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Kant and Strawson on the Content of Geometrical Concepts[1]

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Introduction

This paper considers Kant's understanding of conceptual representation in light of his view of geometry.

It is widely acknowledged that the view of geometry drives his conception of our capacity for sensible representation (intuition). Kant supposes that independently of experience, geometry is known to apply to physical space. To explain our *a priori* knowledge of its applicability, he takes us to be able to represent formal conditions that govern all intuition. He holds that we know geometry to hold of all objects represented through sensibility because it describes these conditions, which manifest in a special kind of representation, called "pure intuition".

Subsequent advances in mathematics and physics appeared to discredit Kant's view of intuition. They showed that no geometry can be known *a priori* to apply to physical space. In their light, Kant's view appeared to rest on a confusion between pure geometry, which is known *a priori* but empty, and its applications. While conceding that Kant confuses pure and applied geometry, P. F. Strawson tries to preserve the interest of his view. Strawson seeks to explain how the application of geometry can be independent of experience. Kant holds that the applicability of geometrical concepts to all objects represented in *empirical intuition* (ordinary sense-perception) is proved by our ability to represent objects falling under them in *pure* intuition. Strawson interprets Kant's "pure intuition" as the capacity to give ourselves "pictures" in imagination. He takes Kant to argue that because all use of geometrical concepts involves picturing, what holds of all pictures we can give ourselves must hold of all objects represented through sensibility.

A preliminary goal of this paper is to defend Strawson's view that Kant intends to explain our ability to formulate the criteria (marks) by which we recognize instances of a concept. For the most part, later scholars of Kant's philosophy of mathematics have not adopted this interpretation. (Because they focus on the validity of geometrical inference, for them Kant's view of geometrical concepts must be an account of how the concepts' definitions ground the theorems that are proved.[2]) In §2, I draw on Kant's logic lectures to show that for him, the aim of definition is indeed to articulate a concept's marks and prove the legitimacy of its use (that is, its "objective reality": roughly, the in-principle possibility of perceiving objects falling under it). It follows that for Kant, a concept is not definable unless its use is legitimate.

Strawson is also right that on Kant's view, concepts acquire content of this kind (criteria of application) through an exercise of the imagination. But Strawson misunderstands the imaginative activity through which concepts are defined. He regards it as an application of concepts, to pictures, which is *a priori* in the sense that it mediates all use of the concepts in sense-experience. But I argue (in §3) that for Kant, it is *a priori* in the stronger sense that it is required even to possess the concepts. Kant holds that mathematical concepts are formed by defining them. Since we cannot possess them without grasping definitions, which assure the universal applicability of the concepts, no experience (even that of inspecting pictures) can provide any further guarantee of their applicability. I explain why, according to Kant, experience is not needed either to formulate or to ascribe the marks included in mathematical concepts.

Strawson points us toward a key element of Kant's theory of geometry, but it does not fit the place he gives it in Kant's catalogue of kinds of representation. I sketch a way of reconciling Strawson's interpretation of "pure intuition" (on which it represents objects as we imagine, or are prepared to picture, them) with Kant's view that it proves the applicability of concepts independently of experience. Pure intuition can be taken, in the spirit of Strawson's interpretation, to represent *procedures for constructing* objects that fall under the concepts. I argue that on Kant's view, the representation of such procedures indeed yields *a priori* knowledge of the applicability of concepts. But because these procedures must be represented as general, and intuition represents particulars, it would be wrong to understand pure intuition as the representation of these procedures. I explain that on Kant's view, procedures for constructing objects are represented as "schemata", which are distinct from concepts and intuitions.

My main objection to Strawson is that he overlooks the implications of Kant's view of definition. Kant uses the definition of geometrical concepts to illustrate the sensible faculty's role in cognition. He thinks it is distinctive of mathematical concepts that they are formed by defining them. He also contrasts geometrical concepts, as already having schemata, with those that originate in the understanding alone (as I show in §4). He thus appears to hold that in defining a concept, we formulate its schema.[3] By constraining the formulation of schemata, the sensible faculty enters into the formation of concepts themselves. Strawson not only fails to acknowledge the full extent of this faculty's role in cognition, but also underestimates the importance of geometry for the *Critique*'s argument. He fails to see how (as I explain in §5) Kant uses geometry to fill a gap in his account of the applicability of pure concepts in general.

If Kant's theory of geometry is defunct, we should join Strawson's effort to detach from it insights that can claim to last. So to defend the relevance of Kant's theory (even for interpreting the *Critique*), I must deal with the objections that lead Strawson to minimize its role. If Kant's view that mathematical concepts have schemata rested merely on an inability to conceive them more abstractly, he could still be charged with confusing pure and applied geometry. But Kant holds that outside of mathematics, concepts can be formed independently of the constraints imposed by sensibility. Since we can form (but not give schemata to) concepts that do not accord with (Euclidean) geometry, the formation of geometrical concepts involves a kind of choice (as I explain in §6).

Kant's view of geometry thus has an affinity, overlooked by Strawson, with "conventionalist" views. Yet this choice is not arbitrary for Kant in the way it is on later views. Kant holds that the choice of concepts intended for mathematical use is informed by cognition of their applicability (as I explain in §7). Specifically, we voluntarily restrict the understanding to forming only those concepts whose schemata we can represent (which entails that objects answering to them can be represented in accordance with the conditions on our perception). By choosing to form only applicable concepts, we license ourselves to ascribe their marks independently of experience: in particular, without regard to our experience of how concrete material objects fall short of geometry's specifications. So our prerogatives to fix criteria for the application of mathematical concepts, and stipulate their satisfaction, ultimately rest on a more fundamental ability to perceptually represent objects that answer to the concepts. Kant's account of geometry's applicability to the physical world thus stands as a genuine alternative to the positivists' conventionalism, as well as to the picture of the "mind making nature" repudiated by Strawson.

1. Strawson on "Phenomenal Space"

I begin by explaining why, on Strawson's view, the theory of geometry has only limited significance.

In the *Critique*, Kant claims that intuition is distinguished by its "singularity" and the "immediacy" with which it relates to the object (A320/B377). He maintains that our intuition is sensible (meaning that it relates us to objects only insofar as they are given to us) and has two forms, space and time. What Strawson calls the "peculiarly intimate connexion between space and time, on the one hand, and the idea of the particular item, the particular instance of the general concept, on the other" ((1966), 48) is a signal feature of Kant's thought.

On Strawson's interpretation, spatiality is important solely or primarily as the mode in which particulars are represented. It is through our ability to locate particulars in space that we can represent their numerical identity (the difference between particulars that fall under the same concepts) and reidentifiability on different occasions of reference. Because space is in this way necessary for the representation of particulars,[4] and the latter is a necessary element of anything we can conceive of as experience, spatial ordering becomes "a necessary element in any conception of experience which we can render intelligible to ourselves" ((1966), 50–1).

An equally prominent feature of Kant's treatment of space's role in cognition is the reliance of his argument on geometry. Kant appeals to the role played by spatial representation in geometrical reasoning to clarify its role in, and establish its importance for, cognition in general. As a body of certain truths which derive their justification from intuition, geometry shows how the representation of particulars can be a basis for general conclusions, valid for all experience.

Strawson acknowledges that geometry has this role in Kant's own theorizing.[5] But Strawson finds geometry far less useful than the connection between spatiality and particularity for understanding intuition's role in cognition. He claims that the distinction between intuition and concept is, like the rest of the *Critique*'s "main structure", "relatively independent" of the theory of geometry ((1966), 23). Strawson dismisses both the conclusion of Kant's "argument from geometry" and the problem he thinks it is intended to solve.

The conclusion, as Strawson understands it, is that space is subjective: that spatial ordering is no more than an aspect of our cognitive organization. For Strawson, this thesis belongs to the "imaginary subject" of "transcendental psychology". He unequivocally rejects Kant's attempt to penetrate beyond experience, to the faculties responsible for its structure, on the grounds that it is unclear how derivations of features of experience from particular faculties could be confirmed or refuted. (In contrast to the subjectivity thesis, the particularity thesis (that the representation of particulars must be spatial) is established by the legitimate means of whittling down experience to just the features without which it cannot intelligibly be conceived.)

Strawson regards the subjectivity thesis as Kant's solution to the problem of how geometry can have both content (or "synthetic character") and the necessity that marks it as *a priori* knowledge.

> Might [geometrical propositions] not owe their synthetic character to their being based upon *some* kind of sensible intuition, and their necessary character to this kind of intuition's being non-empirical? Kant claims that if we accept the doctrine of the subjectivity of space as a mere form of intuition belonging to

> our cognitive constitution, then there is no difficulty in seeing how such a kind of spatial intuition can exist, and how the synthetic propositions which it yields can be known to apply with absolute necessity to the ordinary spatial objects of outer intuition. ((1966), 277–8)

Strawson labels this "a solution to a problem that does not really exist". He takes the side of Kant's positivist critics, who charge that Kant fails to distinguish pure and applied geometry. They hold that insofar as geometry is known *a priori*, it can be no more than an uninterpreted system. As deductive consequences of a system of axioms, mathematical judgments may be necessary, but have as yet no content. Geometry is not *synthetic* knowledge until its terms are defined in such a way as to uniquely determine their referents within an interpretation. The positivists deny that mathematical properties can be meaningfully ascribed to objects beyond our ken. On their view, a mathematical term can be associated with an object only by means of an operational criterion that determines the term's application. So content is conferred on geometrical judgments only by specifying the observations that make their truth known.[6]

For Strawson, Kant's view retains its interest, but as the solution to a different problem: of how geometrical primitives can be given meaning of a certain kind. Strawson claims that geometrical axioms can be regarded as "true solely in virtue of the meanings attached to the expressions they contain", where these meanings are "essentially phenomenal, visual," or picturable ((1966), 283).[7] For instance, we might "satisfy ourselves of the truth of" the axiom that "not more than one straight line can be drawn between two points" by considering "an actual or imagined figure". This exercise would make it "evident that we cannot, either in imagination or on paper, give ourselves a picture such that we are prepared to say of it both that it shows two distinct straight lines and that it shows both these lines as drawn through the same two points".

Strawson suggests that "the curious facility with which phenomenal picture-patterns can be elaborated to exhibit an extensive system of relations" between spatial concepts encourages the view that geometry holds necessarily for objects in space. He emphasizes that, to establish relationships between propositions of geometry, Kant had no alternative but to elaborate patterns of phenomenal figures. Derivability within an uninterpreted sentential calculus, in particular, was not an option for him. Thus, only concepts with picturable content could belong to a "systematic" mathematical discipline, a body of purportedly necessary propositions. So on Kant's view, no concept can belong to a geometry unless we can give ourselves appropriate pictures of what falls under it. Hence, anything we can conceive of as a geometrical object must conform to the conditions under which it can be pictured; consequently, any geometry we can conceive must hold of picturable (phenomenal) objects. Then, according to Strawson, Kant identifies "phenomenal space" (the space in which these pictures are presented) with both the domain of geometry and physical space,[8] with the consequence that geometry must hold for all *physical* objects. Thus, for Kant anything that counts for us as falling under a spatial concept must conform to the conditions on our ability to give ourselves pictures: "the geometry of phenomenal space embodies, as it were, the conditions under which alone things can count as things in space for us" (285).

Strawson has considerable sympathy for the view that geometry is about "phenomenal" objects, which, he suggests, explains the historical development of geometry (see my note 5) as well as the temptation to regard it as necessarily true of objects in space. But he strenuously objects to Kant's assumption that the conditions on phenomenal space also hold of physical space. He claims that Kant, "at [his] stage of the history of science, can scarcely be reproached" for the "fundamental error" of confusing the application of Euclidean geometry to phenomena with its interpretation in physical space ((1966), 285). But he insists that Kant would have done better to more explicitly mark the scope of geometry's necessity. The word "synthetic",

> if we agree to tolerate it here as qualifying 'necessity', signifies no more than that we could not do without this phenomenal exhibition of meanings in developing *this* geometry. Perhaps it would be preferable to use the phrase 'phenomenally analytic', where the adverbial qualification 'phenomenally' serves just the same purpose. (284)

Strawson is entirely right that Kant's theory of geometry is intended to explain how geometrical concepts acquire meaning, and thus our grasp of the necessity of geometrical truths. He grants that Kant infers, from the necessary applicability of geometry, that the capacity for representing particulars has a certain "pure" employment. But he does not recognize the full extent to which this use of the capacity differs from its empirical use. Strawson takes Kant to hold that "we have the faculty of giving ourselves, in imagination, individual exemplars of figures answering to certain spatial concepts (e.g. that of a triangle)", even in the absence of "actual objects" of experience ((1966), 61). Thus, "we do not depend upon the results of empirical observation of the characteristics and relations of objects actually encountered in experience in order to" reach certain conclusions about objects falling under these concepts (62). But Strawson does not acknowledge that from the necessary applicability of geometry, Kant infers that the application of certain concepts does not depend on "empirical observation" at all, whether of actual objects encountered in experience or exemplars given in imagination. We will see that Kant uses geometry to illustrate how the concepts he classifies as "factitious *a priori*" find application through a use of the capacity for sensible representation that is pure in this stronger sense.

