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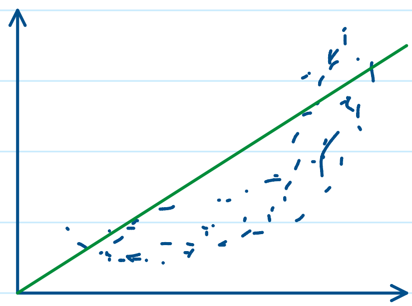
# Polynomial Regression



← Underfitting if we use linear regression

$$y_1 = \beta_0 + \beta_1 x_1 + \beta_2 x_1^2 + \beta_3 x_1^3 \ldots$$
$$y_2 = \beta_0 + \beta_1 x_2 + \beta_2 x_2^2 + \beta_3 x_2^3 \ldots$$
$$\vdots$$
$$y_n = \beta_0 + \beta_1 x_n + \beta_2 x_n^2 \ldots$$

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} = \begin{bmatrix} 1 & x_1 & x_1^2 \\ 1 & x_2 & x_2^2 \\ \vdots & & \\ 1 & x_n & x_n^2 \end{bmatrix} \begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \end{bmatrix}$$

$$\hat{y}_i = \beta_0 + \beta_1 x_i + \beta_2 x_i^2$$

$$E = \sum (y_i - \hat{y}_i)^2$$
$$= \sum \left(y_i - \beta_0 - \beta_1 x_i - \beta_2 x_i^2\right)^2$$

i) $$\frac{\partial E}{\partial \beta_0} = 2\left(\sum_{i=1}^{n} y_i - \beta_0 - \beta_1 x_i - \beta_2 x_i^2\right) \cdot (-1) = 0$$

$$\frac{\partial E}{\partial \beta_0} = 2\left(\sum_{i=1}^{n} y_i - \beta_0 - \beta_1 x_i - \beta_2 x_i^2\right)(-1) = 0$$

$$\Rightarrow \; -\sum_{i=1}^{n} y_i + \beta_0 \sum_{i=1}^{n} 1 + \beta_1 \sum_{i=1}^{n} x_i + \beta_2 \sum_{i=1}^{n} x_i^2 = 0$$

$$\left\{ \Rightarrow \beta_0 n + \beta_1 \sum_{i=1}^{n} x_i + \beta_2 \sum_{i=1}^{n} x_i^2 = \sum_{i=1}^{n} y_i \right\} \text{ — } ①$$

ii) $$\frac{\partial E}{\partial \beta_1} = 2\left(\sum y_i - \beta_0 - \beta_1 x_i - \beta_2 x_i^2\right)(-x_i) = 0$$

$$\Rightarrow \left\{ \beta_0 \sum x_i + \beta_1 \sum x_i^2 + \beta_2 \sum x_i^3 = \sum y_i x_i \right\} \text{ — } ②$$

iii) $$\frac{\partial E}{\partial \beta_2} = 2\left(\sum y_i - \beta_0 - \beta_1 x_i - \beta_2 x_i^2\right)(-x_i^2) = 0$$

$$\Rightarrow \left\{ \beta_0 \sum x_i^2 + \beta_1 \sum x_i^3 + \beta_2 \sum x_i^4 = \sum x_i^2 y_i \right\} \text{ — } ③$$

from ①, ②, ③ $\beta_0, \beta_1, \beta_2 \rightarrow$ unknowns

Matrix Vector form!

$$\begin{bmatrix} n & \sum x_i & \sum x_i^2 \\ \sum x_i & \sum x_i^2 & \sum x_i^3 \\ \sum x_i^2 & \sum x_i^3 & \sum x_i^4 \end{bmatrix} \begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{n} y_i \\ \sum_{i=1}^{n} x_i y_i \\ \sum_{i=1}^{n} x_i^2 y_i \end{bmatrix}$$

$n \rightarrow$ no. of training examples.

$\hookrightarrow$ Use Gaussian Elimination to find $\beta_0, \beta_1, \beta_2$