POPULATION DISPLACEMENT OF UKRAINIAN REFUGEES

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Abstract. In this paper, we attempt to model emigration out of Ukraine since the beginning of the war through systems of SIR-based differential equations. We extend our models 36 months as a potential means of prediction, as well. Our goal is to provide a representation of how quickly people moved out of Ukraine and examine the possibility of the population returning back to pre-war numbers. We surveyed a variety of data sources to determine coefficients and initial conditions in our models. We address a variety of assumptions through different models and other oversimplification in our models. Finally, we provide potential solutions and areas for further exploration.

1. Background/Motivation

The world is currently in a state where people are being displaced on a daily basis. The Russian-Ukrainian war has been a large reason for an influx of refugees seeking asylum in the US and throughout Europe. In this paper we attempt to model how the war has impacted the population of Ukrainian citizens, and the flow of asylum seekers in and out of the country. We begin with a basic SIR model with a few basic assumptions that we will lay out, and a constant population in Ukraine. In later iterations we address the issue of birth and death rates, whether by natural causes or casualties of war, a carrying capacity on the population of Ukraine, and eventually the return of displaced persons back to Ukraine. This combines a logistic growth model with a carrying capacity (k) and the structure of an SIR model. Initial conditions of the population of Ukraine before the war, and total displaced persons at the moment were found through the UN's migration portal that has been tracking the population flux since February of 2022, when the war began. We attempt to answer the questions of how fast the Ukrainian population dropped at the beginning of the war, if it will ever reach a stability point or if it will return to the levels pre-wartime.

Research on refugee immigration and how policy should be implemented to handle the population trends has been very prevalent in the last decade. Current events such as the 10 year Syrian Civil war, Russia-Ukraine war, the current events regarding Israel and Palestine have forced people to confront how to integrate asylum seekers into the population of neighboring countries and what resources can be provided to refugees in such locations.

Research done by Professors at Tsinghua University attempted to address a similar issue caused by the conflict in Asia and Northern Africa [6]. Their model looks at how population accumulates in certain areas based off the flow of refugees and the path presented to them due to impact of their environment. A more recent study done in 2021 posed a similar question about where refugees go and how long it will take them to arrive at their new destination[7]. The research presented incorporates how the environment, considering it as a porous media, restricts the flow of refugees to a certain location. Our model, though much simpler in its assumptions, attempts to address the impact on population in the specific country where refugees are emigrating from, rather than where they're going, and predicts if the population post-wartime will return to pre-wartime levels. Based on our primary objectives our approach hopes to solve questions not frequently asked or focused on in other relevant research.

2. Modeling

2.1. **Model 1.** Our first model is based off a SIR model with the following system of ordinary differential equations:

$$\dot{C}(t) = -\beta CM$$
$$\dot{M}(t) = \beta CM - \gamma M$$
$$\dot{\mathcal{O}}(t) = \gamma M$$

In this instance, C represents the proportion of current citizens of Ukraine who are still in Ukraine. M represents the proportion of citizens that are in the process of moving or emigrating from Ukraine. $\mathcal{O}$ represents the proportion of the citizens who have now emigrated and reside outside of Ukraine. The parameters, β and γ will further be explained.

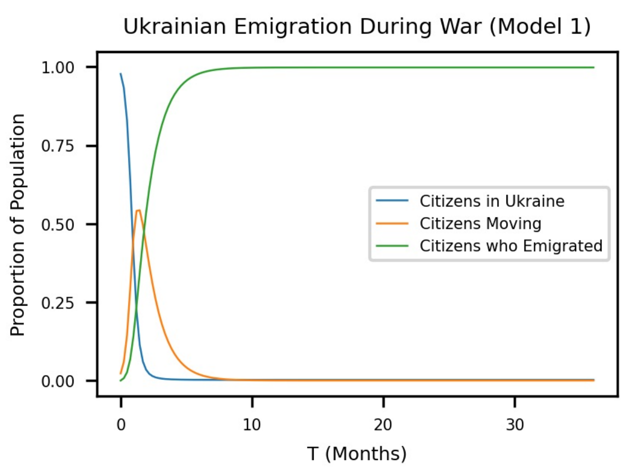
In this model we make several basic assumptions. The assumptions including the following:

- The total population of Ukrainian citizens stays constant over time
- Every citizen will eventually go through this process of moving and leaving the country
- Each citizen is as likely to interact with sets of the populations as others

We will later address many of the flaws in these assumptions with updates to our model. However, we first will show what this model predicts and describe our initial conditions and constants used in this model. To begin, we will model what is going on over the course of 36 consecutive months. Our model will begin in the month of February 2022, right before the war began. By examining the next 36 months we will be able to see our model fits the historical data since 2022, while also predicting into 2024 and 2025. At the beginning of the war it was estimated that there were 44 million people living in Ukraine[9]. Statistics from January and February 2022 show that of those, 100 people were actively documented to be emigrating [5]. While this number may likely be larger, we will stick with it as it has been factually documented. This allows us to set our initial conditions for the population of Ukrainians who were in the country, moving and out of the country at the start of February 2022.

