HW6 Mengchu Li

2020年10月17日 星期六 15:46

§ 3.4

1. (a)

x	$f(x)$	$f'(x)$
$x_0=8.3$	17.56492	3.116256
$x_1=8.6$	18.50515	3.151762

z_i	$f(z_i)$	1st	2nd	3rd
$z_0=8.3$	17.56492	$f[z_0,z_1]=f'(x_0)=3.116256$	$f[z_0,z_1,z_2]=\frac{3.1341-3.116256}{8.6-8.3}=0.05948$	$f[z_0,z_1,z_2,z_3]=\frac{0.058873-0.05948}{8.6-8.3}=-0.00202$
$z_1=8.3$	17.56492	$f[z_1,z_2]=\frac{18.50515-17.56492}{8.6-8.3}=3.1341$	$f[z_1,z_2,z_3]=\frac{3.151762-3.1341}{8.6-8.3}=0.058873$	
$z_2=8.6$	18.50515	$f[z_2,z_3]=f'(x_1)=3.151762$		
$z_3=8.6$	18.50515			

$H_3(x)=17.56492+3.116256(x-8.3)+0.05948(x-8.3)^2-0.00202(x-8.3)^2(x-8.6)$

(c)

	x	$f(x)$	$f'(x)$
x_0	-0.5	-0.02475	0.751
x_1	-0.25	0.3349375	2.189
x_2	0	1.101	4.002

z_i		$f(z_i)$	1st	2nd	3rd	4th	5th
z_0	-0.5	-0.02475	$f[z_0,z_1]=f'(x_0)=0.751$	$f[z_0,z_1,z_2]=2.751$	$f[z_0,z_1,z_2,z_3]=1$	$f[z_0,z_1,z_2,z_3,z_4]=0$	0
z_1	-0.5	-0.02475	$f[z_1,z_2]=1.43875$	$f[z_1,z_2,z_3]=3.001$	$f[z_1,z_2,z_3,z_4]=1$	$f[z_1,z_2,z_3,z_4,z_5]=0$	
z_2	-0.25	0.3349375	$f'(x_1)=2.189$	$f[z_2,z_3,z_4]=3.501$	$f[z_2,z_3,z_4,z_5]=1$		
z_3	-0.25	0.3349375	$f[z_3,z_4]=3.06425$	$f[z_3,z_4,z_5]=3.751$			
z_4	0	1.101	$f'(x_2)=4.002$				
z_5	0	1.101					

$H_5(x)=-0.02475+0.751(x+0.5)+2.751(x+0.5)^2+(x+0.5)^2(x+0.25)$

3. (a) $H_3(8.4)=17.56492+3.116256(8.4-8.3)+0.05948(8.4-8.3)^2-0.00202(8.4-8.3)^2(8.4-8.6)$
$=17.877144$

absolute error $=|H_3(8.4)-f(8.4)|$
$=|17.877144-8.4\ln(8.4)|$
$=1.88469\times10^{-6}$

(c) $H_5(-\frac{1}{3})=-0.02475+0.751(-\frac{1}{3}+0.5)+2.751(-\frac{1}{3}+0.5)^2+(-\frac{1}{3}+0.5)^2(-\frac{1}{3}+0.25)$
$=0.1745185$

$f(-\frac{1}{3})=(-\frac{1}{3})^3+4.001(-\frac{1}{3})^2+4.002(-\frac{1}{3})+1.001$
$=0.1745185185$

absolute error $=|H_5(-\frac{1}{3})-f(-\frac{1}{3})|$
$=1.85\times10^{-8}$

5.

	x	$f(x)$	$f'(x)$
x_0	0.30	0.29552	0.95534
x_1	0.32	0.31457	0.94924
x_2	0.35	0.34290	0.93937

(a)

z_i		$f(z_i)$	1st	2nd	3rd	4th	5th
z_0	0.30	0.29552	0.95534	$f[z_0,z_1,z_2]$ -0.142	$f[z_0,z_1,z_2,z_3]$ -1.05	$f[z_0,z_1,z_2,z_3,z_4]$ 20.732	-431.412
z_1	0.30	0.29552	0.9525	$f[z_1,z_2,z_3]$ -0.163	$f[z_1,z_2,z_3,z_4]$ -0.0134	$f[z_1,z_2,z_3,z_4,z_5]$ -0.8386	
z_2	0.32	0.31457	0.94924	$f[z_2,z_3,z_4]$ -0.16367	$f[z_2,z_3,z_4,z_5]$ -0.05533		
z_3	0.32	0.31457	0.94433	$f[z_3,z_4,z_5]$ -0.16533			
z_4	0.35	0.34290	0.93937				
z_5	0.35	0.34290					

