THE MATHS + STUFF

LET'S CALCULATE THE COMMUTATOR

$$[\hat{X}, \hat{P}] = \hat{X}\hat{P} - \hat{P}\hat{X}$$

WE HAVE DISCUSSED

$|\psi\rangle$

AND $\langle\psi|\psi\rangle$

AND EVEN $\langle\psi|\psi\rangle = \int_{-\infty}^{\infty} \psi^*(x)\psi(x)\,dx$

BUT NOT SO MEASURED OR "EXPECTATION VALUES" OF OBSERVABLES

...

WELL EXCEPT FOR PERHAPS

$\hat{H} |\psi\rangle = E |\psi\rangle$

OR

$\hat{H} |\psi_i\rangle = E_i |\psi_i\rangle$

$\langle \psi_i | \hat{H} | \psi_i \rangle = \langle \psi_i | E_i | \psi_i \rangle = E_i$

BUT WHAT ABOUT OTHER OBSERVABLES

AKA HERMITIAN OPERATORS

SAY POSITION IN 1-D

OBSERVABLE $\hat{X}$

EXPECTATION VALUE OF $\hat{x}$ IN $|\psi\rangle$

$$\langle \psi | \hat{x} | \psi \rangle = \int_{-\infty}^{\infty} \psi^*(x) \, x \, \psi(x) \, dx$$

YEP, JUST X

BUT WHAT IS $\hat{P}$?

WAVE THE HANDS !

$$\psi \sim e^{i(kx - \omega t)} \Leftrightarrow \psi(x,t)$$

JUST FOR GRINS

$$\frac{\partial \psi}{\partial x} = ik\, e^{i(kx-\omega t)} = ik\psi$$

$p = \hbar k$ (DE BROGLIE)

$$\frac{\partial \psi}{\partial x} = i\frac{p}{\hbar}\, e^{i(kx-\omega t)} = \frac{ip}{\hbar}\psi$$

$$\boxed{p = -i\hbar \frac{\partial \psi}{\partial x}} \quad (1)$$

MANY MORE FORMAL WAYS TO SEE THIS

SO BACK TO $[\hat{x}, \hat{p}]$?

$$[\hat{x}, \hat{p}] = \hat{x}\hat{p} - \hat{p}\hat{x}$$

$$[\hat{x}, \hat{p}]|\psi\rangle$$

$$[x, p]\psi = x\left(-i\hbar\frac{\partial}{\partial x}\right)\psi - \left(-i\hbar\frac{\partial}{\partial x}(x\psi)\right)$$

$$= -i\hbar\left(x\frac{\partial\psi}{\partial x} - \psi - x\frac{\partial\psi}{\partial x}\right)$$

$$[x, p]\psi = i\hbar\psi$$

$$[x, p] = i\hbar$$