HW2 Mengchu Li

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§2.1

1. $f(x)=\sqrt{x}-\cos x=0$ on $[0,1]$

$f(0)=\sqrt{0}-\cos 0=0-1=-1<0$

$f(1)=\sqrt{1}-\cos 1=1-\cos 1>0$

$f(0)\cdot f(1)<0$

$p_1=\frac{0+1}{2}=\frac{1}{2}$

$f(p_1)=f(\frac{1}{2})=\sqrt{\frac{1}{2}}-\cos\frac{1}{2}$

$=\frac{\sqrt{2}}{2}\cos\frac{1}{2}=-0.170475781<0$

$f(p_1)\cdot f(1)<0$

$\therefore p_2=\frac{1+\frac{1}{2}}{2}=\frac{3}{4}$

$f(p_2)=f(\frac{3}{4})=\sqrt{\frac{3}{4}}-\cos(\frac{3}{4})$

$=\frac{\sqrt{3}}{2}-\cos(\frac{3}{4})$

$=0.134336535>0$

$\therefore f(\frac{1}{2})\cdot f(\frac{3}{4})<0$

$\therefore p_3=(\frac{1}{2}+\frac{3}{4})/2=\frac{5}{8}=\boxed{0.625}$

2. $f(x)=3(x+1)(x-\frac{1}{2})(x-1)$

(a) $[-2,1.5]$

$f(-2)=3(-2+1)(-2-\frac{1}{2})(-2-1)<0$

$f(\frac{3}{2})=3(\frac{3}{2}+1)(\frac{3}{2}-\frac{1}{2})(\frac{3}{2}-1)>0$

$p_1=(-2+\frac{3}{2})/2=-\frac{1}{4}$

$f(-\frac{1}{4})=3(-\frac{1}{4}+1)(-\frac{1}{4}-\frac{1}{2})(-\frac{1}{4}-\frac{1}{4})>0$

$\therefore f(-2)\cdot f(-\frac{1}{4})<0$

$p_2=(-2-\frac{1}{4})/2=-\frac{9}{8}$

$f(-\frac{9}{8})=3(-\frac{9}{8}+1)(-\frac{9}{8}-\frac{1}{2})(-\frac{9}{8}-1)<0$

$\therefore f(-\frac{9}{8})\cdot f(-\frac{1}{4})<0$

$p_3=(-\frac{9}{8}-\frac{1}{4})/2=-\frac{11}{16}=\boxed{-0.6875}$

(b) $[-1.25,2.5]$

$f(-\frac{5}{4})=3(-\frac{5}{4}+1)(-\frac{5}{4}-\frac{1}{2})(-\frac{5}{4}-1)<0$

$f(\frac{5}{2})=3(\frac{5}{2}+1)(\frac{5}{2}-\frac{1}{2})(\frac{5}{2}-1)>0$

$f(-\frac{5}{4})\cdot f(\frac{5}{2})<0$

$\therefore p_1=(-\frac{5}{4}+\frac{5}{2})/2=\frac{5}{8}$

$f(\frac{5}{8})=3(\frac{5}{8}+1)(\frac{5}{8}-\frac{1}{2})(\frac{5}{8}-1)<0$

$f(\frac{5}{8})\cdot f(\frac{5}{2})<0$

$\therefore p_2=(\frac{5}{8}+\frac{5}{2})/2=\frac{25}{16}$

$f(\frac{25}{16})=3(\frac{25}{16}+1)(\frac{25}{16}-\frac{1}{2})(\frac{25}{16}-1)>0$

$f(\frac{25}{16})\cdot f(\frac{5}{8})<0$

$\therefore p_3=(\frac{25}{16}+\frac{5}{8})/2=\frac{35}{32}=\boxed{1.09375}$

5(c)

