Large SVARs: A Gibbs Sampler for Efficient Bayesian Inference in Sign-Identified SVARs

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SVARs with Sign Restrictions and Large Models

Increasing availability of large datasets $\rightarrow$ growing need for large VARs

- There are many applications in forecasting, but not so many in structural analysis beyond very simple identification schemes

Sign restrictions are widely used in practice to identify structural shocks (e.g.,Canova and De Nicolo, 2002; Uhlig, 2005)

The conventional approach to sign-identified SVARs is accept-reject methods (e.g., Rubio-Ramirez, Waggoner, and Zha, 2010), which fail when

- identification is tight;
- the number of variables is large

Need: Efficient inference methods under tight identification and large models

Gibbs Sampler with Elliptical Slice Sampling

Recent work tries to solve the problem: Chan, Matthes, Yu (2025) (more efficient accept-reject methods); Read and Zhu (2025) (approximating inequalities using smooth functions)

- The bottlenecks still appear when the model is large and/or identification is tight

We propose **elliptical slice sampling within a Gibbs Sampler**

- Eenables tractable inference even under tight identification
- Supports several priors for VARs
  - Natural conjugate
  - Independent
  - Asymmetric
- Allows for general form of sign restrictions

Result: Substantial computational gains

Applications and Benchmarking

Application 1: Killian and Murphy (2014) oil market SVAR

- Combination of sign and ranking restrictions to identify
  - Flow supply
  - Flow demand
  - Speculative demand
- They use the a variant of accept-reject methods
- As we add ranking restrictions the method becomes impractical

Application 2: Chan et al. (2025) large SVAR with 35 variables and 8 shocks

- Their accept-reject methods become impractical as the number of shocks increases
- Our ESS-based Gibbs sampler scales well with the number of shocks

Our algorithm: General, efficient, scalable

One line summary

Our ESS-within-Gibbs sampler breaks with the accept–reject tradition, delivers large speed gains, and turns previously infeasible applications into feasible ones

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Our ESS-within-Gibbs sampler breaks with the accept–reject tradition, delivers large speed gains, and turns previously infeasible applications into feasible ones

Working with sign-restricted SVARs? You should consider our methods that are scalable and fast

Road map

A simple model of demand and supply

- identification and sign-restrictions
- estimation and inference
- accept-reject methods
- elliptical slice sampling within a Gibbs sampler

A general framework and method for SVARs

- extension to general SVARs

Two empirical applications

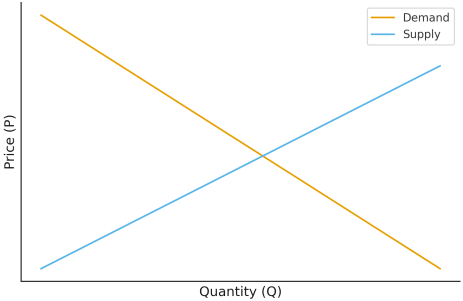
A Simple Model of Demand and Supply

A Simple Demand and Supply Model

Consider a system without lags

$$P_t = \alpha Q_t + \sigma_D \varepsilon_t^D, \quad \varepsilon_t^D \sim N(0,1)$$
$$Q_t = \beta P_t + \sigma_S \varepsilon_t^S, \quad \varepsilon_t^S \sim N(0,1)$$

Demand and supply curves



A Simple Demand and Supply Model

Consider a system without lags

$$
\begin{aligned}
P_t &= \alpha Q_t + \sigma_D \varepsilon_t^D, \quad \varepsilon_t^D \sim N(0,1) \\
Q_t &= \beta P_t + \sigma_S \varepsilon_t^S, \quad \varepsilon_t^S \sim N(0,1)
\end{aligned}
$$

Equivalently,

$$
\begin{pmatrix} 1 & -\alpha \\ -\beta & 1 \end{pmatrix} \begin{pmatrix} P_t \\ Q_t \end{pmatrix} = \begin{pmatrix} \sigma_D \varepsilon_t^D \\ \sigma_S \varepsilon_t^S \end{pmatrix} \iff \begin{aligned} P_t &= \quad = \tfrac{\sigma_D}{1-\alpha\beta}\varepsilon_t^D + \tfrac{\alpha\sigma_S}{1-\alpha\beta}\varepsilon_t^S \\ Q_t &= \quad = \tfrac{\beta\sigma_D}{1-\alpha\beta}\varepsilon_t^D + \tfrac{\sigma_S}{1-\alpha\beta}\varepsilon_t^S \end{aligned}
$$

This is a structural VAR with four unknown parameters: $(\alpha, \beta, \sigma_D, \sigma_S)$.

