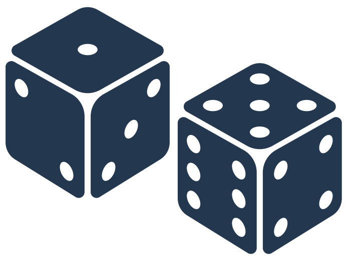
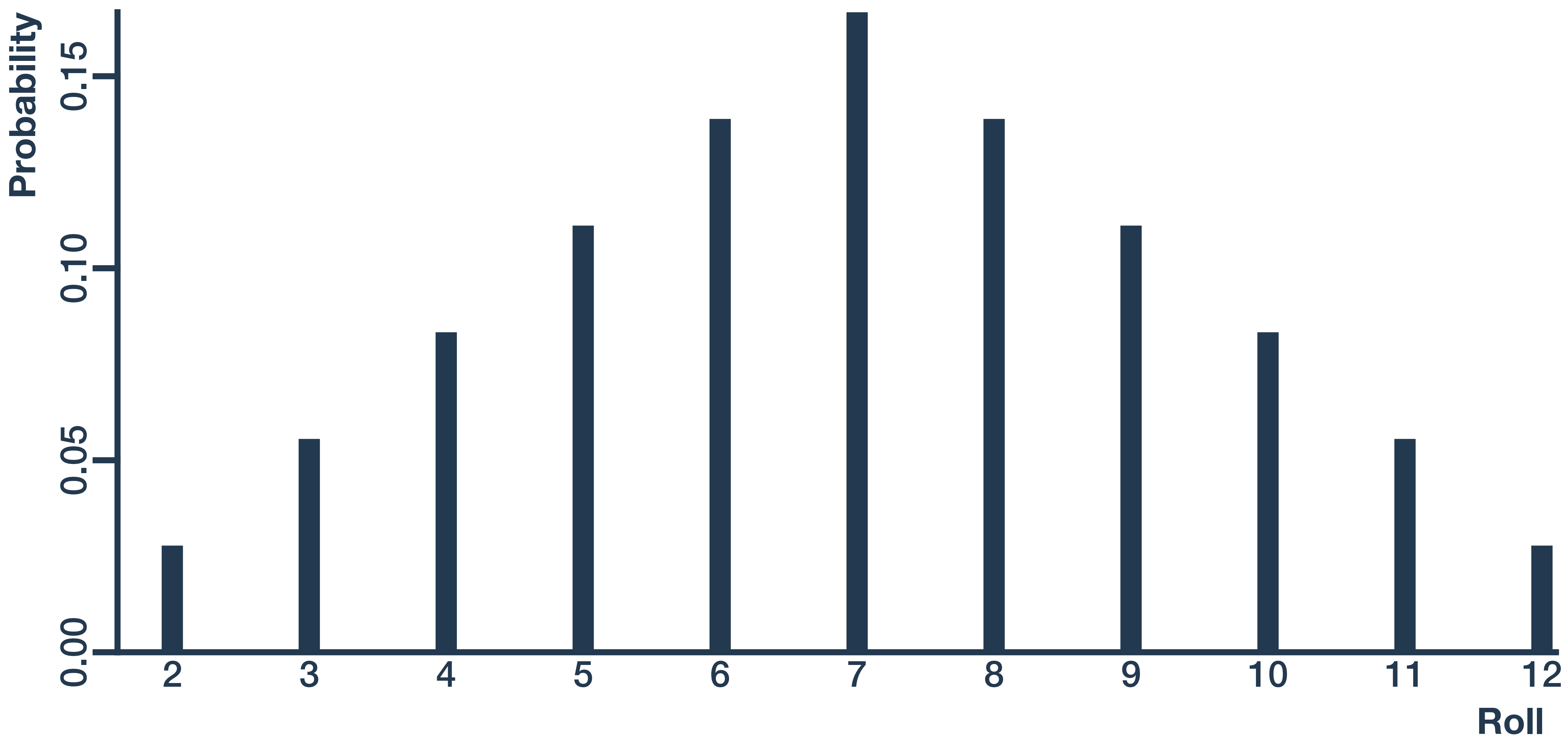
Agenda

