

## 1 Heuristic Analysis

Initially, we consider several different types of heuristics. Custom, Custom\_2, Next\_Step, and Stage all use a function `check_future_steps()`, which looks ahead to consider how long a run the player would have if they chose this move. This is similar to the overall minimax search, except that it ignores moves the opponent might make, and it also does not make a copy of the board at each step. This is an effort to make a fast estimate as to whether a move would leave open several future moves or would be near a dead end.

Next\_Step uses this heuristic for the entire game (I expected this would perform badly, since early on few moves would be near a dead end, and it would give little insight how to pick a good one). Custom, Custom\_2, and Stage are all similar where they start out using a Distance From Center metric, then as the game progresses they use the number of legal moves (for the player minus for the opponent) and then end with this `check_future_steps` heuristic. Progression through the game is determined by the number of empty spaces on the board.

Diff\_Dist, Dist\_Diff, and Diff\_dist\_sum all use combinations only of Dist (distance from the center) and Diff (number of legal moves for the player minus for the opponent). Distance is measured either by traditional Pythagorean (the square of x and square of y) or by counting the number of steps from the middle square (number of squares up or down, plus number of squares left or right).

Diff\_Dist uses Diff for the first half of the game, and Dist for the second. Dist\_Diff is the opposite. Diff\_dist\_sum uses a weighted sum of the two throughout the whole game.

Prior to these tests, we also had tried variations of weighting own legal moves remaining vs opponent legal moves remaining (inside of what we call Diff). We did not find any repeatable advantage over a simple `OWN_MOVES - LEGAL_MOVES`

Because we found a large amount of variation in each tournament run, we changed the number of games to be 100 or 80 per face-off, and in some cases we ran the whole tournament twice.

Two different tournaments of 100 games per match. Win rates:

	First tournament	Second tournament
AB_Improved	66.7%	67.3%
Custom	68.0%	67.4%
Custom_2	67.1%	65.4%
Diff_Dist	58.3%	59.6%
Dist_Diff	66.6%	64.7%
Diff_dist_sum	68.0%	68.4%
Next_Step	62.3%	63.1%
Stage	68.9%	67.4%

At this point, we stop considering the worst performing heuristics (Diff\_Dist, and Next\_Step). We try out a variation of Stage. The main heuristics stay the same, but we adjust weightings in each.

One tournament of 100 matches per face-off

AB_Improved	64.7%
Custom	64.1%
Diff_Dist_sum	67.0%
Mod_Stages	66.3%

We consider if it is better to use squared distance (Pythagorean distance) as in the heuristic given to us in sample\_players rather than counting the number of steps away from the center.

One tournament of 60 matches per face-off

AB_Improved	66.9%
Diff_Dist_sum	70.2%
Diff_SqDist	63.3%

At this point, Diff\_Dist\_sum seems to be the only heuristic that is consistently out-performing AB\_Improved. We now focus on the relative weights for Diff and Dist. Initially we used a positive version of Dist (meaning a higher score is given to squares further from the center). While this is counter-intuitive, we noticed that AB\_Center consistently performed well, so we aimed to include some of that success. (Since the minimax algorithm looks several steps ahead, we thought perhaps looking for a corner square

many steps ahead pushes one to choose a central square early on.) Now in these variations, we consider weighting Dist negatively (giving a higher score to squares closest to the center).

60 Matches

Diff+ (w\*Dist)

Name	w	Win Rate
AB_Improved		68.1%
DD_pt_8	.8	65.5%
DD_half	.5	65.5%
Diff_dist_sum	.3	66.7%
DD_subt	-.3	68.6%
DD_full_subt	-.6	72.4%

Now we explore making Dist even more heavily negative. We also add a variation of the Stages algorithm (where we use diff, dist, and check\_future\_steps() at different stages of the game).

60 Matches

AB_Improved	64.5%	
DD_full_subt	66.2%	(w = -.6)
DD_dbl_sub	66.7%	(w = -2)
Adv_stage	71.2%	

From this, Adv\_stages looks very promising. However, when we run two back-to-back comparisons to confirm this finding, we see this was just a fluke:

80 Matches each tournament

	First tournament	Second tournament
AB_Improved	68.0%	69.6%
Adv stages	65.2%	67.1%

