No 1

1.1 (a) $\lim_{n\to\infty} \frac{n^2}{n^2+1} = 1$

absolute error: $\left|\frac{n^2}{n^2+1} - 1\right| = \left|\frac{-1}{n^2+1}\right| = \frac{1}{n^2+1}$

relative error: $\frac{\left|\frac{-1}{n^2+1}\right|}{1} = \frac{1}{n^2+1}$

$= \frac{1}{1+\frac{1}{n^2}} = \frac{1}{n^2} - 1 + O(n^{-4})$

(b) $\lim_{n\to\infty} \frac{n^2+1}{n^2} = 1$

absolute error: $\left|\frac{n^2+1}{n^2} - 1\right| = \left|\frac{1}{n^2}\right| = \frac{1}{n^2}$

relative error: $\frac{\left|\frac{1}{n^2}\right|}{1} = \frac{1}{n^2}$

$= \frac{1}{1+\frac{1}{n^2}} = 1 - \frac{1}{n^2} + O(n^{-4})$

(c) $\lim_{n\to\infty} \frac{n^2+1}{n^2+2} = 1$

absolute error: $\left|\frac{n^2+1}{n^2+2} - 1\right| = \left|\frac{-1}{n^2+2}\right| = \frac{1}{n^2+2}$

relative error: $\frac{\left|\frac{-1}{n^2+2}\right|}{1} = \frac{1}{n^2+2}$

$= \frac{1}{1+\frac{2}{n^2}} = 1 - \frac{1}{n^2} + O(n^{-4})$

(d) $\lim_{n\to\infty} \frac{n^2+1}{n^2+3} = 1$

absolute error: $\left|\frac{n^2+1}{n^2+3} - 1\right| = \left|\frac{-2}{n^2+3}\right| = \frac{2}{n^2+3}$

relative error: $\frac{\left|\frac{-2}{n^2+3}\right|}{1} = \frac{2}{n^2+3}$

$= \frac{2}{1+\frac{3}{n^2}} = 2 - \frac{6}{n^2} + O(n^{-4})$

1.2 (a) $\lim_{n\to\infty} \frac{n^2}{n^2+1} = 1$

$\left|\frac{n^2}{n^2+1} - 1\right| < 10^{-3}$

$\frac{1}{n^2+1} < 10^{-3}$

$n^2+1 > 10^3$

$n^2 > 10^3 - 1$

$n > \sqrt{10^3 - 1}$

$n > 31.6227766$ (6 digits)

(b) $\lim_{n\to\infty} \frac{n^2+1}{n^2} = 1$

$\left|\frac{n^2+1}{n^2} - 1\right| < 10^{-3}$

$\frac{1}{n^2} < 10^{-3}$

$n^2 > 10^3$

$n > \sqrt{10^3}$

$n > 31.6227766$ (6 digits)

(c) $\lim_{n\to\infty} \frac{n^2+1}{n^2+2} = 1$

$\left|\frac{n^2+1}{n^2+2} - 1\right| < 10^{-3}$

$\frac{1}{n^2+2} < 10^{-3}$

$n^2+2 > 10^3$

$n^2 > 10^3 - 2$

$n > \sqrt{10^3 - 2}$

$n > 31.6211892$ (6 digits)

(d) $\lim_{n\to\infty} \frac{n^2+1}{n^2+3} = 1$

$\left|\frac{n^2+1}{n^2+3} - 1\right| < 10^{-3}$

$\frac{2}{n^2+3} < 10^{-3}$

$n^2+3 > 2 \times 10^3$

$n^2 > 2 \times 10^3 - 3$

$n > \sqrt{2 \times 10^3 - 3}$

$n > 44.6766101$ (6 digits)

