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Necessity, Analyticity, and the *A Priori*

> We are in possession of certain modes of *a priori* knowledge, and even the common understanding is never without them.
>
> Kant, *Critique of Pure Reason* (B3)

> Necessity and strict universality are thus sure criteria of *a priori* knowledge. (B4)

> In all theoretical sciences of reason synthetic *a priori* judgments are contained as principles. (B14)

> Now the proper problem of pure reason is contained in the question: How are *a priori* synthetic judgments possible? (B19)

A. Three Distinctions

In the introduction to the *Critique of Pure Reason* Kant draws three important distinctions: *a priori* versus empirical, necessary versus contingent, and analytic versus synthetic. Although some philosophers lump them together, we should not assume at the outset that the three distinctions divide things up in the same way. Even if it should turn out that they do, each of the distinctions must be given its own explanation.

A priori/Empirical. Kant distinguishes *a priori* from empirical knowledge as follows:

> [K]nowledge that is thus independent of experience and even of all impressions of the senses . . . is entitled *a priori*, and distinguished from the *empirical*, which has its sources *a posteriori*, that is, in experience.(B2)

The term *a priori* is in the first instance an adverb modifying verbs of cognition: person S knows proposition p *a priori* iff S knows p in a way that is independent of experience. We may then go on to define a related sense in which *a priori* is a predicate of judgments or propositions: a proposition p is *a priori* iff it is possible for someone to know p *a priori*. I leave aside the interesting question of whether it is possible for some beings to know *a priori* things that other beings cannot. (E.g., might an infinite intelligence have *a priori* knowledge of propositions about the distribution of primes in the number series that is not available to a finite intelligence?)

The notion of independence that figures in the primary definition of *a priori* must not be misunderstood. In saying that we know a given proposition independently of experience, Kant is not saying that we would still have known it even if we had never had any experience. On the contrary, he allows that experience may be requisite for the knowledge even of an *a priori* proposition in either of two ways. First, it may well be that if we had never had any experience, our cognitive faculties would never have developed to the point that we could entertain any propositions or do any thinking at all. Still, once our faculties are up and running, there are some propositions that we can know to be true without any further need of experience. That is the point of Kant's famous remark that "though all our knowledge begins with experience, it does not follow that it all arises out of experience" (B1). Second, it may be that some of the constituent concepts in a given proposition are concepts that can only be acquired through experience, such as the concept of *red* or the concept of an *event*. In that case, experience would be necessary for us to grasp the proposition or get it before our minds, but once we have framed it in our consciousness, we may be able to ascertain that it is true without any further aid from experience. Kant acknowledges this possibility when he distinguishes (within the class of *a priori* propositions) between the pure and the impure, an impure proposition being one some of whose constituent concepts are derivable only from experience (B3). He gives the example 'every alteration has a cause': the concept of an alteration (or event) is one that can be got only through experience, but the proposition as a whole Kant takes to be *a priori*. For another example (in which the *a priori* status of the proposition is less controversial), I cite 'nothing is simultaneously red and blue'. In the *Prolegomena*, Kant gives 'gold is a yellow metal' as an example of a proposition that is *a priori* (because analytic) even though it contains empirical concepts (p. 14).

The point of the previous paragraph may be made by invoking the familar tripartite analysis of knowledge. Someone knows a proposition only if (i) he believes it, (ii) it is true, and (iii) he is adequately justified in believing it. (See Kant's discussion of opining, believing, and knowing at A822/B850 for an account roughly along these lines.) Experience may be necessary in either of the two ways I have mentioned—in many cases or even in all—for the obtaining of condition (i), the belief condition of knowledge. But if experience is not necessary in a given case for the obtaining of condition (iii), the justification condition, the knowledge will still qualify as *a priori*.

The relevant points here were nicely summed up by Frege:

> [When we classify a proposition as *a priori*,] this is not a judgment about the conditions, psychological, physiological, and physical, which have made it possible for us to form the content of the proposition in our consciousness; nor is it a judgment about the way in which some other man

> has come . . . to believe it to be true; rather, it is a judgment about the ultimate ground upon which rests the justification for holding it to be true.[1]

Necessary/Contingent. The second distinction relates not to how a proposition is known, but to its manner or mode of being true. Among all the things we recognize as true, there are many that (as far as we can see) need *not* have been true—for example, that stones fall when released near the earth, or that the sun is shining as I write these lines. Such truths are contingent. There are others, however, that *had* to be true. That two and three together make five and that a thing never both has and lacks a given property are truths of this sort. They are necessary truths, the necessity of which may be characterized in any of the following ways:

> p could not have been false;
> p not only *is* true, but *must* be true;
> the opposite of p is impossible;
> p holds in every possible world.

I will not try to elucidate these notions further, as they are among the most familiar in philosophy, and I doubt that anything much can be done to explain them to anyone who does not already have a grasp of them anyway. (Such, indeed, is one ground for Quine's misgivings about necessity, discussed later.) Here I simply note that the sense of 'necessary' that is now at issue is what Plantinga calls the "broadly logical" sense.[2] It is logical as opposed to merely physical necessity (i.e., the necessity with which stones fall when released), so laws of nature are not necessary in this sense. And it is broadly logical as opposed to narrowly logical necessity, so laws of formal logic are not the *only* things that are necessary in this sense.

Though the two distinctions drawn so far differ in intension (one relating to the manner of being known and the other to the manner of being true), Kant believes they that they coincide in extension—that they divide up the field of true propositions in the same way. He believes that propositions are necessary iff they are *a priori*, and contingent iff they are empirical or *a posteriori*. As he puts it, "Necessity and strict universality are . . . sure criteria of *a priori* knowledge, and are inseparable from one another" (B4).[3] But Kant famously—and in my opinion correctly—thinks that the next distinction runs at right angles to the first two.

Analytic/Synthetic. I come now to the distinction in this trio with which Kant is most associated. Kant's predecessors drew distinctions in the same general area as the analytic/synthetic distinction; for example, Leibniz distinguished between truths of reason and truths of fact, and Hume distinguished between relations of ideas and matters of fact. But Kant is often regarded as the first major thinker to draw the analytic/synthetic distinction in a way that exhibits it as clearly different from the necessary/contingent and *a priori*/empirical distinctions.[4]

My concern in what follows is not so much with elucidating exactly what Kant meant by the analytic/synthetic distinction as with exhibiting connections between his account of it and more recent accounts. I do not think it requires an undue amount of squinting to see Kant's distinction as essentially agreeing with its twentieth-century descendants. When Kant affirms that there are synthetic *a priori* truths and Ayer or Quine denies it, I think they are engaged in the same debate.

The Containment Characterization. Kant gives two different accounts of the analytic/synthetic distinction, one in terms of conceptual containment and the other in terms of contradiction. The first and better known runs as follows:

> Either the predicate B belongs to the subject A, as something which is (covertly) contained in this concept A; or B lies outside the concept A, although it does indeed stand in connection with it. In the one case I entitle the judgment analytic, in the other synthetic.(A6/B10)

Note that Kant does not merely say that the predicate in an analytic judgment belongs to its *subject*, that is, to that which the judgment is about—that much is presumably the case in *any* true subject-predicate judgment. Instead, he says the predicate is contained in the *concept* of the subject. He goes on to explain that in an analytic judgment, the predicate concept is one of the "constituent concepts that have all along been thought in the subject, although confusedly," whereas a synthetic judgment has a predicate "which has not been in any wise thought in [the concept of the subject], and which no analysis could possibly extract from it" (A7/B11).[5] In another place, he equates "what I am actually thinking in my concept of a triangle" with "nothing more than the mere definition" (A718/B746). If we put these two passages together, we arrive at the result that the judgment that S is P is analytic iff the property of being P is included by definition in the concept of S. In fact, Kant would not have said this, owing to special views he held about the nature of definition.[6] But this gloss at least has the virtue of bringing out that whether a judgment is analytic or synthetic depends on what we mean by the terms used to express it.

I now turn to Kant's own examples for illustration. 'All bodies are extended' expresses an analytic judgment, because by 'body' we mean among other things an extended, impenetrable thing. The proposition we express is therefore equivalent to 'all extended and impenetrable things are extended', in which the inclusion of the predicate in the subject concept is visible to the mind's eye. 'All bodies are heavy,' on the other hand, expresses a synthetic judgment, since being heavy is not part of what we mean by 'body'. Our concept of a body allows us to acknowledge the possibility of bodies (e.g., those placed outside all gravitational fields) that do not have any heaviness at all.

