Agenda
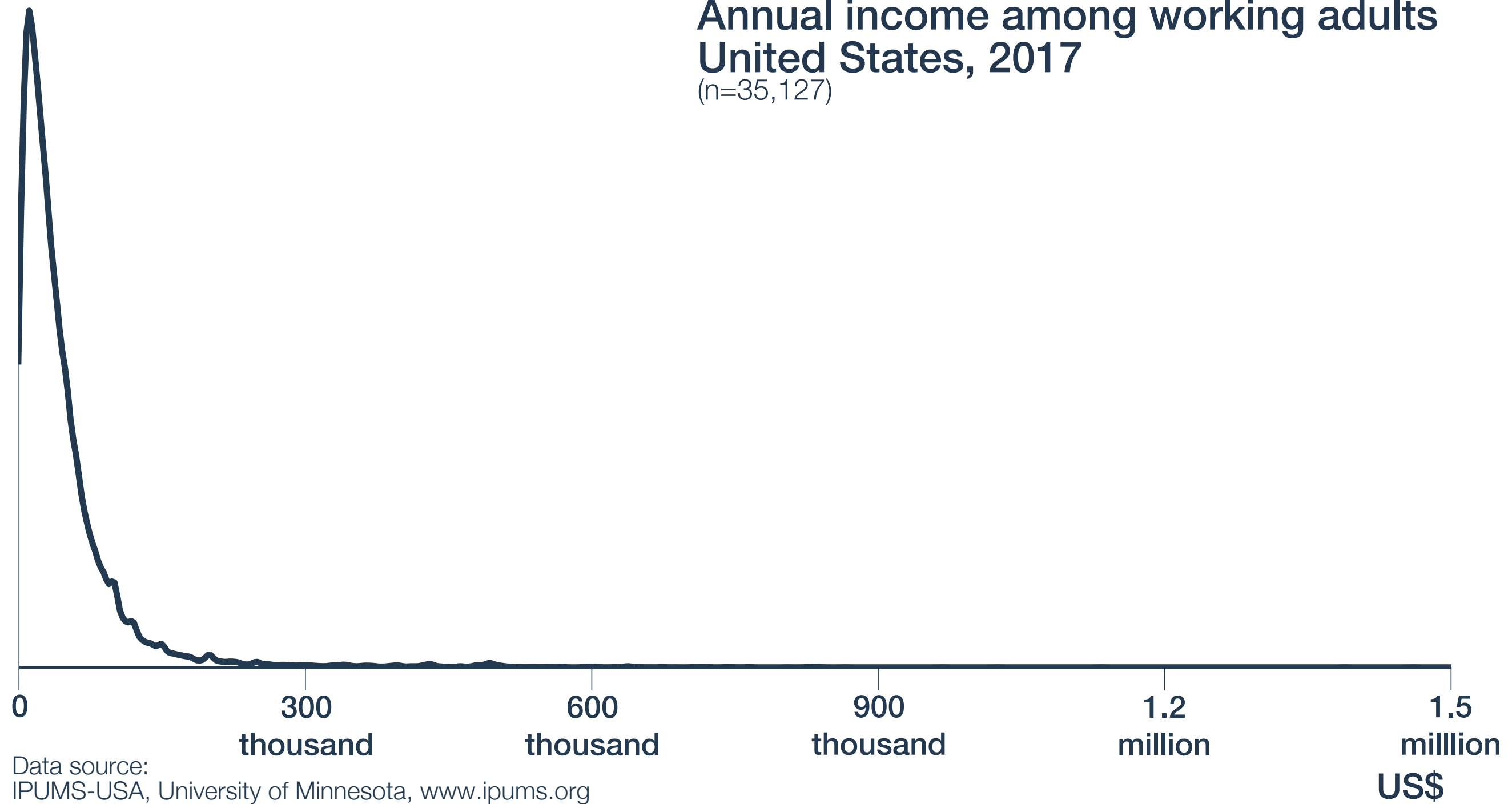
1. **Modeling income**
2. **Two-parameter models**
3. **MAP estimation**