2. The Definability of Factitious Concepts

Since Kant conceives definitions as a ground of all geometrical cognition (A726/B754), they are a focus of his theory of geometry.

The view of definition on which his theory of geometry relies is elaborated in his lectures on logic. It is set out clearly and in detail by Strawson's contemporary Lewis White Beck (in (1955) and (1956)). As Beck notes, the logic lectures "far exceed the bounds [Kant] set around the field of general logic in the *Critique*" ((1956), 65), namely that it abstract "from all content of cognition, *i.e.* from any relation of it to the object" and concern "nothing but the mere form of thinking" (A54/B78). Because practical logic presupposes "a particular kind of object, to which it is applied" (9:17–18), on Kant's view "there really is no practical logic" (24:779).[9] Yet the lectures follow tradition in treating logic as though it had a "practical" aspect or "organon" (*cf.* 9:13). The project of defining concepts belongs to the organon. Since the search for definitions is part of logic, which concerns "not how we do think, but how we ought to think" (9:14), the conditions for the application of a concept that are stated in its definition are not psychological, but justificatory. The definition does not explain what goes on in our minds when we apply a concept, but what makes us right to do so (*cf.* Capozzi (1980), 427–8). Thus, a concept can be defined only if its use is legitimate, that is, if we are justified in bringing particulars under it. Since definability is so significant for epistemology, it should not be surprising that it pertains only to concepts of a certain special kind (as we will see; *cf.* Beck (1956), 68).

The definition of a concept explains how it relates to the particulars that fall under it. To define a concept is to "further its significance" with respect to its "content" (9:140, §98). Kant is often thought to identify the content of a concept with the collection of predicates, or representations of properties, that are contained in the concept (as for instance by Longuenesse (1998), 128. See Young (1992), 110.) But Kant understands these as "marks", which are properties *of things* through which they are cognized (9:58; *cf*. Beck (1956) and Smit (2000)), specifically, cognized to fall under the concept. Thus Kant glosses "content" as "how [the defined concept] determines an object through a mark" (9:94, §5) and claims that it pertains to a concept insofar as that concept is "contained in the representation of things" (9:95, §7). So the definition of a concept sets out, not the particulars falling under it, but the marks or characteristics by which they are identified as things of that kind.[10] Kant thus shares Leibniz's view of distinctness as "like the notion an assayer has of gold, that is, a notion connected with marks and tests sufficient to distinguish a thing from all other similar bodies" ((1684), 24). To define a concept, on Kant's view, is to articulate or explain the ability to recognize objects as instances[11] of that concept.

To see why statements that explain these cognitive capacities count as "definitions", we should think of concepts in terms of abilities, rather than as representations. While Kant defines a concept as a "general or reflected representation" (9:91; *cf.* A320/B377), he also claims a concept is "always something general ... that serves as a rule" (A106). He contrasts intuitions, as "resting on affections", with concepts, which "rest on functions", that is, on "the unity of the action of ordering different representations under a common one" (A68/B93). Strawson and many commentators since, notably Jonathan Bennett and R. P. Wolff, hold that concepts are for Kant ultimately rule-governed activities (or capacities thereto).[12] They point out that since Kant does not permit ("Cartesian") appeal to introspectible mental items, for him possession of a concept can only be a matter of behavior and dispositions to it. Concepts represent, then, in the loose sense in which a rule represents how an action should be performed.[13] The actions in question are, in the first instance, judgments: according to Kant the understanding "can make no other use of [its] concepts than that of judging by means of them". But they are also cognitions of objects, for judgment is "the mediate cognition of an object". Kant's example indicates that they relate objects to kinds:

> In every judgment there is a concept that holds of many, and that among this many also comprehends a given representation, which is then related immediately to the object. So in the judgment, *e.g.*, "All bodies are divisible", the concept of the divisible is related to various other concepts; among these, however, it is here particularly related to the concept of body, and this in turn is related to certain appearances ... These objects are therefore mediately represented by the concept of divisibility. (A68/B93)

Note that the capacities required for concept-possession are all on the side of the subject, so that I can possess a concept (be able to relate it to objects) even if it is not possible in principle for objects falling under it to exist. The concept of a vehicle that accelerates to beyond the speed of light would be an example (from the point of view of modern physics).

On Kant's view, all definitions are complete: a concept is not distinct until all criteria for its application are made explicit. "Definition" is best understood as the ideal at which we aim when we try to make concepts distinct (*cf.* Capozzi (1980), 429). Thus Kant frequently refers to the "definition" of concepts of a certain kind, only to deny that they can truly be defined (at, *e.g.,* A729/B757, 24:270, and 24:915). When Kant speaks strictly, he distinguishes definition from "exposition", the "distinct (even if not complete) representation of that which belongs to" a concept (A727/B755). Both definitions and expositions "exhibit" concepts, by listing the marks belonging to them, but only definitions purport to exhaust these marks.[14]

Since the marks by which particulars count as instances of a concept are themselves represented as concepts,[15] definitions as well as expositions contain representations that are themselves subject to being defined. In many cases, a concept *c* can have only an exposition because (as we will see) the application of both *c* and the concepts contained in its putative definition is determined by further concepts which the understanding is compelled to seek. Thus, the understanding can never claim to have exhausted the marks of such concepts. So they cannot be defined. Hence we cannot know their definitions; nonetheless, we can possess them, by being able to recognize objects as instances of them.

Kant classifies concepts along two dimensions: *a priori* or empirical, and "given" or "made" (factitious) (9:141, §101). In the *Critique* (A50/B74 and A320/B377) and the logic lectures (§3, 9:92 and 24:905) he uses "pure" to refer to concepts that originate in the understanding alone. He does not provide sustained argument for the possibility of such concepts. In contrast to this genetic notion, apriority is a matter of justification. In the *Critique* Kant makes it his project to explain how concepts originating in the understanding can relate to objects *a priori* (see A57/B81 and A79/B105).[16] But he does not precisely articulate a notion of justification that applies to subpropositional items of cognition. I think the best way to make sense of Kant's examples is that he counts as *a priori* a concept whose application is justified if experience does not suffice to justify recognition of a particular as an instance of it. (Compare the recognition of an event as causally necessitated with the recognition of an object as a dog, which rests entirely on what is learned about that object in experience.) Otherwise it is empirical.

Given concepts are those which, as J. Michael Young puts it, "we find ourselves employing, and of whose partial concepts [representations of their marks] we are therefore aware, even though we may not be conscious of these partial concepts and hence may not (yet) be able to identify them" ((1994), 337). The explicit representation of marks is a mere heuristic, or at most a psychological necessity, in recognizing particulars as instances of the concept.

In contrast, a particular can be recognized as an instance of a factitious concept only through marks that are explicitly represented by the user of the concept. Kant contrasts given concepts to those that are made *by* stipulating the marks or properties in virtue of which something is to be an instance of them. He claims that a factitious concept "only comes into existence as a result of" the act of defining it (2:276).[17] Specifically, a factitious concept must be defined "synthetically" (24:918), that is, by joining marks that do not already lie in the concept, rather than extracting marks that do. Because there is no such concept prior to this act, the possessor of the concept must also grasp its (putative) definition, in which its marks are rendered explicit. Because there is no such kind of thing before the concept is drawn up, its instances, the objects of that kind, can be stipulated to have a certain mark: "I say I *want* to represent a thinking being that is not combined with any body. Here I have made the concept [of a spirit] for the first time through the definition" (24:914–5).

In fact, not every factitious concept can be defined, as I will shortly explain.[18] (We must keep in mind that it is presupposed in the logic lectures that the cognitions under study have a justified use, and thus that the act of defining a concept always succeeds at its aim.) But only factitious concepts can be defined. Kant holds that in a definition, marks are represented as properties of the object that falls under the concept. But as explications of cognitive capacities, definitions can attach marks only to concepts, not to extant objects that fall under them. It would be, as Kant puts it in an early work, only a "happy coincidence" (2:277) if a mark that is stipulatively added to a given concept also pertains to its instances. To be sure, marks can also be affixed to a concept empirically, through observation of what is common to its instances. Kant refers to the latter as the "exposition of appearances" and claims that if the concepts of natural things (such as water) could be defined, it would be only in this way.[19] But a collection of these marks can only be an exposition, not a definition. On Kant's view, a definition must exhibit its concept "exhaustively" and "within boundaries". But the exposition of natural-kind concepts are neither exhaustive nor bounded, because the concepts themselves are not fixed. Kant claims that experience can "take some [marks] away and add some" by presenting instances which lack the features we include in the concepts, or have features we do not include (A728/B756).

The marks belonging to *a priori* concepts, on the other hand, remain within bounds set by the understanding.[20] Since what we learn of objects in experience does not justify recognition of them as instances of *a priori* concepts, any new marks added in experience will not belong to the concept in the relevant sense. So we might suppose that *a priori* concepts are definable in the strict sense, whether they are given or made. Yet Kant denies that given *a priori* concepts can be defined (A728/B756). He argues that we cannot complete the analysis of such concepts because what we learn in experience has a necessary role in their application.

Kant's view belongs to an Aristotelian tradition, on which concepts are ordered in a hierarchy of species to genus. Every concept ranks above more specific, or "lower" concepts, and below those general concepts under which it stands. Thus, the concepts "anthracite" and "bituminous" rank below the concept of coal, while the concept of mineral ranks above it. These more specific concepts mediate the application of even the abstract concepts produced by the faculties themselves. Just as we recognize something as a canine (and thus a mammal, an animal, and so forth) by recognizing it as a dog, we recognize an action as a duty by recognizing it as what it is owed to oneself or to another. It is not possible for us to possess all the lower concepts that apply to the instances of a given concept, because every concept can be divided to yield others below it.[21] Kant holds that we must possess some of these lower concepts, and that the understanding is subject to a "transcendental principle of specification" (imposed by the faculty of reason, in its drive to expand cognition) which forbids it from halting its search for the rest.[22] The understanding is thus compelled (by Reason) to regard the exposition of given *a priori* concepts as incomplete.[23]

The categories—the *a priori* concepts whose objective reality is proved in the *Critique*'s central arguments—are abstract in another sense, which entails their indefinability. They originate in the understanding alone, and express only the most general features imposed by it. So they omit the (sensible) features in virtue of which concepts apply to given particulars.[24]

In both respects, mathematical concepts differ from given *a priori* concepts. Mathematical concepts include the sensibly represented features through which particulars instantiate them. The synthesis that defines such a concept extends its content "through what is added" (or "accrues") "as a mark . . . in intuition. The mathematician [makes use of] this synthetic procedure in making distinctness in concepts" (9:63). And the expositions of mathematical concepts are genuine definitions. Their expositions do not require, for their completeness, the further concepts that the understanding is bound to seek. For these lower concepts do not belong in their definitions. As I discover that triangles can be scalene, isosceles, or equilateral, I do not gain insight into my ability to apply the concept "triangle". To possess a factitious concept is to explicitly represent *all* of its criterial marks.

Kant holds that a concept has a definition only if it is both factitious and *a priori*. His examples of empirical factitious concepts are concepts of artifacts, but the concept of spirit would also belong to this category.[25] Kant's argument for the indefinability of empirical factitious concepts reveals an important facet of his view of definition. Kant grants that the exposition of an empirical factitious concept may be exhaustive and bounded. But he denies that it is a definition, because it does not *exhibit* the concept by showing it to have a use: "if the concept depends upon empirical conditions, *e.g.* a chronometer,[26] then the object and its possibility are not given through this arbitrary concept; from the concept I do not even know whether it has an object, and my definition could better be called a declaration (of my project) than a definition of an object" (A729/B757). So on his view, the legitimacy of a concept's use is not only required for its definability, but must be apparent from its definition.

Factitious *a priori* concepts meet the standard for definability because we can know from their marks that the ascription of these marks to particulars is justified independently of experience. Kant's discussion of the definition of a circle (as the collection of points in the plane equidistant from a center point) shows that the definition of a mathematical concept does not include marks that are ascribed on the basis of experience. To say that a circle is "constructed" or exhibited in intuition by means of this definition is to say that

> I may always draw a circle free hand on the board and put a point in it, and I can demonstrate all properties of the circle just as well on it . . . even if this

> circle is not at all like one drawn by rotating a straight line attached to a point. I assume that the points of the circumference are equidistant from the center point. (Letter to Marcus Herz, May 26, 1789 (11:53))

In making the concept, we put into its definition only marks that can be "assumed" and leave out those—such as the perceived "likeness" of a figure to the result of a certain constructive operation—which are known through the empirical investigation of particulars.

