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ELEC 341: Systems and Control

Lecture 17

Bode plot

Course roadmap

Modeling

Laplace transform

Transfer function

Models for systems
- Electrical
- Electromechanical
- Mechanical

Linearization, delay

Analysis

Stability
- Routh-Hurwitz
- Nyquist

Time response
- Transient
- Steady state

Frequency response
- Bode plot

Design

Design specs

Root locus

Frequency domain

PID & Lead-lag

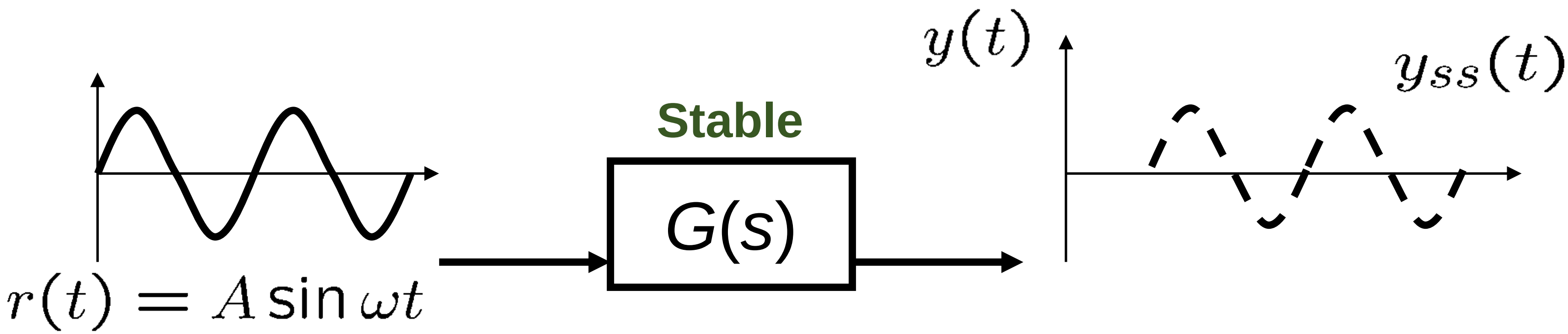
Design examples

Matlab simulations

Frequency response (review)

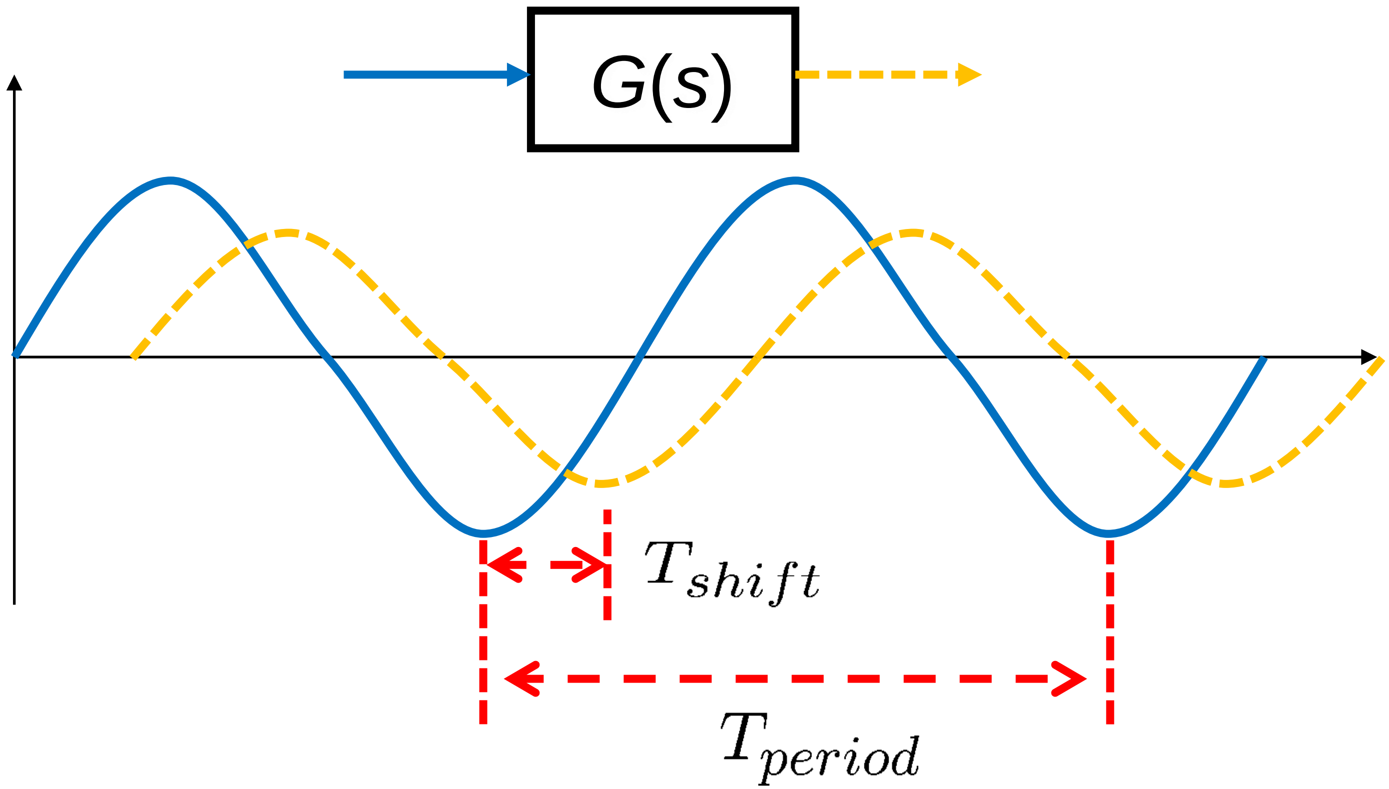
- Steady state output $y_{ss}(t) = A\,|G(j\omega)|\,\sin(\omega t + \angle G(j\omega))$
  - Frequency is same as the input frequency ω
  - Amplitude is that of input (*A*) multiplied by $|G(j\omega)|$
  - Phase shifts $\angle G(j\omega)$

Gain



- Frequency response function (FRF): $G(j\omega)$
- Bode plot: Graphical representation of $G(j\omega)$

Phase shift (review)

