RV Curve Proof

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To determine the general equation for the radial velocity, we start from the x and y components of the orbital velocity of a body on an elliptical orbit:

$$v_x = -\frac{2\pi a}{P\sqrt{1-e^2}}\sin(f)$$

$$v_y = \frac{2\pi a}{P\sqrt{1-e^2}}(e+\cos(f))$$

We can calculate the x and y velocities after a rotation about the argument of periastron (ω) by multiplying the v_x and v_y vectors by the rotation matrix:

$$\begin{pmatrix} v_x^\omega \\ v_y^\omega \end{pmatrix} = \begin{pmatrix} \cos(\omega) & -\sin(\omega) \\ \sin(\omega) & \cos(\omega) \end{pmatrix} \begin{pmatrix} v_x \\ v_y \end{pmatrix}$$

For the radial velocity of the star, we only need the velocity of the star along our line of sight, so we take y-component of the velocity which is:

$$v_y^\omega = v_x \sin(\omega) + v_y \cos(\omega)$$

Accounting for a rotation around the orbital inclination angle (i), the radial velocity is expressed as:

$$v_r = \sin(i)(v_x \sin(\omega) + v_y \cos(\omega))$$

Subbing in v_x and v_y we get:

$$v_r = \sin(i)\left(\left(-\frac{2\pi a}{P\sqrt{1-e^2}}\sin(f)\right)\sin(\omega) + \left(\frac{2\pi a}{P\sqrt{1-e^2}}(e+\cos(f))\right)\cos(\omega)\right)$$

Rearranging to get the general equation for radial velocity in terms of ω, i and f:

$$v_r = \frac{2\pi a}{P\sqrt{1-e^2}}\sin(i)(\cos(\omega)(e+\cos(f)) - \sin(f)\sin(\omega))$$

We can rearrange this using the angle sum trigonometric identity:

$$v_r = \frac{2\pi a}{P\sqrt{1-e^2}}\sin(i)(e\cos(\omega) + \cos(f+\omega))$$

Now showing that K is independent of ω. The RV curve semi-amplitude is defined as:

$$K = \frac{1}{2}(max(v_r)) - min(v_r))$$

Since f is a function of time, we get a maximum when $\cos(f + \omega) = 1$ and a minimum when $\cos(f + \omega) = -1$. Factoring out the terms, we get:

$$K = \frac{\pi a}{P\sqrt{1-e^2}} \sin(i)(e\cos(\omega) + 1 - e\cos(\omega) + 1)$$

Cancelling out the $e\cos(\omega)$, we get

$$K = \frac{2\pi a}{P\sqrt{1-e^2}} \sin(i)$$

This shows that K is independent of ω. Different ω change the shape of the RV curve, but it doesn't effect the RV semi-amplitude. By subbing in Kepler's 3rd law for a, we can get K in terms of the planet and star mass. Measuring K allows one to calculate $M_p \sin(i)$.