3. The Apriority and Particularity of Pure Intuition

The freedom we exercise in bringing particulars under mathematical concepts is, for Kant, underwritten by our freedom to exclude marks from their definitions. We can exclude the lower concepts sought by the understanding, on the grounds that they have no role in the concept's application, and marks whose ascription is empirically justified.

Our choice to include or exclude marks appears, on a casual reading, to be the only constraint on the formation of mathematical concepts. Kant suggests that factitious concepts are distinguished by their dependence on the faculty of choice [*Willkür*],[27] which he signals by calling them *willkürlich*, usually translated "arbitrary".[28]

Kant's view of mathematical concepts as factitious *a priori* appears wide open to the positivists' objection: that the marks we put into them cannot be justifiably ascribed to objects of perception. If nothing constrains our selection of marks or ascription of them to particulars, Kant cannot explain how or guarantee that the concepts apply to perceived objects. Nothing seems to prevent us from "exhaustively defining" the concept of a biangle, for instance, as a plane figure enclosed by exactly two straight lines and predicating these marks of whatever we like. But this definition clearly cannot convince us that what we represent in perception is appropriately related to the concept.

To see how well Kant's view withstands the objection, we must consider intuition's role in the making of mathematical concepts. Kant holds that the definition of a mathematical concept proves its "objective reality" (A242*n.*), thus that something answering to the concept can be represented under the conditions that govern our perception. In particular, such a thing can be represented in the space that contains the objects we perceive. On his view, mathematical concepts can be "determined" (meaning, in this case, applied to objects in space[29]) through an *a priori* activity called "construction in pure intuition" (A723/B751; *cf.* A240/B299). Now to construct a concept is to "exhibit" or "give" (A722/B750) *a priori* an intuition corresponding to it, and Kant claims that the object of a mathematical concept is "first given" through the synthesis that generates the concept (A234/B287). So, on Kant's view, the very act of making the concept is also the exhibition of a *purely* *intuited* object. The concept's definition is likewise an exhibition. To show that mathematics has definitions, Kant argues that "the object that [mathematics] thinks it also exhibits *a priori* in intuition", which "surely can contain neither more nor less than the concept, since through the explanation of the concept the object is originally given" (A729–30/B757–8). Construction in pure intuition thus, as Joëlle Proust notes, "offers the paradigm of" the "closure" that distinguishes *a priori* concepts from those defined in terms of the "inexhaustible" genus-species hierarchy ((1989), 41). In a letter (to Marcus Herz, May 26, 1789) Kant clarifies that the possibility of a circle is "*given* in the definition of the circle, since the circle is actually constructed by means of the definition, that is, it is exhibited in intuition, not actually on paper (empirically) but in the imagination (*a priori*)" (11:53).

However, it is not clear what this "proof of objective reality" really accomplishes. For Kant sharply distinguishes ordinary perception (as empirical intuition) from pure intuition. The positivists would question our right to speak of objects so remote from experience. Even if we reject their strictures on linguistic content, and grant meaning to the notion of an object of pure intuition,[30] it is not clear how these objects can guarantee objective reality. For it is unclear how the conformity of *imperceptible* objects to mathematical concepts can guarantee the conformity of objects of perception.

Strawson gives us a way to understand pure intuition that is not subject to these objections. More recent interpreters share his view that the function of pure intuition is to mediate the application of concepts to empirically represented objects, not to represent objects. I will argue that this way of understanding the representation that establishes a mathematical concept's applicability conflicts with Kant's most closely held tenets. In the following section, I present another interpretation of the construction that proves a mathematical concept's applicability.

On Strawson's interpretation, we apply concepts to perceived objects by comparing those objects with "phenomenal" prototypes, which are represented in pure intuition. A purely intuited object is not an object of a special kind, but a look that an empirically represented object can have: when, namely, we recognize it as an instance of a geometrical concept. Strawson does not conceive these looks as universals in which perceptible objects somehow participate. Looks are no more general than the objects to which geometry applies, for they *are* objects to which geometry applies: the "phenomenal figures" of which geometry is true. Because a physical thing must have some look or other in order to be recognized as an instance of a concept, our ability to recognize instances is constrained by the conditions on the representation of "looks". But since looks are just the looks of perceived objects, these conditions are conditions on perception. So, since a concept's definition explains the ability to recognize its instances, on this interpretation we are constrained to define only concepts whose instances can be represented under the conditions on perception. We cannot define biangles, for instance, because

> any picture we are prepared to give ourselves of the meaning of ‘two straight lines’ is different from any picture we are prepared to give ourselves of the meaning of ‘two distinct lines which are drawn through the same points’ in a way which we count as essential to having pictured what these expressions mean. (283)

It is by comparing the prototypes of “straight line” and “lines intersecting at exactly two points” that we realize that the marks of the biangle cannot both be applied to the same object.

On Strawson’s interpretation, the applicability of defined concepts to perception is guaranteed. But Strawson’s account cannot explain the apriority of mathematical concepts.[31] On it, geometrical concepts are applied to instances by visually inspecting those instances. Strawson makes clear that instances we construct, just like physical objects we do not create, are recognized by means of the “visual effects” that the constructive operations produce. But Kant explicitly denies, in the letter to Herz of May 26, 1789 (quoted above), that there is some way a figure must look in order to be recognized as an instance of a geometrical concept.[32] This passage presents a challenge to anyone who takes Kant to be a “prototype theorist”:[33] to identify one or more respects in which an object must resemble a prototype in order to count as instance, when the freehand figure counts as a circle even though it is “not at all like” the one drawn with a compass.

Another way to understand pure intuition’s role in proving objective reality is to suppose that the conditions on perceptual space govern a kind of activity, rather than a kind of object. Since the act (“synthesis”) through which a mathematical concept is generated is also a construction of the concept, the activity of combining marks is governed by the constraints on constructive activity. So any concept that we can define necessarily applies to what we can construct in pure intuition. Now to ground the applicability of mathematical concepts to objects of perception on our right to apply them to what we make in pure intuition, we must show that the conditions on purely intuited objects also apply to objects of perception. Howard Duncan and Lisa Shabel have argued, in a Strawsonian spirit, that to purely intuit an object is just to represent an empirical object in a special way: as one that is or could be *constructed by us*. This interpretation draws support from Kant’s claim that when a triangle is constructed “by exhibiting an object corresponding to this concept … on paper, in empirical intuition”, the drawn figure “is empirical, and nevertheless serves to express this concept without damage to its universality, for in the case of this empirical intuition we have taken account only of the action of constructing the concept” (A713–4/B741–2). In Shabel’s words, intuition that represents an “actually drawn” object of sense-perception "functions as" pure intuition when it permits the object to be "considered in conjunction with the procedure for the construction of such objects" ((2003), 94). Duncan emphasizes that the "objective reality" of an "arbitrarily invented" concept is proved by "the act whereby we construct the concept", where the object is always empirical ((1987), 39).

Since pure intuition's role (thus understood) is to represent an act or procedure rather than the object that results from it, I can represent, in it, a procedure I am not now executing. I can thus apply concepts to empirical objects I do not construct, as when I recognize a plate as a circle by purely intuiting the process of tracing its outline with a compass.

If (as on this view) even the ascription of marks to *empirically* intuited particulars involves constructive activity, Kant's view of definition can be developed in an appealing way. I have argued that definitions in general articulate recognitional capacities. Our only clear examples of definitions, which come from mathematics, express constructive activity. This indicates that recognition in general involves constructive activity. Now, since constructive activity is constrained by the form of intuition, so are recognitional capacities (and the definitions that explain them).

Duncan and Shabel express an important insight: if recognition is explained in terms of the activity of constructing particulars, it cannot require the execution of this activity. Since the activity produces new particulars that fall under the concept, this view of recognition can explain only the application of concepts to these new particulars, not to objects of perception that exist before they are recognized as instances. If the application of concepts to instances that are *not* constructed as such is to be understood in terms of constructive activity, then the new particulars must serve as prototypes. So to avoid the problem of specifying respects in which perceived objects must resemble prototypes, we must suppose that the constructive activity involved in recognition can be represented without being executed.

The problem with this interpretation is that Kant does not claim that in order to purely intuit an object, I must represent the process by which it is constructed. At A713/B741 (quoted above), he holds that (the concept of) a triangle can be exhibited *either* by attending to the action and "on paper, in empirical intuition" *or* "through mere imagination, in pure intuition". In fact, he seems to deny that intuition involves the representation of a specific procedure or occasion of construction. He writes (in a letter to Reinhold, May 17, 1789) that

> Science [*sc.* mathematics] has to do with the properties of an object, not with the manner in which the object may be produced under given conditions. If a circle is defined as a curve all of whose points are equidistant from a mid-point, is not this concept *given in intuition* [my emphasis]? And this even though the practical proposition that follows, *viz, to describe a circle* (as a straight line is rotated uniformly about a point), is not at all considered. (11:43)

Duncan and Shabel can reply that pure intuition involves consideration of *a* procedure, but not a *specific* procedure: neither a specific method, nor the conditions under which it is applied. But since performances constitute the particular tokens of a procedure-type, a procedure can be represented as particular only by representing a performance of it. If representation omits the conditions under which a procedure is implemented, perhaps it can still pick out the performance demonstratively, as *this action*. But to represent a performance in this way, an agent must have some conception of *how* she is carrying out the procedure. Thus, representation that includes neither a method nor an occasion of construction is not rich enough to represent a performance. Since the representation of a procedure can omit both, it is not particular, and so cannot be intuition by Kant's lights.[34]

I will not pursue the question of how the notion of pure intuition should be understood because, as we are about to see, it is less important for his view of geometry than is representation of another kind.

4. The Problem of Heterogeneity

The example of mathematical definitions indicates that recognition involves constructive activity. Since recognition cannot be understood in terms of the comparison of perceived objects with a prototype, constructive activity must involve the representation of a pattern or rule for construction, rather than the generation of a particular. The representation of this activity cannot be pure intuition. As other commentators have noted,[35] Kant's taxonomy of representations has an entry specifically for it: the *schema*, or "representation of a general procedure ... for providing a concept with" the intuition of a particular that falls under it.[36] Because they are distinct from the intuitions produced by the imagination's synthesis, schemata can play the roles in recognition that (on my view) intuitions cannot. They can determine the application of concepts without themselves serving as prototypes to which particulars are compared, and give us a way to represent constructive activity without executing it.

The notion of a schema, and the circumstances of its introduction, are notoriously obscure. The "Schematism" adduces "the sensible conditions under which alone" concepts can be "employed" (A136/B175). The application of concepts to intuition requires what is represented in intuition to be in some way like, or "homogeneous with", that which the concept contains. Insofar as a concept does not share the character of sensible representation, its application must thus be mediated by representation that "stands in homogeneity with" both concept and intuition. This mediating representation is the schema (A138/B177). Kant does little to explain the heterogeneity between concept and intuition, or how the schema overcomes it.

For commentators such as Strawson, who emphasize the particularity of intuition over its other defining features, the Schematism is apt to seem especially mysterious. For they must understand it as an account of how *general* concepts apply to their *instances*. It is then not clear what is gained by the interposition of mediating representation. Worse, on the view of concepts that I (and Strawson) attribute to Kant, there seems be no room for such an account. Thus Jonathan Bennett dismisses the Schematism as "a doomed attempt to satisfy an unsatisfiable demand". Given that I possess a concept, there can be no further explanation of my ability to use it, since "having a concept involves being able ... under favourable sensory circumstances, to apply it to its instances" ((1966), 146). But in his own work, Strawson acknowledges and tries to solve something like the problem of the Schematism, and he finds in the *Critique* a related (but importantly different), intelligible problem and solution. Consideration of Strawson's treatment will show exactly how much importance he accords to the exercise of imagination that supplies geometrical concepts with "phenomenal" meaning.

Strawson's (1953–4) traces a line of argument elaborated in Part II of (1959). In the earlier piece, Strawson observes: "When it has been said that ... there must be criteria of distinctness and (where applicable) of identity for individual instances of a general thing, something of central importance still remains unsaid". For to mention or give these criteria is to speak generally, not to bring particulars into discourse or "indicate how" to introduce them. What remains is to "take account of the means by which" we determine or select a "point of application" for the criteria, *i.e.* "mention, refer to, something to which these criteria are to be applied", *as* such a thing (36). Strawson thus proposes to explain the application of concepts to instances, in cases of identifying reference to those particulars (*cf*. (1959), 181).