Too Subjective? It is a perennial objection to the containment account that it yields a distinction that is merely subjective and variable. What one person includes in his or her concept or definition of the metal gold may include more attributes than another does; in consequence, whether a judgment about gold is analytic or synthetic may vary from person to person. This objection was raised against Kant in his own day by J.G. Maass.[7]

It was also answered in Kant's own day. It is quite true that one person's definition of a term may include more than another's, but this turns out to be harmless. Kant's disciple J.G. Schulze, replying to Maass, explained why:

> Now, suppose that I find, in a judgment which two philosophers express in the same words, that one of them connects the subject with a rich concept in which the predicate is already contained, while the other, on the other hand, connects it with a concept in which the predicate in question is not contained. I would then be entirely correct in saying that the judgment of the first one is analytic, and of the second one synthetic. For although their judgments seem to be one and the same, since they are expressed with the same words, they are nevertheless in this case in fact not one but two different judgments.[8]

Schulze's point is that if it is judgments or propositions that we are classifying as analytic or synthetic, then nothing is ever analytic for one person and synthetic for another. The fact that one person uses a more inclusive definition would only show that he or she operates with a different subject concept and therefore frames a different judgment.

On the other hand, if it is *sentences* that we wish to classify as analytic or synthetic, we do indeed get relativity: the same sentence may be analytic for Smith and synthetic for Jones. But such relativity will not undermine Kant's project in the slightest. As I argue below, there are cases in which a sentence that is synthetic for a given person expresses a proposition that is known *a priori* by that person. Such cases still present us with the central question of the *Critique of Pure Reason*, though now it needs to be phrased as follows:

> How are judgments that are known *a priori* by a given person and expressed by sentences that are synthetic for that person possible?

That lacks the fanfare of Kant's "How are *a priori* synthetic judgments possible?" (B19), but it raises the same profound issues.

Too Narrow? There is another common objection to Kant's containment definition that has more bite—namely, that it is too narrow. In the first place, it applies only to judgments of subject-predicate form, whereas Kant wishes to classify as analytic or synthetic many judgments not of that form. For example, it does not apply to existential judgments (such as 'there are lions'), which (if we accept the dictum that existence is not a predicate) are not of subject-predicate form. Nor does it apply to compound judgments, such as disjunctive judgments, which need not have a single subject. In the second place, even restricting our attention to judgments of subject-predicate form, the containment definition does not classify as analytic all the judgments that Kant himself would wish to classify as analytic. Consider a judgment of the form 'all ABCD is A'; under the containment definition, this will certainly count as analytic. The equivalent contrapositive judgment, 'all non-A are non-(ABCD)', should therefore also count as analytic, yet clearly one might think of something as non-A without taking any thought of B, C, or D, and therefore not of non-(ABCD).[9] Nor need non-(ABCD) be part of the definition of non-A. So, the contrapositive judgment is not analytic by the containment account. It is not even obvious that the containment account properly classifies as analytic those judgments in which the predicate is extractable from the concept of the subject, but only after a great many steps of definitional replacement.[10]

The Contradiction Characterization. Fortunately, Kant has another characterization of analyticity that is less open to the charge of narrowness. There is a first hint of it at B12, where Kant speaks of extracting the predicate of extension from the concept of body "in accordance with the principle of contradiction." The role of contradiction is explicitly recognized later in the section of the *Critique of Pure Reason* entitled "The Highest Principle of All Analytic Judgments," in which Kant says that the truth of any analytic judgment "can always be adequately known in accordance with the principle of contradiction" (A151/B190). Later still, in his critique of the ontological argument for the existence of God, he says that the following feature is "found only in analytic judgments, and is indeed precisely what constitutes their analytic character": their predicates cannot be rejected without contradiction (A598/B626).

What all of this suggests is the following commonly given definition of an analytic judgment:

> A is analytic iff its opposite, –A, is a contradiction.

But what is a contradiction? Many so-called contradictions are not official or formal contradictions in the hard objective sense that they have the logical form 'P & –P'.[11] If we say that a statement is analytic only if its negation is a formal contradiction, nothing will count as analytic except the law of contradiction itself ('–(P & –P)') and its instances—and they will count only with the aid of the ruling that the double negation of a contradiction is itself a contradiction. Not even that paradigm of analyticity, 'all bachelors are unmarried', has an opposite that is contradictory in the formal sense. So, what is intended in the definition above is presumably that –A either is or *implies* a formal contradiction.

Now we need to ask another question: "imply" in accordance with which rules and with the help of which auxiliary premises? *Anything* can be shown to imply a contradiction if there is no limit on what rules and premises we may use in the proof. So, which rules and premises may we use? If we look at the transformations to which we must subject the negation of 'all bachelors are unmarried' to arrive at a formal contradiction, the following answer suggests itself: we may appeal to laws of logic, and we may appeal to definitions. This gives us the following revised account:

> A is analytic iff from its negation, –A, a formal contradiction may be derived, using in the derivation only laws of logic and substitutions authorized by definitions.[12]

Twentieth-Century Accounts. What we have just arrived at is perhaps the most common twentieth-century conception of analyticity. It is equivalent to each of the following:

> *Frege*: "If (in tracing the proof of a proposition) we come only on general logical laws and on definitions, then the truth is an analytic one."[13]
>
> *Carnap*: "The first type of theorem can be deduced from the definitions alone (presupposing the axioms of logic, without which no deduction is possible at all). These we call *analytic* theorems."[14]
>
> *C.I. Lewis*: "Every analytic statement is such as can be assured, finally, on grounds which include nothing beyond our accepted definitions and the principles of logic."[15]
>
> *Quine*: "Statements which are analytic by general philosophical acclaim . . . fall into two classes. Those of the first class are logical truths; those of the second class can be turned into logical truths by putting synonyms for synonyms."[16]

These accounts of analyticity apply in the first instance most readily to sentences—for it is sentences that have meanings, that contain terms that can be interchanged with others having the same meaning, and so on. But we can define a related sense of 'analytic' that applies to propositions: a proposition is analytic iff any sentence expressing it would be analytic.

The resulting conception of analyticity is not open to the charge discussed above of excessive narrowness. It applies to propositions of any logical form, not just those of subject-predicate form. Moreover, it classifies as analytic propositions of the form 'all non-A are non-(ABCD)', whereas the containment definition does not. It also classifies as analytic propositions in which the predicate is extractable from the concept of the subject only after a great many steps of logical or definitional recasting.

I believe also that the resulting notion agrees tolerably well with Kant's own, at least in extension if not in intension. In any case, it is the conception I operate with in this book. If I sometimes use "the opposite implies a contradiction" as my short gloss on analyticity, the reader should remember the unstated provisos that bring the gloss into line with the Fregean conception: from the negation of the statement it must be possible to derive a formal contradiction using only definitions and laws of logic.

B. Synthetic *A Priori*

As I said above, Kant believes that his first two distinctions, *a priori*/empirical and necessary/contingent, make the same divide in the field of judgments, while the third distinction, analytic/synthetic, cuts across the field at right angles to the first two. That makes four compartments in all. Kant believes three of the four to be occupied: analytic *a priori*, synthetic *a posteriori*, and (the famous new possibility) synthetic *a priori*.[17] *A priori* knowledge is perhaps not unduly mysterious when it is of analytic truths, for it is explained in that case by whatever explains our knowledge of logic and our knowledge of our own meanings.[18] But if our *a priori* knowledge extends also to some synthetic truths, what could explain that? Disdain of having to invoke an *oculis rationis* or some other mysterious faculty led the positivists and others to maintain that *a priori* knowledge is to be had only of analytic propositions. But Kant was convinced otherwise, and that set for him the central problem of the *Critique of Pure Reason*: How are synthetic *a priori* judgments possible (B19)?

In the introduction to the *Critique*, Kant asserts that there are three important classes of synthetic *a priori* propositions: the truths of arithmetic, the truths of geometry, and certain framework principles of natural science, such as the principle that every event has a cause. (He also lists as synthetic and putatively *a priori* certain propositions of metaphysics, such as that the world had a beginning, but these turn out for him not to be *a priori* because not knowable at all.) Few agree with Kant nowadays about the synthetic *a priori* status of the propositions in these three classes. Arithmetic is generally thought to be *a priori* but (given the work of Frege, Russell, and Whitehead) not synthetic. Geometry is generally thought to be synthetic but (given the rise of non-Euclidean geometries in the nineteenth century and their subsequent incorporation into physical theory in the twentieth) not *a priori*. Nor are the framework principles of natural science generally thought to be *a priori*.

Although I cannot do justice here to the status of arithmetic and geometry, I offer below a few words in defense of what Kant has to say about a specimen from each subject matter. Then I move on to two more favorable examples of the synthetic *a priori*. Even if Kant is wrong about arithmetic and geometry at large, I believe he is emphatically right in thinking that there are synthetic *a priori* propositions.