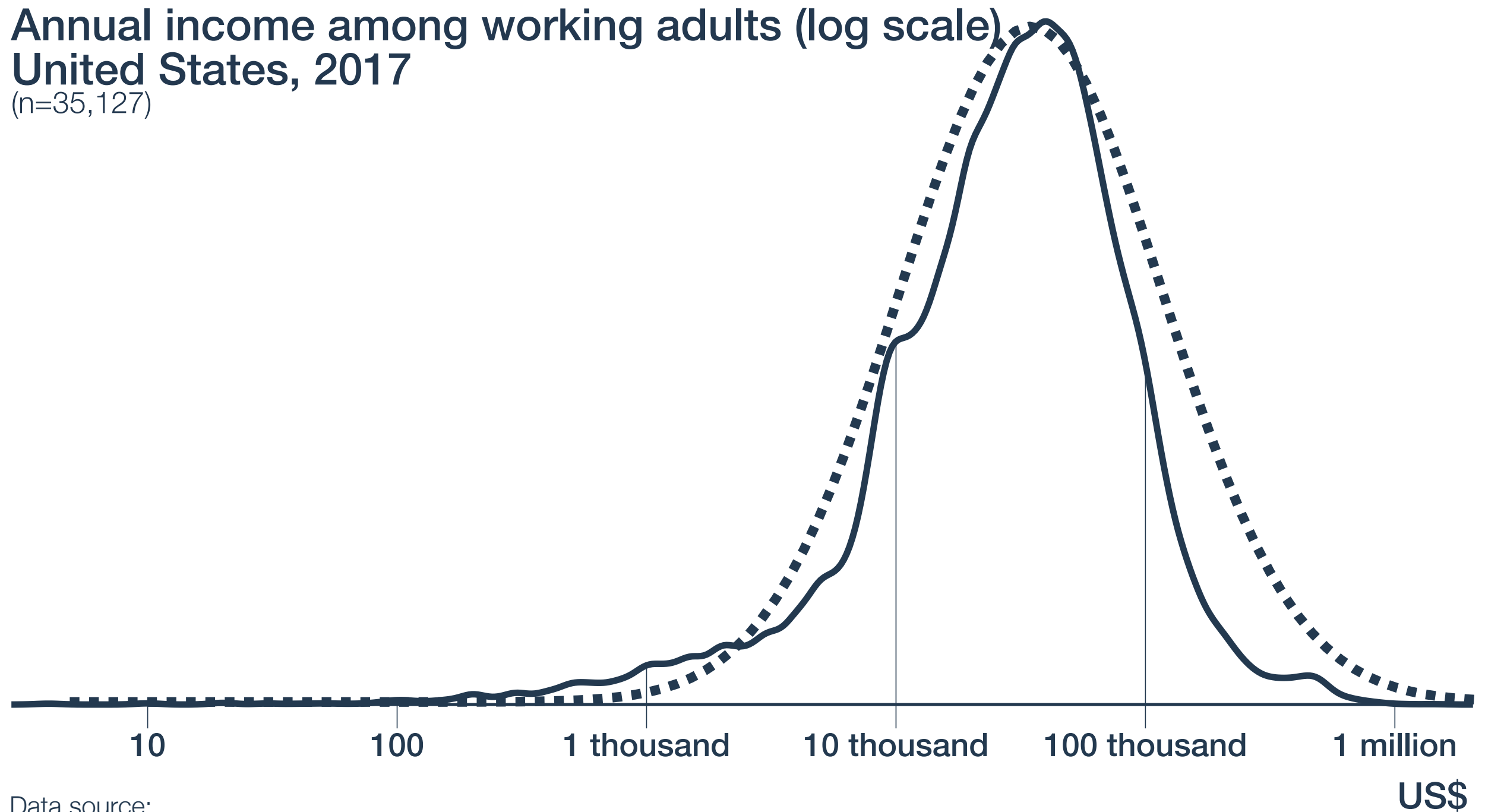
Annual income among working adults
United States, 2017
(n=35,127)

