Data Storage and Retrieval
Lecture 6
Sets and Set Theory

Dr. Graham McDonald

Graham.McDonald@glasgow.ac.uk

Overview

This lecture

- Sets and Set Theory
- Relations and the Cartesian Product

Next lecture (lecture 7)

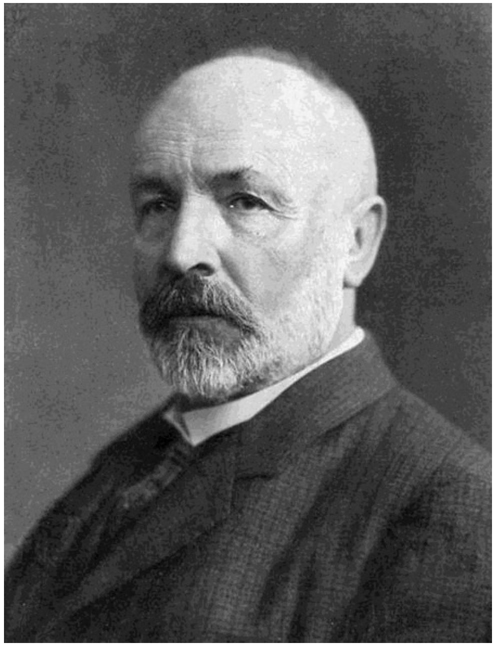
- Relational Algebra
  - The foundations for SQL

Sets and Set Theory

Set theory is the branch of mathematics that studies **sets**

Sets are collections of objects

- The set of all numbers
- the set of all animals with tails
- the set of letters in the alphabet
- the set of all students in a class
- …………………

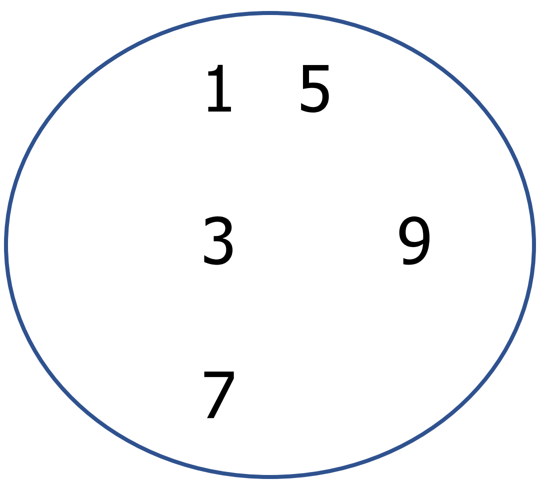


Georg Cantor (1845-1918)

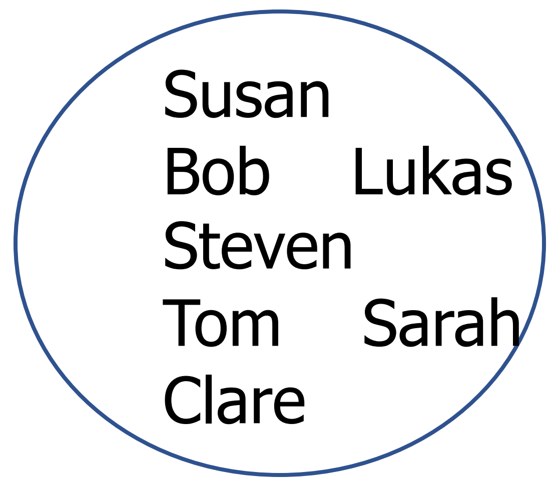
University of Glasgow

Sets

Often all members of a set have similar properties



Odd numbers less than 10



Students in a Tutorial Group

Set Theory - Vocabulary

- Objects in a set are called ***'elements'*** or '***members***' of a set
- A set is said to 'contain' its elements

- In databases
  - all exam scores make up a '***set***' of exam scores.
  - all employees of a company make up a '***set***' of employees

Describing Sets

- Describing a set
  - List all the members between braces
    - E.g. **{a, b, c, d}**
    - Represents the set with the four elements a, b, c, and d.

