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§4.1

1 (a) $f(x)=\sqrt{x}$, $n=3$, $x_0=4$

Taylor series Polynomial formula

$$p_3(x)=\frac{f(x_0)}{0!}(x-x_0)^0+\frac{f'(x_0)}{1!}(x-x_0)^1+\frac{f''(x_0)}{2!}(x-x_0)^2+\frac{f'''(x_0)}{3!}(x-x_0)^3$$

$f(x)=\sqrt{x}$

$f'(x)=\frac{1}{2\sqrt{x}}$

$f''(x)=-\frac{1}{4x\sqrt{x}}$

$f'''(x)=\frac{3}{8x^2\sqrt{x}}$

$$p_3(x)=\sqrt{4}+\frac{1}{2\sqrt{4}}(x-4)-\frac{1}{8\sqrt{4}}(x-4)^2+\frac{3}{48\sqrt{4}}(x-4)^3$$

$$=2+\frac{1}{4}(x-4)-\frac{1}{32}(x-4)^2+\frac{3}{384}(x-4)^3$$

$$=2+\frac{1}{4}(x-4)-\frac{1}{32}(x-4)^2+\frac{1}{128}(x-4)^3$$

$$=1+\frac{x}{4}-\frac{(x-4)^2}{32}+\frac{(x-4)^3}{128}$$

1 (b) $f_n(x)=\frac{1}{x}$, $n=1,2,3$

$f_1(x)=\frac{1}{x}$, $f_2(x)=-\frac{1}{x^2}$, $f_3(x)=\frac{2}{x^3}$

$$p_3(x)=\frac{f_1(x_0)}{0!}(x-x_0)^0+\frac{f_2(x_0)}{1!}(x-x_0)^1+\frac{f_3(x_0)}{2!}(x-x_0)^2$$

$$p_1(x)=\frac{\frac{1}{(x-1)}-\frac{(x-1)^0}{0!}}{0!}=\frac{\frac{1}{(x-1)}-1}{1}$$

$$=\frac{\frac{1}{(x-1)}-1}{1}=\frac{\frac{1}{(x-1)}-\frac{(x-1)}{(x-1)}}{1}$$

$$=\frac{\frac{1-(x-1)}{(x-1)}}{1}=\frac{\frac{2-x}{(x-1)}}{1}$$

$$p_2(x)=\frac{f_1(x_0)}{0!}(x-x_0)^0+\frac{f_2(x_0)}{1!}(x-x_0)^1+\frac{f_3(x_0)}{2!}(x-x_0)^2$$

$$=\frac{1}{1}+\frac{-1}{1}(x-1)+\frac{2}{2}(x-1)^2$$

$$=1-(x-1)+(x-1)^2$$

$$=1-x+1+x^2-2x+1$$

$$=3-3x+x^2$$

$$p_3(x)=\frac{f_1(x_0)}{0!}(x-x_0)^0+\frac{f_2(x_0)}{1!}(x-x_0)^1+\frac{f_3(x_0)}{2!}(x-x_0)^2$$

$$=\frac{1}{1}+\frac{-1}{1}(x-1)+\frac{2}{2}(x-1)^2+\frac{-6}{6}(x-1)^3$$

$$=1-(x-1)+(x-1)^2-(x-1)^3$$

2 (a) $f_n(x)=\frac{1}{x}$, $n=1,2,3$

$f_1(x)=\frac{1}{x}$, $f_2(x)=-\frac{1}{x^2}$, $f_3(x)=\frac{2}{x^3}$

$$p_3(x)=\frac{f_1(x_0)}{0!}(x-x_0)^0+\frac{f_2(x_0)}{1!}(x-x_0)^1+\frac{f_3(x_0)}{2!}(x-x_0)^2$$

$$p_1(x)=\frac{\frac{1}{(x)}-\frac{(x-0)^0}{0!}}{0!}=\frac{\frac{1}{(x)}-1}{1}$$

$$=\frac{\frac{1}{(x)}-1}{1}=\frac{\frac{1}{(x)}-\frac{(x)}{(x)}}{1}$$

$$=\frac{\frac{1-(x)}{(x)}}{1}=\frac{\frac{1-x}{(x)}}{1}$$

$$=\frac{1-x}{x}$$

$$p_2(x)=\frac{f_1(x_0)}{0!}(x-x_0)^0+\frac{f_2(x_0)}{1!}(x-x_0)^1+\frac{f_3(x_0)}{2!}(x-x_0)^2$$

$$=\frac{1}{1}+\frac{-1}{1}(x-0)+\frac{2}{2}(x-0)^2$$

$$=1-x+x^2$$

$$p_3(x)=\frac{f_1(x_0)}{0!}(x-x_0)^0+\frac{f_2(x_0)}{1!}(x-x_0)^1+\frac{f_3(x_0)}{2!}(x-x_0)^2$$