7 + 5 = 12. '7 + 5 = 12' is Kant's well-known example of a synthetic *a priori* proposition in arithmetic. He argues for its synthetic character as follows: "[I]f we look more closely we find that the concept of the sum of 7 and 5 contains nothing save the union of the two numbers into one, and in this no thought is being taken as to what that single number may be which combines both" (B15). Here Kant is invoking the containment characterization of analyticity, and he is clearly right about the results of applying it, at least to some cases if not to this one. There are true arithmetic equalities of the form '7 + 5 = m − n', where m and n are numbers so large no human being has ever thought of them; here, of course, one may entertain the subject concept without having any thought of the predicate.[19] Nor need the concepts of m and n enter into the definition of 7 + 5, or else no concept would have a finite definition, there being infinitely many (m − n) pairs that similarly yield true equations. Thus, by the containment criterion, much of arithmetic is synthetic.

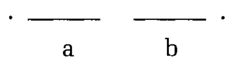
But we know that the containment criterion is too narrow, and that Kant has a better criterion in terms of contradiction. Might it be that arithmetical propositions are analytic by the superior standard? Such, of course, is precisely the contention of the logicism of Frege, Russell, and Whitehead. If they are right, there are definitions of arithmetical concepts in purely logical terms that permit the derivation of any truth of arithmetic from purely logical axioms. That would make arithmetic analytic in Kant's wider sense. But whether the logicists are right is unclear for at least two reasons. First, the status of some of their axioms (e.g., the axiom of infinity, which guarantees the existence of infinitely many objects) as purely logical is controversial. Second, Gödel's incompleteness theorem shows that there cannot be any finite and consistent set of axioms from which every arithmetical truth may be derived, thus apparently dooming the logicist project.[20]

The Straight Line Between Two Points Is the Shortest. 'The straight line between two points is the shortest' is Kant's official example of a synthetic *a priori* proposition in geometry.[21] As with the previous example, he thinks its *a priori* status will be generally conceded. He supports its syntheticity with the following observation: "For my concept of *straight* contains nothing of quantity, but only of quality" (B16). In an interesting discussion of the same proposition, Hume observes, "In common life 'tis establish'd as a maxim, that the streightest way is always the shortest; which wou'd be as absurd as to say, the shortest way is always the shortest, if our idea of a right line was not different from that of the shortest way betwixt two points."[22]

Geometers nowadays often define a straight line in just the way Hume and Kant say we should not: as a geodesic, or shortest line (in the space in question) between two points. That makes analytic the sentence Hume and Kant take to express a synthetic proposition.[23] In the bargain, it threatens to undermine the *a priori* status of another of Kant's favored examples: two straight lines cannot enclose a space. In Riemannian geometry, two lines that are geodesics *can* enclose a space.

Of course, if rival geometries undo the apriority of Kantian theses only by redefining terms, they do not really undo it; they make the same sentence express a different proposition, and it need come as no surprise that the new proposition is not *a priori*. Indeed, on first exposure to the non-Euclidean properties of straight lines in Riemannian geometry, many students claim that the meaning of 'straight' has been changed, in which case there is no challenge to Kant. The impression of meaning change may be heightened when the student is told that great circles on a sphere may serve as models of straight lines. "Those lines aren't really straight," the student may be tempted to say, "for it is obvious that there are straighter lines, so you haven't really shown me how *Euclidean* principles might fail." The standard reply to the student is that *in the space in question*, the line really is straight, for there is no straighter. The student is imagining a more inclusive space in which the line would not be straight, but that is not the space that counts.

I think it is significant that to reply to the student we must use a relativized notion of straightness, and I would like to exploit this fact in defending Kant. For a preliminary illustration, let me switch the example from straightness to congruence. Consider the figures a and b in the one-dimensional space of Lineland:



Are a and b congruent or not? Many writers would say no, for there is no motion in the space that will enable one to be superimposed on the other. But Wittgenstein claimed to the contrary that a and b are in fact completely congruent. "It is quite irrelevant," he said, "that they cannot be made to coincide."[24] I agree with Wittgenstein: because there are *possible* spaces (of two or more dimensions) in which a and b could be made to coincide, they are congruent absolutely speaking, even if they cannot be made to coincide in the limited space to which they are actually confined.[25]

In effect, we are now distinguishing two notions of congruence. The relativized notion may be defined thus: two figures are congruent *in S* iff there is a motion in S by which the figures may be brought into coincidence. This notion must be what the writers who deny the congruence of a and b have in mind. But we may also define an absolute notion of congruence as follows: two figures are *absolutely* congruent iff there is a possible space in which they would be congruent in the relative sense. Wittgenstein's remark is defensible in light of the absolute concept.

To return now to the concept of straightness, any geodesic in a space S may be said to be straight *in S*, for no line is shorter or straighter in S. But whether a line is straight *absolutely* is a function not of the space it happens to inhabit, but of what spaces are possible, and if there are possible spaces in which a line would not be the shortest, then it is not straight in the absolute sense.[26]

We are now in a position to defend what our student says on behalf of Euclid and Kant. When she says, "Your so-called straight lines that return on themselves, enclose spaces, and behave in other non-Euclidean ways aren't really straight," she is right—the lines are not absolutely straight. The necessity of Euclid's principles as governing absolutely straight lines is not put in doubt by the fact that non-Euclidean principles govern lines that are straight relative to one or another special space. At the same time, we are not securing the necessity of Euclidean principles simply by making them analytic. That Euclid holds sway over the absolutely straight is a matter not of definition but of intuition.[27]

I pass now to two putative examples of synthetic *a priori* truths that should be less controversial than the two just discussed: 'nothing is red and green all over' and 'every cube has twelve edges'.

Nothing Is Red and Green All Over. Nearly everyone will concede that 'nothing is red and green all over (at the same time)' expresses a necessary truth that is known *a priori*; the question is whether it is synthetic.[28] Given the account of analyticity described above, the question comes down to this: is the statement true by virtue of definitions plus logic? Anyone who says yes must answer the question: true by *what* definitions and *what* logic?

For starters, we may consider a definition that looks in part like this:

x is red =Df . . . & x is not green & . . .

A definition containing the displayed conjunct would let us derive 'nothing is both red and green' from 'nothing is both green and not green', which is a truth of logic. But what would the rest of the definition look like? How are we to fill in the blanks? Sticking with the "definition by exclusion" strategy, we would have

x is red =Df x is not blue & x is not green & . . . ,

with one conjunct for each color not overlapping redness.[29] A possible drawback of the resulting definition is that it would contain infinitely many conjuncts. Setting that problem aside, there is the following difficulty, which was raised by Arthur Pap: not all colors can be defined in this purely negative way, on pain of circularity in the total system of color definitions (for we would be defining 'x is green' as 'x is not red & . . . ']. Enough of the color terms will have to be defined positively to give us some purchase on the negatives. A strong case can be made that the positive definitions will have to be *ostensive*—'to be red is to be the color of *this object*'. Now consider two colors F and G that have been defined in this ostensive way: it will be an *a priori* truth that nothing is both F and G, but nothing in the present strategy will show that such truths are analytic.

Let's try another tack. Why not define colors in terms of physical magnitudes, perhaps along the following lines:

x is red =Df x reflects light of wave length l.

x is green =Df x reflects light of wave length m.

The incompatibility of colors would then simply be a consequence of the incompatibility of certain physical magnitudes.[30]

I leave aside the objection that we understand color terms perfectly well before we know any physics, and that definitions like those above are not true to our naive understanding. The objection I wish to press is that we have only relocated our difficulty. This was pointed out by F.P. Ramsey in his review of the *Tractatus*, in which Wittgenstein had tried a similar strategy: he sought to reduce color incompatibilities to the impossibility of a particle having two velocities at once. Ramsey commented:

> But even supposing that the physicist thus provides an analysis of what we mean by 'red', Mr Wittgenstein is only reducing the difficulty to that of the *necessary* properties of space, time, and matter or the ether. He explicitly makes it depend on the *impossibility* of a particle being in two places at the same time.[31]

What we would need now is the basis in definitions and logic for saying that a particle cannot have two velocities, or that an object cannot have two different reflectance profiles, and so on.

I relegate discussion of one more strategy for deriving statements of color incompatibility from definitions and logic to appendix A.

Every Cube Has Twelve Edges. This is a nice example for illustrating Kant's views on the epistemology of geometry.[32] Unlike many geometrical propositions, this one (which is actually a proposition of topology rather than of geometry proper) holds in Euclidean and non-Euclidean geometries alike, so there is no challenge to its apriority from non-Euclidean geometry. The question, as with the previous example, is whether it is analytic or synthetic. It is not immediately or obviously analytic, for the standard definition of 'cube' is 'regular solid (or polyhedron) with six square faces', in which there is no mention of the number of edges. How, then, do we know that the proposition is true? Most people verify it simply by visualizing a cube and counting its edges—an exercise in what Kant calls "pure intuition." If we are to exhibit the proposition as analytic, we must deduce the number of edges in a cube using just logic and any relevant definitions. Readers may wish to try their hands (or heads) at this.

