

COMMUTATORS

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MATH SENSE

MEASURES THE DEGREE TO WHICH TWO OPERATIONS
FAIL TO COMMUTE

PHYSICS

OPERATIONS \rightarrow OPERATORS

$$[\hat{A}, \hat{B}] = \hat{A}\hat{B} - \hat{B}\hat{A}$$

REPRESENT THE OPERATORS IN
SOME BASIS

IF THE BASIS IS FINITE \Rightarrow MATRICES

OLD FRIEND

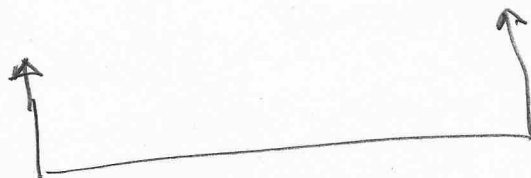
SPIN IN $+z$ BASIS

$$|+\rangle \rightarrow \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$|-\rangle \rightarrow \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

"PHYSICAL" SPIN SOMETHING THAT CAN BE MEASURED

$$S_z = \frac{\hbar}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \triangleq \frac{\hbar}{2} \sigma_z$$



COMMUTATOR IN A DISCRETE BASIS

$$\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad (2)$$

$$[\sigma_x, \sigma_z] = \sigma_x \sigma_z - \sigma_z \sigma_x$$

$$= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} - \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} - \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & -2 \\ 2 & 0 \end{pmatrix}$$

COMMUTATOR HOMEWORK

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OPERATORS \hat{X}, \hat{P}_x

$$[\hat{X}, \hat{P}_x] = \hat{X} \hat{P}_x - \hat{P}_x \hat{X}$$

APPLY $|4\rangle$ REPRESENTED IN THE "x" BASIS

$$|4\rangle \rightarrow \psi(x,t) \quad \hat{X} \rightarrow x \quad \hat{P}_x \rightarrow -i\hbar \frac{\partial}{\partial x}$$

$$[\hat{X}, \hat{P}_x] |4\rangle \rightarrow$$

$$x \left(-i\hbar \frac{\partial}{\partial x} \psi(x,t) \right) - (-i\hbar \frac{\partial}{\partial x} (x \psi(x,t)))$$

$$[x, p_x] \psi(x,t) = -i\hbar x \frac{\partial \psi(x,t)}{\partial x} + i\hbar \psi(x,t) + i\hbar x \frac{\partial \psi(x,t)}{\partial x}$$

$$[x, p_x] \psi(x,t) = i\hbar \psi(x,t)$$

$$[x, p_x] = i\hbar$$