# Agenda

1. **Measuring unemployment two ways**
2. **Bayes' rule**
3. **Grid approximation**

## Unemployment rate in Newfoundland and Labrador

- How do we say something about the proportion of residents who have no employment?
- Full census impractical for our purposes
- Instead we use a probability model to *estimate* the proportion based on a sample ($S$)

## Building a model

- Pretend we already know the proportion, call it $p$
- Probability model tells a story about what $S$ might look like, assuming we know $p$
- **Reverse the logic of your question:**
  *In reality we know S and want to learn about p*
  *In our model we know p and want to describe S*

# Unemployment

## The story of our sample

- The real proportion of the population that is unemployed is $p$
- We choose people **uniformly at random** from that population
- Each person we pick will have a probability $p$ of being unemployed
  “Data generating process”

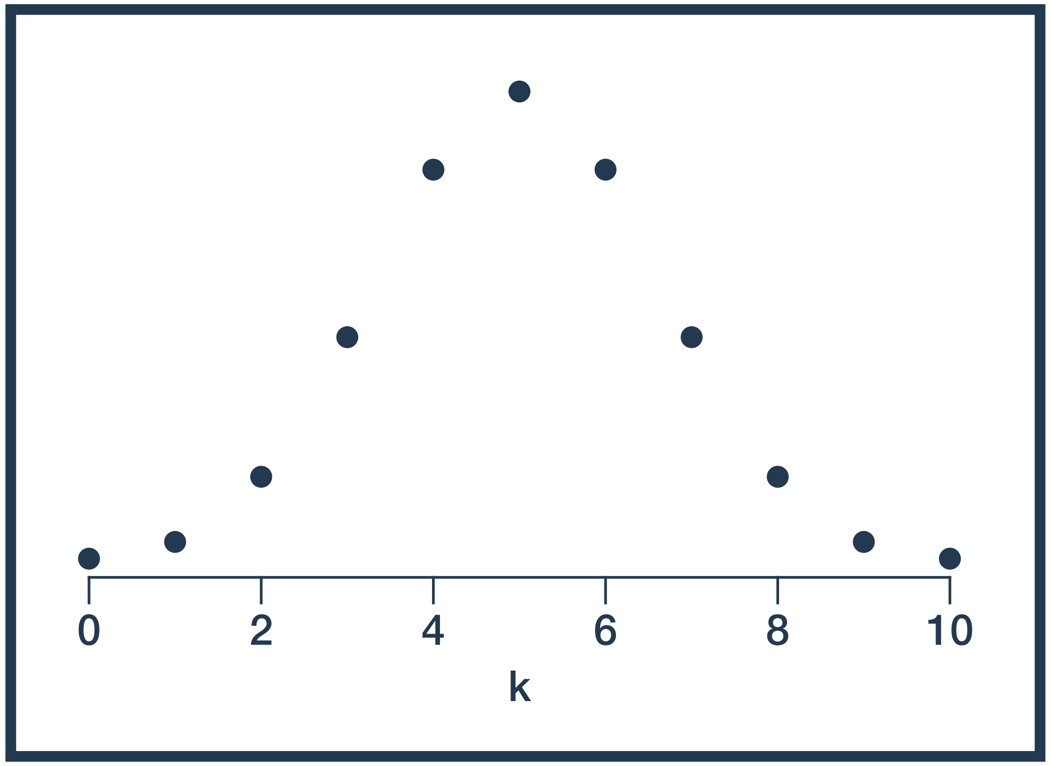
## There is an off-the-shelf probability distribution for exactly this scenario: the binomial distribution

# Binomial distribution

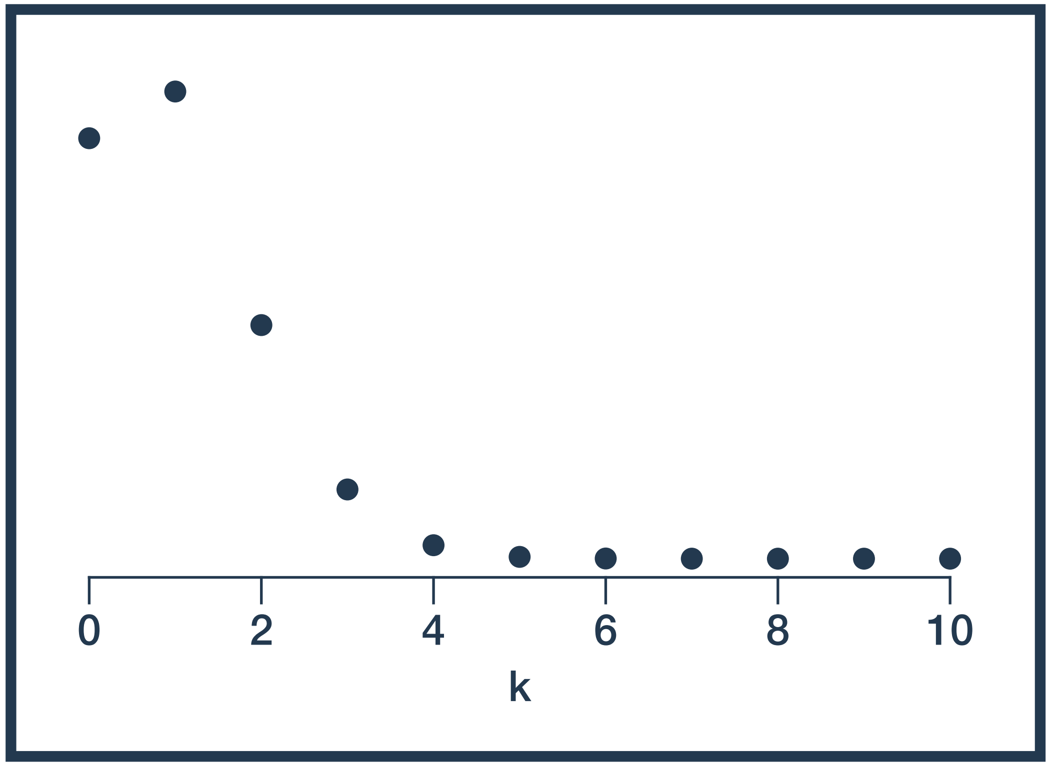
**Binomial distribution**
The probability of getting *k* 'successes' in *n* trials if the probability of success is *p*.

