04-03-2022

Friday, March 4, 2022 3:04 PM

Hard margin SVM

$$y_i(wx + b) \geq 1$$

Soft Margin

Slack variable

$\epsilon_i \geq 0$

$$y_i(w^T x + b) \geq 1 - \epsilon_i$$

$$\min_{w,b} \frac{1}{2}||w||^2 + \sum_{i=1}^{N} \epsilon_i$$ such that

$$\max_{\alpha} \sum_{i=1}^{N} \alpha_i - \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} \alpha_i \alpha_j x_i x_j y_i y_j$$

s.t.

$0 \leq \alpha_i \leq C$

$\sum_{i=1}^{N} \alpha_i y_i = 0$

$$\min_{w,b} \frac{1}{2}||w||^2 + C \sum_{i=1}^{N} \epsilon_i$$

Class Lecture

<Graph for Low C & Large C>

Soft margin can be used for 1 or 2 outliers

Now,

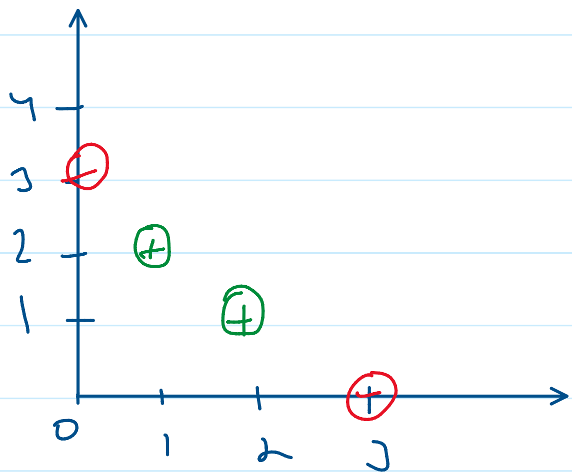
Now,

2D space to Higher dimension

Eg:

$$\begin{pmatrix} 0 \\ 3 \end{pmatrix} \quad \begin{pmatrix} 1 \\ 2 \end{pmatrix} \quad \begin{pmatrix} 2 \\ 1 \end{pmatrix} \quad \begin{pmatrix} 3 \\ 0 \end{pmatrix}$$

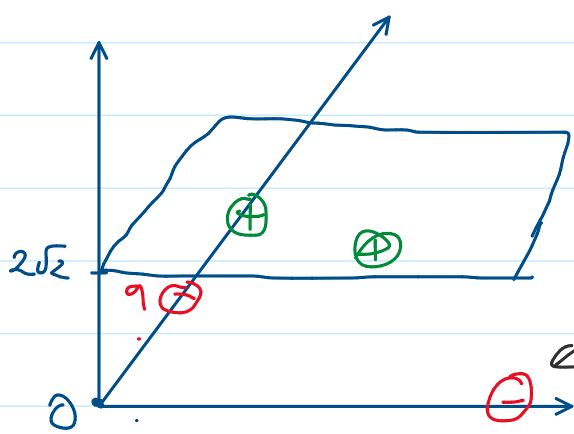
-ve   -ve



$$\phi(x_i) = \begin{pmatrix} x_{i1}^2 \\ x_{i2}^2 \\ \sqrt{2}\, x_{i1} x_{i2} \end{pmatrix}$$

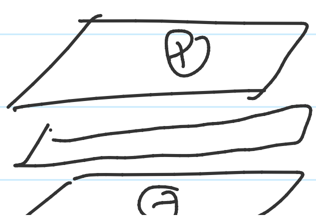
$$\begin{pmatrix} 0 \\ 9 \\ 0 \end{pmatrix} \begin{pmatrix} 1 \\ 4 \\ 2\sqrt{2} \end{pmatrix} \begin{pmatrix} 4 \\ 1 \\ 2\sqrt{2} \end{pmatrix} \begin{pmatrix} 9 \\ 0 \\ 0 \end{pmatrix}$$

Plot in 3d space

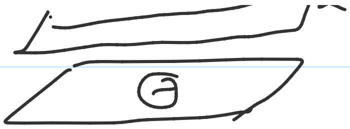


All ⊕ on upper plane

All ⊖ on lower plane



hyperplane



Dual form of obj function

$$\max_{\alpha} \sum_{i=1}^{n} \alpha_i - \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{N} \alpha_i \alpha_j \, x_i \cdot x_j \, y_i \, y_j$$

Kernel function (Kernel Trick)

original vectors $\longrightarrow$ Dot product of transformed vectors in higher dimension

for eg Quadratic Kernel ; $\langle x_i \cdot x_j \rangle^2$