# Agenda

1. Counts as outcomes
2. The Poisson distribution and its assumptions
3. Log link function
4. Priors through a log link
5. Intercept-only Poisson regression in R
6. Poisson regression with covariates in R

# Counts as outcomes

## Kinds of counts

**Average and deviation**

An event that is technically a count, but the scale of process means we can treat it as continuous
E.g. immigration rate, unemployment rate, etc.

**Normal distribution**

**Trials and probability of success**

Outcome could have happened at most N times, our data measures how many times it did happen
E.g. *individual* unemployment, “how many days per week…”, etc.

**Binomial / Bernoulli distribution**

**Rate of occurrence**

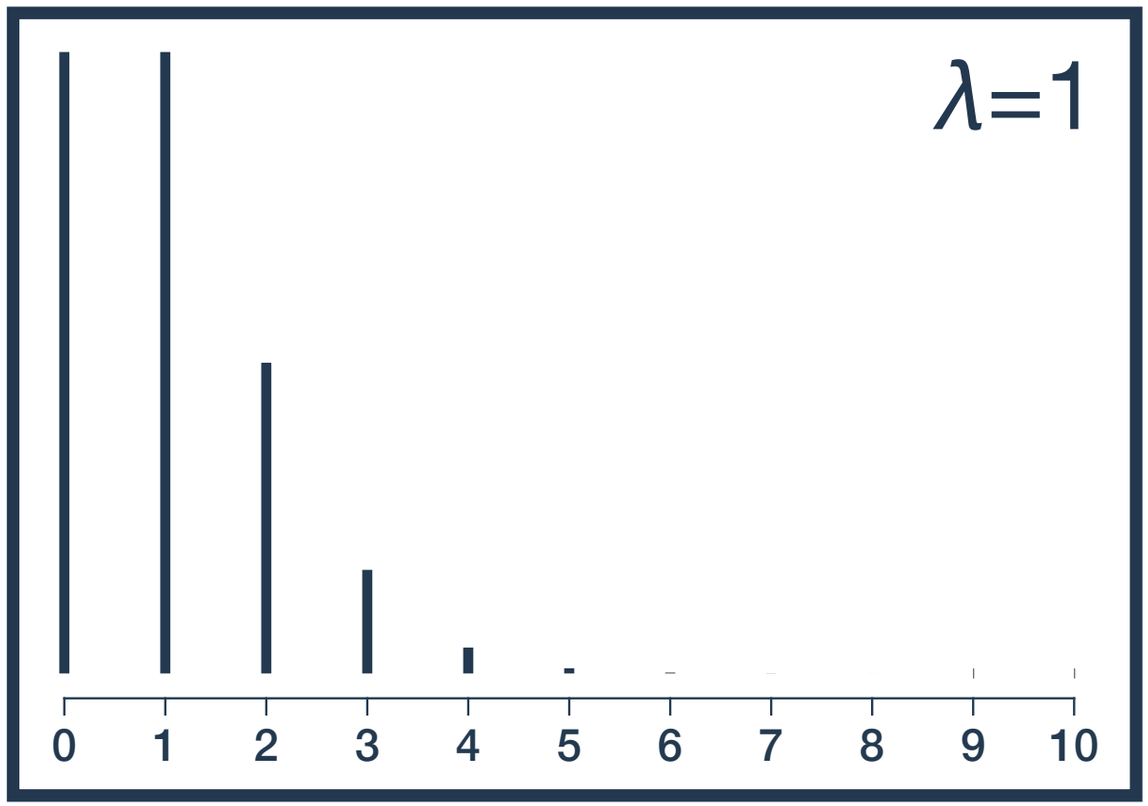
An event that has no (practical) upper limit, but tends to happen at a relatively low rate
E.g. number of friends, grocery stores in a neighbourhood, etc.