A Reduced-Form Model

Observed price-quantity pairs can be statistically modeled as,

$$\begin{aligned} P_t &= u_{1,t} \\ Q_t &= u_{2,t} \end{aligned} \quad , \quad u_t = [u_{1,t}, u_{2,t}]' \sim N(0, \Sigma)$$

Applying Cholesky factorization $\Sigma = \Sigma^{tr}\Sigma^{tr\prime}$

$$\Sigma = \begin{pmatrix} \Sigma^{tr}_{11} & 0 \\ \Sigma^{tr}_{21} & \Sigma^{tr}_{22} \end{pmatrix} \begin{pmatrix} \Sigma^{tr}_{11} & \Sigma^{tr}_{21} \\ 0 & \Sigma^{tr}_{22} \end{pmatrix}$$

This is a reduced-form VAR with three unknown parameters: $(\Sigma^{tr}_{11}, \Sigma^{tr}_{21}, \Sigma^{tr}_{22})$.

Orthogonal Reduced-form Representation

A simple structural model

$$\begin{pmatrix} P_t \\ Q_t \end{pmatrix} = A\varepsilon_t$$

where $\varepsilon_t = [\varepsilon_t^D, \varepsilon_t^S]' \sim N(0, I_2)$.

For example,

$$\begin{aligned} P_t &= \tfrac{\sigma_D}{1-\alpha\beta}\varepsilon_t^D + \tfrac{\alpha\sigma_S}{1-\alpha\beta}\varepsilon_t^S \\ Q_t &= \tfrac{\beta\sigma_D}{1-\alpha\beta}\varepsilon_t^D + \tfrac{\sigma_S}{1-\alpha\beta}\varepsilon_t^S \end{aligned}$$

Four parameters

A reduced-form model

$$\begin{pmatrix} P_t \\ Q_t \end{pmatrix} = u_t$$

$u_t = [u_{1,t}, u_{2,t}]' \sim N(0, \Sigma)$

For example,

$$\Sigma = \begin{pmatrix} \Sigma_{11}^{tr} & 0 \\ \Sigma_{21}^{tr} & \Sigma_{22}^{tr} \end{pmatrix} \begin{pmatrix} \Sigma_{11}^{tr} & \Sigma_{21}^{tr} \\ 0 & \Sigma_{22}^{tr} \end{pmatrix}$$

Three parameters

Orthogonal Reduced-form Representation

There are infinitely many structural models that are observationally equivalent to a reduced-form model with Σ

$$\begin{pmatrix} P_t \\ Q_t \end{pmatrix} = A\varepsilon_t \quad \Longleftrightarrow \quad \begin{pmatrix} P_t \\ Q_t \end{pmatrix} = \begin{pmatrix} \Sigma_{11}^{tr} & 0 \\ \Sigma_{21}^{tr} & \Sigma_{22}^{tr} \end{pmatrix} \underbrace{\begin{pmatrix} q_{11} & q_{12} \\ q_{21} & q_{22} \end{pmatrix}}_{=Q} \begin{pmatrix} \varepsilon_t^D \\ \varepsilon_t^S \end{pmatrix}$$

where Q is an orthogonal matrix (i.e., $QQ' = I_2$ and $Q \in \mathcal{O}(2)$).

- Q is not identified by the data
- In the 2×2 case, Q can be parameterized by θ and s:

$$Q = \begin{pmatrix} \cos(\theta) & -\sin(\theta)s \\ \sin(\theta) & \cos(\theta)s \end{pmatrix}, \quad \theta \in [0, 2\pi], \quad s \in \{+1, -1\}$$

- (Σ, Q) is called the orthogonal-reduced-form parameterization

Traditional Approach to Identification

The traditional approach makes a strong assumption about the non-identified part of the model.

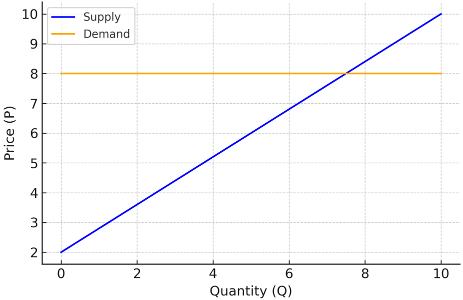
For example, $Q = I_2$,

$$\begin{pmatrix} P_t \\ Q_t \end{pmatrix} = \begin{pmatrix} \Sigma^{tr}_{11} & 0 \\ \Sigma^{tr}_{21} & \Sigma^{tr}_{22} \end{pmatrix} \underbrace{\begin{pmatrix} q_{11} & q_{12} \\ q_{21} & q_{22} \end{pmatrix}}_{=I_2} \begin{pmatrix} \varepsilon^D_t \\ \varepsilon^S_t \end{pmatrix}$$

$$= \begin{pmatrix} \Sigma^{tr}_{11} & 0 \\ \Sigma^{tr}_{21} & \Sigma^{tr}_{22} \end{pmatrix} \begin{pmatrix} \varepsilon^D_t \\ \varepsilon^S_t \end{pmatrix}$$

Supply shocks do not affect price

$\Rightarrow$ The demand curve is horizontal.