| 0 | 300 thousand | 600 thousand | 900 thousand | 1.2 million | 1.5 milllion |

US$

Data source:
IPUMS-USA, University of Minnesota, www.ipums.org

Modeling income

Annual income among working adults (log scale)
United States, 2017
(n=35,127)

| 10 | 100 | 1 thousand | 10 thousand | 100 thousand | 1 million |

US$

Data source:
IPUMS-USA, University of Minnesota, www.ipums.org

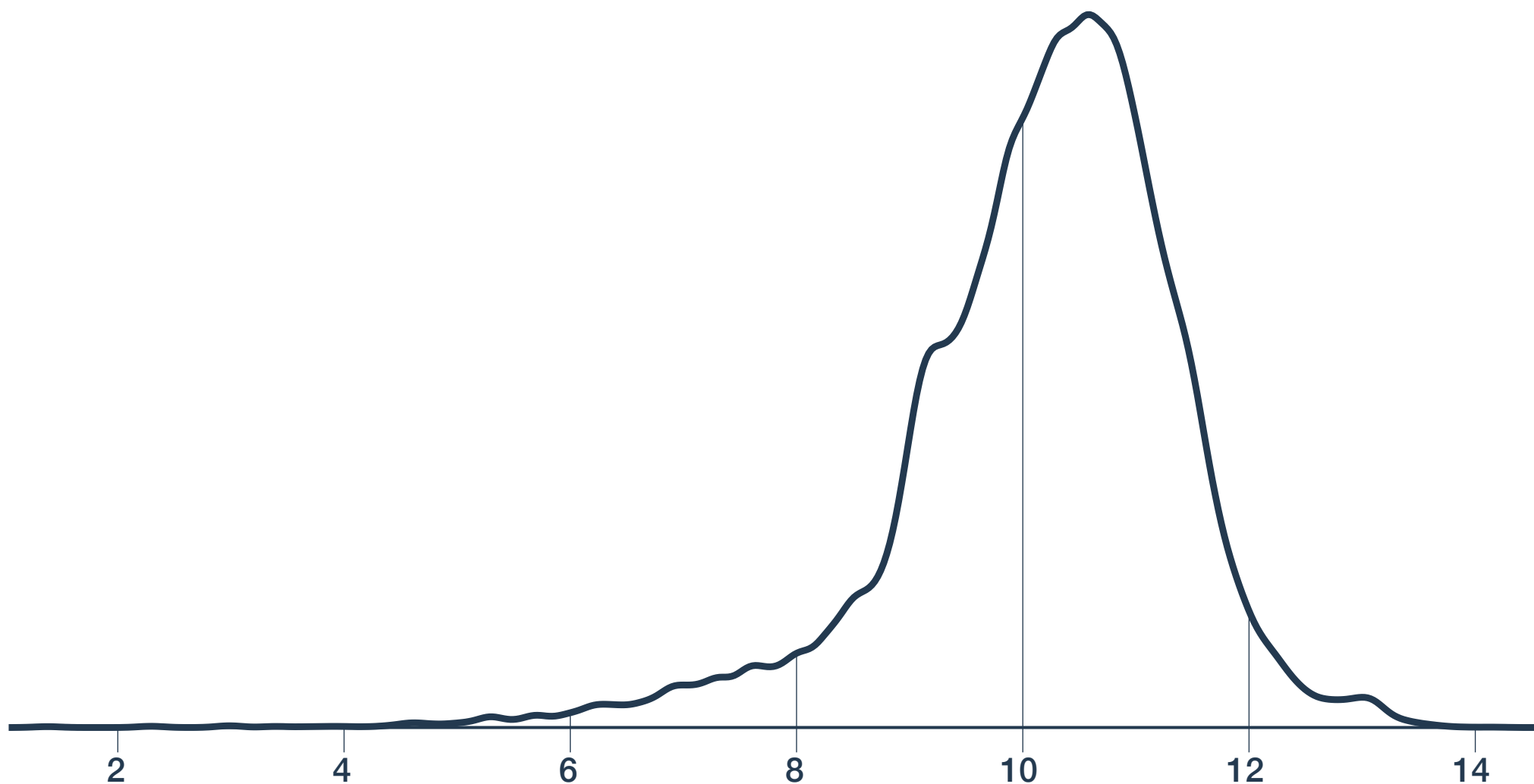**Plan**: model *log* income as normally distributed

A two-parameter model

Model of normally distributed log income:

$$y_i \sim \mathrm{Norm}(\mu, \sigma)$$

Two parameters:

μ determines location
σ determines width

A two-parameter model

Model of normally distributed log income:
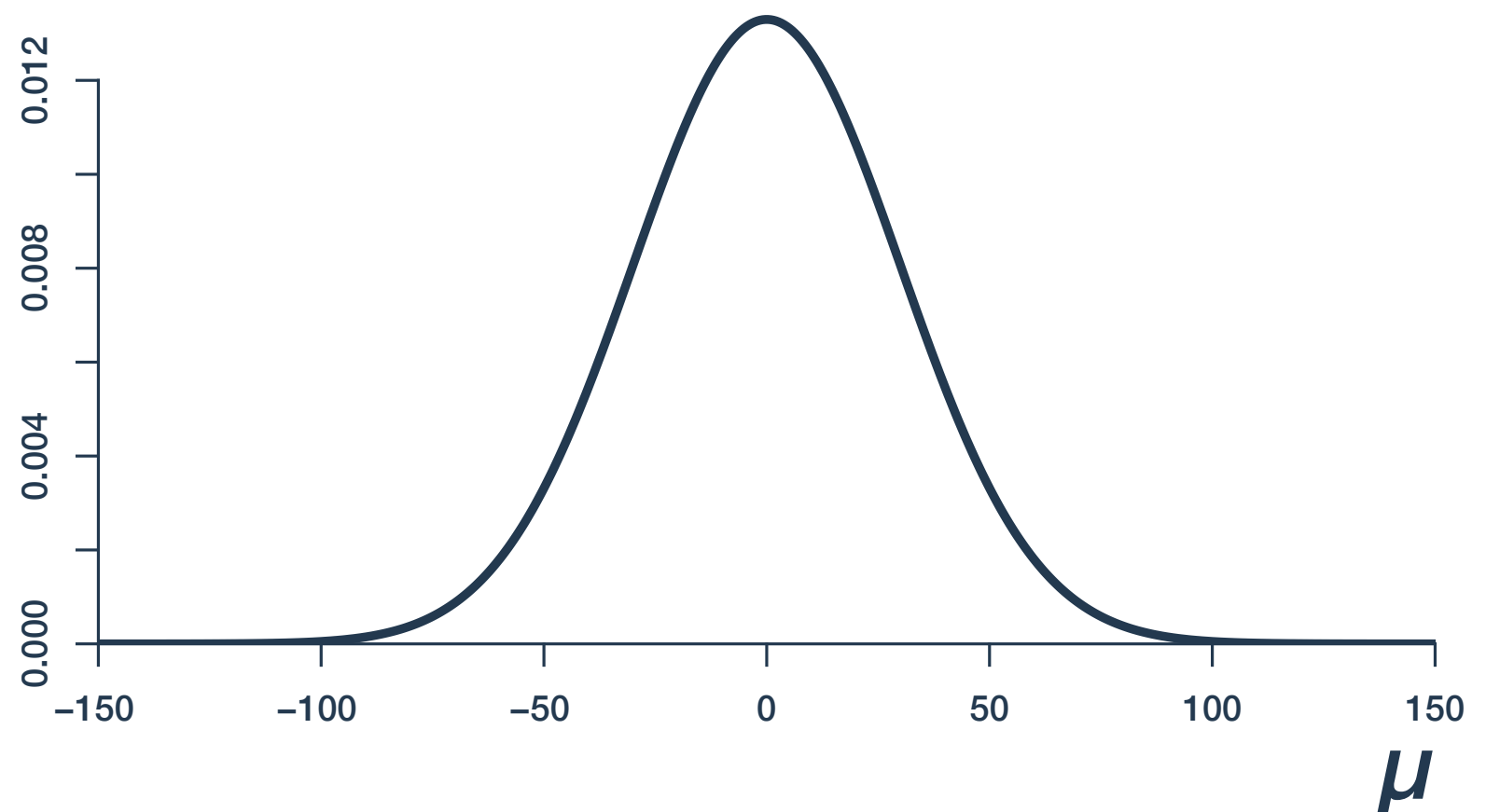
$$y_i \sim \text{Norm}(\mu, \sigma)$$

$$\mu \sim \text{?}$$

A two-parameter model

Model of normally distributed log income:

$$y_i \sim \text{Norm}(\mu, \sigma)$$

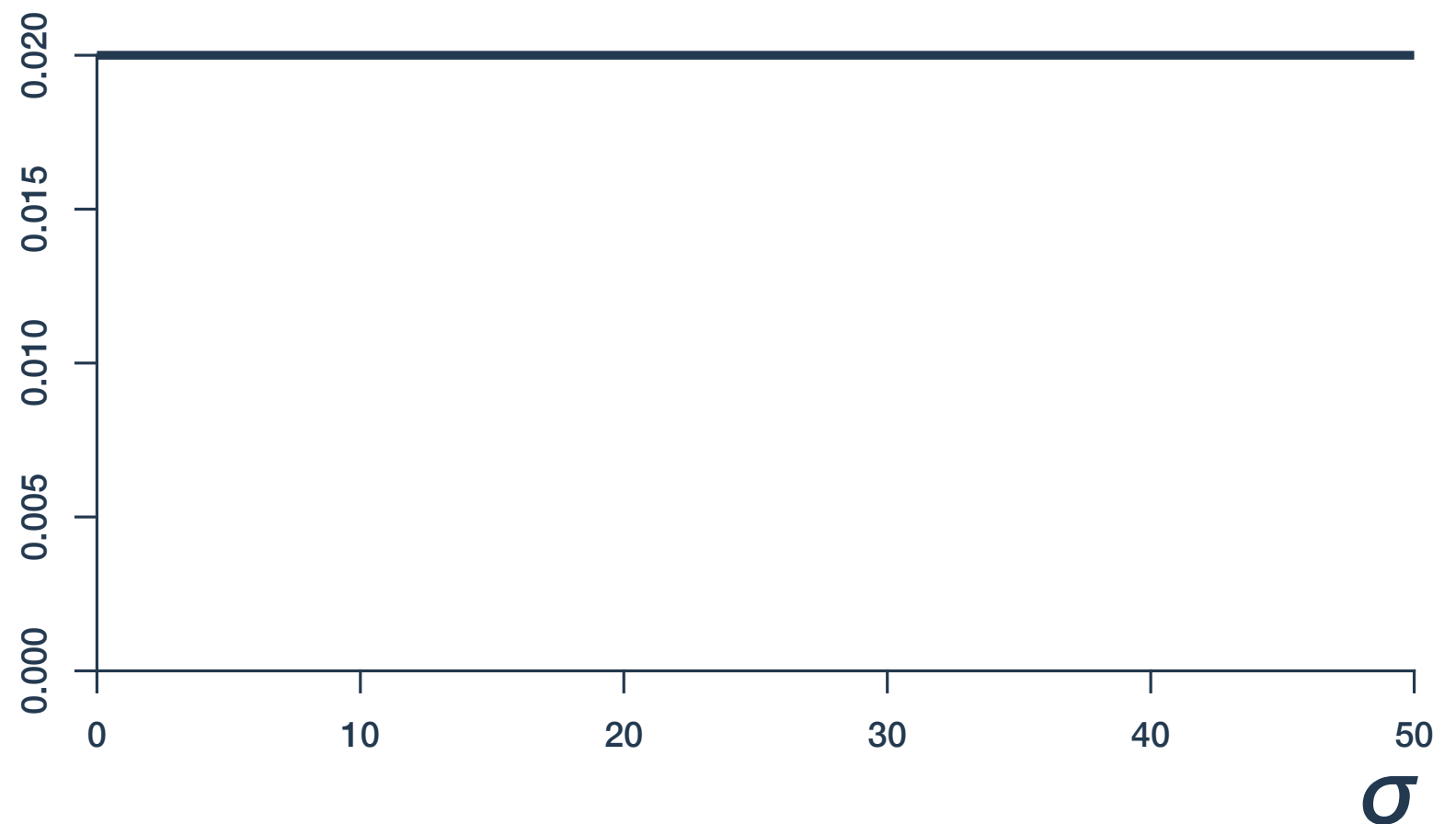$$\mu \sim \text{Norm}(0, 30)$$