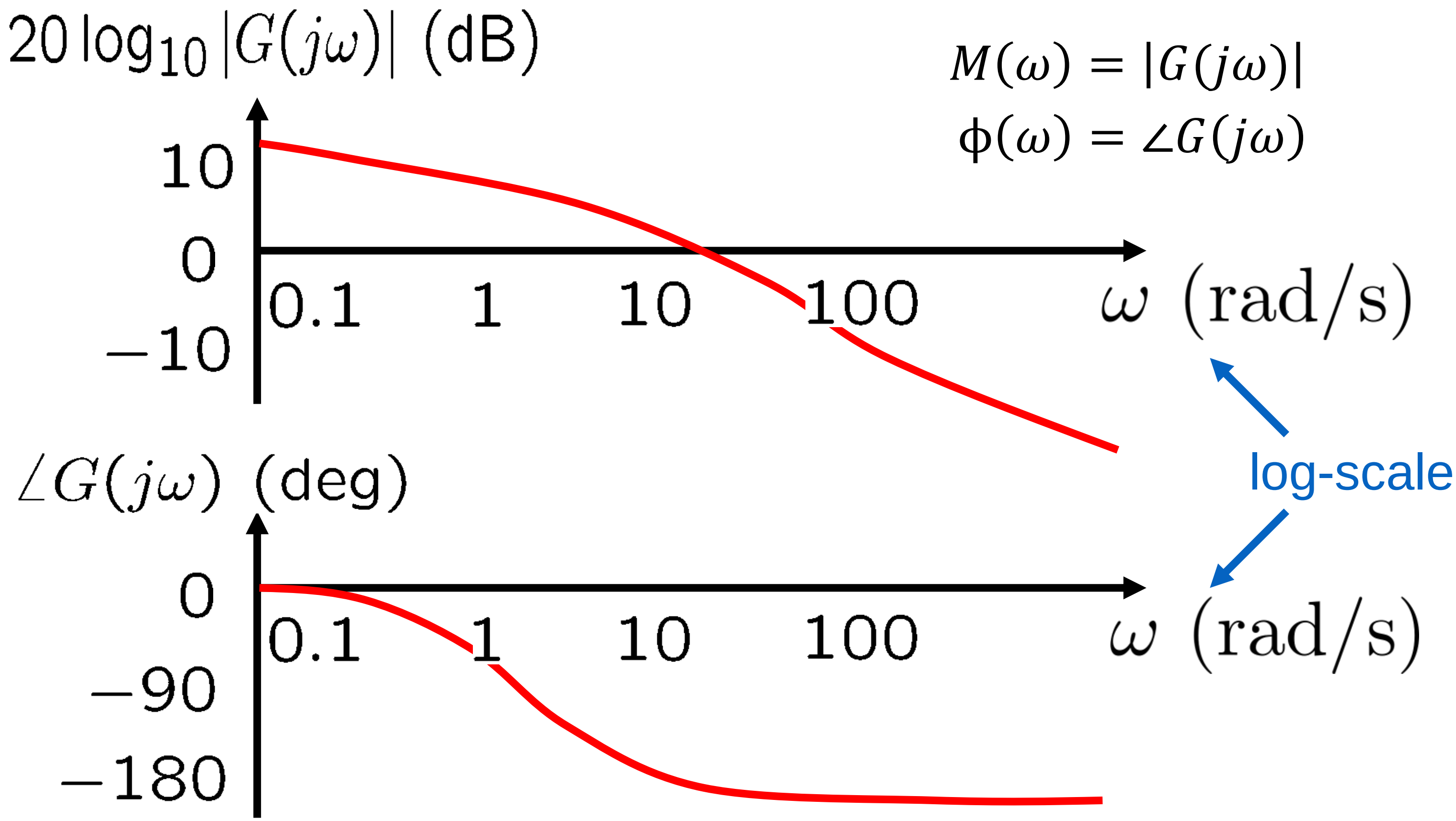


$$\frac{T_{shift}}{T_{period}} = \frac{-\angle G(j\omega)}{360^\circ} \quad \Rightarrow \quad \angle G(j\omega) = -\frac{T_{shift}}{T_{period}} \times 360^\circ$$

Bode plot of $G(j\omega)$ (review)

- Bode diagram consists of gain plot & phase plot

$$M(\omega) = |G(j\omega)|$$
$$\phi(\omega) = \angle G(j\omega)$$



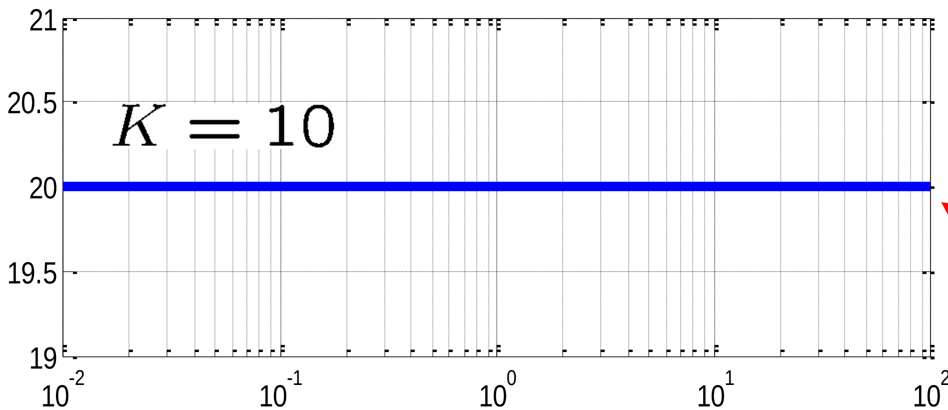
Sketching Bode plot

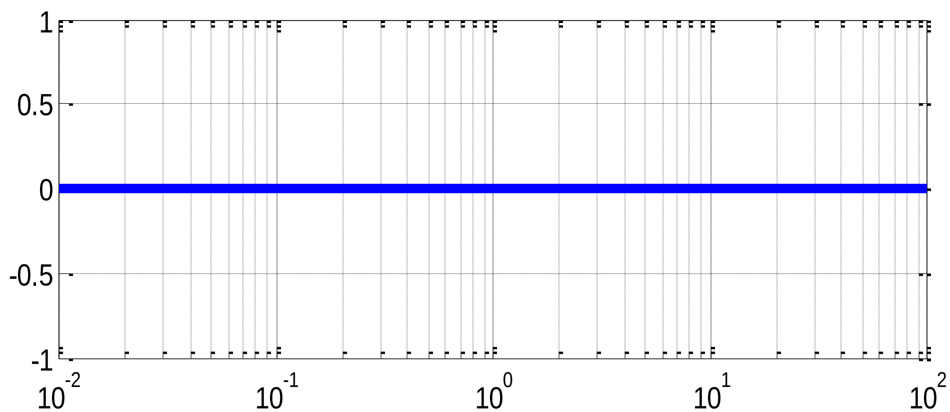
- Basic functions
  - Constant gain
  - Differentiator and integrator
  - Double integrator
  - First order system and its inverse
  - Second order system
  - Time delay
- Product of basic functions
  1. Sketch Bode plot of each factor, and
  2. Add the Bode plots graphically.

Bode plot of a constant gain

$$G(s) = K \Rightarrow |G(j\omega)| = K, \;\; \angle G(j\omega) = 0^\circ, \;\; \forall\omega$$

(for all)





K	$20\log_{10} K$
100	40 dB
$\boxed{10}$	$\boxed{20 \text{ dB}}$
2	≈ 6 dB
1	0 dB
0.1	-20 dB
0.01	-40 dB

