# Problem Set 2

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## 1 Problem 1

Question: Consider the same environment as Huggett (1993, JEDC) except assume that there are enforceable insurance markets regarding the idiosyncratic shocks to earnings and that there are no initial asset holdings. Solve for a competitive equilibrium. What are prices? What is the allocation? (Hint: think about the planner's problem and then decentralize).

The maximization problem is the following:

$$\max_{{c_t}^{u,t-1},{c_t}^{e,t-1}} \Sigma_{t=0}^{\infty}\beta(\mu(U({c_t}^{e,t-1})) + (1-\mu)(U({c_t}^{u,t-1})))$$
$$s.t. \qquad {c_t}^{e,t-1} + {q_t}^{e,t-1} * {a_t}^{e,t-1} = 1$$
$${c_t}^{u,t-1} = 0.5 + {q_t}^{u,t-1} * {a_t}^{u,t-1}$$
$$\mu * {a_t}^{e,t-1} + (1-\mu) * {a_t}^{u,t-1} = 0$$

Where $\mu$ is the probability that one is employed, $\beta$ is the discount factor and ${c_t}^{u,t-1}$ is consumption when one is unemployed and ${c_t}^{e,t-1}$ is consumption when one is employed.

We set up our Lagrangian maximization:

$$\max_{{c_t}^{u,t-1},{c_t}^{e,t-1}} \{\mathcal{L} = \Sigma_{t=0}^{\infty}\beta(\mu(U({c_t}^{e,t-1})) + (1-\mu)(U({c_t}^{u,t-1})))\}$$
$$+\lambda * (0 - (1-\mu)[\frac{{c_t}^{u,t-1} - 0.5}{{q_t}^{u,t-1}}] - \mu[\frac{{c_t}^{e,t-1} - 1}{{q_t}^{e,t-1}}])\}$$

We then take first order conditions:

$$[{c_t}^{u,t-1}] : \beta * \mu * ({c_t}^{u,t-1})^{-\alpha} - \frac{\lambda * (1-\mu)}{q_t^u} = 0$$
$$[{c_t}^{e,t-1}] : \beta * \mu * ({c_t}^{e,t-1})^{-\alpha} - \frac{\lambda * (1-\mu)}{q_t^e} = 0$$

Rearranging these values gives us:

$$\frac{\beta * \mu * ({c_t}^{e,t-1})^{-\alpha}}{\beta * (1-\mu) * ({c_t}^{u,t-1})^{-\alpha}} = \frac{\frac{\lambda * \mu}{q_t^e}}{\frac{\lambda * (1-\mu)}{q_t^u}}$$

Which then simplifies to:

$$\frac{({c_t}^{e,t-1})^{-\alpha}}{({c_t}^{u,t-1})^{-\alpha}} = \frac{q_t^u}{q_t^e}$$

This implies that if we wish to achieve the planner's outcome, then the optimal prices are:

$$q_t^u = q_t^e$$

Which necessarily yields:

$$({c_t}^{e,t-1})^{-\alpha} = ({c_t}^{u,t-1})^{-\alpha}$$

Finally implying that:

$${c_t}^{e,t-1} = {c_t}^{u,t-1} \quad \forall t \geq 0$$

# 2 Problem 2

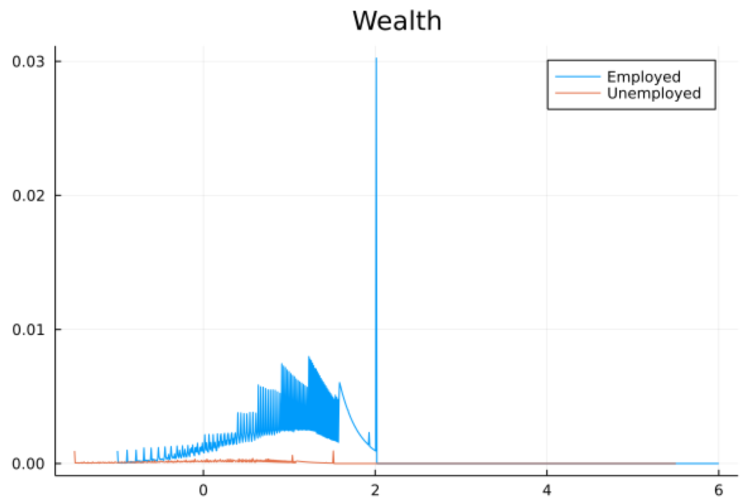
See code for details

a. Plot the policy function

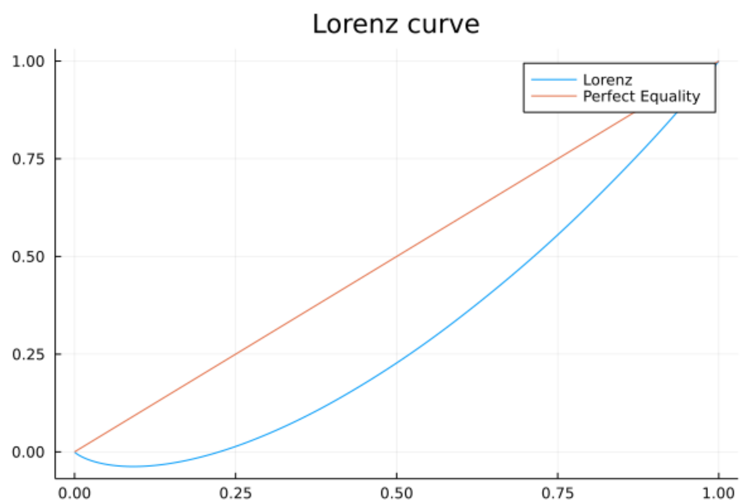

While hard to tell, the green 45 degree line does cross the blue "employed" line. Thus there is a $\hat{a}$ such that $g(\hat{a}, s) < \hat{a}$, meaning that even the employed dissave, leaving a fixed point.

b. What is the equilibrium bond price? Plot the cross sectional distribution of wealth for those employed and those unemployed on the same graph.

The equilibrium bond price is $q = 0.99408$. See the cross sectional distribution of wealth below:




c. Plot a Lorenz curve. What is the gini index for your economy? Compare them to the data.



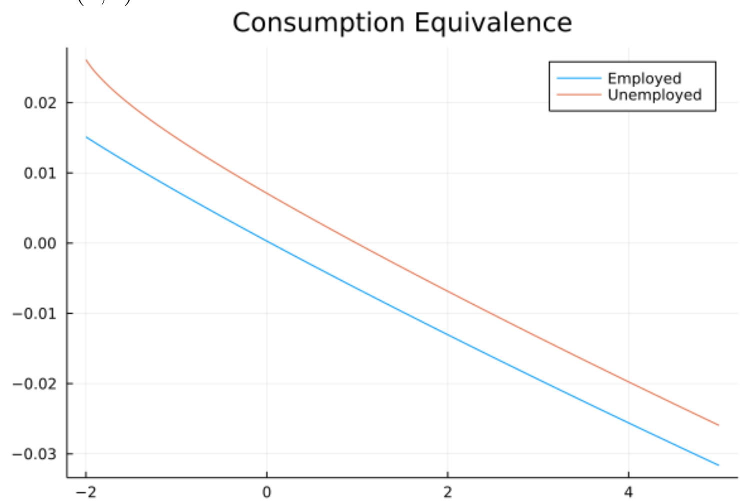

The gini coefficient of wealth for our economy is 0.3818. This is more than half the inequality measured in the data (which is generally around 0.80, Credit Suisse reported .852 in 2019 [1]). This is because, as discussed in class, incomplete markets can only account for some of the wealth inequality seen in American, or really any society.

[1] The source is: https://www.credit-suisse.com/media/assets/corporate/docs/about-us/research/publications/global-wealth-databook-2019.pdf. Warning, going there will automatically download a pdf

# 3 Problem 3

Question: What are the welfare effects of changing to an economy with full insurance?

a. Plot $\lambda(a; s)$ across $a$ for both $s = e$ and $s = u$ on the same graph.




b. What is $W^{FB}$? What is $W^{INC}$? What is $WG$?
$W^{FB} = -4.25$, $W^{INC} = -4.37$,and $WG = 0.000782$

Differences in the values $W^{INC}$ and $W^{G}$ from those given in class seem to be a result of differences in the calculation of our wealth distribution, possibly resulting from a different discretization of the asset vector or in the allowed tolerance resulting in something just a bit different.

c. What fraction of the population would favor changing to complete markets?

0.495 percent of the population would favor changing to complete markets, meaning the proposition would narrowly fail. This of course is a different outcome than seen in class, though only just barely so. Like above, this difference is likely due to a slightly different $\mu(a, s)$ for reasons discussed above. It likely is not due to a mistake in the $\lambda(a, s)$ calculations as the consumption equivalence graphs come out very similar and that crucially does not depend on $\mu$.