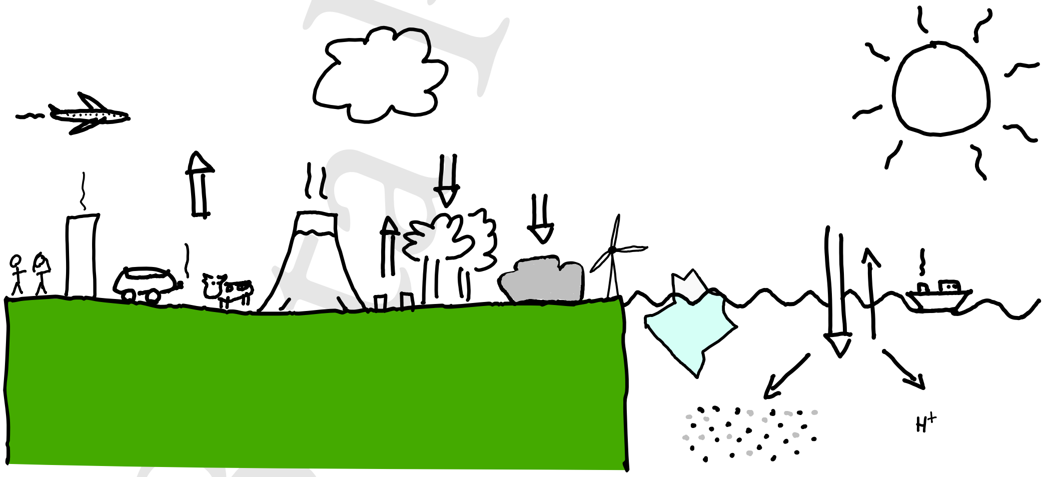
Back-of-the-envelope climate change



Marco Cosentino Lagomarsino

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To Barbara.
Tu sei rimasta una regina,
L'unica che nel male e nel malissimo mi è rimasta vicina.
(Emis Killa)

In memory of Daniele Colombaroli

Contents

Preface

What is this book about, who is writing it, and who is it for? I, the author, am not a specialist of climate change. So why this book? We live in the era of data, but this is also a time where data are often used instrumentally to impress and to bend reality in order to support *a priori* positions, and not to interrogate reality. Often even genuinely honest attempts to present data end up overwhelming us, because we lack the proper reference points. On one hand, we'd like to be able to "call bullshit" on this body of political and scientific pressure (and there's a very nice book about this, entitled "Calling Bullshit: The Art of Skepticism in a Data-Driven World", by Carl Bergstrom and Jevin West). On the other hand, if we became very good at this, and we spent our lives calling bullshit on each other, we would not achieve much. There is also a constructive, creative part of interpreting the data, and attempting to build a coherent picture from different sources and measurements. Trying to get a grasp on this positive part is what drove me to attempt writing this book. Often, this constructive part is more difficult than the negative one. Even if I am no climate scientist, I am a scientist, that sort of scientist that uses plots and mathematical models to understand data. My background is in theoretical physics, but the reader should not be mislead by this. The job of a physicist is not only to know and apply a certain set of mathematical laws, but also to extract minimal "phenomenological" and operative descriptions and predictions ("models") from data. The same approach is valuable in many contexts ranging from other scientific disciplines to consulting or planning and logistics, and “data science”. So this book is a project on data literacy, trying to convey simple techniques for “reading” the data and constructing positive interpretations, while being aware of possible caveats, biases and limitations. As far as I know there isn’t much work of this kind around, which was my main motivation.

If you are worrying about the climate science side of this book, you should know that so did I at some point. Many climate scientists are physicists like me, but I did not specialize or work in their area, and I did not acquire the same specific knowledge. In order to reassure the reader (and me) about this, my colleague Prof. Maurizio Maugeri at the University of Milan[1] kindly agreed to revise the manuscript regarding the climate-science aspects. In any case, a disclaimer for the reader is that this book should not be seen as a popular science book on climate change: it is more about data literacy than anything else. It does not aim to be exhaustive, and

[1]Maurizio Maugeri is a Full Professor at the Department of Environmental Science and Policy.

it does not aim (or claim) to be exact. Note also that a full understanding of climate change is an enormous challenge even for the specialized scientists who dedicate their lives to this goal. Some of the estimates and considerations you will read could be based on debatable assumptions, simplifications or approximations, or biased by my own lomited knowledge, and have to be taken as they are: efforts to extract meaning from data, with the main purpose of stimulating informed critical thinking and making climate change data something we can all begin to grasp by ourselves.

Indeed, my reasons for using climate data were that I thought this could be a very motivating example for a reader and also that I wanted to look into these data myself. Greenhouse emissions and climate change are big in the media, where gigatonnes of greenhouse gases, mass extinctions, impending climate disasters, are thrown at us by unclear numbers, which we need to connect to our everyday experience of life. This is not to say that things are not as bad as they seem. They are very bad, as far as I can judge. But the point is that, regarding climate data as well as other kinds of data, we should be clear-minded about our reactions. My feeling is that there is a huge divide between the narratives about what is happening produced by science, which are difficult to grasp, and typically filtered by the oversimplified translations produced by media, and distorted by politics, and what we experience every day around us. This divide is a big problem. We need a better intuition on the numbers that are involved, and we need to be empowered with the ability to produce rough common-sense estimates just based on the basic facts, as we do with other aspects of our lives (but typically don't indulge and attempt on matters at this scale).

If we had these skills, we would perhaps improve our understanding of the changing world around us. This could also enhance our perception of how we should contain the negative changes and how we should adapt. For example, one key question in the mind of a reader may be whether the climate change we observe is really anthropogenic. Many popular works implicitly say "but the recent change coincides with industrialization and this cannot be a coincidence". However, one can argue that this is not completely convincing and crave for more arguments to support a cause-effect link between atmospheric greenhouse gases of anthropogenic origin and climate. There are tools to argue in support of causality, and there are ways to gather different lines of correlative evidence in support of the idea that this cannot be a coincindence[2].

Hence, this book is an attempt to give some useful tools for reading the data constructively. The book should be readable by anyone who would like to be empowered by a data-driven approach to reflect on worldâĂŹs issues, and in particular on climate change. The mathematical tools used in the book are elementary (middle-school level), but

[2]As we will learn in chapter III, the concerns about climate change don't just come from the observations of warming, but they are above all linked to the scientific knowledge of the radiative balance of our planet and the role that greenhouse gases play in it.

some of the reasoning and analysis techniques parallel those used professionally within quantitative sciences. As a consequence, it is possible that, although the math is elementary, some parts appear unintuitive to some readers who do not have a college or high-school level technical education[3]. In my experience, younger people tend to be the most aware (and arguable the most affected) by our "era of data". Hence, a narrower target for this book could be early college and late high-school students and teachers, particularly in science and engineering. I use some of the material from the book myself as a teaching tool for a 3rd-year college course I teach to physics students, with hands-on projects on the data.

The questions that I aimed to answer concern the simplest things we can establish about climate change *directly from plots or data* that are available surfing the web. The chapters attempt to present the data gradually, and interpret them by straightforward reasoning, and by using the "Fermi approach"[4] of estimates relying on minimal knowledge (or no knowledge at all) of the physics and chemistry that are relevant to climate science. Breaking each question

[3] Most of these parts can be safely skipped.

[4] Enrico Fermi, the famous Italian-American Nobel-prize physicist, was a master of this art. He could always surprise his peers churning out solutions to difficult problems with a set of simple quick-and-approximate techniques. Fermi also had an extraordinary insight into which problems were important and which ones were not . I try to apply some of these techniques professionally (in the interdisciplinary area between statistical physics and biology), and over my career I have been striving to imitate Fermi and others in the insightful art of providing simple descriptions of complex problems, which is what makes physics so beautiful in my eyes.

into smaller sub-questions, and focusing on global trends, I tried to achieve a feeling of the "order of magnitude" of the quantities and processes involved in climate change, and to use this knowledge to draw simple conclusions based on straightforward reasoning.

To improve readability, instead of showing data directly through plots, I decided to use hand-drawn and annotated simplifications, where I could highlight what I thought could be the most important take-home features. The end of each chapter contains a section entitled "Data science take-home messages", which recapitulates the essential tools introduced and used in that chapter. Additionally, all the relevant sources and procedures are described in the "Sources" sections at the end of each chapter. If you are curious about the real plots, the best way is to look at these sources. However, if you are in a hurry, I made a "Sketch2Plot" folder in my GitHub page, where you can take a look at those plots. The sources include scientific publications as well as non-academic sources, such as Wikipedia or data projects (mainly the "Our World in Data" project, by Max Roser and others), where data sets and ready-made plots are more easily accessible to a wide audience[5]). Finally, we have said that the book does not require a prior knowledge of physics. However, if you are a physicist, it should be an easy read, because you will find that it is based on most of the cultural practices of this discipline. In one case, I have also added a short "Physics track" section, because knowing some ele-

[5] In any case, I cite also the original academic sources in the Sources sections.

mentary physics would add precision or substance to some of the arguments.

SOURCES. The book "Enrico Fermi. The last man who knew everything.", by David Schwartz (Basic Books, 2017) is an interesting biography of the famous physicist. The book "Calling Bullshit: The Art of Skepticism in a Data-Driven World", by Carl Bergstrom and Jevin West, is edited by Penguin Random House, and now at its second edition (2021). The web site https://www.callingbullshit.org/ describes the initiative, which includes a university course and a set of videos and tools.

Chapter I

Global temperature is rising

Is global temperature increasing? And how much? If you read newspapers and you are trying to be a conscious citizen, you are probably willing to accept that global temperature has been visibly increasing for some decades now. But you are equally likely not to have actually looked at the data. And if you were to look at these data, you would have to make sure that this increasing trend is meaningful. Admitted that we can establish that there is a trend, a further question is whether this trend is not a fortuitous coincidence, a “fluctuation” as in, “yes, the temperature has been rising for the past 50 years, but it's equally likely to decrease for the next 50". Therefore, at some point, I decided to download the data and look at this record myself, in order to dive directly into these questions and return to you with some representation of the data that we could look at together, as a useful interpretation exercise.

If we think about temperature, we immediately think of a thermometer. This is the instrument by which we measure temperatures in our daily life, but it was not always available to us. Thermometers were known to the ancient Greeks, but a systematic approach to their calibration was achieved only across the 17th and 18th centuries. Hence, we have a reliable temperature record from measurements with thermometers only starting from the mid 19th century. Starting from the 1950s, the record becomes precise and systematic. The period covered by this record gives us very precise and reliable data, but (although it starts a bit after the beginning of the industrial revolution). If our goal is to check if temperatures are rising, we can hope that we can see a significant change over this period [1].

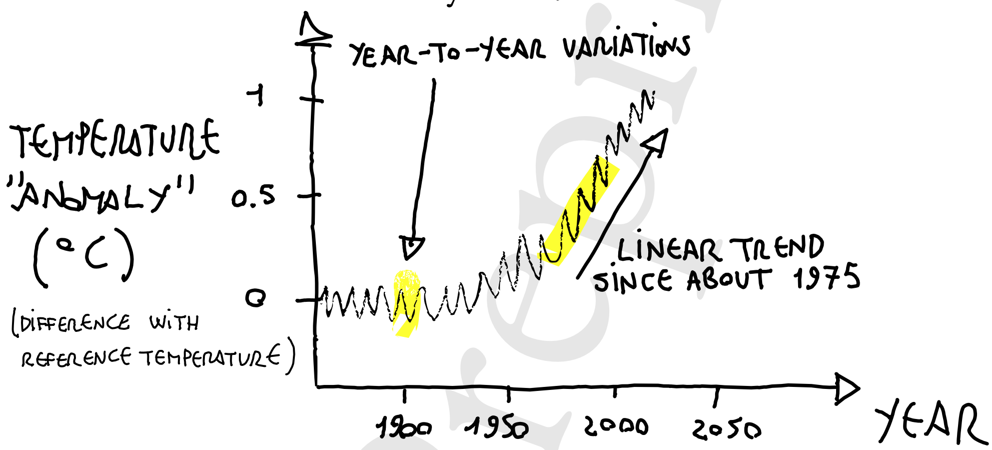
The other problem that we need to face when trying to assess if temperatures are increasing is that temperatures vary a lot. They vary between day and night, from place to place, and they vary across seasons. Many of these variations follow specific trends, mostly due to the illumination from

[1]Specifically should be able to see a change corresponding to when humans burned enough coal and other hydrocarbons to manifectly affect the atmospheric pools of grenhouse gas, beyond our uncertainty in the measurement, but this will be the subject of the next two chapters.

the sun. We all know that July and August are hotter (by many degrees) than December and January in Italy, and vice versa in Brazil; and, quite obviously, we also know that days are warmer than nights. Other changes are more subtle and we might perceive them as random. A detailed understanding of all these oscillations and changes is way beyond our scopes. For us, these changes are a disturbance, as we'd like to capture overall temperature changes beyond any trend with latitude, ocean currents, daily and seasonal oscillations and other fluctuations.

