Problem Set 3

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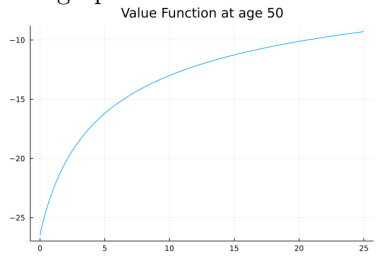
1 Problem 1

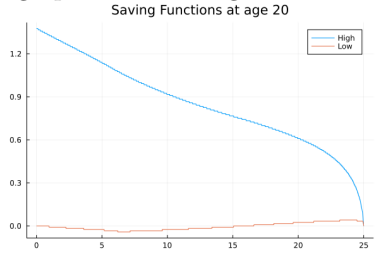
Question: Solve the dynamic programming problem of retirees and workers. Plot the value function over a for a retired agent at the model-age 50. Is it increasing and concave? Plot the savings function for a worker at the model-age 20, $a'_{20}(z, a)$. Is saving increasing in a? Is it increasing in z?

The graph of the value function is the following:



The value function is increasing and concave in asset holdings, reflecting the concavity of the utility function in consumption as well as the tension between the value of present consumption and the discounted value of future consumption.

The graph of the savings function for a worker who is 20, is the following:



The savings function is increasing in Z, with high productivity workers saving more than low productivity workers at all levels of assets. For high productivity workers, savings decrease slowly in assets for most levels of asset holdings, until the upper end of the asset grid where savings decrease quite rapidly. For low-productivity workers, savings decrease markedly at lower levels of asset holdings,

and then increase steadily until the upper end of the asset grid.

2 Problem 2

Please see code.

3 Problem 3

1 Q: First, solve for the benchmark model with social security. Is this economy dynamically efficient (compare the interest rate with the implicit return from social security, which is equal to the population growth rate)? Now eliminate social security by setting $\theta = 0$. Observe how aggregate capital accumulation and labor supply change as a result of the tax reform. Provide intuition in terms of insurance and output efficiency. How does aggregate welfare change? Who benefits and who loses due to this reform? How does the reform affect cross-sectional wealth inequality? You can use table 1 to support your answers

This economy is not currently dynamically efficient as the interest rate is higher than the population growth rate. We can still improve the welfare of some generations without making others worse off by having more saving, and therefore, a lower interest rate. Aggregate capital accumulation and aggregate labor increase as a result of the tax reform. Workers want to have more money for retirement and therefore they both work, and save, more of their earnings. These additional savings fund additional capital purchases, which lead to greater demand for labor as a complementary input in production. Aggregate welfare decreases as a result of the tax reform, with younger high productivity workers benefiting the most with higher wages, and older workers and retirees and low productivity workers losing. Inequality decreases as a result of the tax reform as indicated by a decrease in the coefficient of variation of wealth when moving from the first column to the second.

2 Q: In the second experiment, there is no idiosyncratic risk. Assume that at each age j, $z^L = z^{=}0.5$. First, compute the aggregate variables for the case with social security. How does the aggregate capital stock change relative to the benchmark model? Provide intuition in terms of capital as a buffer stock. Then, eliminate social security. How does the aggregate welfare change? What can you conclude about social security as an insurance device against idiosyncratic risk? Comment on the extent, to which these welfare comparisons across steady states are meaningful or misleading.

The aggregate capital stock decreases considerably relative to the benchmark model. With no idiosyncratic risk there is less need to save against possible low states in the future, and therefore, agents save less. This

reduces the amount of available capital. With no idiosyncratic risk there is less need for additional capital to be held as a buffer against potential shocks, so this also contributes to the decrease in capital levels. Aggregate welfare decreases slightly when we eliminate social security. The decrease in welfare is notably smaller than in the case with idiosyncratic uncertainty. This implies that social security generally succeeds in its role of smoothing consumption and well-being across idiosyncratic states. When a significant portion of idiosyncratic variation disappears, the main benefits of social security disappear as well. These welfare comparisons are useful insofar as they give us baselines to which we can compare different policies. With the comparisons for the 3rd and 4th experiments for example, we see that the value of social security is in large part determined by the the prevalence and magnitude of idiosyncratic shocks. These comparisons can be misleading because even though we know the direction of changes in welfare, we don't have a practical sense of how these ordinal changes correspond to the magnitude of changes in well-being for the population.

3 Consider the case, when labor supply is exogenous ($\gamma = 1$). Compare the distortionary effect of social security on the aggregate labor supply. How does the support for social security change with exogenous labor supply?

The distortionary effects of social security on labor supply are essentially not felt when labor is supplied inelastically. Workers supply close to the same amount of labor with social security as without when $\gamma = 1$. This is because one of the main ways social security impacts the labor supply decision is through the tax rate. With this mechanism gone, the presence of social security does not have a significant impact. With exogenous labor supply, support for social security is higher than in the other experiments. The magnitude of the difference in welfare between the two states is larger than in the other cases, with there being higher welfare when social security is present. With taxes no longer impacting labor decisions, the primary drawback of social security is no longer a factor for households to consider, while the benefits remain. Therefore, it makes sense that support for social security would be higher in the case where labor is supplied inelastically.

	Benchmark model		No risk, $z^H = z^L = .5$		$\gamma = 1$	
	with SS	w/o SS	with SS	w/o SS	with SS	w/o SS
K	3.35895	4.59423	1.127	1.387152	6.34937	8.17873
L	0.34312	0.36507	0.17337	0.1791955	0.75352	0.75389
wage, w	1.45499	1.59265	1.255178	1.33718	1.37857	1.50979
r	0.0236	0.01119	0.0487069	0.037139	0.03201	0.01828
b	0.22507	0	0.09918	0	0.4682	0
welfare W	-35.76894	-37.41719	-44.8284	-45.1414	-23.50867	-25.62865
cv(wealth)	1.67	1.49164	1.2805	1.0998	1.42263	1.15951

Table 1: Results of Policy Experiments