Agenda

1. **Course evaluations**
   https://horizon.mcgill.ca/pban1/twbkwbis.P_WWWLogin?ret_code=f
   (Minerva > Mercury)
2. **Specifying models in brms (continued)**
3. **Multilevel models of time**

Models of time

Common models of time

Autoregression models	Model outcome at time *t* as a function of covariates and outcome at time *t*–1. $y_t = y_{t-1} + \beta X_{t-1} + \varepsilon_t$ $y_{\Delta t} = \beta X_{t-1} + \varepsilon_t$
Survival / event-history models	Model the timing of a one-time event (graduation, job acquisition, death). $\lambda(t \mid X) = \lambda_0 \exp(\beta X)$
Age–period–cohort models	Demographic models aiming to differentiate between effects of individuals' *age*, the date of measurement (*period*), and birth *cohort*.
***Ad hoc* models**	Countless context-specific ways to model a randomly varying or functionally defined effect of time on outcomes.

Student scores over time

Correlation between scores from the same student.

	1985	1986	1987	1988
Student 1	540	531	564	563
Student 2	479	487	505	510
Student 3	503	505	501	503
Student 4	461	471	—	—
Student 5	525	—	506	541
	…	…	…	…

Student scores over time

	1985	1986	1987	1988
Student 1	540	531	564	563
Student 2	479	487	505	510
Student 3	503	505	501	503
Student 4	461	471	—	—
Student 5	525	—	506	541
	…	…	…	…

Scores tend to increase over time.

Student scores over time

	1985	1986	1987	1988
Student 1	540	531	564	563
Student 2	479	487	505	510
Student 3	503	505	501	503
Student 4	461	471	—	—
Student 5	525	—	506	541
	…	…	…	…

→

Student	Year	Score
1	1985	540
1	1986	531
1	1987	564
1	1988	563
2	1985	479
2	1986	487
2	1987	505
2	1988	510
3	1985	503
…	…	…

Student random effects

Score for student i at time t.

$$\boxed{S_{ti}} \sim \text{Norm}(\mu_{ti}, \sigma)$$

Average score for student i.

$$\mu_{ti} = \boxed{\beta_{0i}} + \beta_1 CSize_{ti}$$

$$\beta_{0i} = \boxed{\gamma_{00}} + \eta_{0i}$$

Average score across all students.

