Math 324/524: Linear Algebra and Matrix Theory
Induction Example

Proposition For any positive integer n,

$$1 + 2 + \cdots + n = \frac{n(n+1)}{2}.$$

Proof. **Base Case:** If $n = 1$, then the LHS is $1 + 2 + \cdots + n = 1$ and the RHS is $\frac{n(n+1)}{2} = 1$.

Inductive Step: Assume the statement is true for $n = k$. That is $1+2+\cdots+k = \frac{k(k+1)}{2}$. Consider now the case $n = k + 1$. Then the LHS is

$$\begin{aligned} 1 + 2 + \cdots + k + (k+1) &= \tfrac{k(k+1)}{2} + (k+1) && \text{inductive hypothesis} \\ &= \tfrac{k(k+1)+2(k+1)}{2} && \text{common denominator} \\ &= \tfrac{(k+1)(k+2)}{2} && \text{common factor} \end{aligned}$$

On the other hand, the RHS is $\frac{(k+1)((k+1)+1)}{2} = \frac{(k+1)(k+2)}{2}$ which is exactly the LHS. Therefore the equality holds when $n = k + 1$ and the claim is true by induction. $\square$