Vector Spaces

Vector spaces are fundamental in linear algebra and have applications in various fields such as physics, engineering, and economics.

Linear Independence

A set of vectors

$$\{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_k\}$$

in a vector space

$$V$$

is said to be *linearly independent* if the only scalars

$$c_1, c_2, \ldots, c_k$$

that satisfy

$$c_1\mathbf{v}_1 + c_2\mathbf{v}_2 + \ldots + c_k\mathbf{v}_k = \mathbf{0}$$

are

$$c_1 = c_2 = \ldots = c_k = 0$$

.

Example of a Linearly Independent Set

Consider the vectors

$$\mathbf{v}_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

,

$$\mathbf{v}_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

, and

$$\mathbf{v}_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

in

$$\mathbb{R}^3$$

. These vectors are linearly independent because there are no non-trivial combinations of these vectors that result in the zero vector.

Matricese

A matrix is a rectangular array of numbers, symbols, or expressions, arranged in rows and columns.

Example of a Matrix

The following is a

$$3 \times 3$$

matrix:

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

Where

$$a_{ij}$$

represents the element in the

$$i$$

-th row and

$$j$$

-th column.