$$\eta_{0i} \sim \text{Norm}(0, \phi_0)$$

Student random effects

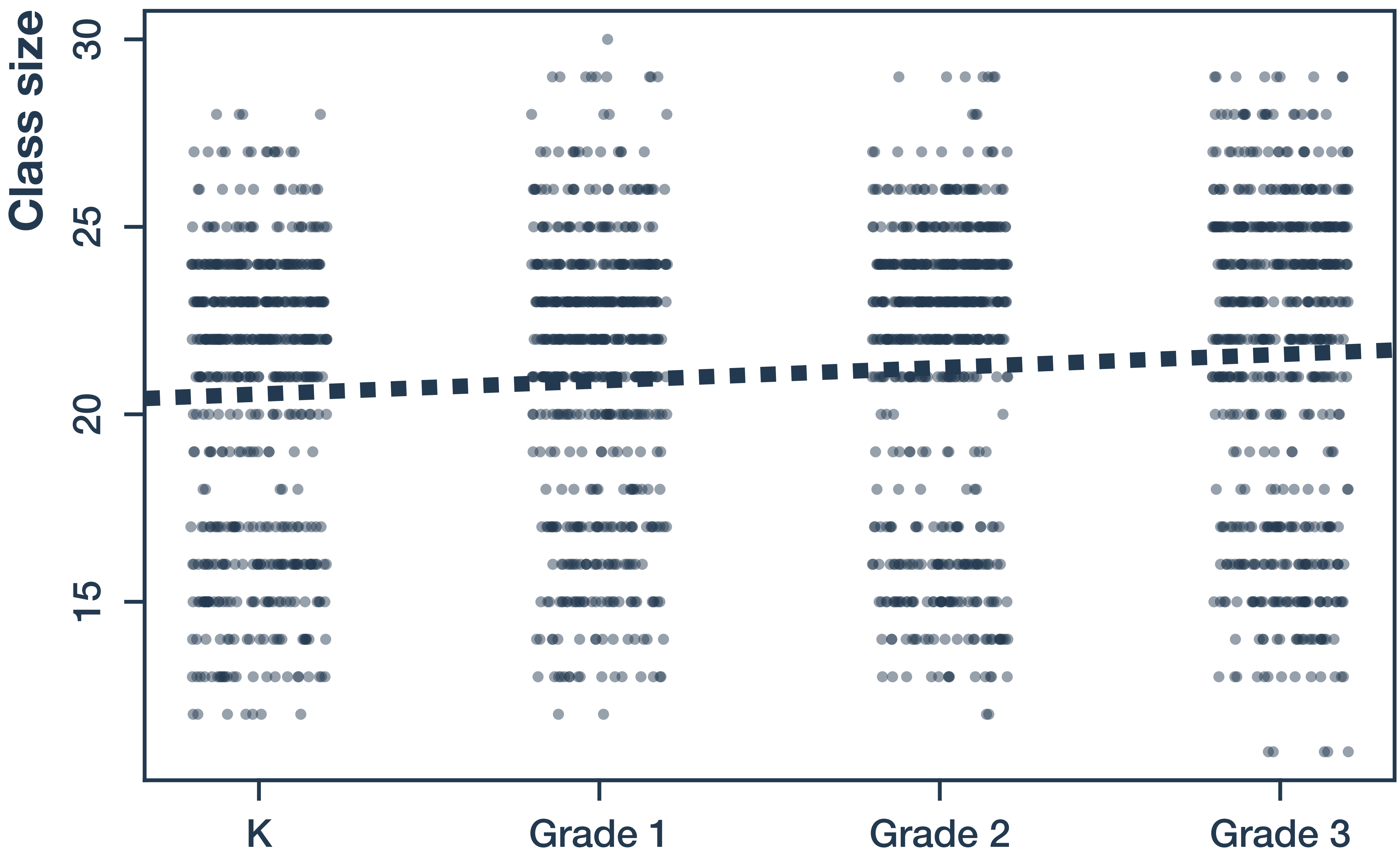
$$S_{ti} \sim \text{Norm}(\mu_{ti}, \sigma)$$
$$\mu_{ti} = \beta_{0i} + \beta_1 CSize_{ti}$$
$$\beta_{0i} = \gamma_{00} + \eta_{0i}$$

$$\eta_{0i} \sim \text{Norm}(0, \phi_0)$$

$$\gamma_{00} \sim \text{Norm}(500, 100)$$
$$\beta_1 \sim \text{Norm}(0, 50)$$
$$\sigma \sim \text{HalfCauchy}(0, 50)$$
$$\phi_0 \sim \text{HalfCauchy}(0, 50)$$

	Mean	*90% credible interval*	
$\boldsymbol{\gamma_{00}}$	547.06	544.05	549.94
$\boldsymbol{\beta_1}$	1.23	0.58	1.87
$\boldsymbol{\sigma}$	56.94	54.97	58.93
$\boldsymbol{\phi_0}$	33.91	30.16	37.60

Student random effects



Linear time trend

$$S_{ti} \sim \text{Norm}(\mu_{ti}, \sigma)$$
$$\mu_{ti} = \beta_{0i} + \beta_{1i} Year_{ti} + \beta_2 CSize_{ti}$$
$$\beta_{0i} = \gamma_{00} + \eta_{0i}$$
$$\beta_{1i} = \gamma_{10} + \eta_{1i}$$