Identification via Sign Restrictions

Sign restrictions may be more natural

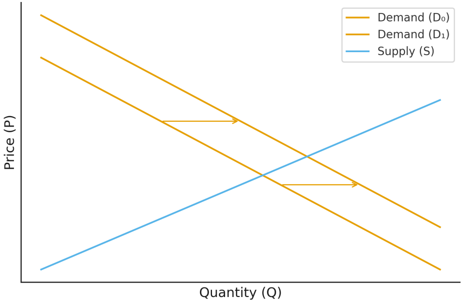
For example, demand shocks increases both price and quantity

$$\begin{pmatrix} P_t \\ Q_t \end{pmatrix} = \begin{pmatrix} \Sigma_{11}^{tr} & 0 \\ \Sigma_{21}^{tr} & \Sigma_{22}^{tr} \end{pmatrix} \begin{pmatrix} q_{11} & q_{12} \\ q_{21} & q_{22} \end{pmatrix} \begin{pmatrix} \varepsilon_t^D \\ \varepsilon_t^S \end{pmatrix}$$

The restrictions are

$$\Sigma_{11}^{tr} q_{11} > 0 \quad \text{and} \quad (\Sigma_{21}^{tr} q_{11} + \Sigma_{22}^{tr} q_{21}) > 0$$

$\Rightarrow$ The supply curve is positively sloped



Identification via Sign Restrictions

Assume $\Sigma^{tr}_{11} = \Sigma^{tr}_{22} = 1$ and $\Sigma^{tr}_{21} > 0$

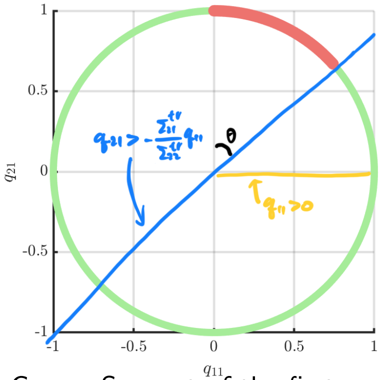
The previous sign restrictions

$\Sigma^{tr}_{11} q_{11} > 0$ and $(\Sigma^{tr}_{21} q_{11} + \Sigma^{tr}_{22} q_{21}) > 0$

imply

$$\theta \in \left[\arctan(\Sigma^{tr}_{21}),\ \frac{\pi}{2}\right]$$

where $q_{11} = \cos(\theta)$ and $q_{21} = \sin(\theta)$



- Green: Support of the first column of Q
- Red: Identified set for the first column of Q

Bayesian Estimation and Inference

Let the data be $Y_t = [P_t, Q_t]'$.

Likelihood

$$Y_t \sim N(0, \Sigma), \quad t = 1, 2, ..., T$$

Prior

$$p(\Sigma, Q | S_R(\Sigma, Q) > 0) \propto p(\Sigma)p(Q)1\{S_R(\Sigma, Q) > 0\}$$

where

- $p(\Sigma)$: Standard prior such as an inverse Wishart distribution
- $p(Q)$: Uniform prior (Haar measure) over $\mathcal{O}(n)$
- $1\{S_R(\Sigma, Q) > 0\}$: General sign restrictions where $S_R(\Sigma, Q)$ can be nonlinear

Standard Approach to Posterior Simulation

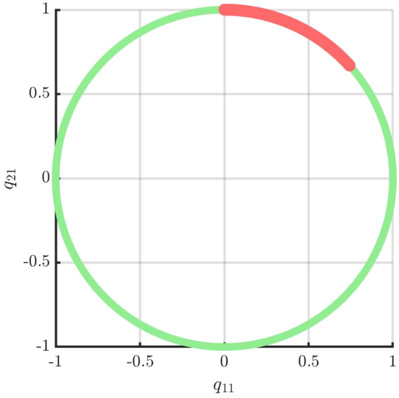
Joint posterior distribution

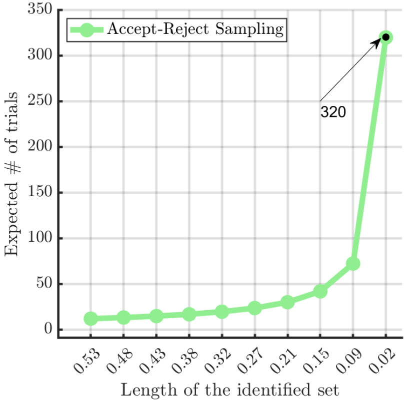
$$p(\Sigma, Q | Y_{1:T}, S_R(\Sigma, Q) > 0) \propto p(Y_{1:T}|\Sigma)p(\Sigma)p(Q)1\{S_R(\Sigma, Q) > 0\}$$