$H_5(x)=0.29552+0.95534(x-0.3)-0.142(x-0.3)^2-1.05(x-0.3)^2(x-0.32)+20.732(x-0.3)^2(x-0.32)^2$
$-431.412(x-0.3)^2(x-0.32)^2(x-0.35)$

$H_5(0.34)=0.29552+0.95534(0.34-0.3)-0.142(0.34-0.3)^2-1.05(0.34-0.3)^2(0.34-0.32)$
$+20.732(0.34-0.3)^2(0.34-0.32)^2-431.412(0.34-0.3)^2(0.34-0.32)^2(0.34-0.35)$
$=0.33348883$

(b) absolute error $=|H_5(0.34)-\sin(0.34)|$
$=1.738\times10^{-6}$

$|f(x)-H_5(x)|=\left|\frac{f^{(6)}(\xi)}{6!}(x-0.3)^2(x-0.32)^2(x-0.35)^2\right|$
$=\left|\frac{-\sin\xi}{720}(x-0.3)^2(x-0.32)^2(x-0.35)^2\right|$ $\xi\in[0.30,0.35]$

when $x=0.34$
$|f(0.34)-H_5(0.34)|=\left|\frac{-\sin\xi}{720}(0.04)^2(0.02)^2(0.01)^2\right|$
$\le\left|\frac{-\sin0.35}{720}(0.04)^2(0.02)^2(0.01)^2\right|$
$\le 3.05\times10^{-14}$

∴ the actual error 1.738×10^{-6} is bigger than this error bound.
This error bound is not consistent.

(c)

	x	$f(x)$	$f'(x)$
x_0	0.30	0.29552	0.95334
x_1	0.32	0.31457	0.94924
x_2	0.33	0.32404	0.94604
x_3	0.35	0.34290	0.93937

z_i		$f(z_i)$	2nd	3rd	4th	5th	6th		
z_0	0.30	0.29552	0.95534	$f[z_0,z_1,z_2]$ -0.142	$f[z_0,z_1,z_2,z_3]$ -1.05	$f[z_0,z_1,z_2,z_3,z_4]$ 20.732	-431.412	-503380.93	52967742.11
z_1	0.30	0.29552	0.9525	$f[z_1,z_2,z_3]$ -0.163	$f[z_1,z_2,z_3,z_4]$ -0.0134	$f[z_1,z_2,z_3,z_4,z_5]$ -0.8386	-15532.84	1085631.3	
z_2	0.32	0.31457	0.94924	$f[z_2,z_3,z_4]$ -0.16367	$f[z_2,z_3,z_4,z_5]$ -0.05533	-156.167	17036.7		
z_3	0.32	0.31457	0.94433	$f[z_3,z_4,z_5]$ -0.16533	-1.617	14.2			
z_4	0.35	0.34290	0.93937	-0.1815	-1.475				
z_5	0.35	0.34290	0.943	-0.152					
z_6	0.33	0.32404	0.94604						
z_7	0.33	0.32404							

$H_7(x)=0.29552+0.95534(x-0.3)-0.142(x-0.3)^2-1.05(x-0.3)^2(x-0.32)+20.732(x-0.3)^2(x-0.32)^2$
$-431.412(x-0.3)^2(x-0.32)^2(x-0.35)-503380.93(x-0.3)^2(x-0.32)^2(x-0.35)^2$
$+52967742.11(x-0.3)^2(x-0.32)^2(x-0.35)^2(x-0.33)$

$H_7(0.34)=H_5(0.34)-503380.93(0.34-0.3)^2(0.34-0.32)^2(0.34-0.35)^2$
$+52967742.11(0.34-0.3)^2(0.34-0.32)^2(0.34-0.35)^2(0.34-0.33)$
$=0.33501683$