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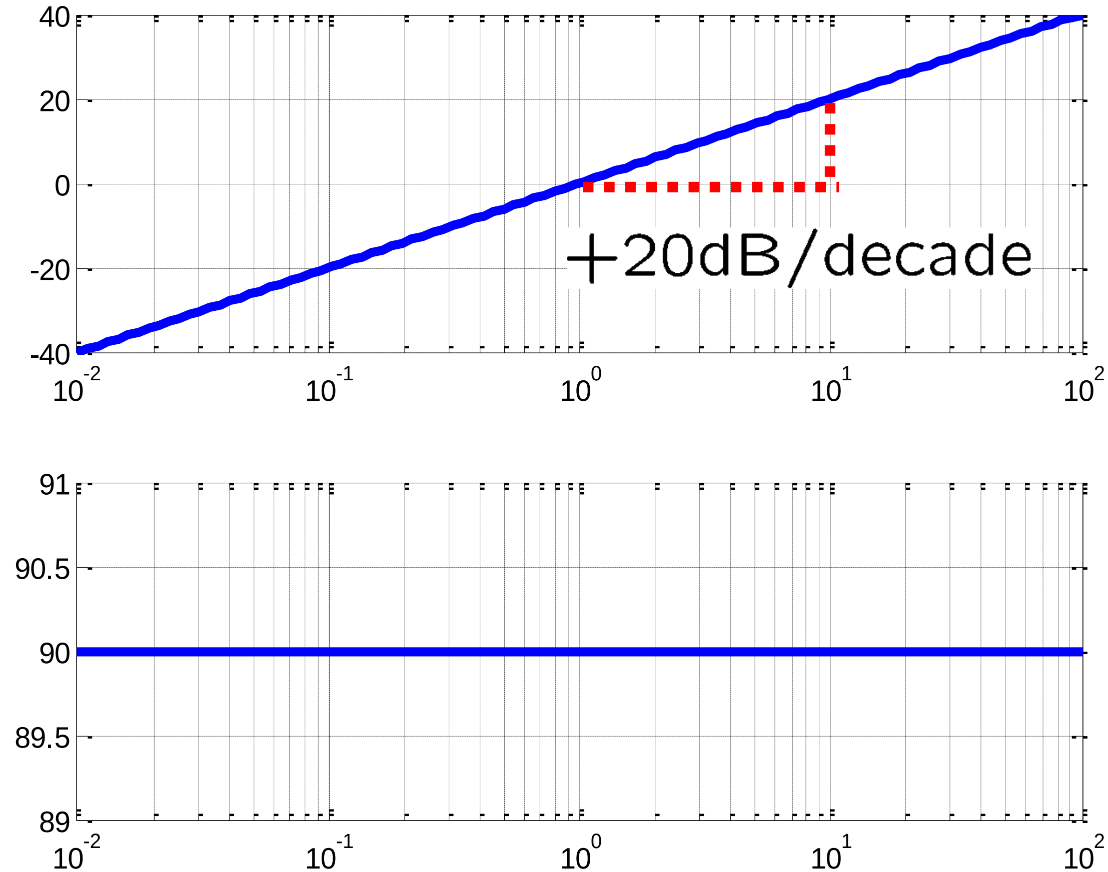
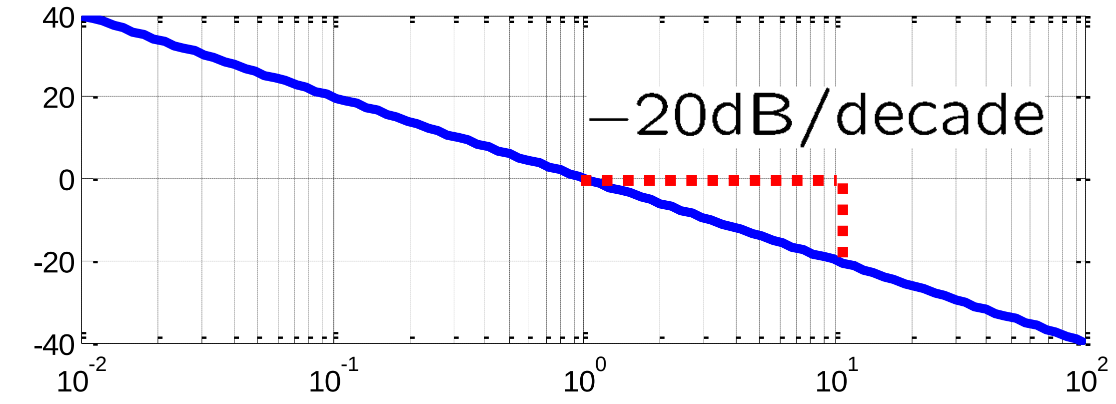
Bode plot of a differentiator

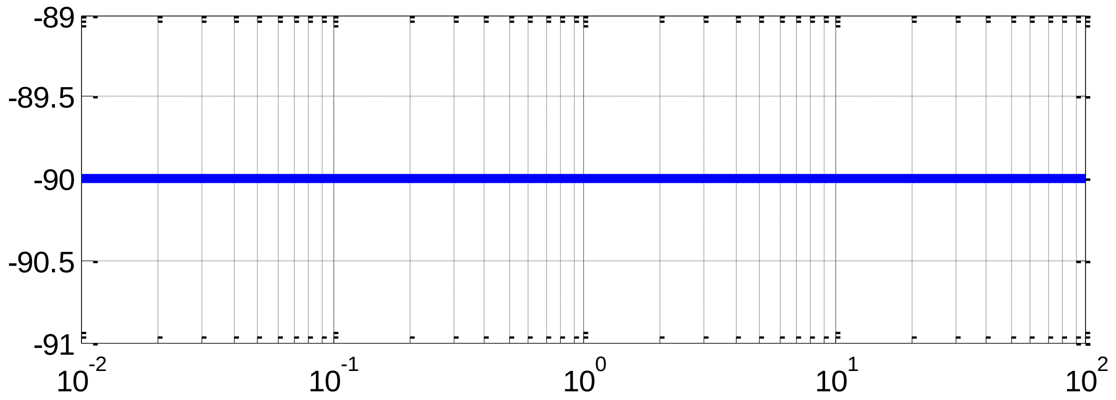
$$G(s) = s \Rightarrow |G(j\omega)| = \omega, \ \angle G(j\omega) = \angle j\omega = 90^\circ, \ \forall\omega$$



Bode plot of an integrator

$$G(s) = \frac{1}{s} \Rightarrow |G(j\omega)| = \frac{1}{\omega},\ \angle G(j\omega) = \angle \frac{1}{j\omega} = -90^\circ,\ \forall\omega$$

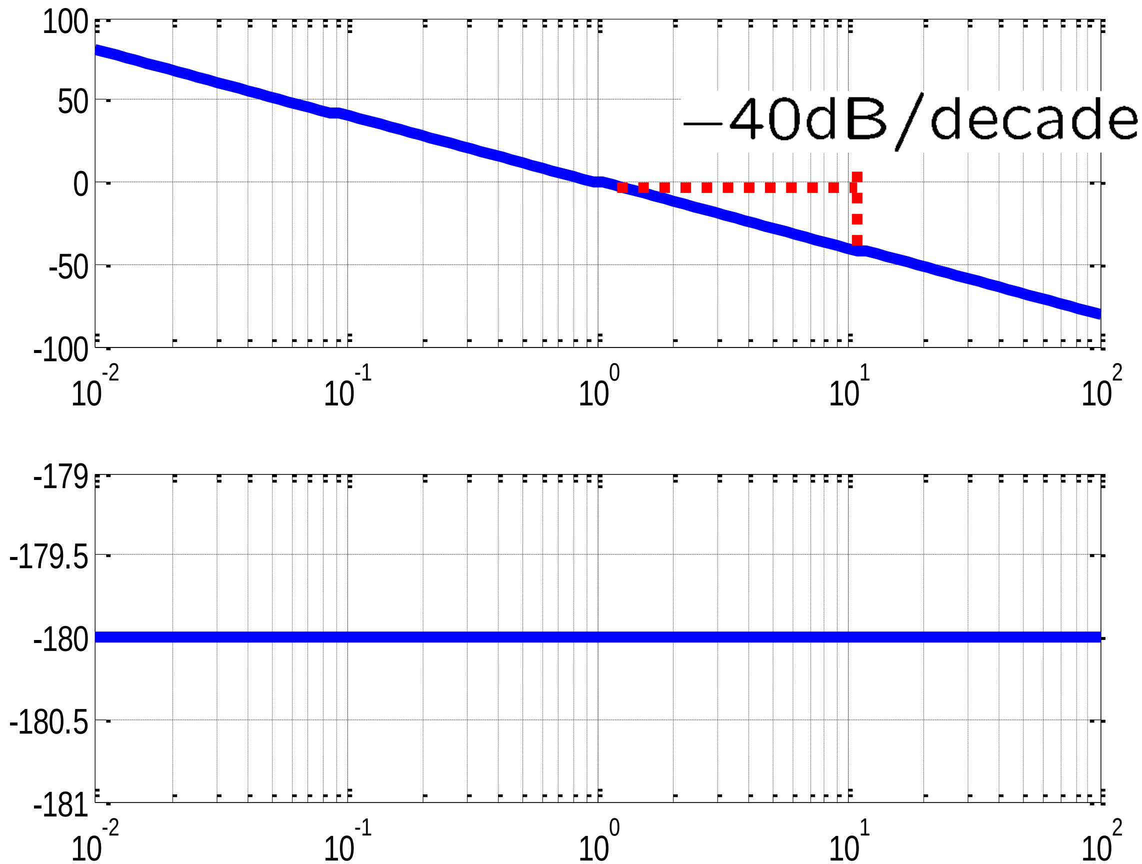




Mirror image of the Bode plot of $G(s) = s$ with respect to ω-axis.