$$\Pr(k|n,p) = \frac{n!}{k!(n-k)!}p^k(1-p)^{n-k}$$

***Bin*(*n*=10, *p*=0.5)**




***Bin*(*n*=10, *p*=0.1)**



# Estimating unemployment

$$S = (E, E, E, U, U, E, E, E, U, E)$$

## Frequentist estimation

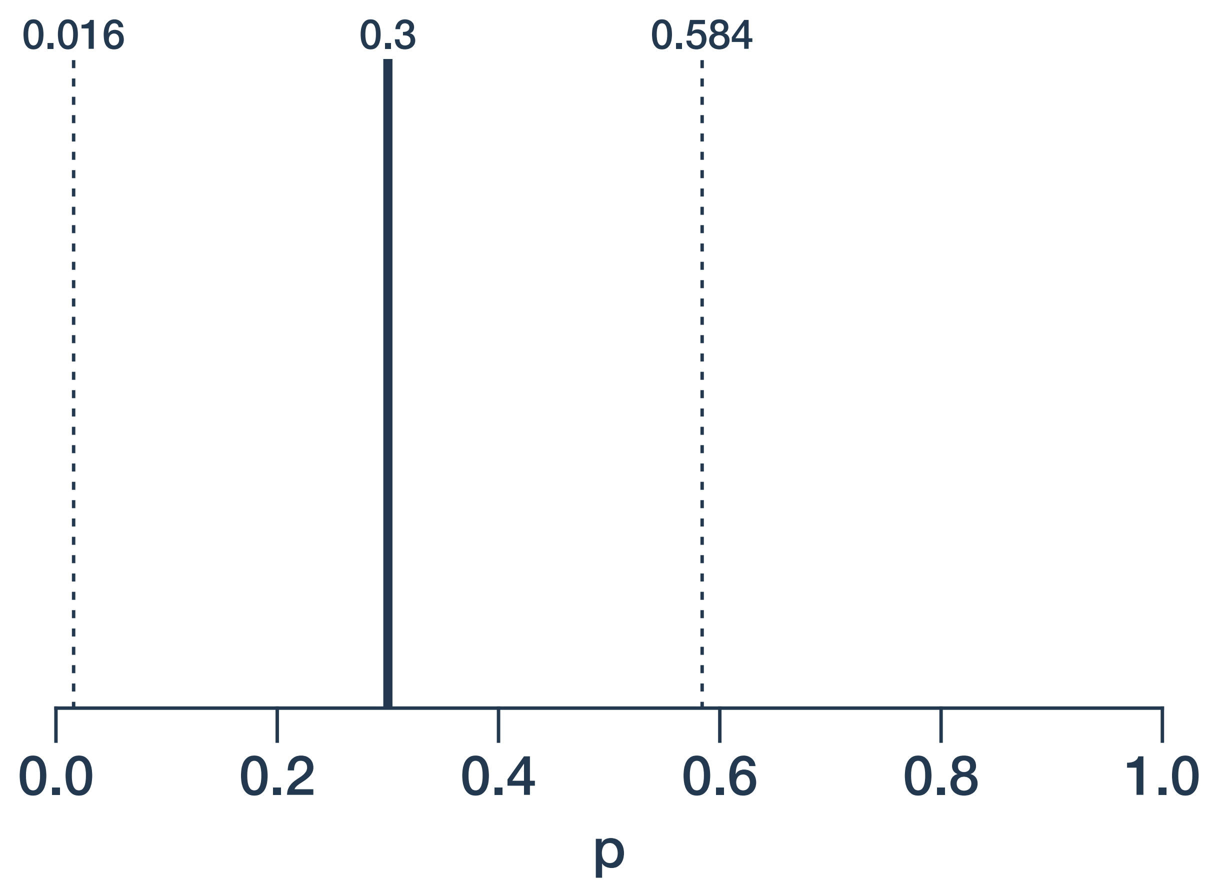
1. Pick an "estimator" (such as sample proportion)