[Accept-Reject Algorithm]: The following algorithm independently draws

1. Draw Σ independently from $p(\Sigma|Y_{1:T}) \propto p(Y_{1:T}|\Sigma)p(\Sigma)$
2. Draw Q independently from the uniform distribution over $\mathcal{O}(n)$
3. Keep (Σ, Q) if the sign restrictions are satisfied
4. Return to Step 1 until the required number of draws has been obtained

The Accept-Reject Algorithm Eventually Fails





The size of the set depends on

- the tighteness and number of restrictions
- the number of shocks

Large SVARs 1: Restrictions on Many Variables

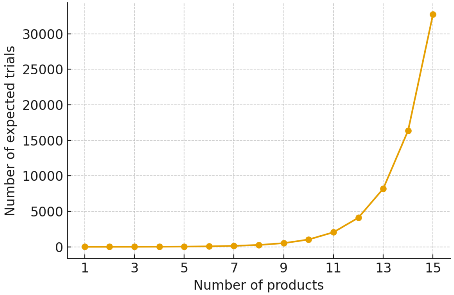
Consider m products,

$$\begin{pmatrix} P_{1t} \\ Q_{1t} \\ \vdots \\ P_{mt} \\ Q_{mt} \end{pmatrix} = \Sigma^{tr} \times Q \times \begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \vdots \\ \varepsilon_{2m-1,t} \\ \varepsilon_{2m,t} \end{pmatrix}$$

Aggregate demand shocks raise prices and quantities for all products.

- ε_{1t} increases P_{it} and Q_{it} for $i = 1, 2, ..., m$.

Computation time increases as the number of products increases.



Large SVARs 2: Identifying Many Shocks

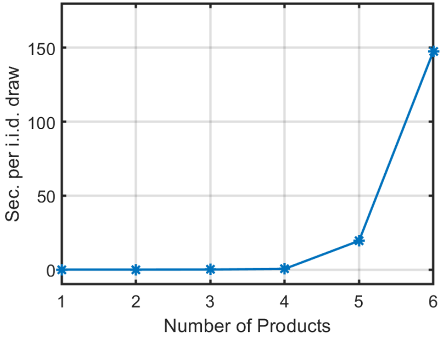
Consider m products,

$$\begin{pmatrix} P_{1t} \\ Q_{1t} \\ \vdots \\ P_{mt} \\ Q_{mt} \end{pmatrix} = \Sigma^{tr} \times Q \times \begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \vdots \\ \varepsilon_{2m-1,t} \\ \varepsilon_{2m,t} \end{pmatrix}$$

Product-specific demand shocks raise the prices and quantities of their own products:

- ε_{jt} increases P_{jt} and Q_{jt} for $j = 1, 2, ..., m$.

Computation time increases as the number of products increases.



Our Proposal: Elliptical Slice Sampling within Gibbs

The joint posterior distribution

$$p(\mathbf{\Sigma}, \mathbf{Q} \,|\, \mathbf{y}_{1:T}, \mathbf{S}_R(\mathbf{\Sigma}, \mathbf{Q}) > \mathbf{0}) \propto p(\mathbf{y}_{1:T} \,|\, \mathbf{\Sigma}) p(\mathbf{\Sigma}) p(\mathbf{Q}) 1\{\mathbf{S}_R(\mathbf{\Sigma}, \mathbf{Q}) > \mathbf{0}\}$$

A Gibbs sampling algorithm that iterates two conditionals

1. $p(\mathbf{Q} \,|\, \mathbf{\Sigma}, \mathbf{y}_{1:T}, \mathbf{S}_R(\mathbf{\Sigma}, \mathbf{Q}) > \mathbf{0}) \propto p(\mathbf{Q}) 1\{\mathbf{S}_R(\mathbf{\Sigma}, \mathbf{Q}) > \mathbf{0}\}$
2. $p(\mathbf{\Sigma} \,|\, \mathbf{Q}, \mathbf{y}_{1:T}, \mathbf{S}_R(\mathbf{\Sigma}, \mathbf{Q}) > \mathbf{0}) \propto p(\mathbf{y}_{1:T} \,|\, \mathbf{\Sigma}) p(\mathbf{\Sigma}) 1\{\mathbf{S}_R(\mathbf{\Sigma}, \mathbf{Q}) > \mathbf{0}\}$