A two-parameter model
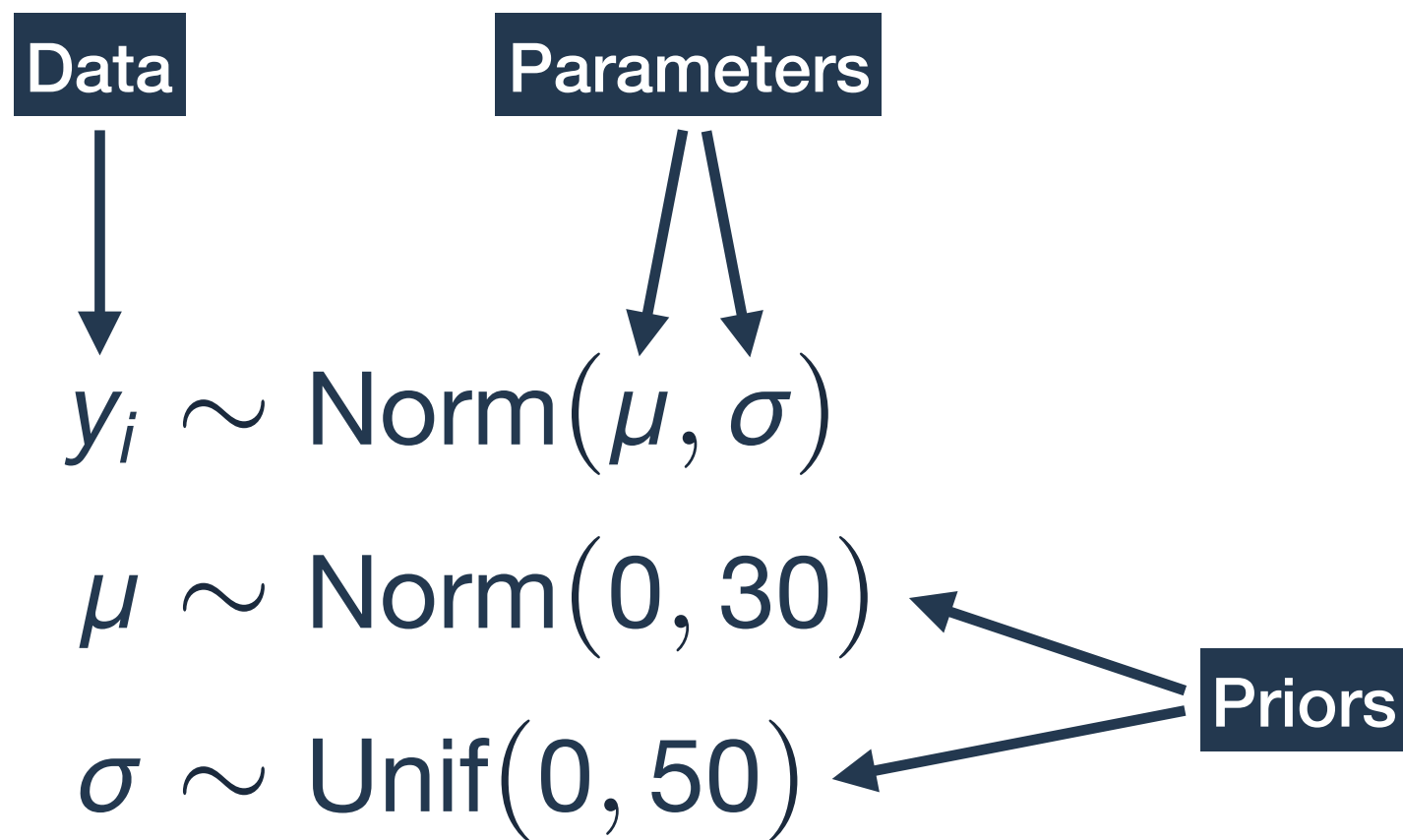
Model of normally distributed log income:

$$y_i \sim \text{Norm}(\mu, \sigma)$$

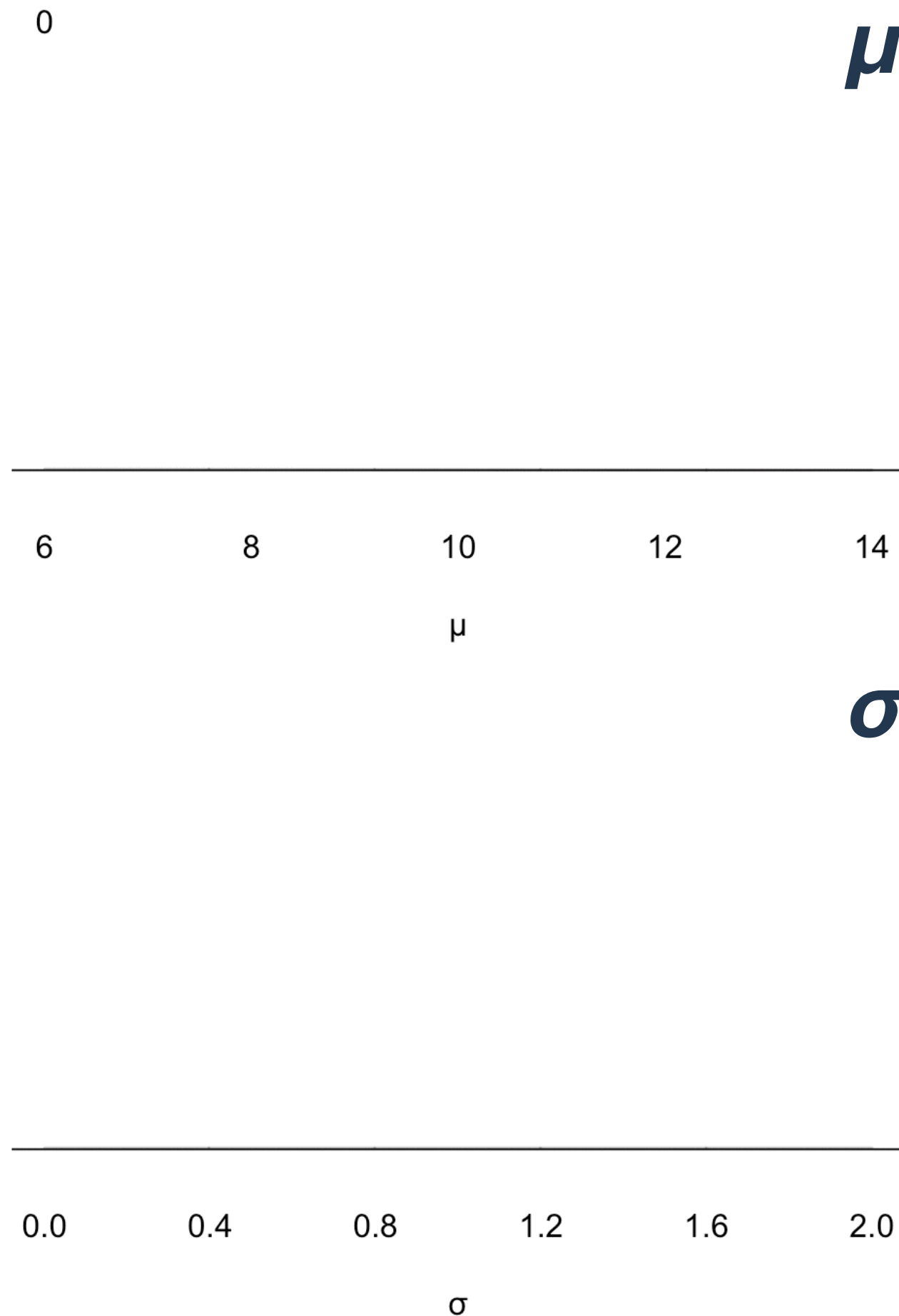$$\mu \sim \text{Norm}(0, 30)$$

$$\sigma \sim \text{Unif}(0, 50)$$

A two-parameter model

$$y_i \sim \text{Norm}(\mu, \sigma)$$
$$\mu \sim \text{Norm}(0, 30)$$
$$\sigma \sim \text{Unif}(0, 50)$$