In our ordinary differential equations systems we also have two constants, β and γ. The first of these, (β) represents the relationship between citizens in Ukraine and those moving out of Ukraine. β should represent the number of people that someone interacts with in a month that could possibly influence them to move from Ukraine. Based on monthly interaction and seriousness of moving out of your home-country, we estimate this number to be 5.

On the other hand, γ represents the relationship between those moving out of Ukraine and once they are officially out of Ukraine residing in another country. In essence, this number reflects the amount of time that it takes to move out of Ukraine. This number varies significantly depending on several factors including where you are moving to, how many people you are moving with and where you are moving from. The average household in Ukraine is around 3 people[10]. Most data points suggest that moving this amount of people takes 3-6 months[11]. However, that is a general move that is not a result of war. In this instance, we estimate that is takes an average household around 5 weeks to move or 1.25 months. With our initial conditions set and our constants we can now examine the model.



This might initially seem surprising. The model itself shows that nearly everyone would be out of Ukraine within 5 months. A closer analysis shows that over 50 percent of the country was moving during the second and third month. However, based on our assumptions this makes sense. We have not factored into our model how the population itself changes. So we may be missing information based on birth and death rates (both natural and those caused from the war). On top of this, we have assumed that everyone will eventually move, in other words we have assumed that no one has a reason to stay in Ukraine. In the absence of a reason to stay, it makes sense that everyone would leave a war-torn country as quickly as possible. However, this does not reflect reality as millions of people stay for varying reasons. These include but are not limited to the following:

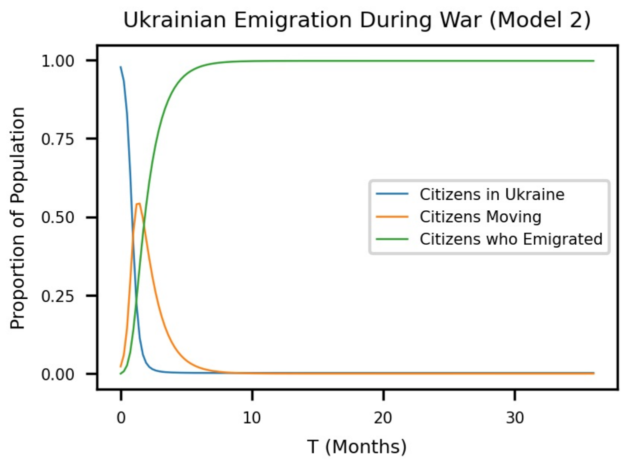
- Staying to help fight in the war
- Having no realistic option (whether from physical or monetary restrictions) to leave the country
- Wanting to stay in your home nation

Now that we have examined our first and most simplistic model we will now consider the birth and death rates effecting the citizens in Ukraine.

2.2. **Model 2.** In our second model we adjust one of the assumptions that we had in our first model. We assumed that the total population of Ukrainian citizens was staying constant. This helped us to understand how the population was emigrating after the war, but had many inaccuracies. In order to adjust for that, we will first take into account the birth and date rates for citizens who are still in Ukraine. This means that we will add a term to account for both birth rate b and death rate d into the first equation of our system of ordinary differential equations. This will result in the following equations:

$$\begin{aligned}
\dot{C}(t) &= -\beta CM + bC - dC \\
\dot{M}(t) &= \beta CM - \gamma M \\
\dot{O}(t) &= \gamma M
\end{aligned}$$

As seen in the above equations, the only changes are the additional two C terms in the first equation. We will assume the second equation M is not affected as citizens are only in that subset of the population for such a small duration of time. Then since we are mainly focusing on the population that is remaining in Ukraine we also do not add equivalent terms to our last equation. The additional two terms in the first equation relate to the death and birth rates within Ukraine. The first term, with the b coefficient is relating to the birth rate. Sources show that the birth rate in Ukraine in 2023 was 8.6 per 1000[8]. This rate was given in a yearly time period so we divided by 12 to get it into our monthly time periods. This birth rate then directly relates to the population that is still residing in Ukraine. The second term, with the d coefficient is relating to the death rate. Sources show that the death rate in Ukraine in 2023 was 18.5 per 1000[8]. This rate was given in a yearly time period so we divided by 12 to get it into our monthly time periods. This death rate then directly relates to the population that is still residing in Ukraine. This explains the additional coefficients and altercations to our model. The rest of the initial conditions and constants from our first model remain the same. Now we can consider what the model looks like.



