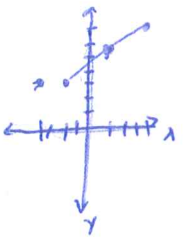
Key

Math 109: Algebra for Calculus

Exam 2 Review
Chapter 2: Functions and Relations

1. Given the points $(-2, 3)$ and $(4, 7)$.



(a) Compute the distance between these points.

Distance Formula
$\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$

$\Rightarrow d = \sqrt{(-2-4)^2+(3-7)^2}$
$= \sqrt{(-6)^2+(-4)^2} = \boxed{\sqrt{52} = 2\sqrt{13}}$ both ok

52, 2, 26, 2, 13

(b) Find the midpoint of the line segment connecting them.

Midpoint Formula
$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$\Rightarrow \left(\frac{-2+4}{2}, \frac{3+7}{2}\right) = \boxed{(1, 5)}$

(c) Compute the slope of the line connecting the points.

Slope Formula
$\frac{\Delta y}{\Delta x} = \frac{y_2-y_1}{x_2-x_1}$

$\Rightarrow m = \frac{7-3}{4-(-2)} = \frac{4}{6} = \boxed{\frac{2}{3}}$

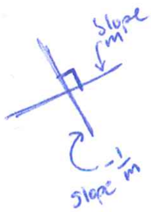
(d) Write an equation for the line containing these two points.

$\boxed{y-7 = \frac{2}{3}(x-4)}$ or $\boxed{y-3 = \frac{2}{3}(x+2)}$ or $\boxed{y = \frac{2}{3}x + \frac{13}{3}}$

(e) Find the $x-$ and $y-$ intercepts of this line.

x-intercepts: when $y=0$
$0 = \frac{2}{3}x + \frac{13}{3} \rightarrow x = \frac{-13}{2}$ $\boxed{\left(\frac{-13}{2}, 0\right)}$

y-intercepts: when $x=0$
$y = \frac{2}{3}\cdot 0 + \frac{13}{3} = \frac{13}{3}$ $\boxed{\left(0, \frac{13}{3}\right)}$



(f) Find the slope of a line perpendicular to this line.

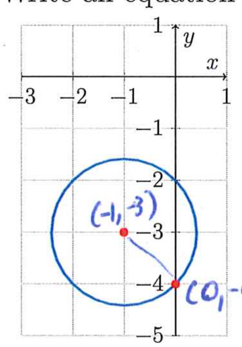
negative reciprocal
$-\frac{1}{m}$

$\boxed{-\frac{3}{2}}$

(g) Write an equation for a line perpendicular to this line passing through the point $(3, 0)$.

$y - 0 = \frac{-3}{2}(x-3) \rightarrow \boxed{y = \frac{-3}{2}x + \frac{9}{2}}$

2. Write an equation for the circle graphed here.



Center: $(-1, -3)$
Radius: $d = \sqrt{(-1-0)^2+(-3-(-4))^2} = \sqrt{2}$

Formula center (h,k) radius r is
$(x-h)^2+(y-k)^2 = r^2$

$\Rightarrow \boxed{(x+1)^2+(y+3)^2 = 2}$

3. What is the center and the radius of a circle with equation $x^2 + y^2 + 8x + 14y + 1 = 0$?
$-1 \quad -1$

Complete the squares

$x^2 + 8x + \left(\frac{8}{2}\right)^2 + y^2 + 14y + \left(\frac{14}{2}\right)^2 = -1 + \left(\frac{8}{2}\right)^2 + \left(\frac{14}{2}\right)^2 = -1 + 16 + 49$

$4^2 \quad 7^2$

$(x-4)^2 + (y-7)^2 = 64$
$= 8^2$

$\boxed{\text{Center: } (4, 7) \quad \text{Radius: } 8}$

4. Find the domain of the function. Write your answer in interval and set builder notation.

(a) $f(x) = \sqrt{3x-2}$

~~[illegible]~~ Bad: $3x-2<0$

$\rightarrow x < \frac{2}{3}$

Good $x \geq \frac{2}{3}$

$\boxed{\text{Interval } [\tfrac{2}{3}, \infty) \quad \text{Set } \{x \in \mathbb{R} \mid x \geq \tfrac{2}{3}\}}$