Describing Sets

- E.g. The set V of all vowels in the English alphabet

- E.g. The set O of positive integers less than 10

Describing Sets

- E.g. The set V of all vowels in the English alphabet
  - **V = {a, e, i, o, u}**
- E.g. The set O of positive integers less than 10
  - **O = {1, 3, 5, 7, 9 }**
- | | denotes the *cardinality* of a set
  - **|V| = 5, |O| = 5**

Set Equality

- Two sets are ***equal*** if and only if they have the same elements

  - **Order doesn't matter**
    - {1,3,5} = {1,5,3} = {3,1,5} = {3,5,1}= {5,1,3} ={5,3,1}
  - **Repetition doesn't matter**
    - {1,2} = {1,1,2} = {1,2,2,2,2}

Set Equality

A = {1,2,3}

B = {3,2,1} C = {1,1,2,2,2,3} D = {1,2,3}

Which set(s) are equal to A?

Set Equality

$A = \{1,2,3\}$

$B = \{3,2,1\}$ $C = \{1,1,2,2,2,3\}$ $D = \{1,2,3\}$

A = B, C and D

Sets

- Sets *usually* group together elements with associated properties
  - but seemingly unrelated properties can also be listed as a set

  - {2, e, Fred, Paris} is also a set
    - We just don't know much about exactly how they are related to each other

Predicates and Sets

- It is sometimes inconvenient or impossible to describe a set by listing all of its elements

  - What is the set of all integers less than 1 million?

Predicates and Sets

- It is sometimes inconvenient or impossible to describe a set by listing all of its elements

  - What is the set of all integers less than 1 million?
    - {1,2,3,4,5…………………………………..!!!!!!!}

Set Builder Notation

- Characterise all those elements in the set by stating the properties they must have to be members

E.g.

- The set O of all positive integers less than 10 in set builder notation is:
  - $O = \{X \mid X \text{ is an odd integer less than } 10\}$
- More mathematical definitions are also OK:
  - $O = \{X \mid X \in \mathbb{N} \land x < 10 \land x \% 2 == 1 \}$

Predicates and Sets

- A **predicate** is sometimes used to indicate **set membership**
- A predicate *F(x)* will be true or false, depending on whether *x* belongs to a set

Predicates and Set Membership

An example

$\{x \mid x \text{ is a positive integer less than } 4\}$

is the set $\{1,2,3\}$

If t is an element of the set $\{x \mid F(x)\}$

then the statement $F(t)$ is *true*

So if F(x) is defined as x % 2 = 0

$\{x|F(x)\}$ contains.... the set of all even numbers

Here, $F(x)$ is referred to as the ***predicate***, and x the *subject* of the *proposition*

Some Notation

- **$a \in A$**
  - a is an element of set A
- **$a \notin A$**
  - a is not an element of set A
- **∅**
  - The empty or null set
  - Also represented by { }

Graphical Representations

- Sets can be represented graphically using ***Venn diagrams***

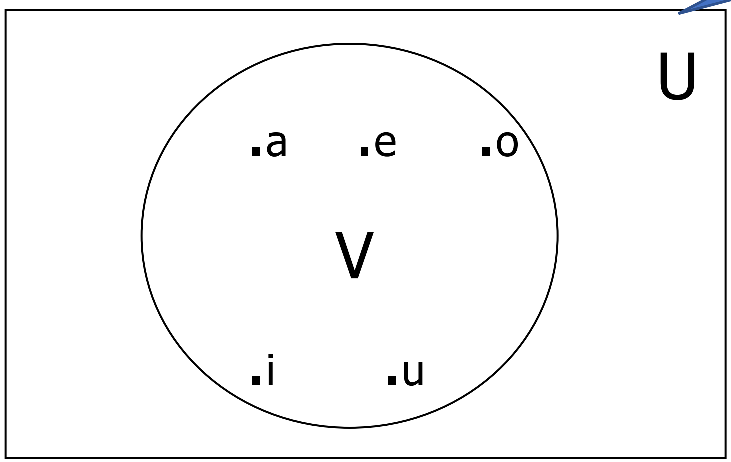
- The ***universal set U*** (which contains all of the objects under consideration) is represented by a rectangle

