Lattice QCD Computation of the Proton Isovector Scalar Charge

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Abstract

The isovector scalar charge (g_s) describes the strength of certain short-distance nucleon interactions in Quantum Chromodynamics (QCD) [1]. In order to calculate g_s with precision, Lattice QCD is simulated on high-performance computers and statistical analyses are performed on the data. Calculations of g_s are essential for understanding nucleon substructure and physics beyond the Standard Model. This poster presents computations of the scalar charge at two different unphysical quark masses. To obtain the physical value of g_s, an extrapolation is made down to the physical quark mass.

Introduction

Protons and neutrons are each composed of fundamental particles called quarks and gluons which interact via the strong force. The theory describing the strong force interactions is called Quantum Chromodynamics (QCD). The dynamics of quarks and gluons are determined by the QCD Lagrangian [2].

$$\mathscr{L}_{QCD} = \sum_{f=u,d,s,c,b,t} \bar{\Psi}_f(x)(i\not{D} - m_f)\Psi_f(x) - \frac{1}{4}TrF^{\mu\nu}F_{\mu\nu} \tag{1}$$

The summation term over all quark flavors contains the quark fields Ψ, while the second term describes the gluons $F^{\mu\nu}$. In order to calculate physical results in QCD, one must evaluate operator expectation values

$$\langle\mathcal{O}\rangle = \frac{1}{Z}\int DA_\mu \mathcal{O} e^{-\int \mathrm{d}^4x \mathscr{L}} \tag{2}$$

where Z is the partition function and the integral is over all possible gauge field configurations A_μ. Since it is impossible to exactly evaluate an infinite-dimensional integral, we must discretize spacetime onto a four-dimensional lattice.

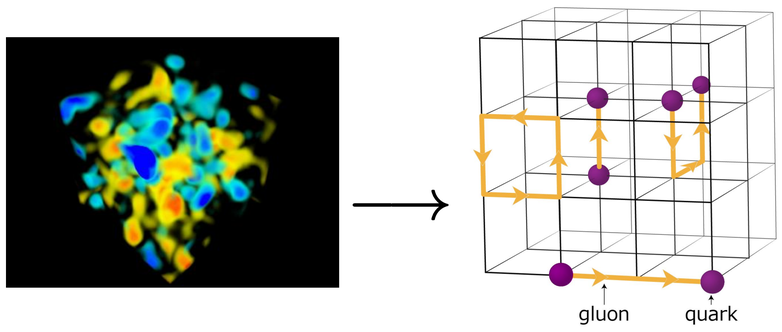

Figure 1: The QCD action is transcribed onto a discrete spacetime lattice. Interactions between quarks are mediated by gluons.

Now, it is possible to evaluate integrals in the form of eq.(2) via numerical Monte Carlo (MC) integration. Unfortunately, MC methods introduce correlations between sampled values and careful statistical techniques are needed to maintain a desired level of precision.

Jackknife Statistics and All-Mode Averaging

This research employs the use of jackknife statistics. The procedure, developed to correct for bias, is well-suited for calculations involving correlated samples [3]. To illustrate the technique, we let $\{\mathcal{O}\}$ denote some set containing n sampled values of an observable $\mathcal{O}$. Then, we construct a jackknife set of size n from the original sample, where the ith element is an ensemble average over all except the ith (original) element.

$$\left\langle\hat{\mathcal{O}}\right\rangle_i = \frac{1}{n-1}\sum_{j\neq i}^{n}(\mathcal{O}_j) \tag{3}$$

We now have a set of n estimators which can be used for further computations. This procedure also results in simple error propagation. After a computation on each of the n jackknife estimators is performed, the final estimator is simply the set average, and the associated errors can be calculated directly using the n jackknife estimators and the final averaged estimator. To further reduce statistical errors, we implement all-mode averaging (AMA). Here this means relaxing the stopping condition of the conjugate gradient method when calculating the inverse of the Dirac operator so that all eigenmodes are taken into account [4]. The final result is the improved estimator

$$\left\langle\hat{\mathcal{O}}\right\rangle^{(imp)} = \frac{1}{N_{appx}}\sum_{i}^{N_{appx}}(\mathcal{O}_i^{(appx)}) + \frac{1}{N_{exact}}\sum_{i}^{N_{exact}}(\mathcal{O}_i^{(exact)} - \mathcal{O}_i^{(appx)}) \tag{4}$$

where $N_{appx} >> N_{exact}$ and $\mathcal{O}_i^{(appx)}$ in the second summation is restricted access to only the type of gauge configurations used by $\mathcal{O}_i^{(exact)}$.