Strawson finds a basis for such an account in "the possibility of making statements" (let us call them "unloaded") which neither make use of, nor presuppose statements that use, "the notion of individual instances" ((1953–4), 37), but "provide the materials for" introducing particulars into discourse. Examples of such statements in ordinary language are "feature-placing", for example, "Snow is falling" or "There is gold here". Strawson suggests that their use involves "a level of thought at which we recognize the presence of", or "signs of the past or future presence of", a feature, without yet thinking identifyingly of particulars with that feature ((1959), 205). The notion of a particular, say, patch of snow can then be seen as "something logically complex in relation to the two simpler notions of the feature and of placing" ((1953–4), 38). In (1959), Strawson acknowledges that such particulars "can scarcely be said to constitute a fair or reasonable selection from the class of" basic particulars (*i.e.* material bodies). For they are instances of sortal universals "whose names incorporate, as a part, the names of kinds of stuff (*water, gold*) which seem supremely ... well adapted to be introduced as universal terms into feature-placing sentences". But "the sortal universals of which basic particulars are more characteristically instances (*e.g.* men, mountains, apples, cats, trees) do not thus happily separate into indications of a particularizing division, such as *pool* or *lump*, on the one hand, and general features, such as *water* or *gold*, on the other" (205).[37] The program of introducing particulars on the basis of unloaded statements thus threatens to abort. It is "surely more difficult"

> to envisage a situation in which, instead of operating with the notion of the sortal universal, *cat* or *apple*, and hence with the notion of particular cats and apples, we operate with the notion of a corresponding feature and of placing . . . Is it not perhaps the essential difference between, say, cats and snow, that there *could* be no concept of the cat-feature such as the theory seems to require, that any general idea of cat *must* be the idea of *a* cat, *i.e.* must already involve criteria of distinctness and reidentification for cats as particulars? (205)

Strawson's response to the difficulty is to concede that any general concept of cat "must already include in itself the *basis* for the criteria of *distinctness* which we apply to particular cats. Roughly, the idea of the cat-feature, unlike that of snow, must include the idea of a characteristic shape, a characteristic pattern of occupation of space". Since this idea "leads naturally enough to that of a continuous path traced through space and time by such a characteristic pattern", which "in its turn provides the core of the idea of particular-identity for basic particulars", the concept of the cat-feature "provides a basis for the idea of reidentification of particular cats" as well. But Strawson insists that there is still a gap between having the concept of the cat-feature and having the idea of reidentifiable particular cats. For proof, he envisages a "language-game" in which "the general name for a kind of thing" is uttered in the presence of things of that kind, and changes in the quantity of kind present ("More cat", "Cat again") are registered, but no distinction is made between "Another cat" and "The same cat again" (206–7).[38]

Strawson's conclusion, that general concepts such as *cat* are associated with shapes, coincides with Kant's (as we see in the following section). But this is not how Strawson understands the Schematism. For Strawson here assumes that we are able to represent mind-independent particulars in space, while he reads the Schematism as (part of) an account of this ability. Strawson takes it as obvious that intuition can make particulars available to the mind only in conjunction with concepts: for "any item even to enter our consciousness, we must be able to classify it in some way, to recognize it as possessing some general characteristics", *i.e.*, we must have some general concepts ((1966), 20). According to Strawson, Kant argues (in the Transcendental Deduction) that the concepts employed must "suffice to yield" the conception of "a unified objective world, through which [a subject's] series of experiences . . . forms just one subjective or experiential route", for the subject even to be concious of experience as his (26). Certain special concepts would find employment in any conception of such a world (and so are essential to any experience that we could make sense of as our own).

Kant identifies these as the categories, but Strawson, unpersuaded by this move, claims only that they must be of objects "existing within an *abiding* framework within which they can enjoy their own relations of coexistence and succession" and can be encountered by us at different times (27). The Schematism establishes the possibility of representing mind-independent particulars by filling in the account of their application.

The Schematism's role is specifically to fill a gap in the account. Strawson emphasizes that the categories' schemata pertain exclusively to *temporally* organized intuition, "for it is the temporal character of experience that is invoked in" Kant's premises (while it "emerges in the course of the argument" that the framework required for the categories' application must be conceived as spatial ((1966), 31)). Before the necessary applicability of the categories is shown, this intuition can be conceived as "fleeting" perception or "subjective events". But, Strawson maintains, the "natural and unforced account of my perceptions" has to be "a description involving the mention of something not fleeting at all, but lasting, not a subjective event at all, but a distinct object" ((1971), 87). This raises "the question of what is necessarily involved in this being the case". The puzzle is not how we could have such concepts (86), but how they can thus "infuse" or "soak" our perceptions, so that the realist view of the world implicit in them should not be a *theory* with respect to which sensible experience is "interpreted", but rather embodied in the experience itself (92–3; *cf*. (1979)).

To solve it, Strawson reflects on what is involved in taking an object to endure over time, as the same object, and in recognizing it as an object of a certain kind. He argues that "there would be no question of counting any transient perception as a perception *of an enduring and distinct object* unless we were prepared or ready to count some different perceptions as perceptions of one and the same" object ((1971), 88, emphasis added). Since the present perception "would not be just the perception it is but for" these other perceptions, the thought of them "has . . . a peculiarly intimate relation to our counting or taking [it] as the perception of such an object", so that they can be said to be "*alive* in" it (88–9). Similarly, to see something *as an object of a certain kind* "is implicitly to have the thought of other possible perceptions". These may be related to the actual perception as perception of the same object, as when something seen "as a dog, silent and stationary," is necessarily seen "as a possible mover and barker", even without "giving yourself actual images of it as moving and barking". Or they may relate to the actual perception as perception of other objects of the same kind.[39] In either case, thought of these perceptions can be said to "soak" or "infuse" the present perception (89).

So the Kantian insight, as Strawson understands it, is that picturing or something very like it is required for the use of empirical as well as geometrical concepts. But the crux of Strawson's disagreement with Kant is that the ability to give ourselves pictures has no role in the application of geometrical concepts to "physical" space (the space described by scientific theories). It is necessary only for their application to "phenomenal" space (and thus, in Kant's context, for them to belong to a geometry). Since this exercise of the imagination conditions only one antiquated (and thus familiar—but not central) use of the concepts, it has no special relevance for an overall theory of concepts. The primary use of empirical concepts, in contrast, is "phenomenal": to ascribe "phenomenal properties" (sensible qualities). (Of course, "ordinary concepts of objects", such as "our concepts of cabbages, roads, tweed coats, horses, the lips and hair of the beloved", may also have a "physical" use, within or according to the principles of a scientific theory. But, Strawson insists, "[s]urely we mean by a cabbage a kind of thing of which most of the specimens we have encountered have a characteristic range of colours and visual shapes and felt textures; and not something unobservable" ((1979), 54–5).[40]) We have seen that it is integral to this (ordinary, unreflective) use of the concepts to see the objects falling under them as capable of certain behaviors (as "possible movers"). The activity of imagination required for their deployment now solves the puzzle of how concepts enter so intimately into experience. The "infusion" of the present perception with thought of the other*s is the answer* to the puzzle. For Strawson, laying weight on Kant's claim that the "combination" of perceptions required for experience is "such as they cannot have in sense itself", concludes that *the concept* serves to link these perceptions (87).

This may not seem like much of a solution. To say that perception has generality of its own does not answer, but only postpones, the question of how something with the generality of a concept can materialize (be "embodied") in the perception of a particular. But Strawson could reply that the demand for explanation can be pushed only so far, because only so much distance can be put between perception and concepts of enduring obejcts. To consider the contribution of the sensible faculty in isolation from these concepts is to conceive it as fleeting impressions and images. Kant's advance beyond his phenomenalist-idealist predecessors was to realize that such a conception of experience betrays our "plain reports" ((1971), 86) and cannot stand on its own.

I agree that the Schematism does not so much attempt to show *how* heterogeneous representation is linked as to show *that* it is, but I dispute Strawson's intepretation of the problem. On Kant's view, there is room for a more abstract account of how sensibility and the understanding relate in general, because the categories are not just concepts of enduring objects.[41] The problem of the Schematism, of how they can be joined to sensible representation, can therefore be raised without supposing that discursive and sensible components can be separated in the experience of objects falling under such concepts as *dog* and *tree*. Kant indicates that the problem arises only with respect to the categories: they alone are "so different and heterogeneous from" the representations of their objects *in concreto* that "a special discussion of the application of the former to the latter is necessary".

Although the problem of the Schematism is specific to the categories, its solution sheds light on the application of pure concepts in general. Kant indicates that the need to provide the categories with schemata is due to the same factors that make them indefinable. They cannot be defined because *apart from* their schemata, they express only logical functions of judgment, which already structure all definitions.[42] But it is because they "can contain nothing but the logical function for bringing the manifold under a concept" that they omit "the general *sensible condition*" by which a concept has "determinate significance and relation to any object" (A245). In linking the indefinability of categories with the *exclusion* from them of sensible conditions, Kant at least suggests that concepts that *can* be defined *include* their schemata (*cf.* Capozzi (1980)). Since (as we have seen) a concept's definition sets forth the sensible features in virtue of which particulars fall under it, it is plausible to suppose that the same content that is expressed discursively in a definition is also prescribed to the imagination, in the form of a schema. Similarly, the exposition of a given empirical concept expresses the features which, as far as our experience shows, hold of all objects falling under it. Kant's identification of both schemata and concepts as rules, together with his admission that (on pain of regress) rules cannot be demanded to explain the application of rules, provides further evidence that on his view concepts typically include their own schemata.[43] But the categories cannot acquire sensible content in the same way as factitious *a priori* concepts, because they cannot be defined—or as given empirical concepts, because no experience can determine, of objects sharing a sensible feature, that they fall under a category. The problem of the Schematism is thus to explain how categories can have the sensible content that empirical and factitious *a priori* concepts already include.

5. Geometry's Role in the Schematism

Kant solves the problem by providing schemata for the categories. He argues that there must be a "third thing", the schema, that is "homogeneous" with both the category and the intuitively represented instance (A138/B177). As rules for the imagination, schemata must be consistent with the conditions on perception. To possess such a rule is to be able to generate intuition displaying the features that license the concept's application.

Geometry is the model for Kant's solution to the problem of heterogeneity between category and intuition. He uses the representation of a triangle to illustrate the notion of the "third thing". He assumes that geometry involves not only intuition, but representation that captures what a particular has in common with other instances of a concept. He makes clear that the latter is not intuition. The sensible representation that "grounds" the concept of a triangle is not an "image", where an image is what we would ordinarily think of as an intuition: the perceptual representation of a particular.[44]

> No image of a triangle would ever be adequate to the concept of it. For it would not attain [*erreichen*] the generality of the concept, which makes this valid for all triangles, right or acute, *etc.*, but would always be limited to one part of this sphere. The schema of the triangle can never exist anywhere except in thought, and signifies a rule of the synthesis of imagination with regard to pure shapes in space. (A141/B180)

Kant indicates that the schema of a triangle differs from any image in that it is not determined with respect to such features as the size of its angles. It is thus suited to express the universal applicability of a pure concept. But while the schema attains to, or matches, the concept's generality, it is not represented as a concept. Kant claims that mathematics "considers the universal" *in* an individual that "corresponds to" the object of the concept "only as its schema", so that the object "must ... be thought as universally determined" just as the individual is "determined under certain general conditions of construction" (A714/B742). Here, then, he uses the assumption that geometry bases conclusions valid for all experience on the representation of particulars, to secure the applicability to experience of pure concepts in general.

All of the concepts in Kant's taxonomy—given, factitious, empirical, and *a priori*—have schemata. Kant's example of the schema of an empirical concept brings out the distinction between schema and image still more clearly. The schema associated with the concept of a dog "signifies a rule in accordance with which my imagination can specify [*verzeichnen*] the shape of a four-footed animal in general" (A141/B180). While the specification of a triangle's shape need not be a triangle, the specification of a dog's shape cannot be a dog.

The introduction of schemata for empirical concepts fills a gap in the account of the *a priori* conformity between the pure concepts of the understanding and the intuition produced by the sensible faculty. The provision of schemata for the categories is only a partial answer to the fundamental question of why intuition should conform to pure concepts. This question cannot be answered *a priori* just by giving rules for generating intuition corresponding to the categories themselves, because their application to intuition is essentially mediated by more specific, and ultimately empirical, concepts. By motivating the understanding to seek the requisite "lower" (more specific) concepts in experience, the transcendental principle of specification ensures that some of the conditions on the categories' application are fulfilled. But their application is subject to conditions on the sensible faculty as well as the understanding: because intuition must conform to the lower concepts in order to conform to the categories, the intuition that displays features licensing the application of the latter must also display features that license that of the former. Because the lower concepts are empirical, their applicability to intition is assured. But because they are empirical, it is not clear why intuition displaying the features in virtue of which particulars fall under *them* should also display features in virtue of which *categories* apply to it. While experience justifies the ascription of empirical concepts, it cannot justify the ascription of a category.

