CAN SAY MORE ABOUT "CANNONICALLY CONJUGATE"

VARIABLES MIGHT ALSO BE VIEWED AS

FOURIER TRANSFORM "DUALS"

REVIEW

FOURIER SERIES

EXPRESS A PERIODIC FUNCTION AS

$$f(t) = \sum_{n=-\infty}^{\infty} c_n e^{i\omega_n t} \qquad \omega_n = n\frac{2\pi}{T_0}$$

$$c_n = \frac{1}{T_0}\int_{t_0}^{t_0+T_0} dt\, e^{-i\omega_n t} f(t)$$

$$\sum \longrightarrow \int \qquad \Longrightarrow \text{DERIVATION}$$

SERIES $\longrightarrow$ TRANSFORM

$$f(t) = \frac{1}{A}\int_{-\infty}^{\infty} \tilde{f}(\omega)\, e^{i\omega t}\, d\omega = \mathscr{F}^{-1}(\tilde{f}(w))$$

$$\tilde{f}(\omega) = \frac{1}{B}\int_{-\infty}^{\infty} f(t)\, e^{-i\omega t}\, dt = \mathscr{F}(f(t))$$

$\tilde{f}(\omega)$ = FOURIER TRANSFORM OF $f(t)$

A + B ARE "ANNOYING"

$$AB = 2\pi$$

IN AN ENGINEERING APPROACH TO UNCERTAINTY, LET'S GO WITH

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{F}(\omega)\, e^{j\omega t}\, d\omega$$

FULL ON E.E.

$$\tilde{F}(\omega) = \int_{-\infty}^{\infty} f(t)\, e^{-j\omega t}\, dt$$

$$j = i$$

WHY THE MADNESS

TIME SIGNALS SOMEWHAT EASIER TO UNDERSTAND ???

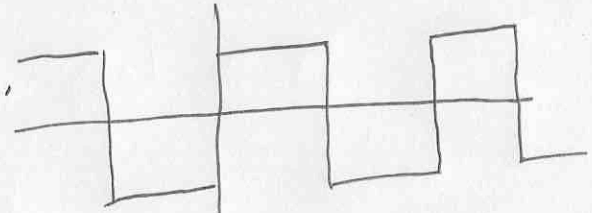
AND BY THE WAY t, ω FOR AN E.E. WOULD BE A FOURIER TRANFORM DUAL

ATTEMPT TO GET A "FEEL" FOR UNCERTAINTY

BACK TO FOURIER SERIES FOR A BIT

$$f(t) = \begin{cases} +1 & 0 \le t \le \frac{T_0}{2} \\ -1 & \frac{T_0}{2} \le t \le T_0 \end{cases}$$

ETC.



ODD FUNCTION

$$f(t) = \sum_n a_n \cos\omega_n t + b_n \sin\omega_n t$$

$$b_n = \frac{2}{T_0}\left[\int_0^{T_0/2} (1)\sin\left(\frac{2\pi n}{T_0}t\right)dt + \int_{T_0/2}^{T_0} (-1)\sin\left(\frac{2\pi n t}{T_0}\right)dt\right]$$

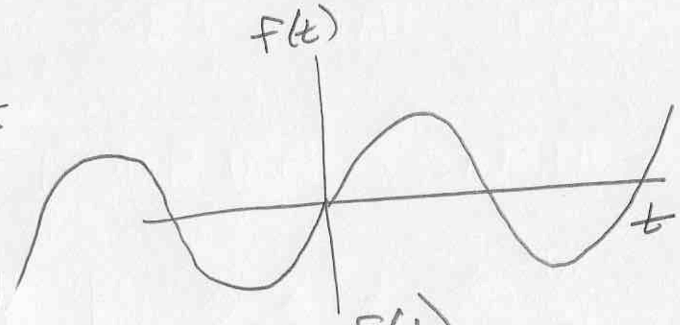
$$d(\sin\omega t) = \omega\cos\omega t \qquad d(\cos\omega t) = -\omega\sin\omega t$$

$$b_n = \frac{-2}{T_0}\left(\frac{1}{\frac{2\pi n}{T_0}}\right)\left[\cos\frac{2\pi n t}{T_0}\Big]_0^{T_0/2} - \cos\frac{2\pi n t}{T_0}\Big]_{T_0/2}^{T_0}\right]$$