Some will try to make the statement immediately analytic after all, saying that *their* definition of 'cube' *does* include the having of twelve edges. To them I say: very well, it is analytic that cubes as you define them have 12 edges. But consider now the following proposition: every cube as defined by me (in terms of six faces) is a cube as defined by you (in terms of twelve edges). That is, every regular solid with six square faces is a solid with twelve edges. I submit that this proposition may be seen to be true *a priori* as easily as the first, but it is *not* true by definition. Or if it is, that has yet to be made out. So, we may as well expend our efforts on the original statement.[33]

Here is one good try at extracting twelve edges from the original definition of a cube:[34]

1. Every cube is a polyhedron with six square faces (by definition).
2. Six separate squares have twenty-four edges, since each square (by definition) has four edges and $6 \times 4 = 24$.
3. When six squares are assembled into a cube, each edge of a square coincides with the edge of another square to form one edge of the cube, and each edge of the cube is an edge of exactly two squares (no square edge coinciding with more than one cube edge). Thus, the number of edges in the original group of squares decreases by one half.
4. Therefore, every cube has twelve edges ($24 \div 2$). Q.E.D.

That apparently does it—at least if we may correctly assume (in opposition to Kant) that arithmetic is analytic, and that for counting purposes coincidence is as good as identity.

Even granting these assumptions, however, there is a further hitch. Where does step 3 come from? I readily concede its *truth*, for I see that things work as step 3 says when I picture squares coming together to form a cube. But unless step 3 is itself true by definition, we have not yet shown that the conclusion of the argument is analytic.

Well, perhaps step 3 is true by virtue of the definition of a polyhedron. A topology textbook offers this:

> By a *polyhedron* we mean any system of polygons arranged in such a way that (1) exactly two polygons meet (at an angle) at every edge, and (2) it is possible to get from every polygon to every other polygon by crossing edges of the polyhedron.[35]

The first clause of this definition delivers the essential part of our step 3.

That may end the argument over 'every cube has twelve edges', but there are other apparent instances of the synthetic *a priori* in the vicinity. Let's turn our attention to 'every cube has eight corners (i.e., vertices)'. That proposition is also *a priori*, but its analyticity is not brought out by any of the definitions so far considered. We could make it analytic by adding to the definition of a polyhedron a clause from which we could deduce something analogous to step 3 above: each vertex of a square coincides with two other such vertices to form one vertex of the cube, and each vertex of the cube is a vertex of exactly three squares (no square vertex coinciding with more than one cube vertex).[36] Then we could conclude that every cube has eight vertices (since six squares have twenty-four, which divided by 3 is 8).[37]

My rejoinder is by now predictable: the foregoing strategy would make analytic a statement that was formerly synthetic and whose content we knew to be true *a priori* even when the statement was synthetic. That is, we knew *a priori* that cubes in the original "thinner" sense (i.e., polyhedra with six square faces, with polyhedra defined as in the textbook definition above) have eight vertices. We can, if we like, make analytic the statement that formerly expressed what we knew about cubes, but what would that show? The knowledge was there before the analyticity. Indeed, the analytic statement does not even express our old knowledge, but a more highly articulated proposition instead.

Is It Analytic That All A Priori *Truths Are Analytic?* The thesis that there are synthetic *a priori* truths need not be supported by examples alone. What is the status of the opposing thesis that all *a priori* truths are analytic? It is not an empirical thesis, so if true at all, it had better be analytic.[38] But is it? Defenders of the thesis have seldom risen to the challenge of demonstrating its own analyticity.[39]

C. Quine's Attack on Analyticity

Analyticity and associated notions play such a large role in Kant's philosophy that it behooves me to say something here about Quine's famous criticisms of them—even at the risk of saying little that has not been said before.

Every student of philosophy knows that Quine denies the analytic-synthetic distinction. But what exactly is it to deny a distinction? Well, what is it to *uphold* a distinction? Upholding a distinction (in robust fashion) between the As and the Bs involves maintaining each of the following: (i) there are As, (ii) there are Bs, (iii) nothing is both, (iv) nothing is neither, (v) nothing is a borderline case, (vi) nor is it otherwise indeterminate whether something is an A or a B, and finally (vii) we have a clear idea what we are talking about when we talk about As and Bs. Quine's denial of the analytic-synthetic distinction presumably involves denying one or more of (i)–(vii), but which?

I take it that Quine denies the first and the last of the theses in this sequence.[40] That is, he denies that there are any analytic statements, for reasons developed in the second part (sections 5 and 6) of "Two Dogmas of Empiricism," and he denies that we have any tolerably clear idea of what can be meant by the terms 'analytic' and 'synthetic', for reasons developed in the first part (sections 1–4) of the same article.[41]

One might wonder how these two opinions can be combined. (Compare: "I don't think anyone really knows what a hippoglub is; moreover, there aren't any.") But that difficulty can be circumvented by construing Quine's argument in the second part of "Two Dogmas" as *ad hominem*: whatever exactly 'analytic' means, believers in it associate it with a certain privileged status (irrevisability), and Quine argues that nothing has that status.

The best-known criticism from the first part of "Two Dogmas" is that the term 'analytic' is a member of a small family of terms any of which can be explained in terms of the others, but none of which can be explained from the outside. This is not an uncommon circumstance. Kant provides us with another example of it: the *chiral* family of terms, which include 'left', 'right', 'clockwise', 'counterclockwise', and names for the various points of the compass. We can give a verbal definition of 'right' if we are permitted to use other chiral terms, but we cannot define any chiral term without using other chiral terms. Does it follow that no one understands the chiral terms? Not at all, for it is possible that some of them are understood without need of any verbal definition. Kant's own view is that some of them (it does not matter which) can and must be understood ostensively, or through intuition, and may then be used to define the rest.[42]

One key member of the 'analyticity' family is 'synonymy': if we can understand the latter, we can understand 'analytic'. The notion of synonymy was implicitly involved in our characterization of analyticity above, since in a correct definition, the definiens must be synonymous with the definiendum. Now, synonymy is simply sameness of meaning; what is so problematic about that? In defense of this notion, I cite three points, all of which have been well made by Strawson or by Grice and Strawson. (1) If there is such a thing as significance, or a sentence's meaning something at all, must there not be such a thing as synonymy, or two sentences' meaning the same thing?[43] (2) Quine admits that there are cases of synonymy induced by stipulative definition or abbreviation. That is to admit that there is such a state of affairs as two expressions' having the same meaning; if so, may it not come about by other means, for example, convergence of usage?[44] (3) Quine does not think there is any unclarity about the notion of logical truth. (The logical truths are the unproblematic subclass of the class of truths that are said in "Two Dogmas" to be "analytic by general philosophical acclaim.") But the notion of logical truth already presupposes synonymy—'all banks are banks' is a logical truth only if the first occurrence of 'banks' is synonymous with the second.[45]

Quine's later writings make clear that his difficulties over synonymy stem largely from his demanding a behaviorist account of it—some way of determining from a person's linguistic responses and dispositions whether two expressions are synonymous for him or not.[46] To this might not the proper response be: so much the worse for behaviorism?

I say no more about the first part of "Two Dogmas," but turn to the second part, from which (as noted above) we may extract an argument against the existence of analytic truths. (A parallel argument could be given to discredit the notion of *a priori* truths; much of what I say below applies equally to the parallel version.) The argument I have in mind runs as follows:

1. If a statement is analytic, it is immune to revision.
2. No statement is immune to revision.
3. Therefore, no statement is analytic.

Premise 1 is implicit rather than explicit in the article; premise 2 is a direct quote.[47]

To evaluate this argument, we must begin by asking what Quine means by 'statement' and what he means by 'immune to revision'. As for 'statement', the main possibilities are obviously (a) sentence and (b) proposition.[48] As for 'immune to revision', the main possibilities are (c) such that no one *could* ever revise it (i.e., reject it or change one's mind about its truth value) and (d) such that no one could ever *reasonably* reject it. These possibilities intersect to give us four possible readings of each premise. It is my contention that there is no combination that gives plausible readings to both premises simultaneously.

The four possible readings of the first premise are the following:

1ac: If a sentence is analytic, no one could ever reject it.
1ad: If a sentence is analytic, no one could ever reasonably reject it.
1bc: If a proposition is analytic, no one could ever reject it.[49]
1bd: If a proposition is analytic, no one could ever reasonably reject it.

The first two theses, 1ac and 1ad, have no plausibility at all. One could reject any sentence, even one that is now analytic, provided an appropriate change in its meaning takes place first. By the same token, one could reject any sentence reasonably, provided an appropriate change in its meaning takes place first. If 'all bachelors are unmarried' came to mean that all triangles are four-sided, it would be reasonable to reject it.

