

## Feedback — Mergesort

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You submitted this quiz on **Mon 21 Sep 2015 7:14 PM EDT**. You got a score of **2.60** out of **3.00**. You can [attempt again](#), if you'd like.

To specify an array or sequence of values in an answer, separate the values in the sequence by whitespace. For example, if the question asks for the first ten powers of two (starting at 1), then the following answer is acceptable:

1 2 4 8 16 32 64 128 256 512

If you wish to discuss a particular question and answer in the forums, please post the entire question and answer, including the seed (which can be used by the course staff to uniquely identify the question) and the explanation (which contains the correct answer).

### Question 1

(seed = 547624)

Give the array that results immediately after the 7th call (and return) from `merge()` when top-down mergesorting the following array of size 12:

35 79 11 95 78 68 33 83 27 39 28 52

Your answer should be a sequence of 12 integers, separated by whitespace.

**You entered:**

11 35 68 78 79 95 27 33 83 39 28 52

**Your Answer**

**Score**

**Explanation**

11 35 68 78 79 95 27 33 83 39 28 52



1.00

Total

1.00 / 1.00

### Question Explanation

The correct answer is: 11 35 68 78 79 95 27 33 83 39 28 52

Here is the array immediately after each call to merge():

```

                35 79 11 95 78 68 33 83 27 39 28 52
merge(0, 0, 1): 35 79 11 95 78 68 33 83 27 39 28 52
merge(0, 1, 2): 11 35 79 95 78 68 33 83 27 39 28 52
merge(3, 3, 4): 11 35 79 78 95 68 33 83 27 39 28 52
merge(3, 4, 5): 11 35 79 68 78 95 33 83 27 39 28 52
merge(0, 2, 5): 11 35 68 78 79 95 33 83 27 39 28 52
merge(6, 6, 7): 11 35 68 78 79 95 33 83 27 39 28 52
merge(6, 7, 8): 11 35 68 78 79 95 27 33 83 39 28 52

```

## Question 2

(seed = 16382)

Give the array that results immediately after the 7th call (and return) from merge() when bottom-up mergesorting the following array:

21 71 52 98 49 18 50 29 11 15

Your answer should be a sequence of 10 integers, separated by whitespace.

**You entered:**

21 52 71 98 18 29 49 50 11 15

Your Answer	Score	Explanation
21 52 71 98 18 29 49 50 11 15	✓ 1.00	
Total	1.00 / 1.00	

**Question Explanation**

The correct answer is: 21 52 71 98 18 29 49 50 11 15

Here is the array immediately after each call to merge():

```

                21 71 52 98 49 18 50 29 11 15
merge(0, 0, 1): 21 71 52 98 49 18 50 29 11 15
merge(2, 2, 3): 21 71 52 98 49 18 50 29 11 15
merge(4, 4, 5): 21 71 52 98 18 49 50 29 11 15
merge(6, 6, 7): 21 71 52 98 18 49 29 50 11 15
merge(8, 8, 9): 21 71 52 98 18 49 29 50 11 15
merge(0, 1, 3): 21 52 71 98 18 49 29 50 11 15
merge(4, 5, 7): 21 52 71 98 18 29 49 50 11 15

```

**Question 3**

(seed = 526958)

Which of the following statements about mergesort are true? Check all that apply. Unless otherwise specified, assume that mergesort refers to the pure recursive (top-down) version of mergesort (with no optimizations), using the merging subroutine described in lecture.

Your Answer	Score	Explanation
<input checked="" type="checkbox"/> The number of compares of mergesort on an array of $N/2$ 1s interleaved with $N/2$ 0s (e.g.	✗ 0.00	In general, each merge involves two subarrays of the form 00001111 (i.e., $1/2 N$ 0s followed by $1/2 N$ 1s). This takes $\sim 3/4 N$ compares because the left subarray is exhausted with $1/4 N$ 1s remaining in the right subarray. Thus, the total number of compares satisfies the recurrence $T(N) = 2 T(N/2) + 3/4 N$ , which yields $T(N) \sim 3/4 N \lg N$ .

```
, 1 0 1 0 1
0 1 0 1 0) i
s ~ 1 N lg N
.
```



0.20

This can be proved by induction - in either version of mergesort, all of the subarray sizes are powers of 2.

For any array of  $N$  distinct keys with  $N$  a power of 2, top-down mergesort and bottom-up mergesort compare exactly the same pairs of keys (but possibly in a different order).




0.20


Similar to the lower bound argument with 2-way compares, but now the height of the tree is at least  $\log_6(N!)$  since each node has as many as 6 children, corresponding to the 3! possible outcomes for each super-compare.

Suppose we have a sorting algorithm that in addition to regular compares, is also allowed super-compares: take three keys and return those three keys in sorted order. Then, any compare-based sorting algorithm requires at least  $\lg(N!)$  compares or super-compares (i

n the worst case) to sort an array of  $N$  items.

☐  0.20 Any merging algorithm requires at least  $2N-1$  compares in the worst case to merge two sorted arrays of the form  $x = [0, 2, 4, \dots, N-2]$  and  $y = [1, 3, 5, \dots, N-1]$  because  $x[i]$  must be compared with both  $y[i-1]$  and  $y[i+1]$ . Alternatively, such a merging algorithm would lead to a compare-based sorting algorithm that guarantees to make no more than  $\sim \frac{3}{4} N \lg N$  compares, which would violate the sorting lower bound.

It is possible to design a compare-based algorithm to merge two sorted arrays, each of size  $N$ , with no more than  $\frac{3}{2} N \lg N$  compares.

☒  0.00 The number of compares ranges from  $\sim \frac{1}{2} N \lg N$  (sorted array) to  $\sim N \lg N$  (random array).

The number of compares in mergesort depends only on the size of the array  $N$  (and not on the items in the array).

Total 0.60 / 1.00

#### Question Explanation