(b) $g(x) = \dfrac{2x+1}{x^2-9}$

Bad: $x^2-9=0$

opt 1: $x^2 = 9 \rightarrow x = \pm 3$

opt 2: $(x+3)(x-3)=0 \rightarrow x=3$ or $x=-3$

$x+3=0 \quad x-3=0$

Interval: $(-\infty,-3)\cup(-3,3)\cup(3,\infty)$

Set: $\{x \in \mathbb{R} \mid x \neq \pm 3\}$

5. For each function compute the given items then determine if the function is even/odd/neither.

(a) $f(x) = 3x^2 - 2$

i. $f(0) = 3(0)^2 - 2 = \boxed{-2}$

ii. $f(4) = 3(4)^2 - 2 = \boxed{46}$

iii. $f(-2) = 3(-2)^2 - 2 = \boxed{10}$

iv. $f(a) = \boxed{3a^2 - 2}$

v. $f(x+h) = 3(x+h)^2 - 2$ ← ok

$= 3x^2 + 6xh + 3h^2 - 2$ ← better

vi. $f(t^2+1) = 3(t^2+1)^2 - 2$ ← ok

$= 3t^4 + 6t^2 + 1$ ← better

vii. x-intercepts

$y=0$

$0 = 3x^2 - 2 \rightarrow x^2 = \frac{2}{3} \rightarrow x = \pm\sqrt{\frac{2}{3}} \rightarrow \boxed{(\pm\sqrt{\tfrac{2}{3}}, 0)}$ ok with me

viii. y-intercepts

$x=0$

$\boxed{(0,-2)}$

ix. the average value of $y = f(x)$ from $x = 1$ to $x = 3$

$\dfrac{f(3)-f(1)}{3-1} = \dfrac{(3(3)^2-2)-(3(1)^2-2)}{2}$

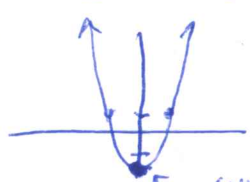
$= \dfrac{24}{2} = \boxed{12}$

x. Is f even/odd/both/neither?

f is even b/c

$f(-x) = 3(-x)^2 - 2 = 3x^2 - 2 = f(x)$

Graph:



(b) $g(x) = 2|x-1| - 4$

i. $g(0) = 2|0-1| - 4 = 2|-1| - 4$

$= 2 \cdot 1 - 4 = \boxed{-2}$

ii. $g(4) = 2|4-1| - 4 = 2|3| - 4 = 2\cdot 3 - 4 = \boxed{2}$

iii. $g(-2) = 2|-2-1| - 4 = 2|-3| - 4 = 2\cdot 3 - 4 = \boxed{2}$

iv. $g(a) = \boxed{2|a-1| - 4}$ ← This is it.

v. $g(x+h) = \boxed{2|x+h-1| - 4}$

vi. $g(r^2+1) = 2|r^2+1-1| - 4 = 2|r^2| - 4 = \boxed{2r^2 - 4}$

$r^2 \geq 0$ always

vii. x-intercepts

$0 = 2|x-1| - 4$

$2 = |x-1|$

$2 = x-1$ or $-2 = x-1$

$3 = x \qquad -1 = x$

$\boxed{(3,0), (-1,0)}$

viii. y-intercepts

$y = 2|0-1| - 4$

$\boxed{(0,-2)}$

ix. the average value of $y = g(x)$ from $x = 1$ to $x = 3$

$\dfrac{g(3)-g(1)}{3-1} = \dfrac{(2|3-1|-4)-(2(|1-1|)-4)}{2}$

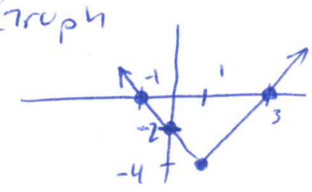
$= \dfrac{0-(-4)}{2} = \boxed{2}$

x. Is g even/odd/both/neither?

g is neither

$g(-x) = 2|-x-1| - 4$ ← not $g(x)$ and not $-g(x)$

Graph



Not symmetric across y-axis or 180° rotation about (0,0)

6. Is the relation $\{(2,3),(-1,3),(5,3)\}$ a function? What is the domain of the relation? What is the range of the relation?

$\boxed{\text{Yes}}$ this is a function — no two y's have the same x

$\boxed{\text{Domain: } \{2,-1,5\}}$ $\boxed{\text{Range: } \{3\}}$

x-values y-values

7. If $H(t)$ describes the height of a tree that is t years old, then what does the average rate of change of H from $t=1$ to $t=5$ represent?

The average rate of change represents the average number of feet the height of the tree changes by each year.

8. A company that makes thing-a-ma-bobs has a start up cost of $16936. It costs the company $1.54 to make each thing-a-ma-bob and the company charges $4.27 for each thing-a-ma-bob. Let x represent the number of thing-a-ma-bobs made.

per item price fixed per item cost

(a) Write a cost function for this company.

