Practical Algorithms - Logical Equivalences

Identity laws:

- $P \wedge \texttt{true} \equiv P$
- $P \vee \texttt{false} \equiv P$

Domination laws:

- $P \vee \texttt{true} \equiv \texttt{true}$
- $P \wedge \texttt{false} \equiv \texttt{false}$

Idempotent laws:

- $P \wedge P \equiv P$
- $P \vee P \equiv P$

Double negation law:

- $\neg(\neg P) \equiv P$

Commutative laws:

- $P \wedge Q \equiv Q \wedge P$
- $P \vee Q \equiv Q \vee P$

Associative laws:

- $(P \wedge Q) \wedge R \equiv P \wedge (Q \wedge R)$
- $(P \vee Q) \vee R \equiv P \vee (Q \vee R)$

Distributive laws:

- $P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$
- $P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$

De Morgan laws:

- $\neg(P \wedge Q) \equiv \neg P \vee \neg Q$
- $\neg(P \vee Q) \equiv \neg P \wedge \neg Q$

Contradiction and tautology laws:

- $P \wedge \neg P \equiv \texttt{false}$
- $P \vee \neg P \equiv \texttt{true}$

Implication law:

- $P \to Q \equiv \neg P \vee Q$

Exclusive or and biconditional laws:

- $P \oplus Q \equiv (P \vee Q) \wedge \neg(P \wedge Q)$
- $P \leftrightarrow Q \equiv (P \to Q) \wedge (Q \to P)$

Quantifier laws:

- $\forall x. \forall y. Q(x, y) \equiv \forall y. \forall x. Q(x, y)$
- $\exists x. \exists y. Q(x, y) \equiv \exists y. \exists x. Q(x, y)$
- $\neg(\exists x. \neg P(x)) \equiv \forall x. P(x)$
- $\neg(\forall x. \neg P(x)) \equiv \exists x. P(x)$
- $\forall x. (P(x) \wedge Q(x)) \equiv \forall x. P(x) \wedge \forall x. Q(x)$
- $\exists x. (P(x) \vee Q(x)) \equiv \exists x. P(x) \vee \exists x. Q(x)$