Data

Parameters

Priors

Multi-parameter posteriors

Updating marginal posterior distributions for *μ* and *σ*
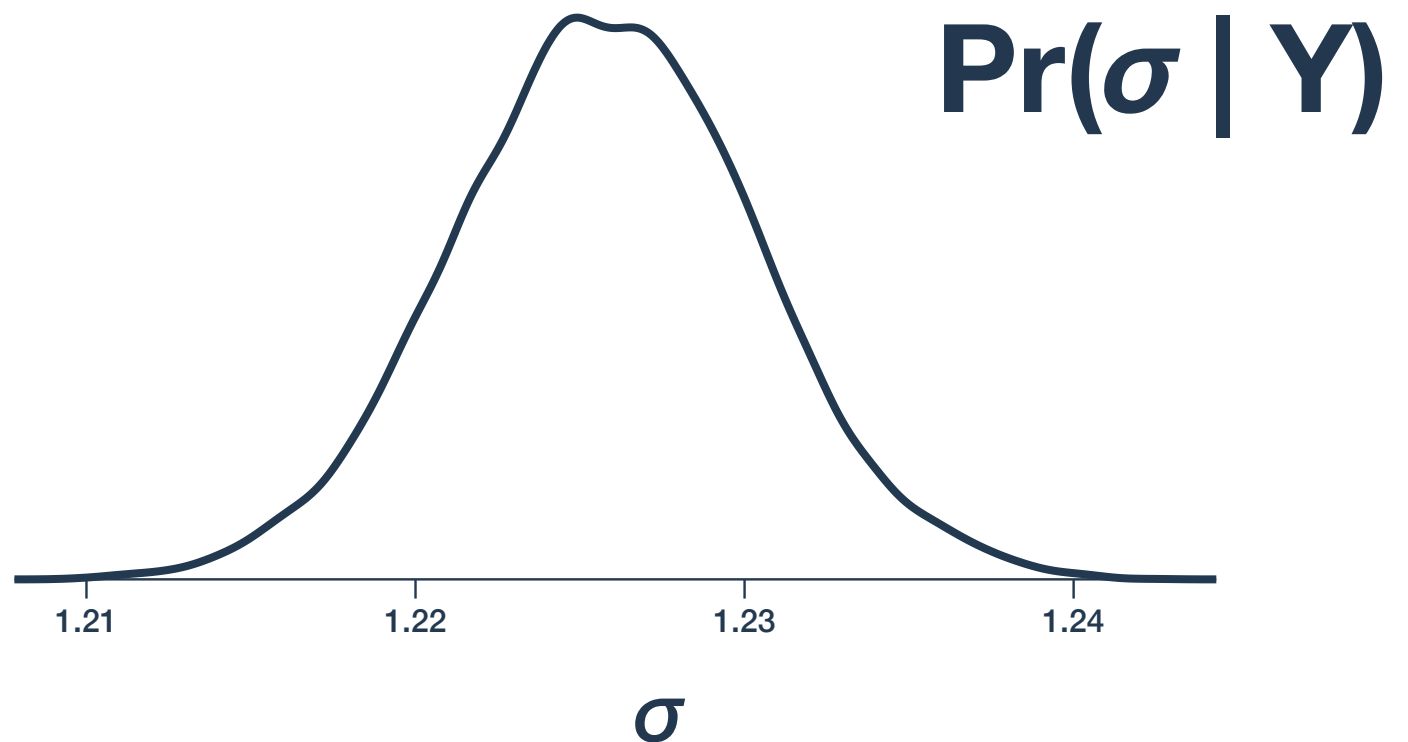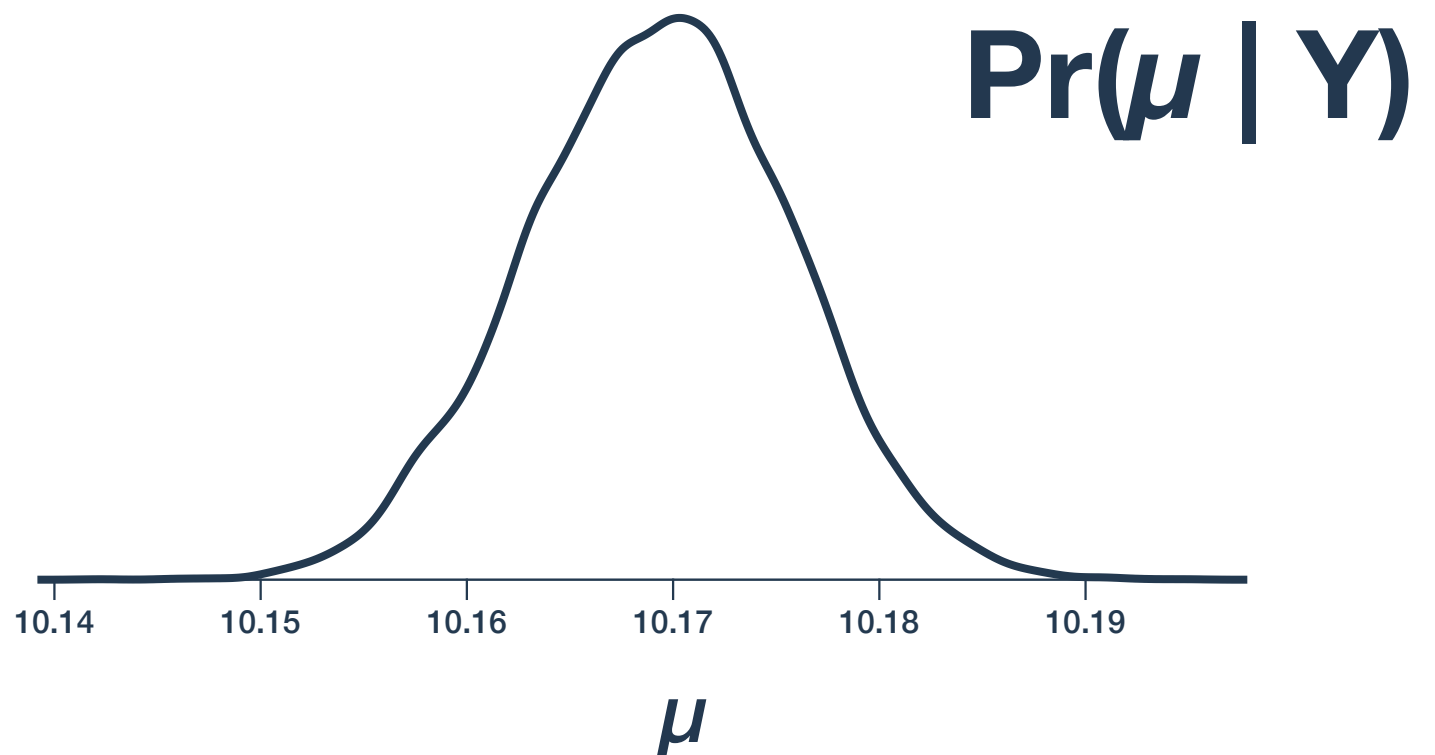
First 100 samples

μ

0

6   8   10   12   14

μ

σ

0.0   0.4   0.8   1.2   1.6   2.0

σ

Multi-parameter posteriors

Updated marginal posterior distributions for *μ* and *σ*

Full sample (*n=35,127*)

Pr(*μ* | Y)

10.14   10.15   10.16   10.17   10.18   10.19

μ

- **Nearly normal in shape**
- **Very narrow bounds**
- **Lumpy because built from posterior sample**

Pr(*σ* | Y)

1.21   1.22   1.23   1.24

σ

**Updated joint
posterior distribution
for μ and σ**

Full sample (n=35,127)

Pr(μ, σ | Y)

σ

μ