$$[\eta_{0i}, \eta_{1i}] \sim \text{MVNorm}([0, 0], \Phi, R)$$

Linear time trend

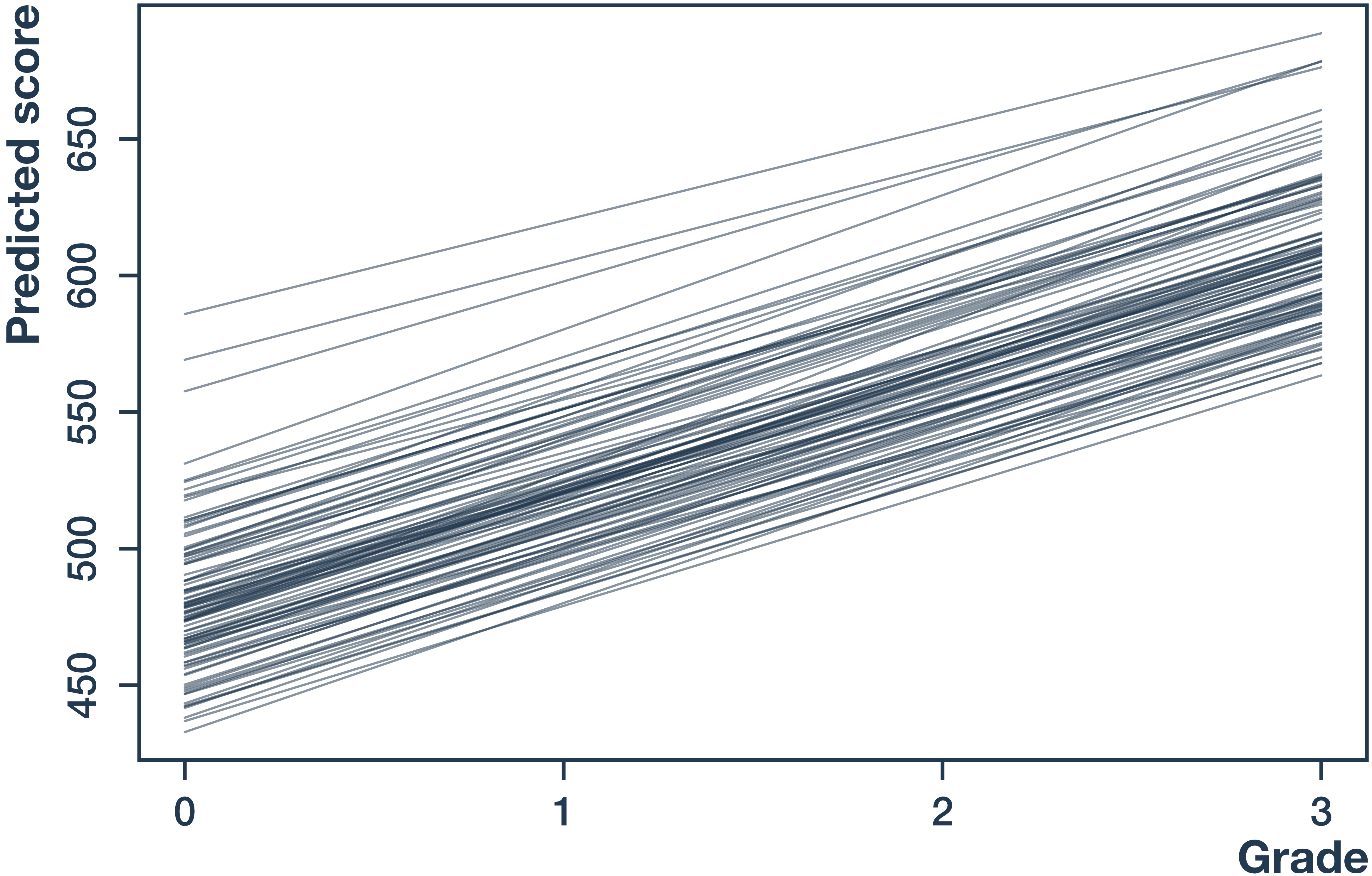
$$S_{ti} \sim \text{Norm}(\mu_{ti}, \sigma)$$
$$\mu_{ti} = \beta_{0i} + \beta_{1i} Year_{ti} + \beta_2 CSize_{ti}$$
$$\beta_{0i} = \gamma_{00} + \eta_{0i}$$
$$\beta_{1i} = \gamma_{10} + \eta_{1i}$$