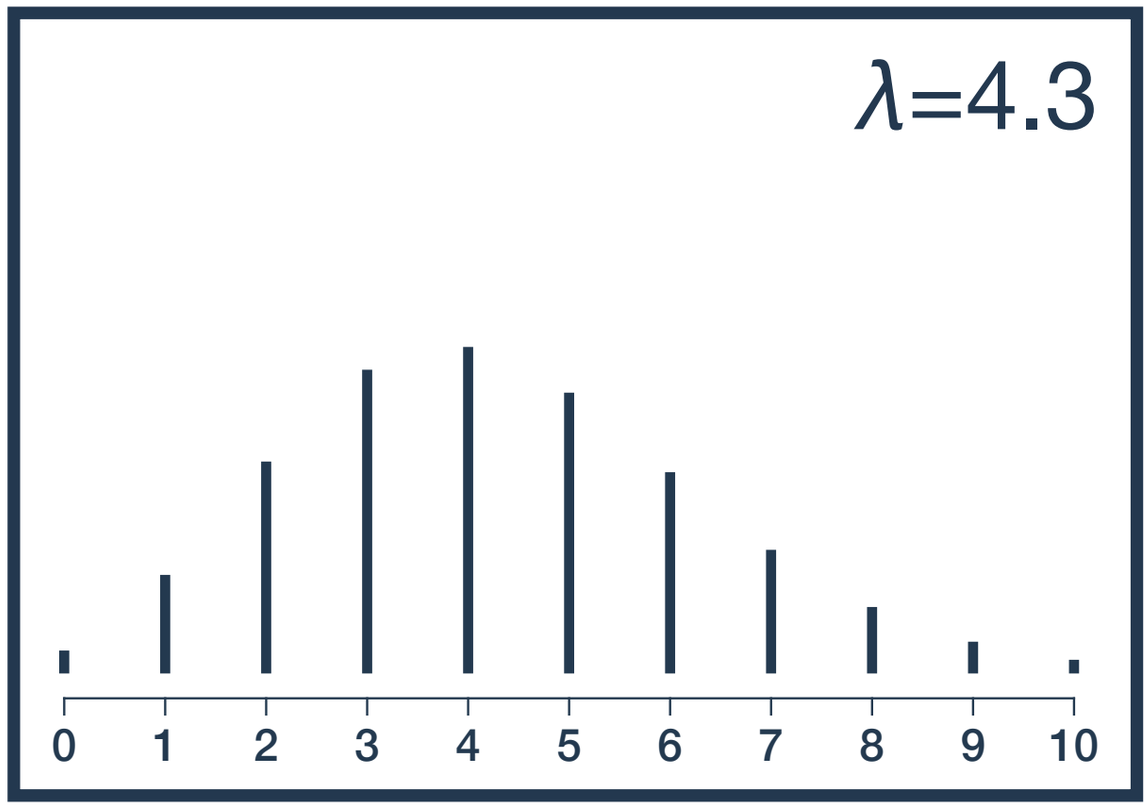
**Poisson distribution**

# Poisson distribution

The ***Poisson distribution*** gives the probability that an event will happen $k$ times in a particular unit of time or space if it has an average rate of occurrence of $\lambda$ in that unit of time or space.