1. Probability distributions
2. Summarizing random variables
3. Sampling from distributions

A discrete distribution

Probability mass function (PMF)
Sum of two fair dice



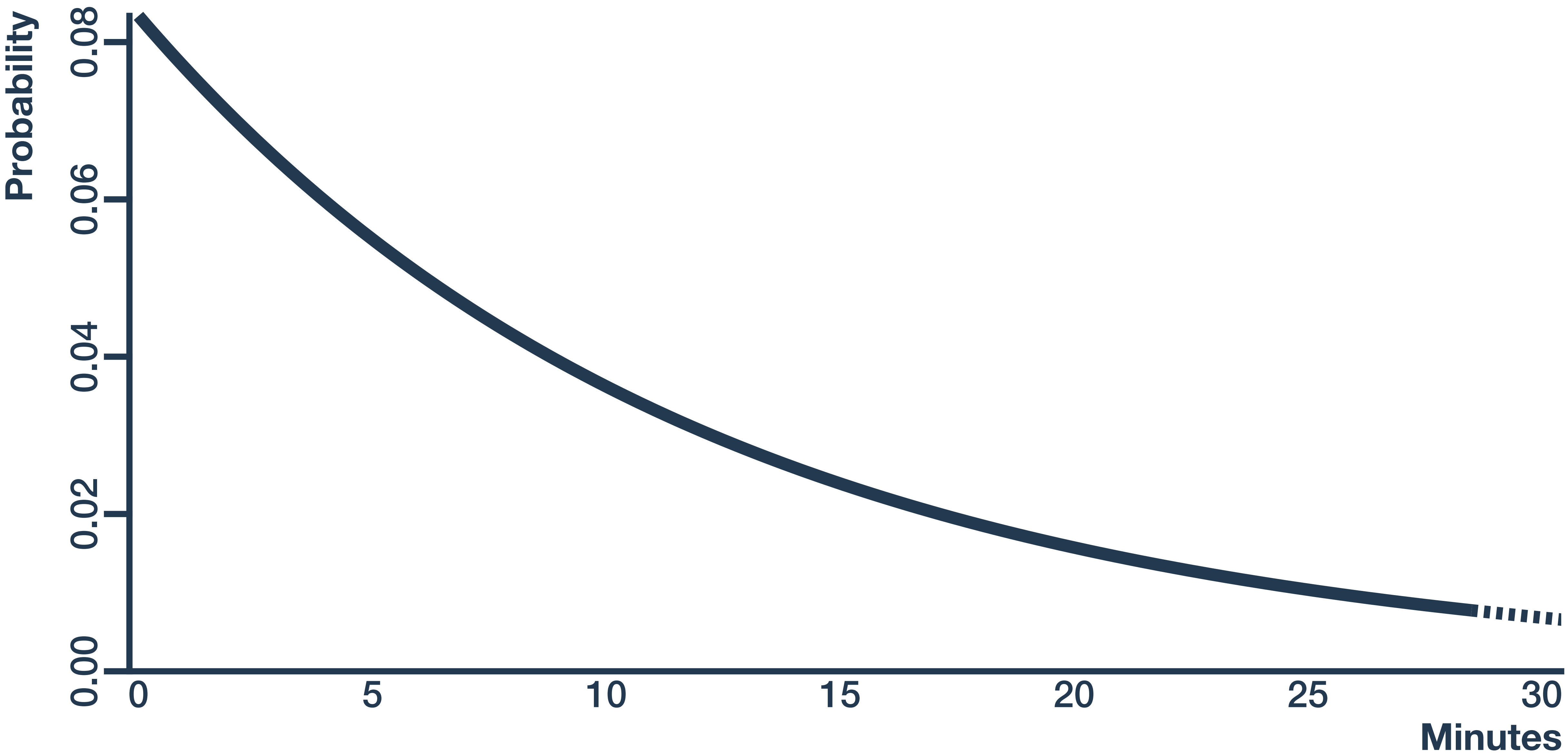


Support: integers from 2 to 12 (discrete)

