Previously.

- Relations
- Equivalence Relations
- Equivalence Classes

This Section.

- Induction

Principle of Mathematical Induction. Let $P(n)$ be a statement for each integer $n \geq m$. Suppose the following conditions are satisfied,

1. $P(m)$ is true, and

2. $P(k) \Rightarrow P(k+1)$ for every $k \geq m$.

Then $P(n)$ is true for every $n \geq m$.

Example. Claim: For all $n \geq 4$,

$$2^n < n!.$$

Example. Show that $3^{3n} + 1$ is a multiple of 7 for all odd $n \geq 1$.

Induction in Algebra.. We assume the distributive property $a(b+c) = ab + ac$.
Prove that this property can be extended, that is show that for all integers $n \geq 2$, for all $a, b_1, \ldots, b_n \in \mathbb{Z}$, we have

$$a(b_1 + b_2 + \cdots + b_n) = ab_1 + ab_2 + \cdots + ab_n.$$

Exercise 1. Consider the statement, for all $n \geq 1$,

$$n^3 + (n+1)^3 + (n+2)^3$$

is a multiple of 9.
Use Induction to prove this statement.

(a) What is the base case?

(b) What is the inductive hypothesis?

(c) Use the inductive hypothesis to complete the proof.

Hint: Do NOT do any expanding of the terms $(k+1)^3$ and $(k+2)^3$.

Exercise 2. Let $a_n = 2^{3n} - 1$ for $n \geq 0$. Guess a common divisor for each a_n and prove your assertion. (The common divisor should be greater than 1.)

(a) Use a calculator or math software co compute a_1, a_2, a_3, a_4. What common divisor do you notice?

(b) Use induction to prove your claim.

Another Induction Principle. Let $P(n)$ be a statement for each integer $n \geq m$. Suppose the following conditions are satisfied,

1. $P(m)$ and $P(m+1)$ are true, and

2. If $k \geq m$ and both $P(k)$ and $P(k+1)$ are true then $P(k+2)$ is true.

Then $P(n)$ is true for every $n \geq m$.

Exercise 3. Let a_n denote a number for each integer $n \geq 0$ and assume that $a_{n+2} = a_{n+1} + 2a_n$ holds for every $n \geq 0$.
Show that if $a_0 = 1$ and $a_2 = 2$, then $a_n = 2^n$ for each $n \geq 0$.