3120homework03

Class	CPSC 3120
Due Date	@April 2, 2025
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R-13.4 Bob loves foreign languages and wants to plan his course schedule to take the following nine language courses: LA15, LA16, LA22, LA31, LA32, LA126, LA127, LA141, and LA169. The course prerequisites are:

- LA15: (none)
- LA16: LA15
- LA22: (none)
- LA31: LA15
- LA32: LA16, LA31
- LA126: LA22, LA32
- LA127: LA16
- LA141: LA22, LA16
- LA169: LA32.

Find a sequence of courses that allows Bob to satisfy all the prerequisites.

LA15, LA16, LA22, LA31, LA32,

~~LA126, LA127, LA141, LA169~~

He will be done with all prerequisites after LA 15,16,22,31, and 32

C-14.2 Give an example of a weighted directed graph, $\vec{G}$, with negative-weight edges, but no negative-weight cycle, such that Dijkstra's algorithm incorrectly computes the shortest-path distances from some start vertex v.

Dijkstra's algorithm fails on graphs with negative weight edges because it assumes that once a shortest path is found, it cannot be improved. But in graphs that have negative weights, a shorter path can still be found after a vertex is finalized.

Goal: find shortest path from A to C

Start A

↓

C shortest (1). no outgoing, backtrack to A, finalize processing C

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B shortest (4)

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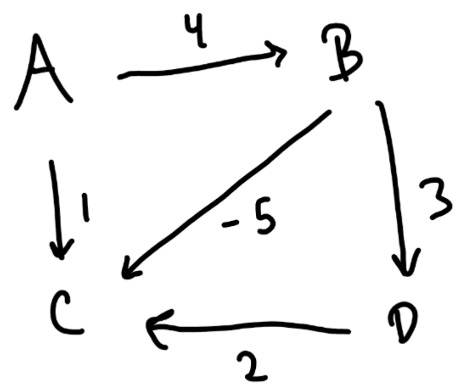
C shortest (-5), 4-5=-1 from A→B→C where we already found A→C=1



R-17.1 Professor Amongus has shown that a decision problem L is polynomial-time reducible to an ***NP***-complete problem M. Moreover, after 80 pages of dense mathematics, he has also just proven that L can be solved in polynomial time. Has he just proven that $\boldsymbol{P} = \boldsymbol{NP}$? Why, or why not?

The professor has not proven that P=NP. He has shown that L is easier or as easy as M, and L happens to be in P. but this does not mean that M (or any other NP complete problem) is in P.

A-17.1 Imagine that the annual university job fair is scheduled for next month and it is your job to book companies to host booths in the large Truman Auditorium during the fair. Unfortunately, at last year's job fair, a fight broke out between some people from competing companies, so the university president, Dr. Noah Drama, has issued a rule that prohibits any pair of competing companies from both being invited to this year's event. In addition, he has shown you a website that lists the competitors for every company that might be invited to this year's job fair and he has asked you to invite the maximum number of noncompeting companies as possible. Show that the decision version of the problem Dr. Drama has asked you to solve is ***NP***-complete.

Given a set of n companies C={C1,...,Cn} and a competition map/graph G=(C,E) where an edge (Ci, Cj) → E means that companies Ci and Cj are competitors cannot both be invited. Integer k is a subset S contained in C of at least k companies such that no two companies in S are competitors (no two companies in S share an edge in G).

This is equivalent to finding an independent set of size at least k in the competition graph k.

Given a set of invited companies S of size k, we can verify in polynomial time whether S is independent or not by checking whether two companies in S are competitors (connected in G). Since this can be done in polynomial time, it is in NP.