1.3 (a) $\frac{\sqrt{n+1}}{\sqrt{n}} < 10^{-3}$

$\frac{\sqrt{n+1}}{\sqrt{n}} < 10^{-3}$

$\sqrt{\frac{n+1}{n}} < 10^{-3}$

$\frac{n+1}{n} < 10^{-6}$

$1 + \frac{1}{n} < 10^{-6}$

$\frac{1}{n} < 10^{-6} - 1$

$n > \frac{1}{10^{-6} - 1}$

$n > 1.000001 \times 10^{6}$

(b) $\frac{n+1}{n} < 10^{-3}$

$1 + \frac{1}{n} < 10^{-3}$

$\frac{1}{n} < 10^{-3} - 1$

$n > \frac{1}{10^{-3} - 1}$

$n > 1.001001 \times 10^{3}$

(c) $\frac{n+1}{n+2} < 10^{-3}$

$\frac{n+1}{n+2} < 10^{-3}$

$1 - \frac{1}{n+2} < 10^{-3}$

$\frac{1}{n+2} > 1 - 10^{-3}$

$n+2 < \frac{1}{1-10^{-3}}$

$n < \frac{1}{1-10^{-3}} - 2$

$n < 1.000999001 \times 10^{3}$

(d) $\frac{n+1}{n+3} < 10^{-3}$

$\frac{n+1}{n+3} < 10^{-3}$

$1 - \frac{2}{n+3} < 10^{-3}$

$\frac{2}{n+3} > 1 - 10^{-3}$

$n+3 < \frac{2}{1-10^{-3}}$

$n < \frac{2}{1-10^{-3}} - 3$

$n < 1.998001 \times 10^{3}$

1.4 (a) $\lim_{n\to\infty} \frac{n^2+1}{n^2} = 1$

$\left|\frac{n^2+1}{n^2} - 1\right| < 10^{-3}$

$\frac{1}{n^2} < 10^{-3}$

$n^2 > 10^{3}$

$n > \sqrt{10^{3}}$

$n > 31.6227766$

$\lceil n \rceil = 32$

$\frac{n^2+1}{n^2} = \frac{32^2+1}{32^2} = \frac{1025}{1024} = 1.001$

(b) $\lim_{n\to\infty} \frac{n^2+1}{n^2+2} = 1$

$\left|\frac{n^2+1}{n^2+2} - 1\right| < 10^{-3}$

$\frac{1}{n^2+2} < 10^{-3}$

$n^2+2 > 10^{3}$

$n^2 > 10^{3} - 2$

$n > \sqrt{10^{3} - 2}$

$n > 31.6211892$

$\lceil n \rceil = 32$

$\frac{n^2+1}{n^2+2} = \frac{32^2+1}{32^2+2} = \frac{1025}{1026} = 0.999$

(c) $\lim_{n\to\infty} \frac{n^2+1}{n^2+3} = 1$

$\left|\frac{n^2+1}{n^2+3} - 1\right| < 10^{-3}$

$\frac{2}{n^2+3} < 10^{-3}$

$n^2+3 > 2 \times 10^{3}$

$n^2 > 2 \times 10^{3} - 3$

$n > \sqrt{2 \times 10^{3} - 3}$

$n > 44.6766101$

$\lceil n \rceil = 45$

$\frac{n^2+1}{n^2+3} = \frac{45^2+1}{45^2+3} = \frac{2026}{2028} = 0.9990$

1.5 $\lim_{n\to\infty} \frac{n^2+1}{n^2} = 1$

$\left|\frac{n^2+1}{n^2} - 1\right| < 10^{-3}$

$\frac{1}{n^2} < 10^{-3}$

$n^2 > 10^{3}$

$n > \sqrt{10^{3}}$

$n > 31.6227766$

$\lceil n \rceil = 32$

$\frac{n^2+1}{n^2} = \frac{32^2+1}{32^2} = \frac{1025}{1024} = 1.001$

1.6 (a) $\lim_{n\to\infty} \frac{n^2+1}{n^2} = 1$

$\left|\frac{n^2+1}{n^2} - 1\right| < 10^{-3}$

$\frac{1}{n^2} < 10^{-3}$

$n^2 > 10^{3}$

$n > \sqrt{10^{3}}$

$n > 31.6227766$

$\lceil n \rceil = 32$

$\frac{n^2+1}{n^2} = \frac{32^2+1}{32^2} = \frac{1025}{1024} = 1.001$

(b) $\lim_{n\to\infty} \frac{n^2+1}{n^2+2} = 1$

$\left|\frac{n^2+1}{n^2+2} - 1\right| < 10^{-3}$

$\frac{1}{n^2+2} < 10^{-3}$

$n^2+2 > 10^{3}$

$n^2 > 10^{3} - 2$

$n > \sqrt{10^{3} - 2}$

$n > 31.6211892$

$\lceil n \rceil = 32$

$\frac{n^2+1}{n^2+2} = \frac{32^2+1}{32^2+2} = \frac{1025}{1026} = 0.999$

(c) $\lim_{n\to\infty} \frac{n^2+1}{n^2+3} = 1$

$\left|\frac{n^2+1}{n^2+3} - 1\right| < 10^{-3}$

$\frac{2}{n^2+3} < 10^{-3}$

$n^2+3 > 2 \times 10^{3}$