$|f(x)-H_7(x)|=\left|\frac{f^{(8)}(\xi)}{8!}(x-0.3)^2(x-0.32)^2(x-0.33)^2(x-0.35)^2\right|$
$=\left|\frac{\sin\xi}{40320}(x-0.3)^2(x-0.32)^2(x-0.33)^2(x-0.35)^2\right|$

plug in $x=0.34$
$|f(0.34)-H_7(0.34)|=\left|\frac{\sin\xi}{40320}(0.04)^2(0.02)^2(0.01)^2(-0.01)^2\right|$
$\le\left|\frac{\sin0.35}{40320}(0.0016)(0.0004)(0.0001)^2\right|$
$\le 5.44\times10^{-20}$

§4.1

1. (a)

x	$f(x)$	$f'(x)$
0.5	0.4794	0.852
0.6	0.5646	0.852/0.796
0.7	0.6442	0.796

choose $h=0.1$

forward-difference: $f'(x_0)\approx\frac{f(x_0+h)-f(x_0)}{h}$

backward-difference: $f'(x_0)\approx\frac{f(x_0)-f(x_0-h)}{h}$

$f'(0.5)=\frac{f(0.6)-f(0.5)}{0.1}=\frac{0.5646-0.4794}{0.1}=0.852$

$f'(0.6)\approx\frac{f(0.7)-f(0.6)}{0.1}=\frac{0.6442-0.5646}{0.1}=0.796$ / $f'(0.6)\approx\frac{f(0.6)-f(0.5)}{0.1}=\frac{0.5646-0.4794}{0.1}=0.852$

$f'(0.7)=\frac{f(0.7)-f(0.6)}{h}=\frac{0.6442-0.5646}{0.1}=0.796$

(b)

x	$f(x)$	$f'(x)$
0.0	0.00000	3.707
0.2	0.74140	3.152/3.707
0.4	1.3718	3.152

choose $h=0.2$

$f'(0.0)=\frac{f(0.2)-f(0)}{0.2}=\frac{0.74140-0}{0.2}=3.707$

$f'(0.2)=\frac{f(0.4)-f(0.2)}{0.2}=\frac{1.3718-0.74140}{0.2}=3.152$ forward

$f'(0.2)=\frac{f(0.2)-f(0)}{0.2}=\frac{0.74140-0}{0.2}=3.707$ backward

$f'(0.4)=\frac{f(0.4)-f(0.2)}{0.2}=\frac{1.3718-0.74140}{0.2}=3.152$

3. (a) $f(x)=\sin x$ $f''(x)=-\sin x$
$f'(x)=\cos x$

for $x=0.5$, actual error: $|0.852-\cos(0.5)|$
$=|0.852-0.8776|$
$=0.0256$

for forward-difference $|error|=|\frac{h}{2}f''(\xi)|$ $\xi\in[0.5,0.6]$
$=|0.05\cdot(-\sin\xi)|$
$=|0.05\sin\xi|$
$\le0.05|\sin(0.6)|$
≤0.0282

∴ actual error for $f'(0.5)$ is 0.0256, $|error|_{max}$ is 0.0282

for $x=0.6$ i) forward-difference
actual error: $|0.796-\cos(0.6)|$
$=|0.796-0.8253|$
$=0.0293$

error bound: $|error|=|\frac{h}{2}f''(\xi)|$ $\xi\in[0.6,0.7]$
$=|0.05\sin(\xi)|$
$\le0.05|\sin(0.7)|$
≤0.0322

ii) backward-difference
actual error: $|0.852-\cos(0.6)|$
$=|0.852-0.8253|$
$=0.0267$

error bound: $|error|=|\frac{h}{2}f''(\xi)|$ $\xi\in[0.5,0.6]$
$\le|0.05\sin(0.6)|$
≤0.0282

for $x=0.7$, actual error: $|0.796-\cos(0.7)|$
$=|0.796-0.7648|$
$=0.0312$

error bound: $|error|=|\frac{h}{2}f''(\xi)|$ $\xi\in[0.6,0.7]$
$\le|0.05\sin(0.7)|$
≤0.03221

5. (a)

x	$f(x)$	$f'(x)$
1.1	9.025013	17.7697
1.2	11.02318	22.193635
1.3	13.46374	27.10735
1.4	16.44465	32.51085