$$[\eta_{0i}, \eta_{1i}] \sim \text{MVNorm}([0, 0], \Phi, R)$$

$$\gamma_{00} \sim \text{Norm}(500, 100)$$
$$\gamma_{10} \sim \text{Norm}(0, 50)$$
$$\beta_2 \sim \text{Norm}(0, 50)$$
$$\sigma \sim \text{HalfCauchy}(0, 50)$$
$$\phi_0, \phi_1 \sim \text{HalfCauchy}(0, 50)$$
$$R \sim LKJ(2, 2)$$

	Mean	***90% credible interval***	
$\boldsymbol{\gamma_{00}}$	484.64	481.84	487.39
$\boldsymbol{\gamma_{10}}$	42.88	41.75	44.02
$\boldsymbol{\beta_2}$	–0.33	–0.73	0.070
$\boldsymbol{\sigma}$	24.27	23.27	25.32
$\boldsymbol{\phi_0}$	38.25	35.84	40.65
$\boldsymbol{\phi_1}$	8.89	7.17	10.56
$\boldsymbol{\rho_{01}}$	–0.37	–0.48	–0.23

Linear time trend



Quadratic time trend

$$
\begin{aligned}
S_{ti} &\sim \text{Norm}(\mu_{ti}, \sigma) \\
\mu_{ti} &= \beta_{0i} + \beta_{1i} Year_{ti} + \beta_{2i} Year_{ti}^2 + \beta_3 CSize_{ti} \\
\beta_{0i} &= \gamma_{00} + \eta_{0i} \\
\beta_{1i} &= \gamma_{10} + \eta_{1i} \\
\beta_{2i} &= \gamma_{20} + \eta_{2i}
\end{aligned}
$$

$$
[\eta_{0i}, \eta_{1i}, \eta_{2i}] \sim \text{MVNorm}([0, 0, 0], \Phi, R)
$$

Quadratic time trend

$$S_{ti} \sim \text{Norm}(\mu_{ti}, \sigma)$$
$$\mu_{ti} = \beta_{0i} + \beta_{1i} Year_{ti} + \beta_{2i} Year_{ti}^2 + \beta_3 CSize_{ti}$$
$$\beta_{0i} = \gamma_{00} + \eta_{0i}$$
$$\beta_{1i} = \gamma_{10} + \eta_{1i}$$
$$\beta_{2i} = \gamma_{20} + \eta_{2i}$$

$$[\eta_{0i}, \eta_{1i}, \eta_{2i}] \sim \text{MVNorm}([0, 0, 0], \Phi, R)$$

$$\gamma_{00} \sim \text{Norm}(500, 100)$$
$$\gamma_{10} \sim \text{Norm}(0, 50)$$
$$\gamma_{20} \sim \text{Norm}(0, 50)$$
$$\beta_3 \sim \text{Norm}(0, 50)$$
$$\sigma \sim \text{HalfCauchy}(0, 50)$$
$$\phi_0, \phi_1, \phi_2 \sim \text{HalfCauchy}(0, 50)$$
$$R \sim LKJ(2, 3)$$

	Mean	*90% credible interval*	
γ_{00}	482.32	479.37	485.30
γ_{10}	49.28	45.77	52.88
γ_{20}	–2.10	–3.14	–1.09
β_3	–0.36	–0.74	0.02
σ	21.91	20.76	23.04
ϕ_0	40.70	38.16	43.43
ϕ_1	31.48	26.35	36.38
ϕ_2	7.29	5.62	8.96
ρ_{01}	–0.45	–0.55	–0.35
ρ_{02}	0.40	0.25	0.53
ρ_{12}	–0.98	–1.00	–0.96

Quadratic time trend