Allowed Steps. When Solving for Variables in Inequalities

- **Add** something to both sides.
- **Subtract** something from both sides.
- **Multiply** both sides by a **positive** number.
- **Divide** both sides by a **positive** number.
- **Multiply** both sides by a **negative** number and **reverse** the inequality.
- **Divide** both sides by a **negative** number and **reverse** the inequality.

$$\begin{array}{cc} < & \leq \\ \updownarrow & \updownarrow \\ > & \geq \end{array}$$

Not Allowed. When Solving for Variables in Inequalities
Multiply or divide both sides by an expression containing variables.

Notation Conversions.

Inequality	$\leftrightarrow$	Interval
$2 < x < 7$	$\leftrightarrow$	$(2, 7)$
$2 \leq x < 7$	$\leftrightarrow$	$[2, 7)$
$2 < x \leq 7$	$\leftrightarrow$	$(2, 7]$
$2 \leq x \leq 7$	$\leftrightarrow$	$[2, 7]$
$-3 \leq x$	$\leftrightarrow$	$[-3, \infty)$
$-3 < x$	$\leftrightarrow$	$(-3, \infty)$
$x < 5$	$\leftrightarrow$	$(-\infty, 5)$
$x \leq 5$	$\leftrightarrow$	$(-\infty, 5]$
$x \neq 5$	$\leftrightarrow$	$(-\infty, 5) \cup (5, \infty)$
$-3 \leq x < 2$ or $x > 4$	$\leftrightarrow$	$[-3, 2) \cup (4, \infty)$
all real numbers	$\leftrightarrow$	$(-\infty, \infty)$

Example. Solve $-6x + 4 < 34$ for x.

- Subtract 4 from each side:

$$-6x+4-4 < 34-4 \quad \Rightarrow \quad -6x < 30$$

- Divide both sides by -6 **and** reverse the inequality:

$$\frac{-6x}{-6} > \frac{-30}{-6} \quad \Rightarrow \quad x > -5$$

- Write in interval from:

$$x > -5 \quad \Rightarrow \quad (-5, \infty)$$

Example. Solve $5 < -2x + 7 \leq 11$ for x.

- Subtract 7 from each section:

$$5-7 < -2x+7-7 \leq 11-7 \quad \Rightarrow \quad -2 < -2x \leq 4$$

- Divide each section -2 **and** reverse the inequalities:

$$\frac{-2}{-2} > \frac{-2x}{-2} \geq \frac{4}{-2} \quad \Rightarrow \quad 1 > x \geq -2$$

- Write in interval from:

$$1 > x \geq -2 \quad \Rightarrow \quad [-2, 1)$$

Example. Solve $\frac{x+1}{3} - \frac{2x-4}{6} \leq -\frac{x}{2}$ for x.

- Clear the denominators by multiplying both sides by $\frac{6}{1}$:

$$\frac{6}{1}\left(\frac{x+1}{3} - \frac{2x-4}{6}\right) \leq \frac{6}{1}\left(-\frac{x}{2}\right) \quad \Rightarrow \quad \frac{2\cdot\cancel{3}}{1}\cdot\frac{x+1}{\cancel{3}} - \frac{\cancel{6}}{1}\cdot\frac{2x-4}{\cancel{6}} \leq \frac{\cancel{2}\cdot 3}{1}\cdot -\frac{x}{\cancel{2}}$$

$$\Rightarrow \quad 2(x+1) - (2x-4) \leq -3x$$

- Distribute the 2 and $-$:

$$2(x+1) - (2x-4) \leq -3x \quad \Rightarrow \quad 2x + 2 - 2x + 4 \leq -3x$$

- Simplify as much as possible before moving forward:

$$2x + 2 - 2x + 4 \leq -3x \quad \Rightarrow \quad 6 \leq -3x$$

- Divide both sides by -3 **and** reverse the inequality:

$$\frac{6}{-3} \geq \frac{-3x}{-3} \quad \Rightarrow \quad -2 \geq x$$

- Write in interval from:

$$-2 \geq x \quad \Rightarrow \quad (-\infty, -2]$$