Math 425 Assignment 1

Purpose

- Assess objectives and additional exercises from sections 0.1-0.3, namely
  - Intro-1: Prove a map is or is not one-to-one/onto/bijective.
  - Supplemental exercises Supp-1, Supp-2, and Supp-3.
- Build a strong foundation of creative critical thinking and proof-writing techniques.

Task

- Complete the exercises listed on the following page(s) of this document and submit your solutions as a pdf to Canvas.
- I strongly recommend you use LaTeX to typeset your proofs.
- You may work in groups but everyone should submit their own assignment written in their own words. Do NOT copy your classmates.
- Allowed resources: our textbook, classmates, your notes, videos linked in Canvas.
- Unacceptable resources: anything you find on an internet search. Do NOT use a homework help website (e.g., Chegg). Their solutions are often wrong or use incorrect context. I want you to practice making arguments that are yours. Take some ownership.

Criteria

All items will earn a score using the following scale:

- **Exceptional** - Solution is succinct, references the correct theorems and definitions, and is entirely correct.
- **Satisfactory** - Solution is nearly correct. It still references the correct theorems and definitions. It may be longer than necessary, have minor errors, or have some grammatical mistakes.
- **Unsatisfactory** - Solution has major errors, references content not covered in class or in the textbook, or is incomplete in some major way.

Recall from the syllabus

- If you earn either an **Exceptional** or **Satisfactory** mark on an objective exercise (labeled Intro-, Group-, or Ring-) then you may consider that item complete.
- If you earn an **Unsatisfactory** mark on a an objective exercise (labeled Intro-, Group-, or Ring-) then you have not yet completed this objective.

- You may submit a new attempt at completing that objective on a future Wednesday. You must select a new exercise listed under the given objective, you cannot resubmit a version you have attempted previously. The only limit you have on number of attempts is the number of exercises available for the objective.
- If you earn an **Exceptional** mark on an additional exercise (labeled A-) then you will earn one point toward the fifteen total points in that section of your overall grade. You can consider the exercise complete.
- If you earn an **Satisfactory** mark on an additional exercise (labeled Supp-) then you will earn 0.5 points toward the fifteen total points in that section of your overall grade. You may submit a new attempt at this exercise on a future Wednesday. Unlike the objective exercises, you must submit an attempt at the exact same exercise rather than another in a similar theme.
- If you earn an **Unsatisfactory** mark on an additional exercise (labeled Supp-) then you will earn 0 points toward the fifteen total points in that section of your overall grade. You may submit a new attempt at this exercise on a future Wednesday. Unlike the objective exercises, you must submit an attempt at the exact same exercise rather than another in a similar theme.

Name: _____

(Intro-1.1) Let $A \xrightarrow{\alpha} B \xrightarrow{\beta} C$ be mappings. If $\beta\alpha$ is one-to-one and α is onto, show that β is one-to-one.

Name: _____

(Supp-1) Prove that $A \times (B \cap C) = (A \times B) \cap (A \times C)$.
Note: You must use the principle of set equality and the definitions of $\times$ and $\cap$.

Name: _____

(Supp-2) Prove the result by splitting into cases, or give a counterexample.
If n is any odd integer, then $n^2 = 8k + 1$ for some integer k. (Hint: Start by testing several n values to determine whether or not the statement is true.)

Name: _____

(Supp-3) Use proof by contradiction to show that if x and y are positive real numbers, then

$$\sqrt{x+y} \neq \sqrt{x} + \sqrt{y}.$$