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The Ideality of Space

Incongruent Counterparts

> Those who cannot yet rid themselves of the
> notion that space and time are actual quali-
> ties inherent in things in themselves may ex-
> ercise their acumen on the following para-
> dox. When they have in vain attempted its
> solution and are free from prejudices at least
> for a few moments, they will suspect that the
> degradation of space and time to mere forms
> of our sensuous intuition may perhaps be
> well founded.
>
> Kant, *Prolegomena to Any Future*
> *Metaphysics* (sec. 13)

Incongruent counterparts are objects that are perfectly similar except for being mirror images of each other, such as left and right human hands. Kant was ev-idently the first major thinker to notice the philosophical significance of such objects. He called them "counterparts" because they are alike in every intrin-sic spatial respect,[1] "incongruent" because, despite their similarity, one could never be put in the place of the other. The left glove, as Kant points out, will not fit the right hand.

Kant called on incongruent counterparts during the course of his career to establish three very different conclusions. In an essay published in 1768, he cited them as proving that space is absolute (existing in its own right and in-dependent of matter), thus vindicating Newton against Leibniz.[2] In the Inaugural Dissertation of 1770, he cited them as proving that our knowledge of space and spatial configurations is by way of intuition, not concept.[3] Finally, in the *Prolegomena* of 1783, he cited them as proving that space is ideal, be-longing only to our form of intuition and not to things in themselves.[4] I have dealt at length with Kant's first two uses of incongruent counterparts else-where[5] and will not do so here (but, for a brief recap of the 1768 argument, see appendix C). In this chapter I discuss Kant's third use of incongruent coun-terparts—the so-called paradox that he offers in proof of idealism. I consider three different reconstructions of Kant's argument and three associated con-

ceptions of things in themselves, seeking to determine which conception of them affords the best argument.

A. The Argument from Intelligibility

Norman Kemp Smith interprets the argument given in Kant's *Prolegomena* along the following lines:[6]

1. The difference between two incongruent counterparts cannot be known conceptually. (A purely conceptual description of a hand will not determine whether it is left or right.)
2. Things in themselves *can* be adequately known conceptually—all their features can be captured by purely conceptual descriptions.
3. Therefore, objects like left and right hands are not things in themselves.

From this conclusion we may presumably generalize and say *no* spatial objects are things in themselves.[7]

The argument above is not explicit in the *Prolegomena*. The materials for it are rather to be found in the Inaugural Dissertation, where Kant does indeed assert each of the premises and the conclusion. Curiously, however, he does not assemble them all into one argument; he asserts the conclusion on independent grounds, and the only lesson he draws from the facts about incongruent counterparts is that our knowledge of space is intuitive rather than conceptual.

Could the argument above be the argument Kant meant to give in the *Prolegomena*? The first premise is indeed there, and there is a hint of the second in Kant's use of the phrase "things as they are in themselves *and as some pure understanding would know them*" (emphasis mine). It is extremely puzzling, however, to find Kant using such an argument in 1783. As Kemp Smith notes, the second premise is pre-Critical. It amounts to saying that things in themselves are noumena in the positive sense (things adequately knowable by nonsensible means), not just in the negative sense (things not adequately knowable by sensible means), and this is a doctrine Kant rejects in the *Critique of Pure Reason* (at least as far as human beings are concerned).[8] Kemp Smith thus concludes that the argument is a remnant of a repudiated view, taking this to explain why there is no trace of the argument in the second edition of the *Critique* in 1787.