A continuous distribution

Probability density function (PDF)

Time between Metro arrivals, (λ=1/12)



Support: non-negative, real [0,∞)

A discrete bivariate distribution

Contingency table

Questions measuring authoritarian attitudes

			X_2: Women should have to promise to obey their husbands when they get married.	
			Agree	**Disagree**
X_1	Gays and lesbians are just as healthy and moral as anybody else.	**Agree**	0.05	0.53
		Disagree	0.33	0.09

Joint distributions measure probability across multiple variables *and* the association between those variables

$\Pr(X_1{=}\text{A}, X_2{=}\text{A}) = 0.05$ $\Pr(X_1{=}\text{A}, X_2{=}\text{D}) = 0.53$

$\Pr(X_1{=}\text{D}, X_2{=}\text{A}) = 0.33$ $\Pr(X_1{=}\text{D}, X_2{=}\text{D}) = 0.09$

A discrete bivariate distribution

		X_2	
		Women should have to promise to obey their husbands when they get married.	
		Agree	**Disagree**
X_1 — Gays and lesbians are just as healthy and moral as anybody else.	**Agree**	0.05	0.53
	Disagree	0.33	0.09

Conditional probability: measures probability of one variable in a joint distribution, holding the other constant

$\Pr(X_2 \mid X_1{=}D)$		**Agree**	**Disagree**
	X_1 = **Disagree**	$\frac{0.33}{0.33+0.09} = 0.79$	$\frac{0.09}{0.33+0.09} = 0.21$