A simple approach is to average temperatures globally, and also average temporally over a year. This process should remove the effect of all the daily and seasonal oscillations, and should "iron out" all the effects that we wish to disregard in order to capture our trend. Hence, it's a good idea to consider these global averages (and climate scientists often do this). This book will only consider global trends, also when we will move to the other quantities that are relevant for climate change. Since I am no climate scientist, I started from processed data - trusting the work that has been performed by expert scientists to derive this information from raw data (and, I guess, trying to cross-check information whenever possible). As you can imagine, performing reliable estimates of these global averages may be a lot of work. For example, the spatial locations of data points are be spread out, and measurements vary over time and location from 1850 to now, and so do the sparsity of the coverage of the planet by measurements and the precision of measured temperatures.

We would like to use these data to quantify how much global temperature has increased compared to pre-industrial times. Here's my sketch, after looking at these plots, of what the data for global temperature roughly look like in the last almost 200 years[2],



The first thing that hits the eye is perhaps that this plot does not report the actual temperatures, but a temperature anomaly, which is a temperature difference. In other words, it subtracts a reference value (here the global average of the data in the 1850-1900 period, but sometimes this can be the average over anotther reference period, such as 1991-2020, or another convenient reference) to obtain what is called

[2]Note that all the sketches in this book are rough representations, meant to convey what I judged as the most important aspects, to show how I would represent the data if I had to draw them on the back of an envelope. Obviously, they likely contain some deviation from the original. You are encouraged to look at the real data to get a more precise idea. There are several reasons why I did not use the real data. First, I wanted to integrate the salient trends from different sources. Second, I wanted to try and simplify the essential information, making it more accessible. Third, I wanted to focus on the interpretation part and not on all the tecniques associated to producing plots. Fourth, real plots sometimes look very technical and I did not want to discourage the reader.

the "temperature anomaly". This expedient is necessary to average effectively the data across space. Weather stations on land are found at different elevations, they may take temperatures at different times of the day, and stations in different countries may calculate average monthly temperatures using different formulas. To avoid biases that could result from these differences, monthly average temperatures are reduced to anomalies. Empirically, anomalies tend to be constant over hundreds of kilometers, while absolute temperatures can vary over much shorter length scales (this is why we know the absolute temperature of earth with much higher uncertainty than its average teperature anomaly). Averaging the anomaly avoids possible offsets due to calibration and eases the comparison of data from different land and sea areas, as well as time spans[3].

Let's look a bit more closely into how these data are averaged. The procedures differ across projects, and the global average data mostly come from weather stations. For example, HadCRUT5 is a global temperature dataset from the Climatic Research Unit in the UK (see the Sources section at the end of this chapter), and provides gridded temperature anomalies across the world as well as global averages. It starts from monthly-mean temperatures data from over 10000 stations. The number of stations was small during the 1850s, but increases to over 2000 by 1900 and to more than 5000

[3] Although it can also be a source of confusion, since if you look at this plot from different sources, you will see different values in the y axis, depending on whether the anomaly has been computed using the average from the period, say, 1991-2020, or 1951-1980, or another period of choice.

after 1951. The stations are not evenly distributed across the earth's surface. There are many in North America for example, and much fewer in North Africa. Over the ocean areas, the data mostly consist of sea-surface measurements, which need to be averaged together with the air temperature available from land measurements. Combining these data implies assuming the anomalies of sea surface temperature agree with those of marine air temperature. This assumption is based on previous studies comparing large-area averages of both quantities. Today, the spatial averages are updated every month and performed dividing the earth surface in cells using a 5^o latitude by 5^o longitude grid (different projects use different grids and sometimes the same project has data for different grid choices). They use anomalies referenced to the 1961-1990 period, over which they have the best coverage. For each cell, the averaging procedure weighs the data based on a "statistical model", a mathematical model that treats the data as they were generated randomly with some prescriptions[4]. In the model, nearby locations are expected to vary in a similar way, and distant locations are expected to vary more weakly. This model also keeps into account uncertainties in the measurements. For each hemisphere they consider a weighted average, where the weights are the cosines of the central latitudes of each grid cell (this gives the same weight to each fixed latitude "ring" around

[4] In other words, a statistical model assumes that the data are random variables and formulates a set of assumptions about the "probability distribution" that generated the observed data. We will define a probability distribution more precisely in chapter VI.

the planet) taking all the non-missing grid-cell anomalies. Finally, they average the northern and southern hemisphere data.

The goal of such elaborate procedures is to reduce "biases", which are systematic errors in the data due to different sources of error, including time and location of observation, instrumentation type and changes in measurement practice over time, the fact that urban areas are hotter, use of sea surface temperatures (probably the strongest potential source of bias, which also depends on the historical changes in the use of engine-room sensors or buoys) and variations in the number of data points ("sampling") available in a grid cell. So the procedure is quite complex, and there are many choices. How important is the procedure, if different projects use different procedures? On one hand, the experience of many researchers and projects has identified a series of crucial steps to improve the accuracy and avoid important biases, so it is very important. On the other hand, there is theoretical knowledge that tells us that as long as we average enough data (and we manage to avoid systematic biases), we should get close to estimating the underlying global average. The law of large numbers, in probability and statistics, is a theorem that states that as a sample size (in our case, the number of sampled stations) grows, its mean gets closer to the underlying expected value, which we could measure if we had complete information on the data. Another theorem, the central limit theorem, also tells us how big a sample needs to be to represent a population distribution with prescribed accuracy. Therefore, one could also compare methods by verifying how the means change with the number of sampled weather stations (but we do not attempt it here).

Since the plot can be produced with data coming from different projects, one way to be confident that these data are trustworthy is that several independent studies, made by different sets of scientists, doing things differently in terms of the choices described above, and also using different (though overlapping) data, arrive to very similar results for the time series of yearly global temperatures, with discrepancies across data sets that are typically about one tenth of a degree Celsius for a given year. We can take this discrepancy as an indication of the uncertainty in the data. There are several studies that quantify these uncertainties in more direct ways, and give similar figures. Naturally, the error decrease with time of measurement. For example, HadCRUT5 gives "confidence intervals", a very stringent evaluation of the error defined as the interval where we are 95% sure that the true average would fall into (in a scenario where history is repeated many times), decreasing from 0.9 ^{o}C in the 1850s to 0.16-0.2 ^{o}C today. In order to estimate these uncertainties they generate 200 random realizations of the data from all individual weather stations using the statistical model involved in the averaging process, where data are drawn keeping into account the known ranges of uncertainty that emerge from the gridding process.

Back to our global trend of temperature anomaly, since we want to quantify the increase compared to pre-industrial time, an important question is that one should define what exactly is meant by "pre-industrial". By convention, in reports from the IPCC (Intergovernmental Panel on Climate Change), warming is expressed relative to the global mean temperature of the period 1850-1900, taken an approximation of pre-industrial temperatures. So the figures discussed in the news typically refer to this definition (but as I said the plots may not). I tried to make the plot that I sketched comply to this definition. Today (in 2022), global temperatures have risen by about 1.1^{o}C above the 1850-1900 reference.

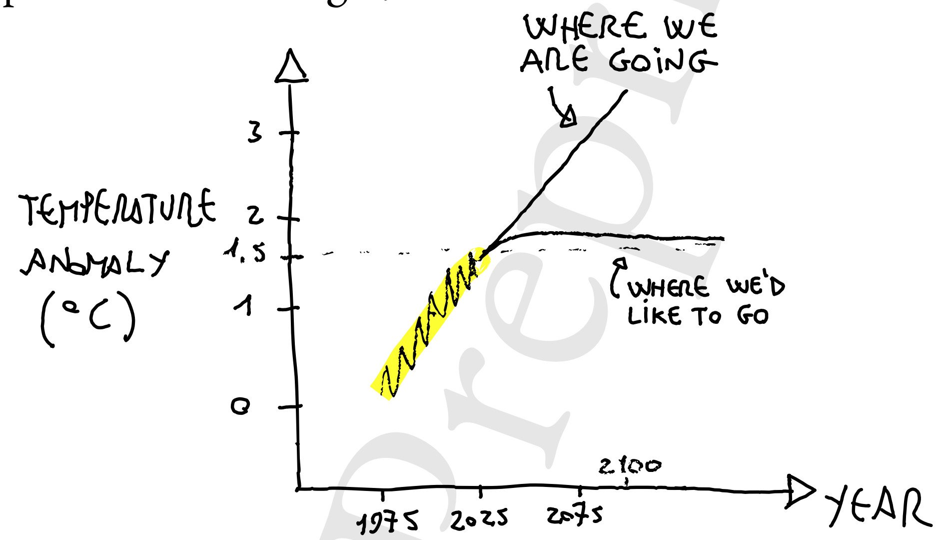
Looking more closely at the plot sketched above, the second thing one could notice is that, despite of the averaging over all Earth locations and all times over the year, there are interesting "noisy" year-to-year variations. Those are the zigzags in the plot; the ones in my drawing are not the real ones (because they were way too difficult to draw precisely for me), but they represent more or less the real wiggle. What is important is to look at how big these zigzags are compared to the trend of the plot. Understanding the source of these changes (whose extent is above the uncertainty that we have defined above) is difficult (and beyond our scopes), but as we will see in a minute, these variations can be our "ruler". Looking at the trend beyond these year-to-year variations, the temperature increase seems to be quite sluggish or absent between 1850 and 1900 (so this is a good period against which to define the anomaly), then it picks up increasingly faster over the next 100 years, and finally it becomes really marked after 1975. After that year, the roughly linear increase hits the eye, with a slope of almost 0.5^{o}C change over 25 years, and with a total increase of almost 1^{o}C between 1975 and today. Surely this change is well beyond our uncertainty in the data, which we defined as the discrepancy of estimated global temperatures across different studies, hence we are quite sure that we are not getting this trend because of measurement errors.

So we have established that there is a rising trend in the global temperature beyond our measurement errors, during the industrial era[5]. But this is not enough. In order to claim that temperature has been changing significantly over this period, we need a meaningful term of comparison for the "natural variation" of our variable, beyond experimental error. For example, we could ask whether the global average yearly temperature has varied more over the years than local temperatures vary from day to day, or, perhaps more fairly, than the global value for, say, the northern hemisphere varies across seasons. Since we are familiar with daily and seasonal temperature variations, we know that the answer to both of these questions would clearly be no, but it is also easy to argue that this comparison is not fair, because these local or faster changes cancel out when we consider global averages. Then how can we conclude that the temperature increase that we see in the past 45 years or so is

[5]Note that in this chapter we are just asking whether there is a trend during the industrial era. This is consistent with the interpretation that this trend is caused by human activity, but it is not a proof. Firmly establishing causality is very difficult, and in this case relies on our direct experimental knowledge of the physics of the greenhouse effect, as we will discuss in chapter III.

considerable? One way is to compare them with the year-to-year variations, (the “fluctuations”) of the same global average. As we noticed before, these are about one tenth of a degree, and the trend of the increase we are seeing in the data since 1975 is way above this amount. Hence, just looking at global temperature data we conclude that we cannot dismiss the 1 ^{o}C increase between 1975 and now as an irrelevant or purely coincidental change, based on what we know on temperature variations from the past two-hundred years or so (as I could have guessed, all those newspaper headlines had a meaning!). So the increase in global temperatures is small compared to the temperature changes we experience between day and night, but very big compared to its natural year-to-year variation, which is a better ruler, because it considers that local and fast variations cancel out in our global averages. We’ll get a confirmation of this point looking at more ancient (indirectly) recorded temperatures.