Bode plot of a double integrator

$$G(s) = \frac{1}{s^2} \Rightarrow |G(j\omega)| = \frac{1}{\omega^2},\ \angle G(j\omega) = \angle\frac{1}{(j\omega)^2} = -180^\circ,\ \forall\omega$$



Sketching Bode plot

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Bode plot of a 1st order system

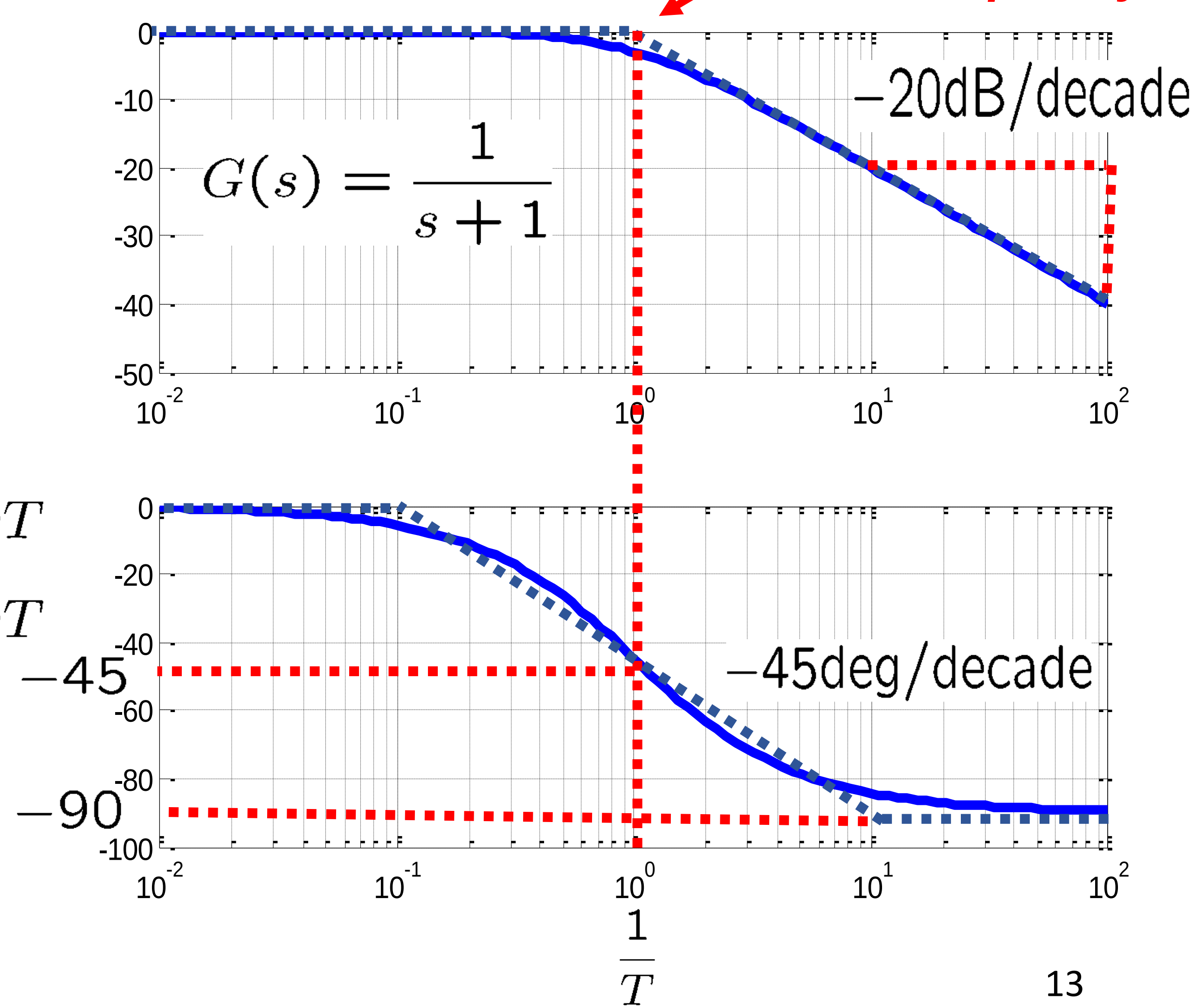
Straight-line approximation ······

Corner frequency

$$G(s) = \frac{1}{Ts+1}$$

$$G(j\omega) = \frac{1}{j\omega T+1}$$

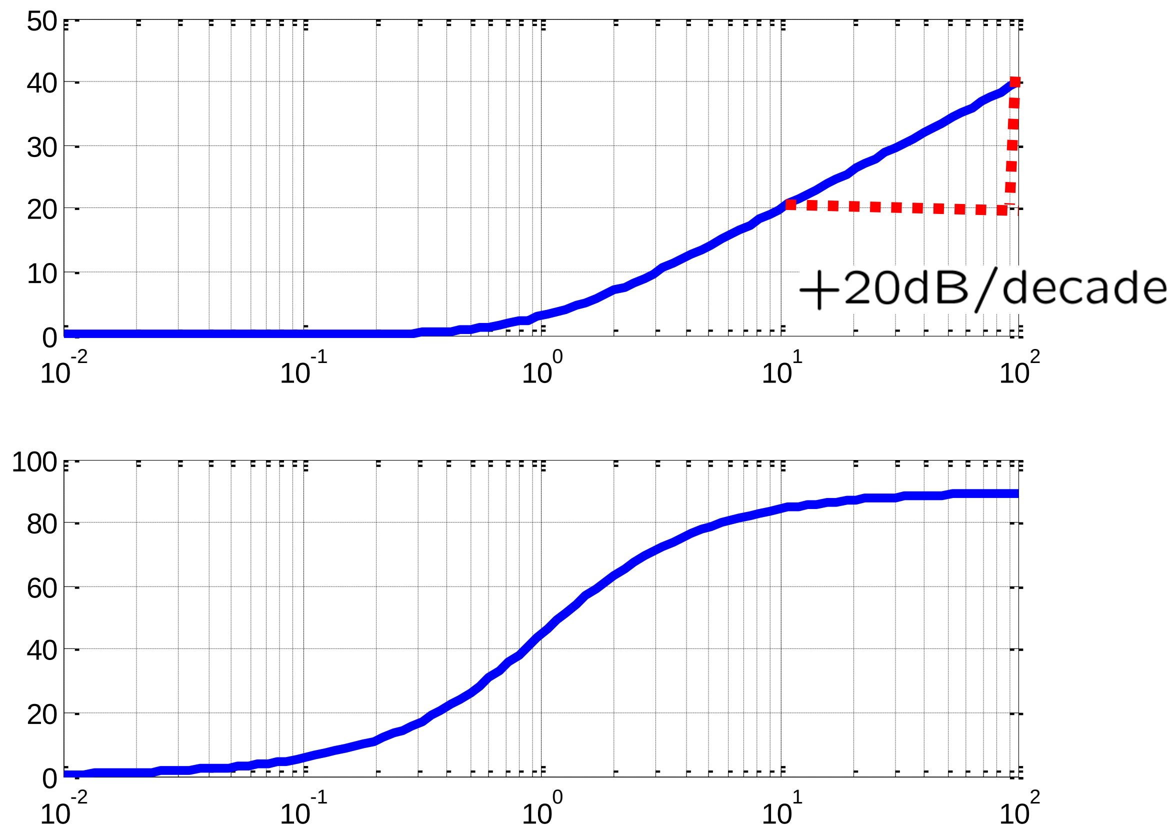
$$\approx \begin{cases} 1 & \text{if } 1 \gg \omega T \\ \frac{1}{j\omega T} & \text{if } 1 \ll \omega T \end{cases}$$



Bode plot of an inverse system

$$G(s) = Ts + 1 = \left(\frac{1}{Ts+1}\right)^{-1}$$

Mirror image of the original Bode plot with respect to ω-axis.



