University of Glasgow

Wednesday 9 February 2022
Expected duration: 2 hours
Start time: 2.00 pm
Time allowed: 4 hours

BSc Software Engineering (Graduate Apprenticeship)

PRACTICAL ALGORITHMS

COMPSCI1021

Answer All Questions

Show complete working wherever applicable

This examination paper is worth a total of 100 marks, and worth 60% of the course

Question 1 (30 Marks)

Algorithms, Searching & Sorting, and Complexity

1a. Consider the following algorithm described in pseudo-code. It takes an array **A** of **n** positive integers as input. It locally uses a stack data structure called `Stack`. It creates and returns a list called **`output`**.

```
define some_algo (A, n)
    output = []
    for i from 0 to n-1
        if A[i] is not equal to 0 then
            Stack.push(A[i])
        else
            while Stack is not empty
                output.append(Stack.pop())
    return output
```

Answer each of the following concerning this algorithm:

(i) Describe in one sentence what this algorithm does.

[3]

(ii) What is the output of this algorithm for the array `A = [1, 2, 3, 4, 0, 5, 6, 7, 8, 0, 9, 10, 11, 0]`

[3]

(iii) Using the big-O notation, characterize the running time of the above algorithm in terms of **n**, the number of integers in **A**. Show your working.

[4]

1b. Explain in your own words why the *binary search algorithm* requires the input to be in a sorted form. Does the input list have to be in ascending order, or can the algorithm also work with lists sorted in descending order? If so, does it need any modification to make it work with descending-order lists?

[4]

1c. Consider the following input array being sorted into ascending order using the quicksort algorithm.

`A = [4, 6, 30, 12, 7, 19, 0, 20, 16]`

Using the *last* element as the pivot, show a step-by-step application of the PARTITION function on this entire array. That is, show what happens when PARTITION(A, 0, 8) is applied. You should show the state of the entire array on each step, and identify the location of the relevant pointers. Also write a short comment to indicate what has happened in each step.

[6]

1d. The insertion sort has a best-case complexity of $\Omega(n)$, and a worse case complexity of $O(n^2)$. Show and briefly explain one example of an 8-element list of integers where you get the best-case complexity. Similarly, show and explain another example of an 8-element list of integers where you get the worst-case complexity.

[4]

1e. Analyse the PARTITION algorithm shown below to find its big-Oh complexity.

```
PARTITION (A,start,end)
      x = A[end]
      lrbi = start – 1
      for ti from start to end-1
            if A[ti] less than or equal to x then
                  lrbi := lrbi + 1
                  SWAP(A[lrbi],A[ti])
      SWAP(A[lrbi+1],A[end])
      return lrbi + 1
```

[6]

Question 2 (25 Marks)

Data Collections

2a. Given A = {a, b, c, d}, B = {x, y, z} and C = {{a}, b, c, y, z}, P(X) denotes the power set of X, and |X| denotes the cardinality of set X, determine:

(i) $P(A \cap C)$

(ii) $|P(A \cup B \cup C)|$

(iii) $|A \times B|$

(iv) $A - C$

[4]

2b. Explain in your own words what is meant by *inverse* of a function*,* and why invertible functions need to be *bijective.*

[4]

2c. Explain in your own words how the addition of a tail-pointer in a *linked-list* data structure converts *tail insertion* from an O(n) to O(1) operation. Draw a figure to explain your answer. You are not expected to write pseudo-code as part of your answer.

[4]

2d. The following pseudo-code shows the DELETE operation in a *doubly-linked list*. Explain this pseudo-code by commenting the code. You may find it useful to draw figures to explain your answer.

```
# list1: the linked list
# item: the element to be deleted from the linked list
# prev_pntr: the pointer pointing to the previous element of a node
# next_pntr: the pointer ponting to the next element of a node
# first: the pointer to the first (or head) element of the list
define Delete (list1, item)
      if item.prev_pntr != NIL then
            item.prev_pntr.next_pntr = item.next_pntr
      else
            list1.first:= item.next_pntr
      if item.next_pntr != NIL then
            item.next_pntr.prev_pntr := item.prev_pntr
```

[4]

2e. A binary search tree (BST) is a binary tree T such that each internal node v of T stores an item e. Items stored at nodes in the left subtree of v are less than or equal to e, and items stored in the right subtree of T are greater than e.

(i) Insert into an initially empty binary search tree the following items in the order shown: 29, 10, 39, 22, 26, 11, 1, 36. Draw the tree after the insertions have been completed.

[3]

(ii) Give the preorder, inorder and postorder traversals of the tree.

[3]

(iii) If we had to insert 255 items into a binary search tree, what could be the minimum and the maximum height of the binary search tree? Suggest what property a data set might have to create *maximum* height? In your answer explain what we mean by the height of a tree.

[3]

Question 3 (15 Marks)

Sequences, Induction, and Recurrence

3a. Let $\{a_n\}$ be a sequence that satisfies the recurrence relation $a_n = \frac{a_{n-1}}{2}$ for n = 1,2,3,4,… . Suppose that $a_0 = 1024$. What are the terms a_0, a_1, a_2 and a_3 of the sequence $\{a_n\}$?

[2]

3b. Solve this Recurrence Relation of part 3a by finding its closed formula, using the iterative method.

[4]

3c. Using mathematical induction, prove that the sum of the cubes of first n even positive integers is n(n+1). That is, show that: $2 + 4 + 6 + \cdots + 2n = n(n + 1)$

[5]

3d. What are the two basic properties that a recursive method must satisfy? Explain using an example.

[4]

Question 4 (15 Marks)

Counting and Probability

4a. The Computing Science department of a University has 12 members, and the Maths department has 8 members. How many ways are there to select an interdepartmental committee of 6 members such that:

(i) At least one mathematician must be on the committee?
(ii) At most 4 computer scientists can be on the committee?

[6]

4b. Suppose every telephone number in the country is assigned a number that contains:

- a 3-digit country code, where the first digit must be a 0, and the other two can be any other digit.
- followed by a 10-digit number such that:
    - the first five digits are from the set {2,3,4,5}
    - the remaining five digits are from the set {1,6,7,8,9}

How many different telephone numbers would be available worldwide under this numbering scheme? (Show the complete working).

[4]

4c. What is the conditional probability that a randomly generated bit string of length five contain *at least* three consecutive 1's, given that the first bit is a 0? Explain your answer.

[5]

Question 5 (15 Marks)

Formal Reasoning

5a. Using either truth tables or laws of logical equivalences, show that the following equivalence is valid:

$$\big(q \wedge (p \rightarrow \neg q)\big) \rightarrow \neg p \equiv true$$

[Hint: Consider using the implication law to start things off: $p \rightarrow q \equiv \neg p \vee q$]

[4]

5b. Suppose we have the following predicates:

- $P(x)$: x is prime
- $E(x)$: x is even
- $L(x, y)$: x < y

(i) Express this English statement in logical formulae using the predicates given above: "No prime greater than 2 is even."

[3]

(ii) Express the following proposition in concise English without using variables, and determine its truth value:

$$\forall x \in \mathbb{Z}^{+}. \big(E(x) \wedge P(x)\big) \rightarrow L(x, 100)$$

[3]

5c. Using the definition of odd and even integers*, prove the following statement is true for all integers n: "if n is even, then n^2+1 is odd". Use either direct proof, or proof by contradiction.

[5]

**Hint:*
if n is even, then n = 2k for some integer k.
if n is odd, then n = 2k+1 for some integer k.