

# Math 324: Linear Algebra

## Contradiction

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**Last Time.**

- If and only If Statements
- Contrapositive, Converse, and Inverse

**Today.**

- Proof by contradiction

### Proposition.

If  $x$  is a real number in  $[0, \pi/2]$  then  $\sin x + \cos x \geq 1$ .

### Exercise 1.

Complete the proof in the handout.

The general format of proof by contradiction for a statement of the form  $P \Rightarrow Q$  is as follows:

1. Assume  $P$  is true.
2. Suppose **toward contradiction**, that  $Q$  is false.  
Yes, including “toward contradiction” is important, don't be lazy.
3. Derive any contradiction.
4. Conclusion: Therefore is impossible for both  $P$  to be true and  $Q$  to be false, so the claim is true.  
Yes, write this line too, don't be lazy.

**Exercise 2.**

Use proof by contradiction to prove:

If  $a$  and  $b$  are integers then  $a^2 - 4b \neq 2$ .

You may use the fact that if  $a^2$  is even then  $a$  is even.

**Exercise 3.**

Use proof by contradiction to prove:

If  $a$ ,  $b$  and  $c$  are integers satisfying  $a^2 + b^2 = c^2$ , then  $a$  or  $b$  is even.

### Exercise 4.

Often for contradiction proofs, we may be negating statements of the form  $\neg(A \text{ or } B)$ , which is logically equivalent to  $\neg A \text{ and } \neg B$ . Consider the statements  $A = \text{my cat is not orange}$  and  $B = \text{my dog has whiskers}$ .

- (a) In an English sentence, how would you write  $\neg(A \text{ or } B)$ ?
- (b) How would you write  $\neg A$  and  $\neg B$ ?
- (c) Why are these logically equivalent.

## Exercise 5.

### Claim.

If  $AB$  is singular then  $A$  is singular or  $B$  is singular.

- (a) Represent this statement as  $P \Rightarrow Q$ . What is  $P$ ?  $Q$ ?  $\neg Q$ ?
- (b) What are the first two sentences of your contradiction proof of this claim?
- (c) Find a contradiction. You may want to consider citing Theorem 2.9.
- (d) Conclude the proof.

## Exercise 6.

### Claim.

There are no positive integers  $a$  and  $b$  such that  $18a + 6b = 1$ .

- (a) Write this statement as: if  $P$  then  $18a + 6b \neq 1$ .
- (b) What are the first two sentences of your contradiction proof of this claim?
- (c) Find a contradiction.
- (d) Conclude the proof.

## Exercise 7.

### Claim.

The sum of a rational number and an irrational number is irrational.

- (a) Write this statement as: if  $P$  then  $Q$ .
- (b) What is  $P$ ?  $Q$ ?  $\neg Q$ ?
- (c) What are the first two sentences of your contradiction proof of this claim?
- (d) Find a contradiction.
- (e) Conclude the proof.