Before we do that, let’s take a look at what we would predict if we extend this trend to future temperatures. Making meaningful predictions for the future is difficult, but the pressure is much less if we clarify our assumptions. There is a very simple assumption that we can make, which is that things in the near future will continue to behave as they did in the past years (which is a good approximation of the scenario where we will do nothing about climate change)[6]. In this case, since the trend has been increasing as a straight line for the past forty-something years, it seems reasonable to assume that it will keep doing the same. We can then keep drawing the almost straight line we see from 1975 to now, extending the trend to future years. If I assume this "linear extrapolation", the sketch below roughly depicts trend that I get,



The highlighted part is basically the current trend. To show how radical a change of trend we need, compared to the current state of things, in order to reach our internationally set climate change targets, I have also drawn an imaginary trend line that reaches this target. Clearly, where we want to go[7] is radically different to where we are currently going. Looking at the literature, it seems that the actual climate models are more pessimistic than my linear extrapo-

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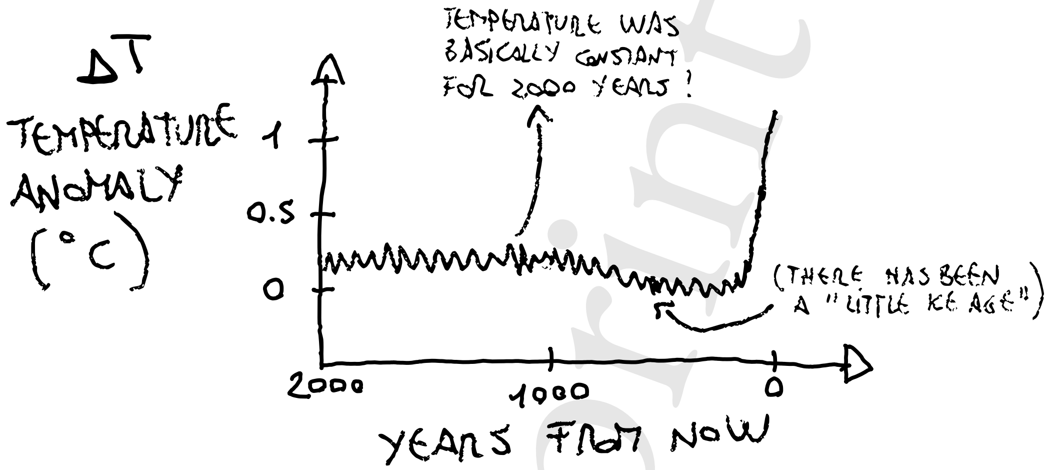
6 There are, of course, better approximations, and there are connections between these global averages and extreme events. Please hold on until subsequent chapters for more information on these points.

7 At least referred to those people who agree that increasing global temperatures are a concern, the rest of this book will provide more arguments for why they should be.

lation, as they tend to predict that with no intervention, global temperature would increase even faster (the current estimates say that the peak increase, reached before 2100, could easily be almost 3 degrees Celsius above pre-industrial levels). But what these models really want to establish is how much we need to reduce our greenhouse gas emissions (see chapter II and III) in order to stay within the targets of 1.5 or 2 degrees Celsius above pre-industrial levels that we are trying to set ourselves. We will come back to this problem later, and I'll try to address it with some back-of-the-envelope arguments. Why we set these targets is also something that we would like to understand more directly, as we go through the data. For now we can say that the current consensus among the scientists that study these things is that an increase of 1.5 to 2 degrees Celsius above pre-industrial levels could already be dangerous, but given the situation we are in we cannot really hope to do any better.

BEFORE THERMOMETERS WERE INVENTED, temperature changes left a set of different "footprints" that we can measure. These footprints give rise to many possible indirect measurements of temperatures that evaluate diverse sources of information, including the chemical composition of ice cores, the width of tree rings, ocean sediments, fossil pollen and corals. These data give a very instructive perspective on climate change, so at some point I reached the conclusion that I should look myself into these data as well. Here I try to hand-draw global temperature data from the past 2000 years that integrate multiple sources, and that are calibrated with high-precision contemporary measurements.



The plot sketched above is very definite in telling us that the temperature changes we are seeing today are exceptional, and other than that global temperature stayed remarkably constant since people like Julius Cesar and Cleopatra strolled around the Mediterranean[8]. What is curious and remarkable is that when the industrial revolution started, and humans began pumping tonnes of CO_2 in the atmosphere by the millions, temperature was actually slightly decreasing. This "Little Ice Age" began in the 14th century and the coldest period ends around 1850 (possibly thanks to our intervention). Glaciers were expanding all around the planet, and Londoners would skate on the Thames during Winter. The reasons for this climate change are not known precisely, but probably lay in volcanic eruptions and/or re-

[8]There is a "Medieval Warm Period" between 900 and 1300 AD which I have not emphasized in my sketch, and is not clearly visible in the original plot. The warming is well documented across the North Atlantic region, and the current agreement is that peak warmth occurred at different times for different regions, hence the event was not uniform across the globe.

duced activity of the Sun. The Little Ice Age is another important ruler. Interestingly, we know that it had severe consequences in different areas of the planet. There are records of bad harvests and famines in northern and central Europe, Japan and the Mississippi valley, and of a reduction in North Atlantic cod fishing[9]. Yet, the temperature change observed during the Little Ice Age is quite small compared to what we witnessed in the last 50 years. These considerations, grounded on data, confirmed again my growing opinion that it is a good idea today to pay attention to climate changes.

Ice cores are a precise way to measure temperatures. They are part of the data set I just discussed, but they can go much further back in time, so by looking at these data we can venture into the more distant past of our global temperature. The way scientists can obtain these temperature estimates relies on the quantification of the ratio between heavier and lighter hydrogen and oxygen isotopes that form the water molecules in the ice. An isotope is a version of an atom that is identical in terms of chemical reactions, but has a nucleus with a different weight. Intuitively, the principle behind this measurements is that heavier isotopes precipitate faster and need more energy to evaporate. Hence, the relative amounts of heavier and lighter isotopes. This justifies a correlation[10] of isotope ratios with temperature. The

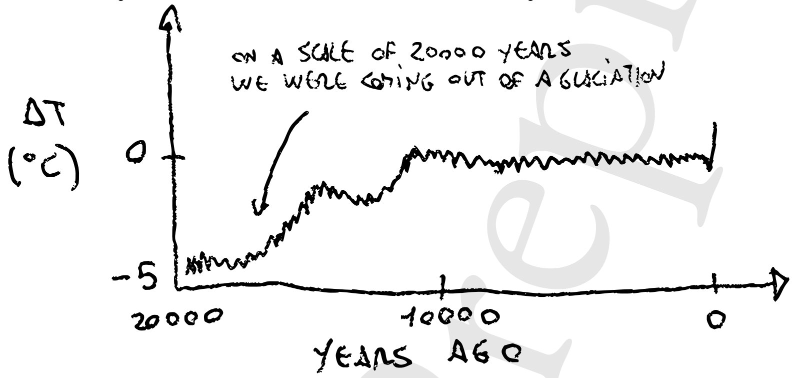
[9] I did not check myself the significance of these findings. Let us take them as "anecdotal evidence" that the global temperature change during the Little Ice Age had an impact. The question, however, may be subject to debate.

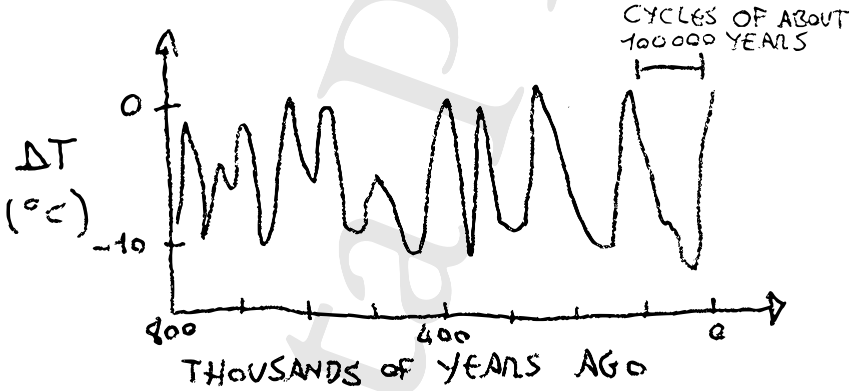
[10] We will define more rigorously the concept of correlation in chapter III.

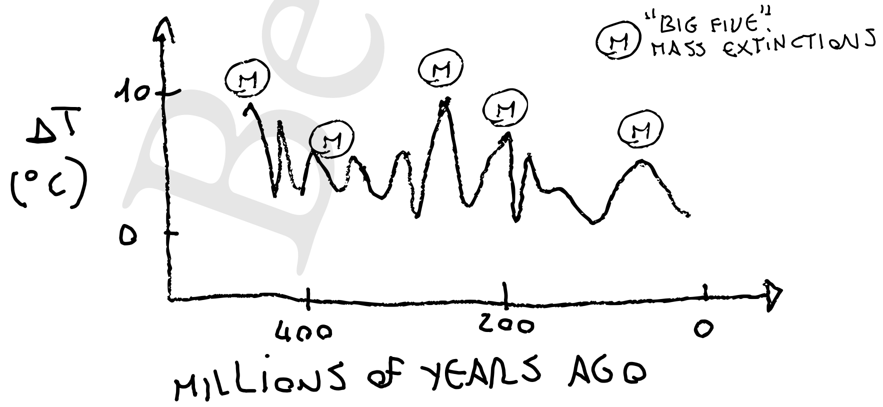
question arises of whether this method is precise. As a newbie, I initially thought that this method could be sloppy, but it is not. In order to check, I looked a study where they calibrate the measurement using yearly seasonal temperature variation (the exact 1997 study by van Ommen and Morgan where I found these data is cited in the "Sources" section at the end of this chapter). This study reports a where in the x axis there is a range of measured temperature (at a given time in the year) and the y axis reports the isotope ratios. The data cluster very tightly on a straight line, meaning that the isotope ratio variation for a given temperature is very small. A 1 percent increase of oxygen isotope ratio corresponds to a change of about 1.8 degrees Celsius. From the plot, the uncertainty appears to be between 1 and 2 degrees. However, the instrumental uncertainty (evaluating the isotope ratio) is very small, and most of the errors are due to relating the ice core layer to a specific period. Hence, measurements could be much more precise on time scales of many years. As I will discuss, ice cores also give us information on the greenhouse gas content of the Earth's atmosphere in the past, just because we can find those gases dissolved in the frozen water.

Amazingly, we can obtain ice core information on temperatures from up to 800000 years ago. Beyond that date it is still possible to have access to temperatures from geological sources, for example looking at the chemical composition of sediments. These records are obviously less precise in terms of dating, but they can still swiftly measure temperatures back to hundreds of millions of years ago, which is tremendously impressive to me. A big warning is that, as we will see, in general data look different on different time scales, just because the time resolution is different. In practice, we can define the time resolution of a time series (or time scale) as the time interval between two subsequent data points, for example two consecutive layers in an ice core. On some time scale, a time series (for example, temperature versus time) follows some trend or fluctuation, or varies with some periodicity, but all this may cancel out if we go to a larger time scale. For example, if we sample the mean temperature in Europe every month, we see that it oscillates because of seasons, but if we take a time point every January each year, or we average the temperature of all the months each year, we will lose this oscillation. Equally, if every first Tuesday of each month the temperature would halve, but just for one day, we would not see it in the monthly records if we average or we measure on any other day. We can think of a change in time scale as an average, where each time we go to a larger time scale the "fast" changes cancel out because of the averaging, and a clear trend might emerge (but obviously we will have fewer data points). In fact, ice core layers contain averages of isotope levels from snow deposits of many years, and it's very reasonable to think of the measurements as averages, or totals, from all those years. The same will happen if we compare recent data on a scale of one year to old data on scales of thousands to millions of years. At each change of scale, we are looking at averages, and the only truly fair comparisons can be made between data taken at the same time resolution.

WARPING BACK TO THESE DISTANT TIMES, with these warnings in mind, we can discover surprisingly dramatic changes in our planet's temperature. The following sketched plots take us on a breath-taking ride to three different scales of the Earth's past, 20000 years, 800000 years and 450 million years.







The authors of these plots[11] made efforts to put all temperatures on the same scale. Clearly this task is difficult and the results are subject to uncertainty, but we will take their calibration for good and assume that the temperatures that we see on the different plots are comparable. The first plot sketched above shows that before the little ice age and the very recent radical increase connected in the industrial age, temperature has been rather stable since about twelve thousands years ago[12]. Remarkably, about twelve thousands years ago is also when we humans began transitioning to farming, after living as hunter gatherers for almost two hundred thousand years. This transition was the key to a set of dramatic changes in our social structure and living standards that made it possible for human populations to start a phase of exponential growth, and eventually build the technological society we have now. Are you thinking that such a remarkably stable climate might have played a role in the dramatic turn in human history during this period? That's also what I thought when I saw this plot for the first time. And, of course, we are not the first, a scientific article by Feynman and Ruzmaikin formulated this hypothesis in 2007 (see Sources section).