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At first glance, it seems like the models are very similar. With further investigating, we see that some of the metrics are slightly different with the proportion moving being slightly delayed. Considering our rates this similarity is actually to be expected and can be seen analytically. The difference between the birth and the death rate is -9.9 per 1000 per year. Given our initial conditions (44 million people) this would result in a difference of approximately 435k people per year. This is less than .01 of the population. While that is certainly still a lot of people, proportionally it is not a huge number. This is even more so considering that the number of citizens is changing throughout the year as they move out of the country leading to a even smaller impact. Overall, this gives us our second model with more realistic assumptions. Now we will address one of the least realistic assumptions we made, that everyone would leave Ukraine. We will now attempt to account for the many reasons that people would stay in Ukraine despite the war.

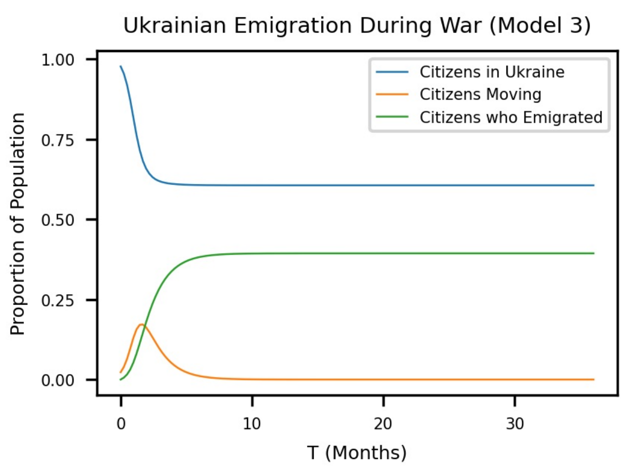
2.3. **Model 3.** This third model looks into solving and accounting for one of the largest assumptions that our initial model made. This assumption was related to all people moving out of Ukraine, or in other words that in the face of war everyone would emigrate. This is most certainly not true due to many reasons that were previously discussed (such as serving in the military or financial reasons).

Understanding how each of these mathematically effects our model is rather difficult and cumbersome. However, we can aggregate these into one constant k in our model. The aggregate effect will result in some sort of a carrying capacity. This is not a carrying capacity in the sense of having limited resources that will only support a certain amount of population. Rather, it is a carrying capacity that limits the amount of people who will move from the country because of various reasons to stay. This is seen in the equations of our third model in our ordinary differential equations. The equations are as follows:

$$\dot{C}(t) = \beta M \left(1 - \frac{C}{k}\right) + bC - dC$$
$$\dot{M}(t) = -\beta M \left(1 - \frac{C}{k}\right) - \gamma M$$
$$\dot{O}(t) = \gamma M$$

The added terms to account for this carrying capacity are seen in the the first and second equation. We see that if C is greater than k, $\dot{C}(t)$ will remain negative, representing the value of C decreasing as people leaving Ukraine. However as C begins to approach k the term $\left(1 - \frac{C}{k}\right)$ gets closer to zero. This will slow the number of people leaving Ukraine as the system plateaus to account for the population of citizens who remain in the country.

The following graph shows our model for when $k = 0.6$ representing about 60% of the population staying in Ukraine[4]. This better fits what is happening in reality and reflects the fact that many Ukrainian citizens have not and will not ever flee the country. In later models, we will perfect this value of k to more closely represent the actual proportion of the population that stayed.



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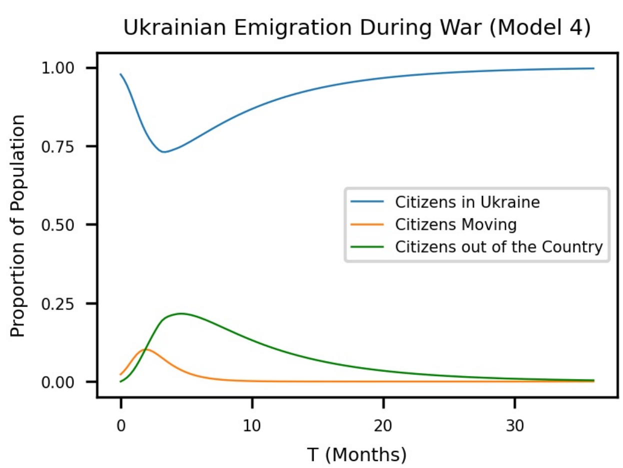
2.4. **Model 4.** Our final model attempts to remedy one more assumption we made: everyone that leaves Ukraine will not return. In this model, we consider the fact that, after approximately three months, people began returning to Ukraine at a rate of about 20,000 people each day or 600,000 each month [4]. We assume that once someone has returned to Ukraine, they will not leave again. We also assume they only return from the "left" category and not the "leaving" category. Thus, we introduce a new categorization: returned. Our new system is as follows:

For $t \geq 3$:

$$\dot{C}(t) = \beta M \left(1 - \frac{C}{k}\right) + bC - dC$$
$$\dot{M}(t) = -\beta M \left(1 - \frac{C}{k}\right) - \gamma M$$
$$\dot{\mathcal{O}}(t) = \gamma M - \alpha \mathcal{O}$$
$$\dot{\mathcal{R}}(t) = \alpha \mathcal{O}$$

In these equations, we introduce a new variable, α, which represents the rate at which people return back to Ukraine. We based this on data previously mentioned. In the following graph, the "Citizens In Ukraine" label now represents the sum of those returned and those who never left Ukraine.



