COMMUTATORS

MATH SENSE

MEASURES THE DEGREE TO WHICH TWO OPERATIONS FAIL TO COMMUTE

PHYSICS

OPERATIONS $\rightarrow$ OPERATORS

$$[\hat{A}, \hat{B}] = \hat{A}\hat{B} - \hat{B}\hat{A}$$

REPRESENT THE OPERATORS IN SOME BASIS

IF THE BASIS IS FINITE $\Rightarrow$ MATRICES

OLD FRIEND

SPIN IN $+Z$ BASIS

$$|+\rangle \rightarrow \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad |-\rangle \rightarrow \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

"PHYSICAL" SPIN SOMETHING THAT CAN BE MEASURED

$$S_z = \frac{\hbar}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \triangleq \frac{\hbar}{2}\sigma_z$$

COMMUTATOR IN A DISCRETE BASIS

$$\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \qquad \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$[\sigma_x, \sigma_z] = \sigma_x \sigma_z - \sigma_z \sigma_x$$

$$= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} - \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} - \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & -2 \\ 2 & 0 \end{pmatrix}$$

COMMUTATOR HOMEWORK

OPERATORS $\hat{X}$, $\hat{P}_x$

$$[\hat{X}, \hat{P}_x] = \hat{X}\,\hat{P}_x - \hat{P}_x\,\hat{X}$$

APPLY $|\psi\rangle$ REPRESENTED IN THE "X" BASIS

$$|\psi\rangle \rightarrow \psi(x,t) \qquad \hat{X} \rightarrow X \qquad \hat{P}_x \rightarrow -i\hbar\frac{\partial}{\partial x}$$

$$[\hat{x}, \hat{P}_x]|\psi\rangle \longrightarrow$$

$$x\left(-i\hbar\frac{\partial}{\partial x}\psi(x,t)\right) - -i\hbar\frac{\partial}{\partial x}\left(x\,\psi(x,t)\right)$$

$$[x, p_x]\,\psi(x,t) = -i\hbar\, x\frac{\partial\psi(x,t)}{\partial x} + i\hbar\,\psi(x,t) + i\hbar\, x\frac{\partial\psi(x,t)}{\partial x}$$

$$[x, p_x]\,\psi(x,t) = i\hbar\,\psi(x,t)$$

$$[x, p_x] = i\hbar$$