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Preface

If the word nature is taken simply in its *formal* meaning, where it means the first inner principle of all that belongs to the existence of a thing,* then there can be as many different natural sciences as there are specifically different things, each of which must contain its own peculiar inner principle of the determinations belonging to its existence. But nature is also taken otherwise in its *material* meaning, not as a constitution,[a] but as the sum total of all things, insofar as they can be *objects of our senses*, and thus also of experience. Nature, in this meaning, is therefore understood as the whole of all appearances, that is, the sensible world, excluding all nonsensible objects. Now nature, taken in this meaning of the word, has two principal parts, in accordance with the principal division of our senses, where the one contains the objects of the *outer* senses, the other the object of *inner* sense. In this meaning, therefore, a twofold doctrine of nature is possible, the *doctrine of body* and the *doctrine of the soul*, where the first considers *extended* nature, the second *thinking* nature.

Every doctrine that is supposed to be a system, that is, a whole of cognition ordered according to principles, is called a science. And, since such principles may be either principles of *empirical* or of *rational* connection of cognitions into a whole, then natural science, be it the doctrine
[468] of body or the doctrine of the soul, would have to be divided into *historical* or *rational* natural science, were it not that the word *nature* (since this

* Essence is the first inner principle of all that belongs to the possibility of a thing. Therefore, one can attribute only an essence to geometrical figures, but not a nature (since in their concept nothing is thought that would express an existence).

[a] *Beschaffenheit*.

signifies a derivation of the manifold belonging to the existence of things from their inner *principle*) makes necessary a cognition through reason of the interconnection of natural things, insofar as this cognition is to deserve the name of a science. Therefore, the doctrine of nature can be better divided into *historical doctrine of nature*, which contains nothing but systematically ordered facts about natural things (and would in turn consist of *natural description*, as a system of classification for natural things in accordance with their similarities, and *natural history*, as a systematic presentation of natural things at various times and places), and natural science. Natural science would now be either *properly* or *improperly* so-called natural science, where the first treats its object wholly according to a priori principles, the second according to laws of experience.

What can be called *proper* science is only that whose certainty is apodictic; cognition that can contain mere empirical certainty is only *knowledge*[b] improperly so-called. Any whole of cognition that is systematic can, for this reason, already be called *science*, and, if the connection of cognition in this system is an interconnection of grounds and consequences, even *rational* science. If, however, the grounds or principles themselves are still in the end merely empirical, as in chemistry, for example, and the laws from which the given facts are explained through reason are mere laws of experience, then they carry with them no consciousness of their *necessity* (they are not apodictally certain), and thus the whole of cognition does not deserve the name of a science in the strict sense; chemistry should therefore be called a systematic art rather than a science.

A rational doctrine of nature thus deserves the name of a natural science, only in case the fundamental natural laws therein are cognized a priori, and are not mere laws of experience. One calls a cognition of nature of the first kind *pure*, but that of the second kind is called *applied* rational cognition. Since the word nature already carries with it the concept of laws, and the latter carries with it the concept of the *necessity* of all determinations of a thing belonging to its existence, one easily sees why natural science must derive the legitimacy of this title only from its pure part – namely, that [469] which contains the a priori principles of all other natural explanations – and why only in virtue of this pure part is natural science to be proper science. Likewise, [one sees] that, in accordance with demands of reason, every doctrine of nature must finally lead to natural science and conclude

[b] *Wissen*. Cf. "science [*Wissenschaft*]" in the previous sentence.

there, because this necessity of laws is inseparably attached to the concept of nature, and therefore makes claim to be thoroughly comprehended. Hence, the most complete explanation of given appearances from chemical principles still always leaves behind a certain dissatisfaction, because one can adduce no a priori grounds for such principles, which, as contingent laws, have been learned merely from experience.