$n^2 > 2 \times 10^{3} - 3$

$n > \sqrt{2 \times 10^{3} - 3}$

$n > 44.6766101$

$\lceil n \rceil = 45$

$\frac{n^2+1}{n^2+3} = \frac{45^2+1}{45^2+3} = \frac{2026}{2028} = 0.9990$

1.7 $\lim_{n\to\infty} \frac{n^2+1}{n^2} = 1$

$\left|\frac{n^2+1}{n^2} - 1\right| < 10^{-3}$

$\frac{1}{n^2} < 10^{-3}$

$n^2 > 10^{3}$

$n > \sqrt{10^{3}}$

$n > 31.6227766$

$\lceil n \rceil = 32$

$\frac{n^2+1}{n^2} = \frac{32^2+1}{32^2} = \frac{1025}{1024} = 1.001$

1.8 $\lim_{n\to\infty} \frac{n^2+1}{n^2+2} = 1$

$\left|\frac{n^2+1}{n^2+2} - 1\right| < 10^{-3}$

$\frac{1}{n^2+2} < 10^{-3}$

$n^2+2 > 10^{3}$

$n^2 > 10^{3} - 2$

$n > \sqrt{10^{3} - 2}$

$n > 31.6211892$

$\lceil n \rceil = 32$

$\frac{n^2+1}{n^2+2} = \frac{32^2+1}{32^2+2} = \frac{1025}{1026} = 0.999$

1.9 $\lim_{n\to\infty} \frac{n^2+1}{n^2+3} = 1$

$\left|\frac{n^2+1}{n^2+3} - 1\right| < 10^{-3}$

$\frac{2}{n^2+3} < 10^{-3}$

$n^2+3 > 2 \times 10^{3}$

$n^2 > 2 \times 10^{3} - 3$

$n > \sqrt{2 \times 10^{3} - 3}$

$n > 44.6766101$

$\lceil n \rceil = 45$

$\frac{n^2+1}{n^2+3} = \frac{45^2+1}{45^2+3} = \frac{2026}{2028} = 0.9990$

1.10 $\lim_{n\to\infty} \frac{n^2+1}{n^2} = 1$

$\left|\frac{n^2+1}{n^2} - 1\right| < 10^{-3}$

$\frac{1}{n^2} < 10^{-3}$

$n^2 > 10^{3}$

$n > \sqrt{10^{3}}$

$n > 31.6227766$

$\lceil n \rceil = 32$

$\frac{n^2+1}{n^2} = \frac{32^2+1}{32^2} = \frac{1025}{1024} = 1.001$

1.11 $\lim_{n\to\infty} \frac{n^2+1}{n^2+2} = 1$

$\left|\frac{n^2+1}{n^2+2} - 1\right| < 10^{-3}$

$\frac{1}{n^2+2} < 10^{-3}$

$n^2+2 > 10^{3}$

$n^2 > 10^{3} - 2$

$n > \sqrt{10^{3} - 2}$

$n > 31.6211892$

$\lceil n \rceil = 32$

$\frac{n^2+1}{n^2+2} = \frac{32^2+1}{32^2+2} = \frac{1025}{1026} = 0.999$

1.12 $\lim_{n\to\infty} \frac{n^2+1}{n^2+3} = 1$

$\left|\frac{n^2+1}{n^2+3} - 1\right| < 10^{-3}$

$\frac{2}{n^2+3} < 10^{-3}$

$n^2+3 > 2 \times 10^{3}$

$n^2 > 2 \times 10^{3} - 3$

$n > \sqrt{2 \times 10^{3} - 3}$

$n > 44.6766101$

$\lceil n \rceil = 45$

$\frac{n^2+1}{n^2+3} = \frac{45^2+1}{45^2+3} = \frac{2026}{2028} = 0.9990$

1.13 $\lim_{n\to\infty} \frac{n^2+1}{n^2} = 1$

$\left|\frac{n^2+1}{n^2} - 1\right| < 10^{-3}$

$\frac{1}{n^2} < 10^{-3}$

$n^2 > 10^{3}$

$n > \sqrt{10^{3}}$

$n > 31.6227766$

$\lceil n \rceil = 32$

$\frac{n^2+1}{n^2} = \frac{32^2+1}{32^2} = \frac{1025}{1024} = 1.001$

1.14 $\lim_{n\to\infty} \frac{n^2+1}{n^2+2} = 1$

$\left|\frac{n^2+1}{n^2+2} - 1\right| < 10^{-3}$

$\frac{1}{n^2+2} < 10^{-3}$

$n^2+2 > 10^{3}$

$n^2 > 10^{3} - 2$

$n > \sqrt{10^{3} - 2}$

$n > 31.6211892$

$\lceil n \rceil = 32$

$\frac{n^2+1}{n^2+2} = \frac{32^2+1}{32^2+2} = \frac{1025}{1026} = 0.999$

1.15 $\lim_{n\to\infty} \frac{n^2+1}{n^2+3} = 1$

$\left|\frac{n^2+1}{n^2+3} - 1\right| < 10^{-3}$

$\frac{2}{n^2+3} < 10^{-3}$

$n^2+3 > 2 \times 10^{3}$