B. The Argument from Interchangeability

What is the "paradox" about incongruent counterparts? Kant states it as follows:

> If two things are quite equal in all respects as much as can be ascertained by all means possible, quantitatively and qualitatively, it must follow that the one can in all cases and in all circumstances replace the other, and this substitution would not occasion the least perceptible difference. This is in fact true of plane figures in geometry; but some spherical figures [e.g., spherical triangles in opposite hemispheres with an arc of the

equator as common base and corresponding sides equal] exhibit, notwith-
standing complete internal agreement, such a difference in their exter-
nal relation that the one figure cannot be put in the place of the other.
(*Prolegomena*, p. 33)

Let me use "internally alike" to abbreviate Kant's "quite equal in all respects
as much as can be ascertained by all means possible, quantitatively and qual-
itatively" (i.e., alike in all nonrelational respects) and "interchangeable" to ab-
breviate his "the one can in all cases and in all circumstances replace the
other." The paradox can then be stated more compactly as follows:

1. Things that are internally alike must be interchangeable.
2. Some spatial figures are internally alike but *not* interchangeable.

This is indeed a paradox, since as they stand 1 and 2 contradict each other.
What is the way out?

According to Kant, we must first of all recognize that the first premise is
true only with a qualification; it must be rewritten as

1′. Things *in themselves* that are internally alike must be interchange-
    able.

This restores consistency—*provided* we do not shrink from drawing the con-
clusion that now follows:

3. Some spatial figures are not things in themselves.

This is just the conclusion Kant draws: the figures in question are not things
in themselves, but only "appearances, whose possibility rests upon the rela-
tion of certain things unknown in themselves to something else, namely to our
sensibility" (*Prolegomena*, p. 33). As before, we may presumably extend the
result to all spatial configurations.

My objection to this argument is that the first premise seems perfectly plau-
sible *without* the qualification. If that is so, the only way to avoid contradiction
is to deny the *second* premise. This would mean either denying that incongru-
ent counterparts are internally alike or holding that they are interchangeable
after all. I mention below a supporting ground for each option.[9]

(i) Suppose we take seriously the idea of *absolute direction*. That is, sup-
pose we maintain that having a part lying in a certain direction from another
part does not tacitly involve a relation to something outside the whole.[10] Then
it would be false that incongruent counterparts are internally alike: Kant's
spherical triangles would differ in that the direction from middle to largest to
smallest angle would be clockwise in one and counterclockwise in the other.

(ii) Suppose we take seriously the idea of a *fourth spatial dimension*. Then
it would be false that incongruent counterparts are noninterchangeable: al-
though we cannot interchange spherical triangles in *our* space, spaces would
be possible in which they *could* be interchanged.[11] One could resist this ob-
jection to the second premise by holding that 'interchangeable' must mean 'in-
terchangeable in the *actual* space', but then the *first* premise would become
questionable.

C. The Argument from Reducibility

According to Jill Buroker, the gist of Kant's incongruent counterparts argument is that "the nature of space is incompatible with the analysis of relations."[12] I agree completely as to gist but would like to give a different account of the details. Here, then, is a third reconstruction of Kant's argument:

1. Incongruent counterparts (e.g., Kant's two spherical triangles) are different in virtue of their differing relations to space as a whole. (Thus, triangle 1 bears R to space and triangle 2 does not.)
2. All relations among things in themselves are reducible to nonrelational characters (qualities) of the relata.
3. Therefore, if space and figures within it are things in themselves, triangle 1 must differ internally from triangle 2.
4. But, in fact, triangle 1 does *not* differ internally from triangle 2.
5. Therefore, space itself and figures within it are not things in themselves.[13]

Now for a series of comments to elucidate the argument.

(a) The premise that comes to the fore in this version of the argument is the *reducibility of relations* among things in themselves. It is a merit of Buroker's discussion to bring out the importance of this Leibnizian element in Kant's philosophy. Kant does not explicitly state this premise in the *Prolegomena* argument, but he does endorse it at least implicitly elsewhere. It underlies two of his assumptions in the "Amphiboly" section of the *Critique of Pure Reason*: things in themselves cannot have their nature exhausted by relations, and they cannot differ numerically without also differing qualitatively.[14]

(b) Strictly speaking, it is not the *reducibility* of relations that is needed in the argument above, but only their *supervenience*. That is, a relational fact aRb need not be *equivalent* to any conjunction Fa & Gb, but there must be some such conjunction that *entails* it. I shall nonetheless stick with the language of reducibility in what follows.