All *proper* natural science therefore requires a *pure* part, on which the apodictic certainty that reason seeks therein can be based. And because this pure part is wholly different, in regard to its principles, from those that are merely empirical, it is also of the greatest utility to expound this part as far as possible in its entirety, separated and wholly unmixed with the other part; indeed, in accordance with the nature of the case it is an unavoidable duty with respect to method. This is necessary in order that one may precisely determine what reason can accomplish for itself, and where its power begins to require the assistance of principles of experience. Pure rational cognition from mere *concepts* is called pure philosophy or metaphysics; by contrast, that which grounds its cognition only on the *construction* of concepts, by means of the presentation of the object in an a priori intuition, is called mathematics.

Properly so-called natural science presupposes, in the first place, metaphysics of nature. For laws, that is, principles of the necessity of that which belongs to the *existence* of a thing, are concerned with a concept that cannot be constructed, since existence cannot be presented a priori in any intuition. Thus proper natural science presupposes metaphysics of nature. Now this latter must always contain solely principles that are not empirical (for precisely this reason it bears the name of a metaphysics), but it can still either: *first*, treat the laws that make possible the concept of a nature in general, even without relation to any determinate object of experience, and thus undetermined with respect to the nature of this or that thing in the sensible world, in which case it is the *transcendental* part
[470] of the metaphysics of nature; or *second*, concern itself with a particular nature of this or that kind of thing, for which an empirical concept is given, but still in such a manner that, outside of what lies in this concept, no other empirical principle is used for its cognition (for example, it takes the empirical concept of matter or of a thinking being as its basis, and it seeks that sphere of cognition of which reason is capable a priori concerning these objects), and here such a science must still always be called a metaphysics of nature, namely, of corporeal or of thinking nature. However,

[in this second case] it is then not a general, but a *special* metaphysical natural science (physics or psychology), in which the above transcendental principles are applied to the two species of objects of our senses.[1]

I assert, however, that in any special doctrine of nature there can be only as much *proper* science as there is *mathematics* therein. For, according to the preceding, proper science, and above all proper natural science, requires a pure part lying at the basis of the empirical part, and resting on a priori cognition of natural things. Now to cognize something a priori means to cognize it from its mere possibility. But the possibility of determinate natural things cannot be cognized from their mere concepts; for from these the possibility of the thought (that it does not contradict itself) can certainly be cognized, but not the possibility of the object, as a natural thing that can be given outside the thought (as existing). Hence, in order to cognize the possibility of determinate natural things, and thus to cognize them a priori, it is still required that the *intuition* corresponding to the concept be given a priori, that is, that the concept be constructed. Now rational cognition through construction of concepts is mathematical. Hence, although a pure philosophy of nature in general, that is, that which investigates only what constitutes the concept of a nature in general, may indeed be possible even without mathematics, a pure doctrine of nature concerning *determinate* natural things (doctrine of body or doctrine of soul) is only possible by means of mathematics. And, since in any doctrine of nature there is only as much proper science as there is a priori knowledge therein, a doctrine of nature will contain only as much proper science as there is mathematics capable of application there.

[1] See the discussion in the Architectonic of Pure Reason in the first *Critique*: "Metaphysics in the narrower sense consists of *transcendental philosophy* and the *physiology* of pure reason. The former considers only the *understanding* and reason itself in a system of concepts and principles that relate to objects in general, without assuming objects that may be *given* (Ontologia). The latter considers *nature* – i.e., the totality of *given* objects . . . and is therefore *physiology* (although only rationalis)" (A845/B873). After explaining that the latter doctrine (rational physiology) consists in turn of "metaphysics of corporeal nature" or "rational physics," and "metaphysics of thinking nature" or "rational psychology" (A846/B874), Kant then continues as follows: "how can I expect an a priori cognition, and thus a metaphysics, of objects insofar as they are given to our senses, and therefore given a posteriori? . . . The answer is: we take no more from experience than what is necessary to *give* us an object – of either outer or inner sense. The former takes place through the mere concept of matter (impenetrable, lifeless extension), the latter through the concept of a thinking being (in the empirical inner representation: I think)" (A847–48/B875–76).