$$=\frac{1}{1}+\frac{-1}{1}(x-0)+\frac{2}{2}(x-0)^2+\frac{-6}{6}(x-0)^3$$

$$=1-x+x^2-x^3$$

2 (b) $f_n(x)=\frac{1}{x}$, $n=1,2,3$

$f_1(x)=\frac{1}{x}$, $f_2(x)=-\frac{1}{x^2}$, $f_3(x)=\frac{2}{x^3}$

$$p_3(x)=\frac{f_1(x_0)}{0!}(x-x_0)^0+\frac{f_2(x_0)}{1!}(x-x_0)^1+\frac{f_3(x_0)}{2!}(x-x_0)^2$$

$$p_1(x)=\frac{\frac{1}{(x)}-\frac{(x-0)^0}{0!}}{0!}=\frac{\frac{1}{(x)}-1}{1}$$

$$=\frac{\frac{1}{(x)}-1}{1}=\frac{\frac{1}{(x)}-\frac{(x)}{(x)}}{1}$$

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$$p_2(x)=\frac{f_1(x_0)}{0!}(x-x_0)^0+\frac{f_2(x_0)}{1!}(x-x_0)^1+\frac{f_3(x_0)}{2!}(x-x_0)^2$$

$$=\frac{1}{1}+\frac{-1}{1}(x-0)+\frac{2}{2}(x-0)^2$$

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$$p_3(x)=\frac{f_1(x_0)}{0!}(x-x_0)^0+\frac{f_2(x_0)}{1!}(x-x_0)^1+\frac{f_3(x_0)}{2!}(x-x_0)^2$$

$$=\frac{1}{1}+\frac{-1}{1}(x-0)+\frac{2}{2}(x-0)^2+\frac{-6}{6}(x-0)^3$$

$$=1-x+x^2-x^3$$

3 $f(x)=\frac{1}{x}$, $n=3$, $x_0=1$, $h=\frac{1}{2}$, $x=\frac{3}{2}$

$$p_3(x)=\frac{f(x_0)}{0!}(x-x_0)^0+\frac{f'(x_0)}{1!}(x-x_0)^1+\frac{f''(x_0)}{2!}(x-x_0)^2+\frac{f'''(x_0)}{3!}(x-x_0)^3+R_3(x)$$

$$p_3(x)=\frac{f(x_0)}{0!}(x-x_0)^0+\frac{f'(x_0)}{1!}(x-x_0)^1+\frac{f''(x_0)}{2!}(x-x_0)^2+\frac{f'''(x_0)}{3!}(x-x_0)^3$$

$$=\frac{1}{1}+\frac{-1}{1}(x-1)+\frac{2}{2}(x-1)^2+\frac{-6}{6}(x-1)^3$$

$$=1-(x-1)+(x-1)^2-(x-1)^3$$

$$R_3(x)=\frac{f^{(4)}(c)}{4!}(x-x_0)^4$$

$$=\frac{-24}{4!}(x-1)^4$$

$$=-(x-1)^4$$

$$=-\left(\frac{3}{2}-1\right)^4$$

$$=-\left(\frac{1}{2}\right)^4$$

$$=-\frac{1}{16}$$

$$=-0.0625$$

4 $f(x)=\frac{1}{x}$, $n=3$, $x_0=1$, $h=\frac{1}{2}$, $x=\frac{3}{2}$

$$p_3(x)=\frac{f(x_0)}{0!}(x-x_0)^0+\frac{f'(x_0)}{1!}(x-x_0)^1+\frac{f''(x_0)}{2!}(x-x_0)^2+\frac{f'''(x_0)}{3!}(x-x_0)^3+R_3(x)$$

$$p_3(x)=\frac{f(x_0)}{0!}(x-x_0)^0+\frac{f'(x_0)}{1!}(x-x_0)^1+\frac{f''(x_0)}{2!}(x-x_0)^2+\frac{f'''(x_0)}{3!}(x-x_0)^3$$

$$=\frac{1}{1}+\frac{-1}{1}(x-1)+\frac{2}{2}(x-1)^2+\frac{-6}{6}(x-1)^3$$

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$$p_3(x)=\frac{f(x_0)}{0!}(x-x_0)^0+\frac{f'(x_0)}{1!}(x-x_0)^1+\frac{f''(x_0)}{2!}(x-x_0)^2+\frac{f'''(x_0)}{3!}(x-x_0)^3+R_3(x)$$

$$p_3(x)=\frac{f(x_0)}{0!}(x-x_0)^0+\frac{f'(x_0)}{1!}(x-x_0)^1+\frac{f''(x_0)}{2!}(x-x_0)^2+\frac{f'''(x_0)}{3!}(x-x_0)^3$$

$$=\frac{1}{1}+\frac{-1}{1}(x-1)+\frac{2}{2}(x-1)^2+\frac{-6}{6}(x-1)^3$$

$$=1-(x-1)+(x-1)^2-(x-1)^3$$

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$$=\frac{-24}{4!}(x-1)^4$$

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6 $f(x)=\frac{1}{x}$, $n=3$, $x_0=1$, $h=\frac{1}{2}$, $x=\frac{3}{2}$