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[11] See the "Sources" section for details.

[12] Note that this statement simplifies the extent of natural climate swings in this period. In view of our goal of extracting the most important information from the data, and in view of the recent global temperature changes, we can consider it to be valid, but it should be seen as a strong approximation and of course we can very well imagine that there are a lot of relevant things that happened climate-wise during twelve thousands years.

The second plot shows that on a scale of hundreds of thousands of years global temperatures underwent dramatic oscillatory cycles, with changes that reached an amplitude of up to 10^{o}C, toward colder temperatures than today. These cycles and (which also change their periodicity going further in the past) were proposed to depend on changes in the Earth's orbit around the Sun, affecting the irradiated heat. They are called Milanković cycles, after the astronomer that first hypothesized their existence in the 1920s. While the link between Milanković cycles and ice ages is not debated today, there are some aspects that are still open.

Milanković believed that variation of the summer exposure in northern high latitudes had the strongest effect, and based on this he deduced a 41 ky (kilo years, 41000 years) cycle. This is observed in older data, from 1 to 3 million years ago (though not visible in my sketches). However, as we see from my sketched plots, the last 800 ky show a clear period of about 100 ky (which matches the eccentricity cycle). Various explanations for this discrepancy have been proposed, but this and other discrepancies indicate that the links between global climate and orbital variations of our planet are complex and not controlled directly by exposure (in the sense that many other factors may play a role). We will return to this question in chapter VI.

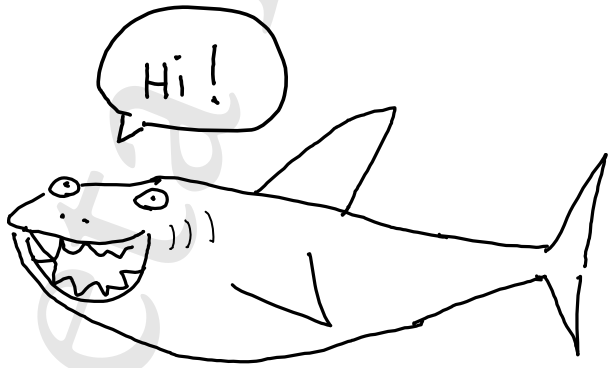
Humans (homo sapiens) have been around for two to three hundred thousand years, so, as a species, we have experienced at least two, maybe three cycles of these dramatic climate changes. This proves that our species can survive radical climate changes. But of course we achieved that feat as small sparse population of ape-like hunter gatherers. Sustaining a large, complex and costly technological civilization such as the one we have today through such changes appears to me like a daunting task. What happened to humans in the other "peak" periods when temperatures where reasonably warm? The most recent of these peaks corresponds more or less to the period when we moved out of Africa to migrate to Europe and Asia. Specifically, there is evidence that 120000 to 90000 years ago, when the climate in Africa may have become too hot, humans started to migrate to the Fertile Crescent and Arabia, although the primary human migration is dated around fifty thousand years ago, in a cold and dry period (suggesting that other local factors played a role in the migration).

The other important consideration that we can make looking at this second plot is that the temperatures at which we thrived as a species are hot for the Earth's standards, within this time range spanning hundreds of thousands of years. However, looking at the third plot, which spans millions of years, we get a completely different impression. Today's global climate appears much colder considering the averages on a scale of hundreds of millions of years in the past. Strikingly, five of the temperature peaks correspond to huge mass extinction events known as "the big five", which were first detected from dramatic reductions in the diversity of the fossil record of key sets of species. Dinosaurs lived between the last two mass extinctions, and a widely accepted theory[13] is that the last mass extinction, which wiped them away, was due to an asteroid impact that took place sixty-six million years ago in Yucatan, Mexico. Scientists estimated what happened to temperature "shortly" after this event by looking at oxygen isotope composition of fish debris. Clearly, for times so far away in the past, the time resolution that can be achieved is limited, but what they found is quite interesting. After a short (a few decades) artificial winter caused by dust blocking sunlight, it seems that temperatures quickly rose by about 5^oC, and this situation lasted for about a hundred thousand years. This is believed to have occurred due to increased CO_2 emissions because the asteroid hit rocks that are rich in carbon, directly releasing it in the atmosphere, as well as triggering vast wildfires. This situation of fast CO_2 increase and fast temperature increase is probably the closest "natural experiment" that we can use as a term of comparison to the situation of the last two hundred years, where CO_2 emissions and temperatures are rising very rapidly. In most other occasions in the historical record these changes occurred very slowly. The following two chapters will deal with CO_2 emissions and their link with temperature changes.

Going back to the wider picture of million-year temperature changes, the visible correlation between temperature

[13] This hypothesis encountered criticism. A leading critic is Gerta Keller, who proposes that volcanism is a likely cause of a gradual extinction. We know about the asteroid impact and about many of its consequences, but establishing causality with a very long-term process of mass extinction is very difficult. More on causality in chapter III.

peaks and extinction events (which does not only involve the big five mass extinctions, but also smaller detectable mass extinction events) at these time scales triggers the idea that global warming is not good for biodiversity, which we will discuss a bit more later on. Curiously, sharks survived all five of these mass extinctions events. This diverse and adaptable species can live from deep oceans to shallow seas and even rivers, and they eat a wide variety of food, from plankton to fish, to mammals like seals. Consequently (if that is of any comfort), we can expect that sharks as a group are likely to survive future oceans changes, including the ones that are triggered by humans. Unfortunately, all this Darwinian fitness did not make sharks a more intelligent or sympathetic species (at least from our standpoint). Sharks remain relatively not fun to hang out with.



DATA SCIENCE TAKE-HOME MESSAGES. This chapter is centered on the concept of "time scale". In order to compare two quantities we need a "scale". You probably remember from high school that it is only possible to compare things that have the same unit of measure (we cannot compare 200 years with 400 meters), but even when we correctly compare things with the same units (in our case, temperature changes), we need a "scale". The scale can be different depending on the question we ask. We have used different terms of comparison: we used a comparison based on the discrepancy between data sets coming from different projects in order to get an estimate of uncertainty in the data, but all the rest was based on comparisons between typical changes on different time scales. For example, looking at our data we have used (symmetric) year-to-year variations to define a time scale and an amplitude for *fluctuations*, which we have distinguished from a *trend* followed by the data in the past 200 years. In data science, the analysis of quantities that vary in time is called time-series analysis. A time series is a sequence of data points collected over time intervals. We have seen that time-series data can report changes over different time scales, milliseconds, days, years, millions of years. And we also noticed that we can understand a change (a reduction) of time resolution as an average. We can say that time series data represent a mixture of variation patterns at different time scales, and we have analyzed the variation of the global temperature time series across different time scales. We found that, for global temperature, different patterns manifest at distinct time scales, as it is typical in many complex systems. For this reason, we cannot compare too stringently time series that "live" on different time scales. If we say that 200 million years ago the temperature was 5 degrees Celsius higher than in 2020, this statement is inaccurate, because we are comparing an average over many thousands of years with an average over a single year. Equally, if we want to compare the temperature change of the past 100 years with the ancient record, we can only do it with past steps of a 100 years (and in data sets that have sufficient resolution to sample data avery 100 years). Incidentally, doing this with a data set that has data every 100 years for the past million years, shows that the increasing trend of about 0.01 ^{o}C/y, is *only* found for the past 100 years. Additionally, we have learned that an average can be seen as an approximation of an underlying typical behavior, and the more data we have the bettwer the approximation (in probability thoery these statements translate into the law of large numbers, and the central limit theorem). Unless our data have systematic errors: in such case the systematic errors will add up and skew our averages with respect to the underlying value. Finally, we have learned the concept of linear extrapolation, if we isolate a region where our data look like a straight line, we can formulate a bare-bone simple testable prediction by assuming that this trend will continue for some time.

SOURCES. The global average temperature datasets 1850-2020 from NASA, NOAA, Berkeley Earth, and meteorological offices of the U.K. and Japan, as well as the temperature record of the last 2,000 years, which comes from the PAGES 2k consortium (Nat. Geosci. 12, 643-649, 2019) were downloaded from Wikipedia at the page "Global temperature record" (link), or directly from the sources: NASA GISS HadCRUT5, from the Met Office Hadley Centre (the web page at this link also provides useful information on how the data are collected and averaged, and how their uncertainty is evaluated), NCDC NOAA National Centers for Environmental Information, National Oceanic and Atmospheric Administration, Japan Meteorological Office, Berkeley Earth Database. Detailed scientific information on the precise estimates of global temperature uncertainties and the establishment of a trend beyond incertainties can be found on the scientific article by Folland et al. Geophys Res Lett 28, 13, 2001. This study used climate models to correct for biases due to the use of sea-surface temperature, and concluded that the change in global temperature between 1861 and 2000 was 0.61 degrees Celsius, with an uncertainty of 0.16 degrees. The uncertainties quoted in the text come from the most recent HadCRUT5 study, Morice et al. JGR Atmospheres 126, 3, 2021. A wide-audence article by Alan Buis describing the correction procedures and why they are necessary can be found on the ASK NASA website at this link. Information on climate models can be found on the article by Jeff Tollefson on Nature 580, 443-445 (2020) The 800 ky Ice core data was downloaded from the NOAA database (`https://www.ncdc.noaa.gov`). It is the EPICA Dome C Ice Core 800 kYr Deuterium Data and Temperature Estimates, and the original publication is Jouzel and coworkers, Science, 317, 5839, 793-797 (2007). The calibration plot for the isotope ratio discussed in the text comes from Figure 3 in the study van Ommen and Morgan, J. Geophys. Res., 102(D8), 9351-9357 (1997). The million-year time scale ice core plot is hand drawn from Song and coworkers Nat Commun 12, 4694 (2021). The hypothesis on a role of climate stability for the development of agricultural societies originally comes from J. Feynman and A. Ruzmaikin, Climatic Change 84, 295-311 (2007). The climate context of the out-of-Africa human migration is investigated by Jessica Tierney and coworkers Geology 45 (11): 1023-1026 (2017). The information on the consequences of the Yucatan meteorite impact comes from MacLeod and coworkers, Science 360 (6396) 1467-1469 (2018), and the cover article by Cristophe Lécuyer on the same issue.

Chapter II

Atmospheric greenhouse gases are rising

A greenhouse gas is a gas that is able to absorb heat from the Earth's surface and reflect it back to the Earth's surface. The news tell us every day that release of carbon in the atmosphere (our "carbon emissions") is the main contributor to temperature change, and naturally I needed to look at the data to obtain some back-of-the envelope estimate of how much gas we need to generate a certain amount of "atmospheric heat". However, before entering that problem, it was necessary for me to get an idea of which amounts we are talking about when we deal with global greenhouse gases; for carbon dioxide, but also for the other relevant greenhouse gases. The present chapter is an account of what I found.

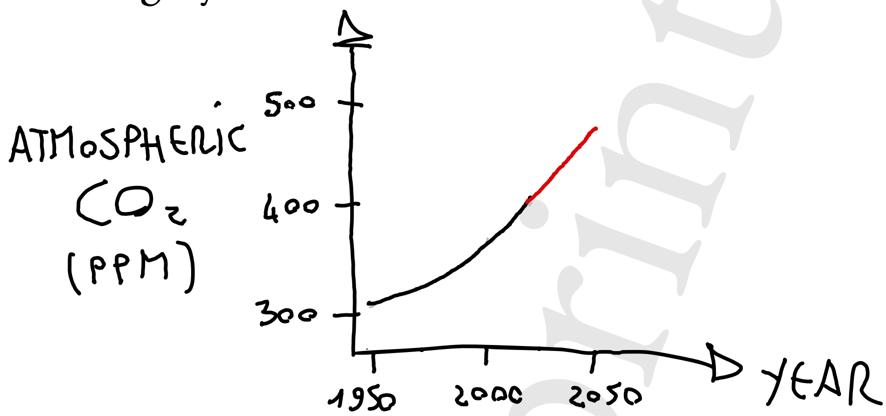
Water vapor is a major greenhouse gas, but it does not stay around very long, because increasing amounts of water vapor lead to condensation to form liquid water (clouds, rain). Because of condensation, temperature change from water vapor is mitigated, and, as it turns out, we do not need to worry too much about water vapor for climate change. Besides, water vapor emissions are largely unrelated to human activity. However, in presence of global warming, water vapor plays an important indirect role, since warmer air holds more vapor and condenses less, hence, it absorbs more heat, triggering a self-reinforcing phenomenon, often called "positive feedback loop", towards increasing temperature. Positive feedback can greatly enhance the effect of a small perturbation and, in brief, makes water vapor enhance the greenhouse effect[1]. Incidentally, this kind of positive feedback can be very dangerous because it also threatens the stability of a system. I will try to address it further in a later stage (Chapter VI).