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3. Results

From these models we can see that the population of Ukraine drops dramatically at the start of the war. In our first two models it appears that everyone moves out of Ukraine and does not move back. As mentioned, this is due to poor assumptions: that everyone would try to leave Ukraine and no one would return. These models suggest that everyone in Ukraine will move as quickly as possible, which is simply not reflected by reality. When we added a term that acts similarly to a carrying capacity, one of those assumptions was seemingly tackled, with Model 3 showing that a large proportion of Ukraine's population will remain there.

We address the second assumption in Model 4, showing that people will return back to their homes after a couple of months. From this we see that while citizens were quick to leave Ukraine they have also been returning and our model predicts that citizens will continue returning back to their home country. In the figure below, we compare true data for emigration from Ukraine to what our third and fourth models predicted. We see that the final model fit the overall shape of the true data fairly well, while the third model seems to follow the data closer at the start, highlighting that each model has different strengths.

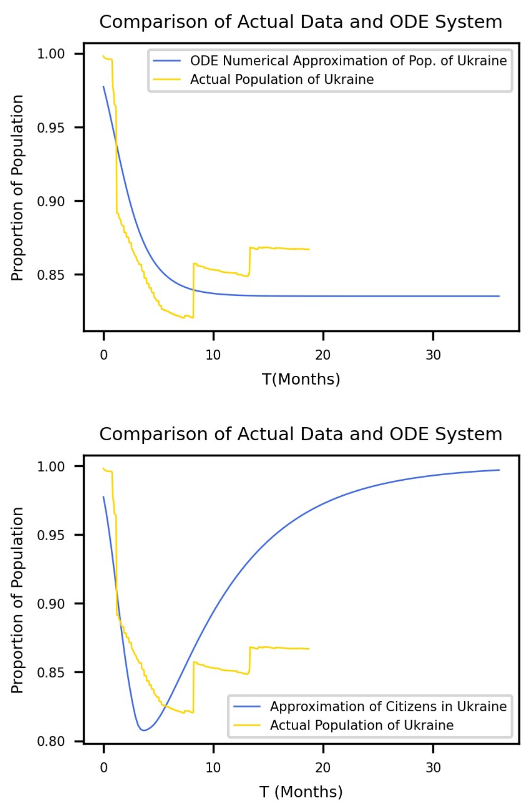

FIGURE 1. Approximations of Ukraine population using Models 3 (top) and 4 (bottom) with better fitting k values.

4. Analysis/Conclusions

With our final model, we seem to predict the answer to the question of if Ukraine will return to its pre-war population. Our prediction is yes. We have seen the population dramatically increase since most migrants left, and if the trends we've modeled continue, it will continue to increase.

Throughout this process, we have attempted to model an intricate system of Ukrainians escaping war and returning to their home country. If we have learned anything, it's that we are always making oversimplifications that can dramatically affect the way our model behaves. Though we attempted to address several of these assumptions, there are many more that leave room for further exploration. For example, our model seems to account for a single event that caused people to leave. Though the beginning of the war caused an initial spike in emigration, there were multiple events that likely caused smaller spikes. So with additional time we would likely explore adding another element to our system of ordinary differential equations that looks to factor in the intensity of the war. It is likely that during periods of high intensity, emigration would increase and citizens returning to the country would decrease. The opposite of this is likely also true, that during low intensity periods, emigration would decrease and citizens returning to the country would increase. This is a factor that certainly impacts emigration that we would explore further. Another assumption that we hold is that everyone in our model is as likely to interact with each other. It is likely that citizens in larger cities interact with more people and hence might be more likely to emigrate. Both of these assumptions are elements of our model that could be further explored and clarified.

Throughout the process of this project, we have learned that SIR based models are really useful for modeling the change of populations for a single event in time or a unique relationship. When a population has many complex factors affecting it, it becomes much more difficult to get an accurate model. We had a challenging time addressing how the conditions and implications of the war change over time, resulting in the rate of people leaving to change over time based on events. We believe that a more complex model would be needed to accurately represent the population of Ukraine as the war with Russia evolves.

Overall, there is much that we have learned through this project including understanding how Ukrainian's emigrated during war. This is an incredibly substantial learning as it can be used by many organizations and governments throughout the world. This could be utilized to help predict refugee crises during a war. Government and humanitarian organizations could use such a model to help allocate resources more efficiently. Forced human migration has been an incredibly relevant occurrence in recent world history.

It has displaced millions and put peaceful citizens in dangers way. We hope this can be used to better care for some of societies' most vulnerable people.