As Kant conceives it, the schema through which an empirical concept is applied to intuition also provides for the application of higher concepts.[45] The schemata of empirical concepts are geometrical. The schema associated with the concept of a dog, for instance, provides the imagination with a rule for the specification of "the shape of a four-footed animal *in general*" (A141/B180, emphasis added). Taken literally, this rule directs the imagination to generate intuition that instantiates the concept *quadruped*, which is more general than *dog*. (In the Linnaean taxonomic trees that constitute the main scientific application of Aristotelian conceptual hierarchies, *quadruped* appears above *canine*, the genus that contains *dog* as a species.[46]) We might think of the rule as prescribing the shape—head and tail joined to a horizontal cylinder, which rests on four vertical cylinders—that is common to all quadrupeds.[47] Since intuition must display additional features to instantiate the concept *dog*, this rule is not a sufficient condition for generation of such intuition; but it is necessary. Kant's point is that the recognition of intuited particulars as dogs requires not just conformity with, but representation of, the rule expressed by the schema. Thus, subjects who are able to bring intuited particulars under the concept *dog* are also prepared to generate intuition that instantiates concepts ranked above it. That is, they are prepared to generate intuition displaying features that license the application of these higher concepts. But since intuition generated according to the rule need not instantiate the concept *dog*, it may display *only* the features that license the application of more general concepts.

We may suppose that more general concepts such as *quadruped* have their own schemata, which express rules governing still more abstract geometrical features. To recognize something as a quadruped, for instance, we must represent it as a (roughly) solid volume whose area and volume satisfy certain conditions. As we ascend through the conceptual hierarchy, the features whose representation is prescribed by schemata become more and more abstract, till ultimately we reach those (quantities and their increase and decrease) which provide for the application of the categories themselves (such as substance and cause). So even though the marks of empirical concepts are ascribed on the basis of experience, intuition generated according to their schemata can instantiate concepts whose marks cannot be prescribed on the basis of experience. The activity that fulfills discursive conditions on the application of given *a priori* concepts—namely, the formation and use of empirical concepts that rank below them—thus provides for the satisfaction of sensible conditions as well.

It is no accident that the properties common to lower and higher concepts are expressed geometrically, as shapes. Kant claims that geometry's axioms, which state the basic determinations common to its concepts, express *a priori* "the conditions of sensible intuition, under which alone the schema of a pure concept of outer appearance can come about" (A163/B204). Geometry thus plays at least two roles in the argument of the Schematism. It affords a general model for representation that captures what is common to a concept's instances, without being conceptual. It also supplies specific rules, for the generation of intuition that falls under empirical concepts, that provide for the application of higher concepts. Geometry serves, in short, to illustrate the notion of a schema and to express, *a priori*, the content that can be represented according to the conditions on perception.

For mathematical cognition to fulfill these roles, it must afford *a priori* cognition of the capacities of the sensible faculty. As we have seen, the activity of making concepts is central to mathematical cognition. Because the activity of making mathematical concepts is constrained by the conditions on perceptual representation, but not by experience, it is a means by which our capacities for sensible representation can be cognized *a priori*. Strawson is thus right to say that for Kant, the activity that gives *a priori* content to geometrical concepts also expresses "the conditions under which alone things can count as things in space". His error is to regard this activity as one application of the concepts, to phenomenal space, when really it is integral to their formation.

6. Two Kinds of Definition

Kant's doctrine of schemata promises to explain how, and thus fill out his view that, the definition of a geometrical concept proves its applicability. We saw in §3 that a definition cannot prove the concept's applicability, as Kant claims it does, unless our ability to "make" (define) concepts is constrained. In particular, the making of concepts must conform to the conditions on perception. On my interpretation, the definition of a concept explains our ability to recognize its instances. So to show that the making (definition) of concepts is constrained by the conditions on perceptual space, Kant must show that recognitional capacities are constrained by them. I hold that recognition involves the representation of rules for constructive activity, which are expressed by schemata. So the conformity of schemata to the conditions on perception ensures the conformity of recognitional capacities, and with them definitions, to these conditions.

At this point, it can be objected that Kant is not entitled to claim that schemata are subject to the conditions on perception. If the schema is represented as a general procedure, nothing seems to distinguish it from a concept.

Kant indeed describes it as "really only [*eigentlich nur*] . . . the sensible concept of an object" (A146/B186). So with his theory of schemata, he seems merely to stipulate that, rather than to explain how, the origination of mathematical concepts is subject to the conditions on perception.

This objection can be seen to be a version of Strawson's (itself, of course, a version of the positivists'). Strawson objects to Kant's identification of "phenomenal" space with physical space that it is based merely on the absence of alternatives. Recall that Strawson takes Kant to identify physical space, the space containing mind-independent objects, with phenomenal space, the domain of the picturable. On this view, whatever we can picture must hold of the objects we perceive. I take Kant to identify the conditions on schematization with the conditions on perception, so that any concept we can schematize must necessarily apply to objects of perception. The latter view may well seem just as naïve as the former.

But Kant's view that the formation of mathematical concepts is constrained by the conditions on schematization, and thus the conditions on perception, does not rest on the absence of alternatives. Or so I will argue. Kant does not just entertain the notion of concepts that do not have schemata, but holds that we can form such concepts. Because they have no legitimate use, these concepts do not belong to any of the classes (given, factitious, empirical, *a priori*) discussed in Kant's logic lectures. To fit them into his taxonomy, we must distinguish two kinds of definition.

In Kant's time, it was customary to distinguish between "real" and "nominal" definitions. According to Kant, a real definition differs from a nominal definition in that it "does not merely make distinct a concept but at the same time its *objective reality*" (A242*n.*). Kant's notion of a real definition, as both the explicit representation of a concept's marks and proof of its objective reality, derives from Leibniz. On an Aristotelian tradition, a nominal definition explains how the name of a thing is applied, and serves as a placeholder for the real definition that gives the thing's essence if the latter is not yet known. Leibniz accepts this distinction, but not the traditional opposition between real definitions, as metaphysical principles concerning things, and nominal definitions, as epistemic or semantic principles. On his view a real definition is also a nominal definition, that is, an explanation or "enumeration" of the marks by which the thing is individuated.[48] A property that serves as a nominal definition "constitutes a real definition" when it "makes known the possibility of the thing" ((1686), 57).

With Leibniz, Kant holds that the properties ascribed to an object in its real definition make its possibility known. While the "external" marks contained in a nominal definition serve only to distinguish the thing from others, a real definition contains "inner" marks or "determinations in the thing itself, by which it can be cognized without comparison with other things" (24:106). To collect marks in a nominal definition is not to ensure that all these determinations belong to one thing. But a real definition cuts at the joints. The defined object is possible insofar as it has the kind of unity that requires explanation and contains its own explanatory ground. By making this ground explicit, the real definition proves the object's possibility, and furthermore "suffices for cognizing and deriving everything that belongs to the thing" or for "explaining the thing internally" (24:919).[49] So for Kant, the reality of definitions consists in a kind of completeness.[50]

I have suggested that the content expressed discursively by a definition is also prescibed, in the form of a schema, to the imagination. This explains why a complete articulation of a concept's marks should also be proof of its objective reality. Kant regards mathematical definitions as paradigmatic of real definitions (A242*n.*). We saw in §2 above that a mathematical concept's definition is complete in the precise sense that it includes all the marks relevant to the application of the concept. Kant claims that because the object of a mathematical concept is "originally given" through "the explanation of the concept", "*i.e.* without the explanation being derived from anywhere else", the object is "exhibited *a priori*" in an intuition that "contains neither more nor less than the concept" (A730/B758). We can now take his point to be that the intuition is synthesized in accordance with the concept's schema. Because the schema conforms to the conditions on sensible representation, it affords sensible representation of what the concept contains. It provides for the sensible exhibition of *all* that the concept contains, because it corresponds to the concept's definition, which exhausts its marks. And the intuition synthesized according to it may contain *only* what the concept does, because intuited particulars need have only the properties expressed in the concept's definition (and no others) to be recognized as instances of the concept. In contrast, a nominal definition does not provide for the exhibition of the concept, because it does not express all the features that an intuited particular must have to be recognized as an instance of it.

The distinction between nominal and real definability and the given/factitious and empirical/*a priori* distinctions overlap to the extent that only factitious *a priori* concepts can have real definitions (*cf.* Capozzi (1980), 427–8). (As we have seen, empirical concepts are not definable. Kant argues that given concepts can have only nominal definitions, for the same reasons that their definitions must be regarded as expositions (*cf.* A728/B756).[51] The marks of a given concept are made explicit by differentiating species within the same genus. But the object of a given concept, in contrast to that of a factitious concept, is not given through a definition of the concept. Because "I cannot compare" the concept's object "with all possible things," "I can never find the marks to distinguish the thing from all possible other things". Thus, I cannot "know that my marks exhaust the whole concept," *i.e.* that my definition is real (24:920).) But the distinction between nominal and real definability applies more widely. For, as Emily Carson argues, by forming a nominal definition we can come to have a concept that cannot legitimately be applied to sensible representation. Such a concept cannot be either given or made.

Carson observes that for Kant, the thought of a two-sided rectilinear plane figure appears to be an example of a concept that "cannot be given an object" ((1997), 505). Kant claims that "in the concept of a figure that is enclosed between two straight lines there is no contradiction, for the concepts of two straight lines and their intersection contain no negation of a figure". The impossibility of the object represented in this way "rests not on the concept in itself, but on its construction in space, *i.e.* on the conditions of space and their determinations" (A221/B268). I have argued that in the notion of a schema, Kant has a way to explain how these conditions bar us from recognizing objects as instances of the concept. They bar us from formulating the rule that would let us synthesize the intuition of a space enclosed by exactly two straight lines. But they do not bar us from joining the relevant marks, represented as concepts in the discursive faculty. So while we cannot *make* this concept, in Kant's technical sense, we can nonetheless *come to have it*. Carson suggests that we do so by giving "a mere nominal definition in accordance with concepts alone," which is "something like a collection of characteristic marks" (504).

Carson's point brings out the liberty we exercise in making concepts. It militates against an interpretation, such as Strawson's, on which any conception of a plane figure must have sensible content. For the example of a two-sided plane figure appears to belie Strawson's view that for Kant, anything we can conceive as a geometrical figure must conform to the conditions under which it can be "pictured". However, we must take care not to overestimate our freedom to form mathematical concepts.[52] For the alternatives among which we choose do not seem to compete on the same footing.[53] While concepts of two-sided plane figures and the like are represented through nominal definitions, it is not clear that the definitions of mathematical concepts can be merely nominal. Rather, it appears that the definition of a mathematical concept must be real, for it provides for sensible exhibition of all the marks that an object must have to be brought under the concept. So we do not choose between alternatives in ignorance of their applicability, but between an alternative whose applicability is certain and one whose is not.

This point fills out Kant's epistemology of geometry in two ways. First, it clarifies the sense in which the definition of a mathematical concept proves its objective reality. It does not seem to prove in the sense of putting to rest doubts about the concept's objective reality—at least, not for anyone who possesses the concept. (What it would be to doubt its objective reality without possessing it is a problem that I will not try to solve.) One who possesses the concept cannot genuinely doubt the legitimacy of its use, for she must grasp its definition, and thus (I have argued) represent the schema by which the imagination can synthesize representation of particulars falling under it. The definition proves, rather, in the sense of making explicit the grounds for what is already rightfully believed.[54]

Secondly, it allows us to distinguish Kant's position from conventionalism.

7. Kant and the Positivists, Again

The positivists grant that the primitives of pure geometry can be "defined" after a fashion, even prior to its application. No geometry can claim to explicate *the* concept of space, since no geometry can be known *a priori* to apply to physical space. Rather, each geometry is concerned with certain initial stipulations. These stipulations can be regarded as definitions of geometrical terms.[55] They express our determination to use the terms in such a way that any objects denoted by them will satisfy the stipulated conditions.[56] This intention of ours makes geometry true. The conventions that express it, like Kant's "pure intuition", provide geometry with a subject matter without subjecting it to empirical refutation. But because they do not provide geometry with *sensible* content, they do not make it synthetic.