$$\text{Prob}(k|\lambda) = \frac{\lambda^k e^{-\lambda}}{k!}$$



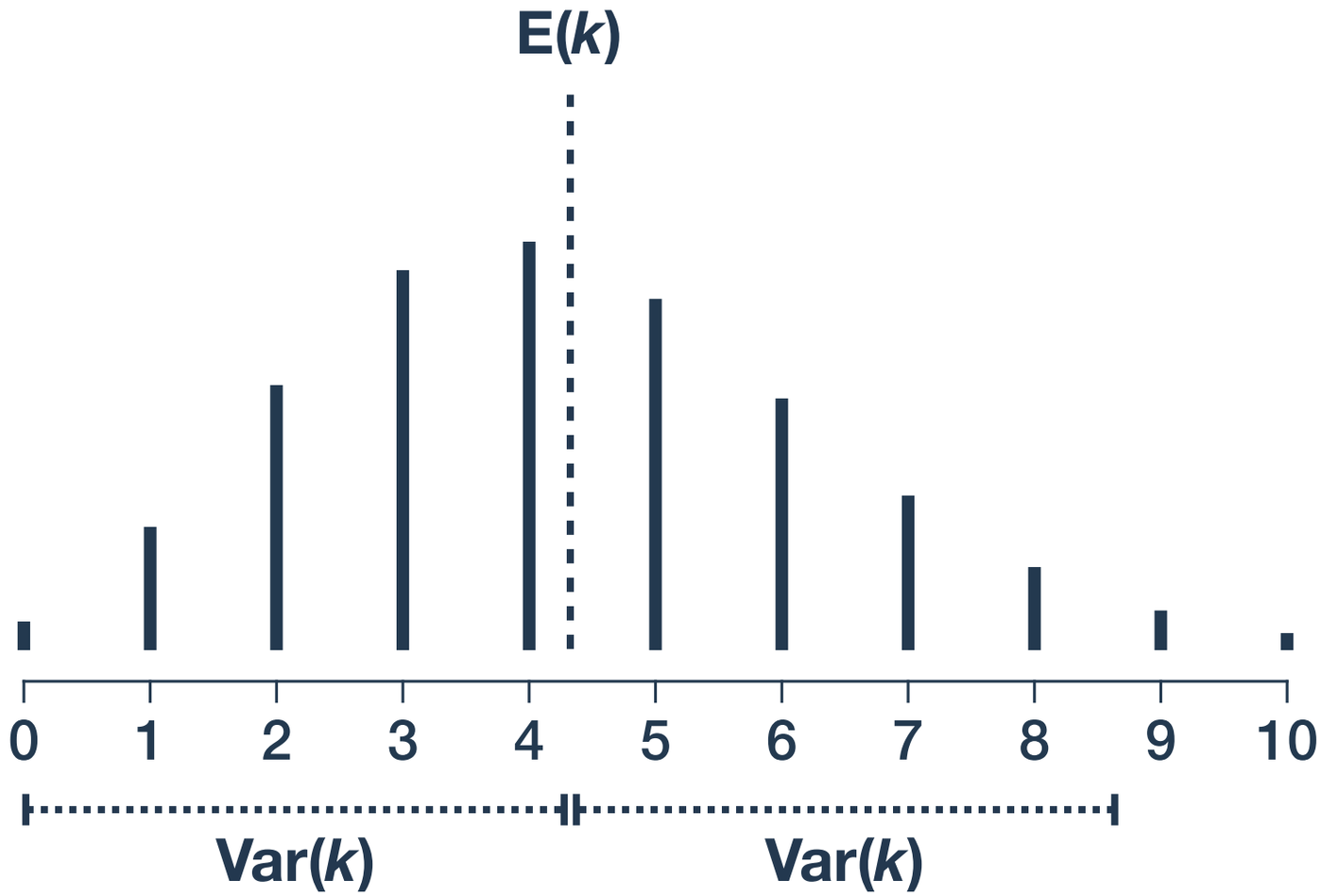



# Poisson distribution

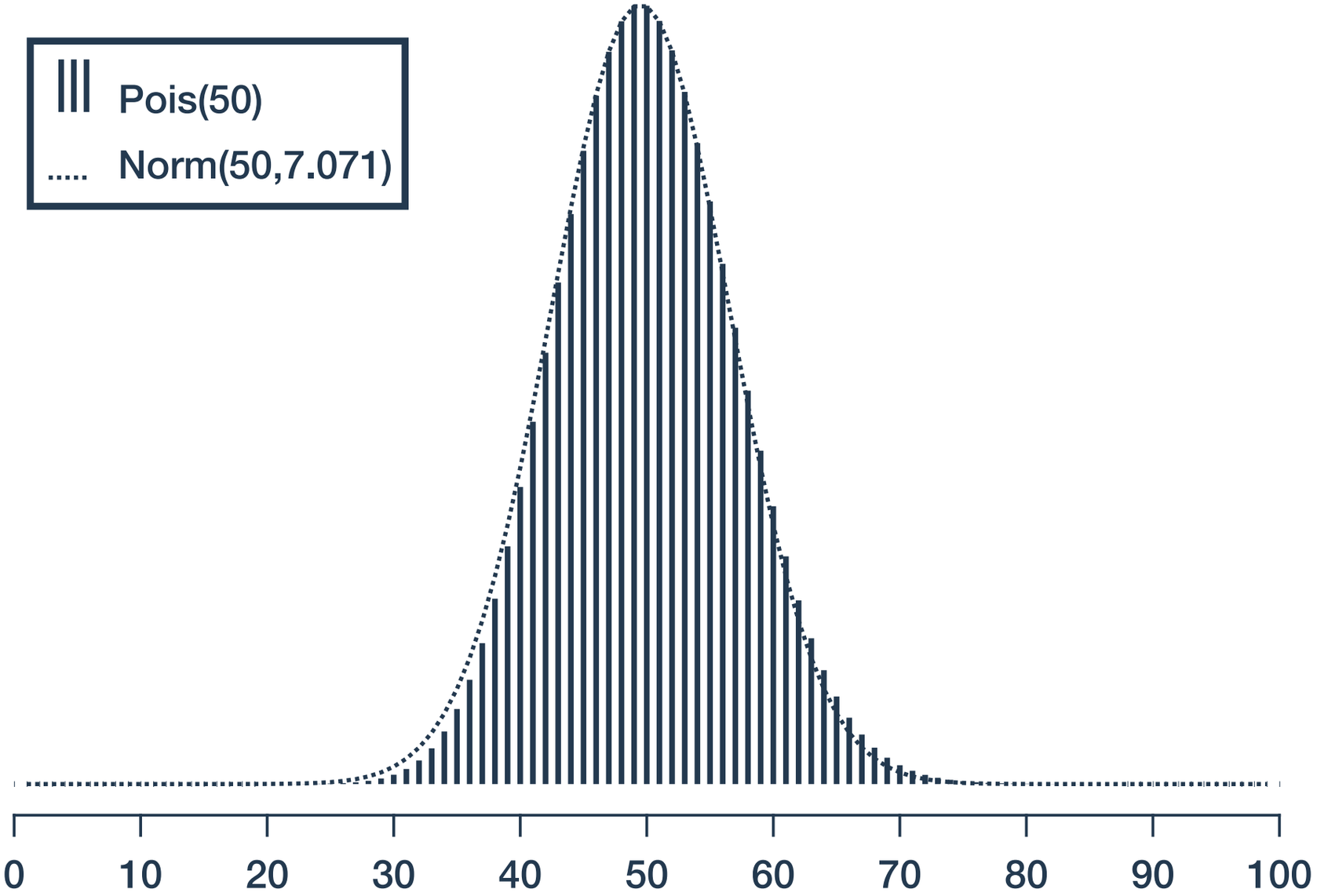
$$k \sim \text{Pois}(\lambda)$$

**Single parameter**

$\lambda$ is both the *mean* and the *variance* (s.d. squared) of the Poisson distribution

