Exercise 1

D = detected, L = lied (Lying)

1. $P(D|L) = 80/100 = .8$

$P(D|\neg L) = 5/100 = .05$

$P(L) = \frac{100}{200} = .5 \quad \therefore P(\neg L) = .5$

$$P(L|D) = \frac{.8 \cdot .5}{.8 \cdot .5 + .05 \cdot .5} \leftarrow \frac{P(D|L)P(L)}{P(D|L)P(L) + P(D|\neg L)P(\neg L)}$$

$P(D|L)P(L) + P(D|\neg L)P(\neg L) = P(D)$

$= 0.941176$

$P(L|D) \approx 0.94$

2. $P(\neg L|\neg D)$

Negate the initial prob for more information... $P(\neg D|L) = .2$

$P(\neg D|\neg L) = .95$

$$P(\neg L|\neg D) = \frac{P(\neg L)P(\neg D|\neg L)}{P(\neg D)}$$

$$\cancel{= \frac{(.5)(.2)}{(.5)(.2)+(.5)(.95)}}$$

$$= \frac{(.5)(.95)}{(.5)(.95)+(.5)(.2)} = .826087$$

$P(\neg L|\neg D) \approx .82$

15% people arrested lie
Similar prob to previous but change $P(L)$

$\therefore P(L) = .15 \quad P(\neg L) = .85$

detected ↙ use previous ↙

$$P(L|D) = \frac{P(D|L)\,P(L)}{P(D)}$$

$$= \frac{(.8)(.15)}{(.8)(.15)+(.05)(.85)}$$

$$= 0.738462$$

$$\boxed{P(L|D) \approx \boxed{0.74}}$$

$$P(\neg L|\neg D) = \frac{P(\neg D|\neg L)\,P(\neg L)}{P(\neg D)}$$

$$= \frac{(.95)(.85)}{(.95)(.85)+(.2)(.15)} = .964179$$

$$\boxed{P(\neg L|\neg D) \approx .96}$$

If only 3% people lie: $P(L) = .03 \quad P(\neg L) = .97$

$$P(L|D) = \frac{(.8)(.03)}{(.8)(.03)+(.05)(.97)} \approx \boxed{0.33}$$

$$P(\neg L|\neg D) = \frac{(.95)(.97)}{(.95)(.97)+(.2)(.03)} \approx \boxed{.99}$$

Note: Did not update probabilities through each of the successive problems $\because$ New probability of lying was given each time.

Exercise 2

1. In my instinct (opinion) Gamble A is better

Calculation:

$E(A) = 100$ million(s)

$E(B) = (.1)(500 \text{ million}) + (.89)(100 \text{ million}) + 0$

$= 50 \text{ million} + 89 \text{ million}$

$= 139 \text{ million}$

$139 \text{ million} > 100 \text{ million}$

$E(B) > E(A)$

interesting difference

$\therefore$ based of expectation, Gamble B should be preferred

2. Instinct: Gamble D (10% → 500 mill)

$E(C) = .11(100 \text{ million}) = 11 \text{ million}$

$E(D) = .10(500 \text{ million}) = 50 \text{ million}$

$E(D) > E(C) \therefore$ Gamble D "should be preferred

3. I think that this difference poses a problem to what utility is defined to be and that utility from one agent can be different to another. For example, w/ money a richer person might be willing to take more of a risk.

4.a $E(\text{Lottery}) = (\$10 \cdot \frac{1}{50} + \$1{,}000{,}000 \cdot \frac{1}{2{,}000{,}000}) - \$1$

$= \$.2 + \$.5 - \$1$

$= -\$.3$

$= -\$.30$

Scanned by CamScanner

$U(\$K) = 0 \therefore \quad , U(\$1) = 1$

$U(\$10) = 10 \because \quad U(\$10) = 10\,U(\$1) = 10$

$U(\$1{,}000{,}000) = ?$

4.b. $0 \leq \frac{U(\$10)}{50} + \frac{U(\$1{,}000{,}000)}{2{,}000{,}000} - U(\$1)$

$0 \leq \frac{1}{5} - 1 + \frac{U(\$1{,}000{,}000)}{2{,}000{,}000}$

$\frac{U(\$1{,}000{,}000)}{2{,}000{,}000} > \frac{4}{5}$

$U(\$1{,}000{,}000) > 1{,}600{,}000$

$\therefore$ It is rational to buy a ticket when the utility of getting a million dollars is greater than 1,600,000.

c. I think that they have a different utility function because large amounts of money could be a game-changer for them. Watching an adventure movie an imagining the possibility of winning a lottery for a person who doesn't have that sort of money of the winning amount are similar because the amount of money, superpowers, or general utility you'd gain is so much greater than what you currently have. It would make a compounding impact on your life.