While Kant's definitions are of concepts rather than linguistic expressions, they are otherwise as the positivists would have them. On Kant's view as well as the positivists', definitions give content to representations by stating criteria for ther application. Both Kant's and the positivists' criteria are stipulative insofar as we decide what perceptual representation will determine the application of a representation (term or concept). So on both views, the necessity conferred on pure geometry (which contains only the defined representations essentially) would seem to be merely conventional: nothing that contravenes these stipulations is a correct use of the representations. And if pure geometry is necessary with respect only to our conventions, it has no more content than the posivists allow it to have. In particular, it has no sensible content. To evade the positivists' dilemma, Kant must explain why the consequences of our conventions should also hold of objects we perceive.

In answer, Kant invokes his idealism. He claims that we are free to ascribe properties to the objects to which our concepts apply because we "make" *the objects*: "we can determine our concepts *a priori* in intuition, for we create the objects themselves in space and time through homogeneous synthesis, considering them merely as *quanta*" (A723/B751). The creation of spatial objects thus appears to be up to us to exactly the same extent as the formation of concepts. Consequently, Kant's resort to idealist metaphysics leaves no way to distinguish his view from conventionalism.

But the notion of a schema is a bulwark by which Kant's view of geometrical concepts can be kept from collapsing into his assertion that spatial objects depend on the mind. It allows us to separate the thesis that *the formation and application of concepts involves constructive activity* from the idealist doctrine that *space and the objects in it are generated by the mind's activity*. The "homogeneous synthesis" of intuition, through which Kant claims we make objects in space, is governed by schemata. So our liberty to create objects is precisely as broad as our freedom to formulate schemata. But, as the example of the two-sided plane figure shows, this is narrower than our freedom to combine the marks of concepts: it is constrained by the conditions on perception. Now, Kant himself would surely deny that anything external to our faculties determines the scope of possible perception. But we need not follow him in holding that the conditions on mathematical concept-formation agree with the conditions on perceptual representation because *we* impose both.

To steer between the horns of the positivists' dilemma, it is enough just to assert this agreement. So long as it is not possible for us to form mathematical concepts that do not agree with the conditions on perception, the sensible content of the concepts we do form is guaranteed, prior to any application.

But as long as the agreement is not explained, we seem to have no grounds to accept this *a priori* guarantee. I believe an authentically Kantian account of the agreement can be given without reliance on idealism. We can retain Kant's view of the arbitrariness with which mathematical concepts are formed. We might suppose, in particular, that concept-formation involves either or both of the kinds of choice that Kant posits: in the use or the selection of criteria for a concept's application. First, we could suppose that in applying a mathematical concept's criteria, we choose to disregard countervailing evidence (such as the inclination of "parallel" lines towards one another). Secondly, we could agree that mathematical concepts differ from empirical concepts in that we may choose not to include lower (more specific) concepts among their criteria. Most importantly, we could accept Kant's contention that in geometry, we restrict ourselves to forming only concepts that agree with the conditions on perceptual representation. (To explain how these conditions can be cognized prior to experience, we would need his notion of pure intuition.) We would thus detach, from Kant's idealism, a view that retains some conventionalist elements. But we would not allow the conditions on perceptual representation to be regarded as stipulative.

Notes

[1] Many intellectual debts were incurred during the writing of this paper. Hannah Ginsborg's lectures on the Schematism at Berkeley in 2003 brought me to see the importance of Strawson's interpretation. I am grateful to audiences at the Central Division APA, Chicago, 2007, and Brandeis University, March 2009. For useful discussion and comments on drafts, I thank Lanier Anderson, Tyler Burge, Emily Carson, John Carriero, David Hills, Joseph Hwang, Charles Larmore, Gavin Lawrence, Tyke Nunez, and Charles Parsons. (Apologies to anyone who was inadvertently left off this list.) I owe special thanks to Ian Proops, Daniel Sutherland, Michael Friedman and Daniel Warren for important criticisms and suggestions.

[2] The role of intuition in proof was a central issue in the interpretation of Kant's philosophy of mathematics from the 1960s through the end of the twentieth century. Evert Beth (1956–7), Jaakko Hintikka (in (1964) and (1967)), and Michael Friedman (1992) argued that Kant introduces the notion of intuition primarily to account for the derivation of theorems from first principles (which are comprised of axioms, definitions, and postulates). Friedman makes especially clear why these inferences could not be expressed in the logic available to Kant. Meanwhile, Lewis White Beck (1955), Charles Parsons (in (1969), (1983), and (1992)), and Emily Carson (1997) hold that intuition's role is primarily to show that the first principles have significance for or within experience. In this debate, what matters about definitions is whether consequences can be extracted from them without recourse to intuition; see for instance Beck (1955), 89, and Hintikka (1964), 127.

[3] *Cf.* Capozzi (1980).

[4] Strawson himself takes seriously the possibility that particulars can be represented other than as spatial. In Chapter 2 of (1959), he considers at length whether we can represent particulars while "confin[ing] ourselves imaginatively to what is not spatial" (specifically to "purely auditory" experience). He suggests, however, that Kant would not allow the possibility to be raised. He comments that Kant "would probably think it tautological to say that we cannot *imagine* ourselves possessing" other forms of sensibility or intuition (than space and time), and "would probably also say that it was impossible to *imagine* ourselves *not* possessing both these forms" (62).

[5] The involvement of intuition in geometrical reasoning has the same "place in Kant's theorizing" that it can plausibly be supposed to have "in the systematic development of geometry in general": namely, to explain why geometry developed "systematically", as "a mathematical discipline" or body of purportedly necessary propositions, when it "owes its beginnings of existence" to concern with practical problems of terrestrial measurement ((1966), 286–7).

[6] This attitude is, as Michael Friedman points out, "epitomized in Einstein's famous dictum (in which he has geometry specifically in mind): 'As far as the laws of mathematics refer to reality, they are not certain; and as far as they are certain, they do not refer to reality'" ((1992), 55–6), quoting Einstein (1921), 28). The dilemma is posed in these terms by Schlick ((1932), 344) and Carnap ((1958), vi).

[7] Strawson sets little store by Kant's notion of the "synthetic *a priori*". Thus he does not resist the implication that geometry is analytic, at least by twentieth-century criteria. See the next paragraph save one of the main text.

[8] Strawson suggests that this identification is made natural by "think[ing] of something's counting as a physical body for us in terms of its *appearing* to us, presenting to us a phenomenal figure, a figure of the kind which phenomenal geometry treats of" ((1966), 285).

[9] See Guyer and Wood's edition of the *Critique*, 637 n.3. Kant regards logic as a mere "canon": it gives "principles for the correct use of certain cognitive faculties in general" (A796/B824). It is because logic "is not an organon, only a canon" that it cannot be practical (24:779). Now a canon is possible only if the relevant faculty has a correct use, *i.e.* if it is capable of producing cognition. So on Kant's view, logic is normative for cognition *per se*, but is required to abstract from cognition's semantic content. (Kant's conception of logic is thus narrower than the view which (as MacFarlane argues in (2002)) dominated his intellectual context. On the received view, logic was understood as a body of laws that normatively govern all cognition. It investigates how the understanding "ought to proceed in thought", rather than how it "is and does think and how it has previously proceeded in thought" (9:14), and can contain an organon.)

[10] Kant claims that a definition "does not merely supply other and more intelligible words for the name of a thing, but rather contains in itself a clear mark by means of which the object can be securely cognized, and that makes the concept that is to be explained usable in application" (A241*n.*).

[11] I use "instance" to refer to an object falling under the concept, not the representation that tokens the concept-type within the user's psychology. There is precedent for this usage in cognitive science (see *e.g.* Margolis and Laurence (1999), 25*n*.30.)

[12] Strawson glosses "general concepts" as "capacities for recognition and classification" ((1966), 48). Bennett describes Kant's "actual working use of 'concept'" as "rather thoroughly Wittgensteinian", meaning that "the interest of concepts lies in the abilities with which they are somehow associated" ((1966), 54). (As he understands Kant, to have a concept is specifically "to be able to cope with—*i.e.* generally to sort out true from false among—judgments" (73). Bennett holds, further, that on Kant's view "I cannot articulate a judgment at all, even to consider it as a possible candidate for acceptance, unless I know something about what intuitions of mine *would*, if they occurred, be relevant to its truth or falsity" ((1974), 27). This "concept-empiricism" is not integral to the understanding of concepts as rule-governed activities or abilities.) Wolff claims that "concepts for Kant cease to be *things* (mental contents, objects of consciousness) and become *ways of doing things* (rules, forms of mental activity)" ((1963), 70). Robert Pippin ((1982), 104–111) and Richard Aquila ((1989), 119–122) agree that for Kant, to have a concept is to be able to do something, though each argues that concepts "represent" in a more robust sense. Much of Béatrice Longuenesse's influential (1998) can be understood as an explication of the two "roles" of the categories in the activity of judging (196): first, guiding the unification of sensible representation; secondly, representing the resulting unities in explicit discursive form.

[13] According to Wolff, when an activity is performed according to a rule "we can formulate a set of prescriptions in conformity with which the activity is done" ((1963), 122). Wolff also emphasizes that the rule gives unity to the "parts or stages of the activity" (123). Since concepts are rules, on his account, the "act of conception" which Kant occasionally conflates with the concept is "the consciousness of the rule" (130). It is not entirely clear whether what is thus represented is the prescriptions or the unity.

[14] A critic can accept the exposition of a concept "as valid to a certain degree while yet retaining reservations about its exhaustiveness" (A729/B757). See also §105 of the *Jäsche Logik* (9:142–3).

[15] Definitions are products of the discursive faculty, as is already clear from the fact that they arise through logical investigation. (Kant's argument that the categories cannot be defined, because they are only "ways of defining" concepts, provides further evidence that he views definitions as discursive representations. The argument explicitly assumes that all definitions are produced by the conceptualization and judgment of the understanding: a definition that relates a concept to "a determinate object", through a mark by which its instances are recognized, "would itself be a judgment and have to contain these functions", namely, the categories (A245–6).) So in the context of a definition, marks are represented conceptually. In other contexts, they may be represented as particulars (property-instances), as Smit argues in (2000).

[16] This usage of "pure" in conjunction with "*a priori*" is different from the way these terms are applied to propositional items of cognition in the *Critique*'s Introduction (B3).

[17] *Cf. Vienna Logic*, 24:919: "with a *conceptus arbitrarius* one always has to begin with the definition, because the very concept is produced through this . . . ; without definition, such a concept would be nothing".

[18] The concept of spirit cannot in fact be defined, according to Kant (see note 25 below). In his "Prize Essay", he gives this as an example of a "determination of the meaning of a word" that fails as a definition (2:277). So his claim that this concept is "made through the definition" must be taken to mean that we acquire it through an act that fails at its aim. What recommends it as an example is presumably its prominence within the Wolffian corpus (see Walford's note in the Cambridge edition of Kant's *Theoretical Philosophy, 1755–1770*). In particular, it appears in G. F. Meier's *Vernunftlehre*. For his logic courses, Kant used Meier's abridgment (*Auszug*), according to which concepts can be made through an arbitrary combination of two concepts, one abstracted from (higher than) a concept *c* and one a mark that distinguishes *c* from other concepts of the same rank (§266). In the full text, Meier indicates that the concept of spirit is

made by combining the concept of "thinking thing" with the mark "combined with a body" (§456).

[19] In this case the concepts themselves would be "made empirically" (9:141, §102). In *Reflexionen* 2910 and 2914, Kant indicates that all empirical concepts are made through synthesis of this kind. But at this stage, he does not seem to recognize *a priori* factitious concepts as different in kind from both empirical and "rational" (*a priori*) given concepts, which suggests that the given/made distinction has not yet become an axis of classification distinct from the rational/empirical one. §102 of the *Jäsche Logic* can thus be taken to address the inadequacy of the earlier taxonomy.

[20] As Kant puts it in the *Wiener Logik*, "the concept concerns an object that I am to cognize through the understanding alone; consequently, whatever I am to say of it must be in my understanding, because we cannot go out of our understanding and seek elsewhere" (24:917).

[21] "[I]n the series of species and genera there is no lowest concept or lowest species, under which no other would be contained, because such a one cannot possibly be determined. For even if we have a concept that we apply *immediately* to individuals, there can still be specific differences in regard to it . . ." (9:97, §11*n.*).

[22] This is a "regulative" principle, by which the faculty of reason compels the understanding to "seek always for further differences, and to suspect their existence even when the senses are unable to disclose them" (A657/B685). Unlike a constitutive principle, which provides for the concrete instantiation of concepts, regulative principles yield only rules to guide the search for instances (A178-9/B122-3; *cf*. A664-6/B692-4). Regulative principles serve to motivate (provide ends for) and unify empirical investigation.