I am inclined to give the same verdict on the third thesis, 1bc. People do the craziest things; couldn't any proposition whatsoever, even if analytic and no matter how obvious, be rejected by some benighted person? Well, that is controversial. It would be denied by Davidson, for whom ascriptions of belief must be constrained by a principle of charity—a principle that places limits on how much craziness there can be.[50] Here I simply note that if 1bc is to be accepted on grounds of charity, the corresponding version of the second premise, 2bc, which says that every proposition is such that someone might reject it, must be rejected on the same grounds. So, there is no hope for a sound argument against analyticity using the b–c combination.

That leaves us with the b–d combination as the only one left in the running. That is to say, we have now to consider the argument whose premises are the following:

1bd: If a proposition is analytic, no one could ever reasonably reject it.
2bd: Every proposition is such that it could be reasonably rejected.

Premise 1bd has more going for it than the other versions of premise 1, but I am not sure that even this version of the premise is true. Might there not be analytic propositions that in certain states of information it is reasonable to reject? I have in mind, for example, the negation of Cantor's naive comprehension principle, which was proved analytic by Russell's paradox, but which before the paradox came to light was arguably reasonable to reject.

(In the parallel argument against *a priori* truths, the first premise would say that if a proposition is *a priori*, no one could ever reasonably reject it. This is indeed a consequence of some accounts of the *a priori*, e.g., Chisholm's.[51] However, there are other conceptions of the *a priori* that are compatible with a thoroughgoing fallibilism, e.g., Pollock's.[52])

What I want to focus on, however, is premise 2bd, which I find highly questionable to say the least. How could it ever be reasonable to reject the proposition we now express by the sentence 'all triangles have three sides'? Or the law of noncontradiction? Or (to take an example of an even harder-to-reject proposition from Putnam) the proposition we now express by 'not every proposition is both true and false'? To reject *that* would not merely be to flout the law of noncontradiction; it would be to hold that every single proposition is both true and false.[53] So, it seems there are at least some propositions that could never be reasonably rejected. The only version of the "Two Dogmas" argument against analytic propositions that has any chance of succeeding fails.

D. The Hume Problem

In a well-known passage in the *Prolegomena,* Kant says, "I openly confess my recollection of David Hume was the very thing which many years ago first interrupted my dogmatic slumber and gave my investigations in the field of speculative philosophy a quite new direction" (p. 8). It is generally agreed that it is Hume's teachings on causation that Kant found so disruptive of his sleep. But *which* of Hume's teachings on causation? There are two main candidates. In the *Treatise*, Hume distinguished the following two questions:

> First, For what reason we pronounce it *necessary*, that every thing whose existence has a beginning, shou'd also have a cause?
>
> Secondly, Why we conclude, that such particular causes must *necessarily* have such particular effects? . . .[54]

In other words, he distinguished the question of the necessity or contingency of *specific causal laws* (e.g., that fire causes smoke) from the question of the necessity or contingency of the *general causal maxim* (i.e., the principle that every event has a cause). These questions are independent of each other. (For a discussion of this, see appendix B.) As he is standardly interpreted, however, Hume's position is that they are to be answered in the same way: specific laws are not necessary, and neither is the general maxim.[55]

Which of these two negative theses about causation roused Kant from his slumber? Despite a few misleading indications from Kant that point toward the first, Norman Kemp Smith has made a convincing case that it was the second.[56] Ample support for Kemp Smith occurs in the following passage:

> That sunlight should melt wax and yet also harden clay, no understanding, [Hume] pointed out, can discover from the concepts which we previously possessed of these things, much less infer them according to a law. Only experience is able to teach us such a law. . . . If, . . . [however], wax, which was formerly hard, melts, I can know *a priori* that *something* must have preceded . . . upon which the melting has followed according to a fixed law, although *a priori*, independently of experience, I could not determine, *in any specific manner*, either the cause from the effect, or the effect from the cause. Hume was therefore in error in inferring from the contingency of our determination *in accordance with the law* the contingency of the *law* itself. (A766/B794)

Taking "the law itself" to refer to the general maxim and bearing in mind that for Kant the empirical coincides with the contingent, we learn four things from this passage. (i) Kant distinguished the question of the necessity or contingency of specific causal laws from the question of the necessity or contingency of the general causal maxim.[57] (ii) He accused Hume (whether fairly or not) of erroneously inferring from the contingency of specific laws the contingency of the general maxim. (iii) He agreed with Hume that specific causal laws are contingent. (iv) In opposition to Hume, he held that the general causal maxim is necessary.

The real point of contention between Kant and Hume, then, was the modal and epistemic status of the general causal maxim. Kemp Smith has suggested that it was through pondering Hume's views on this principle that Kant was led to formulate the central question of the *Critique of Pure Reason*.[58]

We can see how this might have come about by looking at Hume's argument against the apriority of the maxim, which occurs in *Treatise* I.3.iii. Taking a few liberties, we may reconstruct the argument as follows:[59]

1. If a proposition is *a priori*, its denial implies a contradiction.
2. If a proposition implies a contradiction, it is inconceivable.
3. The denial of the causal maxim is conceivable.
4. Therefore, the denial of the causal maxim does not imply a contradiction (from 2 and 3).
5. Therefore, the causal maxim is not *a priori* (from 1 and 4).

We may add to this that the causal maxim is not knowable empirically, either. As a universal proposition (*every* event has a cause), it outruns what experience could ever establish. (Even its individual instances lie beyond the power of experience to verify, for they, too, are implicitly universal: an event E has a cause only if there is some event C such that *whenever* a C-type event occurs, an E-type event follows.) So, if Hume is right, the causal maxim is not knowable at all—a result that Kant thought would be disastrous for science and knowledge. Such is the problem Hume posed for Kant.

The reconstruction above may not be entirely true to Hume, for reasons discussed below. Nonetheless, setting the argument out as I have lets me show how Hume's challenge generates the central problem of the *Critique of Pure Reason*. In Kantian terminology, the short way to say that the denial of a proposition p implies a contradiction is 'p is analytic'. From Kant's point of view, therefore, the argument amounts to this: the causal maxim is not *a priori* because it is not analytic (step 4), and only the analytic is *a priori* (step 1). A similar argument, Kant perceived, would show that not even mathematics is *a priori*—an assertion from which Hume's "good sense would have saved him" (B20). This is why the category of synthetic *a priori* judgments was so important for Kant: if they are possible, the Humean argument above can be evaded.

I believe the argument 1–5 is a plausible reconstruction of how Hume's argument may have struck Kant;[60] I believe it is also usefully pedagogically as a lead-in to the central question of the *Critique of Pure Reason*. To be fair to Hume, however, I note two alternative ways in which his argument might be construed.

First, some scholars have questioned whether Hume was committed after all to denying the existence of synthetic *a priori* propositions. Perhaps when he speaks of a proposition p "implying a contradiction," he does not mean that p by itself (or p augmented only by definitions) logically implies a contradiction, but rather that p in conjunction with one or more self-evident or intuitively certain premises logically implies a contradiction. If the admissible extra premises may include propositions not certified by logic and definitions alone, premise 1 above would not imply that only the analytic is *a priori*.[61]

Second, Gary Rosenkrantz has pointed out to me that premises 1 and 2 in the argument above could be telescoped into the single premise *if a proposition is* a priori, *its denial is inconceivable*. This premise is capable of standing on its own, without need of defense using 'implies a contradiction' as a middle term,[62] and it combines with premise 3 to yield the original conclusion. There is no need in the resulting argument to rely on the questionable assumption that only the analytic is *a priori*.

So much by way of defense Hume; I end here with a criticism of him. On a closer look at *Treatise* I.iii.3, it is not entirely clear that he says the denial of the causal maxim is conceivable. What he says appears rather to be this: for any event e and any supposed cause of it c, we can separate the ideas of e and c in the imagination; that is, we can imagine or conceive e to happen without its being preceded by c. Invoking Hume's principle that whatever is conceivable is possible, we could then conclude that for any events e and c, it is possible that e occurs without being caused by c. But that is not yet to show that an event e might occur without *any* cause; it is only to show that for any particular cause c, e might occur without being caused *by c*.[63] So, perhaps in the end Hume offers cogent reasons only for denying the necessity of specific causal laws, not for denying the necessity of the general maxim.

E. The Herz Problem

Believing that some synthetic propositions are known *a priori* makes Kant a *judgment rationalist* as textbooks define that term. He is also a *concept rationalist*, for he tells us that "*a priori* origin is manifest in certain concepts no less than in judgments" (B5). More precisely, he believes that there are concepts that are *a priori* in the sense that they are not abstracted from experience (or compounded in Lockean fashion out of concepts so abstracted) but that are applicable to objects of experience nonetheless. The most famous of these are his twelve categories, of which the concepts of substance and causation are the most important.