(c) In accordance with the reducibility principle, if we have

triangle 1 bears R to space & triangle 2 does not,

we must also have (for some nonrelational features F and G)

triangle 1 is F & space is G & $-$(triangle 2 is F & space is G),

which implies

triangle 1 is F & triangle 2 is not F.

Thus, the triangles must differ internally. This is the justification for step 3 above.

(d) Note that the first premise in the argument above is the conclusion of Kant's 1768 argument: the difference between incongruent counterparts is a difference in their relations to space itself. This bears out Buroker's contention

that Kant's later arguments incorporate, rather than contradict, his earlier conclusion.[15]

(e) But note as well that it would not have been necessary for Kant to make space a term of relations in the argument. He could simply have noted that the relation of incongruent counterparthood, a relation holding directly between the triangles, is an irreducible relation. If relations among things in themselves must be reducible, it would then follow that the triangles are not things in themselves. Thus, I disagree with Buroker's view that if Kant's conclusion of 1768 were denied, the 1783 version of his argument would be undercut.[16]

(f) Moreover, once armed with the reducibility principle, Kant need not have resorted to anything so recondite as incongruent counterparts to make his point, for spatial relations quite generally fail to be reducible (or "internal," in one leading sense of that term). Take, for example, the relation of *distance*: there is nothing about my pen and my ruler taken separately that would enable anyone to deduce that they are now six inches apart. So, Kant could simply have argued thus: all relations among things in themselves are reducible; distance is not a reducible relation; therefore, nothing in the field of the distant-from relation (which is to say, nothing in space) is a thing in itself.

(g) The big question raised by this interpretation of the argument is this: why should Kant have thought that the Leibnizian reducibility principle holds for things in themselves but not for appearances? In the sections that follow, I consider three possible answers to this question, each invoking a different conception of the contrast between appearances and things in themselves.

D. Things in Themselves as Intelligibilia

The first answer is given by the following syllogism:

1. Intelligible entities (entities adequately knowable by the intellect alone) can stand in none but reducible relations.
2. Things in themselves are intelligible entities.
3. Therefore, things in themselves can stand in none but reducible relations.

Appearances, as sensible entities, would not fall under the major premise.

This answer to our question (which seems in places to be Buroker's[17]) is unsatisfying for two reasons. First, the minor premise is the Dissertation doctrine all over again, so we are thrown back to the argument from intelligibility. Is there an answer to our question the *Critical* Kant can give? Second, why should even the pre-Critical Kant have thought the major premise true? That is, why should he have thought irreducible relations a bar to intelligibility? To use James's famous figure, why may the life of the intellect not include flights as well as perchings?

I note in this connection that according to some philosophers, the nature of a number is exhausted by its relations to other numbers.[18] Yet numbers are intelligible entities *par excellence*.

E. Things in Themselves as Things Apart from Relation

In some contexts, Kant's phrase "in itself" seems to contrast with "in relation to other things (or to us)." If we understand the phrase in this way, the unknowability of things in themselves is no longer the utter unknowability of things of a special kind but rather the unknowability in certain respects (namely, nonrelational respects) of things in general.[19] The way is then open for holding that what present themselves to our senses are the very things that are unknowable "in themselves"; what is denied us is not any access whatever to these things but merely knowledge of their nonrelational or qualitative aspects. In support of such an interpretation one might cite the following passage:

> Now a thing in itself cannot be known through mere relations; and we may therefore conclude that since outer sense gives us nothing but mere relations, this sense can contain in its representation only the relation of an object to the subject, and not the inner properties of the object in itself. (B67)

I discuss the merits of this interpretation of the 'in itself' at greater length in chapter 10, section E. For now, I discuss only its usefulness in answering our present question—why Kant thought the reducibility principle holds for things in themselves. The idea would be that if 'in itself' means 'apart from relation', it becomes a tautology to say that things in themselves stand in no relations (or at any rate, none but reducible relations, which in a sense are nothing in addition to the relata and their qualities).