In any case, the greenhouse gases that we need to worry about the most are the ones that do not condense (and hence can stay around very long), and that are linked to industrial-age human activities. Carbon dioxide (CO_2 in chemical djargon, we will use both terms hereon) is the first of the list. It is, at the concentrations we are considering, a

[1]More precisely, this holds until more vapor leads to more clouds, which can lead to more reflected sun light.

non-poisonous inert gas, formed by respiration of all living systems. For example when we breathe we combine oxygen and glucose and emit CO_2, and the equivalent process exists in the metabolism of microbes. A different microbial process, fermentation, breaks down glucose in absence of oxygen, and can also produce CO_2, for example CO_2 is produced when yeast populations ferment to make wine, beer or bread. Conversely, photosynthesis from plants or plankton takes up CO_2 and water, using light to make glucose. Thus, there is a sort of "natural balance" between emissions and uptake from photosynthetic and non-photosynthetic life forms. Human industrial-age CO_2 emissions perturb this balance, and come mainly from two activities: the burning of fossil fuels and the production of cement. Natural phenomena such as volcanic eruptions may also release a lot of CO_2.

Similarly to what we did for temperature, we can look at the evolution of global CO_2 levels in recent years (for example since the 1950s), where we have direct and precise measurements. We should also bear in mind that how much CO_2 we find in the atmosphere differs from how much we emit. Instead, atmospheric CO_2 is the result of the complex balance between all the "sources" and "sinks" of global atmospheric carbon. Considering all these processes is very complicated (see below), but if we imagine for the time being that all that matters for the greenhouse effect is how much carbon is in the atmosphere, we can consider directly atmospheric CO_2 levels, and worry later about where they come from (and how to reduce them). Here's my sketch of what the data for atmospheric CO_2 since 1950 until recent times roughly look like,



The y axis units of this plot are in "parts per million" (PPM). This quantity tells us how many particles of carbon dioxide there are in one million particles of air (which is made of different substances, including CO_2, nitrogen and oxygen). The sketched data can be considered as global averages by location and yearly fluctuations (as we did for temperatures, and for the same reasons), in order to visualize a gross trend. The plot, whose slope increases steadily since the 1950s, looks more markedly bent or "banana shaped" than we found for temperature in the previous chapter, but more or less follows a similar trend. In the 1950s and 1960s the increase still looked mild (for temperatures it was almost flat). In the late 1960s, the increase of atmospheric carbon dioxide was around 1 PPM/year. Over the next decades, the yearly increment has increased almost invariably, reaching 2.5 PPM/year (and more) in the 2010s. The red line in my sketch is a linear extrapolation of the trend from 2008 to 2018 (about 2.3 PPM/year) to future years, up to 2050. According to this prediction, we would reach about 480 PPM of global atmospheric carbon in 2050. The prediction is conservative, because it assumes that every year we will increase atmospheric CO_2 by the same amount, but so far we've done increasingly worse, adding more PPM than the previous year to our atmospheric CO_2 almost every year since the 1950s. As for temperature, the trend is much beyond the uncertainty in the data, and in this case the seasonal variation of global CO_2 levels is actually small (a few PPM). We will use the historical record to "calibrate" these values below, but before we do that, I need to discuss the other greenhouse gases that we should not ignore.

Our plot shows us that we currently have slightly more than 400 PPM of CO_2 in our atmosphere. But how much is that? Can we put our hands on this quantity? Suppose we had the technology to take up all the carbon from the atmosphere and compress it into a diamond. How big would it be (if we know, we can also plan to place it somewhere nice)? Diamond is one of the densest possible forms of carbon; its density is around 3500 kg/m^3, about half that of steel. We can go on the IPCC web site and find out that 1 PPM of CO_2 corresponds to about 2.12 Gt of carbon (Gt means gigatonnes, or billion tonnes, which is 10^{12} kg, see below for a more direct estimate)

So, we are dealing with 400 PPM times 2.12 Gt, so about 850 Gt carbon, which is 8.5 10^{14} kg. Divided by 3500 kg/m^3 and then by 10^9 m^3/km^3, it gives us a block of diamond of almost 250 km^3. This is a cube whose side is a bit larger than 6 km, or a sphere whose diameter is almost 8 km. This purediamond ornament would not fit in the Tower of London. On the other hand, it would be large enough to embellish our highest mountain ranges, such as the Himalaya, the Andes, or the Alps. Imagine watching the sun set behind a diamond mountain, with reflections of a million colors... how romantic![2]

This was our first example of estimate in the style of Enrico Fermi. As I mentioned in the preface, the physicist Enrico Fermi was famous for his amazing ability in approximate problem-solving, using (seemingly) little or no actual data (a proverbial example where Fermi operated in this way is his estimate of the power of the atomic bomb that exploded in the Trinity test, based on the movement of a few pieces of paper that he dropped from his hand as the shock wave due to the explosion passed through). This has given the name “Fermi problem” or “Fermi question” to situations of this kind. Fermi himself used exercises of this type for teaching purposes in his courses. In a Fermi problem, the goal is not to arrive at an "exact" answer, but to obtain a "good enough" answer, as well as (more importantly) some insight on the structure of the question under exam.

In our Fermi estimate, we used the value of 2.12 Gt per PPM of carbon by getting it from the internet. This could be considered cheating by some (or “standing on the shoulders of giants” by others). Instead, we can try to get it by another Fermi estimate. If we want to do that estimate by

[2] But don’t stay out too late, nights can get chilly on this planet without any carbon in our atmosphere! (see the next chapter.)

ourselves we need to know the volume of the atmosphere, and how many particles of air per unit volume there are in the atmosphere (the number density of air), then by knowing the mass of carbon we can get to our goal. To determine the volume of the atmosphere, we can use the fact that the Earth has a radius of approximately 6300 kilometers and the atmosphere extends up to an altitude of about 40 kilometers above the surface. This gives a volume of the atmosphere of about $20\ 10^{18}$ m^3. The number density of air is not simple to estimate without some physics. It can be estimated roughly by the "ideal gas" law, which relates it to temperature and pressure. At room temperature and pressure (25 ^{o}C and a pressure of 10^5 Pa), this law gives about $2.5\ 10^{25}$ molecules/m^3. In practice, air density changes with height (as pressure and temperature do) and the atmosphere does not have a very well defined height as it does in our estimate. In order to compensate for this effect we will divide the volume by a factor of 5[3]. So there are about $2.5\ 10^{25}$ times $4\ 10^{18}$, which 10^{44} particles of air, and 1 PPM of that is obtained by dividing by one million, or 10^6. This is 10^{38} carbon atoms. The mass of carbon in molecular mass units

[3] This is adjusted, because getting the number of air molecules in the atmosphere right is a bit the weak point of this not-so-simple estimate. There is another way to get to the same number. The total mass of the Earth's atmosphere is about $5\ 10^{21}$ grams. If we take air to be a mixture of about four molecules of nitrogen to one of oxygen, based on their molecular weight, the mass of 1 mole of air will be about 29 grams. One mole of any substance contains by definition about $6\ 10^{23}$, an Avogadro number, of molecules. So there are about 10^{44} molecules in the Earth's atmosphere. Based on this estimate we can also claim that every time we take a breath of air, we inhale some of the atoms breathed by Leonardo da Vinci or Cleopatra

(u) is 12, because it's made of six protons and six neutrons (and its electrons are very light). A molecular mass unit is the mass of a proton, about $1.7\,10^{-27}$ kg. If we multiply this mass by 12 to estimate the mass of a carbon atom, and then by 10^{38} carbon atoms in one PPM, we get about 2 Gt, which is the quantity that we were looking for. If we hadn't adjusted the total number of air molecules we'd be a factor of 5 off. Not too bad, but not sufficiently precise for our scopes. So, getting this directly was a bit laborious (sometimes Googling is easier!) but see the "Physics track" section at the end of this chapter to justify this point with a bit of physics insight.

We said that CO_2 is the main contributor to greenhouse gases. However, other greenhouse gases produced by farming and industry significantly affect the Earth's atmospheric global temperatures. We can break it down into three main contributors: nitrous oxide, methane, and a whole class of fluorinated gases. All these gases are quantitatively much less than CO_2, but unfortunately their potential for causing global warming is much higher, and for this reason we cannot neglect their contribution to climate change. Indeed, different greenhouse gases radiate heat differently. To compare the global warming caused by different gases, scientists have introduced the notion of "global warming potential" (GWP). The GWP is defined as the heat absorbed by any greenhouse gas over a certain period of time, divided by the heat absorbed by the same mass of carbon dioxide over the same period. By definition, GWP is 1 for CO_2. Additionally the GWP of another substance depends on the number of years over which the ratio is calculated. Often the time span is indicated by a subscript, as in GWP_{100} for the GWP for 100 years. This hundred-year period is considered frequently by the practitioners, as this is the time frame over which we need to contain the consequences of industrial-age emissions, so sometimes when the subscript is absent it is implied that we mean a hundred years.

Using the GWP we can compute a "carbon dioxide equivalent" (CO_2e) as the amount of CO_2 which would cause the same greenhouse effect as a given combination of greenhouse gases (over a certain time span), multiplying the amount of each substance in the atmosphere by its GWP and summing over all the substances. Over a hundred years, the GWP for nitrous oxide is about 250-300[4], that of methane is about 25-30[5], and fluorinated gases range from one thousand to tens of thousands. Fortunately they are quite rare.

Fluorinated greenhouse gases are used today as replacements to so-called ozone depleting substances, which were major contributors to stratospheric ozone depletion (as well as to the greenhouse effect), hence since the 1980s they have been first banned and gradually replaced by alternatives (in refrigerants, as aerosol propellers and as foam blowing

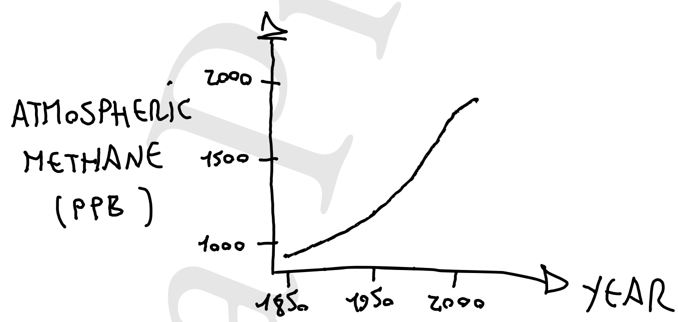
[4] The estimated value in the IPCC sixth assessment report is 273, we will use 300 in the following.

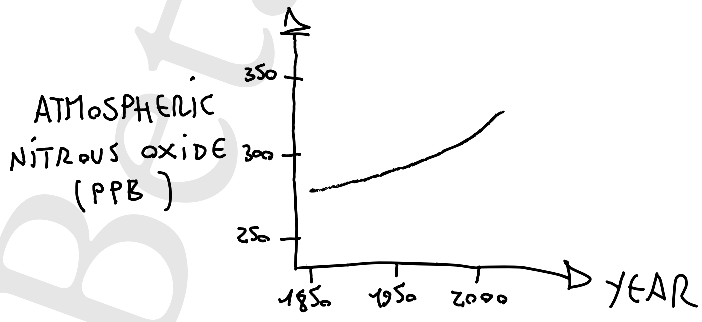
[5] We will use 30 in the following.

agents). Fluorinated greenhouse gases do not damage the atmospheric ozone layer. However, most of them are powerful greenhouse gases, with a very high warming potential (which varies from gas to gas). Fortunately, their concentration is low in the atmosphere. Hence, despite of the huge GWP of fluorinated gases, for the moment nitrous oxide and methane appear to be more important today as contributors to the greenhouse effect. However, we should note that the cumulative warming effect of fluorinated gases and ozone-depleting substances (which are different in terms of effect on the ozone layer, but can be considered as the same family in terms of theor behavior as greenhouse gases) currently still surpasses that of nitrous oxide.