In his Inaugural Dissertation of 1770, Kant had distinguished two sources of representations, sensibility and intelligence.[64] Through the former we have empirical representations, which are caused to arise in us by our encounters with objects; through the latter we have *a priori* representations, which are not caused by any such encounters. The *a priori* representations include several of the concepts that later figure in Kant's list of categories—possibility, necessity, existence, substance, and cause. He had also held in the Dissertation that the representations of sensibility are only representations of things as they appear, whereas the representations of intelligence are representations of things as they are.

The status of such *a priori* concepts came not long thereafter to seem very puzzling to Kant. In a letter to his former pupil Marcus Herz in 1772,[65] he asked how it is possible for representations to represent or refer to objects if (i) the objects do not cause the representations (as happens with empirical concepts) and (ii) the representations do not cause the objects (as happens, Kant says, "when divine cognitions are conceived as the archetypes of all things"). Our *a priori* representations stand in no causal relation, either active or passive, to their putative objects, so how do we know that they *have* objects? Indeed, what could their having objects possibly amount to? As he put it in the *Nachlass*: how can "that which is merely an offspring of my own brain . . . be related to an object as its representation?"[66]

Notice the assumption that Kant is apparently making here: a concept can have the Fs for its denotation only if there is a causal connection between Fs and our employment of the concept. This is an assumption that is also found in some important contemporary writing on reference and intentionality—for example, in the work of Putnam and Fodor.[67]

The problem suggested to Kant by his remembrance of Hume—how are synthetic *a priori* judgments possible?—may be rephrased as follows: how is judgment rationalism possible? In similar fashion, the problem posed by Kant in his letter to Herz may be rephrased this way: how is concept rationalism possible? How, in other words, is it possible for there to be *a priori* concepts with nonempty denotation—concepts that were not derived from experience but that have application to the objects of experience nonetheless?[68]

The answer Kant eventually worked out to the Herz problem is contained in his Transcendental Deduction of the Categories, to which I devote chapter 7. The gist of it is that the categories, though not derived from experience, help to make it possible: their "objective validity" or applicability to objects is necessary if experience is to take place at all. As he puts it in the *Prolegomena*, "they do not derive from experience, but experience derives from them" (p. 60).

unless it were perceived, since strictly speaking there is no *it*. But we can say this: there being an appearance entails the occurrence of certain perceptions.

25. Wilfrid Sellars interprets Kant's distinction between things in themselves and appearances as the distinction between things having "formal reality" and things having "objective reality" (in the medieval and Cartesian senses of these terms). Thus, things in themselves exist *simpliciter* while appearances exist only as contents of thought and awareness. See his *Science and Metaphysics: Variations on Kantian Themes* (London: Routledge & Kegan Paul, 1968), ch. 2, and "Kant's Transcendental Idealism," in *Proceedings of the Ottawa Congress on Kant*, edited by P. Laberge, F. Duchesneau, and B.E. Morrisey (Ottawa: University of Ottawa Press, 1976), pp. 165–81. Similar interpretations are advanced and illuminatingly discussed by Phillip Cummins in "Kant on Outer and Inner Intuition," *Nous*, 2 (1968), 271–92, and Richard Aquila, *Representational Mind: A Study of Kant's Theory of Knowledge* (Bloomington, Ind.: Indiana University Press, 1983), especially ch. 4. For Aquila's views, see also his "Things in Themselves: Intentionality and Reality in Kant," *Archiv für Geschichte der Philosophie*, 61 (1979), 293–307.

26. R.J. Hirst, "Realism," in *The Encyclopedia of Philosophy*, edited by Paul Edwards (New York: Macmillan, 1967), vol. 7, p. 77.

27. This is R.B. Perry's "cardinal principle of idealism" as quoted in Curtis Brown, "Internal Realism: Transcendental Idealism," in *Midwest Studies in Philosophy*, vol. 12, edited by Peter A. French, Theodore E. Uehling, Jr., and Howard K. Wettstein (Minneapolis: University of Minnesota Press, 1988), pp. 145–55.

28. Definitions get tricky at this point. I said above (n. 24) that virtual objects are mind-dependent in the sense that their existence entails the occurrence of cognitive acts. On a Cartesian view of the mind, according to which a mind must be thinking at every moment at which it exists, minds would be mind-dependent in just this sense. If we said 'x is mind-dependent iff x would not exist unless there were a cognitive act *apprehending* x,' then perhaps even Cartesian minds would no longer be mind-dependent. But then, strictly speaking, virtual objects would not be mind-dependent either, since they are not in the range of our quantifiable variables.

29. Michael Dummett cites this as a possible view in *Truth and Other Enigmas* (Cambridge, Mass.: Harvard University Press, 1978), p. 231.

30. Another important contemporary form of antirealism does away with *facts* altogether, holding that there is nothing in the world to make sentences in the target area evaluable as either true or false. Expressivist or noncognitivist theories about ethical discourse are one paradigm here. For a canvassing of possibilities, see Simon Blackburn, *Spreading the Word* (Oxford: Clarendon Press, 1984), ch. 5, and Crispin Wright, *Truth and Objectivity* (Cambridge, Mass.: Harvard University Press, 1992), pp. 1–7.

31. For Dummett's views, see *Truth and Other Enigmas*, especially the preface and essay 10, and *The Logical Basis of Metaphysics* (Cambridge, Mass.: Harvard University Press, 1991). For Putnam's views, see "Realism and Reason," in *Meaning and the Moral Sciences* (London: Routledge & Kegan Paul, 1978), pp. 123–40, and *Reason, Truth, and History* (Cambridge: Cambridge University Press, 1981). It is a seldom-noted point of commonality that both philosophers employ a "BIV" test for realism: do you accept BIValence, and do you allow that we might all be Brains In Vats?

Chapter 2

1. Gottlob Frege, *The Foundations of Arithmetic*, translated by J.L. Austin (Evanston, Ill.: Northwestern University Press, 1980), p. 3.

2. Alvin Plantinga, *The Nature of Necessity* (Oxford: Oxford University Press, 1974), pp. 1–2.

3. I do not discuss here Saul Kripke's suggestions that there are *a priori* truths that are not necessary and necessary truths that are not *a priori*. See his *Naming and Necessity* (Cambridge, Mass.: Harvard University Press, 1980), pp. 53–56 and 97–105.

4. For anticipations of the synthetic *a priori* in Kant's predecessors, however, see Lewis White Beck, "Analytic and Synthetic Judgments Before Kant," in *Essays on Kant and Hume* (New Haven, Conn.: Yale University Press, 1978), pp. 80–100.

5. In the *Prolegomena* Kant puts the point this way: "Analytical judgments express nothing in the predicate but what has been already actually thought in the concept of the subject, though not so distinctly or with the same (full) consciousness" (p. 14).

6. See Lewis White Beck, "Kant's Theory of Definition," in *Studies in the Philosophy of Kant* (Indianapolis, Ind.: Bobbs-Merrill, 1965); reprinted in *Kant: Disputed Questions*, edited by Moltke S. Gram (Chicago: Quadrangle Books, 1967), pp. 215–27.

7. For discussion of this, see Henry E. Allison, *The Kant-Eberhard Controversy* (Baltimore: Johns Hopkins University Press, 1973), pp. 42–43.

8. Quoted in Allison, *The Kant-Eberhard Controversy*, pp. 174–75.

9. To avoid this objection, one might try to unpack Kant's notion of "covert" containment in terms of latency or dispositionality: we say that property P is contained in concept S if anyone who thinks of something as S *and considers whether it is also P* must think that it is P. But this proposal will classify as analytic some propositions that Kant wants to count as synthetic—for example, a straight line is the shortest distance between two points.

10. In chapter 1 of *Kant's Analytic* (Cambridge: Cambridge University Press, 1966), Jonathan Bennett notes the narrowness of Kant's definition and complains that it makes the existence of synthetic *a priori* judgments too easy to establish. He therefore suggests a broader notion according to which a proposition should count as analytic if it may be verified by purely conceptual considerations. I think this threatens to make the *non*existence of synthetic *a priori* judgments too easy to establish: if 'purely conceptual' equals *a priori*, it is trivially true that everything *a priori* is analytic. On the other hand, if conceptual considerations are limited (as I propose below) to those mobilizing definitions or analyses and logic, there is still room for the synthetic *a priori*.

11. Ironically, those who criticize Kant's definition for being subjective or psychologistic often rely on a loose notion of contradiction that is at least as subjective as anything Kant offers. They call a statement contradictory not because it has the logical form of a contradiction, but because it has a contradictory ring to their ears.

