MISCELANEOUS NOTE

F.T. "DERIVATION"

CONSIDER A FUNCTION $f_{T_0}(t)$ PERIODIC WITH PERIOD T_0

IN THE INTERVAL $\frac{-T_0}{2} \le t \le \frac{T_0}{2}$ LET

$$f_{T_0}(t) = f(t)$$

$$f_{T_0}(t) = \sum_{n=-\infty}^{\infty} c_n e^{i\omega_n t} \qquad \omega_n = n\,\text{"}\omega_0\text{"} = n\left(\frac{2\pi}{T_0}\right)$$

CALCULATE c_n BY

$$c_n = \frac{1}{T_0}\int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} f(t)\, e^{-i\omega_n t}\, dt = \text{A NUMBER} = \frac{1}{T_0}\int_{\tau=-\frac{T_0}{2}}^{\frac{T_0}{2}} f(\tau)\, e^{-i\omega_n \tau}\, d\tau$$

CONSIDER RE WRITING TO

$$\omega_{n+1} = \frac{2\pi}{T_0}(n+1) \qquad \omega_n = 2\pi\frac{n}{T_0} \qquad \omega_{n+1} - \omega_n = \frac{2\pi}{T_0}(n+1-n) = \frac{2\pi}{T_0}$$

$$\frac{1}{T_0} = \frac{\omega_{n+1}-\omega_n}{2\pi} \qquad T_0 \to \infty \qquad \omega_{n+1}-\omega_n \to \Delta\omega \sim d\omega \qquad \sum_n e^{i\omega_n} \to \int e^{i\omega}\, d\omega$$

$$\underset{\substack{\downarrow \\ f(t)}}{f_{T_0}(t)} = \underset{\substack{\downarrow \\ \int}}{\sum_{n=-\infty}^{\infty}} \underset{\substack{\downarrow \\ \frac{d\omega}{2\pi}}}{\frac{1}{T_0}} \underbrace{\int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} f(\tau)\, e^{-i\underset{\substack{\downarrow \\ \omega}}{\omega_n}\tau}\, d\tau}_{\tilde{f}(\omega)}\ \underset{\substack{\downarrow \\ e^{i\omega t}}}{e^{i\omega_n t}}$$

BASIS PROJECTION

$|\psi\rangle$

DECOMPOSE INTO A SET OF EIGENVECTORS (WEIGHTED ONES)
IN A DISCRETE BASIS

$$\langle \phi_k | \phi_l \rangle = \delta_{kl} = 1 \quad k = l$$
$$= 0 \quad k \neq l$$

$$|\psi\rangle = \sum \alpha_i |\phi_i\rangle$$

← EIGEN STATES

WEIGHT IN THAT "DIRECTION"

$$|\psi\rangle = \sum \langle \phi_i | \psi \rangle \, |\phi_i\rangle = \sum_i \underbrace{|\phi_i\rangle\langle\phi_i|}_{\text{PROJECTION}} \psi\rangle$$

$\sum \rightarrow \int$ FOR X

$$|\psi\rangle = \int \Big(|x\rangle \langle x| \Big) |\psi\rangle \, dx$$

$$|\psi\rangle = \int |x\rangle \, \psi(x) \, dx$$

$$\langle \psi | \psi \rangle = \int \langle \psi | x \rangle \, \psi(x) \, dx$$

$$= \int_{x=-\infty}^{\infty} \psi^*(x) \, \psi(x) \, dx$$

$$\mathcal{F}\left(\phi'(x)\right) = \frac{i}{\hbar}\int p\,\phi_p\, e^{-\frac{ip}{\hbar}}\, dx$$ F.T 2

$$\left|\mathcal{F}\left(\phi'(x)\right)\right|^2 = \frac{1}{\hbar^2}\int p^2 |\phi(p)|^2 d$$

PROPERTY OF FOURIER XFORM

$$\mathcal{F}\left(\phi(x)\right) = \phi_p(p) = \frac{1}{\sqrt{2\pi\hbar}}\int \phi(x)\, e^{-i\left(\frac{p}{\hbar}\right)x}\, dx$$

$$\mathcal{F}\left(\phi'(x)\right) = \frac{1}{\sqrt{2\pi\hbar}}\int \underbrace{\phi'(x)\, e^{-\frac{ip}{\hbar}x}\, dx}$$

INTEGRATION BY PARTS, ISN'T IT ALWAYS

INTEGRATION BY PARTS

$$u = e^{-ip/\hbar x} \qquad du = \frac{-ip}{\hbar} e^{-ip/\hbar x}$$

$$v = \phi(x) \qquad dv = \phi'$$

$$d(uv) = u\,dv + v\,du$$

$$d\left(\phi e^{-ip/\hbar x}\right) = \underbrace{e^{-ip/\hbar x}\,\phi'} + \phi(x)\frac{-ip}{\hbar}e^{-1}$$

$$\mathcal{F}\left(\phi'(x)\right) = \frac{1}{\sqrt{2\pi\hbar}}\left[\int_{-\infty}^{\infty} d\left(\phi e^{-ip/\hbar x}\right)dx + \frac{ip}{\hbar}\int \frac{ip}{\hbar}\,\underbrace{\phi(p)\, e^{-\frac{ip}{\hbar}}}\, dx\right.$$

$$= \frac{i}{\hbar}\int p\,\phi(p)\, e^{ip/h}\, dx$$