Kant on Mathematics

PHIL 871

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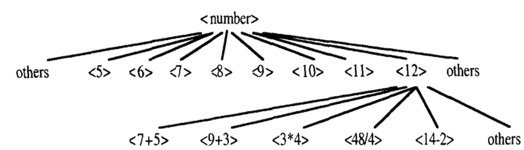
1 *The Rationalist Conception of Mathematical Knowledge*

- All mathematical (including geometrical) knowledge can (in principle) be demonstrated in terms of explicit, step-by-step inferences from the contents of mathematical concepts
  - Mathematical knowledge is achieved via *analysis*
  - Analysis makes explicit the containment relationships between concepts[1]

[1] **Mathematical Analysis**:



2 *Kant's Three Claims*

1. Mathematical knowledge is synthetic
2. Mathematical knowledge is a priori
3. Mathematical method is unique to mathematics

3 *Mathematics as Synthetic*

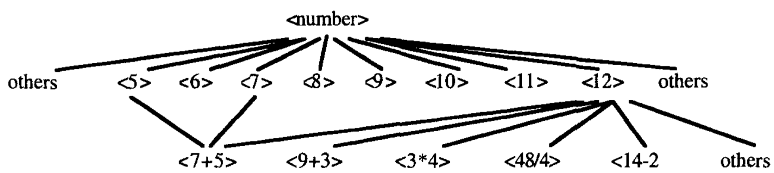
- Early Modern conception of mathematics as consisting fundamentally of the studies of discrete magnitudes (arithmetic) and continuous magnitudes (geometry)
  - If arithmetic and geometry are both synthetic, then so is mathematical knowledge generally.

3.1 *The Arithmetic Case (B15-16)*

- '7 + 5 = 12' is not an analytic truth[2]
  - '7 + 5 = m - n' will be totally unobvious for any large number substituted for m and n
    - * Assumes that analyticity is subjective (what one is aware of in thinking)

[2] To be sure, one might initially think that the proposition "7 + 5 = 12" is a merely analytic proposition that follows from the concept of a sum of seven and five in accordance with the principle of contradiction. Yet if one considers it more closely…The concept of twelve is by no means already thought merely by my thinking of that unification of seven and five, and no matter how long I analyze my concept of such a possible sum I will still not find twelve in it. One must go beyond these concepts… (B15)

- Containment analyticity cannot account for arithmetical relationships[3]



[3] an arithmetic judgment like '7+5=12' expresses two basically different and orthogonal kinds of relation among the numbers 7, 5, and 12, each of which must be separately specified in order to capture the proposition's content. This exceeds the expressive capacities of the one-dimensional hierarchy, and violates the reciprocity of content and extension mandated by the division rules. (Anderson (2004), 518)

3.2 *The Geometric Case (B16)*

- 'The shortest distance between two points is a straight line' is not an analytic truth[4]
  - `<shortest>` is not contained in `<straight>`
    - *quality* vs. *quantity*
- Intuition provides, without any mediating inference, knowledge of the connection between `<straight>` and `<shortest>`

[4] That the straight line between two points is the shortest is a synthetic proposition. For my concept of **the straight** contains nothing of quantity, but only a quality. The concept of the shortest is therefore entirely additional to it, and cannot be extracted out of the concept of the straight line by any analysis. Help must here be gotten from intuition, by means of which alone the synthesis is possible. (B16)

4 *Mathematics as A Priori*

- The status of mathematics as a priori was not much debated
- Knowledge was explained in one of two ways
  - Mathematical knowledge is innate (rationalist)
  - Mathematical knowledge is the result of stipulation/construction of mathematical concepts (empiricist)
- Kant's main contribution is to show how we might explain the a priority of mathematical knowledge in a way that is applicable to all and only the spatio-temporal objects of possible experience

KANT'S DILEMMA FOR TRANSCENDENTAL REALISM (A39-41/B56-8):

Those, however, who assert the absolute reality of space and time, whether they assume it to be subsisting or only inhering, must themselves come into conflict with the principles of experience. For if they decide in favor of the first (which is generally the position of the mathematical investigators of nature), then they must assume two eternal and infinite self-subsisting non-entities (space and time), which exist (yet without there being anything real) only in order to comprehend everything real within themselves. If they adopt the second position (as do some metaphysicians of nature)…then they must dispute the validity or at least the apodictic certainty of *a priori* mathematical doctrines in regard to real things (e.g., in space), since this certainty does not occur *a posteriori*, and on this view the *a priori* concepts of space and time are only creatures of the imagination, the origin of which must really be sought in experience…On our theory of the true constitution of these two original forms of sensibility both difficulties are remedied. (A39-41/B56-8)

1. Transcendental Realists must assume one of two positions about mathematical knowledge: Either (a) mathematical knowledge is innate; (b) mathematical knowledge is acquired via experience
2. Nativists (i.e. the 'mathematicians of Nature' – Descartes & Newton) fail to explain why mathematical knowledge applies to all and only the objects of possible experience
3. Empiricists (i.e. the 'metaphysicians of nature' – especially Leibniz) explain the *applicability* of mathematical knowledge but not its a priori universal and necessary status
4. ∴ We should reject the underlying assumption of Transcendental Realism
5. ∴ Transcendental Idealism is the best explanation of the apriority and applicability of mathematics[5]