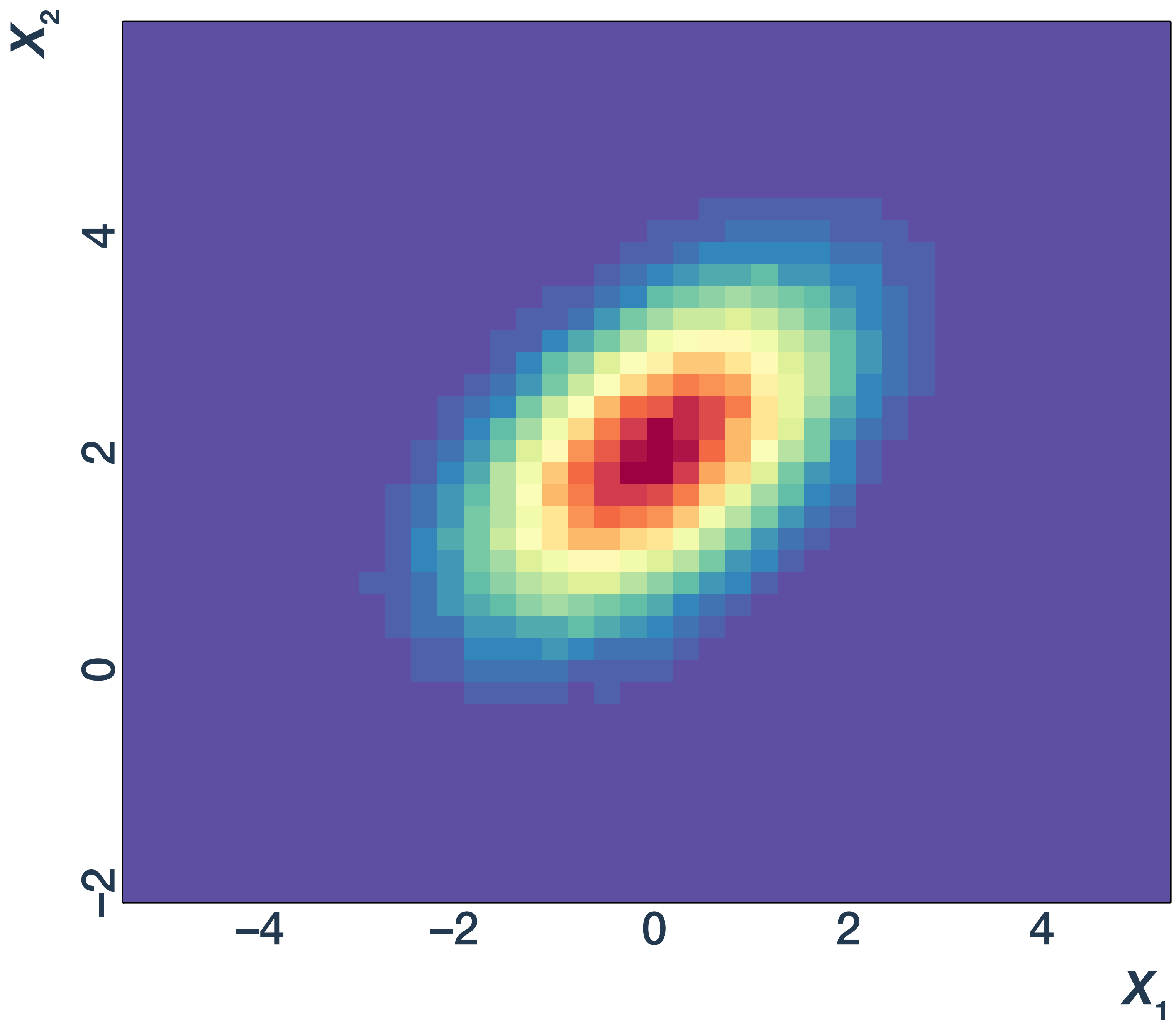
A discrete bivariate distribution

		X_2: Women should have to promise to obey their husbands when they get married.		
		Agree	**Disagree**	
X_1: Gays and lesbians are just as healthy and moral as anybody else.	**Agree**	0.05	0.53	**0.58**
	Disagree	0.33	0.09	**0.42**
		0.38	**0.62**	

Marginal probability: measures probability of one variable in a joint distribution, across all possible values of the other

	Agree	**Disagree**
$\Pr(X_2)$	$0.5 + 0.33 = 0.38$	$0.53 + 0.09 = 0.62$

A continuous bivariate distribution



$$X \sim \text{Norm}\left(\mu = (0, 2), \Sigma = \begin{bmatrix} 1.2 & 0.5 \\ 0.5 & 0.8 \end{bmatrix}\right)$$

Some common distributions

	Type	Parameters	Support
Binomial	*Discrete*	n, p	$\{0,\ldots,n\}$
Poisson	*Discrete*	λ	$\{0,1,2,\ldots\}$
Geometric	*Discrete*	p	$\{0,1,2,\ldots\}$
Normal (Gaussian)	*Continuous*	μ, σ	$(-\infty,\infty)$
Cauchy	*Continuous*	x_0, γ	$(-\infty,\infty)$
Beta	*Continuous*	α, β	$[0,1]$
Exponential	*Continuous*	λ	$[0,\infty)$

(Statisticians have devised and named *many* distributions over time.
See https://en.wikipedia.org/wiki/List_of_probability_distributions for an incomplete list)

Describing models

A language for describing probabilistic models
Using probability distributions to link our (known) data with our (unknown) parameters allows succinct communication

Data

Example from last week:
Estimating the unemployment rate p from count of unemployed (Y) in our sample of n individuals

$$Y \sim \mathrm{Binom}(n, p)$$

$$p \sim \mathrm{Unif}(0, 1)$$

Parameter

Hyperparameters

Describing models

A language for describing probabilistic models
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Example from last week:
Estimating the unemployment rate p from count of unemployed (Y) in our sample of n individuals

$$Y \sim \text{Binom}(n, p)$$
$$p \sim \text{Unif}(0, 1)$$

Changes to model are clear

$$Y \sim \text{Binom}(n, p)$$
$$p \sim \text{Beta}(1.01, 1.01)$$

A note on likelihood

$$\underbrace{\boxed{\Pr(p \mid n, Y)}}_{\text{Posterior probability}} = \frac{\underbrace{\boxed{\Pr(Y \mid n, p)}}_{\text{Likelihood}}\,\underbrace{\boxed{\Pr(p)}}_{\text{Prior probability}}}{\Pr(Y)}$$