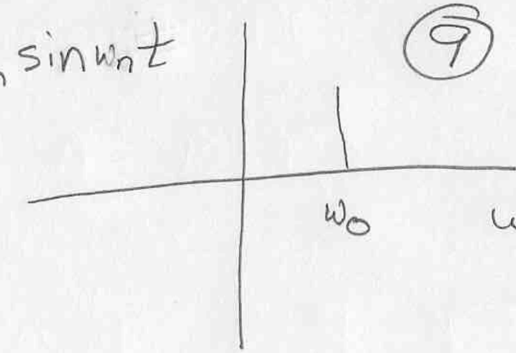
$$b_n = -\frac{1}{n\pi}\left[\cos n\pi - 1 + 1 + \cos n\pi\right]$$

$$b_n = \frac{2}{n\pi}\left(1 - \cos n\pi\right) \Rightarrow \begin{matrix} 0 & N \text{ EVEN} \\ \frac{4}{n\pi} & N \text{ ODD} \end{matrix}$$

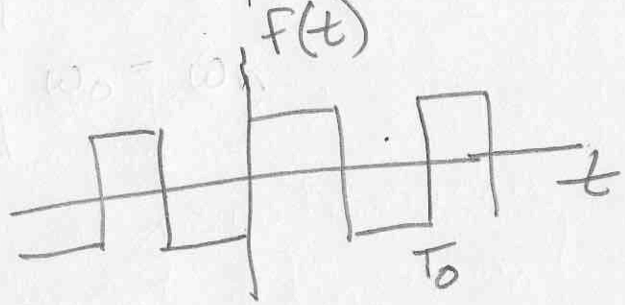
$f(t) = \sin \omega_0 t$

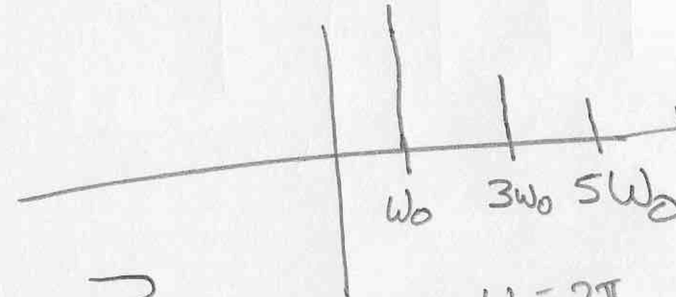


$\sum b_n \sin \omega_n t$



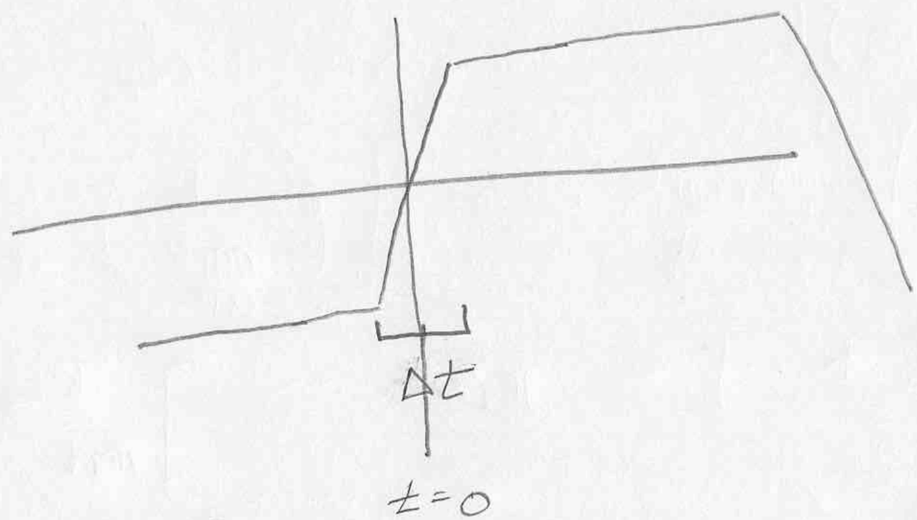
$f(t) =$ SQUARE WAVE





$\omega_0 = \frac{2\pi}{T_0}$

WHERE IS $f(t)$ ILL DEFINED?



