Agenda

1. **Adding predictors to logistic regressions**
2. **Odds (and odds ratios) versus probabilities**
3. **Transforming posterior distributions**
4. **Prior predictive simulation in R**

Cocaine use among adolescents

$$C_i \sim \text{Bernoulli}(p_i)$$

$$\text{logit}(p_i) = \alpha + \beta G_i$$

$$\alpha \sim \text{Norm}(0, 1.5)$$

$$\beta \sim \boxed{\text{Norm}(0, 0.3)}$$

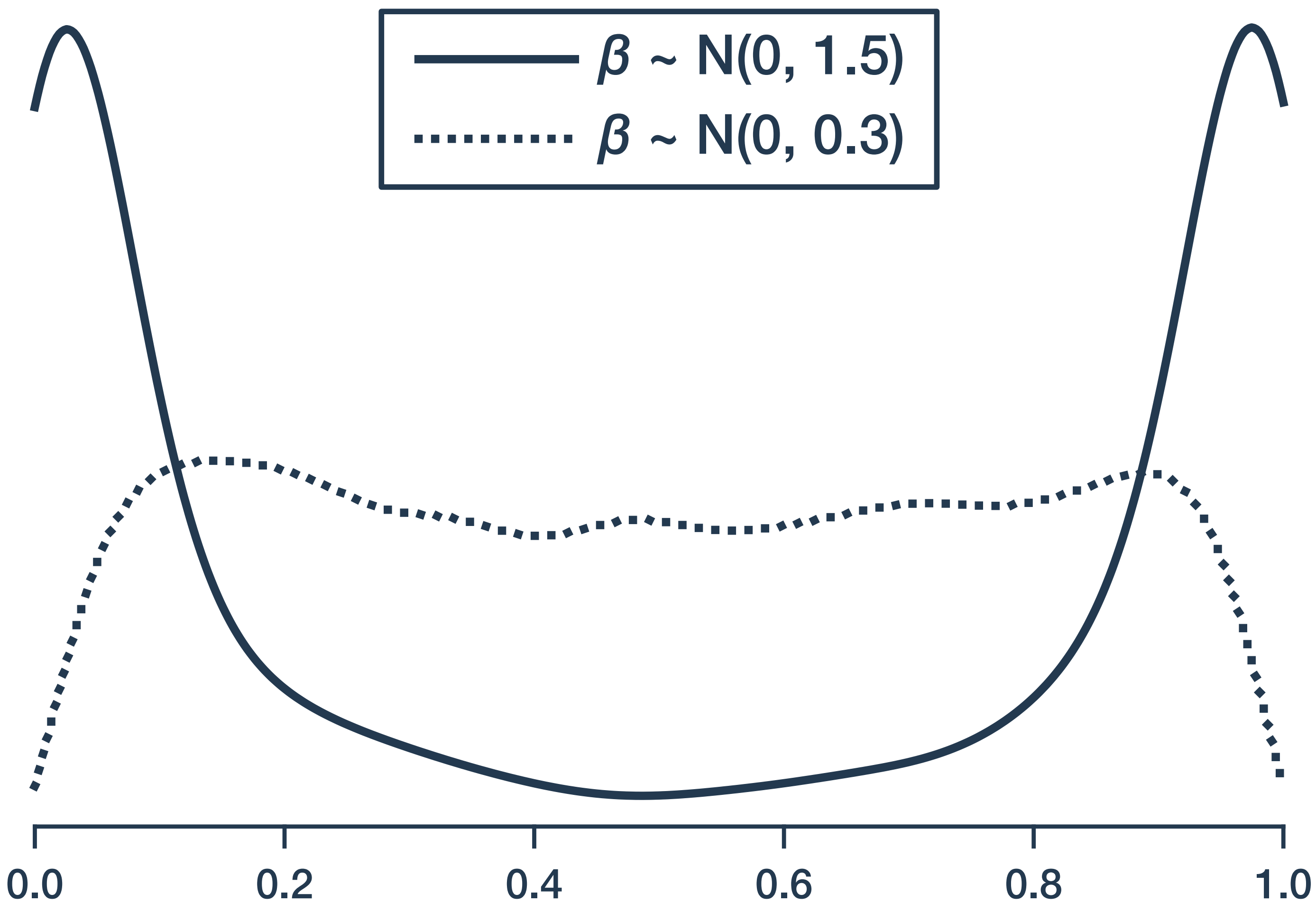
Where did this prior come from?

