## Problem Set 4

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## 1 Exercise 1

Question 1: Use the parametrization from the previous problem set. We continue to assume that labor supply is endogenous. Solve for the stationary equilibrium with social security ( $\theta_0^{SS} = 0.11$ ) without ( $\theta_N^{SS} = 0$ ) following the algorithm described in the lecture notes (Step 1: Calculating the stationary competitive equilibrium). Denote the initial distribution of agents over age, j, asset holdings, a, and productivity levels, z, by  $\Gamma_0^{SS}(z,a,j;\Theta_0^{SS})$ . Denote the welfare of agents alive in the initial steady state by  $V_0^{SS}(z,a,j;\Theta_0^{SS})$ .

Question 2: Compute the transition path of the economy using the algorithm in Step Solving for the transition path in the lecture notes. Try N = 30 for the number of periods it approximately takes to get to the new steady state. Obtain and store the value function for the generations in the initial steady state,  $V_0^{SS}(z,a,j;\Theta_0^{SS},\Theta_N^{SS})$ . Plot the transition paths of interest rate, wage, capital and effective labor. Comment on the results you obtain.

**Solution:** As the new policy is announced, workers know they want to have more money for retirement and save more solving for the new equilibrium conditions each period as they approach the steady state. As the capital increases from the level with Social Security to the level without Social Security, it increases more rapidly in the beginning and levels off to approach the level in the new steady state. People solve for the optimal labor level under the new announcement which will be strictly higher as people have to work more to save more for retirement. Before capital has adjusted accordingly, the labor rises rapidly for one period exceeding the level in the steady state and then falls down gradually to the steady state level. Accordingly interest rises rapidly for a period and wage falls given the increase in labor, before interest rate falls to the new level  $i^{\rm w/o~SS} < i^{\rm SS}$ , and the wage rate rises to the new level  $w^{\rm w/o~SS} > w^{\rm SS}$ .

Capital Transition Path Labor Transition Path 4.25 0.365 0.355 0.345 (a) Capital Path (b) Labor Path Interest Rate Transition Path Wage Transition Path 1.58 1.56 1.54 1.52 1.50 0.0175 1.48 0.0150 1.46 (c) Interest Rate Path (d) Wage Path

Figure 1: Transition Path

Figure 2: Transition Path Figures

Note:

Question 3 What fraction of the overall population would support the reform? Compute and plot the measure of consumption equivalent variation for each age,  $EV_i$  using

$$EV_{j} = \sum_{z} \int_{a} EV(z, a, j) \Gamma_{0}^{S} S(z, a, j; \theta_{0}^{SS}) da$$

**Solution:** Only about 45.3% of the population would vote for the reform given the transition path. The transition marginally increases the welfare gains lost for the young, but greatly decreases the welfare loss for the old as they are able to optimize each period as they transition.

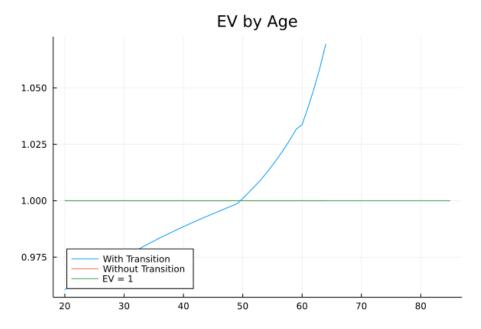


Figure 3: Interest Rate Path

Figure 4: Support for the reform

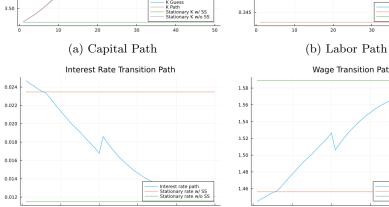
## 2 Problem 2

Instead of considering an unexpected elimination of the social security system, assume that in t=0 the government credibly announces that it is going to abolish the public pension system starting from t=21 onwards. Thus, all individuals retired keep their social security benefits, but future retirees anticipate that they will receive only part or no social security benefits. Repeat steps (1)-(3) of exercise 1 to study how agents readjust their plans and how political support changes for the anticipated reform in 21 years. You will have to increase the number of transition periods (try N=50). Discuss your results.

Solution: Here, the government announces that it is going to abolish the public pension system starting from t=21, we see the capital transitioning to people saving more as before, and similarly with people adjusting their labor supply, but here there is a discontinuity for when the government actually gets rid of the public pension system and people no longer have to pay for it so labor increases sharply at period 21, where interest rate also increases for a period, and wage decreases for a period with the influx of the labor supply, and then the interest rate falls to the steady state level and wage increases to the steady state level. As people anticipated the change in the pension system, and are reoptimizing each period, they do not need to adjust their savings drastically

Labor Transition Path Capital Transition Path 0.365 0.360 0.355 0.350 0.345

Figure 5: Transition Path



(c) Interest Rate Path

Wage Transition Path (d) Wage Path

Figure 6: Transition Path Figures

Note:

4.25

around period 20. Here, as noted, we need more periods for adjusting due to the shock as interest and wage take longer to reach the steady state levels after the shock.