Math 425 Objective Intro-2 Exercises

Purpose

- This document is intended to provide additional opportunities to complete the objective

  Intro-2: Prove that a relation is an equivalence relation then compute the set of unique equivalence classes.

Task

- If you have not yet earned a **Satisfactory** or **Exceptional** mark on an exercise labeled with the objective here, you may submit a single one of the following exercises, that you have not yet attempted via Canvas by 4pm on any following Wednesday.
- I strongly recommend you use LaTeX to typeset your proofs.
- You may work in groups but everyone should submit their own assignment written in their own words. Do NOT copy your classmates.
- Allowed resources: our textbook, classmates, your notes, videos linked in Canvas.
- Unacceptable resources: anything you find on an internet search. Do NOT use a homework help website (e.g., Chegg). Their solutions are often wrong or use incorrect context. I want you to practice making arguments that are yours. Take some ownership.

Criteria

All items will earn a score using the following scale:

- **Exceptional** - Solution is succinct, references the correct theorems and definitions, and is entirely correct.
- **Satisfactory** - Solution is nearly correct. It still references the correct theorems and definitions. It may be longer than necessary, have minor errors, or have some grammatical mistakes.
- **Unsatisfactory** - Solution has major errors, references content not covered in class or in the textbook, or is incomplete in some major way.

Recall from the syllabus

- If you earn either an **Exceptional** or **Satisfactory** mark on an objective exercise (labeled Intro-, Group-, or Ring-) then you may consider that item complete.
- If you earn an **Unsatisfactory** mark on an objective exercise (labeled Intro-, Group-, or Ring-) then you have not yet completed this objective.
- You may submit a new attempt at completing that objective on a future Wednesday. You must select a new exercise listed under the given objective, you cannot resubmit a version you have attempted previously. The only limit you have on number of attempts is the number of exercises available for the objective.

Name: _____

(Intro-2.2) Let $\sim$ on $\mathbb{Z}$ be defined by $a \sim b$ if and only if $a^2 - b^2$ is divisible by 4. Prove that $\sim$ is an equivalence relation and describe the set of unique equivalence classes.

Name: _____

(Intro-2.3) Let $S = \{(a, b) \in \mathbb{Z} \times \mathbb{Z} \mid b \neq 0\}$ Let $\equiv$ be the relation on S defined by $(a, b) \equiv (c, d)$ if $ad = bc$. Prove that $\equiv$ is an equivalence relation and describe the set of unique equivalence classes.

Name: _____

(Intro-2.4) Let A be the set of all people in the world. We define the relation $\sim$ on A by $a \sim b$ if and only if a and b were born in the same month of the year. Prove that $\sim$ is an equivalence relation and describe the set of unique equivalence classes.

Name: _____

(Intro-2.5) Let $\mathcal{L} = \{a + bx \mid a, b \in \mathbb{R}\}$ be the set of linear and constant polynomials over $\mathbb{R}$. You can also think of $\mathcal{L}$ as the set of all lines in $\mathbb{R}^2$. For $\ell_1, \ell_2 \in \mathcal{L}$ we will say $\ell_1 \equiv \ell_2$ if ℓ_1 and ℓ_2 have the same slope. Prove that $\equiv$ is an equivalence relation and describe the set of unique equivalence classes.

Name: _____

(Intro-2.6) Let $\sim$ on $\mathbb{Q}$ be defined by $a \sim b$ if and only if $a - b \in \mathbb{Z}$. Prove that $\sim$ is an equivalence relation and describe the set of unique equivalence classes.