 + 12s^2 + 22s + 20 \longrightarrow \text{now match coefficients w/ polyn. in } K, p, z.$$

$$\therefore \quad Kz = 20$$

$$p = 12$$

$$K = 22$$

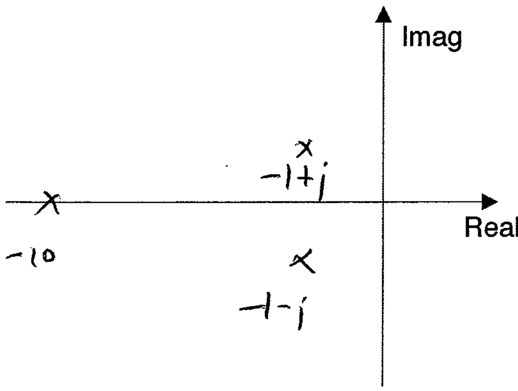
$$\Rightarrow z = \frac{20}{22} = \frac{10}{11}$$

$$\therefore \quad = (s+10)((s+1)^2 + 1)$$

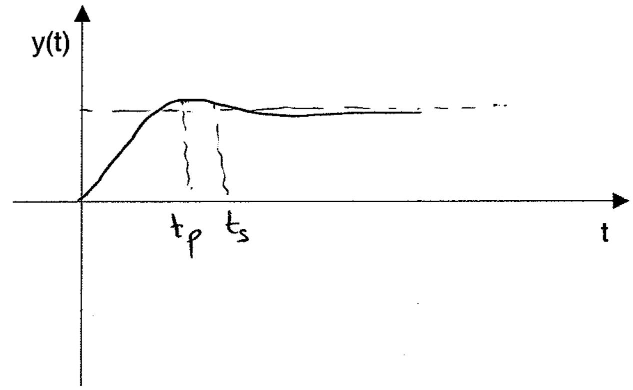
poles at $-10, -1 \pm j$

Use these computed values of K, p, z for the remainder of the problem.

3. Sketch the location of the closed-loop poles in the complex plane, and sketch the corresponding step response in the time-domain. Identify and label the peak time and settling time for your computed values of ζ, ω_n.



(a)



(b)

4. Show that the open-loop and closed-loop systems are either internally stable, marginally stable, or unstable.

open-loop: Unstable since colocated poles on imaginary axis

closed-loop: stable, since all poles have negative real part.

5. What is the type number of the closed-loop system? What is the steady-state response to a unit ramp input?

type # is 2 (= # integrators in $G_c(s)G(s)$)

$\therefore\ e_{ss} = 0$

6. Now consider the case in which there is an additive disturbance $D(s)$ that enters between the controller and the plant. Find the sensitivity function.

$$Y(s) = G(s)\big(D(s) + G_c(s)(R - Y)\big)$$

$$Y(1 + G_c G) = G \cdot D + G \cdot G_c \cdot R$$

$$Y(s) = \underbrace{\frac{G(s)}{1 + G_c(s)G(s)}}_{\text{sensitivity fcn}} D(s) + \frac{G(s)G_c(s)}{1 + G(s)G_c(s)} R(s)$$

$$= \frac{1/s^2}{1 + (1/s^2)\left(k\frac{s+z}{s+p}\right)}$$

$$= \frac{s+p}{s^2(s+p) + k(s+z)}$$

$$S(s) = \frac{s+12}{(s+10)((s+1)^2+1)}$$

7. What is the steady-state output response to a step input in the disturbance (assuming that the reference input is 0)?

$$\lim_{s \to 0} s \cdot S(s) \cdot \frac{1}{s} = \lim_{s \to 0} S(s)$$

$$= \frac{12}{10 \cdot (1+1)} = \frac{3}{5} \qquad \left(= \frac{p}{kz}\right)$$