Math 324/524 Homework 10 - Computational Exercises
YOUR NAME
Due 12/6/2019

1. Section 6.1, Exercise 28
   Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation such that

   $$T(1,0,0) = (2,4,-1),\ T(0,1,0) =\ (1,3,-2),\ \text{and } T(0,0,1) = (0,-2,2).$$

   Find $T(-2,4,-1)$.

2. Section 6.1 Exercise 81
   Let $T : M_{n,n} \to \mathbb{R}$ be the transformation defined by $T(A) = a_{11} + a_{22} + \cdots + a_{nn}$, the sum of the diagonal entries of A—called the *trace* of A. Carefully verify that T is a linear transformation.

   (No this is not one of the **proofs** that you need to type. Though if you're handwriting the solution, be precise.)

3. Consider the linear transformation $T : \mathbb{R}^4 \to \mathbb{R}^3$ represented by $T(\vec{x}) = A\vec{x}$, where $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 3 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$.

(a) What is $T(3, 1, 0, 0)$?

(b) What is $\ker(T)$ what is $\text{range}(T)$? (Give a basis for each.)

(c) What are $\text{nullity}(T)$ and $\text{rank}(T)$?

(d) Is T one-to-one? Explain.

(e) Is T onto? Explain.

(f) Is T an isomorphism? Explain.

4. Section 6.3 Exercise 22
   Find the standard matrix for the linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ that is the reflection through the vector $\vec{w} = (3, 1)$.

   A formula for this is given by $T(\vec{v}) = 2\,\mathrm{proj}_{\vec{w}}\,\vec{v} - \vec{v}$.

5. Section 6.3 Exercise 43
   Let $T : P_2 \to P_3$ be the linear transformation given by $T(p) = xp$. Find the matrix for T relative to the bases $B = \{1, x, x^2\}$ and $B' = \{1, x, x^2, x^3\}$.

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1. **Section 6.1 Exercise 79**
   Let $S = \{\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_n\}$ be a set of linearly dependent vectors in V, and let $T : V \to W$ be a linear transformation. Prove that $S' = \{T(\vec{v}_1), T(\vec{v}_2), \ldots, T(\vec{v}_n)\}$ is a linearly dependent set of vectors in W.

   Proof. Yay linear algebra! □

2. **Section 6.2 Exercise 64**
   Let $T : V \to W$ be a linear transformation. Prove that T is one-to-one if and only if $\text{rank}(T) = \dim(V)$.

   Proof. I like vectors and vectors like me. □