- Inside the rectangle, circles are used to represent sets

- Sometimes points are used to represent the particular elements of the set

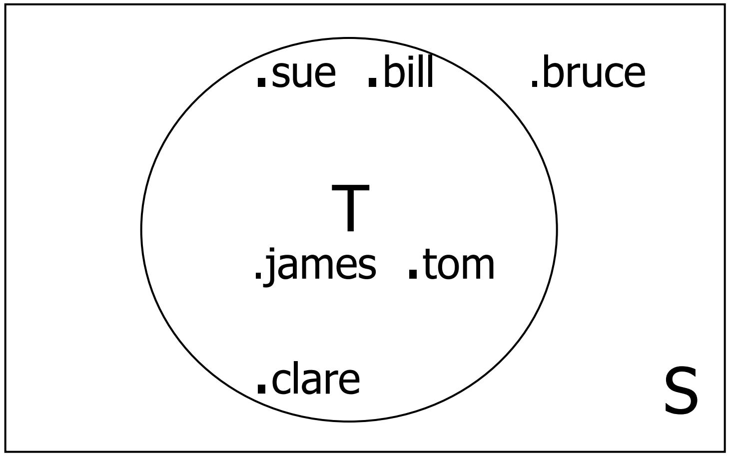
An Example Set

U is the Universe – the set of all possible items



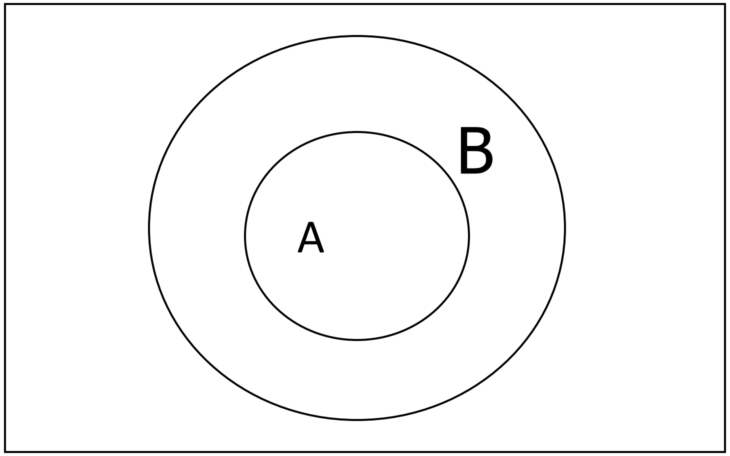
The set V of vowels from all letters U

An Example Set



The set T of people in tutorial group from all Students S

Subsets

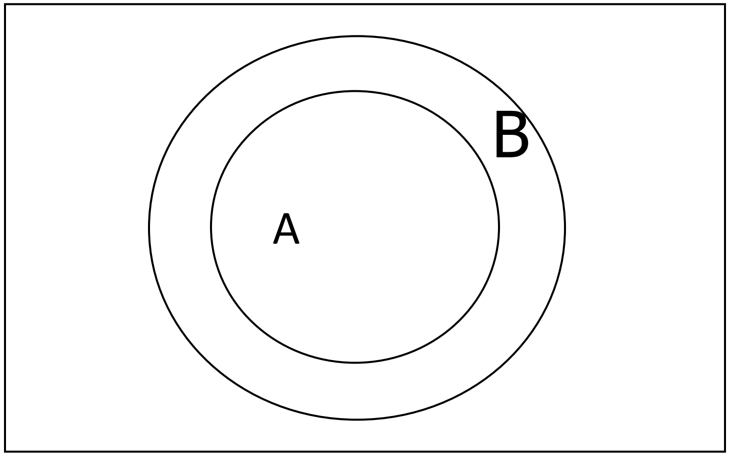


A is a subset of B

$A \subset B$

A test that returns true iff $A \subset B$

Subsets (2)



A is a subset or equal to B

$A \subseteq B$

A test that returns true iff $A \subseteq B$

The Power Set

- Given a set S, the ***power set*** is the set of all subsets of the set S
  - Denoted by P(S) or $\mathbb{P}(S)$