12. Accurately speaking, this is a definition of what it is to be *analytically true*. We need also to recognize the category of the *analytically false*: A is analytically false iff from A itself, a formal contradiction may be derived, and so on. Synthetic statements then comprise all those that are neither analytically true nor analytically false.

13. Gottlob Frege, *The Foundations of Arithmetic*, sec. 3.

14. Rudolf Carnap, *The Logical Structure of the World*, translated by Rolf A. George (Berkeley: University of California Press, 1967), p. 176.

15. C.I. Lewis, *An Analysis of Knowledge and Valuation* (La Salle, Ill.: Open Court, 1946), p. 96.

16. W.V. Quine, "Two Dogmas of Empiricism," in *From a Logical Point of View* (New York: Harper Torchbooks, 1963), pp. 20–46, in sec. 1.

17. Question: why is there no analytic *a posteriori*? Answer: the definition

of '*a posteriori* proposition' that complements our definition of '*a priori* proposition' is that p is such that no one *could* know it save through experience. Although there may be analytic propositions that a given person actually comes to know on the basis of experience (someone looks out the window, sees that it is raining, and deduces 'it is raining or it is not raining' by the law of addition), there are arguably no analytic propositions that could be known *only* through experience.

18. Note, however, that classifying logical principles as analytic does nothing to explain how we know them to be true. As we have defined 'analytic', logical principles are included within the sphere of the analytic simply by courtesy; it is analytic that they are analytic.

19. Compare Kant's letter to Schulz of November 25, 1788, in which he points out that one may form different concepts of the same number by many different additions and subtractions. If equations of arithmetic were analytic, he argues, then in thinking 3 + 4, one would also be thinking 2 + 5, which "does not jibe with my own awareness." See Arnulf Zweig, ed., *Kant: Philosophical Correspondence, 1759–99* (Chicago: University of Chicago Press, 1967), pp. 128–30.

20. For a brief discussion of these two points and related matters, see Karel Lambert and Gordon Brittan, *An Introduction to the Philosophy of Science* (Englewood Cliffs, N.J.: Prentice-Hall, 1970), ch. 2.

21. Kant claims synthetic *a priori* status for geometry at large, making exception only for propositions such as 'a = a' that serve as "links in the chain of method" (B16–17). Other examples of synthetic *a priori* propositions in geometry he cites are the following: space has three dimensions (B41); two straight lines cannot enclose a space (A47/B65); the sum of the angles of a triangle is equal to two right angles (A716/B744).

22. David Hume, *A Treatise of Human Nature*, I.ii.4 (pp. 49–50 in Selby-Bigge's edition). This passage also gives the following reason for regarding the common maxim as synthetic: "A right line can be comprehended alone; but this definition [of a right line as the shortest] is unintelligible without a comparison with other lines. . . ."

23. Accurately speaking, what it makes analytic is 'any straight line between two points is shorter than any nonstraight line between the points'. The additional implication of the Hume-Kant proposition that there is only one such straight line is not a matter of definition, and is indeed not even true in Riemannian geometry.

24. Ludwig Wittgenstein, *Tractatus Logico-Philosophicus*, translated by D.F. Pears and B.F. McGuinness (London: Routledge & Kegan Paul, 1961), proposition 6.36111. In my copy of the *Tractatus*, Wittgenstein's figure b unfortunately contains an erroneous extra hyphen, destroying the congruence of a and b.

25. For more on this point, see section XI of my "Right, Left, and the Fourth Dimension," *The Philosophical Review*, 96 (1987), 33–68, and also in *The Philosophy of Right and Left*, edited by James Van Cleve and Robert E. Frederick (Dordrecht: Kluwer, 1991), pp. 203–34.

26. Compare the following passage from II.xiii.3 of the *New Essays on Human Understanding* in which Leibniz distinguishes between an absolute and a relative sense of distance:

> To put it more clearly, the distance between two fixed things—whether points or extended objects—is the size of the shortest possible line that can be drawn from one to the other. This distance can be taken either absolutely or relative to some figure which contains the two distant things. For instance, a straight line is abolutely the dis-

> tance between two points; but if these two points both lie on the same spherical surface, the distance between them *on that surface* will be the length of the smaller arc of the great circle that can be drawn from one to the other. [*New Essays on Human Understanding*, translated by Peter Remnant and Jonathan Bennett (Cambridge: Cambridge University Press, 1981), p. 146; emphasis mine]

27. It must be confessed that this way of defending Kant affords no guarantee that absolutely straight lines are actually exemplified: they would not be if a curved space were the whole of space. But that is all right, since the synthetic *a priori* status of principles about lines or figures does not depend on whether the lines or figures are actually exemplified. For more on this point, see Gary Rosenkrantz, "The Nature of Geometry," *American Philosophical Quarterly*, 18 (1981), 101–10.

28. This is not one of Kant's own examples. What he says at B44 implies that there are no synthetic *a priori* truths about colors, but I think he was surely wrong about that.

29. Why not simply say 'for each color distinct from redness'? Because determinates of redness (e.g., scarlet) are distinct from it, as are its determinables (e.g., being red-or-orange), yet we do not want our definition to imply that red things cannot be scarlet or red-or-orange.

30. The physical definitions could be more sophisticated than this, perhaps making reference as well to effects on the human nervous system.

31. F.P. Ramsey, "Critical Notice of L. Wittgenstein's *Tractatus Logico-Philosophicus*," *Mind*, 32 (1923), 465–78, at p. 473. In "Some Remarks on Logical Form" [*Proceedings of the Aristotelian Society*, 9 Suppl. (1929), 162–71], Wittgenstein in effect concedes Ramsey's point, allowing that atomic propositions may exclude one another even though they do not contradict one another.

32. It is also the chief supporting example used by C.H. Langford in "A Proof That Synthetic *A Priori* Propositions Exist," *The Journal of Philosophy*, 46 (1949), 20–24. Langford at first appears to make his task too easy by defining an analytic proposition as "one that can be certified solely by reference to logical principles" (p. 22). It emerges, however, that recourse to definitions is also permitted. It will become clear below that I agree with Langford on this point: "[I]t is sufficient for our purposes that there should be at least one adequate definition from which this consequence [having twelve edges] does not follow" (p. 22).

33. This paragraph gives my answer to the title question of Lewis White Beck's "Can Kant's Synthetic Judgments Be Made Analytic?" *Kant-Studien*, 46 (1955), 168–81; reprinted in *Kant: Disputed Questions*, edited by Moltke S. Gram (Chicago: Quadrangle Books, 1967), pp. 228–46. A synthetic sentence 'S is P' can always be made analytic by enriching the meanings of the subject term, but the sentence 'every S in the old sense is an S in the new sense' will be synthetic and *a priori* if the original sentence was. There is no banishing the synthetic *a priori* by this method.

34. Thanks to my former student Jeremy Bernstein.

35. D. Hilbert and S. Cohn-Vossen, *Geometry and the Imagination*, translated by P. Nemenyi from the 1932 German edition (New York: Chelsea, 1956). According to Imre Lakatos, *Proofs and Refutations* (Cambridge: Cambridge University Press, 1976), p. 15, this definition comes originally from Möbius, who used it to prevent two polyhedra with an edge or a vertex in common from counting as a single polyhedron.

36. I do not know what the formulation of this clause generalized to cover all polyhedrons should be.

37. Here is another strategy: if it is analytic that cubes have six faces and twelve edges, we may use Euler's formula ($V - E + F = 2$) to deduce that they have eight vertices. But now we must determine whether Euler's formula is analytic or synthetic. I invite the reader to consult Lakatos's discussion of the proof of this formula in *Proofs and Refutations*, asking himself whether the various lemmas that are needed are true by definition or simply seen to be true on the strength of their intuitive (*anschauliche*) evidence.

38. As pointed out by A.C. Ewing in *The Fundamental Questions of Philosophy* (London: Routledge and Kegan Paul, 1951), ch. 2.

39. A notable exception is Anthony Quinton, "The *A Priori* and the Analytic," *Proceedings of the Aristotelian Society*, 64 (1963–64), pp. 31–54; reprinted in *Philosophical Logic*, edited by P.F. Strawson (Oxford: Oxford University Press, 1967), pp. 107–28. Quinton's purported proof consists in the following chain of claimed implications: *a priori* ⇒ necessary ⇒ not contingent ⇒ not contingent on anything outside ⇒ true in virtue of factors internal to itself ⇒ true in virtue of its meaning. I question the equation of 'contingent' with 'contingent on something'—'contingent' just means 'possibly false'. I also question the significance of the conclusion that necessary truths are true in virtue of meanings. It is by virtue of its meaning that a sentence expressing a necessary truth expresses the truth that it does; it is not by virtue of the meaning of the sentence expressing it that a given truth is necessary.