$$p_3(x)=\frac{f(x_0)}{0!}(x-x_0)^0+\frac{f'(x_0)}{1!}(x-x_0)^1+\frac{f''(x_0)}{2!}(x-x_0)^2+\frac{f'''(x_0)}{3!}(x-x_0)^3+R_3(x)$$

$$p_3(x)=\frac{f(x_0)}{0!}(x-x_0)^0+\frac{f'(x_0)}{1!}(x-x_0)^1+\frac{f''(x_0)}{2!}(x-x_0)^2+\frac{f'''(x_0)}{3!}(x-x_0)^3$$

$$=\frac{1}{1}+\frac{-1}{1}(x-1)+\frac{2}{2}(x-1)^2+\frac{-6}{6}(x-1)^3$$

$$=1-(x-1)+(x-1)^2-(x-1)^3$$

$$R_3(x)=\frac{f^{(4)}(c)}{4!}(x-x_0)^4$$

$$=\frac{-24}{4!}(x-1)^4$$

$$=-(x-1)^4$$

$$=-\left(\frac{3}{2}-1\right)^4$$

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7 $f(x)=\frac{1}{x}$, $n=3$, $x_0=1$, $h=\frac{1}{2}$, $x=\frac{3}{2}$

$$p_3(x)=\frac{f(x_0)}{0!}(x-x_0)^0+\frac{f'(x_0)}{1!}(x-x_0)^1+\frac{f''(x_0)}{2!}(x-x_0)^2+\frac{f'''(x_0)}{3!}(x-x_0)^3+R_3(x)$$

$$p_3(x)=\frac{f(x_0)}{0!}(x-x_0)^0+\frac{f'(x_0)}{1!}(x-x_0)^1+\frac{f''(x_0)}{2!}(x-x_0)^2+\frac{f'''(x_0)}{3!}(x-x_0)^3$$

$$=\frac{1}{1}+\frac{-1}{1}(x-1)+\frac{2}{2}(x-1)^2+\frac{-6}{6}(x-1)^3$$

$$=1-(x-1)+(x-1)^2-(x-1)^3$$

$$R_3(x)=\frac{f^{(4)}(c)}{4!}(x-x_0)^4$$

$$=\frac{-24}{4!}(x-1)^4$$

$$=-(x-1)^4$$

$$=-\left(\frac{3}{2}-1\right)^4$$

$$=-\left(\frac{1}{2}\right)^4$$

$$=-\frac{1}{16}$$

$$=-0.0625$$

8 $f(x)=\sqrt{x}$, $n=3$, $x_0=4$, $h=\frac{1}{2}$, $x=\frac{9}{2}$

$$p_3(x)=\frac{f(x_0)}{0!}(x-x_0)^0+\frac{f'(x_0)}{1!}(x-x_0)^1+\frac{f''(x_0)}{2!}(x-x_0)^2+\frac{f'''(x_0)}{3!}(x-x_0)^3+R_3(x)$$

$$p_3(x)=\frac{f(x_0)}{0!}(x-x_0)^0+\frac{f'(x_0)}{1!}(x-x_0)^1+\frac{f''(x_0)}{2!}(x-x_0)^2+\frac{f'''(x_0)}{3!}(x-x_0)^3$$

$$=\sqrt{4}+\frac{1}{2\sqrt{4}}(x-4)-\frac{1}{8\sqrt{4}}(x-4)^2+\frac{3}{48\sqrt{4}}(x-4)^3$$

$$=2+\frac{1}{4}(x-4)-\frac{1}{32}(x-4)^2+\frac{3}{384}(x-4)^3$$

$$=2+\frac{1}{4}(x-4)-\frac{1}{32}(x-4)^2+\frac{1}{128}(x-4)^3$$

$$=1+\frac{x}{4}-\frac{(x-4)^2}{32}+\frac{(x-4)^3}{128}$$

$$R_3(x)=\frac{f^{(4)}(c)}{4!}(x-x_0)^4$$

$$=\frac{\frac{15}{16}c^{-\frac{7}{2}}}{4!}(x-4)^4$$

$$=\frac{\frac{15}{16}c^{-\frac{7}{2}}}{24}(x-4)^4$$

$$=\frac{5}{128}c^{-\frac{7}{2}}(x-4)^4$$

$$=\frac{5}{128}c^{-\frac{7}{2}}\left(\frac{9}{2}-4\right)^4$$

$$=\frac{5}{128}c^{-\frac{7}{2}}\left(\frac{1}{2}\right)^4$$

$$=\frac{5}{128}c^{-\frac{7}{2}}\left(\frac{1}{16}\right)$$

$$=\frac{5}{2048}c^{-\frac{7}{2}}$$

$$=\frac{5}{2048}\left(\frac{9}{2}\right)^{-\frac{7}{2}}$$

$$=\frac{5}{2048}\left(\frac{2}{9}\right)^{\frac{7}{2}}$$

$$=\frac{5}{2048}\left(\frac{2}{9}\right)^{3}\sqrt{\frac{2}{9}}$$

$$=\frac{5}{2048}\left(\frac{2}{9}\right)^{3}\frac{\sqrt{2}}{3