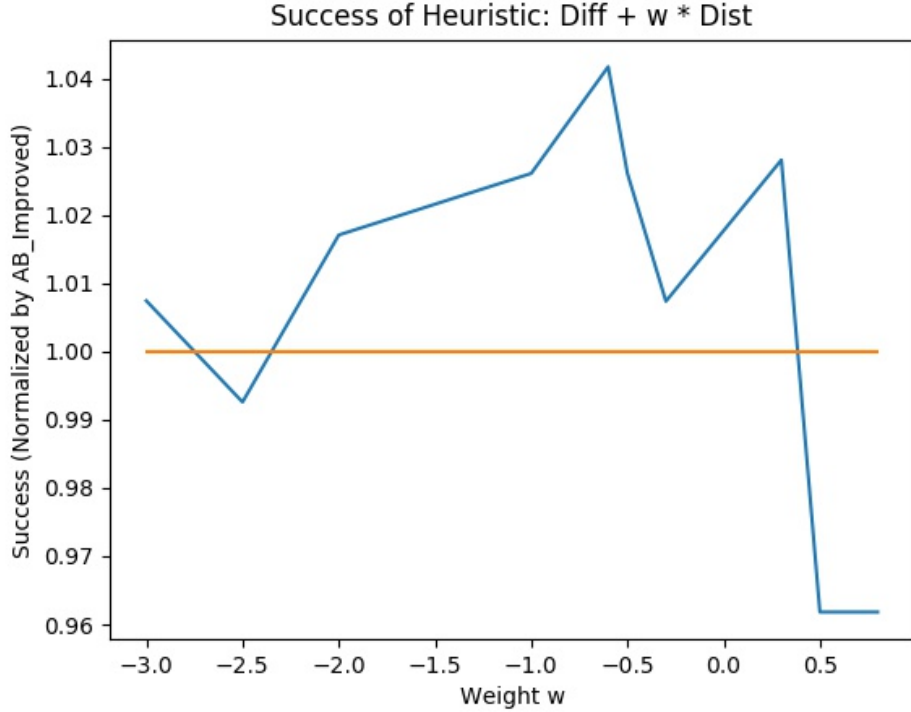


Figure 1: Normalized win rate for different combinations of Diff (difference between own legal moves and opponent legal moves) and Dist (steps from center square).

At this point, we return to a simple combination of Dist and Diff. It seems the added computational time from creating a more complicated look ahead is not worth it. Finally we decide to stay with a simple and fast heuristic. It remains only to determine the best weighting of Dist and Diff. We run one last set looking at a few other weights we haven't yet considered, and then chart average success vs the weight of Dist.

We graph results vs weighting of Dist in Figure 1. Each win rate is normalized by the AB\_Improved win rate for that tournament. If the agent was run in multiple tournaments, we take an average win rate, weighted by number of games in that tournament.

## 2 Tournament Results

These are the full records for the results cited earlier.

*****														
Playing Matches														
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Match #	Opponent	AB_Improved		Custom		Custom_2		Diff_Dist		Dist				
		Won	Lost	Won	Lost	Won	Lost	Won	Lost	Won				
1	Random	97		3	95		5	92		8	83		17	93
2	MM_Open	73		27	70		30	74		26	64		36	73
3	MM_Center	72		28	78		22	77		23	67		33	68
4	MM_Improved	77		23	74		26	74		26	66		34	71
5	AB_Open	50		50	53		47	48		52	47		53	53
6	AB_Center	53		47	55		45	53		47	42		58	60
7	AB_Improved	45		55	51		49	52		48	39		61	48
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Win Rate:		66.7%		68.0%		67.1%		58.3%		66.7%				

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Playing Matches										
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Match #	Opponent	AB_Improved		Custom		Custom_2		Diff_Dist		Dist
		Won	Lost	Won	Lost	Won	Lost	Won	Lost	Won
1	Random	94	6	94	6	95	5	88	12	92
2	MM_Open	74	26	74	26	77	23	72	28	70
3	MM_Center	77	23	78	22	66	34	63	37	71
4	MM_Improved	70	30	66	34	69	31	67	33	70
5	AB_Open	47	53	54	46	50	50	47	53	48
6	AB_Center	60	40	56	44	55	45	42	58	54
7	AB_Improved	49	51	50	50	46	54	38	62	48
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Win Rate:		67.3%		67.4%		65.4%		59.6%		64

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### Playing Matches

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Match #	Opponent	AB_Improved		Custom		Diff_dist_sum		Mod Stages	
		Won	Lost	Won	Lost	Won	Lost	Won	Lost
1	Random	89	11	93	7	92	8	91	9
2	MM_Open	76	24	66	34	68	32	73	27
3	MM_Center	72	28	65	35	74	26	71	29
4	MM_Improved	67	33	69	31	77	23	69	31
5	AB_Open	49	51	50	50	55	45	52	48
6	AB_Center	54	46	57	43	50	50	57	43
7	AB_Improved	46	54	49	51	53	47	51	49
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Win Rate:		64.7%		64.1%		67.0%		66.3%	

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### Playing Matches

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Match #	Opponent	AB_Improved		Diff_dist_sum		Stage	
		Won	Lost	Won	Lost	Won	Lost
1	Random	53	7	58	2	56	4
2	MM_Open	44	16	43	17	44	16
3	MM_Center	46	14	47	13	46	14
4	MM_Improved	39	21	47	13	41	19
5	AB_Open	33	27	31	29	29	31
6	AB_Center	32	28	33	27	33	27
7	AB_Improved	27	33	28	32	27	33
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Win Rate:		65.2%		68.3%		65.7%	