- E.g. the power set of {0,1,2} is …

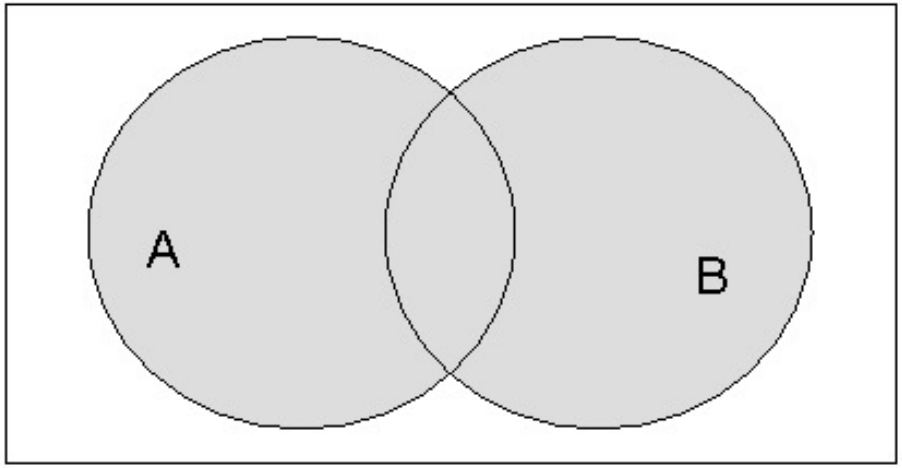
The Power Set

- E.g. the power set of {0,1,2} is
  - P({0,1,2}) = {Ø, {0}, {1}, {2}, {0,1}, {0,2}, {1,2}, {0,1,2} }
  - NB - the **empty set** and the **set itself** are members of this set of subsets
- If a set has n elements, its power set has 2^n elements
- The power set does not contain numbers, it contains SETs

Set Operations

- Two sets can be combined in many different ways
  - The following illustrates some such combinations