2. Generate *point estimate* of *p*

$$\hat{p} = \frac{3}{10} = 0.3$$

3. Use approximation of the sampling distribution to quantify uncertainty

$$\hat{\sigma} = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 0.145$$

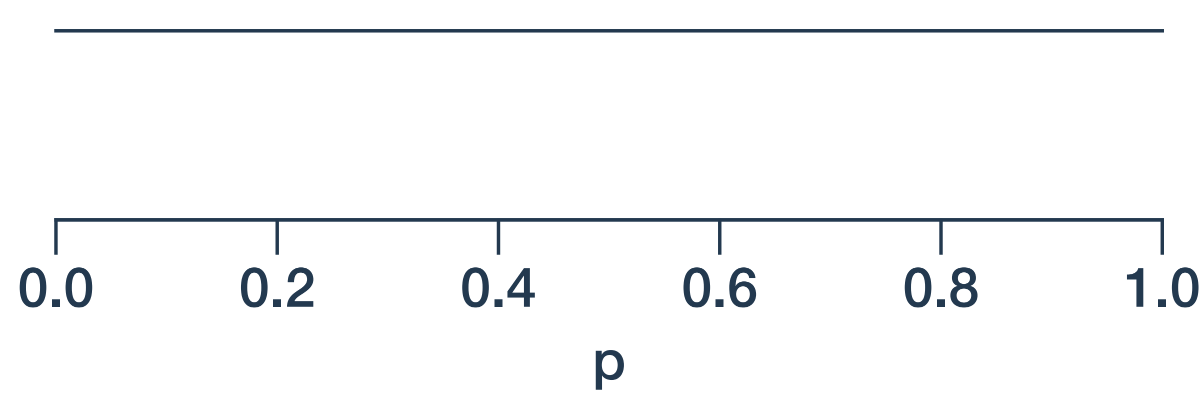


# Estimating unemployment

$$S = (E, E, E, U, U, E, E, E, U, E)$$

**Bayesian estimation**

1. Pick a prior (such as uniform distribution)



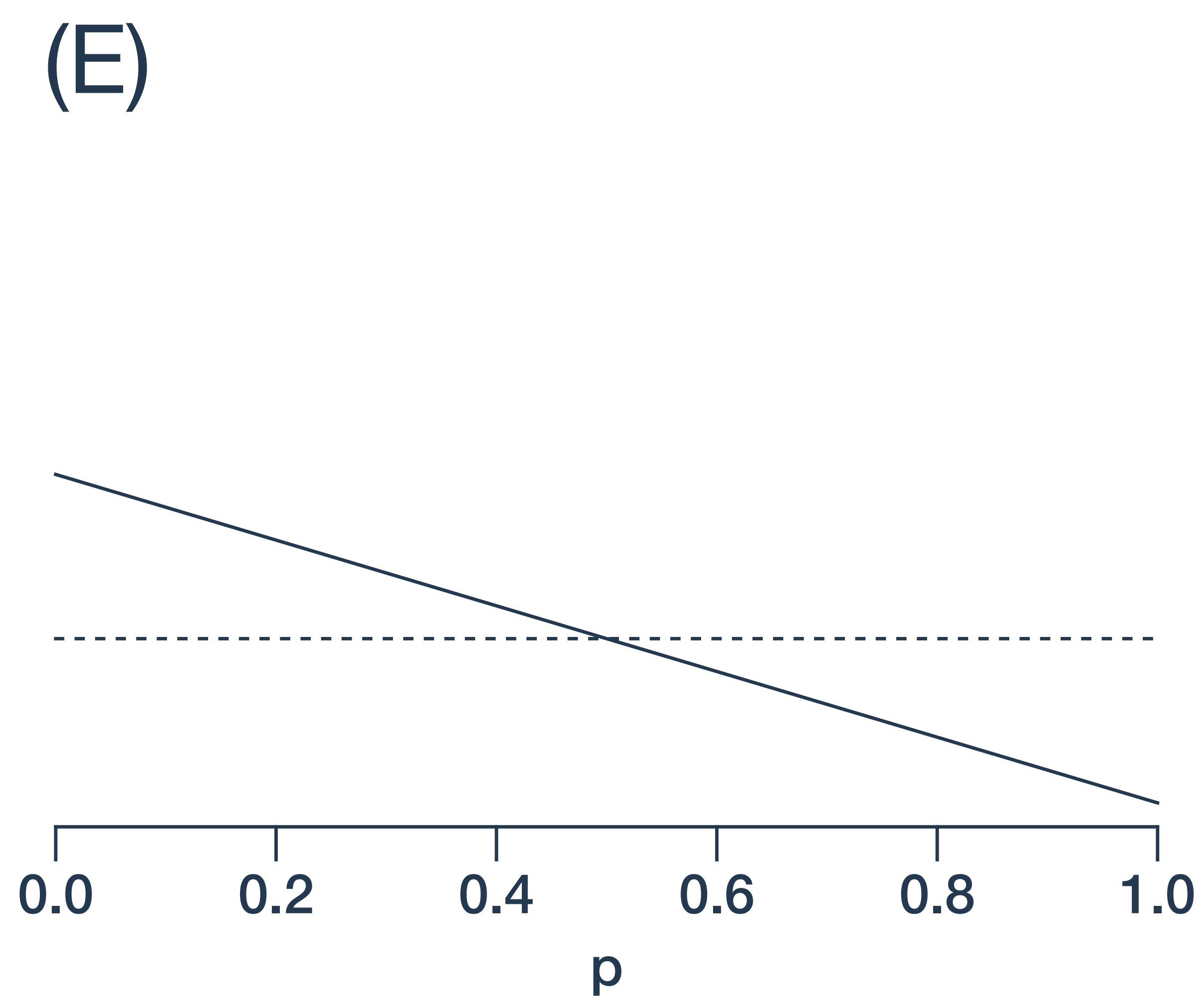
# Estimating unemployment

$$S = (E, E, E, U, U, E, E, E, U, E)$$

## Bayesian estimation

1. Pick a prior (such as uniform distribution)

2. Update prior with data (one at a time or all at once)



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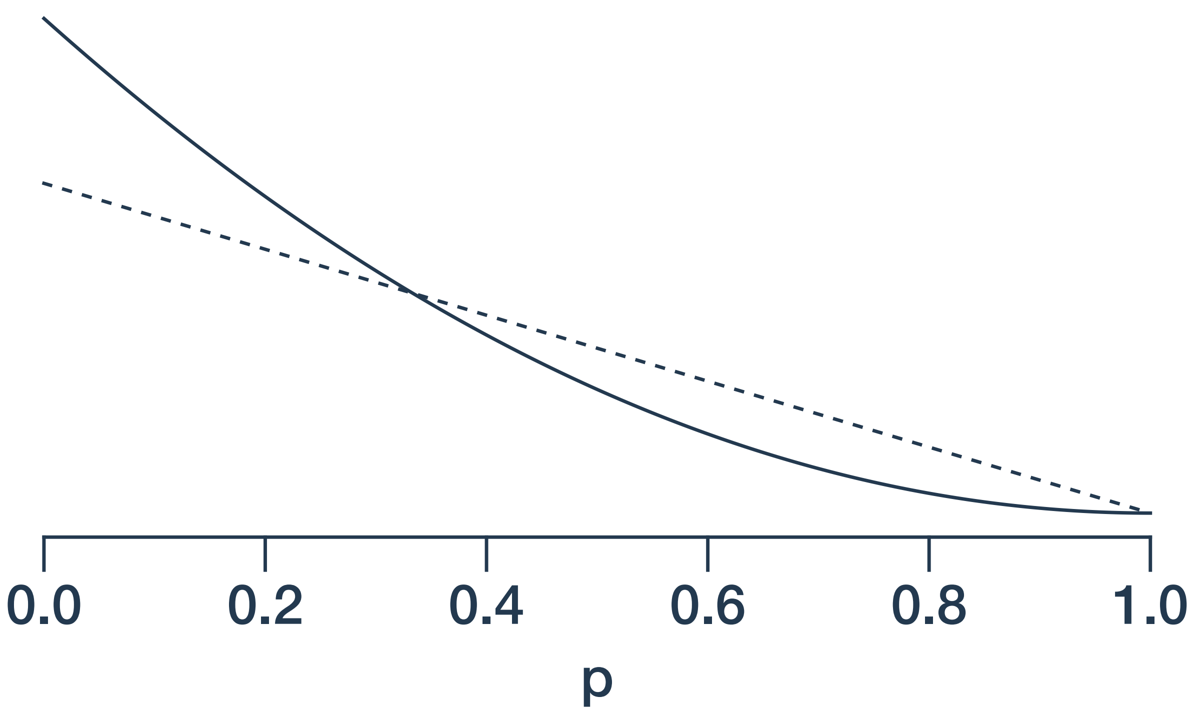
(E,E)



