

13. Note: ⤴ means the pointed material goes
on the circle

$$\begin{array}{c}
 \frac{x \in \text{dom } e}{\langle \text{VAR}(x), \xi, \phi, e \rangle \Downarrow \langle \rho(x), \xi, \phi, e \rangle} \quad (\text{Formal Var}) \\
 \downarrow \\
 \frac{x \in \text{dom } e \quad \langle \text{LIT}(3), \xi, \phi, e \rangle \Downarrow \langle 3, \xi, \phi, e \rangle}{\langle \text{SET}(\text{VAR}(x), \text{LIT}(3)), \xi, \phi, e \rangle \Downarrow \langle 3, \xi, \phi, \rho_2 x \mapsto 3 \rangle} \quad (\text{LIT}) \\
 \downarrow \\
 \frac{\langle \text{SET}(\text{VAR}(x), \text{LIT}(3)), \text{VAR}(x), \xi, \phi, e \rangle \Downarrow \langle 3, \xi, \phi, e, \rho_2 x \mapsto 3 \rangle}{\langle \text{BEGIN}(\text{SET}(\text{VAR}(x), \text{LIT}(3)), \text{VAR}(x), \xi, \phi, e) \rangle \Downarrow \langle \text{VAR}(x), \xi, \phi, \rho \rangle \Downarrow \langle e(x), \xi, \phi, e \rangle} \quad (\text{Formal Var}) \\
 \downarrow \\
 \langle \text{BEGIN}(\text{SET}(\text{VAR}(x), \text{LIT}(3)), \text{VAR}(x), \xi, \phi, e) \rangle \Downarrow \langle 3, \xi, \phi, e, \rho_2 x \mapsto 3 \rangle, \xi
 \end{array}$$

$$\frac{\langle \text{VAR}(x), \xi, \phi, e \rangle \psi \langle x, \xi, \phi, e \rangle \quad x \neq 0 \quad \langle \text{VAR}(x), \xi, \phi, e \rangle \psi \langle x, \xi, \phi, e \rangle}{\langle \text{IF}(\text{VAR}(x), \text{VAR}(x), \text{LIT}(0)), \xi, \phi, e \rangle \psi \langle x, \xi, \phi, e \rangle} \quad (\text{ELITIME}) \quad V_1 = x$$

$$\frac{\langle \text{VAR}(x), \xi, \phi, e \rangle \psi \langle x, \xi, \phi, e \rangle \quad x \neq 0 \quad \langle \text{LIT}(0), \xi, \phi, e \rangle \psi \langle 0, \xi, \phi, e \rangle}{\langle \text{IF}(\text{VAR}(x), \text{VAR}(x), \text{LIT}(0)), \xi, \phi, e \rangle \psi \langle 0, \xi, \phi, e \rangle} \quad (\text{ELITIME}) \quad V_1 = x = 0$$

We know v_e is always $x \therefore V_1 = V_2$

21. (a)

$$\frac{x \notin \text{dom } e \quad x \notin \text{dom } \xi}{\langle \text{VAR}(x), \xi, \phi, \rho \rangle \Vdash \langle \xi, \phi, \rho \rangle \Rightarrow \langle \phi, \phi, \rho \rangle} \quad (\text{Unbound Var})$$

$$\frac{x \notin \text{dom } e \quad x \notin \text{dom } \xi \quad \langle e, \xi, \phi, \rho \rangle \Vdash \langle \nu, \xi, \phi, \rho \rangle}{\langle \text{SET}(\nu, e), \xi, \phi, \rho \rangle \Vdash \langle \nu, \xi, \phi, \rho \rangle} \quad (\text{Unbound Var})$$

21. (b)

$$\frac{x \notin \text{dom } e \quad x \notin \text{dom } \xi}{\langle \text{VAR}(x), \xi, \phi, \rho \rangle \Vdash \langle e(x), \xi, \phi, \rho \rangle \Rightarrow \langle \phi, \phi, \rho \rangle} \quad (\text{Unbound Var})$$

(c) I prefer not because assignment is also included and it covers the chance that I want to use the variable outside of the local scope.

20. Assume we have 2 derivations w/ the same e and same environments on the left, but different v s on the right, v_1 and v_2

Conjecture: if \mathcal{D} is a valid derivation of $\langle e, \xi, \phi, \rho \rangle \Downarrow \langle v_1, \xi, \phi, \rho \rangle$ and $\langle e, \xi, \phi, \rho \rangle \Downarrow \langle v_2, \xi, \phi, \rho \rangle$ then $v_1 = v_2$. (Also induction hypothesis)

1. When the last rule used in \mathcal{D} is literal, the deriv must have form

$$\mathcal{D} = \frac{\langle \text{LITERAL}(v), \xi, \phi, \rho \rangle \Downarrow \langle v, \xi, \phi, \rho \rangle}{\langle e, \xi, \phi, \rho \rangle \Downarrow \langle v, \xi, \phi, \rho \rangle} \quad \text{LITERAL or another form w/ } v \text{ in place of } v_1; \text{ however, } v_1 = v_2 \because \text{the form of } e \text{ is literal and each literal is unique by def.}$$

2. When the last rule used in \mathcal{D} is FORMALVAR the deriv must have the following form

$$\mathcal{D} = \frac{x \in \text{dom } e}{\langle \text{VAR}(x), \xi, \phi, \rho \rangle \Downarrow \langle e(x), \xi, \phi, \rho \rangle} \quad \text{Formal Var or another form w/ } v_2 \text{ in place of } v_1; \text{ however, } v_1 = v_2 \because \text{the form of } e \text{ is variable and each variable is unique by def.}$$

Like 1 and 2 above we repeat the process for the following \mathcal{D} s
Reasoning is similar so don't have to repeat all parts.

$$3. \mathcal{D} = \frac{x \in \text{dom } \rho \quad x \in \text{dom } \xi}{\langle \text{VAR}(x), \xi, \phi, \rho \rangle \Downarrow \langle \xi(x), \xi, \phi, \rho \rangle} \quad \text{Global Var} \quad 4. \mathcal{D} = \frac{x \in \text{dom } \rho \quad \langle e, \xi, \phi, \rho \rangle \Downarrow \langle v, \xi', \phi, \rho' \rangle}{\langle \text{SET}(x, e), \xi, \phi, \rho \rangle \Downarrow \langle v, \xi', \phi, \rho' \rangle \text{ s.t. } x \mapsto v} \quad \text{Formal assign}$$

5. Global Assign (similar to 4) but global

$$6. \mathcal{D} = \frac{\begin{array}{c} \mathcal{D}_1 \\ \langle e_1, \xi, \phi, \rho \rangle \Downarrow \langle v_1, \xi', \phi, \rho' \rangle \end{array} \quad \begin{array}{c} \mathcal{D}_2 \\ \langle e_2, \xi', \phi, \rho' \rangle \Downarrow \langle v_2, \xi'', \phi, \rho'' \rangle \end{array}}{\langle \text{IF}(e_1, e_2, e_3), \xi, \phi, \rho \rangle \Downarrow \langle v_3, \xi'', \phi, \rho'' \rangle} \quad \text{ifTrue}$$

both the antecedent and the consequent must be unique.

7. ifFalse is similar to ifTrue, but false case

Given the different forms we've proved the proved Theoretical Impcore to be deterministic QED