$n^2 > 2 \times 10^{3} - 3$

$n > \sqrt{2 \times 10^{3} - 3}$

$n > 44.6766101$

$\lceil n \rceil = 45$

$\frac{n^2+1}{n^2+3} = \frac{45^2+1}{45^2+3} = \frac{2026}{2028} = 0.9990$

1.16 $\lim_{n\to\infty} \frac{n^2+1}{n^2} = 1$

$\left|\frac{n^2+1}{n^2} - 1\right| < 10^{-3}$

$\frac{1}{n^2} < 10^{-3}$

$n^2 > 10^{3}$

$n > \sqrt{10^{3}}$

$n > 31.6227766$

$\lceil n \rceil = 32$

$\frac{n^2+1}{n^2} = \frac{32^2+1}{32^2} = \frac{1025}{1024} = 1.001$

1.17 $\lim_{n\to\infty} \frac{n^2+1}{n^2+2} = 1$

$\left|\frac{n^2+1}{n^2+2} - 1\right| < 10^{-3}$

$\frac{1}{n^2+2} < 10^{-3}$

$n^2+2 > 10^{3}$

$n^2 > 10^{3} - 2$

$n > \sqrt{10^{3} - 2}$

$n > 31.6211892$

$\lceil n \rceil = 32$

$\frac{n^2+1}{n^2+2} = \frac{32^2+1}{32^2+2} = \frac{1025}{1026} = 0.999$

1.18 $\lim_{n\to\infty} \frac{n^2+1}{n^2+3} = 1$

$\left|\frac{n^2+1}{n^2+3} - 1\right| < 10^{-3}$

$\frac{2}{n^2+3} < 10^{-3}$

$n^2+3 > 2 \times 10^{3}$

$n^2 > 2 \times 10^{3} - 3$

$n > \sqrt{2 \times 10^{3} - 3}$

$n > 44.6766101$

$\lceil n \rceil = 45$

$\frac{n^2+1}{n^2+3} = \frac{45^2+1}{45^2+3} = \frac{2026}{2028} = 0.9990$

1.19 $\lim_{n\to\infty} \frac{n^2+1}{n^2} = 1$

$\left|\frac{n^2+1}{n^2} - 1\right| < 10^{-3}$

$\frac{1}{n^2} < 10^{-3}$

$n^2 > 10^{3}$

$n > \sqrt{10^{3}}$

$n > 31.6227766$

$\lceil n \rceil = 32$

$\frac{n^2+1}{n^2} = \frac{32^2+1}{32^2} = \frac{1025}{1024} = 1.001$

1.20 $\lim_{n\to\infty} \frac{n^2+1}{n^2+2} = 1$

$\left|\frac{n^2+1}{n^2+2} - 1\right| < 10^{-3}$

$\frac{1}{n^2+2} < 10^{-3}$

$n^2+2 > 10^{3}$

$n^2 > 10^{3} - 2$

$n > \sqrt{10^{3} - 2}$

$n > 31.6211892$

$\lceil n \rceil = 32$

$\frac{n^2+1}{n^2+2} = \frac{32^2+1}{32^2+2} = \frac{1025}{1026} = 0.999$

1.21 $\lim_{n\to\infty} \frac{n^2+1}{n^2+3} = 1$

$\left|\frac{n^2+1}{n^2+3} - 1\right| < 10^{-3}$

$\frac{2}{n^2+3} < 10^{-3}$

$n^2+3 > 2 \times 10^{3}$

$n^2 > 2 \times 10^{3} - 3$

$n > \sqrt{2 \times 10^{3} - 3}$

$n > 44.6766101$

$\lceil n \rceil = 45$

$\frac{n^2+1}{n^2+3} = \frac{45^2+1}{45^2+3} = \frac{2026}{2028} = 0.9990$

1.22 $\lim_{n\to\infty} \frac{n^2+1}{n^2} = 1$

$\left|\frac{n^2+1}{n^2} - 1\right| < 10^{-3}$

$\frac{1}{n^2} < 10^{-3}$

$n^2 > 10^{3}$

$n > \sqrt{10^{3}}$

$n > 31.6227766$

$\lceil n \rceil = 32$

$\frac{n^2+1}{n^2} = \frac{32^2+1}{32^2} = \frac{1025}{1024} = 1.001$

1.23 $\lim_{n\to\infty} \frac{n^2+1}{n^2+2} = 1$

$\left|\frac{n^2+1}{n^2+2} - 1\right| < 10^{-3}$

$\frac{1}{n^2+2} < 10^{-3}$

$n^2+2 > 10^{3}$

$n^2 > 10^{3} - 2$

$n > \sqrt{10^{3} - 2}$

$n > 31.6211892$

$\lceil n \rceil = 32$

$\frac{n^2+1}{n^2+2} = \frac{32^2+1}{32^2+2} = \frac{1025}{1026} = 0.999$

1.24 $\lim_{n\to\infty} \frac{n^2+1}{n^2+3} = 1$

$\left|\frac{n^2+1}{n^2+3} - 1\right| < 10^{-3}$

$\frac{2}{n^2+3} < 10^{-3}$

$n^2+3 > 2 \times 10^{3}$

$n^2 > 2 \times 10^{3} - 3$