[23] Strictly speaking, definition furthers the "distinct consciousness" of *content*, what is contained *in* a concept, and thus differs from "logical division", which makes distinct the *extension*, or what is contained *under* it (9:140, §98). Content is comprised of higher (24:910), and extension of lower (24:760), concepts. So we might think that even if we cannot exhaust the concepts contained *under* a given concept, we can *define* it. Kant indeed holds that while "in the series of species and genera there is no lowest concept or lowest species, under which no other would be contained", there can (even must) be a "highest concept" or "*genus* that cannot in turn be a *species*" (9:97, §11*n.*).

But Kant does not in fact restrict the content of concepts to the genera contained in them. For he primarily thinks of content not as the dual of extension, but as what pertains to a concept insofar as it is "contained in the representation of things" (9:95, §7; *cf.* Young (1994), 341). There seems to be no way to articulate the content (understood in this sense) of a concept without reference to the things cognized through it (as their mark). Accordingly, Kant refers to what is in the "extension of a concept" both as things and as concepts, *e.g.* at 24:910 (*cf*. Longuenesse (1998) 92, *n*.23). More importantly, he includes the marks that determine the application of concepts within their content, as when he counts the color black as part of the content of the concept "Negro" (24:911). (For discussion, see Anderson (2005) and de Jong (1995)). Since finer- and finer-grained specific differences may enter in to the identification of a concept's instances, the concept's content may depend intimately on concepts ranked under it. The endlessness of the descent into lower concepts thus renders logical investigation of content incomplete, even though the ascent into higher concepts terminates. (Although definition as I understand it thus concerns the extension of a concept, it still differs from logical division: while a definition includes all and only the lower concepts relevant to application of a given concept, logical division is restricted to concepts that rank immediately below the given concept, and must divide them exhaustively into species.)

Conversely, the extension may depend on the genera contained in it. In the case of logical division, Kant states explicitly that extension can be made distinct only by reference to content (in the strict sense): "We go up from lower to higher concepts, and afterward we can go down from these to the lower ones—through division" (9:146, §110 *n*.2). The relevance of content to logical division is clear when logical division is regarded as a measure of the quantity of extension (or "sphere": 9:96, §8) of a concept. Kant defines logical division as "the complete representation [of lower concepts] insofar as they are considered according to their difference, in which, taken together, they are equal to the sphere of the whole concept" (24:925). On his view, the relative quantity of concepts can be determined only by placing them within a common hierarchy (as at 9:98, §13, and 24:755; for discussion, see Anderson (2005).) So to determine that a given concept is equal to another concept or collection thereof, one must ascend far enough into the hierarchy to ascertain that there is a single concept under which they all stand. This requirement still holds, though it is trivial to satisfy, when the concepts together exhaust the given concept, as the species contained under it.

[24] This argument shows that the "pure", or unschematized (see section 5), categories are not definable. If we regard the schematized categories as distinct concepts, it is not so clear why they cannot be defined. Kant argues that the sensible content of the schematized categories, unlike that of mathematical concepts, is empirical. What we grasp in these concepts essentially involves concepts, such as existence, whose conditions of application cannot be known *a priori*. (Kant holds that even if we could infer "some existence or other" *a priori*, we still could not "cognize it determinately, *i.e.* be able to anticipate that through which its empirical intuition is differentiated from others" (A178/B221; *cf*. A723/B751).) So experience is needed just to formulate the conditions (sensible features) that determine the application of these concepts.

A good example is the concept of causality. To apply it, we must represent particulars as instances of laws whose content is specified empirically (*cf.* A178/B221). The Second Analogy purports to show that this concept is applicable in the weak sense that, to cognize a sequence of events *a*-*b* as successive, I must accept the causal principle that every alteration is preceded by a cause. It is notoriously difficult to see how this argument could establish the repeatability of any such sequence, since I need not cognize *a* and *b* as tokens of the types (*A* and *B*) governed by particular causal laws. Even though experience does not suffice to justify recognition of a sequence "event of type *A*-event of type *B*" as an instance of causation, we need experience to know that "*A*" and "*B*" hold of any instances of the concept, *i.e.* that they are marks of it.

[25] If the concept of spirit had a justified use, it would be empirical, because it is the concept of a thinking being, and particulars are recognized as thinking through experience of them. To be sure, Kant holds that "the transcendental synthesis of the manifold of representations in general" or "original synthetic unity of apperception" involves *a priori* self-consciousness of a thinking subject. But Kant sharply distinguishes the consciousness in which I, as "an intelligence that is merely conscious of its faculty for combination", "think myself" (B158), from representation of a particular object (B157; *cf.* A346/B404). He denies that a thinking subject is represented *a priori* in inner sense. For what is represented *a priori* in it, namely time, "has in it nothing abiding, and hence gives cognition only of a change of determinations, but not of the determinable object" (A381). The intuition that makes "I think" into cognition of an object is rather "an indeterminate empirical intuition, *i.e.* a perception" (B422*n.*). And thinking things are in no way, least of all *a priori*, given in outer sense: "thinking beings, *as such*, can never come before us among outer appearances, [*i.e.*] we cannot intuit their thoughts, their consciousness, their desires, *etc.* externally; for all this belongs before inner sense" (A357). It thus appears that the concept of thinking is applied to other beings by analogy, on the basis of their observed behavior. Kant proclaims it "obvious" that "if one wants to represent a thinking being, one must put oneself in its place, and thus substitute one's own subject for the object one wants to consider" (A353).

Since the concept of spirit is of "a thinking being that is not combined with any body", the evidence for its application is necessarily lacking. So it is for Kant an example of "objects that are . . . perhaps possible in themselves but cannot be given in any experience since in the connection of their concepts something may yet be omitted that yet necessarily belongs to the condition of a possible experience" (A96). It thus differs from typical empirical factitious concepts (*e.g.* of artifacts), which *can* be applied if objects of the specified kind are available. Kant remarks on its specialness in his logic lectures of the 1770s. (His continued use of it is presumably due to its familiarity; see note 18 above.) Although he does not express the difference in terms of applicability, he might be thought to explain why we cannot judge anything to be an instance of this concept, by pointing out that we do not stipulate the marks by which something is to count as an instance. He classifies the invention that gives rise to it as "non-arbitrary", thus as involving "unnoticed fabrication" and "secret inferences of which one often is not aware" (24:263; *cp.* the contemporaneous *Reflexion* 2908).

[26] According to Guyer and Wood's notes, this means "a clock precise enough for the computation of longitude" (752 *n*.10).

[27] "All our concepts are either *given* concepts or ones that are *made*. A concept is *given* insofar as it does not arise from my faculty of choice" (*Wiener Logik*, 28:914).

[28] This adjective appears at, *e.g.*, A729/B757 and 9:142. "Arbitrary" and "*willkürlich*" differ in connotation. Unlike "*willkürlich*" and its own Latin root, "arbitrary" need not connote the involvement of a faculty of choice [Latin: *arbitrium*]. And unlike "arbitrary", "*willkürlich*" does not connote randomness (Beck (1955), *n*. 35). Meerbote indicates that in mathematical contexts, the usage of "*willkürlich"* implies the legislation of standards for how an object is to be, and that "it would be improper to conceive of [this] originary legislating as arbitrary if this means *inconsistent*, *ill-considered*, *random*, or *not binding*" ((1982), 78). As Daniel Sutherland has observed (in conversation), "elective" better translates "*willkürlich*", but in this paper I preserve the custom of 20$^{\text{th}}$-century commentators.

[29] Kant uses "determination" in several senses. Concepts are "determined", in the relevant sense, *through* what is given, as at B158: "for the cognition of" any object I "need an intuition in addition to the thinking of an object in general (through the category), through which I determine that general concept". A "determination", thus understood, is a condition through which a pure concept applies to objects. The "concept of cause as a pure category", for instance, is just the concept of "something that allows an inference to the existence of something else", but "since the possibility of drawing this inference also requires conditions about which I would know nothing [if I leave out the time in which something follows in accordance with a rule], the concept would not even have any determinations through which to apply to any object" (A243/B301).

[30] Kant himself seems reluctant to take this step. What is represented in pure intuition is clearly excluded from the first rank of his ontology. In pure intuition, "we can acquire *a priori* cognitions of objects (in mathematics), but only as far as their form is concerned, as appearances; *whether there can be things* that must be intuited in this form is still left unsettled" (B147, emphasis added). On his view, what is represented in pure intuition cannot be asserted to exist, because it is not represented in empirical intuition (A178/B221). He suggests, further, that it is not a genuine object. Although "we can give" (the concept of) a triangle "an object entirely *a priori, i.e.*, construct it", what we construct is "only the form of an object" and "would still always remain only a product of the imagination, the possibility of whose object would still remain doubtful, as requiring something more" (A224/B271).

[31] My claim is not that Strawson fails to account for the *marks*, namely necessity and universality, of *a priori* cognition. For Strawson's understanding of geometry's subject-matter seems to insulate him from this charge, at least as made in a recent paper. Joongol Kim argues that on Strawson's "phenomenal" interpretation, such facts as "that no two straight lines enclose a space" cannot be known with certainty because we have no way to "decide with absolute certainty that a straight line, *i.e.* the straight *look* of a physical object, and another straight line, *i.e.* the straight *look* of another physical object, do not enclose a space". To the suggestion that "they might *appear* not to enclose a space", Kim replies that "appearances are never a reliable basis for apodictic knowledge" ((2006), 140). But for the lines to appear not to enclose a space just is for it to be true that phenomenal straight lines do not enclose a space. This appearance would thus seem to foreclose all doubt of the truth of the geometrical proposition, as Strawson understands it. (Similarly, for an introspecting

subject to seem to entertain a given mental content just is for the subject to entertain that thought.)

Nonetheless, the way in which these propositions are known differs distinctly in its phenomenology from ways of gaining *a priori* knowledge. (Thus the thinking of a particular thought, which individuates the thinking self, can be predicated of the self only empirically.) Kant draws attention to just this difference when he says of the Greek geometer who first "demonstrated the isosceles triangle" that "in order to know something securely *a priori*", "what he had to do was not to trace what he saw in this figure, or even trace its mere concept and read off, as it were, from the properties of the figure", but rather "to produce the latter from what he himself thought into the concept" (Bxi-xii).

[32] Kant might be thought to commit himself only to the view that the properties of the circle can be "demonstrated on" the freehand figure, not that the concept of circle applies to it. (I owe this suggestion to Ian Proops.) But he refers to the freehand figure as a circle, and in a similar passage from the reply to Eberhard, it seems clear that the object on which a figure's properties are demonstrated falls under the concept of that figure: "In the most general sense, one can call construction all exhibition of a concept through the (spontaneous) *production of a corresponding intuition.* If it occurs through the mere imagination in accordance with an *a priori* concept, it is called pure construction. (These are the constructions which the mathematician must make use of in all his demonstrations.) Hence, *he can demonstrate by means of a circle which he draws with his stick in the sand, no matter how irregular it turns out to be, the attributes of a circle in general,* as perfectly as if it had been etched on a copper plate by the greatest artist" (8:192*n.*, emphasis added). More generally, nowhere (to my knowledge) does Kant suggest an "instrumentalist" account of proof, on which results do not literally hold of the objects used to prove them.

[33] In cognitive science, a "prototype theory" of concepts is any of a family of views based on the work of Eleanor Rosch. In them, the representation of a concept reflects the "typicality" of features common to its instances. (Rosch and Mervis (1975) propose that features found among most of the concept's instances, and not found among instances of other concepts, are privileged.) This commitment does not determine the structure of the representation, which could take the form of a weighted list of features or ranked set of instances rather than a single representative instance. Since Strawson clearly understands prototypes as single instances in his interpretation of Kant, this would be more precisely called a "best example" view.

[34] In fairness to Shabel, she acknowledges that if the pure intuition were "reducible to the act or procedure whereby a figure is constructed", it "would cease to be an intuition in the Kantian sense". But this seems to mean only that the procedure must be represented in conjunction with the drawn object, and since intuition is empirical so long as its content is limited to the drawn figure, it still seems that on her view intuition is pure exactly when it represents (in addition) the procedure for construction. I see no other way to parse her claim that the representation of actually drawn figures, which is in itself empirical, "functions as pure" or "confers *a priority*" when it permits the figures to be "considered in conjunction with the procedure for the construction of those objects" ((2003), 94). To put the point another way, pure intuition seems to consist in nothing other than "that aspect of" an intuition "that distinguishes [it] from an empirical intuition of the same concept, namely, the procedure for constructing that concept according to a rule" (112).

In (2003), Shabel both claims that the capacity of drawn figures "to function purely in the sense described" serves as "an interpretive model for the function of the transcendental schema" (109), and identifies the schema with the aspect (the procedure) that distinguishes a pure intuition (112). In Shabel's (2006), however, she makes clear that the procedure for constructing a figure is represented not by pure intuition, but as a schema. Her later account thus agrees with mine (which was, however, developed independently).