AS $\Delta t \rightarrow 0$

FUNCTION'S "SPREAD IN TIME" NARROWS

$\Delta \omega$ "REQUIRED" EXPANDS

PHYSICISTS WE ARE HOWEVER

$$\psi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \psi_K(K)\, e^{iKx}\, dK$$

SYMMETRIC

$$\psi_K(K) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \psi(x)\, e^{-iKx}\, dx$$

$\psi(x) \longrightarrow$ "INVERSE SPACE" K DIMENSIONALLY $\frac{1}{\text{DISTANCE}}$

$K \propto \frac{2\pi}{\lambda} \leftarrow$ "CHARACTERISTIC" WAVELENGTH

STANDARD SUBSTITUTION

$$\psi_P(P) = \frac{1}{\sqrt{\hbar}}\, \psi_K(K) \qquad P = \hbar K$$

$$\psi_P(P) = \frac{1}{\sqrt{\hbar}} \frac{1}{\sqrt{2\pi}} \int \psi(x)\, e^{-i(P/\hbar)t}\, dx$$

$$= \frac{1}{\sqrt{2\pi\hbar}} \int \psi(x)\, e^{-i(P/\hbar)x}\, dx$$

$$\psi(x) = \frac{1}{\sqrt{2\pi}} \int \psi_K(K)\, e^{iKx}\, dK$$

$$= \frac{1}{\sqrt{2\pi}} \int \sqrt{\hbar}\, \psi_p(p)\, e^{ip/\hbar x}\, \frac{dp}{\hbar}$$

$$\boxed{\begin{aligned} \psi(x) &= \frac{1}{\sqrt{2\pi\hbar}} \int_{-\infty}^{\infty} \psi_p(p)\, e^{ip/\hbar x}\, dp \\ \psi_p(p) &= \frac{1}{\sqrt{2\pi\hbar}} \int_{-\infty}^{\infty} \psi(x)\, e^{-i(p/\hbar)x}\, dx \end{aligned}}$$

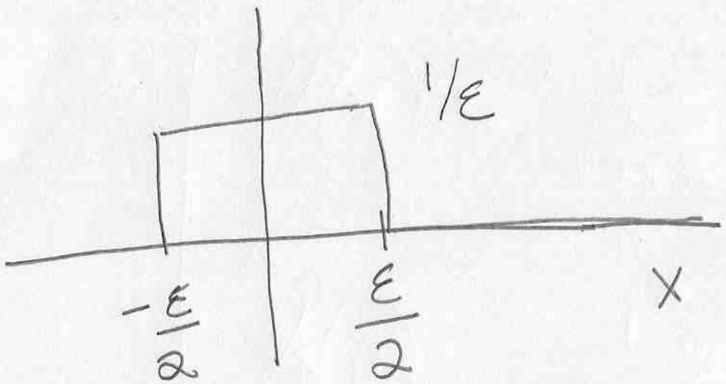
EQ. (1)

(2)

WELL WE HAVE TO DO THIS SOMETIME

MORE MATH TRICKERY

THE DIRAC DELTA FUNCTION $\delta(x)$



$$\delta_\varepsilon(x) = \frac{1}{\varepsilon} \quad -\varepsilon/2 < x < \varepsilon/2$$

$$= 0 \quad \text{ELSEWHERE}$$

$$\varepsilon \longrightarrow 0$$

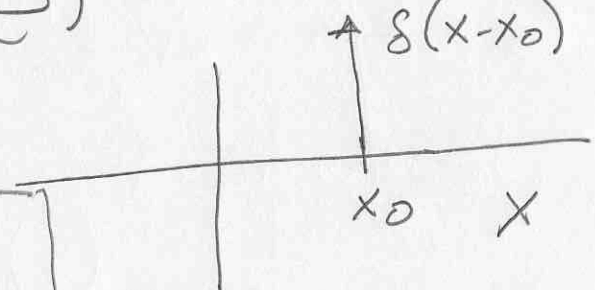
$$\int_{-\infty}^{\infty} \delta(x) \, dx = \int_{-\varepsilon/2}^{\varepsilon/2} \frac{1}{\varepsilon} \, dx = \frac{1}{\varepsilon}\left(\frac{\varepsilon}{2} - \frac{-\varepsilon}{2}\right)$$

