Math 425 Objective Intro-1 Exercises

Purpose

- This document is intended to provide additional opportunities to complete the objective
  Intro-1: Prove a map is or is not one-to-one/onto/bijective.

Task

- If you have not yet earned a **Satisfactory** or **Exceptional** mark on an exercise labeled with the objective here, you may submit a single one of the following exercises, that you have not yet attempted via Canvas by 4pm on any following Wednesday.
- I strongly recommend you use LaTeX to typeset your proofs.
- You may work in groups but everyone should submit their own assignment written in their own words. Do NOT copy your classmates.
- Allowed resources: our textbook, classmates, your notes, videos linked in Canvas.
- Unacceptable resources: anything you find on an internet search. Do NOT use a homework help website (e.g., Chegg). Their solutions are often wrong or use incorrect context. I want you to practice making arguments that are yours. Take some ownership.

Criteria

All items will earn a score using the following scale:

- **Exceptional** - Solution is succinct, references the correct theorems and definitions, and is entirely correct.
- **Satisfactory** - Solution is nearly correct. It still references the correct theorems and definitions. It may be longer than necessary, have minor errors, or have some grammatical mistakes.
- **Unsatisfactory** - Solution has major errors, references content not covered in class or in the textbook, or is incomplete in some major way.

Recall from the syllabus

- If you earn either an **Exceptional** or **Satisfactory** mark on an objective exercise (labeled Intro-, Group-, or Ring-) then you may consider that item complete.
- If you earn an **Unsatisfactory** mark on an objective exercise (labeled Intro-, Group-, or Ring-) then you have not yet completed this objective.
- You may submit a new attempt at completing that objective on a future Wednesday. You must select a new exercise listed under the given objective, you cannot resubmit a version you have attempted previously. The only limit you have on number of attempts is the number of exercises available for the objective.

Name: ____________________________________

(Intro-1.2) Let A and B be nonempty sets with a mapping $\alpha\colon A \to B$. Assume that there is some map $\beta : B \to A$ such that $\alpha\beta = 1_B$. Prove that α is onto.

Name: ________________________________

(Intro-1.3) Let A and B be nonempty sets with a mapping $\alpha\colon A \to B$. Assume that there is some map $\beta : B \to A$ such that $\beta\alpha = 1_A$. Prove that α is one-to-one.

Name: ____

(Intro-1.4) Let $A = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \mid a, b, c, d \in \mathbb{R} \right\}$. Define $\phi \colon \mathbb{C} \to A$ by

$$\phi(a + bi) = \begin{bmatrix} a + bi & 0 \\ 0 & a - bi \end{bmatrix}.$$

Prove that ϕ is one-to-one but not onto.

Name: ______________________________

(Intro-1.5) Let A be a non-empty set. Define $\sigma : A \to A \times A$ by $\sigma(a) = (a, a)$. Is σ one-to-one? onto? a bijection? You must prove each of your answers.

Name: ___________________________________

(Intro-1.6) Write down a solution with proof for all three parts.
Define a map $\tau\colon 2\mathbb{Z} \to 5\mathbb{Z}$ such that.

1. τ is one-to-one but not onto.

2. τ is onto but not one-to-one.

3. τ is a bijection.