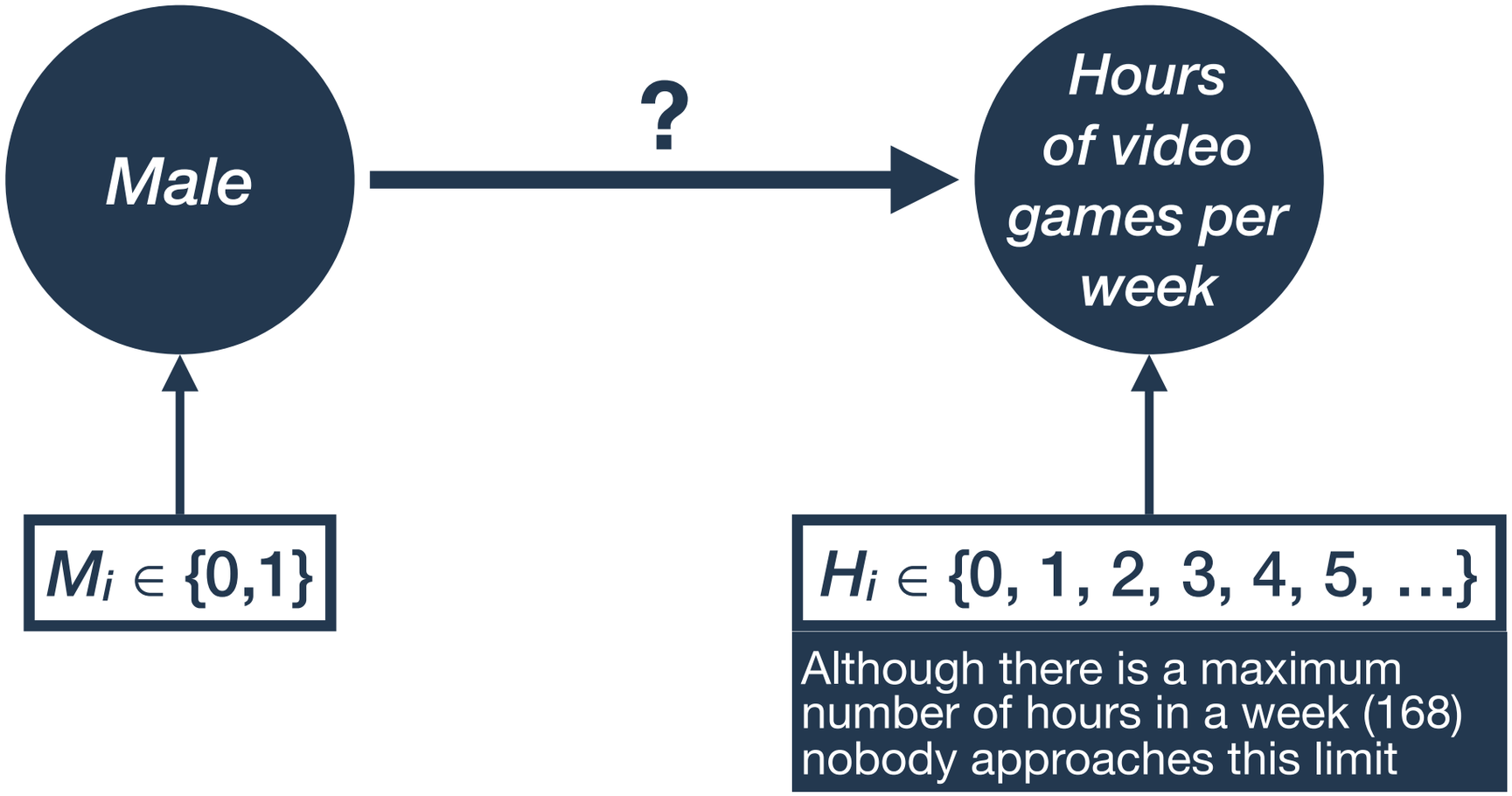


# Poisson distribution




Poisson with large $\lambda$ approximates a normal distribution

# Poisson-distributed outcome




$$H_i \sim \text{Pois}(\lambda)$$

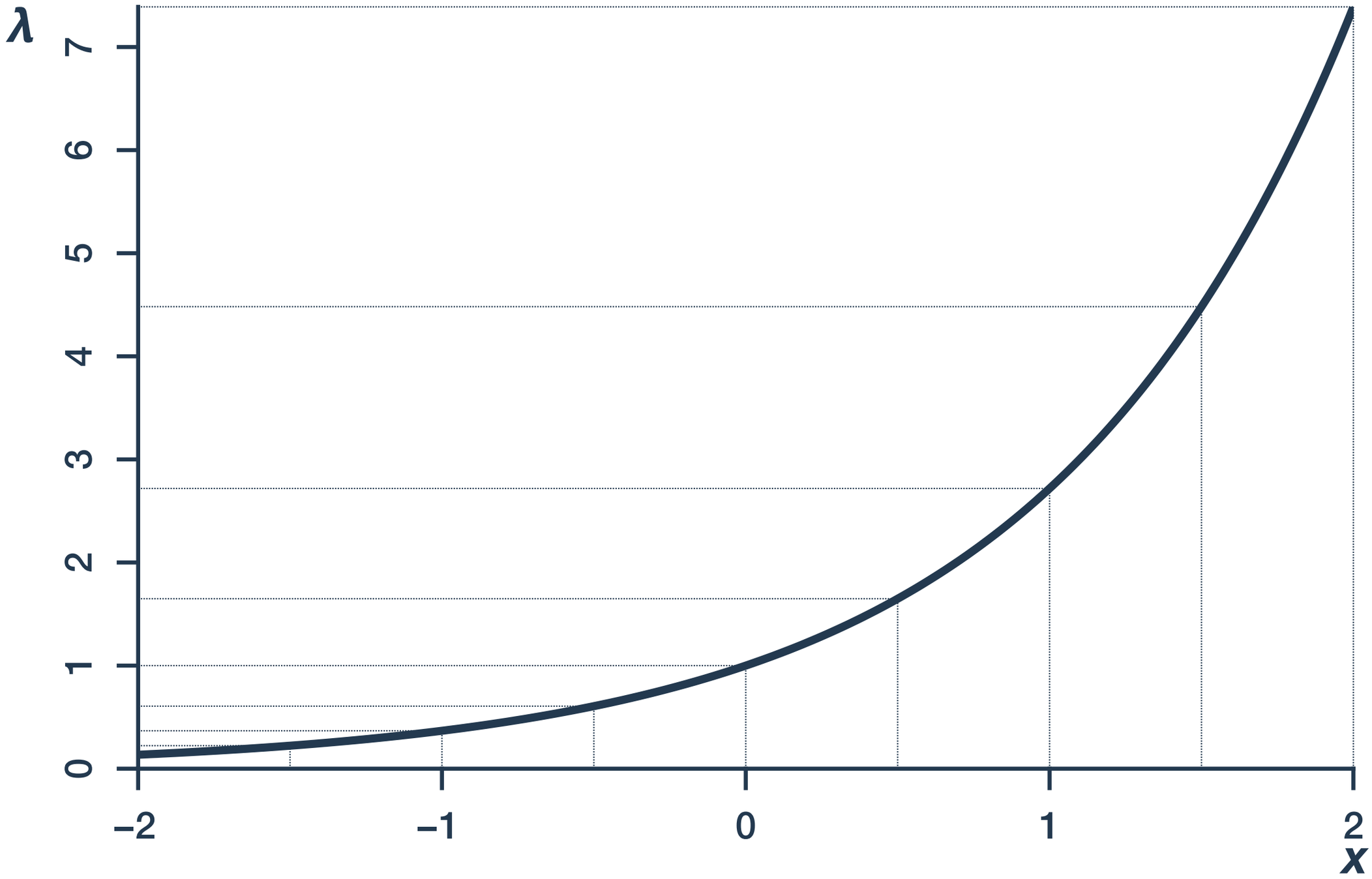
$$f(\lambda) = \alpha + \beta M_i$$