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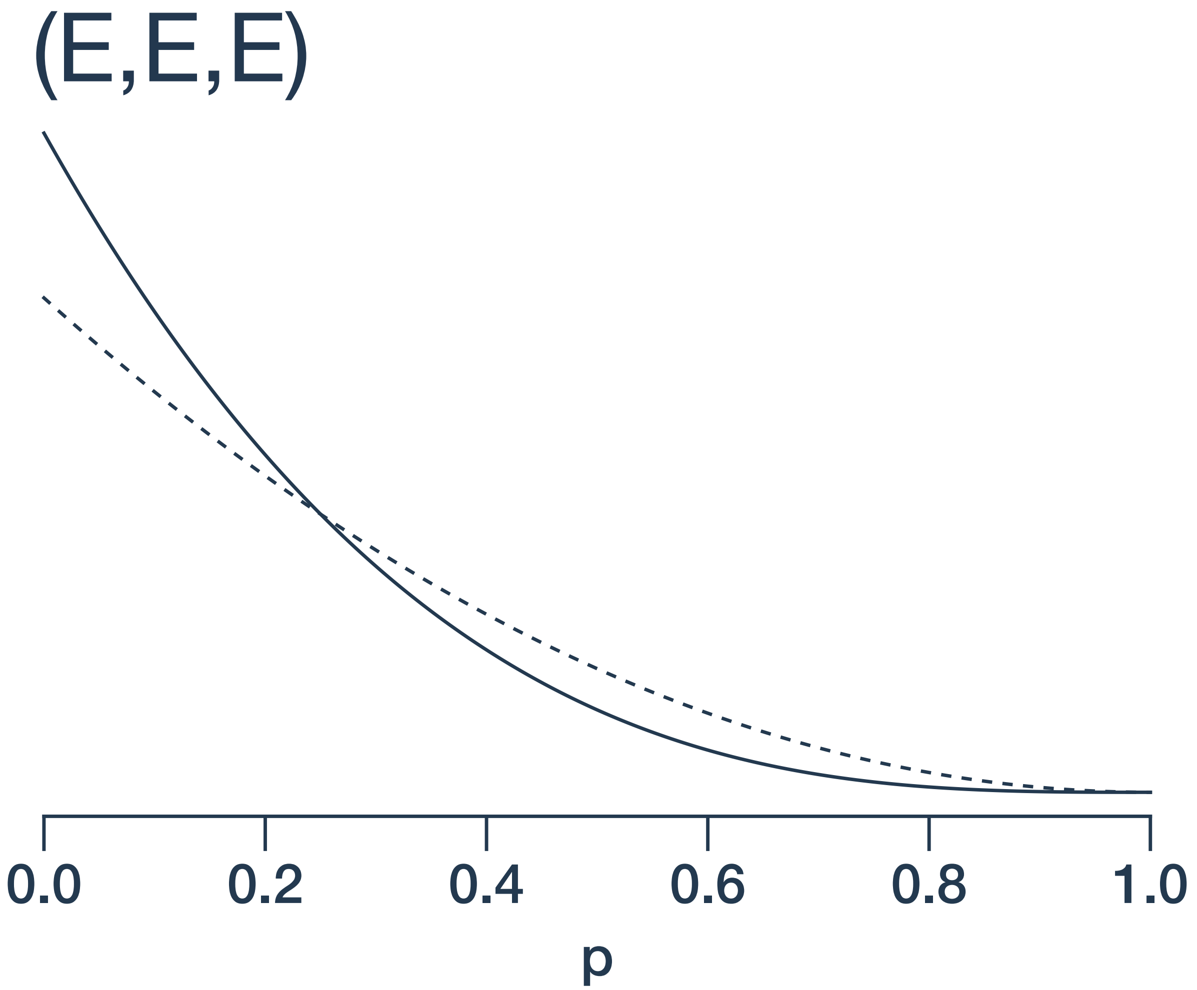

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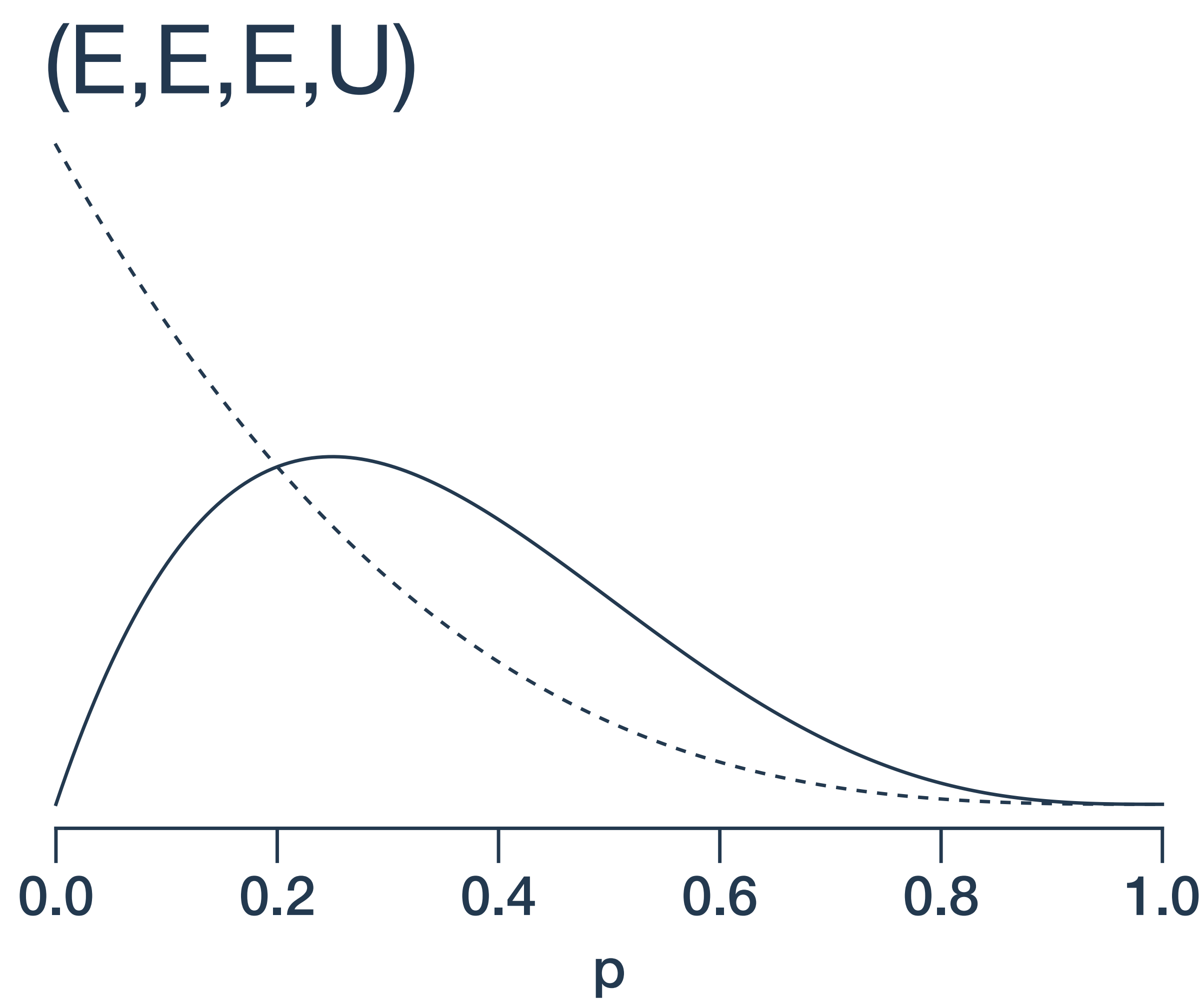

