Math 425: Abstract Algebra 1

Introduction
and
Section 0.1: Proofs

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Last Time.

– Hmmm....

Today.

– Course Introduction
– Direct Proofs
– Cases
– Contrapositive
– Contradiction
– Counterexamples

Exercise 1 (3 Minutes).

At your pods do the following:

(a) Introduce yourself, share your name and any relevant information, such as your preferred toothpaste brand.

(b) Decide on one "class time rule", something that will help strengthen the group's learning experience.

(c) Decide who is going to report back to the group.

Syllabus stuff! Please take time to read it.

Key Notes.

- Group work hours: Wed 10-11 and Fri 11-12 (HHH 311)
- Office Hours: Tu 2-3 (HHH 524/507)
- Welcoming environment
- Grading - Specs!
    - Objective Exercises
    - Supplemental Exercises
    - Project
- Collaboration
- How to use the internet

Objective Exercises.

- 27 Objectives, categorized by topic: Intro, Group, Ring
- About 2 assigned per week, due on Fridays
- Grading scale: Unsatisfactory/Satisfactory/Exceptional
- Credit earned for both S and E marks
- May retry objectives: Must complete a `new` exercise listed for the objective, rewrites due Wednesdays
- Your grade: number of objectives complete/22 (max 100%)

Supplemental Exercises.

- About 2 assigned per week.
- Grading scale: Unsatisfactory/Satisfactory/Exceptional
- 1% earned for E marks
- 0.5% earned for S marks
- Maximum of 15% can be earned in this way
- May retry Supplemental Exercises: Must rewrite the `same` exercise, rewrites due Wednesdays

Benefits.

- Flexibility
- Room for growth and learning
- Focus on the main content
- More time for me to answer your questions
- Faster grading turnaround

Section 0.1: Proofs

Method of Direct Proof.

To prove a statement "if P, then Q", we assume P is true and show that Q must then also be true.

1. Express your statement in the form

$$\forall x \in D, \text{ if } P(x), \text{ then } Q(x).$$

2. Start the proof by letting x be an arbitrary element of D. Assume x satisfies the condition $P(x)$.

3. Show that $Q(x)$ is true.

Example.

Prove that the sum of any two even integers is even.

1. For all

Example.

Prove that the sum of any two even integers is even.

1. For all integers x and y, if x and y are even, then $x + y$ is even.

Proof.

Let x and y be integers. Assume x and y are even.

Method of Reduction to Cases.

Example.

Prove that if n is an integer, then $5n^2 + n - 72$ is even.

Proof.

Let $n \in \mathbb{Z}$. We consider two cases: n is even or n is odd.

Method of Reduction to Cases.

Example.

Prove that if n is an integer, then $5n^2 + n - 72$ is even.

Proof.

Let $n \in \mathbb{Z}$. We consider two cases: n is even or n is odd.

Case 1. Assume n is even. Then there is a $s \in \mathbb{Z}$ such that $n = 2s$. Compute

$$\begin{aligned} 5n^2 + n - 72 &= 5(2s)^2 + (2s) - 72 \\ &= 5(4s^2) + 2s - 72 \\ &= 2(10s^2 + s - 36) \end{aligned}$$

Since s is an integer, so is $10s^2 + s - 36$. Thus $5n^2 + n - 72 = 2(10s^2 + s - 36)$ is even by definition whenever $n = 2s$ is even.

Case 2. Assume now that n is odd. Then there is a $s \in \mathbb{Z}$ such that $n = 2s + 1$. Compute

$$
\begin{aligned}
5n^2 + n - 72 &= 5(2s+1)^2 + (2s+1) - 72 \\
&= 5(4s^2 + 4s + 1) + 2s - 71 \\
&= 20s^2 + 20s + 5 + 2s - 71 \\
&= 2(10s^2 + 11s - 33)
\end{aligned}
$$

Since s is an integer, so is $10s^2 + 11s - 33$. Thus $5n^2 + n - 72 = 2(10s^2 + 11s - 33)$ is by definition even when $n = 2s + 1$ is odd. $\square$