Sketching Bode plot

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Bode plot of a 2nd order system

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Resonant frequency:

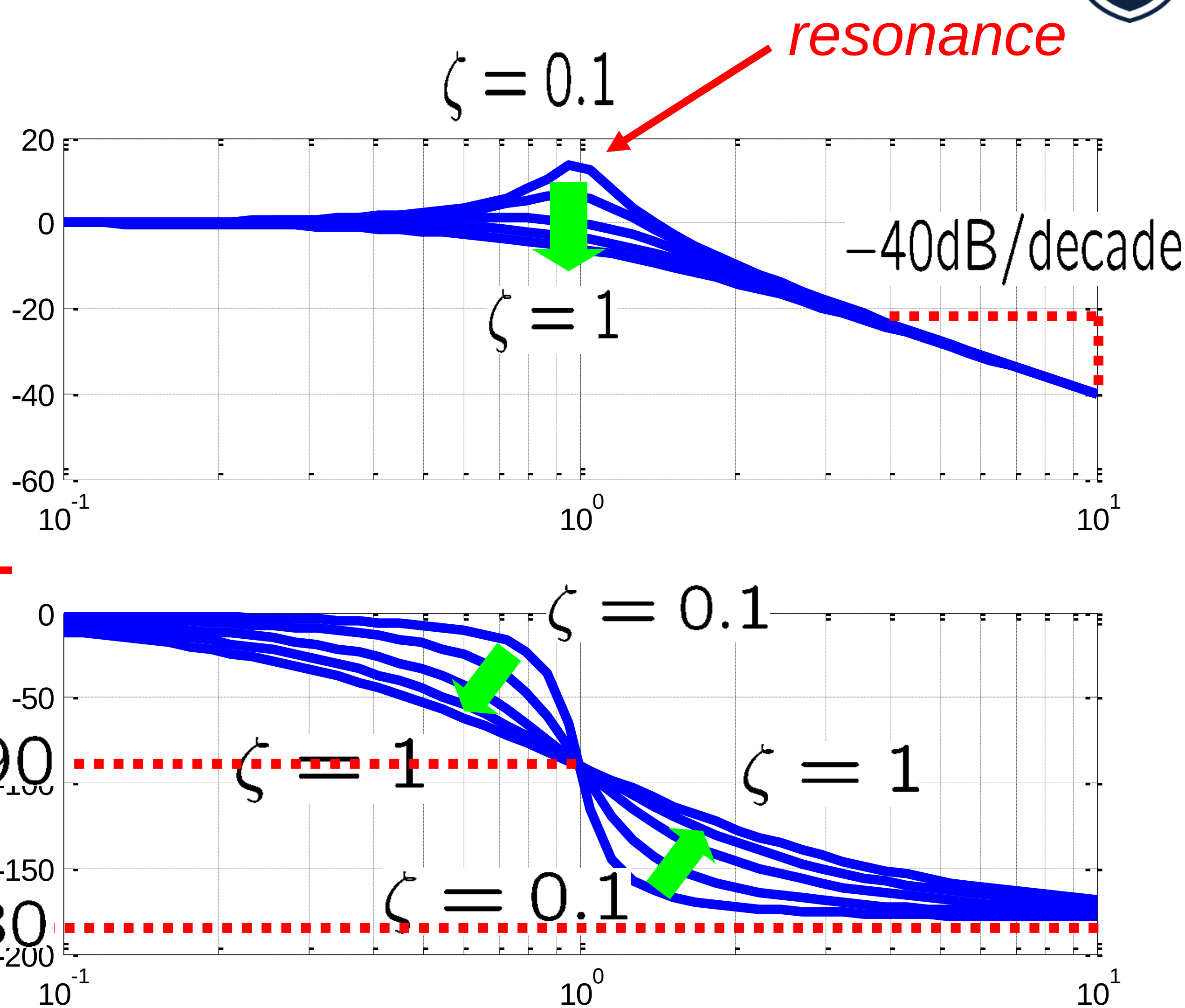
$$\omega_r \;=\omega_n\sqrt{1-2\zeta^2} \approx \omega_n$$

if ζ is small

Peak gain:

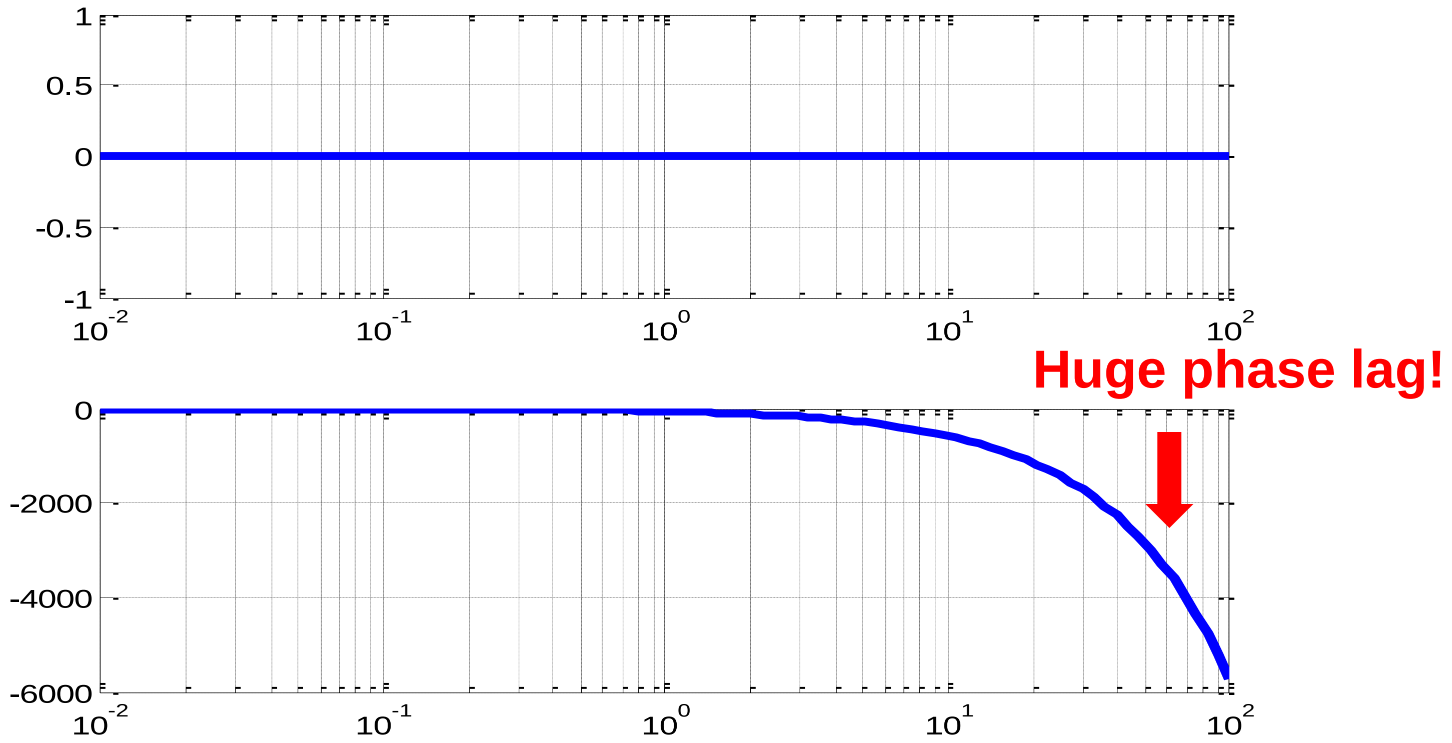
$$\frac{1}{2\zeta\sqrt{1-\zeta^2}} \approx \frac{1}{2\zeta}$$

if ζ is small



Bode plot of a time delay

$$G(s) = e^{-Ts} \Rightarrow |G(j\omega)| = 1, \forall\omega, \ \angle G(j\omega) = -\omega T (\text{rad})$$



The phase lag causes instability of the closed-loop system, and thus, the difficulty in control.

Sketching Bode plot

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Main advantage of Bode plot!

An advantage of Bode plot

- Bode plot of a series connection $G_1(s)G_2(s)$ is the addition of each Bode plot of G_1 and G_2.
  - Gain

$$20\log_{10}|G_1(j\omega)G_2(j\omega)| = 20\log_{10}|G_1(j\omega)| + 20\log_{10}|G_2(j\omega)|$$

  - Phase

$$\angle G_1(j\omega)G_2(j\omega) = \angle G_1(j\omega) + \angle G_2(j\omega)$$

- Later, we use this property to design $C(s)$ so that $G(s)C(s)$ has a "desired" shape of Bode plot.

