MEASUREMENTS

POSTULATE:

WHEN PHYSICAL QUANTITIES ARE MEASURED

IT'S PROBABILISTIC

$$\hat{H}|\psi\rangle = E|\psi\rangle$$

$$\boxed{|\psi\rangle = \sum_i \alpha_i |\psi_i\rangle}$$ WHERE $\hat{H}|\psi_i\rangle = E_i|\psi_i\rangle$

$$P(E_n) = |\langle\psi_n|\psi\rangle|^2$$ $n =$ ONE OF THE i'S

$$|\psi\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$ $\hat{H}|0\rangle = E_0|0\rangle$

IF SINGLE MEASUREMENT YIELDS E_0 IT WILL WITH P

$$|\langle 0|\psi\rangle|^2 = \left|\frac{1}{\sqrt{2}}\langle 0|0\rangle + \frac{1}{\sqrt{2}}\cancel{\langle 0|1\rangle}\right|^2 = \frac{1}{2}$$

MEASURED VALUE OF THE OBSERVABLE $\hat{H}$

$\langle\hat{H}\rangle \simeq \langle E\rangle_{AVERAGE}$ = MEAN VALUE OF ENERGY

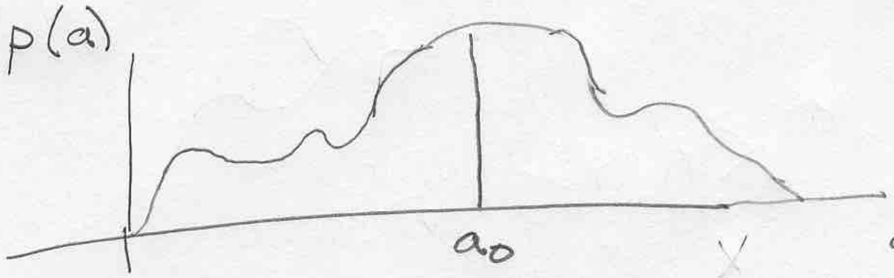
$$\langle\hat{H}\rangle_\psi = \langle\psi|\hat{H}|\psi\rangle$$

$\triangleq$ AVERAGE OF RESULTS OVER LARGE # OF MEASUREMENTS

$\downarrow$

$N \to \infty$

SAY OBSERVABLE $\hat{A}$ (CONTINUOUS)

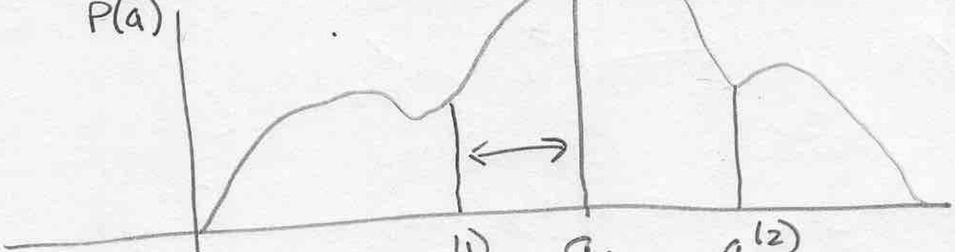


MEASUREMENT OF $\hat{A}$ YIELDS SOME VALUE a

$$\text{MANY MEASUREMENTS} = \frac{\sum \text{MEASURED } a\text{'s}}{N}$$

$$= \text{MEAN} = a_0$$

$$\boxed{\langle A \rangle = \langle \psi | A | \psi \rangle = a_0}$$



INDIVIDUAL MEASUREMENT $a^{(1)}, a^{(2)}, \ldots$

ERROR FOR $a^{(1)} = a_0 - a^{(1)} > 0$

$a^{(2)} = a_0 - a^{(2)} < 0$

BUT ERRORS SHOULDN'T CANCEL

$(\Delta a)^2 = (a - a_0)^2$ OVER LOTS OF MEASUREMENTS

$$= \langle (\hat{A} - \langle \hat{A} \rangle)^2 \rangle$$

$$= \langle \hat{A}^2 - 2\hat{A}\langle A \rangle + \langle \hat{A} \rangle^2 \rangle \qquad \langle A \rangle = a_0$$

$$= \langle \hat{A}^2 \rangle - \langle A \rangle^2$$

$$(\Delta A)^2 = \langle A^2 \rangle - \langle A \rangle^2 = \langle A^2 \rangle - a_0^2$$

$$\sigma_A = \sqrt{(\Delta A)^2} = \sqrt{\left(\langle A^2 \rangle - \langle A \rangle^2\right)}$$

$$\boxed{(\Delta x)^2 = \langle \hat{x}^2 \rangle - x_0^2} = \boxed{\langle \hat{x} - \langle x \rangle \rangle^2}$$

x_0

$$\sigma_x = \sqrt{\left(\langle \hat{x}^2 \rangle - \langle x \rangle^2\right)}$$

SO NOW ON TO THE

PROOFS