References

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[4] Centre for Research & Analysis of Migration. (2023, February). *CReAM*. Retrieved from https://cream-migration.org/ukraine-detail.htm?article=3573

[5] (2023, December). *Humanitarian Data Exchange*. Retrieved from https://data.humdata.org/dataset/ukraine-refugee-situation

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[10] Statista. (2023, September). *Statista*. Retrieved from https://www.statista.com/statistics/1264408/average-size-of-households-in-ukraine

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Source Code

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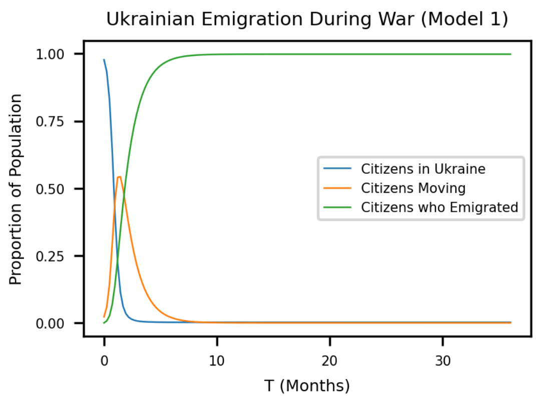
[14]:
```
import numpy as np
import matplotlib.pyplot as plt
from scipy.integrate import odeint, solve_ivp, solve_bvp
from scipy.optimize import minimize
import matplotlib
import pandas as pd
matplotlib.rcParams['mathtext.fontset'] = 'custom'
matplotlib.rcParams['mathtext.rm'] = 'Bitstream Vera Sans'
matplotlib.rcParams['mathtext.it'] = 'Bitstream Vera Sans:italic'
matplotlib.rcParams['mathtext.bf'] = 'Bitstream Vera Sans:bold'
```

0.1 Model 1

[3]:
```
def model_1():
    '''
    Using an SIR based model, we model the change of the Ukrainian population␣
 ↪as the war spreads.
    The main assumptions this model holds are:
    1) population remains constant
    2) people will not come back once the have left
    3) eventually everyone can leave
    '''
    t0 = 0   #represents the starting month (Feb 2022)
    tf = 36   #model the next 3 years after the start of the war
    n = 44000000   #initial population

    beta = 5   #rate of poeple one interacts with
    gamma = 1/1.25   #rate of how long it takes to leave

    # define the ode system as given in the problem
    def ode(t,y):
        return np.array([-beta*y[0]*y[1], beta*y[0]*y[1]-gamma*y[1],gamma*y[1]])

    # define the initial conditions
    y0 = np.array([(n-1000000)/n,1000000/n,0])
    # solve the system
    sol = solve_ivp(ode, (t0,tf), y0, t_eval=np.linspace(t0, tf, 150))
```

```
    # Plot the system and label/title everything
    plt.figure(figsize=(3, 2), dpi=300)
    plt.plot(sol.t,sol.y[0],label='Citizens in Ukraine',linewidth=.6)
    plt.plot(sol.t,sol.y[1],label='Citizens Moving',linewidth=.6)
    plt.plot(sol.t,sol.y[2],label='Citizens who Emigrated',linewidth=.6)
    plt.xlabel('T (Months)',fontsize="6")
    plt.ylabel('Proportion of Population',fontsize="6")
    plt.title('Ukrainian Emigration During War (Model 1)',fontsize="7")
    plt.legend(fontsize="5")
    plt.xticks(fontsize='5')
    plt.yticks(fontsize='5')
    plt.show()
```