Union



The union of A and B

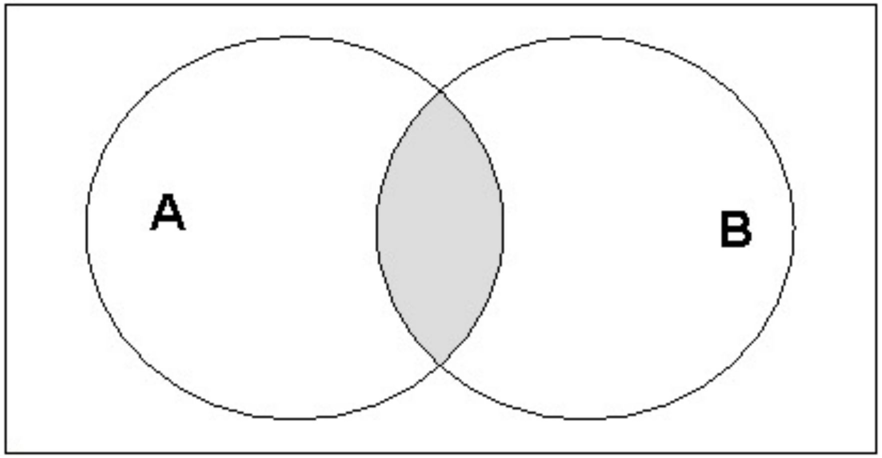
Symbol like Union

$A \cup B$

$$A \cup B = \{x \mid x \in A \vee x \in B\}$$

The set that contains those elements that are either in A, B, or in both

Intersection



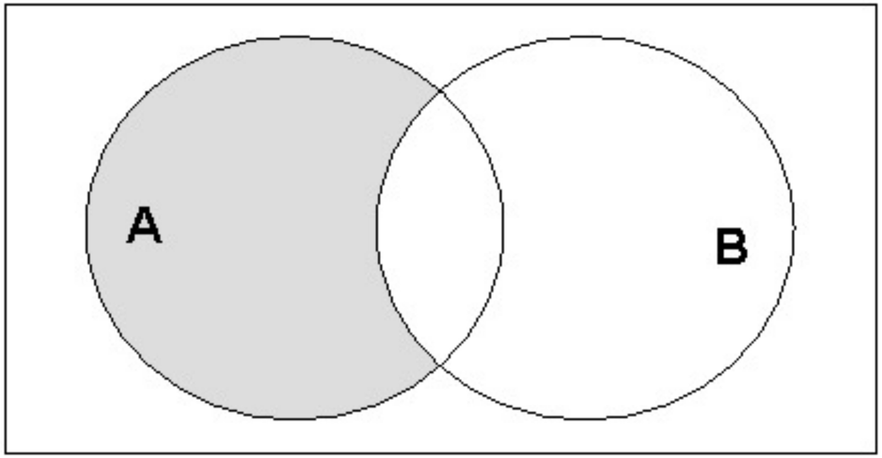
The **intersection** of A and B

$A \cap B$

Symbol like aNd

$$A \cap B = \{x \mid x \in A \wedge x \in B\}$$

Difference

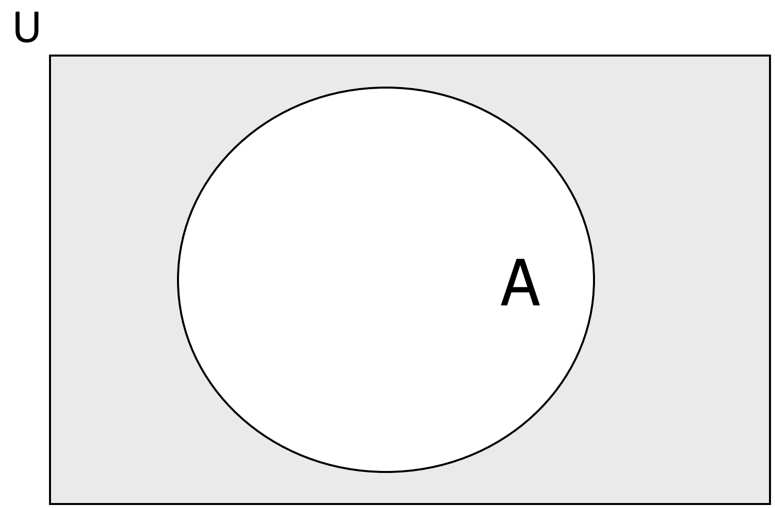


The **difference** of A and B

A - B

$$A - B = \{x \mid x \in A \wedge x \notin B\}$$

Complement

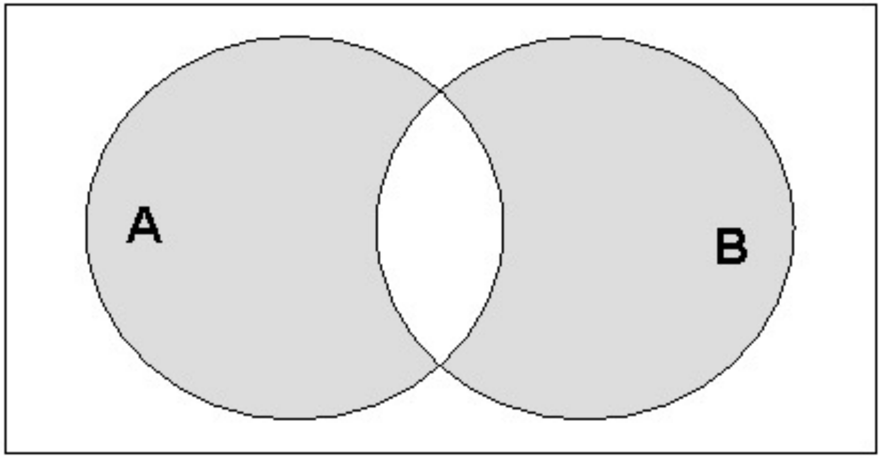


The complement of A

$\overline{A}$

$$\overline{A} = \{x \mid x \notin A\}$$

Symmetric Difference



The symmetric difference of A and B

$$A \oplus B = (A - B) \cup (B - A)$$

$$A \oplus B = \{x \mid (x \in A \wedge x \notin B) \vee (x \in B \wedge x \notin A)\}$$

Summary

- What are sets?
- Notation for making sets, comparing sets
- Operators: making new sets from other sets
  - **$\cup$ union**
  - **$\cap$ intersection**
  - **– difference**
  - **$\overline{A}$ complement**
  - **$\oplus$ symmetric difference**

Example Exam Questions

Given the following sets:

$$A = \{1,3,7,9\},\ B = \{2,4,6\},\ C = \{7,9\}$$

determine the following (assume that P is the powerset operator):

- $|B|$ = 3
- $P(B)$ = $\{\ \{\}, \{2\},\{4\},\{6\}, \{2,4\}, \{4,6\}, \{2,4,6\}, \{2,6\}\ \}$
- $|P(C)|$ = 4
- $A \cup B$ = $\{1,2,3,4,6,7,9\}$
- $A \cap C$ = $\{7,9\}$
- Which of the following are true?
  - $C \subset A$ = TRUE
  - $C \subset B$ = FALSE

Summary of Sets

- What are sets? {1,3,5}
- Set builder notation for making sets; comparing sets
- Operators: making new sets from other sets
  - **$\cup$ union**
  - **$\cap$ intersection**
  - **– difference**
  - **$\overline{A}$ complement**
  - **$\oplus$ symmetric difference**

- So how does this help with databases?