Nitrous oxide is mainly (about 70%) linked to the growing use of nitrogen fertilizers (and is also a depletant of the stratospheric ozone layer). Excess fertilisers based on nitrogen end up into emissions of nitrous oxide essentially because crops cannot use them all. Careful use of fertiliser when crops need it would reduce these emissions. Turning to methane, the first contributor (about 40%) to its emissions is animal agriculture, through livestock, mostly cows and sheep, which produce methane in their digestive processes. There are roughly 1.5 billion cows and 1 billion sheep around the world, and reducing this number would reduce overall methane emissions. Rice production is also an important contributor to agricultural methane emissions, because in flooded fields the water blocks oxygen from penetrating the soil, creating a habitat for bacteria that emit methane. Farming methods that reduce or eliminate flooding could reduce methane. Unintentional leaks of methane from fracking and oil and gas extraction and transportation are the second largest contributor (about 30%) to methane emissions. The third largest contributor (about 15%) is waste landfills, from decomposition of organic materials. Chapter V will attempt a more systematic approach to the numbers of global greenhouse gas emissions, for the moment let's try to understand the global trend of the main greenhouse gases in our atmosphere.

The hand drawn plots below give us the global amounts of atmospheric nitrous oxide and methane since 1850.



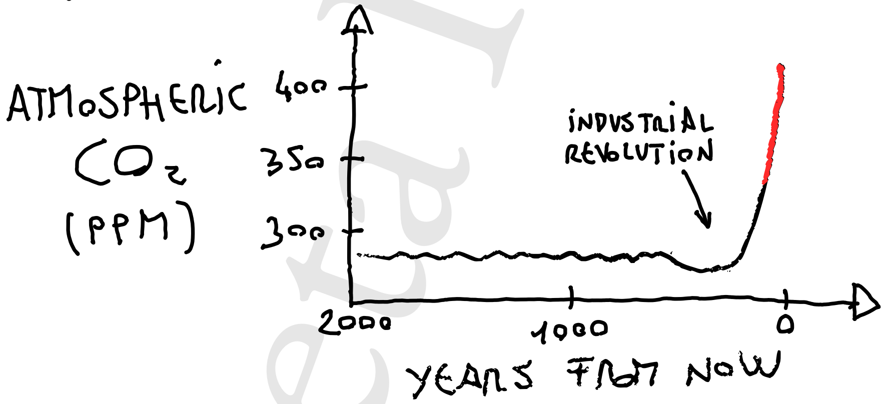


The y axis units are now in parts per billion (PPB). As for CO_2, the plots show clearly that both gases have been increasing steadily over the period we are considering, with methane roughly doubling its amount. There are roughly

2000 PPB of atmospheric methane today, which multiplied by its GWP_{100} give roughly 60 PPM of CO_2e from methane, not negligible compared to the 370 PPM of CO_2. The 325 PPB of nitrous oxide give an ever larger contribution of about 100 PPM CO_2e over a hundred years. Perhaps a better way to compare is by using the concentration differences from pre-industrial levels and the current ones. By this metric, we get a difference of about 150 PPM for CO_2, about 1 PPM for methane and about 0.05 PPM for nitrous oxide. If we now multiply by the GWP of each gas, methane gives about 1/5 of the contribution of CO_2, and nitrous oxide gives about 1/10 of the contribution of CO_2.

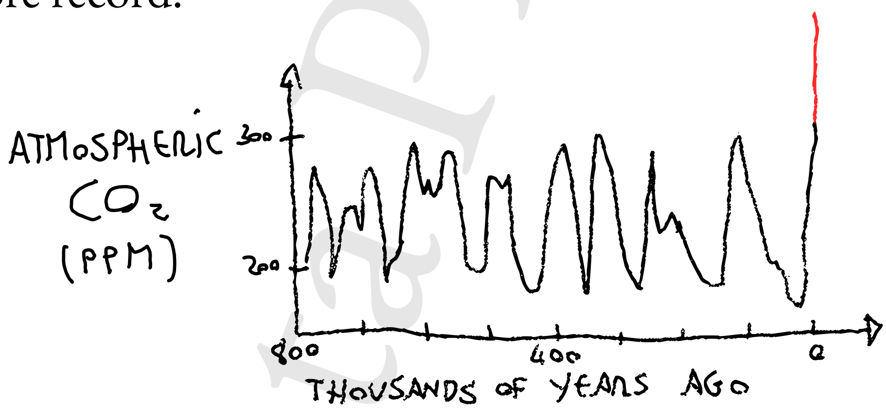
Going back to CO_2, we can look, as we did for global temperatures, at the record of emissions in the more distant and very distant past, to gain some reference that we can use to frame the recent record. As for global temperature, we can use this information to put a scale on the proportion of the changes that we see today. As for temperature, ice-core data give us a fairly precise idea of CO_2 amounts, and in this case the measurement is direct, as it amounts to measuring how much CO_2 is found in a layer of ice corresponding to some time in the past. To be precise, ice cores measure the CO_2 dissolved in ocean waters, since carbon dioxide dissolves in water. Assuming the global amount of dissolved carbon is in chemical equilibrium, scientists can deduce the atmospheric amounts. Roughly, there is about fifty times as much CO_2 dissolved in the oceans as is found in the atmosphere. Dissolved carbon is present in all water basins. The reaction involves three main constituents, free CO_2, the bicarbonate ion and the carbonate ion. Most of the dissolved CO_2 persists as CO_2 molecules, but the dissolved ions are sufficient to induce a decrease in the pH of the oceans, the so-called "ocean acidification". At lower pH, carbonate becomes undersaturated, which means that the equilibrium point between ions dissolved in seawater and attached to crystal (precipitated) is shifted in a way that there are more ions around. In this condition, living beings using shells made of calcium carbonate become vulnerable because their shells dissolve more easily in this acidic environment.

The plot that I sketched below describes the estimated trend in atmospheric CO_2 from ice core data for the past 2000 years,



The y axis is in PPM as above, and the last part of the plot is highlighted, as it corresponds to the roughly two-hundred years period just discussed. As we found for global temperatures, the plot tells us that CO_2 levels have been remarkably constant until the sixteenth century or so, and after that only slightly decreased. Instead, during the industrial revolution atmospheric CO_2 started to increase quickly and radically, and it never stopped since. This plot helps us addressing the important question of where we should aim for atmospheric carbon levels, by an educated guess. Arguably, our target is the pre-industrial level, slightly lower than 300 PPM (around 280 PPM), since we know from thousands of years of human history that it creates temperature ranges that are comfortable enough for the human body to allow (more primitive forms of) our civilization to thrive. Possibly we can get by with slightly more. This reasoning holds as long as we do not drive the climate system irreversibly too far out of its current state (see Chapter VI).

Looking at a more remote past, the following sketch shows a plot of 800000 years of CO_2 levels from the ice-core record.



Once again, the recent record is the last highlighted part of the drawing. This part would barely be visible, but I highlighted it in the drawing, in order to show that the amounts of atmospheric CO_2 that we see today are completely out of range compared to what happened for an enormously long time. This 800000-year record shows multiple variations in CO_2 levels where the atmospheric values of this gas changed by more than 100 PPM, but they stayed in all cases lower than 300 PPM. We were already at a peak when the industrial revolution started. This is at least what we see at the time resolution that is accessible with these data. The time resolution of ice core data depends on the thickness of the layers that can be sampled reliably. For these data, the scientific literature estimates it to vary with time between 200 and 1000 years [6], meaning that we can regard each data point as an average over such a period. In other words, if at some time in the distant past there had been a peak of atmospheric CO_2 that lasted only say 400 years (we can hope that the current peak will not last longer than this), it might not show up in this plot, or be only slightly noticeable. If we remember the equivalent plot for temperature in the previous chapter, we can observe that the oscillations of temperature and CO_2 are remarkably similar. We can conclude that on a time scale of about 10 ky if CO_2 goes up, so does global temperature, and conversely, if CO_2 goes down, global temperature decreases. In brief, these 800000 year-old data seem to be telling us the same story as the latest news: there is a strict correspondence between temperature and atmospheric CO_2. This subject will be the center of the next chapter.

Beyond ice core data, and further in the past, it is possible to estimate (more roughly) CO_2 levels from geochemical proxies in the fossil record, to get information from hundreds of millions years ago. For example, scientists have

[6] See for example Masson-Delmotte and coworkers, Quaternary Science Reviews 29 (2010) 113-128.

looked at stomata (pores) in leaves of fossil plants, and related their amounts to carbon levels. I collected some of these plots, and took a closer look. These more ancient data suggest that the closest time when CO_2 in the atmosphere may have reached the abundance it has today was 20 million years ago. Once again we should bear in mind that the comparison is not completely fair due to the (much poorer) time resolution of these data. Possible transient increases that lasted even thousands of years, would be completely erased out by the averaging in these data. But the geological data still give us a good idea of how exceptional today's atmosphere is in terms of CO_2 content (even if it is compared to these mean values). They also tell us that multiple times tens to hundreds of millions years back, the Earth has seen average CO_2 levels that compare to the levels that we see today, and these increases were correlated with increases in the global temperature.

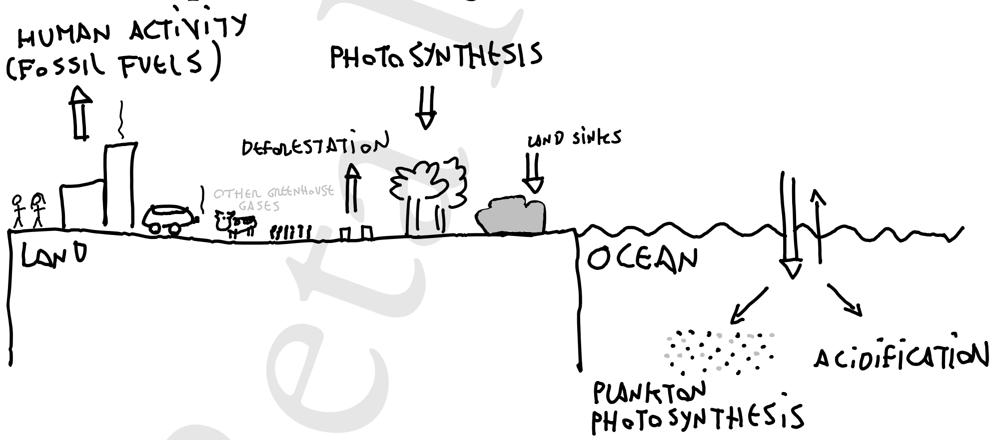
It is mind-blowing to think how dramatically different the climate of our planet has been over its history, but on a more pragmatic level we found out how the question of time resolution is crucial in these data. Let's go over it again. Today's record where CO_2 levels are measured every day with high precision contains day-to-day and seasonal variation. If we plot these data, we can see sinusoidal oscillations during the year, with an amplitude of a few PPM, due to the collective photosynthesis of all plants and plankton (which is seasonal). If we average the yearly data or take one point every year we get a somewhat smoother trend line. Taking data with lower time resolution is equivalent to performing these averages on longer and longer time intervals. Ice core data have varying time resolution, depending on how far in the past they go. Recent data can be compared to the year-to-year averages, but going back by hundreds of thousands years ago each data point (in the current data sets) corresponds to a time step of 4-500 years. In geological records the effective time resolution can become a million years or more. Importantly, the effective time resolution of a data set in the form of time series can also be limited by the precision of the measurements. For example, suppose we can take geological CO_2 data reliably over "slices" of 100000 years and over this period the quantity we measure varies by say 1 PPM. If the resolution of our measurement technique were 30 PPM we'd have to pile up many slices before we are sure we see a change that is above our experimental uncertainty.