[4]:
```
model_1()
```

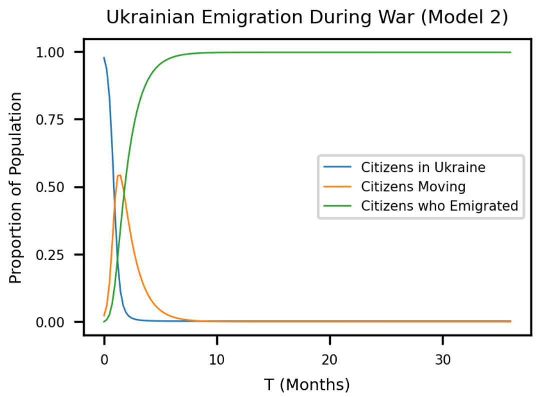


0.2 Model 2

[5]:
```
def model_2():
    '''
    Using an SIR based model, we model the change of the Ukrainian population␣
 ↪as the war spreads.
    The main assumptions this model holds are:
    1) population can change (we now include a death and birth rate)
    2) people will not come back once the have left
    3) eventually everyone can leave
    '''
    #set initial conditions
    t0 = 0   #represents the starting month (Feb 2022)
    tf = 36   #model the next 3 years after the start of the war
    n = 44000000   #initial population

    beta = 5   #rate of poepple one interacts with
```

```
    gamma = 1/1.25   #rate of how long it takes to leave

    birth = (8.6/1000)/12   #birth rate
    death = (18.5/1000)/12   #death rate

    # define the ode system as given in the problem
    def ode(t,y):
        return np.array([-beta*y[0]*y[1]+birth*y[0]-death*y[0],␣
↪beta*y[0]*y[1]-gamma*y[1],gamma*y[1]])

    # define the initial conditions
    y0 = np.array([(n-1000000)/n,1000000/n,0])
    # solve the system
    sol = solve_ivp(ode, (t0,tf), y0, t_eval=np.linspace(t0, tf, 150))

    # Plot the system and label/title everything
    plt.figure(figsize=(3, 2), dpi=300)
    plt.plot(sol.t,sol.y[0],label='Citizens in Ukraine',linewidth=.6)
    plt.plot(sol.t,sol.y[1],label='Citizens Moving',linewidth=.6)
    plt.plot(sol.t,sol.y[2],label='Citizens who Emigrated',linewidth=.6)
    plt.xlabel('T (Months)',fontsize='6')
    plt.ylabel('Proportion of Population',fontsize='6')
    plt.title('Ukrainian Emigration During War (Model 2)',fontsize='7')
    plt.legend(fontsize='5')
    plt.xticks(fontsize='5')
    plt.yticks(fontsize='5')
    plt.show()
```

[6]:
```
model_2()
```

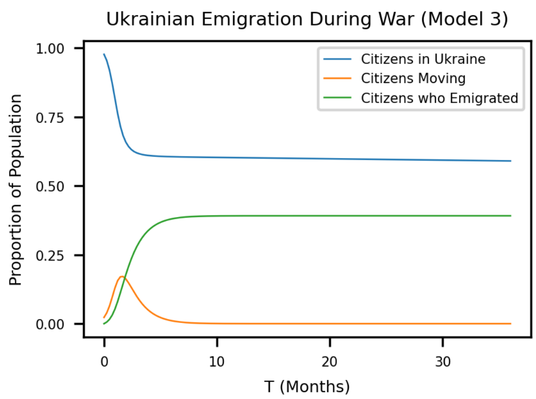


0.3 Model 3

[7]:
```
def model_3():
    '''
    Using an SIR based model,we model the change of the Ukrainian population as␣
 ↪the war spreads.
    The main assumptions this model holds are:
    1) population can change (we now include a death and birth rate)
    2) people will not come back once the have left
    3) only a certain number of citizens will leave (we now include a carrying␣
 ↪capacity (k))
    '''
    #set initial conditions
    t0 = 0   #represents the starting month (Feb 2022)
    tf = 36   #model the next 3 years after the start of the war
    n = 44000000   #initial population

    beta = 5   #rate of poeple one interacts with
    gamma = 1/1.25   #rate of how long it takes to leave

    birth = (8.6/1000)/12   #birth rate
    death = (18.5/1000)/12   #death rate

    k = .6   #carrying capacity--places a limit on how many will leave

    #define ode measuring the spread of disease
    def ode(t,y):
        return np.array([beta*y[1]*(1-y[0]/
 ↪k)+birth*y[0]-death*y[0],-beta*y[1]*(1-y[0]/k)-gamma*y[1], gamma*y[1]])

    #initial conditions
    y0 = np.array([(n-1000000)/n,1000000/n,0])

    #solve ivp and get solution
    sol=solve_ivp(ode,(t0,tf), y0, t_eval=np.linspace(t0,tf,150))

    #plots and labels
    plt.figure(figsize=(3, 2), dpi=300)
    plt.plot(sol.t,sol.y[0],label='Citizens in Ukraine',linewidth=.6)
    plt.plot(sol.t,sol.y[1],label='Citizens Moving',linewidth=.6)
    plt.plot(sol.t,sol.y[2],label='Citizens who Emigrated',linewidth=.6)
    plt.xlabel('T (Months)',fontsize='6')
    plt.ylabel('Proportion of Population',fontsize='6')
    plt.title('Ukrainian Emigration During War (Model 3)',fontsize='7')
    plt.legend(fontsize='5')
    plt.xticks(fontsize='5')
    plt.yticks(fontsize='5')
```

```
    plt.show()
```