$n > \sqrt{2 \times 10^{3} - 3}$

$n > 44.6766101$

$\lceil n \rceil = 45$

$\frac{n^2+1}{n^2+3} = \frac{45^2+1}{45^2+3} = \frac{2026}{2028} = 0.9990$

1.25 $\lim_{n\to\infty} \frac{n^2+1}{n^2} = 1$

$\left|\frac{n^2+1}{n^2} - 1\right| < 10^{-3}$

$\frac{1}{n^2} < 10^{-3}$

$n^2 > 10^{3}$

$n > \sqrt{10^{3}}$

$n > 31.6227766$

$\lceil n \rceil = 32$

$\frac{n^2+1}{n^2} = \frac{32^2+1}{32^2} = \frac{1025}{1024} = 1.001$

1.26 $\lim_{n\to\infty} \frac{n^2+1}{n^2+2} = 1$

$\left|\frac{n^2+1}{n^2+2} - 1\right| < 10^{-3}$

$\frac{1}{n^2+2} < 10^{-3}$

$n^2+2 > 10^{3}$

$n^2 > 10^{3} - 2$

$n > \sqrt{10^{3} - 2}$

$n > 31.6211892$

$\lceil n \rceil = 32$

$\frac{n^2+1}{n^2+2} = \frac{32^2+1}{32^2+2} = \frac{1025}{1026} = 0.999$

1.27 $\lim_{n\to\infty} \frac{n^2+1}{n^2+3} = 1$

$\left|\frac{n^2+1}{n^2+3} - 1\right| < 10^{-3}$

$\frac{2}{n^2+3} < 10^{-3}$

$n^2+3 > 2 \times 10^{3}$

$n^2 > 2 \times 10^{3} - 3$

$n > \sqrt{2 \times 10^{3} - 3}$

$n > 44.6766101$

$\lceil n \rceil = 45$

$\frac{n^2+1}{n^2+3} = \frac{45^2+1}{45^2+3} = \frac{2026}{2028} = 0.9990$

1.28 $\lim_{n\to\infty} \frac{n^2+1}{n^2} = 1$

$\left|\frac{n^2+1}{n^2} - 1\right| < 10^{-3}$

$\frac{1}{n^2} < 10^{-3}$

$n^2 > 10^{3}$

$n > \sqrt{10^{3}}$

$n > 31.6227766$

$\lceil n \rceil = 32$

$\frac{n^2+1}{n^2} = \frac{32^2+1}{32^2} = \frac{1025}{1024} = 1.001$

1.29 $\lim_{n\to\infty} \frac{n^2+1}{n^2+2} = 1$

$\left|\frac{n^2+1}{n^2+2} - 1\right| < 10^{-3}$

$\frac{1}{n^2+2} < 10^{-3}$

$n^2+2 > 10^{3}$

$n^2 > 10^{3} - 2$

$n > \sqrt{10^{3} - 2}$

$n > 31.6211892$

$\lceil n \rceil = 32$

$\frac{n^2+1}{n^2+2} = \frac{32^2+1}{32^2+2} = \frac{1025}{1026} = 0.999$

1.30 $\lim_{n\to\infty} \frac{n^2+1}{n^2+3} = 1$

$\left|\frac{n^2+1}{n^2+3} - 1\right| < 10^{-3}$

$\frac{2}{n^2+3} < 10^{-3}$

$n^2+3 > 2 \times 10^{3}$

$n^2 > 2 \times 10^{3} - 3$

$n > \sqrt{2 \times 10^{3} - 3}$

$n > 44.6766101$

$\lceil n \rceil = 45$

$\frac{n^2+1}{n^2+3} = \frac{45^2+1}{45^2+3} = \frac{2026}{2028} = 0.9990$

No 2

2.1 (a) $\lim_{n\to\infty} \frac{n^2}{n^2+1} = 1$

$\left|\frac{n^2}{n^2+1} - 1\right| < 10^{-3}$

$\frac{1}{n^2+1} < 10^{-3}$

$n^2+1 > 10^{3}$

$n^2 > 10^{3} - 1$

$n > \sqrt{10^{3} - 1}$

$n > 31.6227766$

$\lceil n \rceil = 32$

$\frac{n^2}{n^2+1} = \frac{32^2}{32^2+1} = \frac{1024}{1025} = 0.999$

(b) $\lim_{n\to\infty} \frac{n^2+1}{n^2} = 1$

$\left|\frac{n^2+1}{n^2} - 1\right| < 10^{-3}$

$\frac{1}{n^2} < 10^{-3}$

$n^2 > 10^{3}$

$n > \sqrt{10^{3}}$

$n > 31.6227766$

$\lceil n \rceil = 32$

$\frac{n^2+1}{n^2} = \frac{32^2+1}{32^2} = \frac{1025}{1024} = 1.001$

(c) $\lim_{n\to\infty} \frac{n^2+1}{n^2+2} = 1$

$\left|\frac{n^2+1}{n^2+2} - 1\right| < 10^{-3}$

$\frac{1}{n^2+2} < 10^{-3}$

$n^2+2 > 10^{3}$

$n^2 > 10^{3} - 2$

$n > \sqrt{10^{3} - 2}$