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## Bayesian estimation

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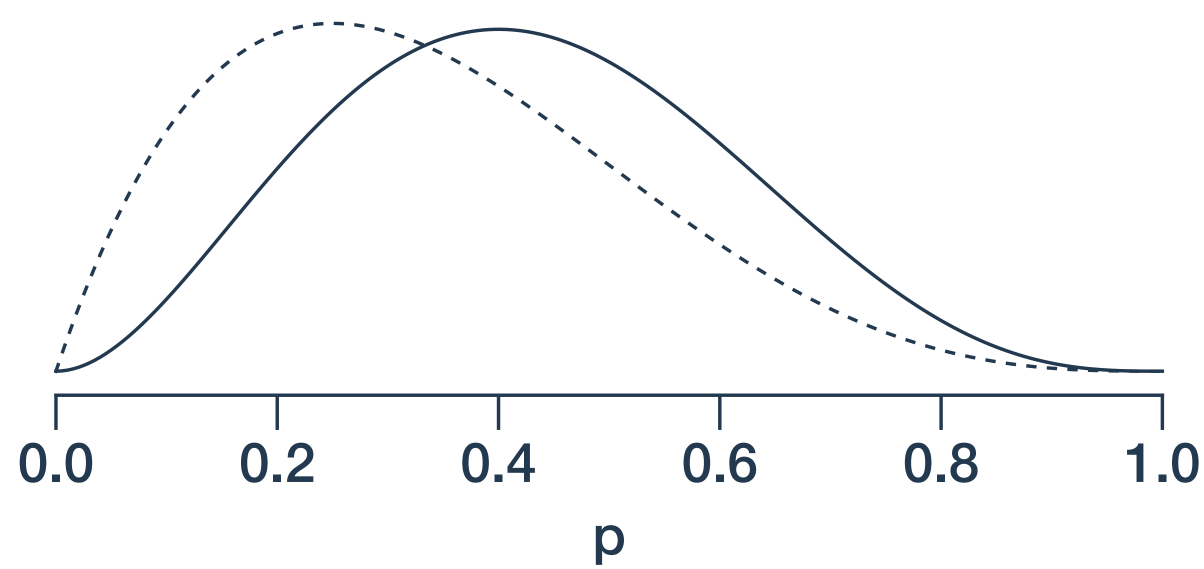
(E,E,E,U,U)



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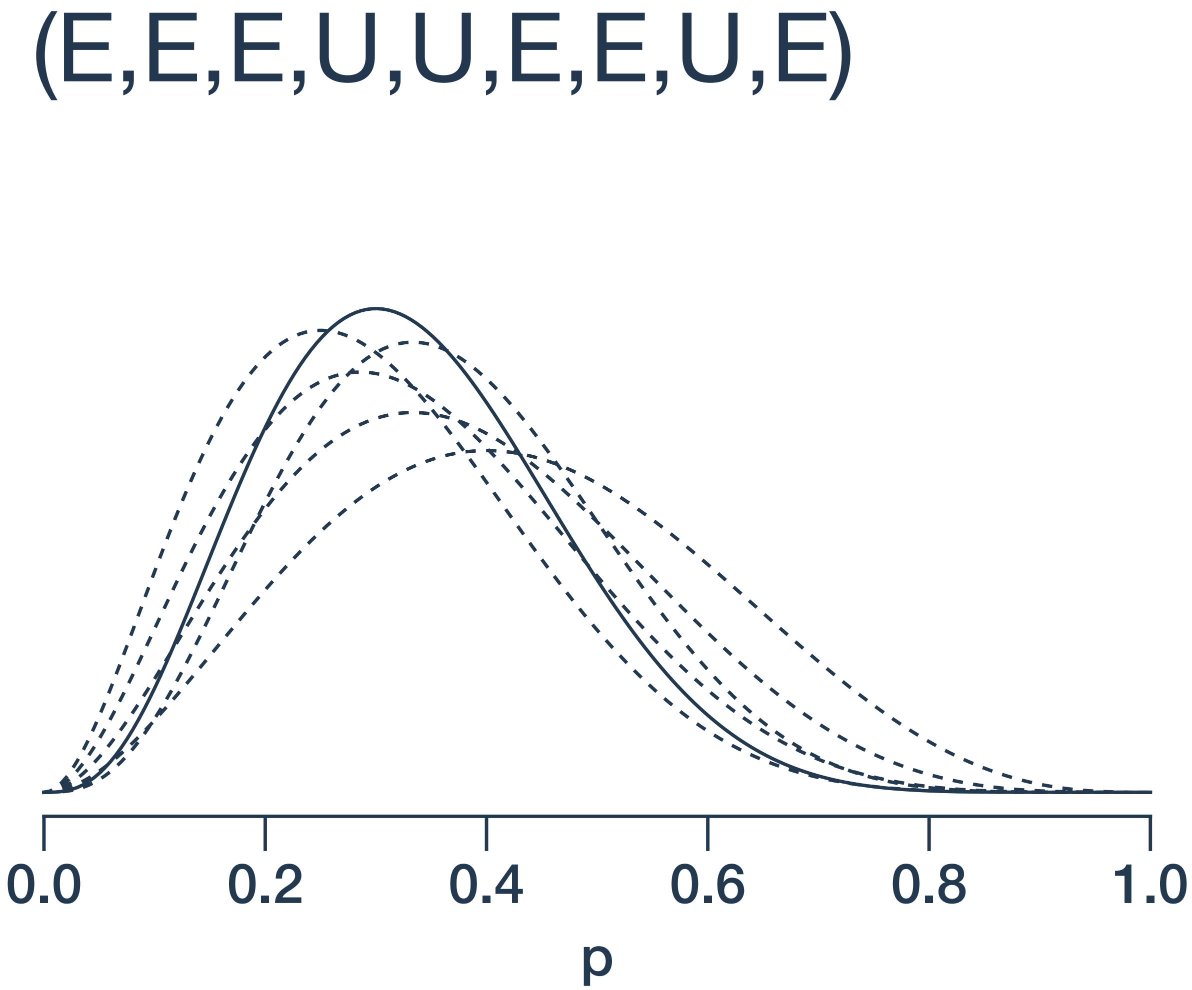