So long, therefore, as there is still for chemical actions of matters on one another no concept to be discovered that can be constructed, that is, no
[471] law of the approach or withdrawal of the parts of matter can be specified according to which, perhaps in proportion to their density or the like, their motions and all the consequences thereof can be made intuitive and presented a priori in space (a demand that will only with great difficulty ever be fulfilled), then chemistry can be nothing more than a systematic art or experimental doctrine, but never a proper science, because its principles are merely empirical, and allow of no a priori presentation in intuition. Consequently, they do not in the least make the principles of chemical appearances conceivable with respect to their possibility, for they are not receptive to the application of mathematics.

Yet the empirical doctrine of the soul must remain even further from the rank of a properly so-called natural science than chemistry. In the first place, because mathematics is not applicable to the phenomena of inner sense and their laws, the only option one would have would be to take the *law of continuity* in the flux of inner changes into account – which, however, would be an extension of cognition standing to that which mathematics provides for the doctrine of body approximately as the doctrine of the properties of the straight line stands to the whole of geometry. For the pure inner intuition in which the appearances of the soul are supposed to be constructed is *time*, which has only one dimension. [In the second place,] however, the empirical doctrine of the soul can also never approach chemistry even as a systematic art of analysis or experimental doctrine, for in it the manifold of inner observation can be separated only by mere division in thought, and cannot then be held separate and recombined at will (but still less does another thinking subject suffer himself to be experimented upon to suit our purpose), and even observation by itself already changes and displaces the state of the observed object. Therefore, the empirical doctrine of the soul can never become anything more than an historical doctrine of nature, and, as such, a natural doctrine of inner sense which is as systematic as possible, that is, a natural description of the soul, but never a science of the soul, nor even, indeed, an experimental psychological doctrine. This is also the reason for our having used, in accordance with common custom, the general title of natural science for this work, which actually contains the principles of the doctrine of body, for only to it

does this title belong in the proper sense, and so no ambiguity is thereby produced.[2]

But in order to make possible the application of mathematics to the [472] doctrine of body, which only through this can become natural science, principles for the *construction* of the concepts that belong to the possibility of matter in general must be introduced first. Therefore, a complete analysis of the concept of a matter in general will have to be taken as the basis, and this is a task for pure philosophy – which, for this purpose, makes use of no particular experiences, but only that which it finds in the isolated (although intrinsically empirical) concept itself, in relation to the pure intuitions in space and time, and in accordance with laws that already essentially attach to the concept of nature in general, and is therefore a genuine *metaphysics of corporeal nature*.

Hence all natural philosophers who have wished to proceed mathematically in their occupation have always, and must have always, made use of metaphysical principles (albeit unconsciously), even if they themselves solemnly guarded against all claims of metaphysics upon their science. Undoubtedly they have understood by the latter the folly of contriving possibilities at will and playing with concepts, which can perhaps not be presented in intuition at all, and have no other certification of their objective reality than that they merely do not contradict themselves. All true metaphysics is drawn from the essence of the faculty of thinking itself, and is in no way fictitiously invented[c] on account of not being borrowed from experience. Rather, it contains the pure actions of thought, and thus a priori concepts and principles, which first bring the manifold of *empirical representations* into the law-governed connection through which it can become *empirical* **cognition**, that is, experience. Thus these mathematical physicists could in no way avoid metaphysical principles, and, among them, also not those that make the concept of their proper object, namely, matter, a priori suitable for application to outer experience, such as the

[c] *erdichtet*.