But there is clearly a fallacy in this attempt to derive the reducibility principle. 'No knowledge of things in themselves' may mean 'no knowledge of things apart from relation', but we cannot infer from this that 'things in themselves' means 'things apart from relation'. Or, if we do, we have no right to assume that there *are* any things in themselves; that would be to make an illicit shift from 'in itself' as adverb to 'in itself' as adjective.[20]

F. Things in Themselves as Real Existents

According to the interpretation of transcendental idealism I am advocating, Kant's distinction between things in themselves and appearances is the distinction between things having genuine independent existence and things existing merely as intentional objects. As Wilfrid Sellars has put the matter, it is the distinction between things that have formal reality and things that merely have objective reality (in the medieval and Cartesian senses of these terms).[21] The idea is that to be a thing in itself is to exist *simpliciter*, whereas to be an appearance is to exist only as a content of thought or awareness. Let us see what light this interpretation throws on the incongruent counterparts argument.

It is useful to begin with a little paradox. Consider the following principle:

1. A genuinely existent entity must be *fully determinate*—that is, one predicate out of every possible pair of contradictorily opposed predicates must belong to it.

To me this principle seems self-evident.[22] Kant endorses it at A573/B601, and I believe it also underlies his assumption in section 7 of his chapter on the antinomies that the spatiotemporal world, if it existed as a thing in itself, would have to be either finite or infinite.

So far, so good, but now let us confront our principle with the following recalcitrant fact:

2. The dragon I dreamt of last night was *not* fully determinate. It had teeth, but no definite number of them.

The paradox is that 1 and 2 both seem true, yet seem to conflict. The solution is obvious: 1 and 2 do *not* conflict but can both be true provided the following is also true:

3. The dragon I dreamt of was not a genuinely existent entity.

The dragon existed only in the dreaming of it and therefore falls outside the intended scope of the determinacy principle.

Now what I suggest is that Kant's incongruent counterparts argument should be viewed as paralleling the argument 1–3:

1'. Genuinely existent entities must stand in none but reducible relations.
2'. Spatial figures stand in some irreducible relations.
3'. Therefore, spatial figures are not genuinely existent entities.

On the suggested interpretation of things in themselves, this is exactly what the argument from incongruent counterparts amounts to. To verify this, please refer back to comment (e) of section C. If you think 3' a most un-Kantian conclusion, please read on.

One thing I wish to suggest with the parallel argument about the dream entities is the following possibility: Kant does not regard the reducibility principle as holding for things in themselves in virtue of some special feature they possess, but instead simply regards it as holding for existents as such. Like the determinacy principle, it is intended as a general logical or ontological principle. That being so, the more pressing half of our question is not why things in themselves must be subject to the reducibility principle, but why appearances are exempt from it.[23]

At one level the answer is that appearances, as things merely having intentional being, do not really exist and hence are not there to be exceptions to any principle. But that cannot be the whole story. There is an important sense in which spatial items *do* exist (they are "empirically real," as Kant insists), and we must inquire how items with their ontological status can be exempt from laws governing the *an sich*.

My answer to this question has two parts. First, *things that exist only in relation to consciousness are logical constructions out of conscious states.* This, of course, is the conception of intentional objects as virtual objects that I set forth in chapter 1. I think this conception is both plausible in its own right and plausibly attributable to Kant. If we do not adopt this conception, we must evidently suppose that intentional being or objective reality is a separate ontological status, a second way of being in the world, and it has always been hard

to understand what such a status could amount to. Moreover, that Kant him-self thinks of intentional objects as constructions is strongly suggested by many of his remarks about "objects" in the Transcendental Deduction and else-where,[24] as discussed in chapter 7.

Second, *if As are logical constructions out of Bs, As need not obey all the same laws as Bs, however self-evident these laws may be.* Here is an example: matrices are logical constructions out of ordinary numbers; ordinary multi-plication is commutative, but matrix multiplication is not.