To make the issue of time resolution more concrete, we can try to address a question that borders with science fiction. Suppose another complex life form lived on Earth many millions of years ago, and started a civilization similar to ours, developing fossil fuel engines and heating systems, building cities and cars, and computers and/or other fancy gadgets. Let's assume that this civilization lasted a few hundreds to a few thousands years. Would we be able to tell? This crazy question occurred to me looking at these data, and curiously it turns out that somebody had actually considered it seriously in the recent past. The study by Gavin

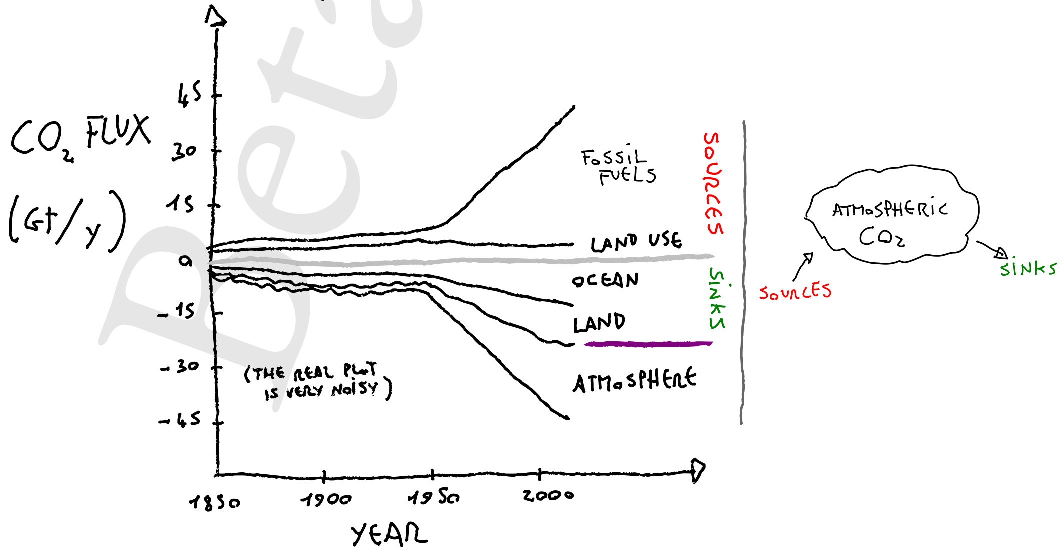
Schmidt Adam Frank is cited in the Sources section at the end of this chapter. The answer they found is intriguing. As could be expected, buildings and cities would be long gone, mainly because of tectonic movements, and it would be extremely unlikely to find artifacts and fossils as, on average, we find one single fossil for every 10000 years of Earth time. One could hope to be able to see some chemical trace, such as residuals in sediments (plastics, for example), and it seems that CO_2 levels from fossil fuels could probably be our best hope to gather such evidence. But the current time resolution would not really allow us to detect short-lived peaks of 300 years, such as the one we are currently producing (in the best of hypotheses). As it turns out, it would be easy to miss carbon emissions from an industrial civilization that lasted 100000 years, 500 times longer than our industrial age so far. Hence, it seems we cannot really exclude that we are not the first technological society on this planet that has faced (and caused) the climate problems we experience today.

There is a "carbon balance", similarly to what happens in a financial budget, and it is a crucial point that we need to discuss in this chapter. Understanding why there is a budget is necessary to get a simplified idea of how much of the CO_2 we emit ends up staying in the atmosphere (affecting climate), and where the rest ends up. We have already mentioned that atmospheric CO_2 is the result of the complex balance between different "sources" and "sinks" of carbon, that plants and some microorganisms carry out photosynthesis, and that CO_2

dissolves in oceans. In this sources/sinks metaphor we can think of carbon fluxes as a plumber thinks of water loss in our house's hydraulic system. In the budget metaphor we can think of it as an accountant collection and tracking of a firm's incomes and expenditures. If we join the two metaphors we probably get the idea that plumbers and accountants are more similar than they actually are in real life, but I guess you get the message. A carbon sink is any identifiable process that absorbs more carbon from the atmosphere than it releases, and vice versa a carbon source has a (net) positive emission towards atmospheric CO_2. The sum of all the fluxes of carbon in and out of the atmosphere (and other earth compartments) is called the carbon cycle. The simplified sketch below illustrates some of the main processes acting as sources and sinks,



An important additional piece of information (which we already touched upon talking about GWP) is that atmospheric carbon, left alone, does not decay or degrade. Methane, by contrast, is removed from the atmosphere by chemical reactions on time scales of tens of years. Thus, although methane is a potent greenhouse gas, its effect is short-lived. Equally, nitrous oxide is destroyed in the stratosphere and removed from the atmosphere, persisting for around a hundred years. Without these spontaneous decay processes, atmospheric carbon levels mainly depend on their emission and absorption by the sources and sinks. Roughly, 70% of atmospheric CO_2 may dissolve into the ocean over a period of 200 years. The other removal processes (for example rock formation) require several hundreds of thousands of years. This means that once in the atmosphere, carbon dioxide can stay there practically indefinitely unless it is removed by sinks. The balance of positive sources and sinks is called the carbon budget. Naturally, if we want to normalize our current situation we can act on both sources and sinks: emit less and help the land and ocean to sequester more carbon. To get a rough impression of the role of sources and sinks, here's a sketched plot of carbon "flux" (in billion tonnes, or gigatonnes, Gt, of CO_2 year) estimated by the Global Carbon Project,



Reasoning on this plot, when I first saw it, was very useful to me. The plot shows two main sources of CO_2. The greater one, rising steadily since the 1950s, is the carbon released from the burning of fossil fuels. The other, which stayed more constant over the years (about 5 Gt/y), is the release of carbon from land use, such as through deforestation (degraded forest emits CO_2, from all the carbon that was stored in the plants), land clearing for agriculture, and degradation of soils. The negative flux part of the plot shows that there are two main carbon sinks, ocean and land. The land sequesters carbon mainly through forests. Plants capture carbon dioxide from the atmosphere for photosynthesis, and some of this carbon is eventually transferred to soil as plants decompose. The ocean captures CO_2 because CO_2 dissolves in it. Phytoplankton in the ocean can absorb some by photosynthesis, and this amount can be stored in oceanic sediments because these organisms tend to sink when they die. It is important to note that the sinks are dynamic. The plot shows that since the 1950s the ocean and land sink have increased their negative fluxes adapting to the increased emissions, but that at the same time atmospheric carbon has also increased steadily. From these data, ocean and land seem to have been able to accommodate (roughly) about one half of the annual budget so far.

However, we also know that this happened at the cost of an increased ocean acidification, which has heavy consequences on ocean ecosystems. At this rate, the estimates tell us that oceanic carbon may saturate in about 100 years, after which the ocean will loose its capacity to dissolve more carbon. Sometimes if you have a clogged drain or sewage system in your home it is not easy to notice it at the beginning; water encounters a partial obstruction, and redirects itself through your plumbing system, which is a complex branched network of pipes, causing initially small reactions. You may experience a slow drain in one or multiple areas of the home, or a funky smell from your sink, and underestimate the situation. But after some point the inner workings of your piping system become saturated, and they are not able anymore to tolerate the perturbation. You flush the toilet and brackish water backs up in your sink, or your yard floods with water from your sewage system. Ideally you want to act before the only solution is hiring a plumber with a honey wagon to pump out your discharge.

With the data collected so far, we can try a back-of-the envelope estimate of how our emissions are linked to the increase of atmospheric carbon. The last plot tells us that today (in 2019-2021), we emit about 35 Gt of CO_2 from fossil fuels, which becomes about 40 Gt including land use. We need to convert this in PPM, which is not completely trivial (we need to know the volume of the atmosphere, its density, and the mass of carbon or CO_2), but a bit of Googling shows that many people have already done this calculation for me. We already mentioned that 1 PPM of CO_2 corresponds to about 2.12 Gt of carbon. Now, since CO_2 is not made only of carbon (there are also two oxygens), we need to further convert the mass of carbon into the mass of CO_2. Chemistry tells us that 1 Gt of carbon is about 3.67 Gt of CO_2. The way to work this out is simple.

CO_2 has a carbon, molecular weight 12 u (six protons and six neutrons, as we said above), and two oxygens, molecular weight 16 u each. So the mass ratio of CO_2 to carbon is just $(16+16+12)/12 \simeq 3.67$. Hence, 1 PPM of CO_2 amounts in about 7.7, say 8 Gt of CO_2. Or vice versa, 1 Gt of emitted CO_2 will make about 1/8 PPM (one eighth of a PPM). However, we know that only a percentage of what we emit stays in the atmosphere. From the plot, this looks like about 40%, probably a bit more, let's say 45%. Therefore we can say that

$$1\ \mathrm{Gt}\ \mathrm{CO_2}\ \mathrm{emission} \simeq \frac{1}{8}\ \mathrm{PPM}\ 45\%\ \mathrm{in\ atmosphere})\ ,$$

and

$$40\,\mathrm{Gt/y}\ \ \mathrm{CO_2}\ \mathrm{emission} \simeq \frac{40}{8}\mathrm{PPM}\,45\%\,\mathrm{in\ atmosphere})\ ,$$

which is 40% of 5 PPM/year, about 2.25 PPM/year which fits reasonably well with the trend we found above, and with the average trend of 2.5 PPM/year found between 2010 and 2020.

To sum up, we have our own rough way to estimate how much of how our emissions end up in the atmosphere. Let's see if our model works for the past. In the period 1950-1955, the global annual CO_2 emission was about 7 Gt, which according to our calculation amounts into 0.35 PPM/year. In reality, it was about 0.5 PPM/year, so we are a bit off but we are not doing too bad. This model is quite naive but tells us something not far from the truth, that if we want to quickly reduce atmopheric greenhouse gases we need to cut our emissions to zero, or nearly so. This should ring a bell, because we just said that until now we have basically just been increasing! At least until very recently; to add an optimistic note, now in 2022 the recent trend seems more sluggish, or weakly decreasing. More precisely, we can only emit as much as plants and microorganisms can absorb, in order to keep the balance. In practice, as our plot shows, these sinks are adaptable (to some extent), and will hopefully work more as long as there is more CO_2 around, helping us a bit.

Our estimates point to two big questions. The first is how fast we need reduce our emissions to nearly zero, in order not to increase too much global temperatures, the second is if we also need to get rid of the excess carbon with additional technologies. Regarding the first question, this is what climate models are all about. Today I think they tell us to cut emissions by 7% every year to be able to stay 2^{o}C above pre-industrial temperatures.

Forse potresti specificare giÃă qui che la prima opzione (cattura alle emissioni) ha prospettive piÃź promettenti della seconda (rimozione dall'atmosfera con metodi diversi dalle piante). In realtÃă dallo spazio che dedichi alla cattura all'emissione giÃă si capisce che ÃÍ cosÃň. Ma forse una frase in piÃź per precisare che "con i sistemi di cattura diffusi (piante a parte)" siamo lontanissimi da soluzioni ragionevoli ci potrebbe stare.

REGARDING the second question, all scenarios where the excess temperature will stay within 1.5^{o}C above pre-industrial levels depend on "carbon capture" technologies that sequester carbon as they are emitted (typically from a power plant or a factory) or from the atmosphere[7], making yearly emissions negative, by the second half of the 21st century. In other words, we would have to remove carbon from the atmosphere using different kinds of large-scale infrastructure. Some technologies, and small scale applications of these technologies exist, but the cost (in terms of design, industrial conversion, workpower, etc. and also in terms of further emissions!) of deploying them to the required scale (Gt per year) may be a formidable challenge. For example, equipping a large-scale coal power plant could cost a billion euros over its lifetime, about 40 years, and result in a 40% use of the energy it produces (about a half). In other words, the equivalent factory with carbon sequestration technology will have 600 MW of usable power. A 1000 MW (1 GW, gigawatts) factory could release about 5 Mt (million tonnes) of CO_2 per year, so a lifetime total of 200 Mt. MW (megawatt, 10^6 watts) is power, or energy per unit time, whereas kWh (kilowatts times hours) is energy. There are 8760 hours in a year, so ideally a 1 MW factory can release about 8.76 10^9 kWh, about 10 GWh, per year. In practice, because of limited efficiency, we can say that, for coal, this is a fifth of the theoretical value, about 2 GWh

[7]Currently the systems that capture CO_2 directly from air (apart from plants) appear to be quite far from providing reasonable solutions.

per year. If the running cost per kWh is 0.01 euros, and we assume that due to the discount rate (economic growth) only of one tenth of the 40 running years (the first 4 years) matter (which might be reasonable for a 10% discount rate), then an "overall cost" of the carbon capture technology can be quantified in

$$\begin{aligned}10^9\ \text{euro} + (400\ 2\ 10^9\ \text{kWh/year})\times\\ (40\ \text{years}\ 0.1)(0.01\ \text{euro/kWh})\ ,\end{aligned}$$

which divided by 200 Mt gives about 150 euros per tonne of CO_2. This reduces to about 45 euros per tonne if the cost of carbon sequestration is only 10% of the produced energy. This very rough reasoning seems compatible with the available estimates (based on very complex reasoning involving economic models and carbon emissions), which say that a carbon price of 100 euros or more per tonne CO_2 would be needed to make industrial carbon sequestration viable. In this estimate, the total cost is nine billion euros, the impact of the one-billion euros construction cost is small (a bit more than 10%), and the energy cost has the most impact. We neglected the carbon emissions related to building the carbon sequestration technology (which might also play a role). I also note that the discount rate (which mirrors how optimistic we are about future economic growth) plays a very important role in this estimate. Roughly, we can say that in absence of the 10% projected economic growth the cost could be 10 times larger.