Each step is carried out using *elliptical slice sampling*

Our algorithm works as long as

- $\mathbf{S}_R(\mathbf{\Sigma}, \mathbf{Q})$ is continuous
- $p(\mathbf{y}_{1:T} \,|\, \mathbf{\Sigma}) p(\mathbf{\Sigma})$ has an inverse-Wishart kernel in $\mathbf{\Sigma}$

Elliptical Slice Sampling (ESS)

ESS is a **rejection-free** Markov chain Monte Carlo (MCMC) algorithm designed to sample from posteriors of the form:

$$p(\boldsymbol{x}) \propto L(\boldsymbol{x})\,\mathcal{N}(\boldsymbol{x}; \boldsymbol{\mu}, \boldsymbol{\Sigma})$$

where $L(\boldsymbol{x})$ is a likelihood function and the prior is Gaussian.

Key Idea

- A proposal lies on an ellipse defined by

  $$\boldsymbol{x}^* = \boldsymbol{v}\sin(\varphi) + \boldsymbol{x}^{old}\cos(\varphi), \quad \varphi \in [0, 2\pi]$$

  where $\boldsymbol{v} \sim \mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$

- Adaptively choose φ in a way such that a) the proposal is always accepted and b) the stationary distribution of the Markov chain is $p(\boldsymbol{x})$

Drawing from $p(\mathbf{Q} \mid \mathbf{\Sigma}, \mathbf{y}_{1:T}, \boldsymbol{S}_R(\mathbf{\Sigma}, \mathbf{Q}) > 0)$

Use a transformation from a matrix normal to $\mathbf{Q}$ via the QR-decomposition.

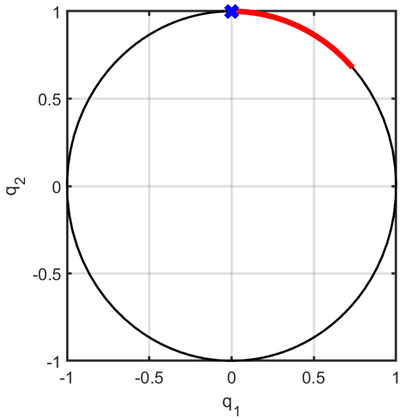
- Let $\mathbf{X} \sim \mathcal{N}_{n\times n}(\mathbf{0}, \mathbf{I}_n, \mathbf{I}_n)$. That is, an $n \times n$ matrix whose entries are i.i.d. $\mathcal{N}(0, 1)$.
- Define the mapping $\mathbf{Q} = \gamma(\mathbf{X})$, where γ extracts the orthogonal matrix from the QR-decomposition of $\mathbf{X}$.
- Then $\mathbf{Q}$ is distributed uniformly with respect to the Haar measure.

Then, the conditional posterior can be written as

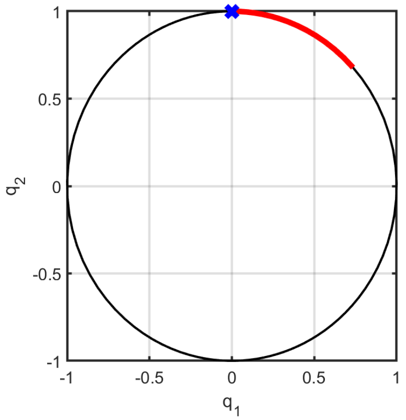
$$[\boldsymbol{S}_R(\mathbf{\Sigma}, \gamma(\mathbf{X})) > 0] N_{(\mathbf{0}, \mathbf{I}_n, \mathbf{I}_n)}(\mathbf{X}).$$

$\mathbf{X}$ is Gaussian. We can use elliptical slice sampling (ESS) to draw from the truncated Gaussian. Then, transform $\mathbf{X}$ back to $\mathbf{Q}$.