# Estimating unemployment

$$S = (E, E, E, U, U, E, E, E, U, E)$$

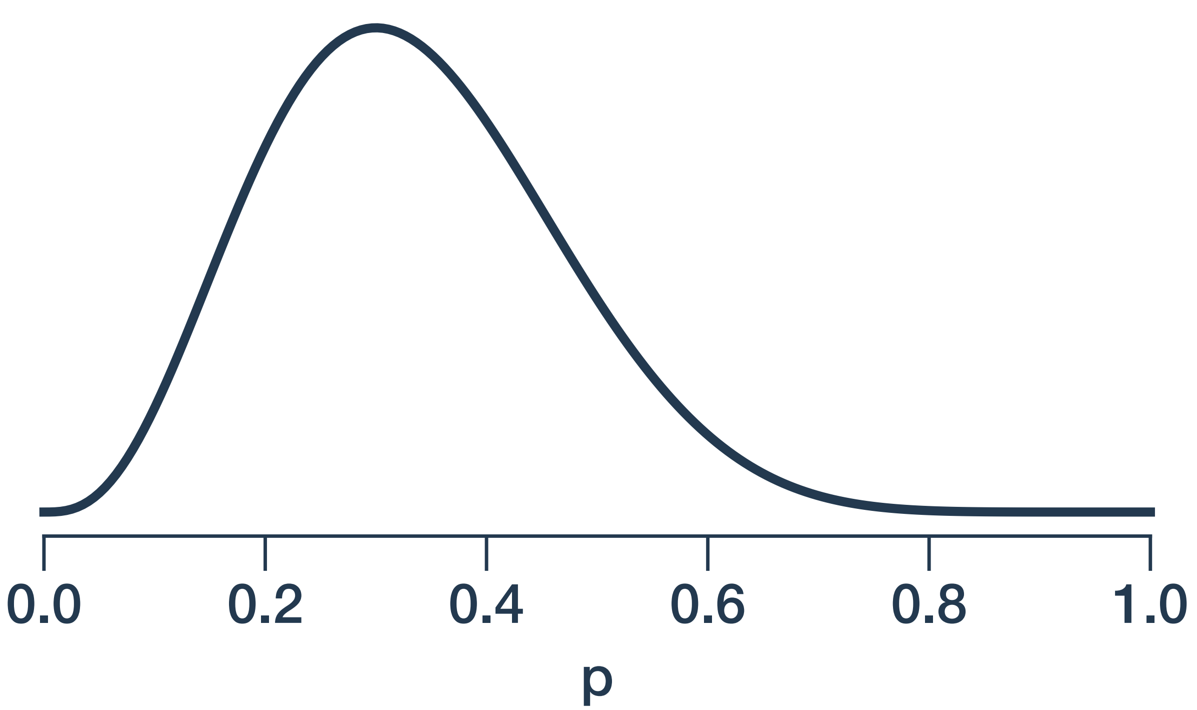
## Bayesian estimation

1. Pick a prior (such as uniform distribution)

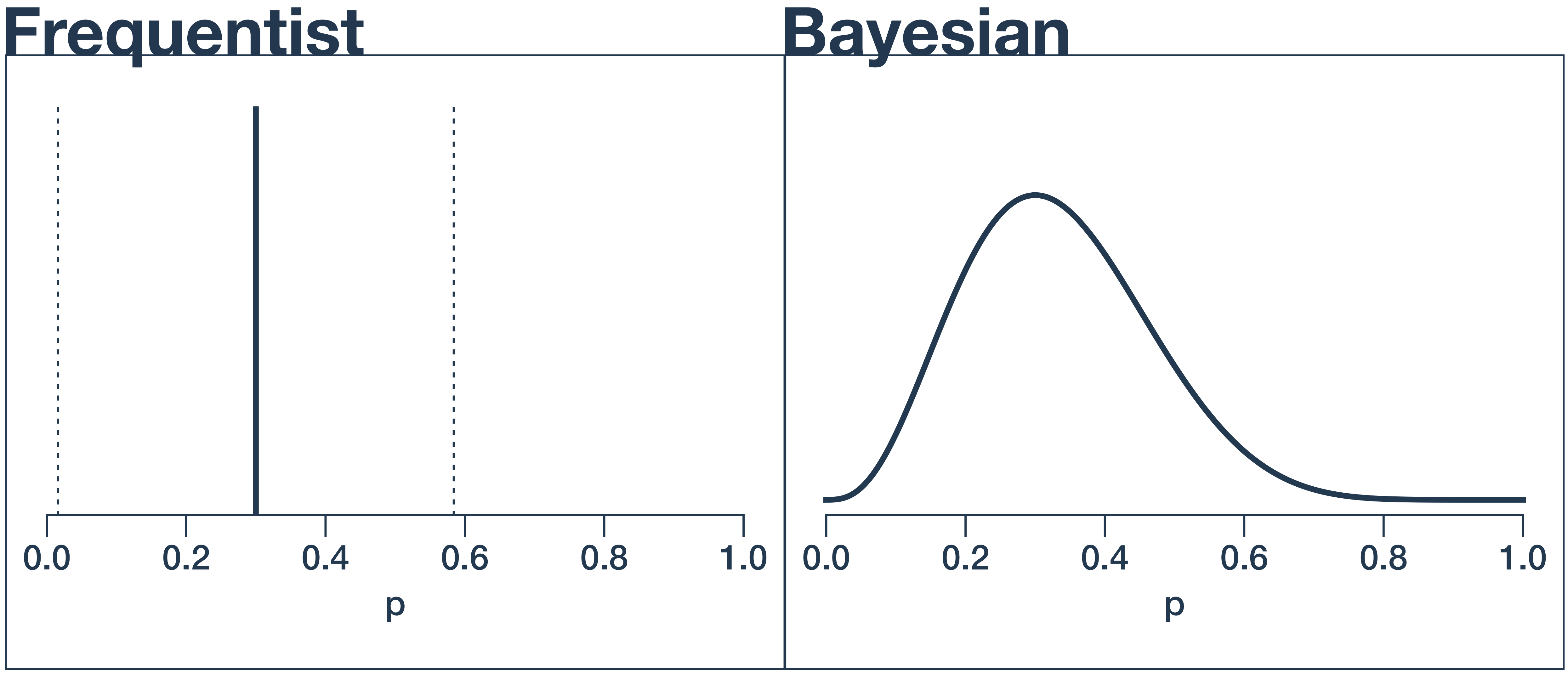
2. Update prior with data (one at a time or all at once)

3. Posterior distribution incorporates all the information we have about *p*

(E,E,E,U,U,E,E,U,E)

