Homework 3

Due date: February 6, 2023, 11:55pm

Learning Objectives

- Express statements given by natural language in symbolic form
- Evaluate logic expressions.
- Identify logically equivalent expressions.
- Apply the laws of logic.

Exercises

1. (6 points)

   Create a truth table for the propositions $\neg p \wedge q \leftrightarrow r \longrightarrow \neg p \vee q$. Determine whether the proposition is a tautology, contradiction, or contingent proposition. Observe the precedence rules!

   Tautology: truth value is always **true** regardless of individual truth value of propositions

   Contradiction: truth value is always **false** regardless of individual truth value of propositions

   Contingent proposition: truth value is true OR false under at least one scenario

   This proposition is a contingent proposition because all possibilities are neither true nor false.

p	q	r	$\neg p$	$\neg p \wedge q$	$\neg p \vee q$	$r \longrightarrow \neg p \vee q$	$\neg p \wedge q \leftrightarrow r \longrightarrow \neg p \vee q$
T	T	T	F	F	T	T	F
T	F	F	F	F	F	T	F
F	T	T	T	T	T	T	T
F	F	F	T	F	T	T	F
T	T	F	F	F	T	T	F
T	F	T	F	F	F	F	T
F	T	F	T	T	T	T	T
F	F	T	T	F	T	T	F

2. (8 points)

Identify the prime propositions in each of the following sentences and represent each prime proposition by a letter. Indicate the prime proposition each letter represents. Then write each of the sentences in symbolic logic using the logic connectives and the letters representing the prime propositions.

a) Inflation does not rise only if excess demand does not rise.

p = Inflation does not rise.

q = excess demand does not rise.

$p \leftrightarrow q$

b) Either excess demand rises or inflation does not rise.

m = Excess demand rises.

n = inflation rises.

$m \vee \neg n$

3. (12 points)
Use equivalence transformations to determine which of the following propositions are logically equivalent to $(p \vee q) \wedge \neg (p \wedge q)$. Mark all that apply. You do not have to show your transformations.

p	q	$p \vee q$	$\neg (p \wedge q)$	$(p \vee q) \wedge \neg (p \wedge q)$
T	T	T	F	F
T	F	T	T	T
F	T	T	T	T
F	F	F	T	F

- ☐ $(p \wedge q) \vee \neg(p \vee q) \Leftrightarrow F \vee F$
- ☐ $(p \wedge \neg q) \vee (\neg p \wedge q) \Leftrightarrow F \vee F$
- ☐ $\neg (p \wedge q) \vee p \vee q \Leftrightarrow T \vee T$
- ☐ <u>$(\neg p \vee \neg q) \wedge (p \vee q) \Leftrightarrow T \wedge T$ This proposition is logically equivalent to $(p \vee q) \wedge \neg (p \wedge q)$</u>

4. (14 points)
Complete the missing steps and annotation in the following proof.

Claim: $p \rightarrow (q \rightarrow r) \Leftrightarrow (p \rightarrow q) \rightarrow (p \rightarrow r)$

Proof:

$p \rightarrow (q \rightarrow r)$

$\Leftrightarrow p \rightarrow (\neg q \vee r)$ <u>implication law</u>

$\Leftrightarrow \underline{\neg p \vee (\neg q \vee r)}$ implication law

$\Leftrightarrow (\neg p \vee \neg q) \vee r$ <u>associative law</u>

$\Leftrightarrow$ (true $\wedge$ ($\neg$p $\vee$ $\neg$q)) $\vee$ r ? where did the True come from?

$\Leftrightarrow$ (($\neg$p $\vee$ p) $\wedge$ ($\neg$p $\vee$ $\neg$q)) $\vee$ r negation law

$\Leftrightarrow$ (($\neg$p $\wedge$ $\neg$p) $\vee$ (p $\wedge$ $\neg$q) $\vee$ $\neg$p) $\vee$ r distributive law (where did ($\neg$p $\wedge$ $\neg$p) go?)

$\Leftrightarrow$ ((p $\wedge$ $\neg$q) $\vee$ $\neg$p) $\vee$ r absorption law

$\Leftrightarrow$ (p $\wedge$ $\neg$q) $\vee$ $\neg$p $\vee$ r associative law

$\Leftrightarrow$ $\neg$ $\neg$(p $\wedge$ $\neg$q) $\vee$ ($\neg$p $\vee$ r) double negation law

$\Leftrightarrow$ $\neg$ ($\neg$p $\vee$ $\neg$ $\neg$q) $\vee$ ($\neg$p $\vee$ r) distributive law

$\Leftrightarrow$ ($\neg$ p $\vee$ q) $\vee$ ($\neg$p $\vee$ r) double negation law

$\Leftrightarrow$ ($\neg$p $\vee$ q) $\longrightarrow$ ($\neg$p $\vee$ r) implication law

$\Leftrightarrow$(p $\longrightarrow$ q) $\longrightarrow$ ($\neg$p $\vee$ r) implication law

$\Leftrightarrow$ (p $\longrightarrow$ q) $\longrightarrow$ (p $\longrightarrow$ r) implication law

Submission

Upload your solution as a PDF file to the course website.