Priors in logistic regressions

$$C_i \sim \text{Bernoulli}(p_i)$$
$$\text{logit}(p_i) = \alpha + \beta G_i$$

$$\alpha \sim \text{Norm}(0, 1.5)$$

Prior predictive simulation
(min. grade)



Logistic regression coefficients

$$C_i \sim \text{Bernoulli}(p_i)$$
$$\text{logit}(p_i) = \alpha + \beta G_i$$

$$\alpha \sim \text{Norm}(0, 1.5)$$
$$\beta \sim \text{Norm}(0, 0.3)$$

	Mean	90% HPDI	
α	-4.59	-5.41	-3.91
β	0.12	0.05	0.20

Interpreting α | "The expected probability of having tried cocaine for a student with G_i=0 is: **logistic(-4.59) = 0.01 = 1%**"

Standardized G_i | "The expected probability of having tried cocaine for a student in grade 9.54: **logistic(-4.59) = 0.01 = 1%**"

Logistic regression coefficients

Interpreting β

Odds ratio

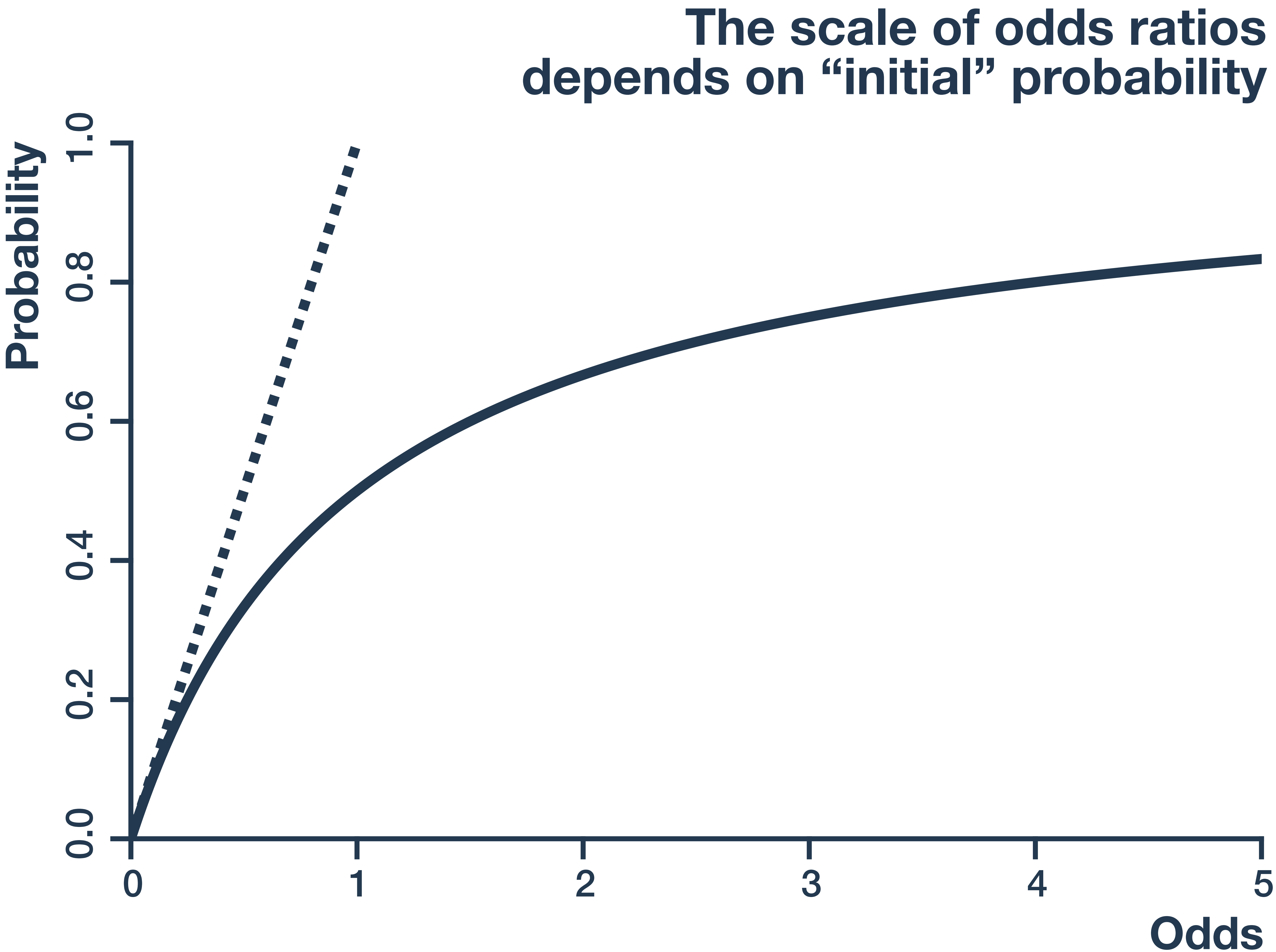
$$\exp(\beta) = \frac{\left(\frac{p^{G=1}}{1-p^{G=1}}\right)}{\left(\frac{p^{G=0}}{1-p^{G=0}}\right)}$$