[35] Frank Leavitt ((1991), 650) and Alfredo Ferrarin ((1995), 143–5) emphasize that the Schematism is intended to explain both the recognition of objects as instances of

concepts and the introduction of particulars within geometrical proof. See also Shabel (2006).

[36] It might be objected that Kant counts only the schemata of "sensible" concepts (pure and empirical), not schemata in general, as rules for the generation of images. For Kant contrasts the former, as that "through and accordance with which the images first become possible", with the schema of a category, as "something that can never be brought to an image at all, but is rather only the pure synthesis, in accord with a rule of unity according to concepts in general, which the category expresses . . ." (A142/B181). But the burden of my §5 is to show that pure sensible concepts function as the paradigm that shows how representation can be both general and sensible, so that the categories figure as exceptions within Kant's overall view.

[37] The asymmetry is evident from the lack, in the latter case, of "a general feature which could be divided in different ways to yield different sortal universals, as the general feature, snow, can be divided in different ways" to yield the universals *patch of snow* or *fall of snow* ((1959), 206).

[38] The ability to register that more, less, or the same amount of a feature is present evidently involves the use of a measure of how much is present. It thus seems to require the use of criteria of distinctness for instances. Strawson indeed holds that even if criteria of distinctness are not "included in" features (as the characteristic shape is included in the cat-feature), they are "provided for" at the feature-placing level. For we may be able to say not only "There is snow here", but also "There is snow *here*—and *here*—and *here*", where each conjunct signals an increase in present quantity of snow. In this event, we make use of "factors which determine multiplicity of placing" that "may become, when we introduce particulars, criteria for distinguishing one particular from another" ((1966), 203–4; *cf*. (1953), 46). It is thus the use of criteria for reidentification that truly distinguishes the introduction of particulars.

[39] To see something as a tree, for instance, requires "the power of recognizing other things as well as trees". Since this power or potentiality gives the momentary perception its character (as, namely, of a tree), it cannot be "external to" the perception, "superadded to it, just as an extra qualification [one] must possess, as it were, if his momentary perception is to count as a case of tree-recognition". Thought of the objects one is "prepared or ready to count as" trees must rather be implicit in the perception itself. Strawson insists that "this is not a matter of [giving] ourselves actual images, either of other trees perceived in the past or wholly imaginary trees not perceived at all", in any actual momentary perception of something recognized as a tree: perception of these objects remains "non-actual (past or possible)" (91).

[40] The contrast between empirical and geometrical concepts is especially marked when the distinction between physical and phenomenal usage is drawn at the level of the shapes associated with empirical concepts. Assuming that it is possible "to abstract the notion of a position in physical space from the phenomenal integuments with which it is . . . associated", "the places where the phenomenally propertied things we seem to perceive seem to be" (and so their shapes) can be assigned to "correlated physically real items" ((1979), 55–6). But even at this level, phenomenal employment is basic: "the way in which a shape" is conceded to physical things is "altogether the wrong way for the common consciousness. The lover who admires the curve of his mistress's lips or the lover who admires the lines of a building takes himself to be admiring features of those very objects themselves; but it is the visual shape, the visually defined shape, that he admires" (54).

[41] Strawson would concede that on Kant's own approach, the problem of heterogeneity can be raised at this abstract level. Strawson acknowledges that since Kant claims to derive the categories "by attending to the requirements of the understanding in abstraction from sensibility," he must suppose them "to have *some* significance . . . apart from sensible intuition". So a purely discursive contribution to experience can be expressed by their means, and the question of how it relates to intuition can be raised. But the problem does not arise on Strawson's own approach because he prefers to understand pure concepts, not as forms imposed by a subjective feature of our constitution, but as what retain their applicability "when we push our

notion of experience to the limits of complete abstraction". He emphasizes that in this approach, there is "nothing ... to suggest that we could detach such a concept altogether from empirical conditions of its application and still use it to make significant assertions" ((1966), 115).

[42] Kant argues that the categories cannot be defined, any more than "the logical functions of judgment in general" (namely "unity and multiplicity, affirmation and negation, subject and predicate"), because "the definition would itself have to be a judgment and therefore already contain these functions of judgment" (A245).

[43] Kant writes that if one "wanted to show generally how one ought to subsume under [rules of the understanding], *i.e.* distinguish whether something stands under them or not, this would not happen except once again through a rule. But just because this is a rule, it would demand another instruction ..." (A133/B172). Major commentaries on the Schematism, including Chipman's (1972) and Guyer's (1987), agree that for Kant concepts other than the categories include or are identical to their schemata. For references, see Allison (2004), 208*n*.9. Allison avoids this conclusion by maintaining that Kant "misstates his own position" by using the term "concept" when "he clearly means the *schema*".

[44] See Chipman ((1972), 111–2). Contrary to Friedman ((1992), 189–90) and Shabel ((2003), 112), I maintain that representations in pure intuition are images. An "image", for Kant, is anything produced by the faculty of imagination, which has an *a priori* as well as empirical employment. Space, for example, is "the pure image of all magnitudes for outer sense in general" (A142/B182). Kant does claim that "the schema of a sensible concept (such as figures in space) is a product and as it were a monogram of pure *a priori* imagination, though which and in accordance with which the images first become possible," while "the image is a product of the empirical faculty of productive imagination" (A141-2/B181). But in calling the faculty responsible for images (the productive imagination) "empirical", Kant is not equating it with the *reproductive* imagination, which "contributes nothing to the explanation of cognition *a priori*" (B152). His point is that we are acquainted with this faculty only by means of the particulars it yields, so our *knowledge of* it is empirical, based on "inner experience" in which "a particular distinction or empirical determination" is given (A343/B401). "Empirical" in this sense is opposed not to "*a priori*" but to "transcendental", which pertains to faculties known through pure apperception.

[45] In (2001), Darius Koriako makes a number of related points. Koriako's concern is to explicate the "barely significant" [*kaum sinnvoll*] difference (294) effected by schematization in (thus the relevance of the Schematism for) empirical concepts. Koriako understands unschematized empirical concepts as "lists" of the marks that compose them. He emphasizes that their marks can be cognized independently of one another, in sharp contrast to those of mathematical concepts, with regard to which "we necessarily must know, how the individual aspects of a figure in space mesh with one another [*ineinander greifen*]: one who does not know that the situation [*Lage*] of the angles of a triangle also determines that of its sides, cannot be said actually to have command of the concept" (297). What schematization adds to an empirical concept is just such a "holistic" representation of the pure sensible aspects of objects falling under it (300). It thus serves (although Koriako does not explicitly draw this consequence) to connect at least some of the concept's marks with one another. In particular (I claim), it connects the concept's proprietary marks with those common to concepts ranked above it.

[46] See Anderson (2005), 30.

[47] This example (and the overall view of how schemata ensure the applicability of pure concepts) was worked out in discussion with Michael Friedman, to whom I am deeply indebted for his assistance.

[48] That real definitions have an epistemic or semantic role, for Leibniz, can be seen most clearly in the *New Essays*. When Locke interprets the traditional notion of real definition as the statement of a "real essence" which serves to "regulate" our classificatory practices (III.vi.25), he departs from the Aristotelian conception, on which real definitions are not necessarily known to us. See Ayers' classic (1981), 260. Although Leibniz

typically criticizes Locke's misunderstandings of the Aristotelian view, he acquiesces in this departure.

[49] *Cf. Jäsche Logik*: Real definitions are those "that suffice for cognition of the object according to its inner determinations, since they present the possibility of the object from inner marks" (9:143).

[50] *Cf. Blomberg Logik*: Real definitions "are ones that contain everything that belongs to the thing in itself" or "whose marks constitute the whole possible concept of the thing" (24:268).

[51] In the *Wiener Logik,* where Kant seems to allow that given *a priori* concepts can be (analytically) defined (24:917), he uses the contrast between factitious and given concepts to show that it is "very hard" to formulate real definitions of the latter. Just because "I can never find the marks to distinguish the [object of a given concept] from all possible things", I can never confidently claim "that my nominal definition is sufficient", *i.e.* also real. Kant seems to leave open that this can be done when he claims that in metaphysics and morals "there must be nothing but real definitions". But his point is not that we can have real definitions of given *a priori* concepts. It is rather that we must continually strive for them: "this is the aim toward which we must direct all our analysis, to bring about a complete concept" (24:920). Kant elaborates that the "whole difficulty" is that "the analysis can still be accomplished, but it is extremely hard to know whether the marks are present completely, and we have cause to be very distrustful about this . . . And since the thing is so hard, I would say that I will define *as an attempt*, that I will establish a definition as an attempt, as it were, . . . and that I intend to regard it as if it were not a definition" (24:923). So these lectures do not contradict the *Critique*'s assertion that "the exhaustiveness of the analysis of my [given *a priori*] concept is always doubtful" and what the analysis yields is properly speaking only an exposition (A728-9/B756-7). *Cf.* 24:918 on the impossibility of "a proper analytic definition" of concepts of the understanding.

[52] I think Carson may overestimate the mathematician's freedom in her attempted resolution of a textual puzzle. In the *Critique*'s "Doctrine of Method", Kant claims that the philosophical method yields only analytic cognition of mathematical concepts. Unlike the mathematical method, it does not license us to go "further than the mere definition" and construct the concept in intuition (A718-9/B746-7). Yet Kant also holds that mathematical definitions "come about" *as* constructions of concepts (A730/B758). On Carson's resolution of the puzzle, the philosopher's definition of the concept is merely nominal. The philosopher grasps the concept without apprehending the "pure intuition" which, Kant claims, such a concept "already contains in itself", and which makes its construction possible. Her grasp of it does not put its objective reality beyond doubt. Carson appears to overlook Kant's view that it is not possible to grasp a mathematical concept without apprehending all that belongs to it. Since the concept is factitious, it cannot be grasped only partially. It is *first* grasped through its definition, which must be real, since it expresses all the marks that an object must have to be brought under the concept. Moreover, in representing the definition, we take ourselves to be justified in ascribing these marks to particulars. So there is no way to grasp the concept without cognizing its objective reality. (This underscores that for Kant, what a concept represents (and its definition articulates) is an ability to recognize particulars. The philosopher cannot fail to cognize a mathematical concept's applicability to intuition because to possess the concept at all, she must take herself to be justified in ascribing its marks to particulars.)

To resolve the puzzle, I would argue that the philosopher apprehends, but cannot exploit, all that belongs to the concept. By grasping its definition, she comes to understand her ability to recognize instances of mathematical concepts. She also understands that she is free to exercise this ability on particulars, that is, to regard them as instances of mathematical concepts, regardless of the features they present in experience. At the same time, she is free to forswear the ability. In choosing to follow the philosophical method, she elects not to apply geometry to the objects she represents in intuition. Possession of the concept demonstrates its objective reality, but does not compel us to effect every step of the proof.

[53] Michael Friedman sharply articulates this point in (1992), 104. His account (as well as Hintikka's and Beth's; see note 2 above) is like Strawson's treatment of the spurious necessity of "picturing" in that it turns on the necessity of intuition for the "systematic" representation of figures. Friedman holds that we cannot possess the concept of an Euclidean figure without cognizing its objective reality because intuition is required even to express the first principles by which it is introduced into "rigorous" reasoning (126). It should be noted that his views have since changed; see his (2010).

[54] Tyler Burge has emphasized (in lectures on the first *Critique*) the importance of this notion of "proof" for Kant.

[55] As Hilbert puts it in his *Grundlagen*, we can think of points, lines, and planes "as having certain mutual relations", which are designated "by means of such words as 'are situated', 'between', 'parallel', 'congruent', 'continuous', etc." and "completely and exactly described" by the axioms of geometry" (3). These conventions will make geometry true in virtue of our determination to use the words "point", "line", and "plane" in such a way that any objects denoted by them will stand in the designated relations.

It is important to note that Hilbert does not claim that the proper use of the primitive terms is up to us. While the primitives have no "preaxiomatic meaning" (as Coffa puts it), the axioms of Euclidean geometry uniquely express the "fundamental facts of our intuition" (*Foundations*, 3). But by supplying a purely formal expression of the relation of implication between a theorem and particular axioms, Hilbert makes possible the identification of theorems with conditionals that do not presuppose the truth of the axioms (Coffa (1986), 35). His program for extirpating geometry's semantic content thus allows the positivists to give it a kind of nonempirical content (Coffa (1991), 140). The positivists suppose that the antecedents of these conditionals can rest on mere convention. Geometry, so understood, is grounded on ("true by") definition even before it is interpreted, *i.e.* applied to physical space.

[56] The "reason why" the propositions of pure geometry "cannot be confuted in experience is that they do not make any assertion about the empirical world. They simply record our determination to use words in a certain fashion." (Ayer (1946), 84).

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