[2] See A381: "When we compare the *doctrine of the soul*, as the physiology of inner sense, with the *doctrine of body*, as a physiology of the objects of the outer senses, we find that, aside from the circumstance that much that is empirical can be cognized in both, there is still this remarkable difference: In the latter science much that is a priori can be synthetically cognized from the mere concept of an extended, impenetrable being, but in the former science nothing at all that is a priori can be synthetically cognized from the concept of a thinking being." And compare the discussion of empirical psychology at A848–49/B876–77.

concept of motion, the filling of space, inertia, and so on.[3] But they rightly held that to let merely empirical principles govern these concepts would in no way be appropriate to the apodictic certainty they wished their laws of nature to possess, so they preferred to postulate such [principles], without investigating them with regard to their a priori sources.

Yet it is of the greatest importance to separate heterogeneous principles from one another, for the advantage of the sciences, and to place each in a
[473] special system so that it constitutes a science of its own kind, in order to guard against the uncertainty arising from mixing things together, where one finds it difficult to distinguish to which of the two the limitations, and even mistakes, that might occur in their use may be assigned. For this purpose I have considered it necessary [to isolate] the former from the pure part of natural science (physica generalis), where metaphysical and mathematical constructions customarily run together, and to present them, together with principles of the construction of these concepts (and thus principles of the possibility of a mathematical doctrine of nature itself), in a system.[4] Aside from the already mentioned advantage that it provides, this isolation has also a special charm arising from the unity of cognition, when one takes care that the boundaries of the sciences do not run together, but rather each takes in its own separated field.

The following can serve as still another ground for commending this procedure. In everything that is called metaphysics one can hope for the *absolute completeness* of the sciences, of such a kind one may expect in no other type of cognition. Therefore, just as in the metaphysics of nature in general, here also the completeness of the metaphysics of corporeal nature can confidently be expected. The reason is that in metaphysics the object is

[3] Compare the definition of matter cited in note 1 above ("impenetrable, lifeless, extension") and the parallel discussion in §15 of the *Prolegomena* – which gives the relevant list of concepts as "the concept of *motion*, of *impenetrability* (on which the empirical concept of matter rests), of *inertia*, and others" (Ak 4:295). (Note that in the Remark to Proposition 3 of the Mechanics, "inertia" is equated with "lifelessness" [544].)

[4] *Um deswillen habe ich für nöthig gehalten, von dem reinen Theile der Naturwissenschaft* (physica generalis), *wo metaphysische und mathematische Constructionen durch einander zu laufen pflegen, die erstere und mit ihnen zugleich die Prinzipien der Construction dieser Begriffe, also der Möglichkeit einer mathematischen Naturlehre selbst, in einem System darzustellen.* This difficult sentence has led to considerable controversy. Plaass (1965) and Schäfer (1966) have made the notion of "metaphysical construction" central to their interpretations, whereas Hoppe (1969) and Gloy (1976) have suggested that "concepts" or "principles" should follow "metaphysical" in the sentence. Here, in any case, one should compare the section on the Discipline of Pure Reason in its Dogmatic Employment from the Doctrine of Method of the first *Critique* (A712–38/B740–66) – which certainly suggests that the construction of concepts is precisely what *distinguishes* mathematics from philosophy.

only considered in accordance with the general laws of thought, whereas in other sciences it must be represented in accordance with data of intuition (pure as well as empirical), where the former, because here the object has to be compared always with *all* the necessary laws of thought, must yield a determinate number of cognitions that may be completely exhausted, but the latter, because they offer an infinite manifold of intuitions (pure or empirical), and thus an infinite manifold of objects of thought, never attain absolute completeness, but can always be extended to infinity, as in pure mathematics and empirical doctrine of nature. I also take myself to have completely exhausted this metaphysical doctrine of body, so far as it may extend, but not to have thereby accomplished any great [piece of] work.