So why is this useful?

Consider a query:

"What are the grades of student 8187491?"

How can we query the database to obtain this information?

So why is this useful?

Two ways of querying a database:

- procedural (relational algebra, Pandas)
  - sequence of operations
  - the output of each operation is the input to the next operation

result ← F4 (F2 (F1(tableA), tableB), F3(tableC))

- declarative (SQL)
  - describes the desired results (in terms of conditions)
  - the DBMS works out the operations

result ← CONDITIONS (tableA, tableB, tabl

This is based on set theory

This is internally implemented as RA operations

RA is key to understanding SQL query processing!

A Relational Instance

STUDENT

name	matric	exam1	exam2
Gedge	891023	12	58
Kerr	892361	66	90
Fraser	880123	50	65

Schema

- This Student relation instance has:
  - Degree 4 and Cardinality 3
- A **relation** is a **set** of **tuples** (or **n-tuples**)
  - As each tuple has n values (same as degree of the relation)
  - E.g. <Fraser,880123,50,65> is a 4-tuple.

N-tuples are not Sets

- The order of elements in a collection is sometimes important
  - But **<u>sets are unordered</u>,** so a different structure is needed

- This is provided by ***ordered n-tuples***
  - *<2,1,5> is an 3-tuple*

N-tuples

- Two ordered n-tuples are equal if and only if each corresponding pair of their elements is equal
  - $<a_1,a_2,....a_n> = <b_1,b_2,.....b_n>$ if and only if $a_i=b_i$ for $i=1,2,...n$
  - {1, 3, 5} = {3, 1, 5} = TRUE for SETS
  - $<1, 3, 5> = <3, 1, 5>$ = FALSE for N-TUPLES
- NB: We can use <> or () to denote tuples, but not {}

Cartesian Product

Let A and B be sets

- The ***cartesian product*** of A and B (A X B) is
    - the set of all ***ordered pairs*** *(i.e. tuples)*

      $\langle a,b \rangle$ where $a \in A$ and $b \in B$

A={0,1}, B={a,b,c}

A X B =

Cartesian Product

Let A and B be sets

- The ***cartesian product*** of A and B (A X B) is
  - the set of all ***ordered pairs*** *(i.e. tuples)*

    $\langle a,b \rangle$ where $a \in A$ and $b \in B$

A={0,1}, B={a,b,c}

A X B =

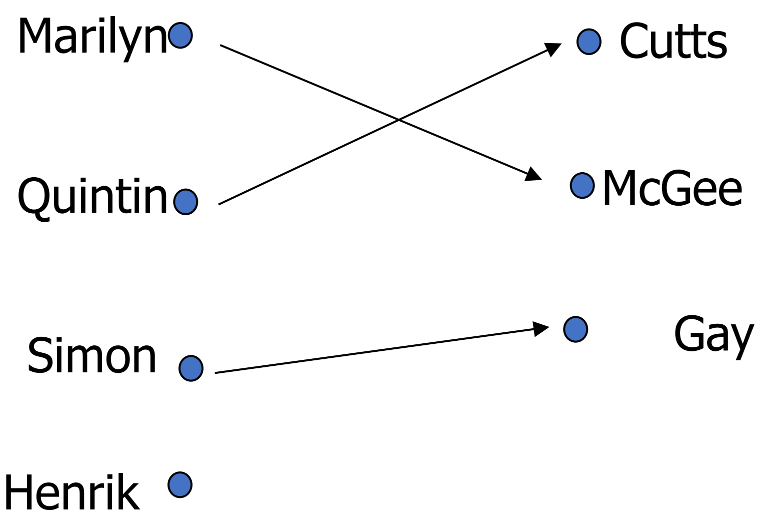
{<0,a>,<0,b>,<0,c>, <1,a>, <1,b> ,<1,c>}

