Name: ____________________________________

(Supp-7) Use proof by induction to show that If $\overline{a}_1, \overline{a}_2, \ldots, \overline{a}_m$ all have inverses in $\mathbb{Z}_n$, show that the same is true for the product $\overline{a}_1\overline{a}_2 \cdots \overline{a}_m$ for all $m \geq 2$.

Proof. Let $n \geq 2$ be an integer. We use induction on the value of m.
Base Case: Let $m = 2$. Assume $\overline{a}_1$ and $\overline{a}_2 \in \mathbb{Z}_n$ have inverses.

Show that $\overline{a}_1\overline{a}_2$ has an inverse. Use the definition of inverse here.

Inductive Step: Let $k \geq 2$ be an integer. Assume the statement holds for $m = k$. Let $\overline{a}_1, \overline{a}_2, \ldots, \overline{a}_{k+1} \in \mathbb{Z}_n$ all have inverses.

Use the base case and the inductive hypothesis to show that $\overline{a}_1\overline{a}_2 \cdots \overline{a}_{k+1}$ has an inverse. Again, use the definition of inverse.

$\square$