Math 324: Linear Algebra

Induction

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Last Time.

- Proof by contradiction

Today.

- Proof by Induction

Exercise 1.

In the video, the speaker talked about what induction is and gave some examples. Here is one.

Proposition.

For any positive integer n, $1 + 2 + \cdots + n = \frac{n(n+1)}{2}$.

Re-read the proof and discuss:

(a) Why does this prove that $1 + 2 + 3 = \frac{3(4)}{2} = 6$?

(b) Why does this prove that $1 + 2 + \cdots + 10 = \frac{10(11)}{2} = 55$?

(c) Why does this prove the statement for $n = 10000$?

(d) What pieces of the proof are essential for induction proofs?

(e) Share your proof from your pre-class exercise.

Exercise 2.

Use proof by induction to prove the statement that follows.

Proposition.

Let $n \geq 1$ be an integer. If A is an $m \times m$ matrix and P is a $m \times m$ invertible matrix, then $(P^{-1}AP)^n = P^{-1}A^nP$.

Exercise 3.

Use proof by induction to prove the statement that follows.

Proposition.

For all integers $n \geq 0$, if $B = \begin{bmatrix} 2 & 0 \\ -1 & 1 \end{bmatrix}$, then $B^n = \begin{bmatrix} 2^n & 0 \\ 1 - 2^n & 1 \end{bmatrix}$.

Exercise 4.

Use induction and Theorem 2.9 to prove the following.

Proposition.

Let $n \geq 2$ be a positive integer. If $A_1, A_2, \ldots, A_n$ are invertible matrices of the same size, then

$$(A_1 A_2 \cdots A_n)^{-1} = A_n^{-1} \cdots A_2^{-1} A_1^{-1}.$$

Exercise 5.

Use induction and Theorem 2.3 Property 2 to prove the following.

Proposition.

Let $n \geq 2$ be a positive integer. If A is an $m \times r$ matrix and $B_1, B_2, \ldots, B_n$ are $r \times s$ matrices, then

$$A(B_1 + B_2 + \cdots + B_n) = AB_1 + AB_2 + \cdots + AB_n.$$

Exercise 6.

Use induction to prove the following.

Proposition.

If n is a positive integer, then

$$1(2) + 2(3) + 3(4) + \cdots + n(n+1) = \frac{n(n+1)(n+2)}{3}.$$

Exercise 7.

Use induction to prove the following.

Proposition.

If $n \geq 4$, then $n! > 2^n$.