Putting the two italicized principles together, we see that intentionalia need not obey all the same laws as the realia out of which they are constructed. (I assume that conscious states are realia, for otherwise we embark on an end-less regress of constructions out of constructions.) For an illustration, let us re-turn to the dragon of my dream. The dragon is a logical construction out of dream states; this explains not only how it is possible for the dragon to have no existence outside the dream, but also how it is possible for it to be indeter-minate. The construction is governed by the following rule: if 'p' states a fact about the dragon internal to the dream, then p iff Dp (i.e., one dreams that p).[25] Now, since reality is determinate, we must have either Dp or −Dp, but we need not have either Dp or D(−p). It follows that the dragon may be indeterminate: there need not be a correct answer to the question, "Did it have more than 100 teeth?"

Lest the dream example mislead, I must enter two caveats. I do not mean to suggest that appearances in general have for Kant exactly the same status as dream objects. They do not: an empirically real beast has relations to the rest of our experience that the dragon does not.[26] But appearances in general are nonetheless *like* dream objects in that they exist only in relation to con-sciousness. Nor do I mean to suggest that the basis in realia for any fact about appearances must always consist in a subject's apprehending that very fact. To take one of Kant's famous examples, the basis for saying that opposite sides of a house exist simultaneously is not that anyone perceives them as existing si-multaneously; it is rather that the perceptions of the sides individually can be obtained in any order. (See A192−93/B237−38; this example is discussed fur-ther in chapter 9.)

I have not addressed the question of how the reducibility principle in par-ticular might fail for appearances, but I hope I have at least made it clear how appearances might fail to obey laws that are deemed valid for things in them-selves. I hope also to have shown how the conception of things in themselves and appearances as real beings and intentional beings, respectively, makes sense of the incongruent counterparts argument for idealism.

Chapter 4

1. The qualification "spatial" is important: two objects that differed in color (e.g., a red left glove and a green right one) could still qualify as counterparts in Kant's sense. Writers who cite incongruent counterparts as a counterexample to Leibniz's principle of the identity of indiscernibles are therefore jumping the gun.

2. Immanuel Kant, "Concerning the Ultimate Ground of the Differentiation of Directions in Space," in *Theoretical Philosophy, 1755–1770*, translated and edited by David Walford in collaboration with Ralf Meerbote. The Cambridge Edition of the Works of Immanuel Kant, Vol. 1 (Cambridge: Cambridge University Press, 1992), pp. 365–72. In some earlier translations, the German word *Gegenden* is misleadingly rendered as "regions" rather than "directions." On the reasons for preferring "directions" to "regions," see ibid., pp. 456–57.

3. Immanuel Kant, *On the Form and Principles of the Sensible and the Intelligible World*, in *Theoretical Philosophy*, pp. 373–405.

4. Kant, *Prolegomena to Any Future Metaphysics*, translated by Lewis White Beck (Indianapolis, Ind.: Bobbs-Merrill, 1950), sec. 13. The *Prolegomena* argument is briefly repeated in *Metaphysical Foundations of Natural Science*, translated by James Ellington (Indianapolis, Ind.: Bobbs-Merrill, 1970), pp. 23–24.

5. James Van Cleve, "Right, Left, and the Fourth Dimension," *The Philosophical Review*, 96 (1987), 33–68, and "Introduction to the Arguments of 1770 and 1783," in *The Philosophy of Right and Left*, edited by James Van Cleve and Robert E. Frederick (Dordrecht: Kluwer, 1991), pp. 15–26 (this volume also includes "Right, Left, and the Fourth Dimension").

6. Norman Kemp Smith, *A Commentary to Kant's 'Critique of Pure Reason'* (New York: Humanities Press, 1962; reprint of 1923 edition), pp. 161–66.

7. Or would anyone suggest that perhaps *symmetrical* objects, which lack incongruent counterparts, might for all the argument shows be things in themselves? To take this suggestion seriously would be to countenance the possibility that while my fingers are things in themselves, my hand as a whole is appearance!

