Homework 6

Morgan Ciliv

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Note: All work was done on scratch paper and the abbreviations used are the first letters of the corresponding phrase in the problem.

1 Basic Probability

$P(Black) = \frac{4}{20}$ because there are four balls in the sample size of twenty balls.
$P(Red \cup Black) = \frac{7+4}{20} = \frac{11}{20}$
$P(Red \cap Black) = \frac{0}{20}$ because there are no balls that are red and black.
$P(Blue \cap WhiteStripes) = \frac{3}{20}$
$P(Striped|Blue) = \frac{3+6}{9} = \frac{9}{9} = 1$
$P(PinkStriped|Blue) = \frac{6}{9} = \frac{2}{3}$

2 Bayes Rule

MM = Has march madness

Using $P(A|B) = \frac{P(A)P(B|A)}{P(B)}$

These are our initial probabilities that we'll plug into Bayes' Rule:
$P(MM) = \frac{1}{13{,}000}$
$P(\neg MM) = \frac{12{,}999}{13{,}000}$
$P(Y|MM) = .99 \qquad P(\neg Y|MM) = .01$
$P(\neg Y|\neg MM) = .95 \qquad P(Y|\neg M) = .05$
We now plug Bayes' Rule and solve:

Therefore,
$P(MM|Y) \frac{\frac{1}{13{,}000} * .99}{(\frac{1}{13{,}000} * .99) + (\frac{12{,}999}{13{,}000} * .05)}$
=0.0015

3 Expected Utility of a Lottery

Let X be a random variable representing the amount of money you win after playing the lottery.

$E(X) = \frac{1}{6} * 15 + \frac{2}{6} * 9 + \frac{3}{6} * 3$
$= \frac{15}{6} + \frac{18}{6} + \frac{9}{6} = \frac{42}{6}$
$= \$7$

Therefore, since you're supposed to double your money, you pay $3.50

4 Using Utilities to make decisions

AA = Aliens Attack
We convert the problem to the mathematical inequality below:

$-1000 * P(AA) \leq -10 * P(AA) - 100 * (1 - P(AA))$
$100 \leq 1090 * P(AA)$
$P(AA) \geq 10/109$