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### Playing Matches

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Match #	Opponent	AB_Improved	Diff_dist_sum	Diff_SqDist
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		Won	Lost	Won	Lost	Won	Lost
1	Random	55	5	59	1	52	8
2	MM_Open	43	17	45	15	44	16
3	MM_Center	48	12	47	13	43	17
4	MM_Improved	46	14	43	17	45	15
5	AB_Open	29	31	31	29	29	31
6	AB_Center	33	27	36	24	27	33
7	AB_Improved	27	33	34	26	26	34
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	Win Rate:	66.9%		70.2%		63.3%	

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Playing Matches  
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Match #	Opponent	AB_Improved	Diff_dist_sum	DD_half	DD_subt
		Won   Lost	Won   Lost	Won   Lost	Won   Lost
1	Random	58   2	58   2	57   3	58   2
2	MM_Open	43   17	44   16	44   16	46   14
3	MM_Center	41   19	44   16	47   13	43   17
4	MM_Improved	44   16	45   15	47   13	48   12
5	AB_Open	28   32	34   26	32   28	32   28
6	AB_Center	38   22	32   28	33   27	35   25
7	AB_Improved	28   32	28   32	32   28	25   35
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	Win Rate:	66.7%	67.9%	69.5%	68.3%

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Playing Matches  
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Match #	Opponent	AB_Improved	Diff_dist_sum	DD_half	DD_subt	DD_ful
		Won   Lost	Won   Lost	Won   Lost	Won   Lost	Won
1	Random	57   3	52   8	58   2	58   2	60
2	MM_Open	46   14	44   16	40   20	43   17	50
3	MM_Center	44   16	42   18	42   18	46   14	45
4	MM_Improved	41   19	45   15	42   18	49   11	46
5	AB_Open	29   31	29   31	28   32	28   32	35

6	AB_Center	36		24	37		23	32		28	32		28	37
7	AB_Improved	33		27	31		29	33		27	32		28	31
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	Win Rate:	68.1%			66.7%			65.5%			68.6%			72

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 Playing Matches  
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Match #	Opponent	AB_Improved		DD_full_subt		DD_dbl_sub		Adv stages	
		Won	Lost	Won	Lost	Won	Lost	Won	Lost
1	Random	52	8	55	5	56	4	57	3
2	MM_Open	38	22	43	17	48	12	48	12
3	MM_Center	44	16	39	21	43	17	55	5
4	MM_Improved	42	18	43	17	42	18	44	16
5	AB_Open	30	30	37	23	32	28	29	31
6	AB_Center	34	26	30	30	32	28	36	24
7	AB_Improved	31	29	31	29	27	33	30	30
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	Win Rate:	64.5%		66.2%		66.7%		71.2%	

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 Playing Matches  
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Match #	Opponent	AB_Improved		Adv stages	
		Won	Lost	Won	Lost
1	Random	74	6	74	6
2	MM_Open	58	22	55	25
3	MM_Center	58	22	52	28
4	MM_Improved	62	18	59	21
5	AB_Open	45	35	39	41
6	AB_Center	46	34	46	34
7	AB_Improved	38	42	40	40
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	Win Rate:	68.0%		65.2%	

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### Playing Matches

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Match #	Opponent	AB_Improved		Adv stages	
		Won	Lost	Won	Lost
1	Random	77	3	76	4
2	MM_Open	59	21	55	25
3	MM_Center	62	18	57	23
4	MM_Improved	52	28	56	24
5	AB_Open	49	31	45	35
6	AB_Center	47	33	48	32
7	AB_Improved	44	36	39	41

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Win Rate:	69.6%	67.1%
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Match #	Opponent	AB_Improved		Diff Late		GameDD_full_subt		DD_dbl_sub		DD_2
		Won	Lost	Won	Lost	Won	Lost	Won	Lost	
1	Random	55	5	56	4	58	2	58	2	56
2	MM_Open	43	17	41	19	41	19	50	10	45
3	MM_Center	48	12	45	15	51	9	41	19	44
4	MM_Improved	41	19	48	12	43	17	45	15	43
5	AB_Open	37	23	29	31	33	27	25	35	29
6	AB_Center	30	30	37	23	35	25	32	28	35
7	AB_Improved	29	31	25	35	32	28	32	28	29

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Win Rate:	67.4%	66.9%	69.8%	67.4%	66.9%
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### Playing Matches

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Match #	Opponent	AB_Improved		DD_half_subt		DD_eq_subt	
		Won	Lost	Won	Lost	Won	Lost
1	Random	53	7	58	2	55	5
2	MM_Open	47	13	43	17	49	11
3	MM_Center	38	22	45	15	45	15
4	MM_Improved	40	20	43	17	39	21
5	AB_Open	30	30	29	31	31	29
6	AB_Center	35	25	34	26	32	28
7	AB_Improved	31	29	29	31	30	30

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Win Rate:	65.2%	66.9%	66.9%
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