$\boxed{C(x) = 1.54x + 16936}$

(b) Write the revenue function for this company.

$\boxed{R(x) = 4.27x}$

(c) Write the profit function for this company.

$P(x) = R(x) - C(x) = 4.27x - (1.54x + 16936) = \boxed{2.73x - 16936 = P(x)}$

(d) What is the minimum number of thing-a-ma-bobs that the company must produce and sell to make a profit?

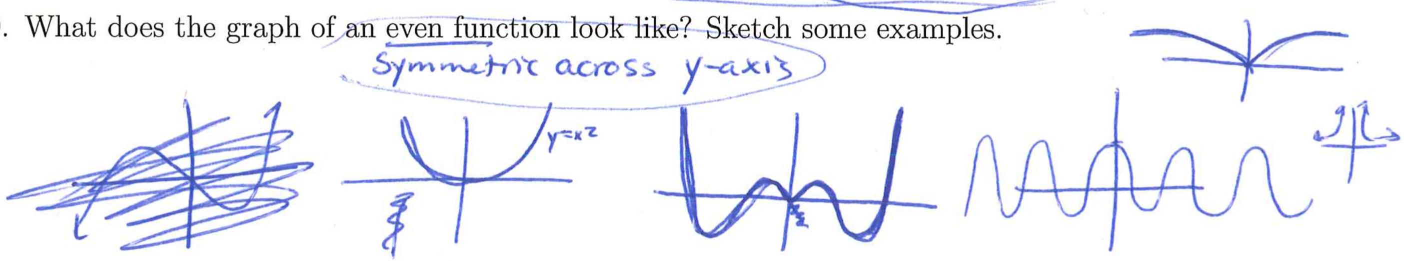
$2.73x - 16936 = 0 \rightarrow x = \frac{16936}{2.73} \approx 6203.663$

up

$\Longrightarrow$ $\boxed{\text{Sell at least } \underline{6204} \text{ units}}$

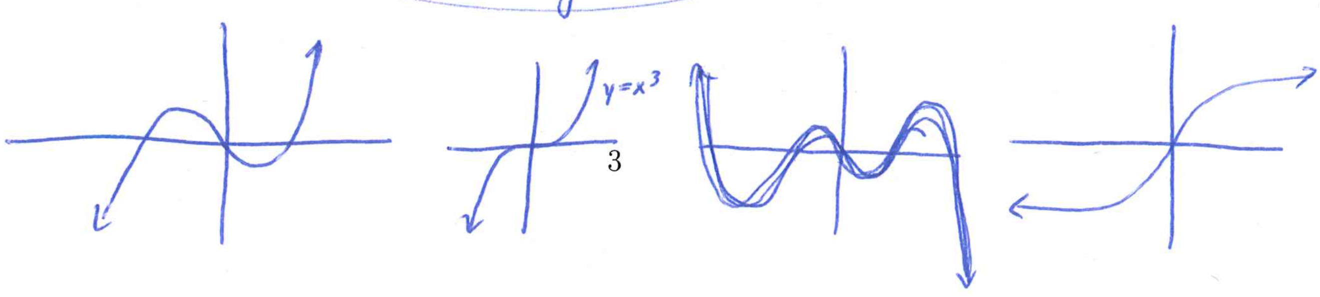
9. What does the graph of an even function look like? Sketch some examples.

Symmetric across y-axis

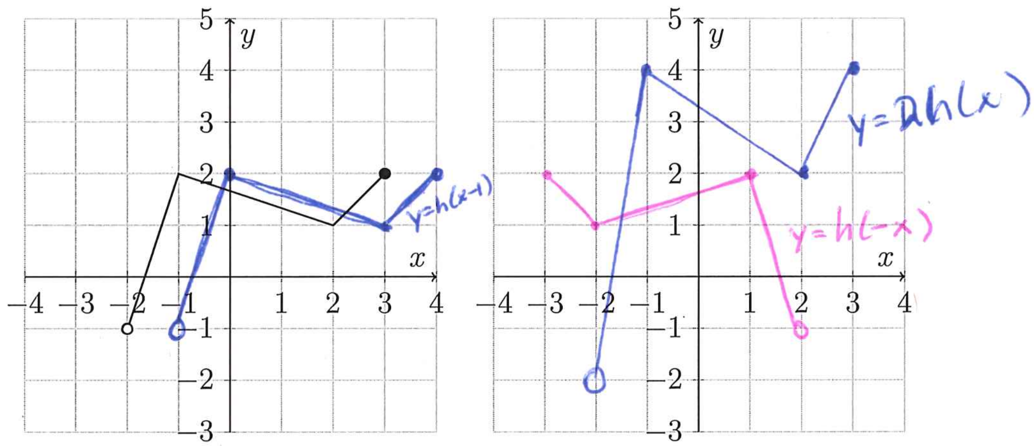


10. What does the graph of an odd function look like? Sketch some examples.

180° rotational symmetry about origin



11. Consider the graph of the function $h(x)$ on the left. The blank grid on the right is for you to possibly use in completing (d)-(f). (Up to you.)