8. See B307–9 for the distinction between the positive and negative senses of 'noumenon'.

9. I discuss the two points that follow in more detail in "Right, Left, and the Fourth Dimension."

10. Some would deny this, maintaining that 'b lies in direction D from a' can only mean something like 'b lies along the line from a to c', where c is the North Pole or some other reference point.

11. Spherical triangles, though themselves objects of two dimensions, can exist only in a space of three dimensions and would require for their interchange a space of four dimensions. Hands, Kant's favorite examples of incongruent counterparts, are three-dimensional objects that cannot be interchanged in a space of three dimensions but could be interchanged in a space of four.

12. Jill Vance Buroker, *Space and Incongruence* (Dordrecht: Reidel, 1981), p. 85.

13. All that strictly follows is that *either* space *or* one of the triangles is not a thing in itself—a conclusion that leaves room for someone to maintain that the triangles are things in themselves although space is not. But this position is ruled out in comment (e) below.

14. The assumption that relations among things in themselves must be reducible may also underlie premise 1 in the argument from interchangeability. For further discussion of the role of the reducibility principle in Kant's think-

ing, see my "Inner States and Outer Relations: Kant and the Case for Monadism," in *Doing Philosophy Historically*, edited by Peter H. Hare (Buffalo, N.Y.: Prometheus Books, 1988), pp. 231–47.

15. Buroker, *Space and Incongruence*, pp. 4 and 69.

16. Ibid., p. 87.

17. See esp. ibid., p. 83.

18. See Paul Benaceraff, "What Numbers Could Not Be," *The Philosophical Review*, 74 (1965), 47–73.

19. See D.P. Dryer, *Kant's Solution for Verification in Metaphysics* (London: Allen & Unwin, 1966), ch. 11, sec. 6, esp. pp. 513–14.

20. See Gerold Prauss, *Kant und das Problem der Dinge an Sich* (Bonn: Grundmann, 1974), for the view that *an sich* functions canonically for Kant as an adverb modifying *betrachtet* (considered). For a brief discussion of Prauss's views, see Richard Aquila, "Things in Themselves: Intentionality and Reality in Kant," *Archiv für Geschichte der Philosophie*, 61 (1979), 293–307.

21. See, for example, Wilfrid Sellars, *Science and Metaphysics: Variations on Kantian Themes* (London: Routledge and Kegan Paul, 1968), ch. 2.

22. At any rate, it seems self-evident to me that every genuine existent must be fully determinate; perhaps it may be questioned whether the law of the excluded middle is a sufficient test for such determinacy.

23. Of course, I still find it perplexing why Kant was so convinced of the reducibility principle. For me it has nothing like the evidence of the determinacy principle—certainly not enough evidence to make me give up the reality of things in space.

24. For example:

> If we enquire what new character *relation to an object* confers upon our representations, what dignity they thereby acquire, we find that it results *only* in subjecting the representations to a rule, and so in necessitating us to connect them in some one specific manner. (A197/ B242, second emphasis mine)

25. Compare Kant's observation in the *Metaphysical Foundations of Natural Science*: "For with regard to what is actual only by its being given in representation, there is not more given than is met with in the representation" (p. 54).

26. More accurately, the experiences constituting the real beast have relations to one another and to other experiences that are not possessed by the experiences constituting the dragon. As Kant puts it:

> [T]he difference between truth and dreaming is not ascertained by the nature of the representations which are referred to objects (for they are the same in the two cases) but by their connection according to those rules which determine the coherence of the representations in the concept of an object, and by ascertaining whether they can subsist together in experience or not. (*Prolegomena*, sec. 13, remark 3, p. 38 in Beck's translation)

Chapter 5

1. At A31/B47, Kant cites two examples of *a priori* principles about time, our knowledge of which is presumably to be explained by the same idealist explanation he has already given for propositions of geometry: time has only one dimension, and different times are not simultaneous but successive. Other putatively necessary topological features of time he might have mentioned here are its unity or connectedness, its continuity, its unboundedness, and its being neither looping nor branching.