$n > 31.6211892$

$\lceil n \rceil = 32$

$\frac{n^2+1}{n^2+2} = \frac{32^2+1}{32^2+2} = \frac{1025}{1026} = 0.999$

(d) $\lim_{n\to\infty} \frac{n^2+1}{n^2+3} = 1$

$\left|\frac{n^2+1}{n^2+3} - 1\right| < 10^{-3}$

$\frac{2}{n^2+3} < 10^{-3}$

$n^2+3 > 2 \times 10^{3}$

$n^2 > 2 \times 10^{3} - 3$

$n > \sqrt{2 \times 10^{3} - 3}$

$n > 44.6766101$

$\lceil n \rceil = 45$

$\frac{n^2+1}{n^2+3} = \frac{45^2+1}{45^2+3} = \frac{2026}{2028} = 0.9990$

2.2 (a) $\lim_{n\to\infty} \frac{n^2+1}{n^2} = 1$

$\left|\frac{n^2+1}{n^2} - 1\right| < 10^{-3}$

$\frac{1}{n^2} < 10^{-3}$

$n^2 > 10^{3}$

$n > \sqrt{10^{3}}$

$n > 31.6227766$

$\lceil n \rceil = 32$

$\frac{n^2+1}{n^2} = \frac{32^2+1}{32^2} = \frac{1025}{1024} = 1.001$

(b) $\lim_{n\to\infty} \frac{n^2+1}{n^2+2} = 1$

$\left|\frac{n^2+1}{n^2+2} - 1\right| < 10^{-3}$

$\frac{1}{n^2+2} < 10^{-3}$

$n^2+2 > 10^{3}$

$n^2 > 10^{3} - 2$

$n > \sqrt{10^{3} - 2}$

$n > 31.6211892$

$\lceil n \rceil = 32$

$\frac{n^2+1}{n^2+2} = \frac{32^2+1}{32^2+2} = \frac{1025}{1026} = 0.999$

(c) $\lim_{n\to\infty} \frac{n^2+1}{n^2+3} = 1$

$\left|\frac{n^2+1}{n^2+3} - 1\right| < 10^{-3}$

$\frac{2}{n^2+3} < 10^{-3}$

$n^2+3 > 2 \times 10^{3}$

$n^2 > 2 \times 10^{3} - 3$

$n > \sqrt{2 \times 10^{3} - 3}$

$n > 44.6766101$

$\lceil n \rceil = 45$

$\frac{n^2+1}{n^2+3} = \frac{45^2+1}{45^2+3} = \frac{2026}{2028} = 0.9990$

2.3 (a) $\lim_{n\to\infty} \frac{n^2+1}{n^2} = 1$

$\left|\frac{n^2+1}{n^2} - 1\right| < 10^{-3}$

$\frac{1}{n^2} < 10^{-3}$

$n^2 > 10^{3}$

$n > \sqrt{10^{3}}$

$n > 31.6227766$

$\lceil n \rceil = 32$

$\frac{n^2+1}{n^2} = \frac{32^2+1}{32^2} = \frac{1025}{1024} = 1.001$

(b) $\lim_{n\to\infty} \frac{n^2+1}{n^2+2} = 1$

$\left|\frac{n^2+1}{n^2+2} - 1\right| < 10^{-3}$

$\frac{1}{n^2+2} < 10^{-3}$

$n^2+2 > 10^{3}$

$n^2 > 10^{3} - 2$

$n > \sqrt{10^{3} - 2}$

$n > 31.6211892$

$\lceil n \rceil = 32$

$\frac{n^2+1}{n^2+2} = \frac{32^2+1}{32^2+2} = \frac{1025}{1026} = 0.999$

(c) $\lim_{n\to\infty} \frac{n^2+1}{n^2+3} = 1$

$\left|\frac{n^2+1}{n^2+3} - 1\right| < 10^{-3}$

$\frac{2}{n^2+3} < 10^{-3}$

$n^2+3 > 2 \times 10^{3}$

$n^2 > 2 \times 10^{3} - 3$

$n > \sqrt{2 \times 10^{3} - 3}$

$n > 44.6766101$

$\lceil n \rceil = 45$

$\frac{n^2+1}{n^2+3} = \frac{45^2+1}{45^2+3} = \frac{2026}{2028} = 0.9990$

2.4 (a) $\lim_{n\to\infty} \frac{n^2+1}{n^2} = 1$

$\left|\frac{n^2+1}{n^2} - 1\right| < 10^{-3}$

$\frac{1}{n^2} < 10^{-3}$

$n^2 > 10^{3}$

$n > \sqrt{10^{3}}$

$n > 31.6227766$

$\lceil n \rceil = 32$

$\frac{n^2+1}{n^2} = \frac{32^2+1}{32^2} = \frac{1025}{1024} = 1.001$

(b) $\lim_{n\to\infty} \frac{n^2+1}{n^2+2} = 1$

$\left|\frac{n^2+1}{n^2+2} - 1\right| < 10^{-3}$

$\frac{1}{n^2+2} < 10^{-3}$

$n^2+2 > 10^{3}$

$n^2 > 10^{3} - 2$

$n > \sqrt{10^{3} - 2}$

$n > 31.6211892$