(m) Local Max Values
When $x=-1$, we have a local max it is $\boxed{y=2}$

(n) Local Min Values
When $x=2$, we have a local min it is $\boxed{y=1}$

(a) Explain why the graph defines y as a function of x.
It passes the vertical line test.

(b) Determine $h(2)$.
When $x=2$, $\boxed{y=1}$

(c) Determine $h(-1)$.
$\boxed{h(-1)=2}$

(d) Sketch a graph of $y = h(x-1)$. on left graph (shift right 1unit)

(e) Sketch a graph of $y = 2h(x)$. on right graph (double y-vals)

(f) Sketch a graph of $y = h(-x)$. on right graph in pink reflect across y-axis

(g) What is the domain of $h(x)$?
x-vals $\boxed{(-2,3]}$

(h) What is the range of $h(x)$?
y-vals $\boxed{(-1,2]}$

(i) What are the $x-$ and $y-$intercepts of the graph?
x: $\boxed{(-\frac{5}{3},0)}$ y: $\boxed{(0,\frac{5}{3})}$

(j) On what intervals is the function increasing?
$\boxed{(-2,-1) \text{ and } (2,3)}$

(k) On what intervals is the function decreasing?
$\boxed{(-1,2)}$

descriptions of x-vals where y-vals are going up/down do not include endpoints

* (l) Write $h(x)$ as a piecewise function by finding equations for each of the three linear portions of the graph.

This one is hard maybe → don't spend tons of time → maybe focus on one line and on piecewise parts

Line 1: slope 3 y-int val: 5 $y=3x+5$ Interval $-2<x\leq -1$

Line 2: slope $\frac{-1}{3}$ y-int val: $\frac{5}{3}$ $y=\frac{-1}{3}x+\frac{5}{3}$ Interval $-1\leq x\leq 2$

Line 3: slope 1 y-intval -1 $y=x-1$ Interval $2\leq x\leq 3$

$$h(x)=\begin{cases} 3x+5 & \text{if } -2<x\leq -1 \\ -\frac{1}{3}x+\frac{5}{3} & \text{if } -1\leq x\leq 2 \\ x-1 & \text{if } 2\leq x\leq 3 \end{cases}$$

12. Given the piecwise function, evaluate the values.

$$g(x) = \begin{cases} x+2 & \text{if } x \leq -5 \\ |x+1|+2 & \text{if } -5 < x < 0 \\ \frac{1}{3x+2} & \text{if } 0 \leq x \leq 2 \\ x^2+x+1 & \text{if } x > 2 \end{cases}$$

(a) $g\cancel{f}(0) = \frac{1}{3\cdot 0+2} = \boxed{\frac{1}{2}}$

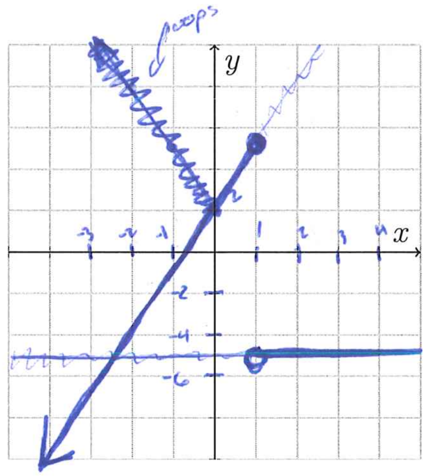
(b) $g\cancel{f}(-5) = -5+2 = \boxed{-3}$

(c) $g\cancel{f}(1) = \frac{1}{3\cdot 1+2} = \boxed{\frac{1}{5}}$

(d) $g\cancel{f}(10) = \boxed{111}$

13. Graph the piecewise function

$$r(x) = \begin{cases} 3x+2 & \text{if } x \leq 1 \\ -5 & \text{if } x > 1 \end{cases}$$



14. Let $f(x) = x^2+1$, $g(x) = |x-2|$, $h(x) = 4x-3$.

(a) Compute $g(h(2))$.