Example 1 $G(s) = \frac{10}{s}$

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- Sketch the Bode plot of the given transfer function

$$\boxed{G(s) = \frac{10}{s}}$$

Step 1: Decompose *G*(*s*) into a product form:

$$G(s) = 10 \cdot \frac{1}{s}$$

Step 2: Sketch a Bode plot for each component on the same graph.

Step 3: Add them all on both gain and phase plots.

Example 1 (cont'd)

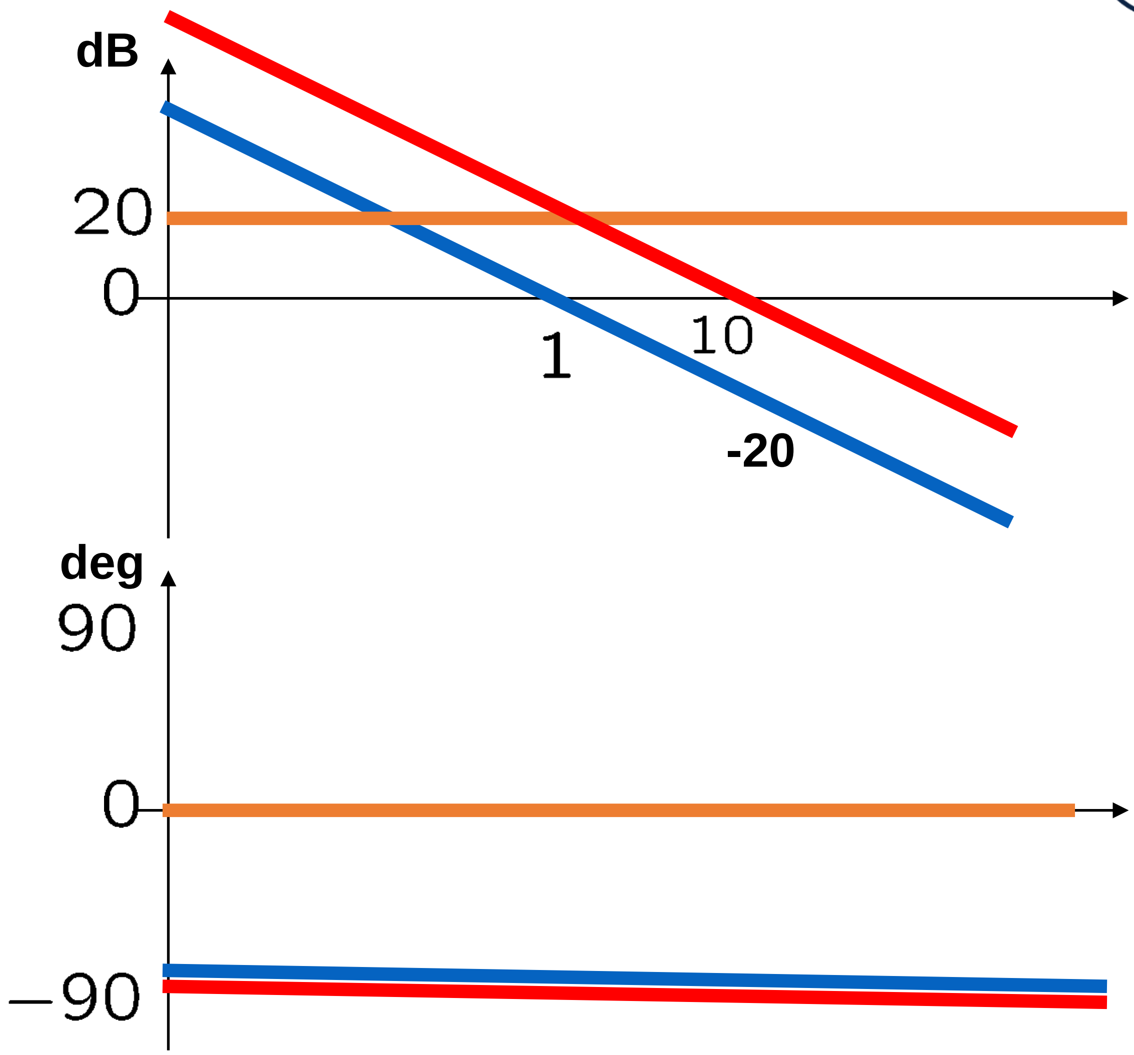
$$G(s) = 10$$

$$\times$$

$$G(s) = \frac{1}{s}$$

$$\Downarrow$$

$$G(s) = \frac{10}{s}$$



Example 2 $G(s) = \frac{0.1}{s}$

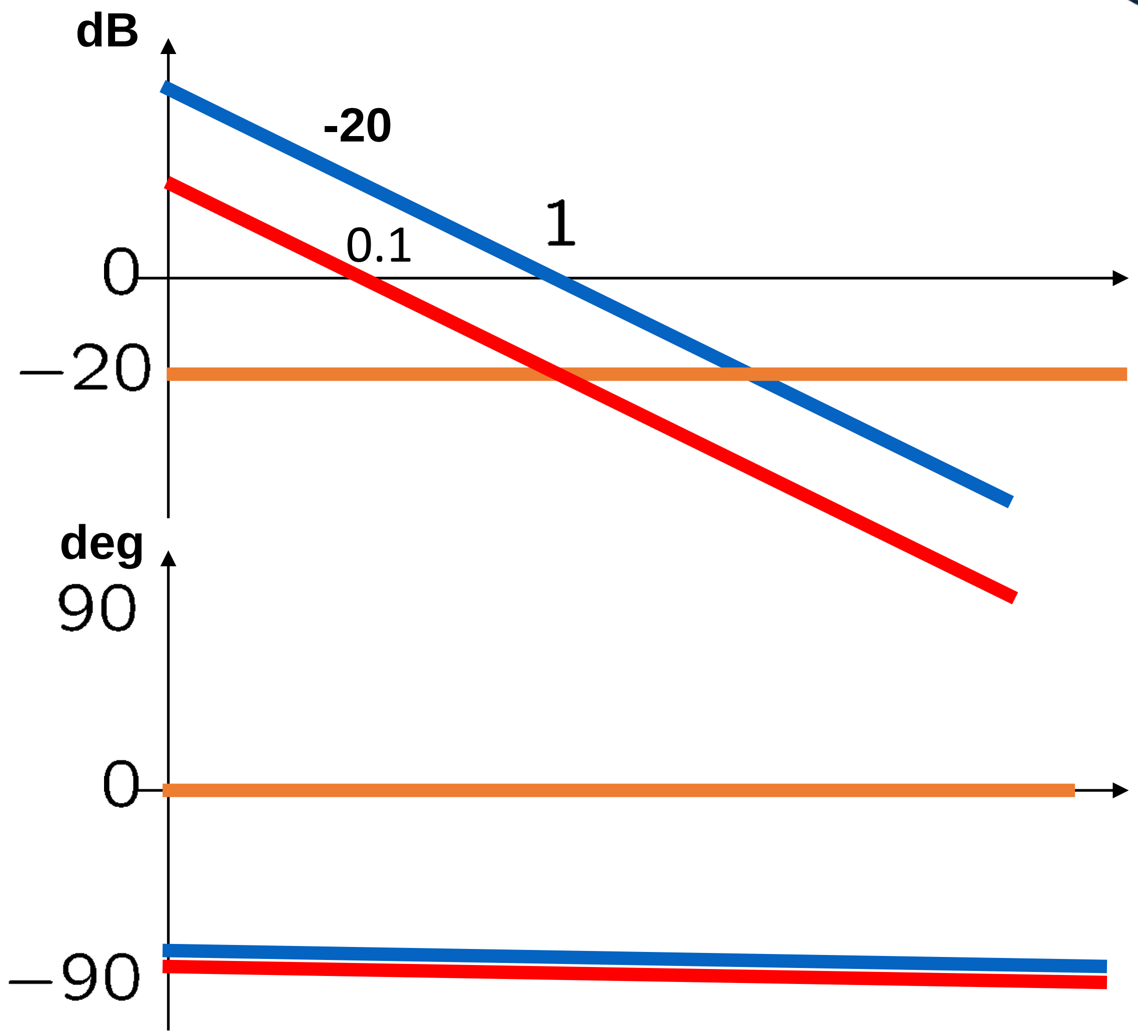
$$G(s) = 0.1$$

$$\times$$

$$G(s) = \frac{1}{s}$$

$$\Downarrow$$

$$G(s) = \frac{0.1}{s}$$



Example 3 $G(s) = \dfrac{1}{s(2s+1)}$