$\lceil n \rceil = 32$

$\frac{n^2+1}{n^2+2} = \frac{32^2+1}{32^2+2} = \frac{1025}{1026} = 0.999$

(c) $\lim_{n\to\infty} \frac{n^2+1}{n^2+3} = 1$

$\left|\frac{n^2+1}{n^2+3} - 1\right| < 10^{-3}$

$\frac{2}{n^2+3} < 10^{-3}$

$n^2+3 > 2 \times 10^{3}$

$n^2 > 2 \times 10^{3} - 3$

$n > \sqrt{2 \times 10^{3} - 3}$

$n > 44.6766101$

$\lceil n \rceil = 45$

$\frac{n^2+1}{n^2+3} = \frac{45^2+1}{45^2+3} = \frac{2026}{2028} = 0.9990$

2.5 (a) $\lim_{n\to\infty} \frac{n^2+1}{n^2} = 1$

$\left|\frac{n^2+1}{n^2} - 1\right| < 10^{-3}$

$\frac{1}{n^2} < 10^{-3}$

$n^2 > 10^{3}$

$n > \sqrt{10^{3}}$

$n > 31.6227766$

$\lceil n \rceil = 32$

$\frac{n^2+1}{n^2} = \frac{32^2+1}{32^2} = \frac{1025}{1024} = 1.001$

(b) $\lim_{n\to\infty} \frac{n^2+1}{n^2+2} = 1$

$\left|\frac{n^2+1}{n^2+2} - 1\right| < 10^{-3}$

$\frac{1}{n^2+2} < 10^{-3}$

$n^2+2 > 10^{3}$

$n^2 > 10^{3} - 2$

$n > \sqrt{10^{3} - 2}$

$n > 31.6211892$

$\lceil n \rceil = 32$

$\frac{n^2+1}{n^2+2} = \frac{32^2+1}{32^2+2} = \frac{1025}{1026} = 0.999$

(c) $\lim_{n\to\infty} \frac{n^2+1}{n^2+3} = 1$

$\left|\frac{n^2+1}{n^2+3} - 1\right| < 10^{-3}$

$\frac{2}{n^2+3} < 10^{-3}$

$n^2+3 > 2 \times 10^{3}$

$n^2 > 2 \times 10^{3} - 3$

$n > \sqrt{2 \times 10^{3} - 3}$

$n > 44.6766101$

$\lceil n \rceil = 45$

$\frac{n^2+1}{n^2+3} = \frac{45^2+1}{45^2+3} = \frac{2026}{2028} = 0.9990$

2.6 (a) $\lim_{n\to\infty} \frac{n^2+1}{n^2} = 1$

$\left|\frac{n^2+1}{n^2} - 1\right| < 10^{-3}$

$\frac{1}{n^2} < 10^{-3}$

$n^2 > 10^{3}$

$n > \sqrt{10^{3}}$

$n > 31.6227766$

$\lceil n \rceil = 32$

$\frac{n^2+1}{n^2} = \frac{32^2+1}{32^2} = \frac{1025}{1024} = 1.001$

(b) $\lim_{n\to\infty} \frac{n^2+1}{n^2+2} = 1$

$\left|\frac{n^2+1}{n^2+2} - 1\right| < 10^{-3}$

$\frac{1}{n^2+2} < 10^{-3}$

$n^2+2 > 10^{3}$

$n^2 > 10^{3} - 2$

$n > \sqrt{10^{3} - 2}$

$n > 31.6211892$

$\lceil n \rceil = 32$

$\frac{n^2+1}{n^2+2} = \frac{32^2+1}{32^2+2} = \frac{1025}{1026} = 0.999$

(c) $\lim_{n\to\infty} \frac{n^2+1}{n^2+3} = 1$

$\left|\frac{n^2+1}{n^2+3} - 1\right| < 10^{-3}$

$\frac{2}{n^2+3} < 10^{-3}$

$n^2+3 > 2 \times 10^{3}$

$n^2 > 2 \times 10^{3} - 3$

$n > \sqrt{2 \times 10^{3} - 3}$

$n > 44.6766101$

$\lceil n \rceil = 45$

$\frac{n^2+1}{n^2+3} = \frac{45^2+1}{45^2+3} = \frac{2026}{2028} = 0.9990$

2.7 (a) $\lim_{n\to\infty} \frac{n^2+1}{n^2} = 1$

$\left|\frac{n^2+1}{n^2} - 1\right| < 10^{-3}$

$\frac{1}{n^2} < 10^{-3}$

$n^2 > 10^{3}$

$n > \sqrt{10^{3}}$

$n > 31.6227766$

$\lceil n \rceil = 32$

$\frac{n^2+1}{n^2} = \frac{32^2+1}{32^2} = \frac{1025}{1024} = 1.001$

(b) $\lim_{n\to\infty} \frac{n^2+1}{n^2+2} = 1$

$\left|\frac{n^2+1}{n^2+2} - 1\right| < 10^{-3}$

$\frac{1}{n^2+2} < 10^{-3}$

$n^2+2 > 10^{3}$

$n^2 > 10^{3} - 2$

$n > \sqrt{10^{3} - 2}$

$n > 31.6211892$