Calculating g_s

The calculation of g_s begins with evaluating the correlation functions, $C_{3pt}^{\mathcal{O}}$ and C_{2pt}, defined as [5]

$$C_{2pt}(t,\vec{P}) = \left\langle N(\vec{p}=\vec{P},t)\bar{N}(\vec{x}=0,0)\right\rangle \tag{5}$$

$$C_{3pt}^{\mathcal{O}}(\tau,T;\vec{P}) = \left\langle N(\vec{p}=\vec{P},T)\mathcal{O}(\vec{p}=0,\tau)\bar{N}(\vec{x}=0,0)\right\rangle \tag{6}$$

If the operator $\mathcal{O}$ is inserted sufficiently far away from the source and sink in time, then

$$g_s \propto \frac{C_{3pt}^{\mathcal{O}}(\tau,T;\vec{P})}{C_{2pt}(t,\vec{P})} = \left\langle N(\vec{P})\middle|\,\bar{u}u - \bar{d}d\,\middle|N(\vec{P})\right\rangle \tag{7}$$

and g_s can be calculated as the ratio of the correlation functions.

Computational Procedure

The following steps outline the computational procedure:

1. Obtain $\{C_{3pt}^{\mathcal{O}}\}$ and $\{C_{2pt}\}$, averaged over source locations, from MC simulations using various gauge configurations.
2. Generate jackknife ensemble-average datasets $\{(C_{3pt}^{\mathcal{O}})_i\}$ and $\{(C_{2pt})_i\}$.
3. Perform element-by-element complex division to obtain new set $\{g_s^J(t)\} \equiv \{\frac{(C_{3pt}^{\mathcal{O}})_i}{(C_{2pt})_i}\}$
4. Fit each plot of $\{g_s^J(t)\}$ in the central timeslice region to obtain jackknife (constant) fit result $\{g_s^J\}$.
5. Calculate the jackknife average and standard error of $\{g_s^J\}$ to obtain constant g_s.
6. Repeat steps 1-6 for different unphysical quark masses and extrapolate results down to the physical quark mass to obtain the true value of g_s.

Results

The following shows the unrenormalized g_s as a function of time (step 5 from above) using an unphysical quark mass of 4.2×10^{-3}.

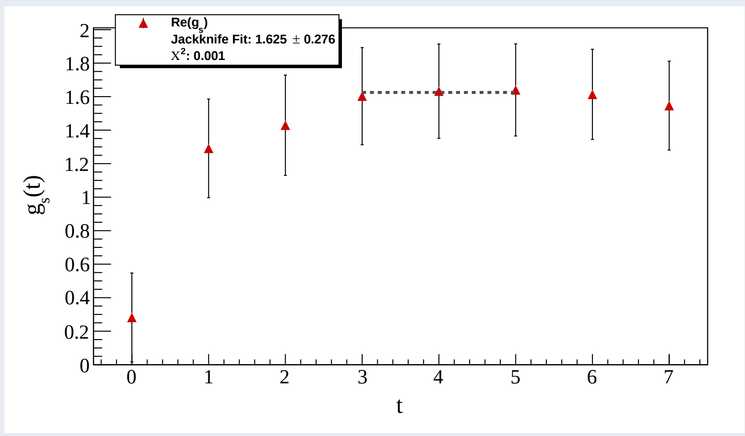

Figure 2: The unrenormalized scalar charge at different timeslices on the lattice. Fluctuations from excited-state contributions are highest at the boundaries. A straight-line fit is applied in the middle plateau region where g_s is most near the actual value.

A fitted line of constant value is shown in the plateau region. This is done because, in reality, g_s is a constant. After obtaining the fitted value of g_s for various quark masses, an extrapolation is made down to the physical quark mass to obtain the desired result, the predicted value of g_s.

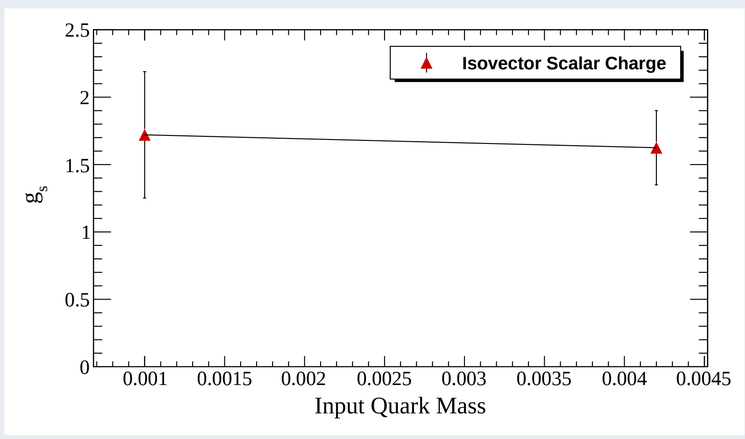

Figure 3: The final computed values of (unrenormalized) g_s at two different (unphysical) quark masses. QCD suggests that a straight-line extrapolation down to the physical quark mass can be made to obtain the true value of g_s.

Conclusions

This research offers the value of the isovector scalar charge (g_s) using the new domain-wall fermion discretization scheme and low quark masses. Such computations are needed to understand experimental measurements of scalar interactions in, for example, neutron beta decay. This interaction is expected to be small compared to the well-known $V-A$ structure of weak interactions, and thus few experiments have been able to measure g_s. However, new high-precision instruments are expected to be able to probe the scales necessary to measure the scalar contribution. Such measurements need precise theoretical constraints in order to convey meaningful information in a physics analysis. The results above both provide new constraints and strengthen former calculations of the isovector scalar charge.

Acknowledgments

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