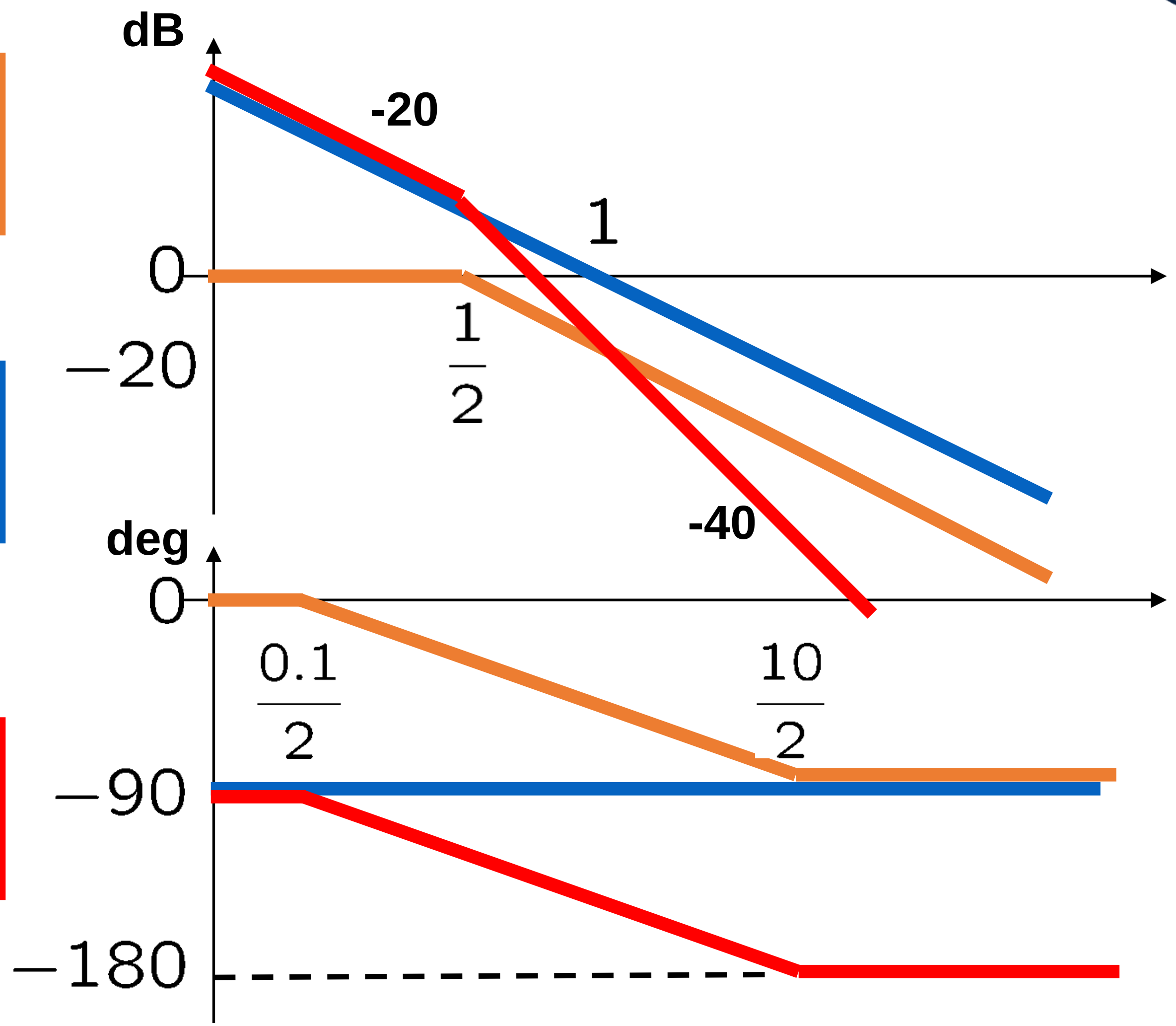
$$G(s) = \frac{1}{2s+1}$$

$$\times$$

$$G(s) = \frac{1}{s}$$

$$\Downarrow$$

$$G(s) = \frac{1}{s(2s+1)}$$



Example 4 $G(s) = \dfrac{10(s+1)}{(s+10)}$

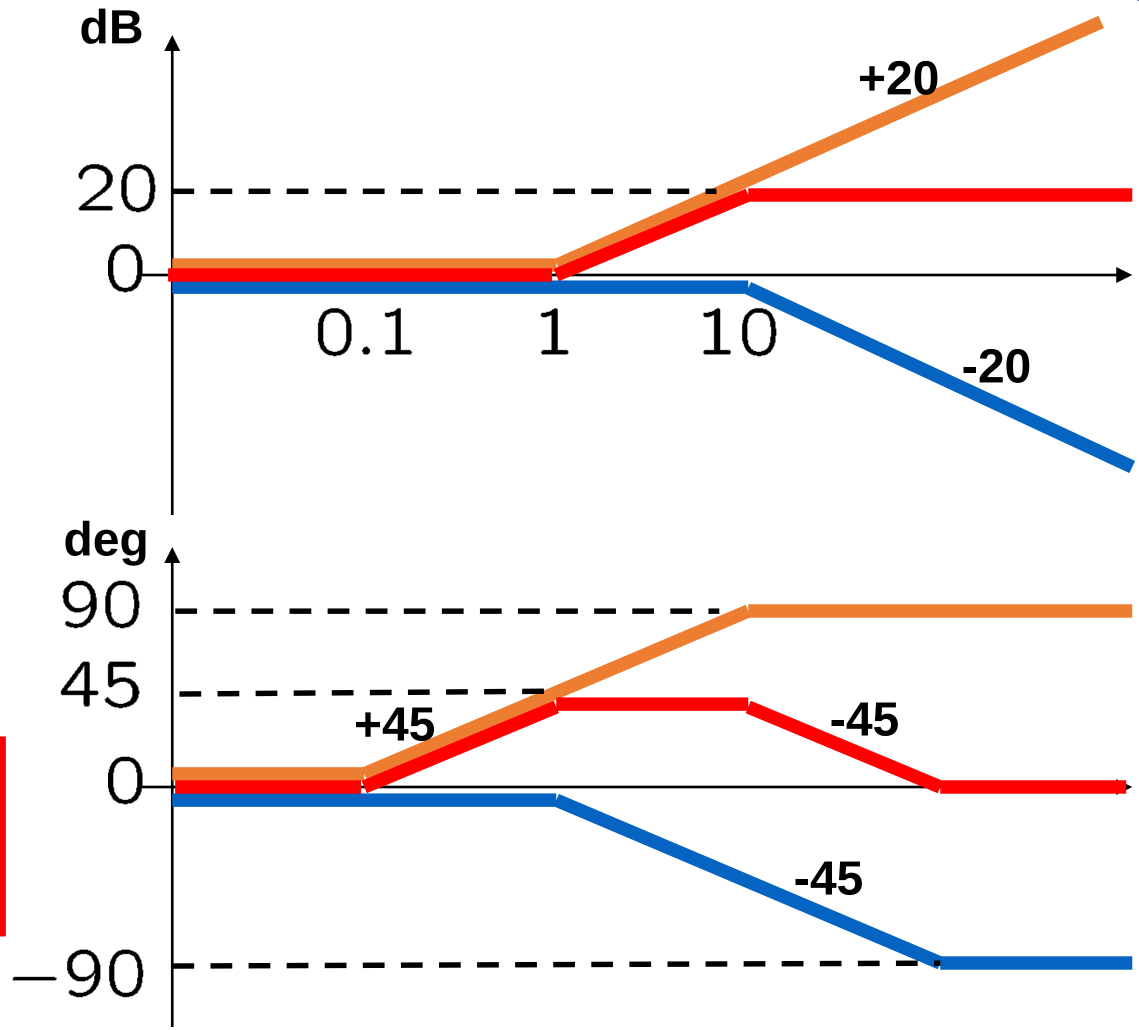
$$\boxed{G(s) = s+1}$$

$$\times$$

$$\boxed{G(s) = \frac{1}{0.1s+1}}$$

$$\Downarrow$$

$$\boxed{G(s) = \frac{10(s+1)}{(s+10)}}$$



Example 5

$$G(s) = \frac{200(s+1)}{(s+10)^2}$$

$$\boxed{G(s) = 2}$$

$$\times$$

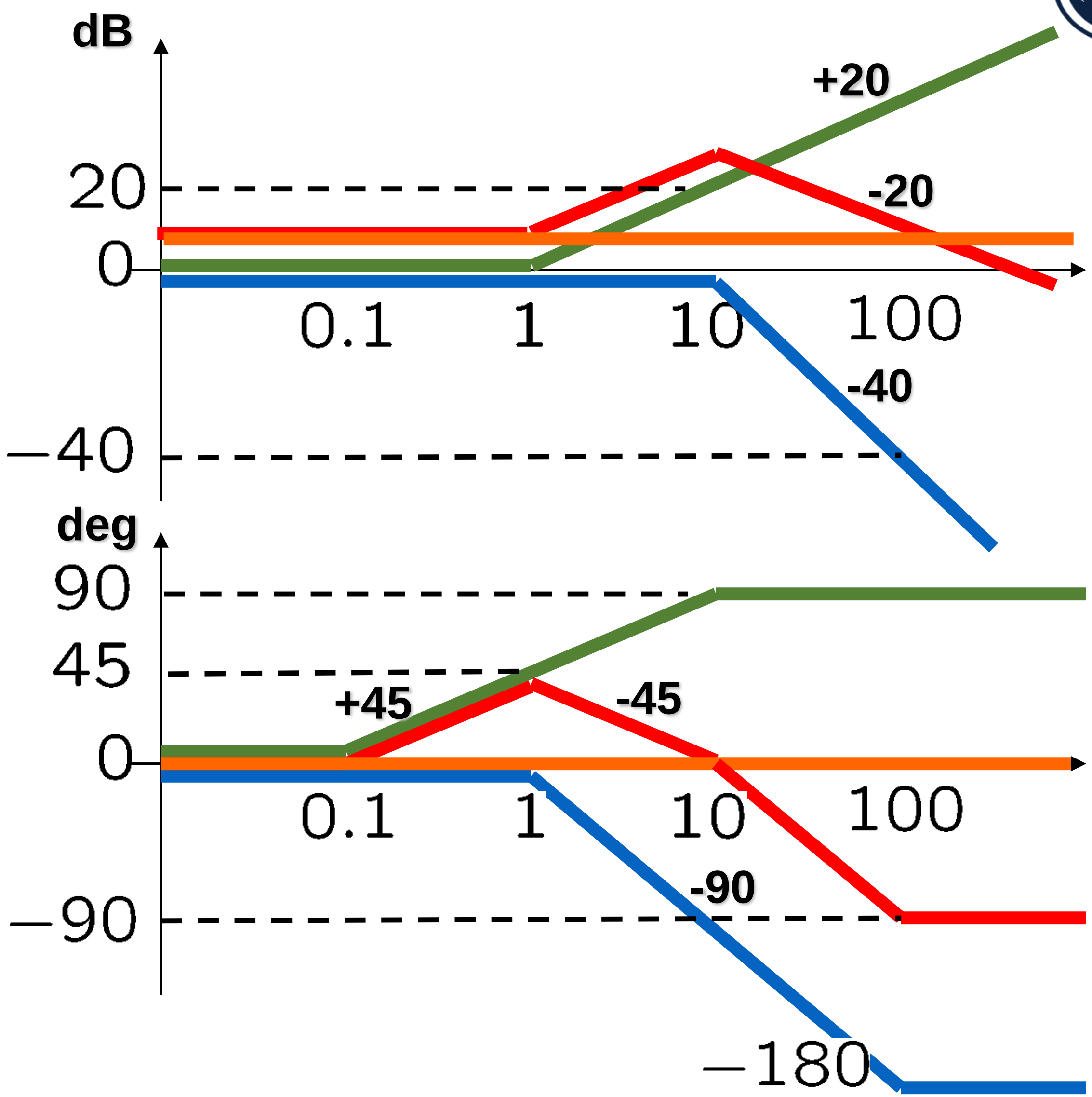
$$\boxed{G(s) = s + 1}$$

$$\times$$

$$\boxed{G(s) = \frac{1}{(0.1s+1)^2}}$$

$$\Downarrow$$

$$\boxed{G(s) = \frac{200(s+1)}{(s+10)^2}}$$



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Remarks

- ALWAYS check the correctness of
