Equations Used in Hybrid Rocket Simulation

Purpose

This document summarizes the key equations used in the hybrid rocket motor simulation, detailing their purpose, involved variables, and includes appropriate units.

1. Oxidizer Mass Flow Rate

Purpose: To calculate the mass flow rate of oxidizer through the injector orifice.

$$\dot{m}_{\mathrm{ox}} = C_d A_{\mathrm{inj}} \sqrt{2 \Delta P \rho_{\mathrm{ox}}} \tag{1}$$

Variables:

- $\dot{m}_{\mathrm{ox}}$: Oxidizer mass flow rate (kg/s)
- C_d: Discharge coefficient (unitless)
- A_{inj}: Injector area (m^2)
- ΔP: Pressure drop across the injector (Pa)
- ρ_{ox}: Oxidizer density ($\mathrm{kg/m}^3$)

2. Fuel Regression Rate

Purpose: To determine the rate at which fuel burns and increases port diameter.

$$\dot{r} = a \left(\frac{G_{\mathrm{ox}}}{1\,\mathrm{kg/(m^2 \cdot s)}} \right)^n \tag{2}$$

Variables:

- $\dot{r}$: Regression rate (m/s)
- a: Regression rate coefficient (m/s)
- n: Regression rate exponent (unitless)
- G_{ox}: Oxidizer mass flux (kg/(m^2·s)), $G_{\text{ox}} = \dot{m}_{\text{ox}}/A_{\text{port}}$
- A_{port}: Fuel port cross-sectional area (m^2)

3. Fuel Port Diameter Update

Purpose: To update the port diameter as fuel burns.

$$D_{\text{port,new}} = D_{\text{port,old}} + 2\dot{r} \cdot \Delta t \tag{3}$$

Variables:

- D_{port}: Fuel port diameter (m)
- $\dot{r}$: Regression rate (m/s)
- Δt: Time step (s)

4. Fuel Burned Mass

Purpose: To calculate fuel mass burned over the time step.

$$m_{\text{fuel}} = \rho_{\text{fuel}} A_{\text{burn}} \dot{r} \cdot \Delta t \tag{4}$$

Variables:

- m_{fuel}: Fuel mass burned (kg)
- ρ_{fuel}: Fuel density (kg/m^3)
- A_{burn}: Burning surface area (m^2), $A_{\text{burn}} = \pi D_{\text{port}} L$
- L: Port length (m)

5. Oxidizer-to-Fuel Ratio (O/F)

Purpose: To compute the mass ratio between oxidizer and fuel.

$$\mathrm{O/F} = \frac{\dot{m}_{\mathrm{ox}}}{\dot{m}_{\mathrm{fuel}}} \tag{5}$$

Variables:

- $\dot{m}_{\mathrm{ox}}$: Oxidizer mass flow rate (kg/s)
- $\dot{m}_{\mathrm{fuel}}$: Fuel mass flow rate (kg/s), $\dot{m}_{\mathrm{fuel}} = m_{\mathrm{fuel}}/\Delta t$

6. Chamber Pressure Estimate

Purpose: To estimate chamber pressure for thrust calculation.

$$P_c = \frac{\dot{m}_{\mathrm{total}} R T_c}{V_c} \tag{6}$$

Variables:

- P_c: Chamber pressure (Pa)
- $\dot{m}_{\mathrm{total}}$: Total mass flow rate (kg/s)
- R: Specific gas constant (J/(kg · K))
- T_c: Chamber temperature (K)
- V_c: Chamber volume (m^3)

7. Thrust Calculation

Purpose: To estimate thrust produced.

$$F = C_f P_c A_t \tag{7}$$

Variables:

- F: Thrust (N)
- C_f: Thrust coefficient (unitless)
- P_c: Chamber pressure (Pa)
- A_t: Nozzle throat area (m^2)