# Log link function

$$\log(\lambda) = x$$

$$\lambda = e^{x}$$

$\lambda \in [0, \infty)$

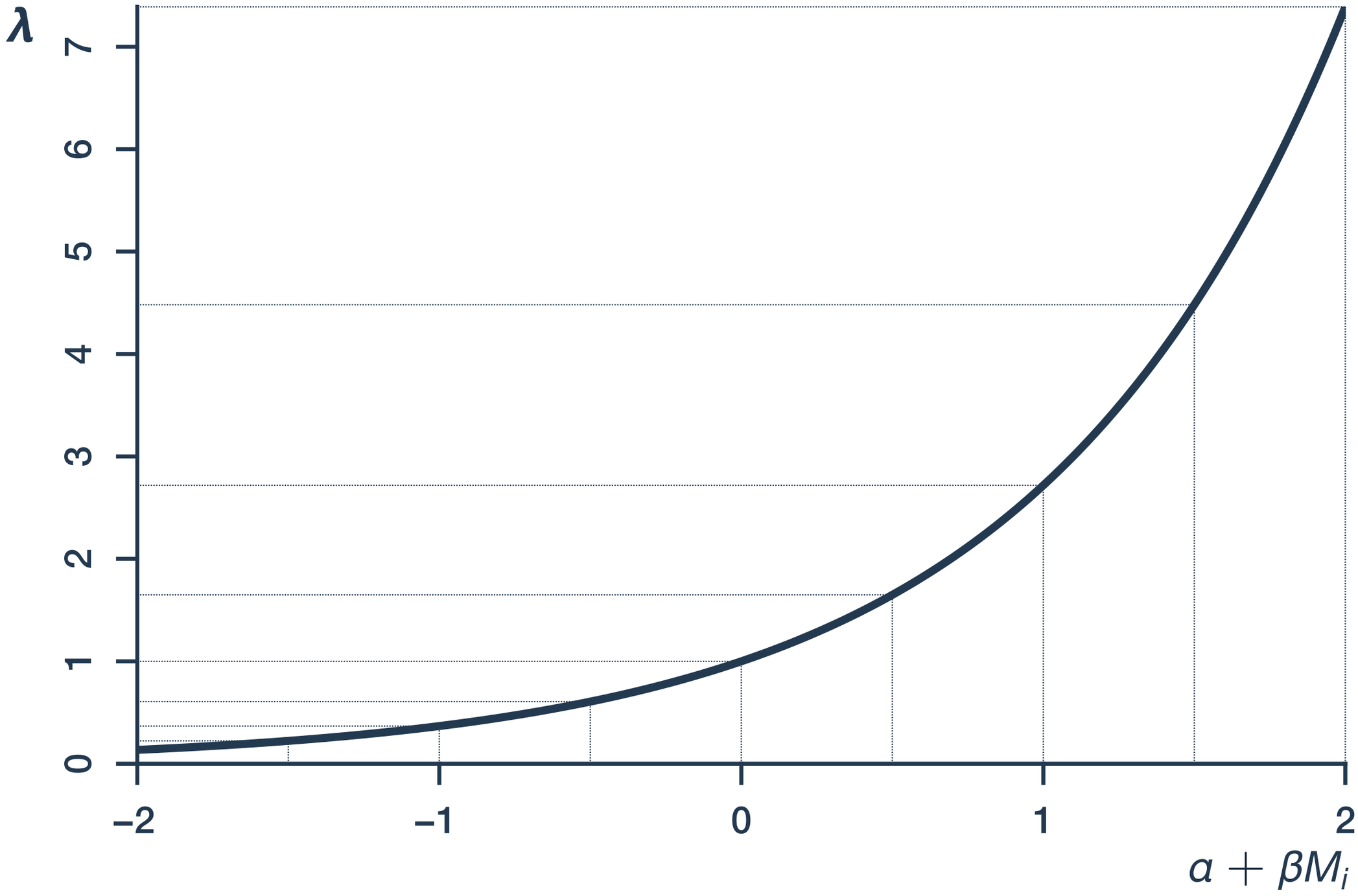



$x \in (-\infty, \infty)$

# Log link function

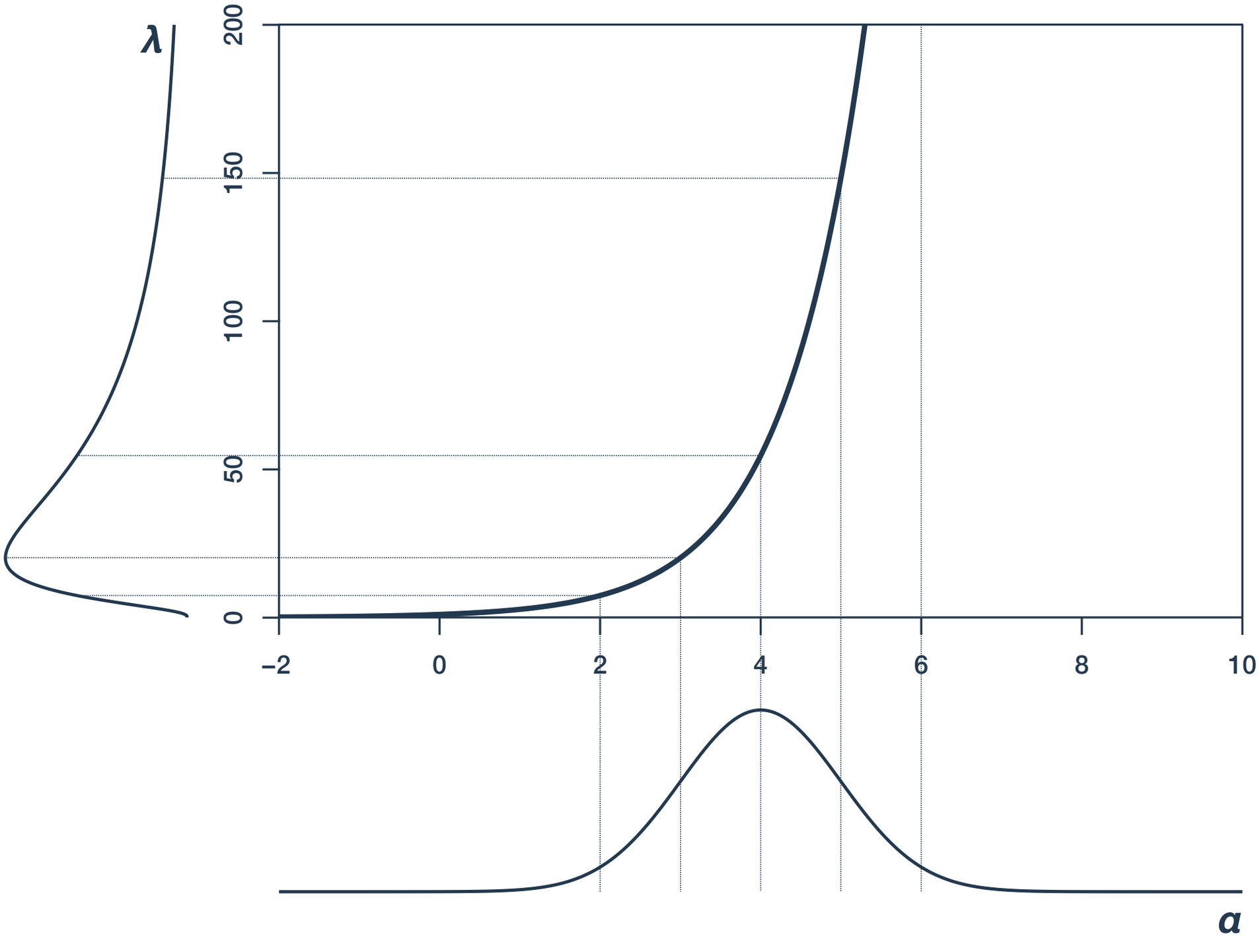
$$H_i \sim \text{Pois}(\lambda)$$

