A Simpler Method for Finding Expected Value in Camel Up

Kyle Eckland

November 2021

1 A New Formulation

Generally, the expected value of a discrete random variable X is defined as $E[X] = \sum_{k=1}^{r} kp(X = k)$, where $p(X = k)$ is the probability that $X = k$. In the context of movement in camel up, if we let X be the random variable representing the distance a camel moved in a given leg, then r would be the furthest distance a camel could move. If we were to try to apply this method to the expected value of a camel moving k spaces in a given leg, we would have to first have to find each way the camel could move a distance of k, which can be difficult. This method is tedious and prone to errors, so instead we'll use a method that paints the same picture with more efficient brush strokes. To do this, we will switch our perspective: rather than looking at ways a camel can move a given distance, we will look at the number of ways a camel moves a given number of times within a leg. We'll denote the random variable that represents the number of times a camel moves in a given turn as Z. Then, the expected number of times a camel move is $\sum_{z=1}^{n} zp(z)$, where n is the total number of times a camel can move in a turn. Notice that each time a camel moves requires a dice roll, and we know that the expected value of a single dice roll here is 2. Since the value of an individual roll is independent from the number of dice rolled or the order they're rolled in, if we multiply the expected value of a dice roll by the expected value of the number of times a camel moves in a turn, we find the expected distance a camel moves in a turn. Thus, $E[X] = 2E[Z]$. This allows for a simpler way to find the expected value by means of calculating the probability of the number of possible movements.