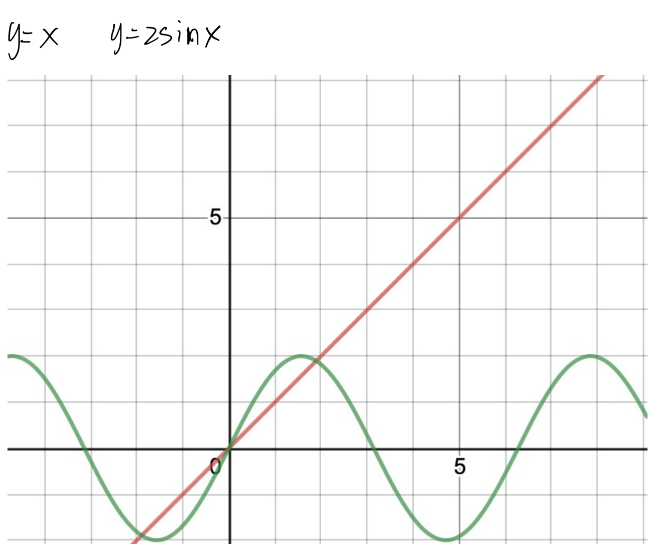
for $[-3,-2]$, root is $-2.19130706787109 38$ } code on the other PDF file
for $[-1,0]$, root is -0.7981643676757812

5(d)

for $[0.2,0.3]$, root is 0.29752807617187504 } Code on the other PDF file
for $[1.2,1.3]$, root is $1.2566223144531 25$

7. $y=x$ $y=2\sin x$

(a)



generated by Desmos

(b) $x=\sin x$

$x-\sin x=0$

From the graph above, the first positive root is between 1.5 and 2

Apply $[1.5,2]$ to the bisection code, then:

The estimating root is 1.895500183105468 8

§2.2

1. $f(x)=x^4+2x^2-x-3$

(a) $g_1(x)=(3+x-2x^2)^{1/4}$

$x=g_1(x)$

$x=(3+x-2x^2)^{1/4}$

$x^4=3+x-2x^2$

$x^4+2x^2-x-3=0$

$f(p)=0 \Rightarrow p^4+2p^2-p-3=0$

$\therefore p$ is the root of $f(x)$

$x=p$ also satisfies $x=g_1(x)$, which is a fixed-point problem

$\therefore$ there exists a fixed point $x=p$, s.t $x=g_1(x)$

(b) $g_2(x)=(\frac{x+3-x^4}{2})^{1/2}$

$x=g_2(x)$

$x=(\frac{x+3-x^4}{2})^{1/2}$

$x^2=\frac{x+3-x^4}{2}$

$2x^2=x+3-x^4$

$x^4+2x^2-x-3=0 \Leftrightarrow f(x)=0$

Given that $f(p)=0$

$\therefore x=p$ also satisfies $x=g_2(x)$

$\therefore$ there exists a fixed point $x=p$, s.t. $x=g_2(x)$

(c) $g_3(x)=(\frac{x+3}{x^2+2})^{1/2}$

$x=g_3(x)$

$x=(\frac{x+3}{x^2+2})^{1/2}$

$x^2=\frac{x+3}{x^2+2}$

$x^2(x^2+2)=x+3$

$x^4+2x^2=x+3$

$x^4+2x^2-x-3=0 \Leftrightarrow f(x)=0$

Given that $f(p)=0$

$\therefore x=p$ also satisfies $x=g_3(x)$

$\therefore$ there exists a fixed point $x=p$, s.t $x=g_3(x)$

(d) $g_4(x)=(3x^4+2x^2+3)/4x^3+4x-1$

$x=g_4(x)$

$x=3x^4+2x^2+3/4x^3+4x-1$

$x(4x^3+4x-1)=3x^4+2x^2+3$

$4x^4+4x^2-x=3x^4+2x^2+3$

$x^4+2x^2-x-3=0 \Leftrightarrow f(x)=0$

Given that $f(p)=0$

$\therefore x=p$ also satisfies $x=g_4(x)$

$\therefore$ there exists a fixed point $x=p$, s.t $x=g_4(x)$

2. (a) $g_1(x)=(3+x-2x^2)^{1/4}$

$p_0=1$

$p_1=g_1(p_0)=(3+1-2)^{1/4}=\sqrt[4]{2}=1.189207115$

$p_2=g_1(p_1)=g_1(1.189207115)=1.080057753$

$p_3=g_1(p_2)=g_1(1.080057753)=1.14967143$

$p_4=g_1(p_3)=g_1(1.14967143)=1.10782053$

$g_2(x)=(\frac{x+3-x^4}{2})^{1/2}$

$p_0=1$

$p_1=g_2(p_0)=(\frac{1+3-1^4}{2})^{1/2}=\sqrt{\frac{3}{2}}=1.224744871$

$p_2=g_2(p_1)=g_2(1.224744871)=0.99366616$

$p_3=g_2(p_2)=g_2(0.99366616)=1.228568645$

$p_4=g_2(p_3)=g_2(1.228568645)=0.98750643$

$g_3(x)=(\frac{x+3}{x^2+2})^{1/2}$

$p_1=g_3(p_0)=(\frac{1+3}{1+2})^{1/2}=\sqrt{\frac{4}{3}}=1.154700538$

$p_2=g_3(p_1)=g_3(1.154700538)=1.11642741$

$p_3=g_3(p_2)=g_3(1.11642741)=1.126052233$

$p_4=g_3(p_3)=g_3(1.126052233)=1.1236388 85$

$g_4(x)=\frac{3x^4+2x^2+3}{4x^3+4x-1}$

$p_0=1$

$p_1=g_4(p_0)=\frac{3+2+3}{4+4-1}=\frac{8}{7}=1.142857143$

$p_2=g_4(p_1)=g_4(1.142857143)=1.12448169$

$p_3=g_4(p_2)=g_4(1.12448169)=1.124123164$

$p_4=g_4(p_3)=g_4(1.124123164)=1.12412303$

(b) For $g_1(x)$, $|p_4-p_3|=0.0418509$

For $g_2(x)$, $|p_4-p_3|=0.241062215$

For $g_3(x)$, $|p_4-p_3|=0.002413348$

For $g_4(x)$, $|p_4-p_3|=0.000000134$

The minimum is for $g_4(x)$, which means $g_4(x)$ is the best approximation

4. $f(x)=x^4+3x^2-2$

(a) $x=\sqrt{\frac{2-x^4}{3}}$, $p_0=1$

$g_1(x)=\sqrt{\frac{2-x^4}{3}}$

$p_1=g_1(p_0)=\sqrt{\frac{2-1}{3}}=\sqrt{\frac{1}{3}}=0.57735$

$p_2=g_1(p_1)=g_1(0.57735)=0.79349$

$p_3=g_1(p_2)=g_1(0.79349)=0.73111$

$p_4=g_1(p_3)=g_1(0.73111)=0.75593$

(b) $x=\sqrt[4]{2-3x^2}$ $p_0=1$

$g_2(x)=\sqrt[4]{2-3x^2}$

$p_1=g_2(p_0)=\sqrt[4]{2-3}$

p_1 not defined

(c) $x=\frac{2-x^4}{3x}$ $p_0=1$

$g_3(x)=\frac{2-x^4}{3x}$

$p_1=g_3(p_0)=\frac{2-1}{3}=\frac{1}{3}=0.3333$

$p_2=g_3(p_1)=g_3(\frac{1}{3})=1.9876$

$p_3=g_3(p_2)=g_3(1.9876)=-2.28218$

$p_4=g_3(p_3)=g_3(-2.28218)=3.67003$

(d) $x=\sqrt[3]{\frac{2-3x^2}{x}}$ $p_0=1$

$g_4(x)=\sqrt[3]{\frac{2-3x^2}{x}}$

$p_1=g_4(p_0)=\sqrt[3]{\frac{2-3}{1}}=-1$

$p_2=g_4(p_1)=\sqrt[3]{\frac{2-3}{-1}}=1$

$p_3=g_4(p_2)=-1$

$p_4=g_4(p_3)=1$

(a) and (c) are appropriate methods since (b) p_1 does not defined, (d) p_n does not converge.