If many power plants were forced to capture all or most of the CO_2 they emit and store it permanently underground (a technology that is currently not well established), and the conversion cost in terms of emitted CO_2 were not too heavy, we could sensibly cut our emissions. The estimated global coal power capacity is currently about 2000 GW (responsible for about 20% of our global emissions); this is 2000 power plants such as the one considered in our example. Hence, the total conversion cost over the lifetime of the plants would be about $2\ 10^{13}$ euros; these are about twenty trillion euros, each of which is a thousand billions. By comparison, the gross natural product (defined as the total value of all goods and services produced by all citizens in a given financial year) of he US and china is around 22-23 trillion euros, and the world gross national product is about 130 trillion ($1.3\ 10^{14}$) euros. An alternative (perhaps more conservative) way to quantify the cost is to look at the investment needed to obtain the same power. We can say that if the carbon sequestration takes 40% of the produced energy in any given time, a 1000 MW factory would have a reduced effective power of 600 MW once the emissions are sequestered. As a consequence of that, instead of 2000 coal power plants we would need about 3300, hence a sheer investment of "only" 3300 billion or 3.3 trillion euros (the current GNP of France). One could repeat similar estimates for oil and gas power plants, but the bottom line is that if these figures are reasonable, global-scale carbon-capture technology would take a significant fraction of all the money we make.

This does not mean that we should not try to deploy carbon capture technologies to a feasible scale in order to improve the global carbon budget, and also it does not mean that we should not try to improve the figures to make them more feasible. Precisely to what scale we should aim to deploy carbon capture technologies seems to me like a very big question, as it involves evaluating carefully delicate tradeoffs between advantages and costs from multiple origins, and also a concerted global planning of course, given the budget. What should be apparent however, is that carbon capture technology (today and in the coming years) alone does not seem to be a feasible global solution to human-induced climate change, and that massive and quick "upstream" cutting of emissions remains a top priority.

As I mentioned above, there are technologies that capture carbon directly from the atmosphere, potentially providing negative emissions, but this is difficult, because CO_2 is very rarefied (a part per million means one gram for each tonne of atmosphere). Essentially, these are huge arrays of fans that filter air and put it through chemical reactions that remove CO_2, after which they let the air go back in the atmosphere (as long as we are emitting CO_2, it makes sense to deploy these technologies as close as possible to where we emit, but this is not simple in the case of cars or planes for example). Similarly to the case of carbon capture from the source, there are challenges for scaling up this technology to the required thoughput, in terms of investments and energy requirements, which lead to considerations that parallel those we attempted above. As a consequence, the current consensus on direct air capture is also similar. Alone, it will not provide a solution, but it might help, especially in the long run. Many are afraid of the risk of planning the near future relying too much on the assumption that carbon capture technologies can be deployed at a global scale, since if they are stopped or delayed by unexpected later hurdles the consequences could be catastrophic.

A simpler (and more ecological) "technology" that removes CO_2 from the air is photosynthesis by trees (and plankton). A fully grown tree can absorb at least 10 Kg of CO_2 per year. How much forest do we need to capture a Gt of CO_2 every year? One Gt is 10^{12} Kg, meaning that we need to plant 10^{11} trees (a hundred billion) to offset one fortieth of our current total emissions. If we suppose that each fully grown tree needs an area of 10 m^2 (roughly a box of 3x3 meters), it means we need 10^{12} square meters of trees, or a million square kilometers for each Gt/year. It's a lot, but on a global scale it does not seem impossible. It is roughly twice the area of France, but only one seventh of the area of the Amazon forest. There is an ongoing project to conserve, restore and grow a trillion trees by 2030 (`https://www.1t.org/`). Importantly, in the long run, a lot of carbon would accumulate in this huge set of trees (mostly in the wood of mature trees), and one would have to make sure that the stored carbon does not go back in the atmosphere too fast (for example by burning the wood), or the effort would be undermined. Once again, my numbers maybe be a bit off, but they give us an idea of the magnitude of the problem, and, to me, they show that planting trees once again can help, but will most probably not work without cutting fossil fuel emissions as well.

What certainly we should *not* do is cut or let die too many trees and let the carbon that makes them up get to the atmosphere, because in those trees there is (roughly speaking) basically all of the carbon absorbed cumulatively over their lifespan, so that they can in principle emit in a very short time much more CO_2 than they can sequester every year. How much CO_2 actually gets released is quite complex, as it depends to what happens to the dead plant (for example it could be degraded by microorganisms, burnt, or the wood could persist for a long time, with different outcomes). On a positive note, wildfires appear to be relatively inefficient in releasing the CO_2 stored in a forest, because they are sparse. Harvesting, instead, causes a higher mortality than wildfire, and consequently (and counterintuitively) it seems that increasing harvest of mature trees to save them from fire may actually increase emissions, rather than reducing them.

To conclude, my back-of-the-envelope arguments suggest that while we should do our best to enhance the carbon sinks, we should definitely cut our emissions. The "where do we start" part is the subject of chapter V; before I get there, the next chapter tries to grasp how atmospheric greenhouse gases translate into temperature changes, and the following one how temperature changes affect weather and extreme events.

DATA SCIENCE TAKE-HOME MESSAGES. In this chapter, we performed our first exercises in the area of quantitative estimates and theoretical modeling. First, we warmed up with the size of a cube mate of the atmospheric carbon. This estimate uses the classic expedient of translating the quantification of something into something that we can relate to (a solid cube). Second, we constructed a more ambitious model to relate CO_2 emissions to atmospheric carbon (in PPM), formulating a testable prediction, which we used to validate the model. Third, we used typical methods from Fermi estimates to reason on the carbon capture costs for a big coal power plant, and for all coal power. During the first estimate, we learned that guessing precisely the conversion from PPM CO_2 to Gt was not very easy. A lesson to learn is that this kind of estimate is very effective to guess the order of magnitude of something, but it may be difficult to refine its precision (or, more precisely, it may require very specific information, scientific insight, and detailed modeling). In terms of methods, here's a few suggestions to construct a good Fermi estimate: (i) break down the problem into sub-problems; (ii) think about bounds and take central values; (iii) think about "sanity checks", to verify the validity of the estimates (iv) dare to approximate, because sometimes it is important to start with an estimate of some kind start getting a grasp on the problem, and sometimes a not-so-good solution opens the way for a better solution; (v) "Google it", if there is no possibility of estimating some value that makes an essential part of our estimate with our knowledge, it's not a crime to rely on the knowledge of others. Other than this, this chapter reiterated the analysis of time series across time scales found in chapter I, this time for atmospheric CO_2 and other greenhouse gases. For example, we learned that because time scales correspond to averages, the carbon or temperature footprint of a hypothetical ancient advanced civilization would be likely be "averaged out", hence not visible in our data.

PYSICS TRACK. We can use hydrostatic equilibrium ("Stevin's law") in order to estimate the height of a constant-density atmosphere as follows. The law states that $\mathrm{d}P == \rho g \mathrm{d}z$, where P is pressure, ρ is the atmospheric density g is gravitational acceleration at the earth's surface, and z stnands for the vertical coordinate. By integrating this relation up to a height h ("the height of the atmosphere"), where we assume that $P = 0$, we get

$$\int_{P_0}^{0} \mathrm{d}P = \int_{0}^{h} \rho g \mathrm{d}z \,,$$

where P_0 is atmospheric pressure at sea level. From this relationship we get

$$h = \frac{P_0}{\rho g} \,.$$

Using $P_0 \simeq 10^5$ Pa, $\rho \simeq 1$ Kg/m^3 and $g \simeq 10$ m/s^2, we get $h \approx 10$ Km, justifying the assumption we used within the chapter. This estimate is itself limited by several aspects, perhaps the principal of which is that temperature, density and pressure are related, and they vary with height. Their relation can be approximated by the ideal gas law $PV = Nk_BT$, or $P = \rho(R/M)T$, where k_B is Boltzmann's constant, T is temperature, R is the universal gas constant and M is the molar mass. The last equation can be solved for density as

$$\rho = \frac{PM}{RT} \,.$$

Assuming constant temperature, substituing ρ into Stevin's law integrating gives

$$P(z) = P_0 e^{\frac{gM}{RT} z} \,.$$

One can further refine the estimate assuming that T is not constant.

SOURCES. The plots on recent greenhouse gas emissions were found on Wikipedia, at the following pages
https://en.wikipedia.org/wiki/Carbon_dioxide_in_Earth's_atmosphere,
https://en.wikipedia.org/wiki/Atmospheric_methane,
https://en.wikipedia.org/wiki/Nitrous_oxide,
and on the curated plots by Ed Dlugokencky and Pieter Tans, NOAA/GML (gml.noaa.gov/ccgg/trends/). To make my own plots of atmospheric CO_2 levels, I downloaded two data sets from the NOAA database (https://www.ncdc.noaa.gov). The recent record (1751-2018) comes from the Oak Ridge National Laboratory, Carbon Dioxide Information Analysis Center, Scripps Institute of Oceanography CO_2 program, and the U.S. Energy Information Administration, International Energy Statistics, accessed December 7, 2020. The 800 ky Ice core data set is called "Antarctic Ice Cores Revised 800KYr CO2 Data" and collects data from several studies. The collection was published as Bereiter and coworkers Geophysical Research Letters, 42(2), 542-549 (2015) (link). The CO_2 data on My time scales come from Retallack Nature 415, 38 (2002), and can be visualized at the web page earth.org. The time series of the global carbon budget was taken from Our World in Data, Ritchie and coworkers (2020), Published online at OurWorldInData.org. Retrieved from this link. It is based on data issued in 2019 by the Global Carbon Project (https://www.globalcarbonproject.org/). For my own analyses, I downloaded the 2020 edition of the Global Carbon Budget. Andrew GCP, 2020. Global Carbon Budget 2020, link. A quantification of Nitrous Oxide sources and sinks can be found in Tian *et al.* Nature 586, 248-256 (2020). This article by Andrew Mose-

man from the MIT climate portal argues that CO_2 levels between 280 and 350 parts per million created the climate where humanity can thrive (link). A NOAA document by Stephen A. Montzka (link) provides a non-technical comparison of the climate-warming influence of different greenhouse gases, including fluorinated gases (grouped as "HFCs") and ozone depleting substances (grouped as "CFCs" and "HCFCs"). The same information can be found in Chapter 7 of the IPCC-AR6 (Cissé *et al.* Climate Change 2022: Impacts, Adaptation, and Vulnerability. Contribution of Working Group II to the Sixth Assessment Report of the Intergovernmental Panel on Climate Change. Cambridge University Press 2022, pp. 1041-1170, doi:10.1017/9781009325844.009, see this link.) The question of the time resolution in ice-core data is discussed in Masson-Delmotte and coworkers Quaternary Science Reviews 29 (2010) 113-128 (2009). The study on the traces left by previous civilizations is discussed by Steven Ashley in Scientific American, Apr 23, 2018, and by Adam Frank in The Atlantic, Apr 13 2018. The original publication is Schmidt & Frank International Journal of Astrobiology, 18(2), 142-150 (2019). The article by Jorge Sarmiento and Nicolas Gruber, Physics Today Volume 55, Issue 8 (2002) discusses very clearly the carbon budget and the the oceanic sink. More recent (and technical) information can be found on Gruber *et al.* Science 363, 6432 1193-1199 (2019). The estimates on coal power plants are derived from figures on the global energy wiki, at the link gem.wiki. They come from the 2021 IEA report "World Energy Outlook" (link), from the 2007 MIT report "The Future of Coal" (link) and from the 2009 report "New Coal-fired Power Plant Performance And Cost Estimates" published by Sargent & Lundy (link). The data on carbon capture can be found on Wikipedia (link). In particular, the data on costs of carbon capture come from Thorbjornsson et al. Energy Strategy Reviews. 7: 18-28 (2015). The article by Realmonte and coworkers, Nature Communications 10, 3277 (2019) discusses feasibility estimates for direct air carbon capture. A brief and clear document on the impact of forests on climate change can be found on the 2003 FAO Forests and Climate Change Working Paper (link). The arguments on wildfires come from Bartowitz and coworkers, Front. For. Glob. Change, 5, 2022.