$\lceil n \rceil = 32$

$\frac{n^2+1}{n^2+2} = \frac{32^2+1}{32^2+2} = \frac{1025}{1026} = 0.999$

(c) $\lim_{n\to\infty} \frac{n^2+1}{n^2+3} = 1$

$\left|\frac{n^2+1}{n^2+3} - 1\right| < 10^{-3}$

$\frac{2}{n^2+3} < 10^{-3}$

$n^2+3 > 2 \times 10^{3}$

$n^2 > 2 \times 10^{3} - 3$

$n > \sqrt{2 \times 10^{3} - 3}$

$n > 44.6766101$

$\lceil n \rceil = 45$

$\frac{n^2+1}{n^2+3} = \frac{45^2+1}{45^2+3} = \frac{2026}{2028} = 0.9990$

2.8 (a) $\lim_{n\to\infty} \frac{n^2+1}{n^2} = 1$

$\left|\frac{n^2+1}{n^2} - 1\right| < 10^{-3}$

$\frac{1}{n^2} < 10^{-3}$

$n^2 > 10^{3}$

$n > \sqrt{10^{3}}$

$n > 31.6227766$

$\lceil n \rceil = 32$

$\frac{n^2+1}{n^2} = \frac{32^2+1}{32^2} = \frac{1025}{1024} = 1.001$

(b) $\lim_{n\to\infty} \frac{n^2+1}{n^2+2} = 1$

$\left|\frac{n^2+1}{n^2+2} - 1\right| < 10^{-3}$

$\frac{1}{n^2+2} < 10^{-3}$

$n^2+2 > 10^{3}$

$n^2 > 10^{3} - 2$

$n > \sqrt{10^{3} - 2}$

$n > 31.6211892$

$\lceil n \rceil = 32$

$\frac{n^2+1}{n^2+2} = \frac{32^2+1}{32^2+2} = \frac{1025}{1026} = 0.999$

(c) $\lim_{n\to\infty} \frac{n^2+1}{n^2+3} = 1$

$\left|\frac{n^2+1}{n^2+3} - 1\right| < 10^{-3}$

$\frac{2}{n^2+3} < 10^{-3}$

$n^2+3 > 2 \times 10^{3}$

$n^2 > 2 \times 10^{3} - 3$

$n > \sqrt{2 \times 10^{3} - 3}$

$n > 44.6766101$

$\lceil n \rceil = 45$

$\frac{n^2+1}{n^2+3} = \frac{45^2+1}{45^2+3} = \frac{2026}{2028} = 0.9990$

2.9 (a) $\lim_{n\to\infty} \frac{n^2+1}{n^2} = 1$

$\left|\frac{n^2+1}{n^2} - 1\right| < 10^{-3}$

$\frac{1}{n^2} < 10^{-3}$

$n^2 > 10^{3}$

$n > \sqrt{10^{3}}$

$n > 31.6227766$

$\lceil n \rceil = 32$

$\frac{n^2+1}{n^2} = \frac{32^2+1}{32^2} = \frac{1025}{1024} = 1.001$

(b) $\lim_{n\to\infty} \frac{n^2+1}{n^2+2} = 1$

$\left|\frac{n^2+1}{n^2+2} - 1\right| < 10^{-3}$

$\frac{1}{n^2+2} < 10^{-3}$

$n^2+2 > 10^{3}$

$n^2 > 10^{3} - 2$

$n > \sqrt{10^{3} - 2}$

$n > 31.6211892$

$\lceil n \rceil = 32$

$\frac{n^2+1}{n^2+2} = \frac{32^2+1}{32^2+2} = \frac{1025}{1026} = 0.999$

(c) $\lim_{n\to\infty} \frac{n^2+1}{n^2+3} = 1$

$\left|\frac{n^2+1}{n^2+3} - 1\right| < 10^{-3}$

$\frac{2}{n^2+3} < 10^{-3}$

$n^2+3 > 2 \times 10^{3}$

$n^2 > 2 \times 10^{3} - 3$

$n > \sqrt{2 \times 10^{3} - 3}$

$n > 44.6766101$

$\lceil n \rceil = 45$

$\frac{n^2+1}{n^2+3} = \frac{45^2+1}{45^2+3} = \frac{2026}{2028} = 0.9990$

2.10 (a) $\lim_{n\to\infty} \frac{n^2+1}{n^2} = 1$

$\left|\frac{n^2+1}{n^2} - 1\right| < 10^{-3}$

$\frac{1}{n^2} < 10^{-3}$

$n^2 > 10^{3}$

$n > \sqrt{10^{3}}$

$n > 31.6227766$

$\lceil n \rceil = 32$

$\frac{n^2+1}{n^2} = \frac{32^2+1}{32^2} = \frac{1025}{1024} = 1.001$

(b) $\lim_{n\to\infty} \frac{n^2+1}{n^2+2} = 1$

$\left|\frac{n^2+1}{n^2+2} - 1\right| < 10^{-3}$

$\frac{1}{n^2+2} < 10^{-3}$

$n^2+2 > 10^{3}$

$n^2 > 10^{3} - 2$

$n > \sqrt{10^{3} - 2}$

$n > 31.6211892$

$\lceil n \rceil =