[8]:
```
model_3()
```

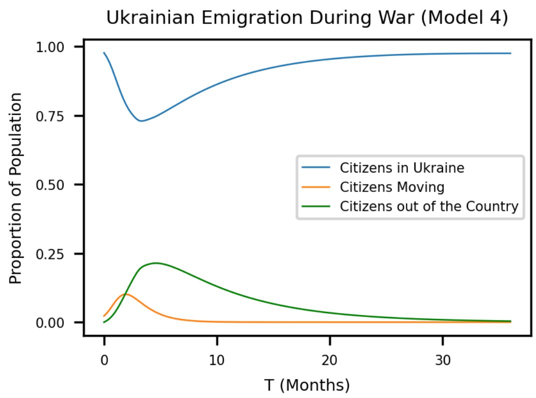


0.4 Model 4

[9]:
```
def model_4():
    '''
    Using an SIR based model, we model the change of the Ukrainian population␣
 ↪as the war spreads.
    The main assumptions this model holds are:
    1) population can change (we now include a death and birth rate)
    2) people can move back to Ukraine after emigrating (we now include a term␣
 ↪to represent people coming back)
    3) only a certain number of citizens will leave (we now include a carrying␣
 ↪capacity (k))
    '''
    #set initial conditions
    t0 = 0  #represents the starting month (Feb 2022)
    tf = 36   #model the next 3 years after the start of the war
    n = 44000000   #initial population

    beta = 5   #rate of poeple one interacts with
    gamma = 1/1.25   #rate of how long it takes to leave

    birth = (8.6/1000)/12   #birth rate
    death = (18.5/1000)/12   #death rate

    k = 29e6/n   #carrying capacity--places a limit on how many will leave
    c = 60e5/n   #rate describing how many people return to Ukraine

    #define ode measuring the spread of disease
```

```
    def ode(t,y):
        # suppose people don't start returning for 3 weeks
        if t < 3:
            return np.array([beta*y[1]*y[0]*(1-y[0]/
↪k)+birth*y[0]-death*y[0],-beta*y[1]*y[0]*(1-y[0]/k)-gamma*y[1],␣
↪gamma*y[1],0])
        else:
            return np.array([beta*y[1]*y[0]*(1-y[0]/
↪k)+birth*y[0]-death*y[0],-beta*y[1]*y[0]*(1-y[0]/k)-gamma*y[1], gamma*y[1] -␣
↪c * y[2],c * y[2]])

    #initial conditions
    y0 = np.array([(n-1e6)/n,1e6/n,0, 0])
    y0[3] = min(y0[3], 1 - y0[0])

    #solve ivp and get solution
    solution=solve_ivp(ode,(t0,tf), y0, t_eval=np.linspace(t0,tf,150))

    #plots and labels
    plt.figure(figsize=(3, 2), dpi=300)
    plt.plot(solution.t, solution.y[0]+solution.y[3],label="Citizens in␣
↪Ukraine",linewidth=.6)
    plt.plot(solution.t, solution.y[1],label="Citizens Moving",linewidth=.6)
    plt.plot(solution.t, solution.y[2],'green',label="Citizens out of the␣
↪Country",linewidth=.6)
    # plt.plot(solution.t, solution.y[3],'purple',label="Returned to Ukraine")
    plt.xlabel('T (Months)',fontsize='6')
    plt.title('Ukrainian Emigration During War (Model 4)',fontsize='7')
    plt.ylabel('Proportion of Population',fontsize='6')
    plt.legend(fontsize='5')
    plt.xticks(fontsize='5')
    plt.yticks(fontsize='5')
    plt.show()
```

[10]:
```
model_4()
```



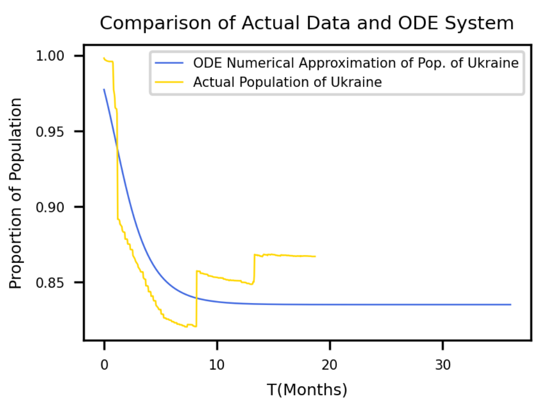
0.5 Comparison Plots

[13]:
```
#clean and load real data
useful = pd.read_csv('Ukraine Explorer Inputs Prod - RefugeesSeries.csv')
data = useful.iloc[1:,:]
data['StartDate'] = data['RefugeesDate'].iloc[0]
data['StartDate'] = pd.to_datetime(data['StartDate'])

data.RefugeesDate = pd.to_datetime(data.RefugeesDate).copy()
data['TimeDifference'] = (data['RefugeesDate'] - data['StartDate']).dt.days /␣
↪30.44
data.set_index(['TimeDifference'],inplace=True)
data['NoRefugees'] = data['NoRefugees'].astype('int').copy()
n = 44e6
data['NoRefugees'] = (n - data['NoRefugees'])/n
```

[22]:
```
def comparison_1():
    '''
    This compares the actual data we see for Ukrainian population throughout␣
↪the war to one of our best models (model 3).
    '''
    #set initial conditions
    t0 = 0   #represents the starting month (Feb 2022)
    tf = 36   #model the next 3 years after the start of the war
    n = 44e6   #initial population

    beta = 5   #rate of poeple one interacts with
    gamma = 1/1.25   #rate of how long it takes to leave

    k = 34e6/n   #carrying capacity--places a limit on how many will leave
    c = 75e5/n   #rate describing how many people return to Ukraine
```