[5] Because, for Kant, mathematics provides us with knowledge of the pure conditions on our sensible representations, mathematical cognition is a priori. Being knowledge of the cognitive conditions on our *having* sensible experience, mathematics is cognition that is necessarily acquired prior to and independent from such experience. Because, for Kant, mathematics provides us with knowledge of the objects that appear to us under such cognitive conditions, mathematical cognition is applicable to the natural empirical world. Being knowledge of the formal features of sensible spatiotemporal objects, mathematics is cognition that is necessarily about the things that inhabit the natural world, at least insofar as we represent them. (Shabel (2005), 48)

5 *The Uniqueness of Mathematical Method (A712-38/B741-66)*

- Mathematics is the only science in which the a priori *construction* of concepts is possible

A MATHEMATICAL CONCEPT is constructed by means of a definition which is sufficient both to :

(i) articulate the content of a distinct concept
(ii) provide the rule or 'recipe' by which one can 'construct' a corresponding figure in pure intuition

The real definition both provides the concept and proves that the concept has an object. In Kant's terms, this means that the object is 'really possible' and that the concept has 'objective validity'.

5.1 Features of Mathematical Construction

1. A priori
2. Construction proceeds via *real* definition
3. The content of the concept is 'elective' [*willkürlich*] & exhaustively defined
4. The content of the concept distinguishes its object as being of a specific kind (e.g. as *round* or *square*)
5. The content of the concept *explains* why the object has the features that it does
6. The concept provides a 'recipe' for generating a corresponding object in pure intuition
7. The pure intuition proves the objective validity of the concept (i.e. that its object is really possible)

- What is the role of pure intuition in construction?
  - *Evidential*: Pure intuition provides evidence (phenomenal or otherwise) for the truth of mathematical statements[6]
  - *Semantic*: Pure intuition provides the objects of reference for mathematical terms[7]

[6] It is, indeed, a necessary logical condition that a concept of the possible must not contain any contradiction; but this is not by any means sufficient to determine the objective reality of the concept, that is, the possibility of such an object as is thought through the concept. Thus there is no contradiction in the concept of a figure which is enclosed within two straight lines, since the concepts of two straight lines and of their coming together contain no negation of a figure. The impossibility arises not from the concept in itself, but in connection with its construction in space, that is, from the conditions of space and of its determination. (A220-1/B268)

[7] One cannot understand "intuition" as "evidence," despite the great number of commentators who do so. The fundamental issue for Kant has little to do with "proof." It is, rather, whether one can "determine" the object of singular reference in mathematics in a purely conceptual or descriptive fashion. That is to say that Kant's motives are primarily semantic, even if they also entail some important epistemic consequences. On my interpretation of his position, singular reference, on which the objectivity and not the truth of mathematics depends, requires intuition. It is not a matter of "verifying" mathematical propositions, but of showing how they are "possible," that is of providing an account of how their subject terms manage to refer. (Brittan (2006), 229)

References

Anderson, R. Lanier. 2004. "It Adds up After All: Kant's Philosophy of Arithmetic in Light of the Traditional Logic." *Philosophy and Phenomenological Research* 69 (3): 501–540.

Brittan, Gordon G. 2006. "Kant's Philosophy of Mathematics." In *A Companion to Kant*, edited by Graham Bird, 222–235. Oxford: Blackwell Publishing Ltd. doi:10.1002/9780470996287.ch16. `http://dx.doi.org/10.1002/9780470996287.ch16`.

Carson, Emily. 1999. "Kant on the Method of Mathematics." *Journal of the History of Philosophy* 37 (4): 629–652. doi:10.1353/hph.2008.0905.

———. 2006. "Locke and Kant on Mathematical Knowledge." In *Intuition and the Axiomatic Method*, edited by Emily Carson and Renate Huber, 70:3–20. Dordrecht: Springer.

Friedman, Michael. 1992. *Kant and the Exact Sciences*. Cambridge: Harvard University Press.

Parsons, Charles. 1969. "Kant's Philosophy of Arithmetic." In *Mathematics in Philosophy: Selected Essays*, 110–42. Ithaca: Cornell University Press.

Shabel, Lisa. 2003. *Mathematics in Kant's Critical Philosophy: Reflections on Mathematical Practice*. New York: Routledge.

———. 2005. "A Priority and Application: Philosophy of Mathematics in the Modern Period." In *The Oxford Handbook of Philosophy of Mathematics and Logic*, edited by Stewart Shapiro. Oxford: Oxford University Press. doi:10.1093/oxfordhb/9780195325928.003.0002. `http://dx.doi.org/10.1093/oxfordhb/9780195325928.003.0002`.

———. 2013. "Kant's Philosophy of Mathematics." In *The Stanford Encyclopedia of Philosophy*, edited by Edward N. Zalta, Fall 2013. `http://plato.stanford.edu/archives/fall2013/entries/kant-mathematics/`.