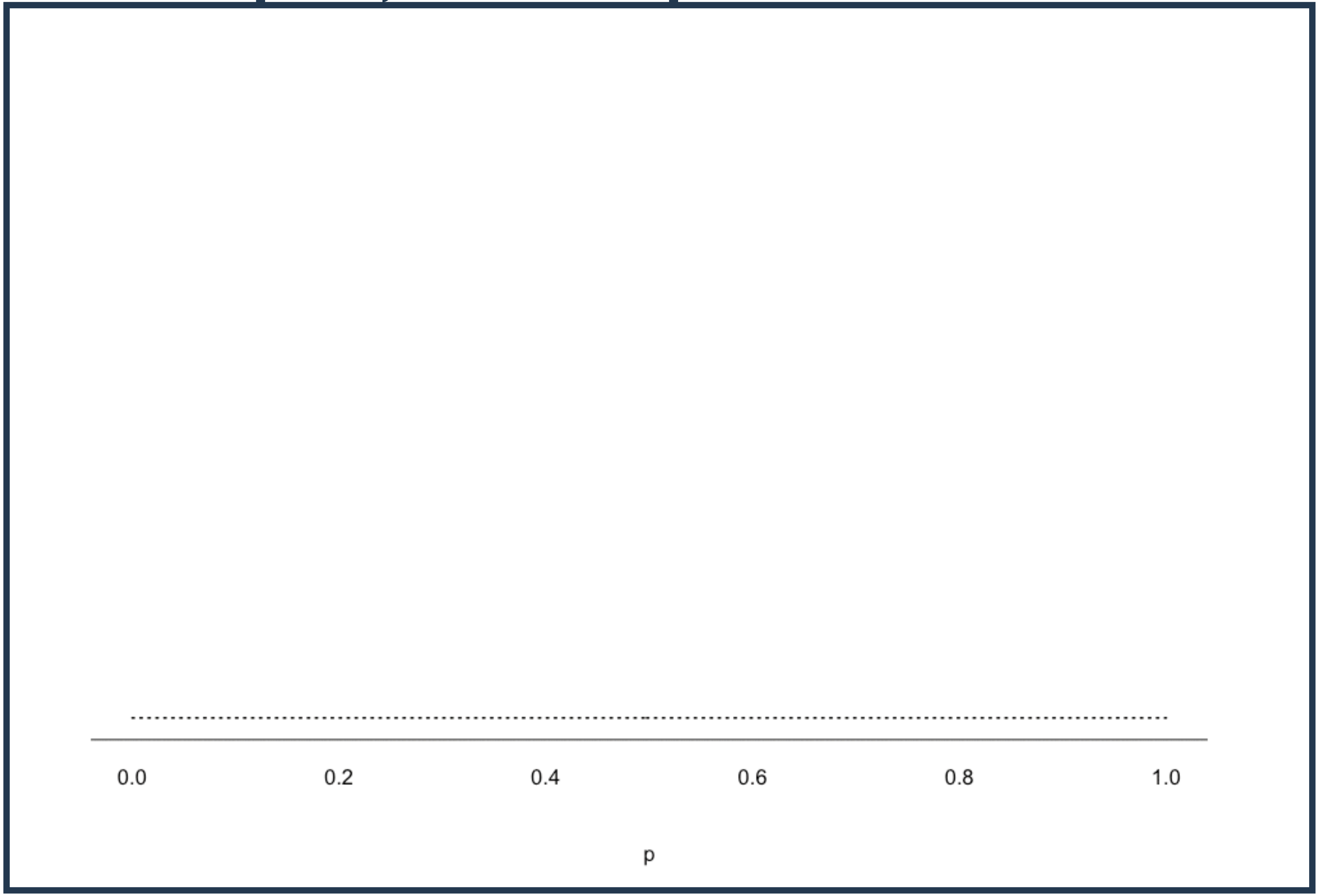


# Comparing estimates



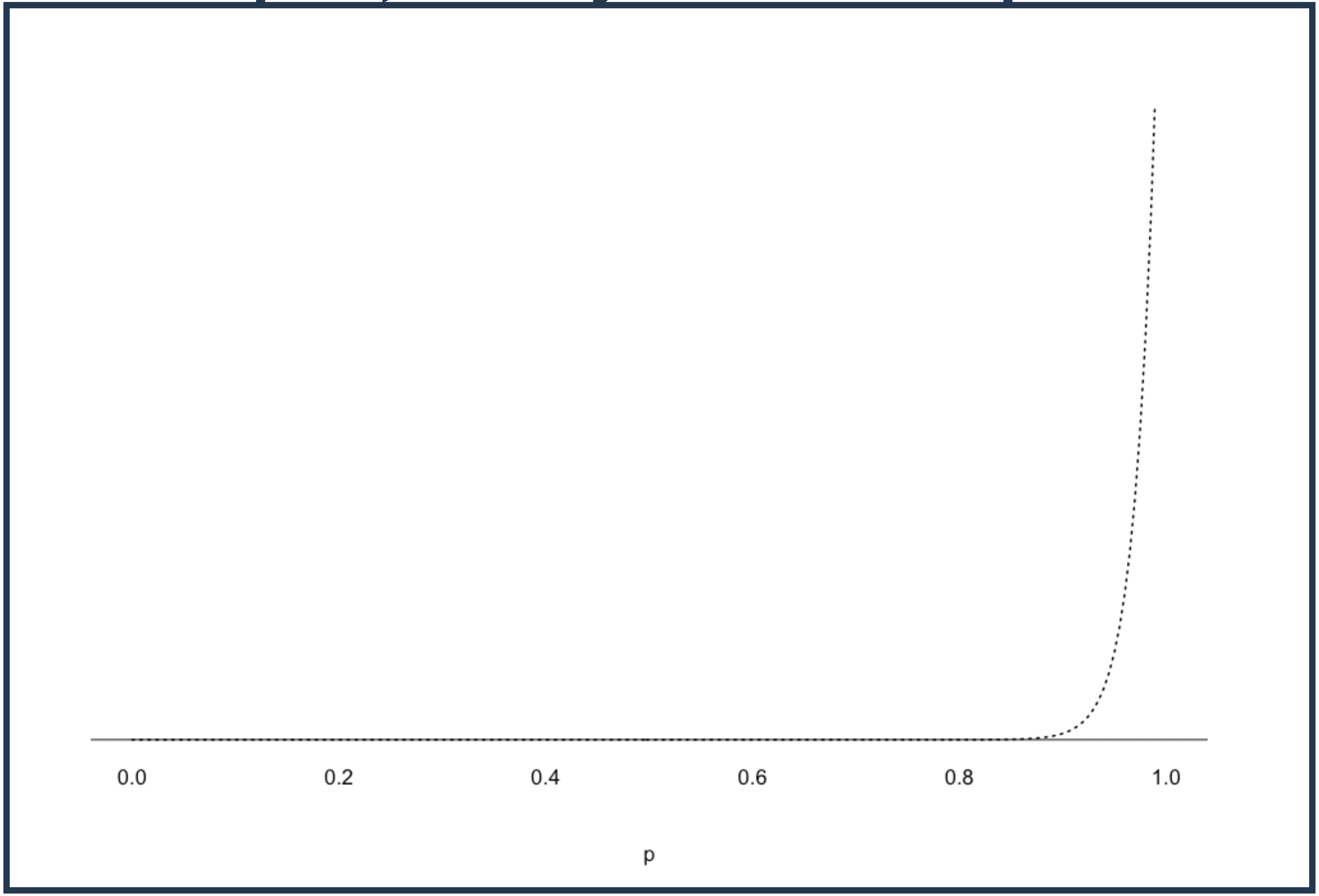
# Bayesian updating

## 500 samples; uniform prior



# Bayesian updating

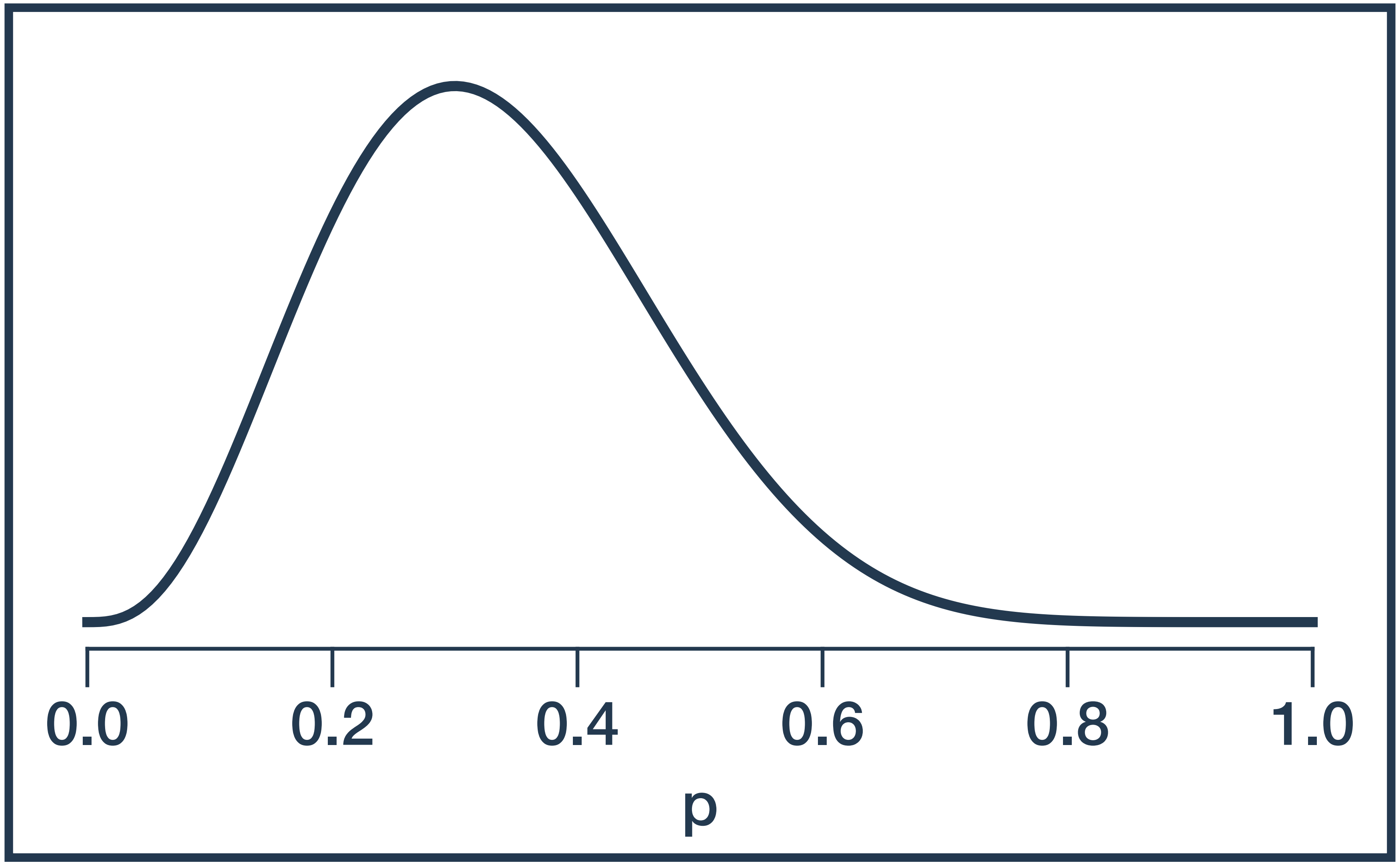
## 500 samples; heavily informative prior



# Conditional probability

Varying Fixed

$$\Pr\left(p \middle| n, S\right)$$



# Bayes' rule

$$\Pr\left(A \middle| B\right) = \frac{\Pr\left(B \middle| A\right)\Pr\left(A\right)}{\Pr\left(B\right)}$$

# Bayes' rule

$$\Pr\left(p \middle| n, S\right) = \frac{\Pr\left(S \middle| n, p\right) \Pr\left(p\right)}{\Pr\left(S\right)}$$

# Bayes' rule

Posterior probability

$$\boxed{\Pr(p \mid n, S)} = \frac{\Pr(S \mid n, p)\Pr(p)}{\Pr(S)}$$

Posterior probability is what we care about. It tells us everything we know about the unemployment rate ($p$) given what we've learned from our sample.

# Bayes' rule

Likelihood

$$\Pr(p \mid n, S) = \frac{\boxed{\Pr(S \mid n, p)}\,\Pr(p)}{\Pr(S)}$$

The likelihood where our model lives. In this case, the probability of getting our sample (*S*), given the actual unemployment rate (*p*) is simply the binomial distribution we saw earlier: ***Bin*(*n*,*p*)**

