Math 425 Objective Intro-3 Exercises

Purpose

- This document is intended to provide additional opportunities to complete the objective **Intro-3:** Prove a property of non-negative integers using induction.

Task

- If you have not yet earned a **Satisfactory** or **Exceptional** mark on an exercise labeled with the objective here, you may submit a single one of the following exercises, that you have not yet attempted via Canvas by 4pm on any following Wednesday.
- I strongly recommend you use LaTeX to typeset your proofs.
- You may work in groups but everyone should submit their own assignment written in their own words. Do NOT copy your classmates.
- Allowed resources: our textbook, classmates, your notes, videos linked in Canvas.
- Unacceptable resources: anything you find on an internet search. Do NOT use a homework help website (e.g., Chegg). Their solutions are often wrong or use incorrect context. I want you to practice making arguments that are yours. Take some ownership.

Criteria

All items will earn a score using the following scale:

- **Exceptional** - Solution is succinct, references the correct theorems and definitions, and is entirely correct.
- **Satisfactory** - Solution is nearly correct. It still references the correct theorems and definitions. It may be longer than necessary, have minor errors, or have some grammatical mistakes.
- **Unsatisfactory** - Solution has major errors, references content not covered in class or in the textbook, or is incomplete in some major way.

Recall from the syllabus

- If you earn either an **Exceptional** or **Satisfactory** mark on an objective exercise (labeled Intro-, Group-, or Ring-) then you may consider that item complete.
- If you earn an **Unsatisfactory** mark on an objective exercise (labeled Intro-, Group-, or Ring-) then you have not yet completed this objective.
- You may submit a new attempt at completing that objective on a future Wednesday. You must select a new exercise listed under the given objective, you cannot resubmit a version you have attempted previously. The only limit you have on number of attempts is the number of exercises available for the objective.

Name: ____

(Intro-3.2) Let $f\colon \mathbb{Z}^+ \to \mathbb{R}$ be defined recursively by $f(1) = 1$ and $f(n+1) = \sqrt{2 + f(n)}$ for all $n \in \mathbb{Z}^+$. Prove that $f(n) < 2$ for all $n \in \mathbb{Z}^+$.

Name: ____________________________________

(Intro-3.3) Prove that for all integers $n \geq 1$

$$1 + 5 + 5^2 + \cdots + 5^n = \frac{5^{n+1} - 1}{4}$$

Name: ________________________________

(Intro-3.4) The Fibonacci numbers f_n, $n = 1, 2, 3, \ldots$ are defined by the formulas $f_1 = 1$, $f_2 = 1$, $f_n = f_{n-1} + f_{n-2}$ for all $n \geq 3$.
Prove by induction that for all $n \geq 1$, $f_1 + f_2 + \cdots + f_n = f_{n+2} - f_2$.

Name: ______________________________________

(Intro-3.5) The Fibonacci numbers f_n, $n = 1, 2, 3, \ldots$ are defined by the formulas $f_1 = 1$, $f_2 = 1$, $f_n = f_{n-1} + f_{n-2}$ for all $n \geq 3$.
Prove by induction that for all $n \geq 1$, $f_1^2 + f_2^2 + \cdots + f_n^2 = f_n f_{n+1}$.

Name: ____

(Intro-3.6) Let A and B be 2×2 matrices with real entries such that $AB = BA$. Use induction to show that for all $n \geq 1$, we have $AB^n = B^n A$.