But the schema for completeness of a metaphysical system, whether it be of nature in general, or of corporeal nature in particular, is the table of categories.† For there are no more pure concepts of the understanding [474]

† In the *Allgemeine Literatur Zeitung*, No. 295, in the review of *Institutiones Logicae et Metaphysicae* by Prof. *Ulrich*, I find doubts, which are not directed against this table of pure concepts of the understanding, but rather against the inferences drawn therefrom to the determination of the limits of the entire faculty of pure reason, and thus all metaphysics, [doubts] with respect to which the deeply delving reviewer declares himself to be in agreement with the no less penetrating author. And, in fact, since these doubts are supposed to concern precisely the principal basis of my system articulated in the *Critique*, they would be grounds for thinking that this system, with respect to its principal aim, does not come close to carrying that apodictic conviction that is required for eliciting an unqualified acceptance. This principal basis is said to be the *deduction* of the pure concepts of the understanding, which is expounded partly in the *Critique* and partly in the *Prolegomena*, and which, however, in the part of the *Critique* that ought to be precisely the most clear, is rather the most obscure, or even revolves in a circle, etc. I direct my reply to these objections only to their principal point, namely, the claim that *without an entirely clear and sufficient deduction of the categories* the system of the *Critique of Pure Reason* totters on its foundation. I assert, on the contrary, that the system of the *Critique* must carry apodictic certainty for whoever subscribes (as the reviewer does) to my propositions concerning the sensible character of all our intuition, and the adequacy of the table of categories, as determinations of our consciousness derived from the logical functions in judgments in general, because it is erected upon the proposition *that the entire speculative use of our reason never reaches further than to objects of possible experience*. For if we can prove **that** the categories which reason must use in all its cognition can have no other use at all, except solely in relation to objects of possible experience (insofar as they simply make possible the form of thought in such experience), then, although the answer to the question **how** the categories make such experience possible is important enough for *completing* the deduction where possible, with respect to the principal end of the system, namely, the determination of the limits of pure reason, it is in no way *compulsory*, but merely *meritorious*. For the deduction is already carried *far enough* for this purpose if it shows that categories of thought are nothing but mere forms of judgments insofar as they are applied to intuitions (which for us are always sensible), and that they thereby first of all obtain objects and become cognitions; because this already suffices to ground with complete certainty the entire system of the *Critique* properly speaking. Thus Newton's system of universal gravitation stands firm, even though it involves the difficulty that one cannot explain how attraction at a distance is possible; but *difficulties are not doubts*. That the above fundamental basis

[475] which can be concerned with the nature of things. All determinations of the general concept of a matter in general must be able to be brought under the four classes of [pure concepts of the understanding], those of *quantity*, of *quality*, of *relation*, and finally of *modality* – and so, too, [must] [476] all that may be either thought a priori in this concept, or presented in

stands firm, even without a complete deduction of the categories, I now prove from the following granted propositions:

[475] 1. *Granted*: that the table of categories contains all pure concepts of the understanding, just as it contains all formal actions of the understanding in judging, from which the concepts of the understanding are derived, and from which they differ only in that, through the concepts of the understanding, an object is thought as *determined* with respect to one or another function of judgment. (Thus, for example, in the categorical judgment *the stone is hard*, the *stone* is used as subject, and *hard* as predicate, in such a way that the understanding is still free to exchange the logical function of these concepts, and to say that something hard is a stone. By contrast, if I represent it to myself as *determined in the object* that the stone must be thought only as subject, but hardness only as predicate, in any possible determination of an object (not of the mere concept), then the very same logical functions now become *pure concepts of the understanding* of objects, namely, as *substance* and *accident*.)

2. *Granted*: that the understanding by its nature contains synthetic a priori principles, through which it subjects all objects that may be given to it to these categories, and, therefore, there must also be intuitions given a priori that contain the conditions required for the application of these pure concepts of the understanding, because *without intuition there can be no object*, with respect to which the logical function could be determined as category, and thus no cognition of any object whatsoever, and hence without pure intuition no principle that determines it a priori for this purpose.

3. *Granted*: that these pure intuitions can never be anything other than mere forms of the *appearances* of outer or of inner sense (space and time), and therefore of the *objects of possible experience* alone.

