A Simpler Method for Finding Expected Value in Camel Up

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Theorem 1 (Expected Value for Stacked Camels)**.** *Suppose it is the start of a new leg and we have a stack of 2 or more racing camels with no camels in front of them. Let X be the discrete random variable representing the distance a given camel from that stack can move in this leg, and assume Z is the random variable that represents the number of times that camel can move in this leg. Then,* $E[X] = 2E[Z]$

Proof. Generally, the expected value of a discrete random variable X is defined as $E[X] = \sum_{k=1}^{r} kp(X = k)$, where $p(X = k)$ is the probability that $X = k$. In the context of movement in camel up, r would be the furthest distance a camel could move. If we were to try to apply this method to the expected value of a camel moving k spaces in a given leg, we would have to first have to find each way the camel could move a distance of k, which can be difficult. Instead we will switch our perspective: rather than looking at ways a camel can move a given distance, we will look at the number of ways a camel moves a given number of times within a leg. We'll denote this with the random variable Z. Then, the expected number of times a camel moves is $\sum_{z=1}^{n} zp(z)$, where n is the maximum number of times a camel can move in a turn. Notice that each time a camel moves requires a dice roll, and we know that the expected value of a single dice roll here is 2. Since the value of an individual roll is independent from the number of dice rolled or the order they're rolled in, if we multiply the expected value of a dice roll by the expected value of the number of times a camel moves in a turn, we find the expected distance a camel moves in a turn to be $E[X] = 2E[Z]$. $\square$