Posterior and prior are distributions over *p*:

Posterior tells us "probability of any p, given the data"

Prior tells us "probability of any p, *a priori*"

When we plot posterior and prior for values of p, we see a valid probability distribution

Likelihood is a distribution over the *data*:

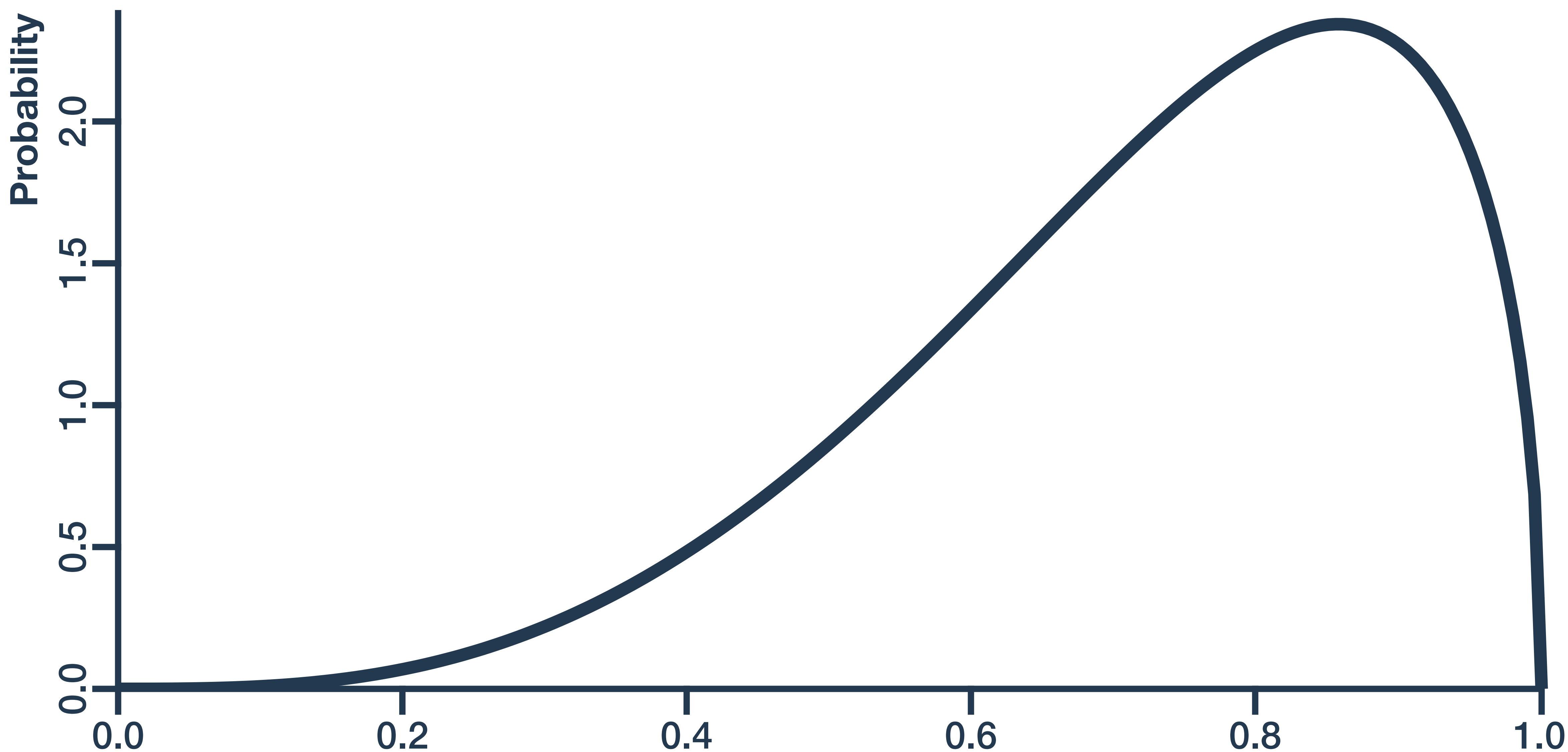
Likelihood tells us "probability of the data, given any p"