$$\begin{aligned}
\operatorname{logit}(p) &= \alpha + \beta G \\
\log\left(\frac{p}{1-p}\right) &= \alpha + \beta G \\
\frac{p}{1-p} &= \exp(\alpha + \beta G) \\
\frac{p}{1-p} &= \exp(\alpha) \times \exp(\beta G)
\end{aligned}$$

"For every unit increase in the covariate, the expected *odds* of the outcome is multiplied by exp(β)"

"For every 1.67 grades a student completes, their expected *odds* of trying cocaine increased by 13%"

Odds ratios



The scale of odds ratios
depends on “initial” probability
Probability
0.0
0.2
0.4
0.6
0.8
1.0
0
1
2
3
4
5
Odds

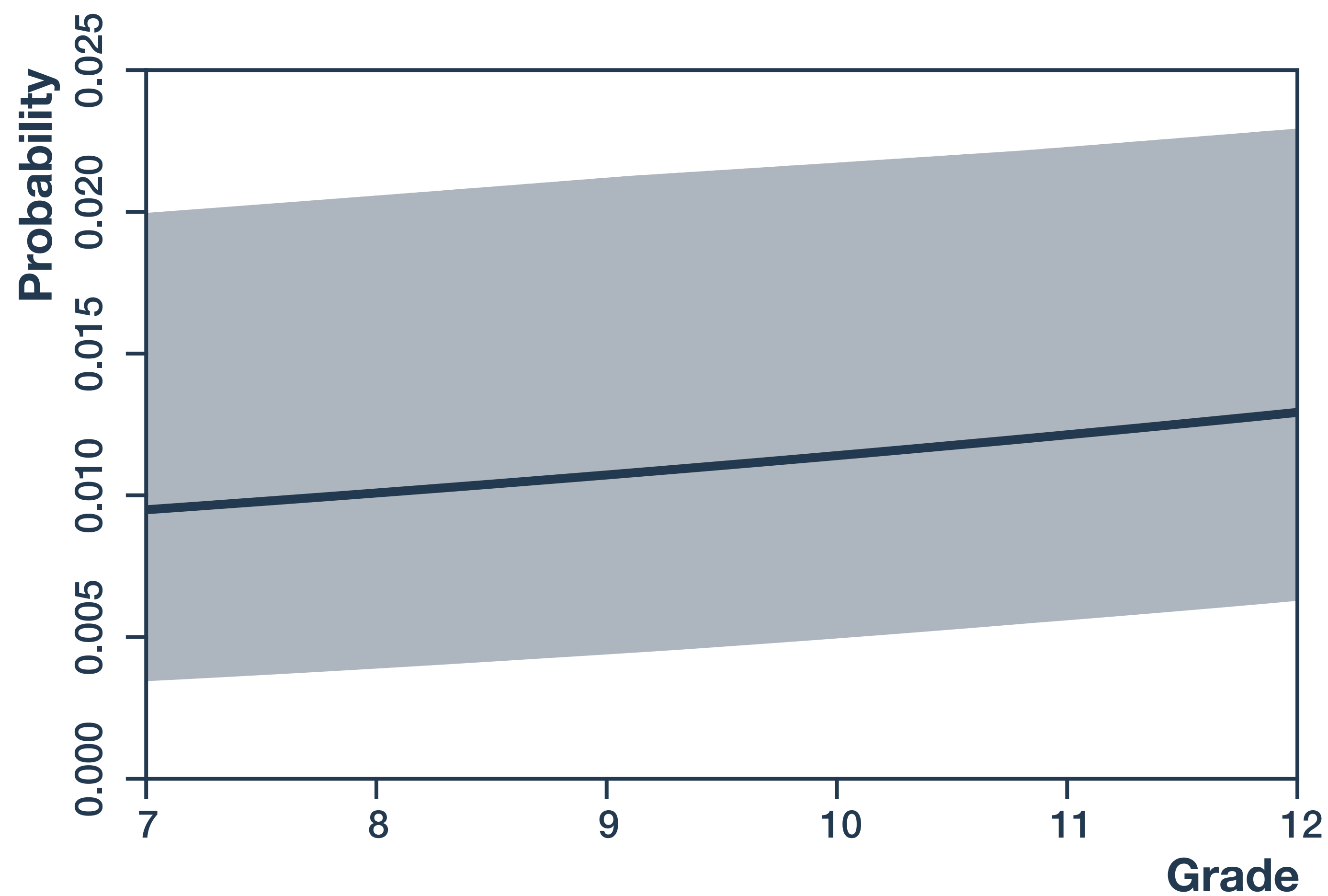
Logistic regression coefficients

Alternatives to odds ratios

Cases

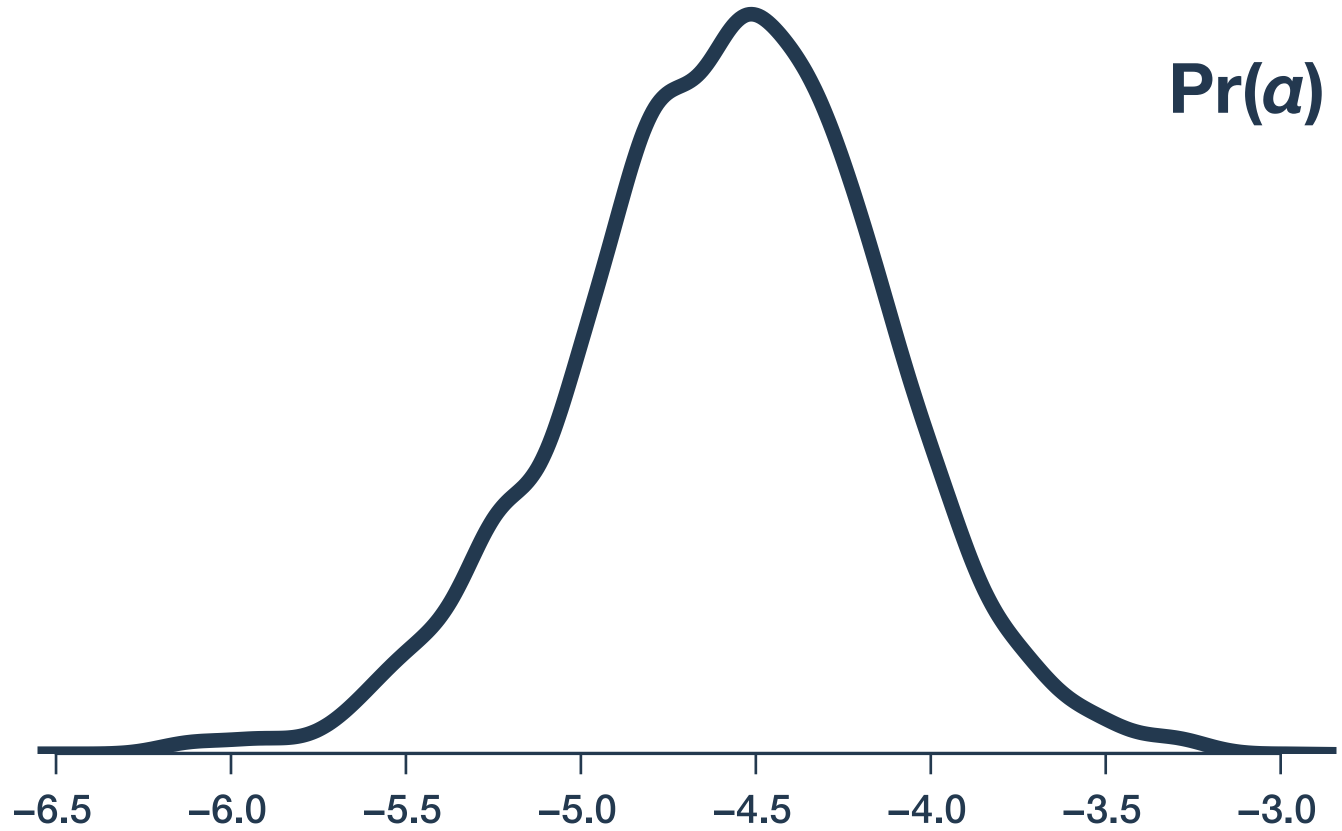
“An average 7th grade student has about a 0.83% chance of having tried cocaine, while for an average 12th grader, that probability is about 1.21%”