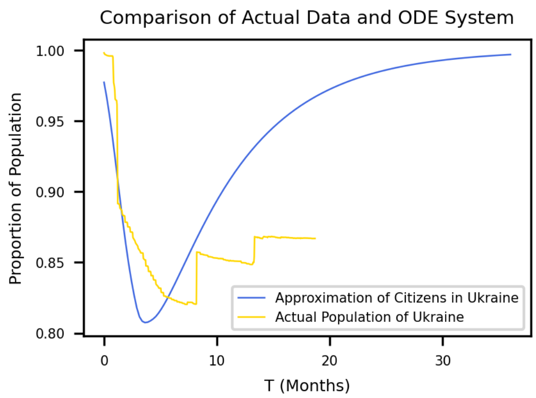
```
    #define ode measuring the spread of disease
    def ode(t,y):
        if t <1:
            return np.array([beta*y[1]*y[0]*(1-y[0]/k),-beta*y[1]*y[0]*(1-y[0]/
↪k)-gamma*y[1], gamma*y[1],0])
        else:
            return np.array([beta*y[1]*y[0]*(1-y[0]/k),-beta*y[1]*y[0]*(1-y[0]/
↪k)-gamma*y[1], gamma*y[1] - c * y[2],c * y[2]])
    #initial conditions
    y0 = np.array([(n-1e6)/n,1e6/n,0, 0])
    y0[3] = min(y0[3], 1 - y0[0])

    #solve ivp and get solution
    solution=solve_ivp(ode,(t0,tf), y0, t_eval=np.linspace(t0,tf,150))

    #plots and labels
    plt.figure(figsize=(3, 2),dpi=300,linewidth=10)
    plt.plot(solution.t, solution.y[0],label="ODE Numerical Approximation of␣
↪Pop. of Ukraine", color = 'royalblue',linewidth=.6)
    plt.plot(data.index, data.NoRefugees, label = 'Actual Population of␣
↪Ukraine', color = 'gold',linewidth=.6)
    plt.xlabel('T(Months)',fontsize="6")
    plt.title('Comparison of Actual Data and ODE System',fontsize="7")
    plt.ylabel('Proportion of Population',fontsize="6")
    plt.legend(fontsize="5")
    plt.xticks(fontsize='5')
    plt.yticks(fontsize='5')
    plt.show()
```

[23]:
```
comparison_1()
```



```
[24]: def comparison_2():
          '''
          This compares the actual data we see for Ukrainian population throughout␣
      ↪the war to one of our best model (model 4).
          '''
          #set initial conditions
          t0 = 0   #represents the starting month (Feb 2022)
          tf = 36   #model the next 3 years after the start of the war
          n = 44e6   #initial population

          beta = 5   #rate of poeple one interacts with
          gamma = 1/1.25   #rate of how long it takes to leave

          k = 31.5e6/n   #carrying capacity--places a limit on how many will leave
          c = 60e5/n   #rate describing how many people return to Ukraine

          #define ode measuring the spread of disease
          def ode(t,y):
              if t <3:
                  return np.array([beta*y[1]*y[0]*(1-y[0]/k),-beta*y[1]*y[0]*(1-y[0]/
      ↪k)-gamma*y[1], gamma*y[1],0])
              else:
                  return np.array([beta*y[1]*y[0]*(1-y[0]/k),-beta*y[1]*y[0]*(1-y[0]/
      ↪k)-gamma*y[1], gamma*y[1] - c * y[2],c * y[2]])
          #initial conditions
          y0 = np.array([(n-1e6)/n,1e6/n,0, 0])
          y0[3] = min(y0[3], 1 - y0[0])

          #solve ivp and get solution
          solution=solve_ivp(ode,(t0,tf), y0, t_eval=np.linspace(t0,tf,150))

          #plots and labels
          plt.figure(figsize=(3, 2),dpi=300,linewidth=10)
          plt.plot(solution.t, solution.y[0]+solution.y[3],label="Approximation of␣
      ↪Citizens in Ukraine", color = 'royalblue',linewidth=.6)
          plt.plot(data.index, data.NoRefugees, label = 'Actual Population of␣
      ↪Ukraine', color = 'gold',linewidth=.6)
          plt.xlabel('T (Months)',fontsize="6")
          plt.title('Comparison of Actual Data and ODE System',fontsize="7")
          plt.ylabel('Proportion of Population',fontsize="6")
          plt.legend(fontsize="5")
          plt.xticks(fontsize='5' )
          plt.yticks(fontsize='5' )
          plt.show()
```

```
[25]: comparison_2()
```


Comparison of Actual Data and ODE System
Proportion of Population
1.00
0.95
0.90
0.85
0.80
0
10
20
30
T (Months)
Approximation of Citizens in Ukraine
Actual Population of Ukraine