Math 324: Linear Algebra

Direct Proofs

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Last Time.

- Statements
- Exploration and Counterexamples
- Quantifiers
- Conditional Statements
- Compound Statements
- Negation

Today.

- Proper Proof Techniques
- Direct Proofs

Exercise 1.

- Read the bolded summaries on the handout, focus on those highlighted here.
- (1) For this class, who is your audience?
- (3) What is the purpose of stating assumptions?
- (4) Why are proofs written using "we" instead of *I*?
- (7) What's the difference between writing a proof and computing an answer?
- (9) Notice that even displayed equations end in a period because they should be considered a sentence.
- (11) No sentence can begin with a mathematical symbol. Why?
- (15) Can we just write something and walk away?

Exercise 2.
Examine your pre-class proof in the context of the handout. What guidelines does it follow? Does it break any of the guidelines? Share your thoughts with your table-mates.

Exercise 3.

Re-write the following "proof" of the claim that follows so that the proof is correct and matches the guidelines from the handout.

Claim.

If the product AB is a square matrix, then the product BA is defined.

Proof.

AB is square so it is size $n \times n$. This means A has n columns and B has n columns. I can now multiply BA. $\square$

Brain Break.
What food are you always running out of at your house or dorm?

Exercise 4.

The proof of the following claim is long, wordy, and hard to understand. Re-write a better proof.

Claim.

If the products AB and BA are both defined then AB is a square matrix.

Proof.

Let A and B be matrices of size $m \times n$ and $r \times s$. Assume that both of the products AB and BA are defined. Since AB is defined the number of columns of A must equal the number of rows of B. Since BA is defined the number of columns of B must equal the number of rows of A. This means that n equals r and m equals s. Since A is size $m \times n$ and B is size $r \times s$, AB is size $m \times s$. But m equals s so AB is square because the number of rows of AB is equal to the number of rows of A which is m and the number of columns of AB is equal to the number of columns of B which is s which is equal to m, the number of rows of AB. $\square$

Definition.

You may have seen previously that if A is a square matrix satisfying $A^2 = A$, then we call A `idempotent`.

Exercise 5.

Follow the steps to prove that if A and B are idempotent matrices satisfying $AB = BA$, then AB is also idempotent.

(a) Start with some scratch work, you want to show $(AB)^2 = AB$. Try multiplying, where do you use the fact that $AB = BA$?

(b) Write down your assumptions.

(c) State what it means for A and B to be idempotent.

(d) Write down a sentence stating what you want to show.

(e) Go through the algebraic steps. Note: You may want to display them as in guideline (9).

(f) Write a concluding sentence as suggested in guideline (13).