It then follows: that all use of pure reason can never extend to anything other than objects of experience, and, since nothing empirical can be the condition of a priori principles, the latter can be nothing more than principles of the *possibility of experience* in general. This alone is the true and sufficient basis for the determination of the limits of pure reason, but not the solution to the problem **how** experience is now possible by means of these categories, and only through these categories alone. The latter problem, although without it the structure still stands firm, has great importance nonetheless, and, as I now understand it, [it can be solved with] just as much ease, since it can almost be accomplished through a single inference from the precisely determined definition of a *judgment* in general (an action through which given representations first become cognitions [476] of an object). The obscurity that attaches to my earlier discussions in this part of the deduction (and which I do not deny), is to be attributed to the common fortunes of the understanding in its investigations, in which the shortest way is commonly not the first way that it becomes aware of. Therefore, I shall take up the next opportunity to make up for this deficiency (which concerns only the manner of presentation, and not the ground of explanation, which is already stated correctly there), so that the perceptive reviewer may not be left with the necessity, certainly unwelcome even to himself, of taking refuge in a preestablished harmony to explain the surprising agreement of appearances with the laws of the understanding, despite their having entirely different sources from the former. This remedy would be much worse than the evil it is supposed to cure, and, on the contrary, actually cannot help at all. For the *objective necessity* that characterizes the pure concepts of the understanding (and the principles of their application to appearances), in the concept of cause in connection with the effect, for example, is still not forthcoming. Rather, it all remains only *subjectively necessary*, but objectively merely contingent, placing together, precisely as Hume has it

mathematical construction, or given as a determinate object of experience. There is no more to be done, or to be discovered, or to be added here, except, if need be, to improve it where it may lack in clarity or exactitude.[d]

The concept of matter had therefore to be carried through all four of the indicated functions of the concepts of the understanding (in four chapters), where in each a new determination of this concept was added. The basic determination of something that is to be an object of the outer senses had to be motion, because only thereby can these senses be affected. The understanding traces back all other predicates of matter belonging to its nature to this, and so natural science, therefore, is either a pure or [477] applied *doctrine of motion*. The *metaphysical* foundations of natural science are therefore to be brought under *four* chapters. The *first* considers *motion* as a pure *quantum* in accordance with its composition, without any quality of the movable, and may be called **phoronomy**. The *second* takes into consideration motion as belonging to the *quality* of matter, under the name of an original moving force, and is therefore called **dynamics**. The *third* considers matter with this quality as in *relation* to another through its own inherent motion, and therefore appears under the name of **mechanics**. The *fourth* chapter, however, determines matter's motion or rest merely in relation to the mode of representation or *modality*, and thus as appearance of the outer senses, and is called **phenomenology**.

Yet aside from the inner necessity to isolate the metaphysical foundations of the doctrine of body, not only from physics, which needs empirical principles, but even from the rational premises of physics that concern the use of mathematics therein, there is still an external, certainly only accidental, but nonetheless important reason for detaching its detailed treatment from the general system of metaphysics, and presenting it systematically as a special whole. For if it is permissible to draw the boundaries of a science, not simply according to the constitution of the object and its specific mode of cognition, but also according to the end that one

when he calls this mere illusion from custom. No system in the world can derive this necessity from anywhere else than the principles lying a priori at the basis of the possibility of *thinking itself*, through which alone the cognition of objects whose appearance is given to us, that is, experience, becomes possible. Even if we suppose, therefore, that the explanation of **how** experience thereby becomes possible in the first place could never be sufficiently carried out, it still remains incontrovertibly certain **that** it is possible solely through these concepts, and, conversely, that these concepts are capable of meaning and use in no other relation than to objects of experience.