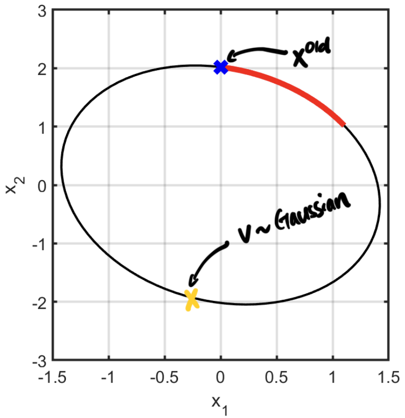
Example of drawing Q, Initial draw



Example of drawing Q, Initial draw

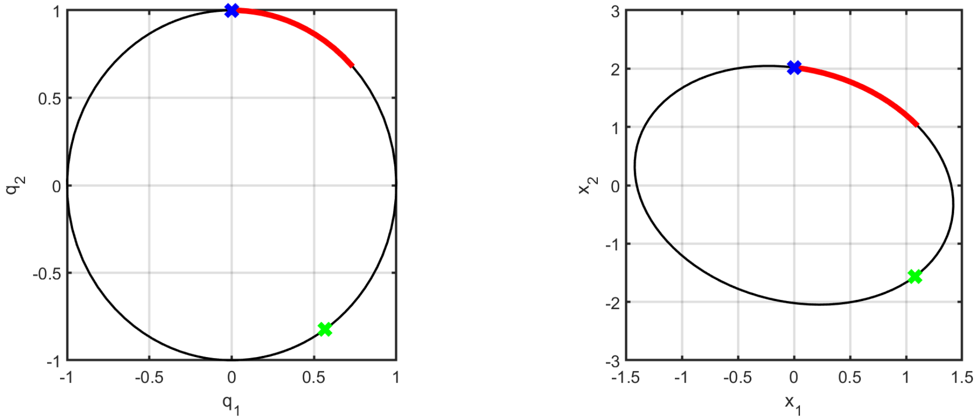


Potential proposals lie on ellipsis:

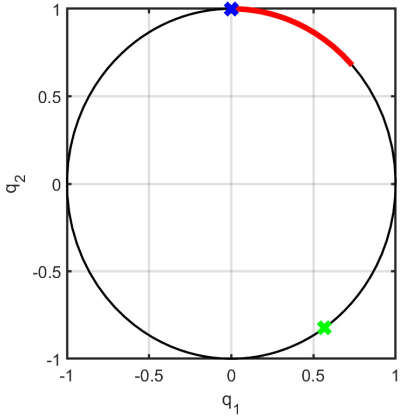


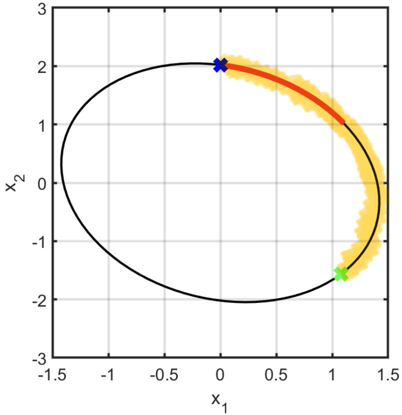
$$\boldsymbol{x}^* = \boldsymbol{v}\sin(\varphi) + \boldsymbol{x}^{old}\cos(\varphi), \quad \varphi \in [0, 2\pi]$$

Example of drawing Q, First trial

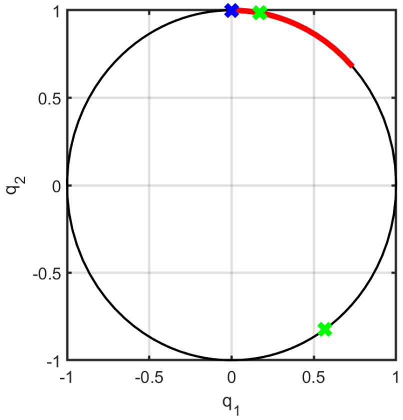


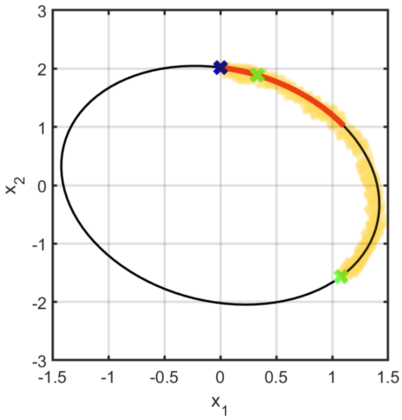
Example of drawing Q, Range for the second trial