When we calculate that probability across values of p, it is not a proper probability distribution

Measure of how surprised we would be by our data for all possible values of p

Summarizing distributions

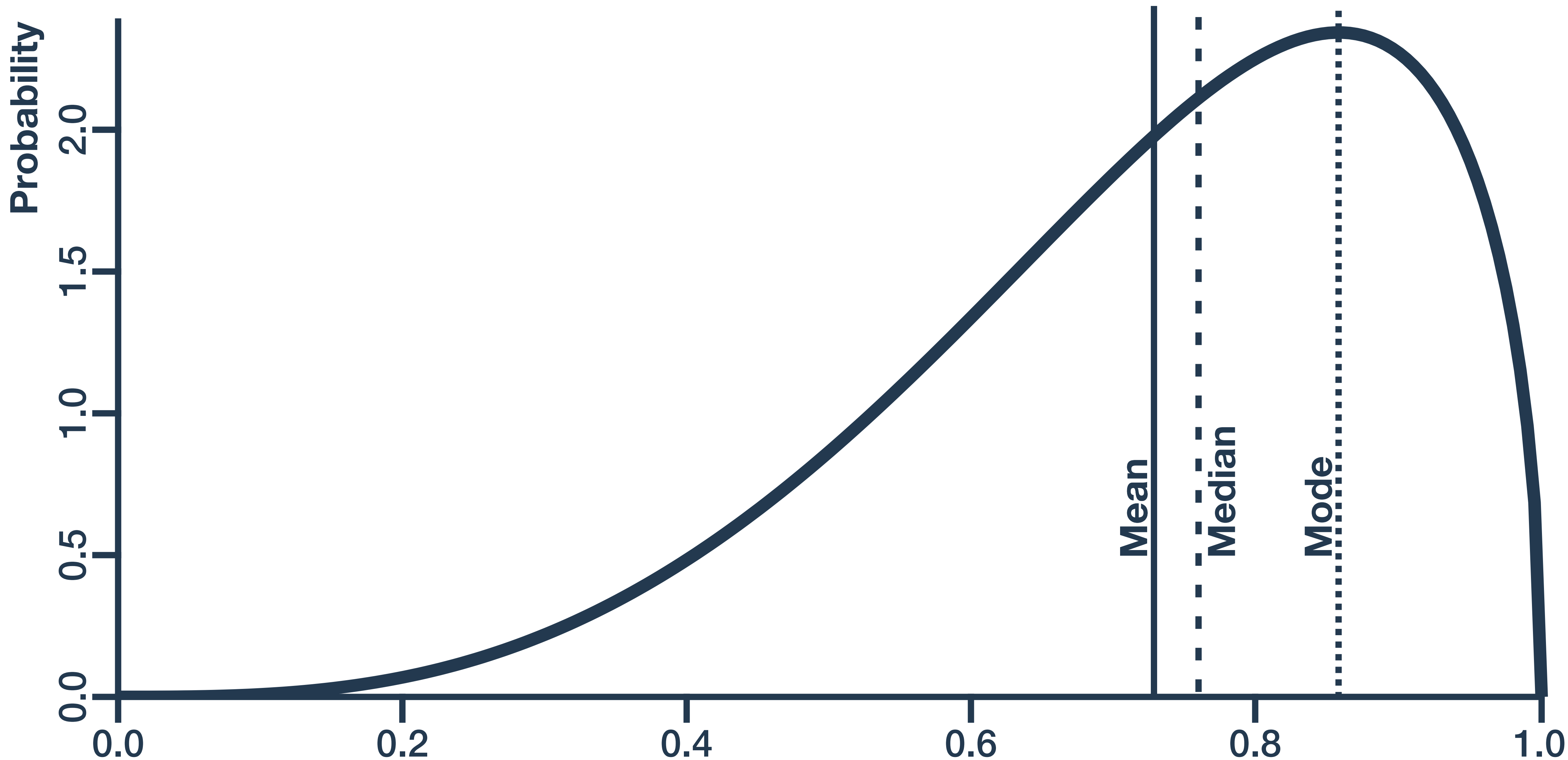
Communicating the shape of a distribution
Probability distributions like those that result from Bayesian analysis are complex