40. I find that some take him to deny (v). But merely to deny (v) is to leave open the possibility that some statements are clearly analytic and others clearly synthetic, which Quine would surely dispute. I find that others take him to deny (vi), holding that there is never a fact of the matter whether a given statement is analytic or synthetic.

41. W.V. Quine, "Two Dogmas of Empiricism," in *From a Logical Point of View*, pp. 20–46.

42. For more on this side of Kant's views, see the selections in Van Cleve and Frederick, *The Philosophy of Right and Left*. Especially relevant are the selections from Kant's Inaugural Dissertation of 1770 and the pieces by Jonathan Bennett and Martin Gardner.

43. H.P. Grice and P.F. Strawson, "In Defense of a Dogma," *Philosophical Review*, 65 (1956), 141–58. One can make this claim without hypostasizing meanings, contrary to what Quine has urged in reply in *Word and Object* (Cambridge, Mass.: MIT Press, 1960), p. 206. Compare: if there is such a thing as an object's having a shape, then there is such a thing as two objects' having the same shape; this is so even if the logical form of 'x has a shape' is 'Fx' and not the hypostasizing 'xRy'.

44. Grice and Strawson, "In Defense of a Dogma."

45. P.F. Strawson, "Propositions, Concepts, and Logical Truths," *Philosophical Quarterly*, 7 (1957), 15–25. In *Word and Object*, Quine says that this is an objection he "cannot claim to have answered anywhere" (p. 65, n. 3).

46. See, for example, Quine, *Word and Object*, ch. 2.

47. The full quote is, "Conversely, by the same token, no statement is immune from revision," coming just one sentence after, "Any statement can be held true come what may, if we make drastic enough adjustments elsewhere in the system." I have always wondered by *what* token Quine deems both of these doctrines true. The Duhemian possibility of holding onto any statement in the face of recalcitrant experience is sufficiently secured if there is always *something* else we can deny to save a given hypothesis; from this it does not follow that to save a given hypothesis we may deny *anything* else we please. That is, it does not follow that *everything* is revisable.

48. Instead of construing statements as propositions, we could to the same end construe them as sentences with their meanings held constant.

49. Here is a way to put this without invoking an ontology of propositions: if a sentence is analytic (as used by us now), then no one could reject it while meaning the same by it as we do now.

50. See Donald Davidson, *Inquiries into Truth and Interpretation* (Oxford: Clarendon Press, 1984), esp. essays 9–11, 13, and 14. Quine himself would also deny that anything whatever can be rejected, thanks to the dictum "deny the doctrine and change the subject," which he thinks governs the standard laws of logic among other things. For further discussion of this dictum, see my "Analyticity, Undeniability, and Truth," *Canadian Journal of Philosophy*, 18 Suppl. (1992), 89–111.

51. Roderick M. Chisholm, *Theory of Knowledge* (Englewood Cliffs, N.J.: Prentice-Hall, 1977; 2nd edition), pp. 42–43. Chisholm makes all *a priori* knowledge depend on axioms, which are defined as propositions that are necessarily certain for any subject who understands them; this implies that they could never be reasonably rejected. So, if *anything* could be reasonably rejected, there are no axioms and hence no items of *a priori* knowledge.

52. John L. Pollock, *Knowledge and Justification* (Princeton, N.J.: Princeton University Press, 1974), ch. 10. The essential idea is that a proposition is *a priori* if having an intuition that it is true gives one a *prima facie* reason for believing it—a reason that normally confers knowledge but that can on occasion be defeated.

53. Hilary Putnam, "Analyticity and Apriority: Beyond Wittgenstein and Quine," *Midwest Studies in Philosophy*, vol. 4, edited by Peter French, Theodore E. Uehling, Jr., and Howard K. Wettstein (Minneapolis: University of Minnesota Press, 1979), pp. 423–41.

54. Hume, *A Treatise of Human Nature*, I.iii.3 (p. 78 of the Selby-Bigge edition).

55. On page 82 of the *Treatise*, Hume says he is going to "sink" the first of these questions into the second. This does not mean that he no longer distinguishes them, but only that he thinks an answer to the second will furnish us with an answer to the first. The connection comes out most clearly on page 172: if we analyze particular causal relationships in the way Hume recommends, we will readily see that the causal maxim is not a necessary truth.

I am ignoring here the view of some scholars that for Hume causal laws are necessary after all. For discussion, see Kenneth Winkler, "The New Hume," *The Philosophical Review*, 100 (1991), 541–79.

56. Norman Kemp Smith, *A Commentary to Kant's 'Critique of Pure Reason'* (New York: Humanities Press, 1962; reprint of 1923 edition), pp. xxv–xxix and 61–64. One way Kant misleads the reader is by discussing only the first of Hume's questions in the introduction to the *Prolegomena*.

57. Here, but not always; he conflates the questions at B5.

58. Kemp Smith, *Commentary*, p. 30.

59. Hume, *Treatise*, I.3.iii (pp. 79–80 in Selby-Bigge). I have also taken a hint or two from section IV of Hume's *Inquiry Concerning Human Understanding*. The argument in the *Treatise* is followed by critiques of four arguments in favor of the maxim.

60. As Kemp Smith points out (*Commentary*, pp. xxviii–xxix), Kant probably had not read the *Treatise*, which was not available in German, but could have learned of its teachings on causation through Beattie.

61. An enlightening interpretation of Hume along these lines has been offered by Georges Dicker in "Hume's Fork Revisited," *History of Philosophy Quarterly*, 8 (1991), 327–42. Dicker goes on to suggest that Hume can admit synthetic *a priori* propositions as long as they concern only "relations of ideas" and not "matters of fact," that is, as long as they are not propositions asserting the existence of nonabstract entities or permitting the inference to some such

propositions from others. The resulting view would let arithmetic and geometry count as synthetic *a priori* but would exclude the causal maxim.

62. According to Arthur Pap, *Semantics and Necessary Truth* (New Haven, Conn.: Yale University Press, 1958), ch. 4, all Hume *means* in saying that a proposition implies a contradiction is that it is inconceivable. If this is correct, premise 1 is already equivalent to the telescoped premise.

63. In other words, the premise Hume needs is not 'for any c, it is conceivable that e occurs without being caused by c', but rather 'it is conceivable that for any c, e occurs without being caused by c'. Could the latter be what is intended by his saying that we can separate the idea of an event e from the idea of "a cause" of it?

64. Immanuel Kant, "On the Form and Principles of the Sensible and Intelligible World," in *Theoretical Philosophy, 1755–1770*, translated and edited by David Walford in collaboration with Ralf Meerbote. The Cambridge Edition of the Works of Immanuel Kant, vol. 1 (Cambridge: Cambridge University Press, 1992), pp. 373–416.

65. The letter may be found in Zweig, *Kant: Philosophical Correspondence, 1759–99*, pp. 70–76.

66. It is sometimes unclear whether in raising questions about "relation to an object" Kant is asking how representations have intentionality or what makes them veridical. I say more about this in chapter 7.

67. Hilary Putnam, *Reason, Truth, and History* (Cambridge: Cambridge University Press, 1981), ch. 1; Jerry Fodor, *A Theory of Content* (Cambridge, Mass.: MIT Press, 1990), esp. ch. 4. Fodor says that the "foundational intuition" of his theory is that "a symbol means *cat* in virtue of some sort of reliable causal connection that its tokens bear to cats" (p. 127).

68. The typical concept empiricist will question whether we even *possess* concepts not derived from experience, not merely whether such concepts have denotation.

Chapter 3

1. For a discussion of this objection, see Henry Allison, *The Kant-Eberhard Controversy* (Baltimore: Johns Hopkins University Press, 1973), pp. 34–35, and *Kant's Transcendental Idealism: An Interpretation and Defense* (New Haven, Conn.: Yale University Press, 1983), pp. 111–14. The "neglected alternative" may be made vivid by the much maligned but highly suggestive "red spectacles" analogy: granted that we see everything as red owing to the red spectacles permanently affixed to our noses, why could it not be the case that the things on the other side of the spectacles happen to be red?

2. See B168, where Kant may be making a similar point.

3. I ignore here the following problem, which is created by the so-called paradox of strict implication: if B is true, it is necessarily true, in which case it follows from any premise whatsoever.

4. If the 1–2 inference is to be valid as it stands (and not just as an enthymeme), the missing premise must itself be a necessary truth. This presents no problem; surely 'cubes exist only in being constructed' is the sort of thing that would be true necessarily if true at all.

5. "Empirical intuition is possible only by means of the pure intuition of space and of time. What geometry asserts of pure intuition is therefore undeniably valid of empirical intuition" (A165/B206). "The formative synthesis through which we construct a triangle in imagination is precisely the same as that which we exercise in the apprehension of an appearance" (A224/B272).

6. For example, at A226/B274 Kant says that the existence of anything too subtle for our senses to perceive consists in the following fact: "[W]e should,