10. Proof: $g(x)=2^{-x}$ has a unique fixed point on $[\frac{1}{3},1]$

$x\in[\frac{1}{3},1]$

$g(\frac{1}{3})=2^{-\frac{1}{3}}=\frac{1}{\sqrt[3]{2}}$

(I) $g(1)=2^{-1}=\frac{1}{2}$

$\therefore g(x)\in[\frac{1}{3},1]$ for all $x\in[\frac{1}{3},1]$

$g'(x)=-\ln 2\cdot 2^{-x}$

Obviously, $g'(x)$ exists on $(\frac{1}{3},1)$

$|g'(x)|=|-\ln 2\cdot 2^{-x}|$

$=\ln 2\cdot 2^{-x}$

(II) $\leq \ln 2\cdot 2^{-\frac{1}{3}}\approx 0.55<1$

Since 2^{-x} is decreasing in $[\frac{1}{3},1]$, maximum is $g(-\frac{1}{3})$ in $x\in[\frac{1}{3},1]$

Hence, $g(x)=2^{-x}$ has a unique fixed point on $[\frac{1}{3},1]$

Codes are the other PDF. With $p_0=\frac{1}{3}$

The iteration to get approximation within 10^{-4} is 12 from Code $p_{12}=0.64166$

From Corollary 2.5

$|p_n-p|\leq\frac{k^n}{1-k}|p_1-p_0|$

$k\approx 0.55$

$\therefore 10^{-4}\leq\frac{0.55^n}{1-0.55}|2^{-\frac{1}{3}}-\frac{1}{3}|$

$10^{-4}\leq\frac{0.55^n}{0.45}|0.46036719|$

$0.55^n\geq 10^{-4}\cdot\frac{0.45}{0.46036719}$

$n\geq\log_{0.55}(10^{-4}\cdot\frac{0.45}{0.46036719})\approx 15.4$

$\therefore n>15$

The actual #iterations is smaller than theoretical estimate.

14. (a) $2+\sin x-x=0$ $[2,3]$

$f(x)=2+\sin x-x$

$f(x)=0 \Leftrightarrow x=2+\sin x$

$\therefore g(x)=2+\sin x$

Use Fixed-Point Iteration Code

Get # iterations: 52

$p_{52}=2.5541921027478667$ with $p_0=\frac{5}{2}$

$g'(x)=\cos x$

$|g'(2)|=0.9994$

$|g'(3)|=0.9899$

Since $|g'(x)|\leq k$ $\therefore k=0.9994$

$|p_n-p|\leq\frac{k^n}{1-k}|p_1-p_0|$

$10^{-5}\leq\frac{k^n}{1-k}|p_1-p_0|$

$10^{-5}\leq\frac{0.994^n}{1-0.994}|2.598472-2.5|$

$0.994^n\geq 10^{-5}\cdot\frac{0.006}{0.098472}$

$n\geq\log_{0.994}(10^{-5}\cdot\frac{0.006}{0.098472})\approx 27689$

$\therefore n>27689$, which is much greater than actual iterations

14. (c) $3x^2-e^x=0$

$3x^2=e^x$

$x^2=\frac{e^x}{3}$

$x=\sqrt{\frac{e^x}{3}}$

$\therefore g(x)=\sqrt{\frac{e^x}{3}}$

$g'(x)=\frac{1}{2}\sqrt{\frac{e^x}{3}}$

Note that $|\frac{1}{2}\sqrt{\frac{e^x}{3}}|\leq k<1$ for all $x\in[0,1]$

Choose $p_0=\frac{1}{2}$

$|g'(0)|=|\frac{1}{2}\sqrt{\frac{1}{3}}|=0.288675$

$|g'(1)|=|\frac{1}{2}\sqrt{\frac{e}{3}}|=0.475945$

$\therefore k=0.475945$

$|p_n-p|\leq\frac{k^n}{1-k}|p_1-p_0|$

$10^{-5}\leq\frac{0.475945^n}{1-0.475945}|0.741332-0.5|$

$10^{-5}\leq\frac{0.475945^n}{0.524055}(0.241332)$

$0.475945^n\geq 10^{-5}\cdot\frac{0.524055}{0.241332}$

$n\geq\log_{0.475945}(10^{-5}\frac{0.524055}{0.241332})\approx 14.5$

$\therefore n>14$, which is equal to actual iterations from Code (# iterations = 14, $p_{14}=0.91000196 74023$)