Math 425 Assignment 2

Purpose

- Assess objectives and additional exercises from sections 0.4 and 1.1, namely
  - Intro-2: Prove that a relation is an equivalence relation then compute the set of unique equivalence classes.
  - Intro-3: Prove a property of non-negative integers using induction.
  - Supplemental exercises Supp-4 and Supp-5.
- Build a strong foundation of creative critical thinking and proof-writing techniques.

Task

- Complete the exercises listed on the following page(s) of this document and submit your solutions as a pdf to Canvas.
- I strongly recommend you use LaTeX to typeset your proofs.
- You may work in groups but everyone should submit their own assignment written in their own words. Do NOT copy your classmates.
- Allowed resources: our textbook, classmates, your notes, videos linked in Canvas.
- Unacceptable resources: anything you find on an internet search. Do NOT use a homework help website (e.g., Chegg). Their solutions are often wrong or use incorrect context. I want you to practice making arguments that are yours. Take some ownership.

Criteria

All items will earn a score using the following scale:

- **Exceptional** - Solution is succinct, references the correct theorems and definitions, and is entirely correct.
- **Satisfactory** - Solution is nearly correct. It still references the correct theorems and definitions. It may be longer than necessary, have minor errors, or have some grammatical mistakes.
- **Unsatisfactory** - Solution has major errors, references content not covered in class or in the textbook, or is incomplete in some major way.

Recall from the syllabus

- If you earn either an **Exceptional** or **Satisfactory** mark on an objective exercise (labeled Intro-, Group-, or Ring-) then you may consider that item complete.

- If you earn an **Unsatisfactory** mark on a an objective exercise (labeled Intro-, Group-, or Ring-) then you have not yet completed this objective.
- You may submit a new attempt at completing that objective on a future Wednesday. You must select a new exercise listed under the given objective, you cannot resubmit a version you have attempted previously. The only limit you have on number of attempts is the number of exercises available for the objective.
- If you earn an **Exceptional** mark on an additional exercise (labeled A-) then you will earn one point toward the fifteen total points in that section of your overall grade. You can consider the exercise complete.
- If you earn an **Satisfactory** mark on an additional exercise (labeled Supp-) then you will earn 0.5 points toward the fifteen total points in that section of your overall grade. You may submit a new attempt at this exercise on a future Wednesday. Unlike the objective exercises, you must submit an attempt at the exact same exercise rather than another in a similar theme.
- If you earn an **Unsatisfactory** mark on an additional exercise (labeled Supp-) then you will earn 0 points toward the fifteen total points in that section of your overall grade. You may submit a new attempt at this exercise on a future Wednesday. Unlike the objective exercises, you must submit an attempt at the exact same exercise rather than another in a similar theme.

Name: ______________________________

(Intro-2.1) Let $U = \{1, 3, 5\}$ and $A = U \times U$. Define $\equiv$ on A by $(a, b) \equiv (c, d)$ if and only if $a + b = c + d$.

1. How many elements are in A and what are the?

2. Prove that $\equiv$ is an equivalence relation on A.

3. Describe the set of unique equivalence classes of A given by $\equiv$.

Name: ______________________________________

(Intro-3.1)

1. Find the smallest positive integer N such that $3^N < N!$ holds.

2. Use induction to prove that for all integers $n \geq N$, the inequality $3^n < n!$ holds.

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(Supp-4) Let $\alpha\colon A \to B$ be a mapping of sets. Define a relation on A by $a_1 \equiv a_2$ if and only if $\alpha(a_1) = \alpha(a_2)$. This is in fact an equivalence relation. Thus there is a set of equivalence classes we will denote by $A_{\equiv}$.
Define $\sigma\colon A_{\equiv} \to B$ by $\sigma([a]) = \alpha(a)$.

1. Show that σ is well-defined. That is, show that if $[a_1] = [a_2]$, then $\sigma([a_1]) = \sigma([a_2])$.

2. Show that σ is one-to-one.

3. Show that if α is onto then σ is also onto.

Name: ____________________________________

(Supp-5) Let p_n denote the statement "$4n - 1$ is divisible by 4".

1. Show that $p_k \Rightarrow p_{k+1}$ for all $k \geq 1$.

2. Show that p_n is in fact false for all n.

3. Why do part (a) and part (b) not contradict one another?