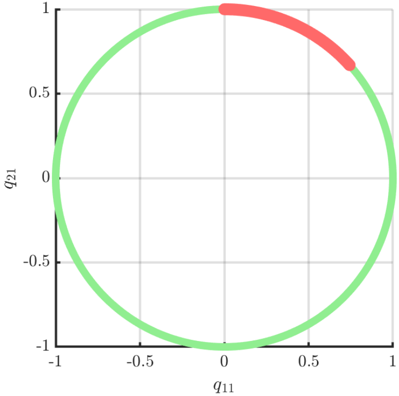


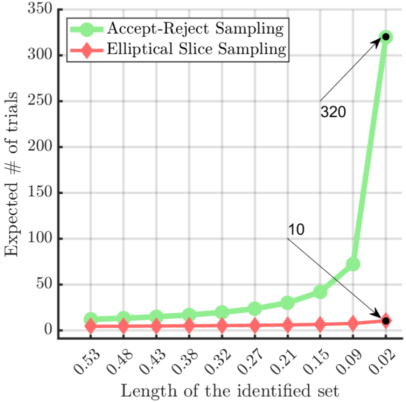
Example of drawing Q, Accepted draw





Performance: Simple Demand-Supply Example





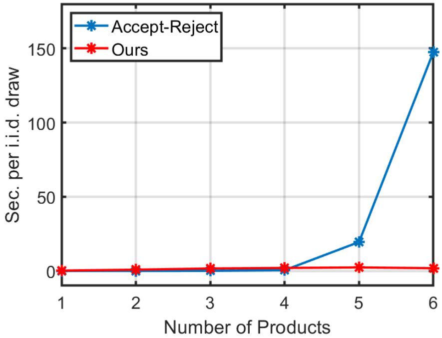
Performance: Large Demand-Supply Example

Suppose we want to identify m product-specific demand shocks:

$$\begin{pmatrix} P_{1t} \\ Q_{1t} \\ \vdots \\ P_{mt} \\ Q_{mt} \end{pmatrix} = \Sigma^{tr} \times Q \times \begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \vdots \\ \varepsilon_{2m-1,t} \\ \varepsilon_{2m,t} \end{pmatrix}$$

- ε_{jt} increases P_{jt} and Q_{jt} for $j = 1, 2, ..., m$.

ESS is much faster than accept-reject:



Drawing from $p(\mathbf{\Sigma} \mid \mathbf{Q}, \mathbf{y}_{1:T}, S_R(\mathbf{\Sigma}, \mathbf{Q}) > 0)$

Use a transformation from a matrix normal to an inverse Wishart via a quadratic mapping:

- Let $\mathbf{R} \sim \mathcal{N}_{n\times\tilde{\nu}}(\mathbf{0}, \tilde{\mathbf{\Phi}}^{-1}, \mathbf{I}_{\tilde{\nu}})$.
- Define the mapping $\mathbf{S} = \varsigma(\mathbf{R}) = (\mathbf{R}\mathbf{R}')^{-1}$.
- Then $\mathbf{S}$ is distributed as an inverse Wishart: $\mathbf{S} \sim \mathcal{IW}(\tilde{\nu}, \tilde{\mathbf{\Phi}})$.

Then, the conditional posterior can be written as

$$[S_R(\mathbf{B}, \varsigma(\mathbf{R}), \mathbf{Q}) > 0] N_{(\mathbf{0}, \tilde{\mathbf{\Phi}}^{-1}, \mathbf{I}_{\tilde{\nu}})}(\mathbf{R}).$$

$\mathbf{R}$ is Gaussian. We can use elliptical slice sampling (ESS) to sample from the conditional posterior distribution. Then, we transform $\mathbf{R}$ back to $\mathbf{\Sigma}$.

So far ...

Using a simple model of demand and supply, we illustrate that a standard method for estimating sign-restricted SVARs can be very inefficient.

We propose a method to learn about the posterior distribution

$$p(\boldsymbol{\Sigma}, \mathbf{Q} \,|\, \mathbf{y}_{1:T}, \mathbf{S}_R(\boldsymbol{\Sigma}, \mathbf{Q}) > \mathbf{0}) \propto p(\mathbf{y}_{1:T} \,|\, \boldsymbol{\Sigma}) p(\boldsymbol{\Sigma}) p(\mathbf{Q}) 1\{\mathbf{S}_R(\boldsymbol{\Sigma}, \mathbf{Q}) > \mathbf{0}\}$$

A Gibbs sampling algorithm that iterates two conditionals

1. $p(\mathbf{Q} \,|\, \boldsymbol{\Sigma}, \mathbf{y}_{1:T}, \mathbf{S}_R(\boldsymbol{\Sigma}, \mathbf{Q}) > \mathbf{0}) \propto p(\mathbf{Q}) 1\{\mathbf{S}_R(\boldsymbol{\Sigma}, \mathbf{Q}) > \mathbf{0}\}$
2. $p(\boldsymbol{\Sigma} \,|\, \mathbf{Q}, \mathbf{y}_{1:T}, \mathbf{S}_R(\boldsymbol{\Sigma}, \mathbf{Q}) > \mathbf{0}) \propto p(\mathbf{y}_{1:T} \,|\, \boldsymbol{\Sigma}) p(\boldsymbol{\Sigma}) 1\{\mathbf{S}_R(\boldsymbol{\Sigma}, \mathbf{Q}) > \mathbf{0}\}$

Each step is done using *elliptical slice sampling*

General Case

The Model

Consider the SVAR with the general form,

$$\mathbf{y}_t' \, \mathbf{A}_0 = \mathbf{x}_t' \, \mathbf{A}_+ + \varepsilon_t', \quad \varepsilon_t \sim N(0, I_n)$$

- Let $1\{\mathbf{S}_S(\mathbf{A}_0, \mathbf{A}_+) > 0\}$ equal 1 if the sign restrictions are satisfied and 0 otherwise.