Beta(4, 1.5)

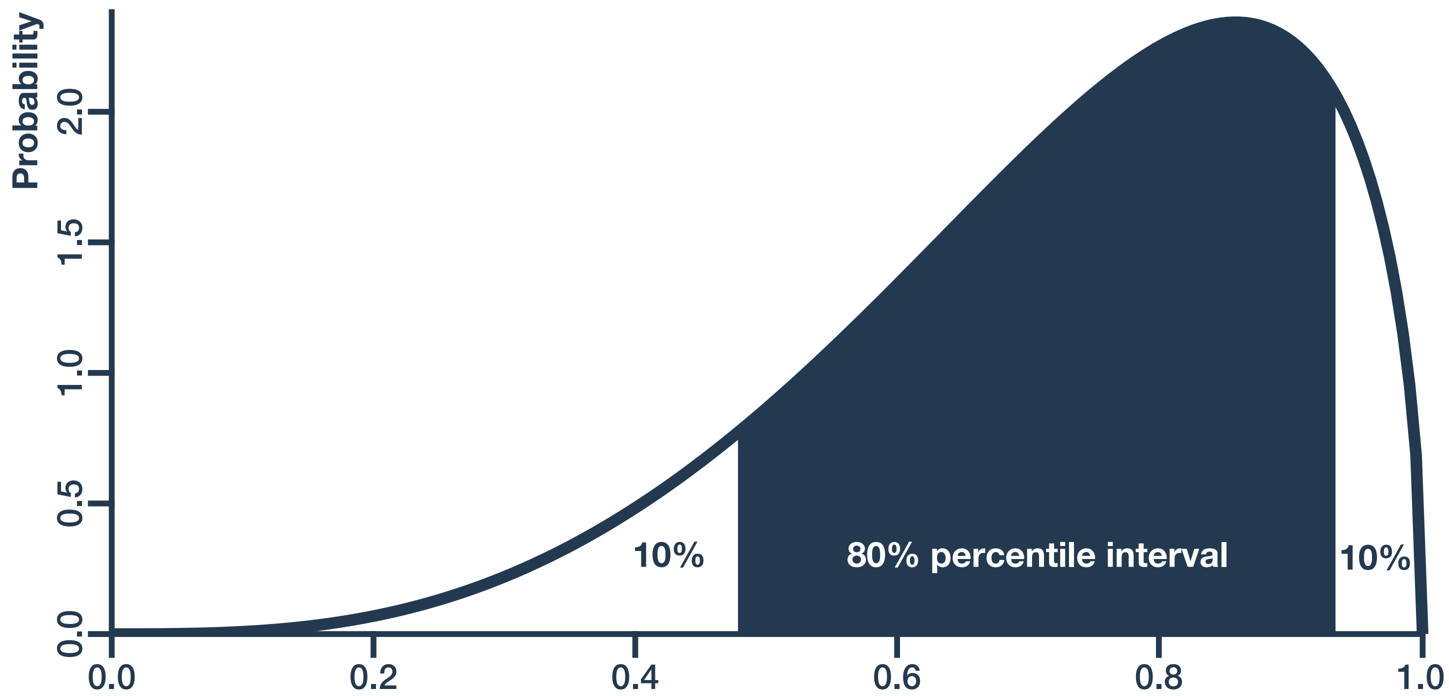
Summarizing distributions

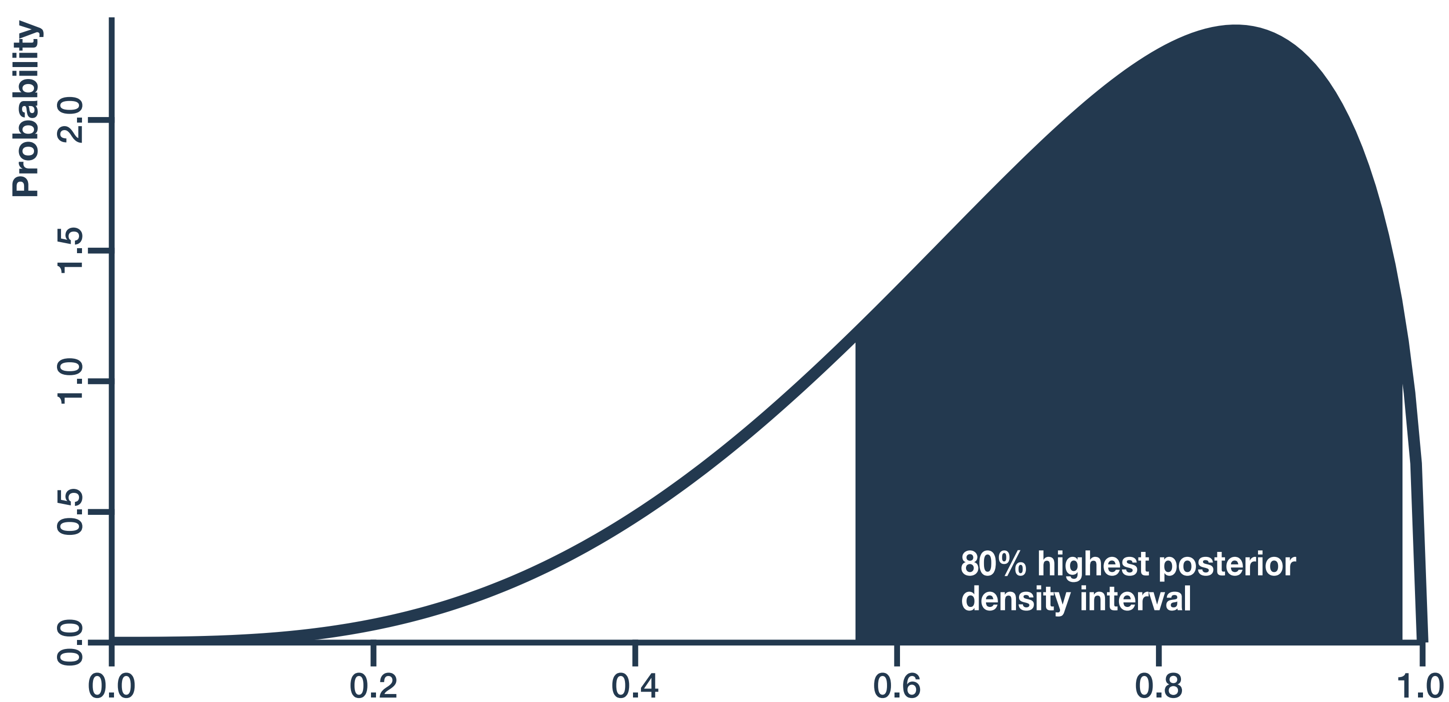
Point summaries
Describe the “center” of the distribution
Mean, median, and mode all have different meanings



Beta(4, 1.5)

Summarizing distributions





Credible intervals

Describe the “spread” of the distribution

Percentile intervals leave the same amount on either end of the distribution

Highest posterior density intervals find the narrowest possible interval