ANSWER IS NO

$\psi(u)$ DOESN'T CONVERGE UNLESS

THE SERIES $\sum c_n u^n$ TERMINATES

WHEN DOES THIS HAPPEN

$$\boxed{c_{n+2} = \left[\frac{2n+1-\mathcal{E}}{(n+1)(n+2)}\right] c_n} \quad (11)$$

IFF

$$\mathcal{E} = 2n+1$$

$n=0$ $\quad \mathcal{E} = 2(0)+1 = 1$

$n=1$ $\quad \mathcal{E} = 2(1)+1 = 3$

$n=2$ $\quad \mathcal{E} = 2(2)+1 = 5$

$n=3$ $\quad \mathcal{E} = \quad 7$

DIMENSIONLESS ENERGY ALWAYS INCREASES BY SAME STEP SIZE

FROM THE "DIMENSIONLESS RELATIONS"

$$\mathcal{E} \stackrel{\Delta}{=} \frac{2E}{\hbar\omega} \Rightarrow E = \frac{\hbar\omega}{2}\mathcal{E}$$

GROUND STATE

$$\mathcal{E}_0 = 1 \Rightarrow E_0 = \frac{\hbar\omega}{2}$$

QUANTUM $E_0 \neq 0$

DIDN'T PROVE LACK OF CONVERGENCE

FIRST

$n=0$ $\quad \psi_0(u) = \sum_{n=0}^{\infty} c_n u^n e^{-u^2/2}$ $\qquad c_{n+2} = \frac{(2n+1-\mathcal{E})}{(n+1)(n+2)}$

$c_{2,4,6,8} = 0$

$c_1 = 0$

$\psi_0(u) = c_0 u^0 e^{-u^2/2}$

$\psi_0(u) = e^{-u^2/2}$

$x = ua \qquad u = \frac{x}{a}$

$a = \sqrt{\frac{\hbar}{m\omega}}$

$u^2/2 = \frac{x^2}{2a^2} = \frac{m\omega x^2}{2\hbar}$

$\psi_0 = c_0 e^{-m\omega x^2/2\hbar}$ $\qquad \int \psi_0^* \psi_0 = 1$

$$\Rightarrow \boxed{\psi_0(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} e^{-\frac{m\omega}{2\hbar}x^2} \qquad E_0 = \frac{\hbar\omega}{2}} \quad (12)$$

IT'S A GAUSSIAN

LET'S LOOK AT WHAT HAPPENS TO $\psi(x)$ IF E_0 IS ONLY SLIGHTLY OFF

(WITHOUT A LOT MORE MATH)

$n = 0 \Rightarrow \mathcal{E}_0 = 1 \quad E_0 = \frac{\hbar\omega}{2}$

$$C_0 = 1 = 2^0$$

$$\boxed{h_0(u) = 1}$$

$n = 1 \Rightarrow \mathcal{E}_1 = 3 \quad E_1 = \frac{3\hbar\omega}{2}$

SET $C_0 = 0$

CHOOSE $= C_1 = 2^1$ ← THE PHYSICIST'S CHOICE

$$h(u) = \sum C_n u^n$$

$$h_1(u) = 2u$$

RECURSION RELATION

$$C_3 = C_{1+2} = \frac{(2n+1-\mathcal{E})\,C_1}{(n+1)(n+2)}$$

$$= \frac{(2(1)+1-3)\,(2)}{(1+1)\,(1+2)} = 0$$

$= C_{3,5,7\ldots}$

IN GENERAL EQUATION (6)

$$\frac{d^2h}{du^2} - 2u\frac{dh}{du} + (\mathcal{E}-1)h = 0$$

BUT $\mathcal{E} = 2n+1$

$$\frac{d^2h}{du^2} - 2u\frac{dh}{du} + (2n+1-1)h = 0$$

(13) $$\boxed{\frac{d^2h}{du^2} - 2u\frac{dh}{du} + 2nh = 0}$$

h(u) SOLUTIONS ARE CALLED HERMITE POLYNOMIALS

A FEW FLAVORS

IF THE COEFFICIENT OF THE HIGHEST ORDER IS $= 2^n$

$\Rightarrow$ PHYSICIST'S HERMITE POLYNOMIALS

$$h_0(u) = 1$$

$$h_1(u) = 2u$$

$$h_2(u) = 4u^2 - 2$$

$$h_3(u) = 8u^3 - 12u$$

....

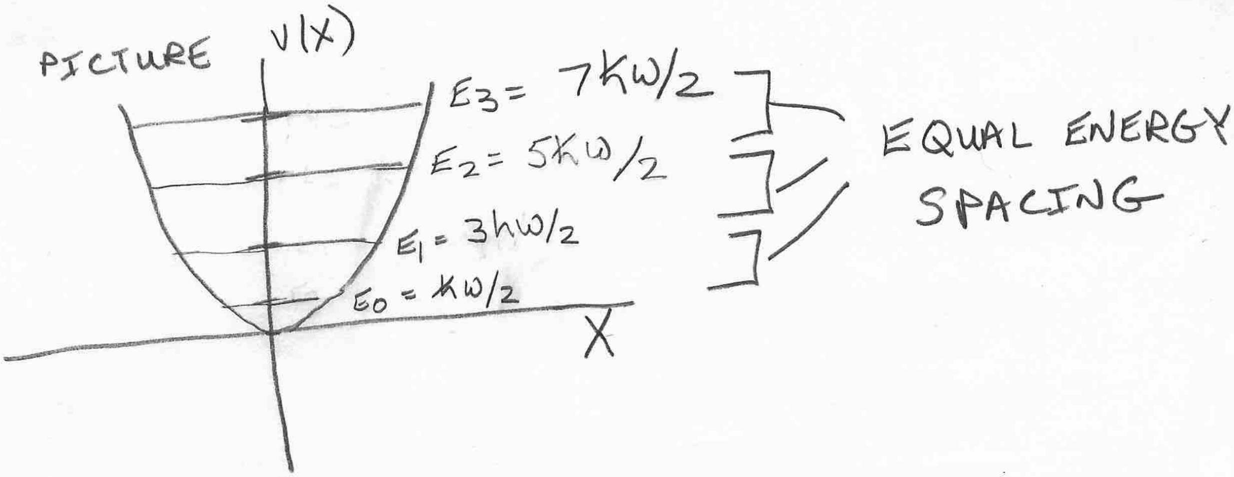
LET'S LOOK @ THEM

WRITTEN AS $H_n(\ )$ $\rightarrow$ SLIDES

TO GET BACK TO X

(14) $$\boxed{\psi_n(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} \frac{1}{\sqrt{2^n n!}} e^{-\frac{m\omega}{2\hbar}x^2} H_n\left(\sqrt{\frac{m\omega}{\hbar}}\, x\right)}$$

COMMON PICTURE



THE RUNGS OF THE ENERGY LADDER ARE EQUALLY SPACED (~EXCEPT E_0)