Connecting to Databases: Representing a Relation

Forename = {Marilyn, Quintin, Simon, Henrik}
Surname = {McGee, Cutts, Gay}

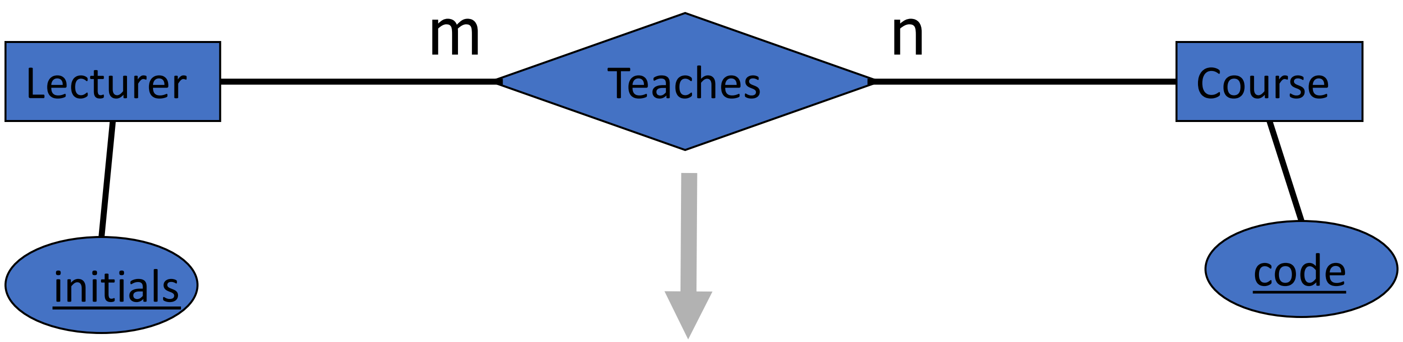
Domains

Names = {<Marilyn,McGee>,<Quintin,Cutts>, <Simon, Gay>}



R	Cutts	McGee	Gay
Marilyn		x	
Quintin	x		
Simon			x
Henrik			

A Relationship and its Equivalent Relation



Lecturer

Initials
MMcG
QC
SGay
HRight
RInnis

Teaches

Lecturer	Course
MMcG	CS1Q
QC	CS1CT
SGay	CS1P
SGay	CS1Q

Course

Code
CS1Q
CS1CT
CS1P

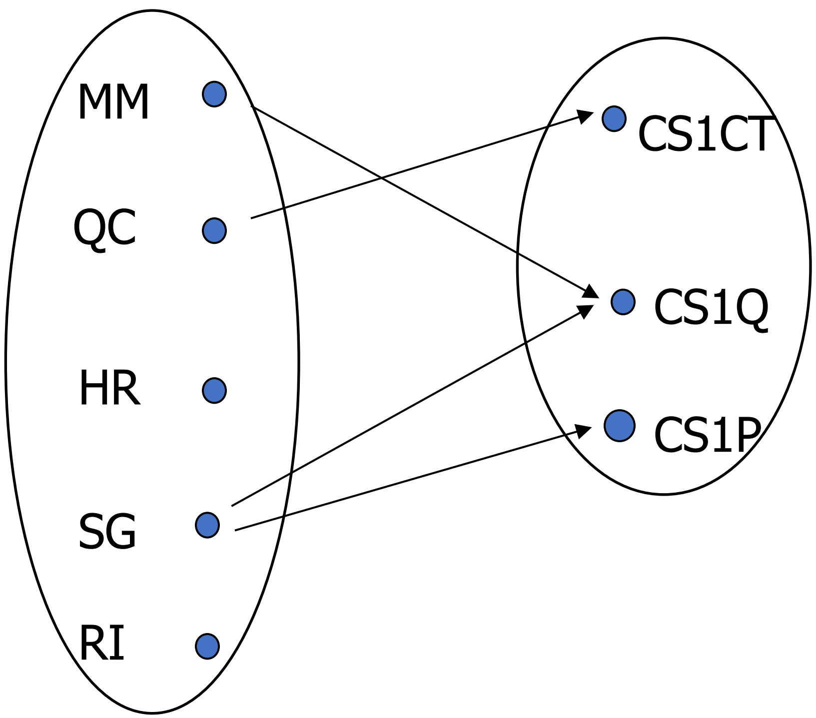
University of Glasgow

The Table as a Relation

Lecturers = {MM, QC, SG, HR, RI}

Courses = {CS1P, CS1Q, CS1CT}

Teaches = {<MM, CS1Q>, <QC, CS1CT>, <SG, CS1P>, <SG,CS1Q>}



Teaches	CS1P	CS1Q	CS1CT
MM		x	
QC			x
SG	X	X	
HR			
RI			