$$= 1 = \text{AREA}$$

$$\int_{-\infty}^{\infty} \delta(x) f(x)\, dx = ?$$

$\varepsilon \to$ SMALL $\qquad f(x) \to f(0)$

$$\boxed{\int_{-\infty}^{\infty} \delta(x) f(x)\, dx = f(0)} \qquad (3)$$



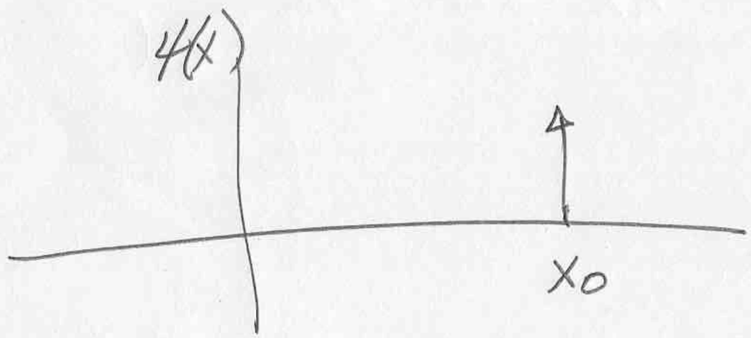
$$\boxed{\int_{-\infty}^{\infty} \delta(x-x_0) f(x)\, dx = f(x_0)} \qquad (4)$$

AN INTERESTING THING TO CONCEPTUALIZE

A FULLY LOCALIZED PARTICLE

$$\psi(x) = \delta(x - x_0) \quad \ldots\ldots$$

YEAH THERE'S A PROBLEM ... FORMALLY



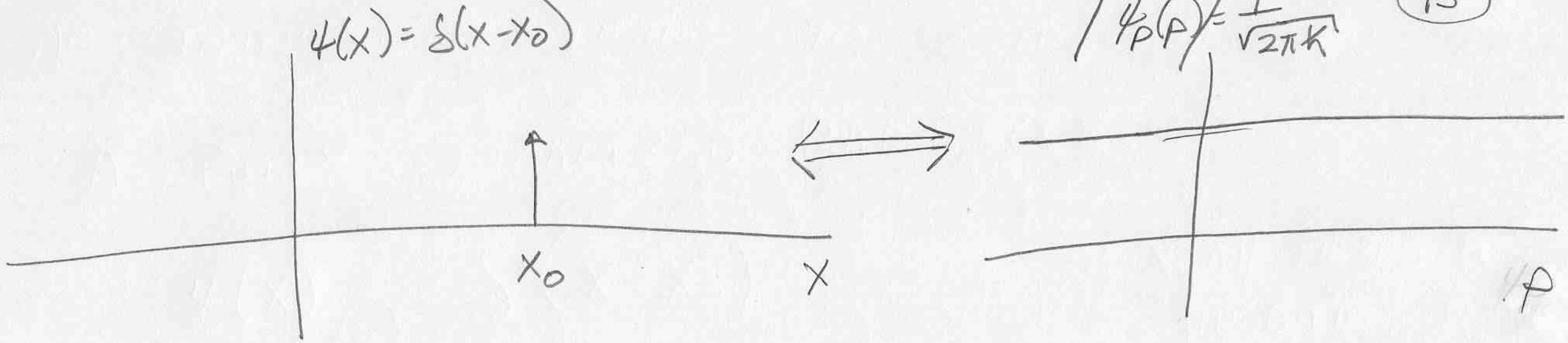
LET'S LOOK AT THE FOURIER TRANSFORM

$$\psi_p(p) = \frac{1}{\sqrt{2\pi\hbar}} \int dx\, e^{-ipx/\hbar}\, \psi(x)$$

$$= \frac{1}{\sqrt{2\pi\hbar}} \int_{x=-\infty}^{\infty} dx\, e^{-ipx/\hbar}\, \delta(x - x_0)$$

(4)

$$\boxed{\psi_p(p) = \frac{1}{\sqrt{2\pi\hbar}} \left(e^{-ipx_0/\hbar} \right)} = \text{CONSTANT } \forall p$$



X DOMAIN $\Longleftrightarrow$ P DOMAIN

INFINITELY CERTAIN — INFINITELY UNCERTAIN