Three-Point Endpoint formula $f'(x_0)=\frac{1}{2h}[-3f(x_0)+4f(x_0+h)-f(x_0+2h)]$

choose $h=0.1$

$f'(1.1)=\frac{1}{0.2}[-3\cdot f(1.1)+4f(1.2)-f(1.3)]$
$=\frac{1}{0.2}[-3\cdot9.025013+4\cdot11.02318-13.46374]$
$=17.7697$

Three-Point Midpoint formula $f'(x_0)=\frac{1}{2h}[f(x_0+h)-f(x_0-h)]$

$h=0.1$ $f'(1.2)=\frac{1}{0.2}[f(1.3)-f(1.1)]$
$=22.193635$

$h=0.1$ $f'(1.3)=\frac{1}{0.2}[f(1.4)-f(1.2)]$
$=27.10735$

$h=-0.1$ $f'(1.4)=\frac{-1}{0.2}[-3\cdot f(1.4)+4f(1.3)-f(1.2)]$
$=\frac{-1}{0.2}[-3\cdot16.44465+4\cdot13.46374-11.02318]$
$=32.51085$

(b)

x	$f(x)$	$f'(x)$
8.1	16.94410	3.09205
8.3	17.56492	3.116150
8.5	18.19056	3.139975
8.7	18.82091	3.163525

Use Three-Point Endpoint formula and choose $h=0.2$

$f'(8.1)=\frac{1}{0.4}[-3f(8.1)+4f(8.3)-f(8.5)]$
$=\frac{1}{0.4}[-3\cdot16.94410+4\cdot17.56492-18.19056]$
$=3.09205$

Use Three-point Midpoint formula and choose $h=0.2$

$f'(8.3)=\frac{1}{0.4}[f(8.5)-f(8.1)]$
$=3.116150$

$f'(8.5)=\frac{1}{0.4}[f(8.7)-f(8.3)]$
$=3.139975$

Use Three-Point Endpoint formula and choose $h=-0.2$

$f'(8.7)=-\frac{1}{0.4}[-3f(8.7)+4f(8.5)-f(8.3)]$
$=-\frac{1}{0.4}[-3\cdot18.82091+4\cdot18.19056-17.56492]$
$=3.163525$

20. $f(x)=3xe^x-\cos x$

second-derivatives Midpoint formula

$f''(x_0)=\frac{1}{h^2}[f(x_0-h)-2f(x_0)+f(x_0+h)]$

$x_0=1.3$ $h=0.1$

$f''(1.3)=\frac{1}{0.01}[f(1.2)-2f(1.3)+f(1.4)]$
$=\frac{1}{0.01}(11.59006-2\cdot14.04276+16.86187)$
$=36.641$

$x_0=1.3$ $h_0=0.01$

$f''(1.3)=\frac{1}{(0.01)^2}[f(1.29)-2f(1.3)+f(1.31)]$
$=\frac{1}{(0.01)^2}[13.78176-2\cdot14.04276+14.30741]$
$=36.5$

$f(x)=3xe^x-\cos x$
$f'(x)=3e^x+3xe^x+\sin x$
$f''(x)=3e^x+3e^x+3xe^x+\cos x$
$=6e^x+3xe^x+\cos x$
$f''(1.3)=6e^{1.3}+3\cdot1.3\cdot e^{1.3}+\cos(1.3)$
≈36.5935

$h=0.1$ $|error|=|36.641-36.5935|$
$=0.0475$

$h=0.01$ $|error|=|36.5-36.5935|$
$=0.0935$

∴ with $h=0.1$, the absolute error is smaller

26. (a) $\tilde{f}(x_0+h)=f(x_0+h)+e(x_0+h)$
$\tilde{f}(x_0)=f(x_0)+e(x_0)$

$f'(x_0)-\frac{\tilde{f}(x_0+h)-\tilde{f}(x_0)}{h}=\frac{e(x_0+h)+e(x_0)}{h}-\frac{h}{2}f''(\xi_0)$

If $|e(x_0+h)|<a$, then $|e(x_0)|<a$ and $f''(\xi_0)\le M$

∴ $\left|f'(x_0)-\frac{\tilde{f}(x_0+h)-\tilde{f}(x_0)}{h}\right|=\left|\frac{2a}{h}-\frac{hM}{2}\right|$
$\le\frac{2a}{h}+\frac{hM}{2}$