# Bayes' rule

**Prior probability**

$$\Pr(p \mid n, S) = \frac{\Pr(S \mid n, p)\,\boxed{\Pr(p)}}{\Pr(S)}$$

The prior probability is everything we claim to know about the unemployment rate ($p$) *before* we ask anybody about their employment.

# Bayes' rule

The evidence is just the average probability of seeing our sample across all possible values of *p*, normalizing our posterior. It is often the hardest part of a model to calculate.

$$\Pr(p|n,S) = \frac{\Pr(S|n,p)\Pr(p)}{\boxed{\Pr(S)}}$$

Evidence

# Bayes' rule

$$\overbrace{\boxed{\Pr(p \mid n, S)}}^{\text{Posterior probability}} = \frac{\overbrace{\boxed{\Pr(S \mid n, p)}}^{\text{Likelihood}}\,\overbrace{\boxed{\Pr(p)}}^{\text{Prior probability}}}{\underbrace{\boxed{\Pr(S)}}_{\text{Evidence}}}$$

# Bayes' rule

$$\Pr\left(p \middle| n, S\right) \propto \Pr\left(S \middle| n, p\right)\Pr\left(p\right)$$

$$\boxed{\text{Posterior probability}} \propto \boxed{\text{Likelihood}} \times \boxed{\text{Prior probability}}$$