Math 324/524 Homework 8
YOUR NAME
Due 11/6/2019

1. For each of the following, determine whether the set spans the given vector space. If not, what is the span? Explain and show your work.

   (a) $S = \{(3,4), (-1,-1), (2,5)\}$, $V = \mathbb{R}^2$

   (b) $S = \{(4,-3,6,2), (1,8,3,1), (3,-2,-1,0)\}$, $V = \mathbb{R}^4$

   (c) $S = \{x^2 + 3x + 1, 2x^2 + x - 1, -3x\}$, $V = P_2$

   (d) $S = \left\{ \begin{bmatrix} 2 & 0 \\ -3 & 1 \end{bmatrix}, \begin{bmatrix} -4 & -1 \\ 0 & 5 \end{bmatrix}, \begin{bmatrix} -8 & -3 \\ -6 & 17 \end{bmatrix} \right\}$, $V = M_{2,2}$

2. For each of the sets S in Exercise 1, determine whether the set is linearly independent. If it is linearly dependent, express one of the vectors as a sum of the others. Explain and show your work.

   (a) $S = \{(3,4), (-1,-1), (2,5)\}$, $V = \mathbb{R}^2$

   (b) $S = \{(4,-3,6,2), (1,8,3,1), (3,-2,-1,0)\}$, $V = \mathbb{R}^4$

   (c) $S = \{x^2 + 3x + 1, 2x^2 + x - 1, -3x\}$, $V = P_2$

   (d) $S = \left\{ \begin{bmatrix} 2 & 0 \\ -3 & 1 \end{bmatrix}, \begin{bmatrix} -4 & -1 \\ 0 & 5 \end{bmatrix}, \begin{bmatrix} -8 & -3 \\ -6 & 17 \end{bmatrix} \right\}$, $V = M_{2,2}$

3. Prove that if $S = \{\vec{u}, \vec{v}\}$ where $\vec{u}$ and $\vec{v}$ are vectors in $\mathbb{R}^2$ that are not scalar multiples of one another, then $\text{span}(S) = \mathbb{R}^2$.

   (Hint: Let $(x, y) \in \mathbb{R}^2$ be a generic vector, and find values for c_1 and c_2 such that $(x, y) = c_1\vec{u} + c_2\vec{v}$.)

   Proof. WORDSSSSSS $\square$

4. Let $S = \{\vec{u}, \vec{v}, \vec{w}\}$ be a linearly independent set. Prove that $T = \{\vec{u}, \vec{u} + \vec{v}, \vec{u} + \vec{v} + \vec{w}\}$ is also linearly independent.

   Proof. MORE WORDSSSSSS $\square$

5. Let $S = \{\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_k\}$ be a spanning set for a vector space V. Prove that if $\vec{v}_k$ can be written as a linear combination of $\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_{k-1}$, then $T = \{\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_{k-1}\}$ is also a spanning set for V.

   Proof. MORE MORE WORDSSSSSS $\square$