Math 324/524: Linear Algebra and Matrix Theory
Induction Example

Proposition For all positive integers n,

$$1 + 2 + \cdots + n = \frac{n(n+1)}{2}.$$

Proof.
Base Case: If $n = 1$, then the LHS is $1 + 2 + \cdots + n = 1$ and the RHS is

$$\frac{n(n+1)}{2} = \frac{1(1+1)}{2} = 1.$$

Since the LHS and RHS are equal, the statement is true for $n = 1$.

Inductive Step: Assume the statement is true for $n = k$. That is, assume

$$1 + 2 + \cdots + k = \frac{k(k+1)}{2}.$$

We want to show that the statement is true for $n = k + 1$. In particular, we want to show

$$1 + 2 + \cdots + k + (k+1) = \frac{(k+1)(k+2)}{2}. \tag{1}$$

Consider the LHS of (1):

$$\begin{aligned}
1 + 2 + \cdots + k + (k+1) &= \frac{k(k+1)}{2} + (k+1), && \textit{inductive hypothesis} \\
&= \frac{k(k+1) + 2(k+1)}{2}, && \textit{common denominator} \\
&= \frac{(k+1)(k+2)}{2}, && \textit{common factor} \\
&= \text{RHS of (1)}.
\end{aligned}$$

Therefore the equality holds when $n = k + 1$ and the claim is true by induction. $\square$