[d] *Gründlichkeit*.

has in mind for this science itself in uses elsewhere; and if one finds that metaphysics has busied so many heads until now, and will continue to do so, not in order thereby to extend natural knowledge (which takes place much more easily and surely through observation, experiment, and the application of mathematics to outer appearances), but rather so as to attain cognition of that which lies wholly beyond all boundaries of experience, of God, Freedom, and Immortality; then one gains in the advancement of this goal if one frees it[e] from an offshoot that certainly springs from its root, but nonetheless only hinders its regular growth, and one plants this offshoot specially, yet without failing to appreciate the origin of [this offshoot] from it,[f] and without omitting the mature plant from the system of general metaphysics. This does not impair the completeness of general metaphysics, and in fact facilitates the uniform progress of this science
[478] towards its end, if, in all instances where one requires the general doctrine of body, one may call only upon the isolated system, without swelling this greater system with the latter. It is also indeed very remarkable (but cannot be expounded in detail here)[5] that general metaphysics, in all instances where it requires examples (intuitions) in order to provide meaning for its pure concepts of the understanding, must always take them from the general doctrine of body, and thus from the form and principles of outer intuition; and, if these are not exhibited completely, it gropes uncertainly and unsteadily among mere meaningless concepts. This is the source of the well-known disputes, or at least obscurity, in the questions concerning the possibility of a conflict of realities, of intensive magnitude, and so on, in which the understanding is taught only by examples from corporeal nature what the conditions are under which such concepts can alone have objective reality, that is, meaning and truth. And so a separated metaphysics of corporeal nature does excellent and indispensable service for *general* metaphysics, in that the former furnishes examples (instances *in concreto*) in which to realize the concepts and propositions of the latter (properly speaking, transcendental philosophy), that is, to give a mere form of thought sense and meaning.

[e] *sie*. The reference is most likely to "metaphysics" – or possibly to "this science."
[f] *jener*. See note e above.

[5] See the General Remark to the System of Principles in the second edition of the *Critique*: "But it is even more remarkable that, in order to understand the possibility of things in conformity with the categories, and thus to verify the *objective reality* of the latter, we require not merely intuitions, but always even *outer intuitions*" (B291).

In this treatise, although I have not followed the mathematical method with thoroughgoing rigor (which would have required more time than I had to spend thereon), I have nonetheless imitated that method[6] – not in order to obtain a better reception for the treatise, through an ostentatious display of exactitude,[g] but rather because I believe that such a system would certainly be capable of this rigor, and also that such perfection could certainly be reached in time by a more adept hand, if, stimulated by this sketch, mathematical natural scientists should find it not unimportant to treat the metaphysical part, which they cannot leave out in any case, as a special fundamental part in their general physics, and to bring it into union with the mathematical doctrine of motion.

Newton, in the preface to his *Mathematical First Principles of Natural Science*, says (after he had remarked that geometry requires only two of the mechanical operations that it postulates, namely, to describe a straight line and a circle): *Geometry is proud of the fact that with so little derived from without it is able to produce so much.*[‡7] By contrast, one can say of [479] metaphysics: *it is dismayed that with so much offered to it by pure mathematics it can still accomplish so little*. Nevertheless, this small amount is still something that even mathematics unavoidably requires in its application to natural science; and thus, since it must here necessarily borrow from metaphysics, need also not be ashamed to let itself be seen in community with the latter.

[‡] "Geometry can boast that with so few principles obtained from other fields, it can do so much." *Newton Princ. Phil. Nat. Math. praefat*.

[g] *Gründlichkeit*.

[6] Again, one should compare A726/B754 from the section of the first *Critique* cited in note 4 above, where Kant explicitly says that philosophy *cannot* imitate mathematics (*Meßkunst*) . See also A735/B763.

[7] Kant quotes the original Latin in the note: Gloriatur Geometria, quod tam paucis principiis aliunde petitis tam multa praestet. The quotation given is from I. Newton, *The Principia*: *Mathematical Principles of Natural Philosophy*, trans. I. B. Cohen and A. Whitman (Berkeley: University of California Press, 1999), p. 382.