$h(2) = 4(2)-3 = 5$ $\quad g(h(2)) = g(5) = |5-2| = |3| = \boxed{3}$

(b) Compute $f \circ g(-3) = f(g(-3))$.

$g(-3) = |-3-2| = |-5| = 5$ $\quad f(g(-3)) = f(5) = 5^2+1 = \boxed{26}$

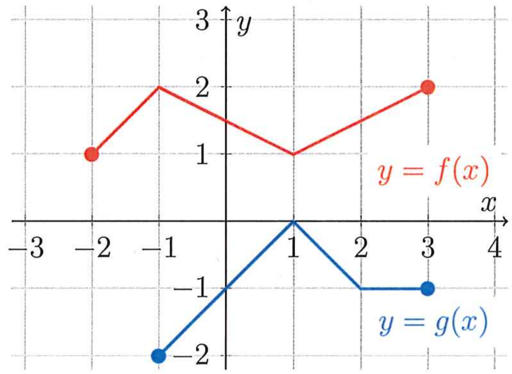
(c) Compute and simplify $f(h(x))$.

$f(h(x)) = f(4x-3) = (4x-3)^2+1 = 16x^2-24x+9+1 = \boxed{16x^2-24x+10}$

(d) Compute and simplify $h \circ h(x) = h(h(x))$.

$h(h(x)) = h(4x-3) = 4(4x-3)-3 = 16x-12-3 = \boxed{16x-15}$

15. Given the graphs below



(a) Compute $g(f(-2))$.

$f(-2)=1 \rightarrow g(f(-2))=g(1)=\boxed{0}$

(b) Compute $f(g(2)))$.

$g(2)=-1 \rightarrow f(g(2))=f(-1)=\boxed{2}$

(c) Compute $f \circ g(0)$).

$f(g(0))=f(-1)=\boxed{2}$

(d) Compute $g \circ f(0)$).

$g(f(0))=g(1.5)=\boxed{-0.5}$

(e) Compute $f \circ f(-1)$

$f(f(-1))=f(2)=\boxed{1.5}$

16. Let $H(x) = 4(x-2)^{10}$. Which of the following pairs of functions $f(x)$ and $g(x)$ will produce $f \circ g(x) = H(x)$? (There are two...) Can you find another decomposition?

- ~~$f(x) = 4x - 2$ and $g(x) = x^{10}$~~
- ~~$f(x) = x^{10}$ and $g(x) = 4x - 2$~~
- $f(x) = 4x^{10}$ and $g(x) = x - 2$
- ~~$f(x) = x - 2$ and $g(x) = 4x^{10}$~~
- $f(x) = 4x$ and $g(x) = (x-2)^{10}$
- ~~$f(x) = (x-2)^{10}$ and $g(x) = 4x$~~

$\boxed{f(x)=x^2 \quad g(x)=2(x-2)^5}$

$\boxed{f(x)=4x^5 \quad g(x)=(x-2)^2}$

⋮

17. For each Section in Chapter 2, write down the key terms and ideas.

(a) Section 2.1: The Rectangular Coordinate System

- distance between two points $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$
- midpoint $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$
- x-intercepts $(a,0)$
- y-intercepts $(0,b)$

(b) Section 2.2: Circles

- standard form center (h,k) radius r
  $(x-h)^2+(y-k)^2=r^2$
- general form
  $x^2+y^2+Ax+By+C=0$

(c) Section 2.3: Functions and Relations

don't have to write

- relation – set of ordered pairs
- domain – set of x-values
- range – set of y-values
- y is a function of x – every x has only one y
- vertical line test
- intercepts of functions
- domain of eqn – $\mathbb{R}$ except those that make denoms = 0 and negatives under even roots

(d) Section 2.4: Linear Equations in Two Variables and Linear Functions

- linear equation
- slope formula
- slope-intercept form
- average rate of change

(e) Section 2.5: Applications of Linear Equations and Modeling

- point-slope formula
- parallell
- perpendicular
- linear cost, revenue, profit functions

(f) Section 2.6: Transformations of Graphs

- vertical traslation
- horizontal translation
- vertical stretch/shrind
- horizontal stretch/shrink
- reflection
- Graphs of $y=x^2$, $y=x^3$, $y=|x|$, $y=\sqrt{x}$, $y=\sqrt[3]{x}$, $y=\frac{1}{x}$

(g) Section 2.7: Analyzing Graphs of Functions and Piecewise Defined Functions

- ~~x~~ axis + origin symmetry
- even/odd functions
- piecewise-defined functions
- intervals of inc/dec
- rel max/min

(h) Section 2.8: Algebra of Functions and Function Composition

- composition
- difference quotient