  - Low frequency gain (DC gain) $G(0)$
  - High frequency gain $G(\infty)$
- Example

$$G(s) = \frac{10(s+1)}{s+5}$$

$G(\infty) = 10 \approx 20\text{dB}$

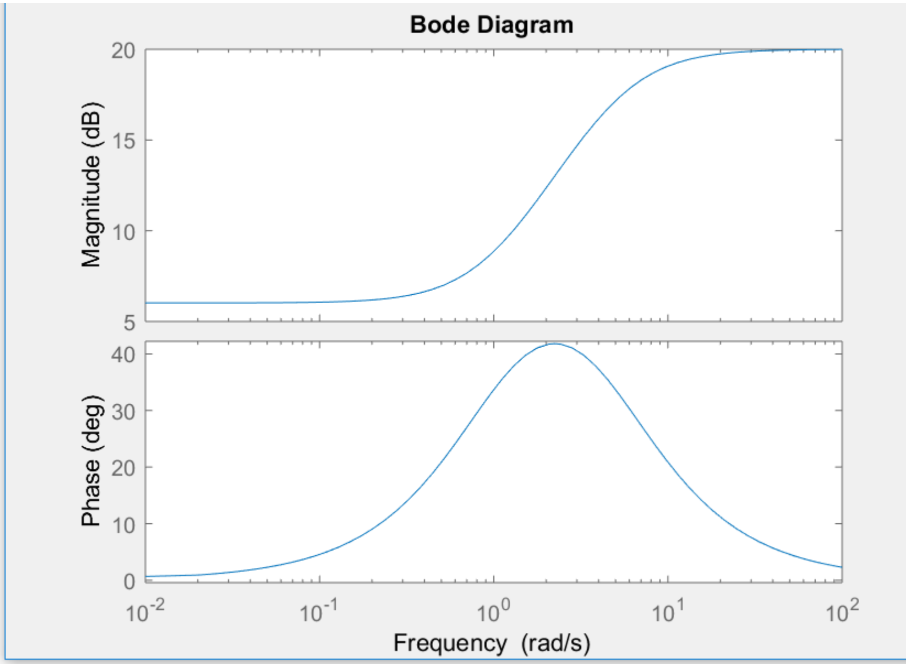
$G(0) = 2 \approx 6\text{dB}$

Remarks (cont'd)

- In Matlab, use "bode" command:

$$G(s) = \frac{10(s+1)}{s+5}$$

```
>> s=tf('s')

s =

  s

Continuous-time transfer function.

>> g=10*(s+1)/(s+5)

g =

  10 s + 10
  ---------
    s + 5

Continuous-time transfer function.

>> bode(g)
>>
```



Summary

- Sketches of Bode plot
  - Basic functions
  - Products of basic functions
- Sketching Bode plot is useful:
  - To get a rough idea of the characteristics of a system.
  - To interpret the result obtained from computer.
- Next
  - *Nyquist stability criterion*