$$\log(\lambda) = \alpha + \beta M_i$$

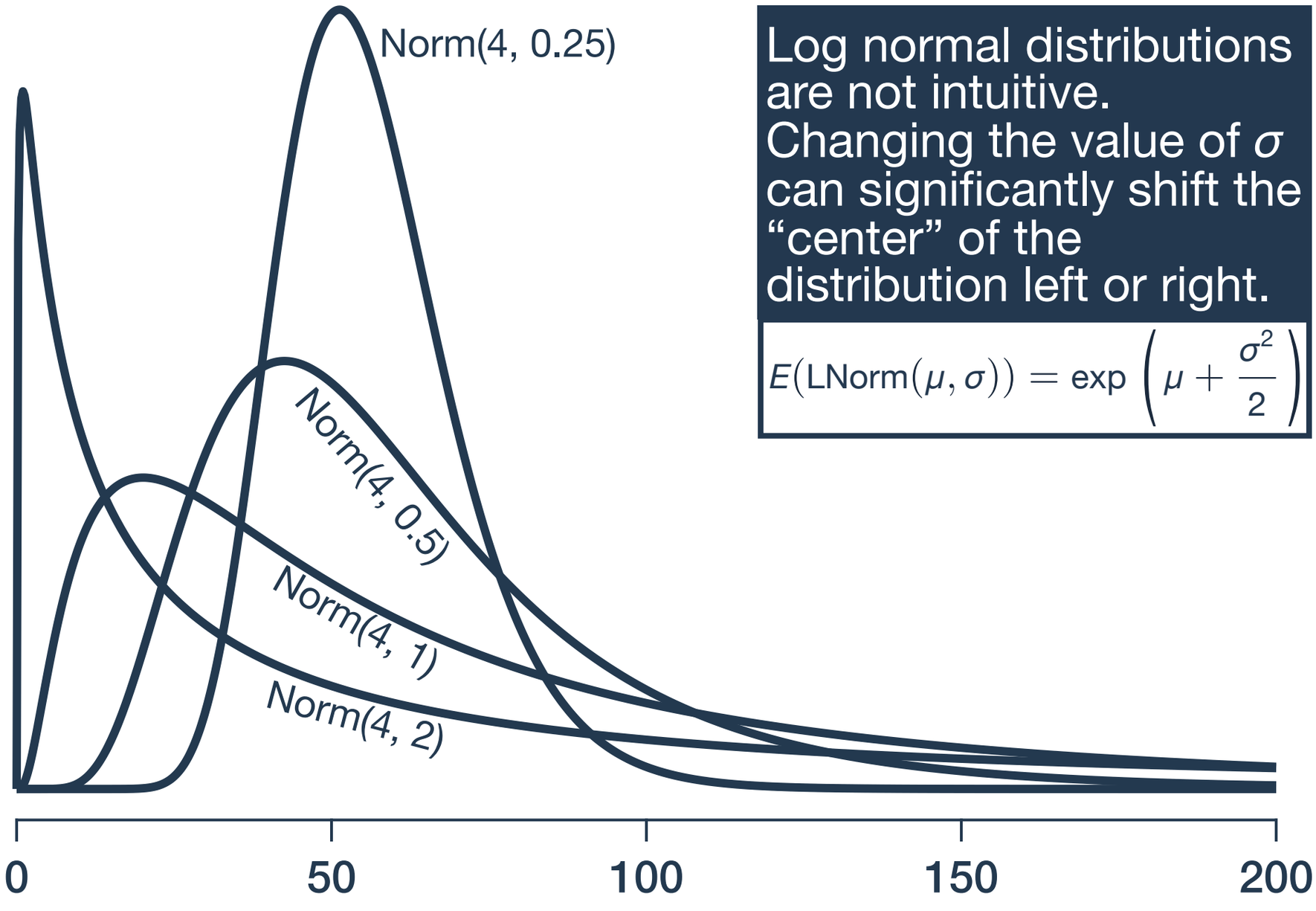


# Log link priors

$$a \sim \text{Norm}(4, 1)$$



# Log link priors

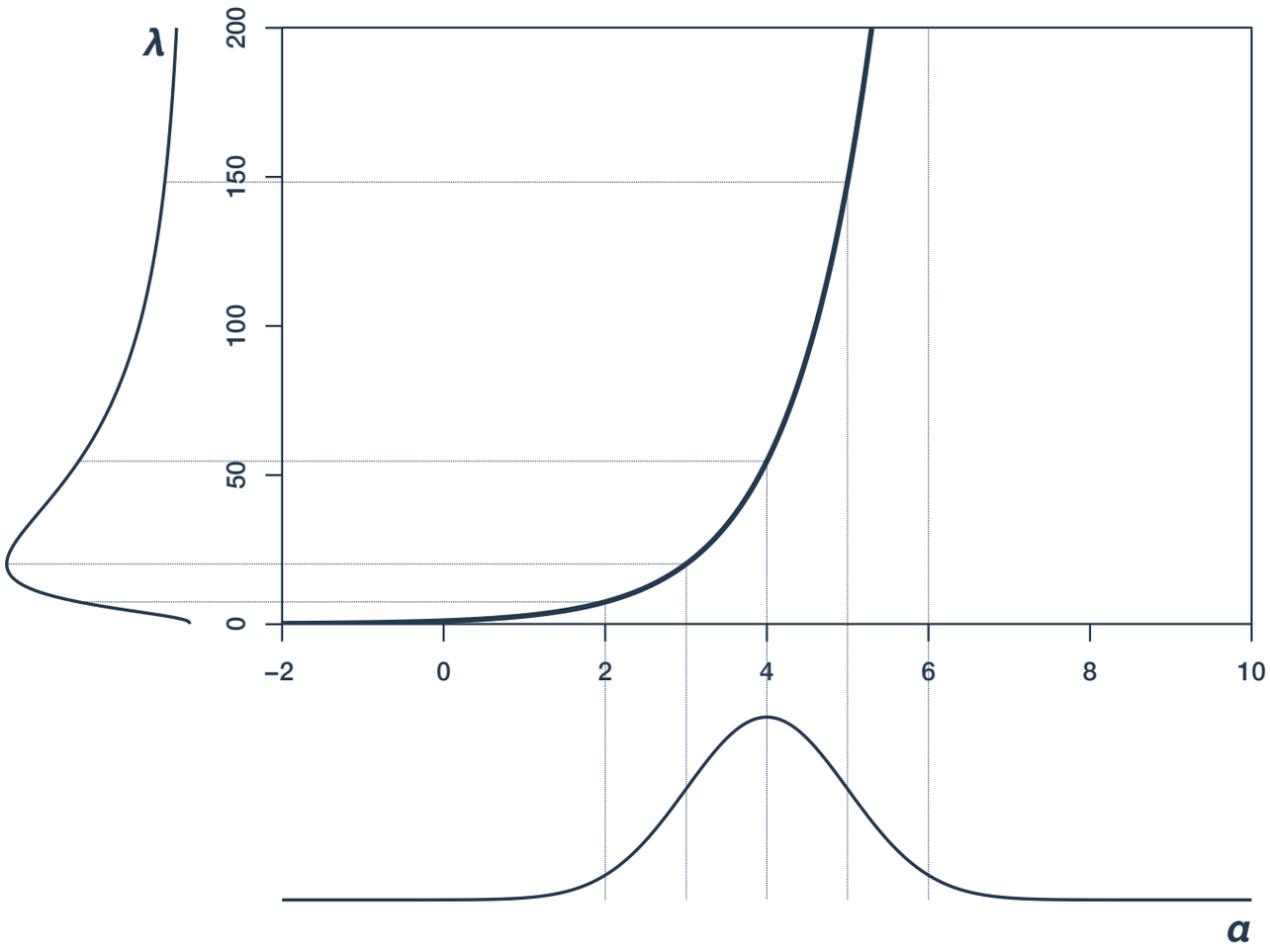



Log normal distributions are not intuitive. Changing the value of $\sigma$ can significantly shift the "center" of the distribution left or right.

$$E(\text{LNorm}(\mu, \sigma)) = \exp\left(\mu + \frac{\sigma^2}{2}\right)$$

# Log link priors

$$a \sim \text{Norm}(4, 1)$$




**Compresses lefthand side of prior** | Log link function maps $a \leq 0$ into the interval $\lambda \in [0,1]$

**Expands righthand side of prior** | Log link function maps $a \geq 6$ into the interval $\lambda \in [400,\infty)$

**Prior depends on what is reasonable in data** | Events you expect to be rare should use a narrower prior