Posterior visualization

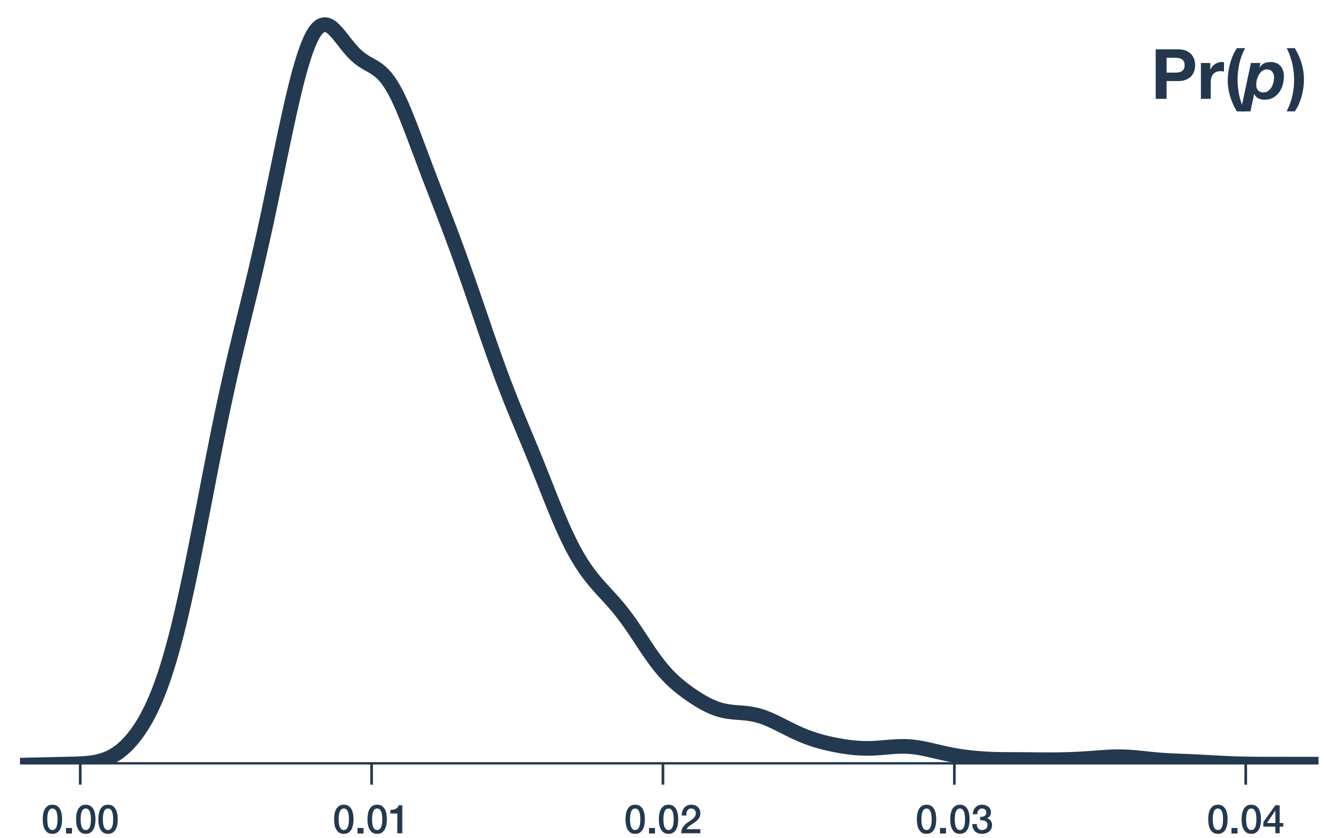


Logistic posteriors

Symmetric posterior distribution for $\alpha + \beta G_i$



Skewed posterior distribution for $p = \text{logit}^{-1}(\alpha + \beta G_i)$



Adding covariates

G_i: Grade in school (standardized)

D_i: Delinquency (standardized)

W_i: White (indicator)

$$C_i \sim \text{Bernoulli}(p_i)$$
$$\text{logit}(p_i) = \alpha + \beta_G G_i + \beta_D D_i + \beta_W W_i$$

$$\alpha \sim \text{Norm}(0, 1.5)$$
$$\beta_G \sim \text{Norm}(0, 0.5)$$
$$\beta_D \sim \text{Norm}(0, 0.5)$$
$$\beta_W \sim \text{Norm}(0, 0.5)$$

	Mean	90% HPDI	
α	-6.26	-7.25	-5.30
β_G	0.18	0.09	0.27
β_D	0.90	0.77	0.99
β_W	0.70	0.40	1.00

$\text{logit}^{-1}(\alpha)$ = 0.0019

$\exp(\beta_G)$ = 1.2022

$\exp(\beta_D)$ = 2.4611

$\exp(\beta_W)$ = 2.0164