The orthogonal reduced-form parameterization is

$$\mathbf{y}_t' = \mathbf{x}_t'\mathbf{B} + \varepsilon_t'\mathbf{Q}' h(\mathbf{\Sigma})$$

- Let $1\{\mathbf{S}_R(\mathbf{B}, \mathbf{\Sigma}, \mathbf{Q}) > 0\}$ in terms of the orthogonal reduced-form parameterization

The Object

The objective is to draw from and transform to parameter of interest

$$p(\mathbf{B}, \mathbf{\Sigma}, \mathbf{Q} \mid \mathbf{y}_{1:T}, \mathbf{S}_R(\mathbf{B}, \mathbf{\Sigma}, \mathbf{Q}) > 0)$$

Which can be decomposed into

$$p(\mathbf{B}, \mathbf{\Sigma}, \mathbf{Q} \mid \mathbf{y}_{1:T}, \mathbf{S}_R(\mathbf{B}, \mathbf{\Sigma}, \mathbf{Q}) > 0) \propto p(\mathbf{y}_{1:T} \mid \mathbf{B}, \mathbf{\Sigma}) p(\mathbf{B}, \mathbf{\Sigma}, \mathbf{Q}) 1\{\mathbf{S}_R(\mathbf{B}, \mathbf{\Sigma}, \mathbf{Q}) > 0\}$$

where

- $p(\mathbf{y}_{1:T} \mid \mathbf{B}, \mathbf{\Sigma})$: Likelihood
- $p(\mathbf{B}, \mathbf{\Sigma}, \mathbf{Q})$: Prior
- $1\{\mathbf{S}_R(\mathbf{B}, \mathbf{\Sigma}, \mathbf{Q}) > 0\}$: Sign-restriction indicator function

The Prior

We consider a *rotational invariant* prior distribution,

$$p(\mathbf{B}, \mathbf{\Sigma}, \mathbf{Q}) = p(\mathbf{B}, \mathbf{\Sigma}) p(\mathbf{Q})$$

where

- $\mathbf{Q}$ is drawn from the Uniform distribution (Haar) on $\mathcal{O}(n)$:

$$p(\mathbf{Q}) = \frac{1}{\int_{\mathcal{O}(n)} d\,\mathbf{Q}}$$

- For $\mathbf{B}$ and $\mathbf{\Sigma}$, we consider various priors
  - Natural Conjugate prior, Independent prior, Asymmetric prior, ...
  - Our algorithm works as long as conditional distribution of $\mathbf{B}$ is Gaussian and $\mathbf{\Sigma}$ is inverse Wishart

- Observationally equivalent $(\mathbf{B}, \mathbf{\Sigma}, \mathbf{Q})$'s have the same prior value

The Accept-Reject Algorithm (Current standard in practice)

Algorithm 1

The following algorithm independently draws from the posterior distribution conditional on the sign restrictions.

1. *Draw* $(\mathbf{B}, \mathbf{\Sigma})$ *independently from* $p(\mathbf{B}, \mathbf{\Sigma} \mid \mathbf{y}_{1:T})$.
2. *Draw* $\mathbf{Q}$ *independently from the uniform over* $\mathcal{O}(n)$.
3. *Keep* $(\mathbf{B}, \mathbf{\Sigma}, \mathbf{Q})$ *if the sign restrictions are satisfied:* $[\mathbf{S}_R(\mathbf{B}, \mathbf{\Sigma}, \mathbf{Q}) > 0] = 1$.
4. *Return to Step 1 until the required number of draws has been obtained.*
5. *Transform to parameterization of interest.*

Our Proposed Algorithm

Algorithm 2

The following algorithm draws from the posterior distribution conditional on the sign restrictions.

1. *Draw* $\mathbf{Q}^i$ *from*

$$p(\mathbf{Q} \mid \mathbf{B}^{i-1}, \boldsymbol{\Sigma}^{i-1}, \mathbf{y}_{1:T}, \mathbf{S}_R(\cdot) > 0) \propto [\mathbf{S}_R(\cdot) > 0] p(\mathbf{Q})$$

2. *Draw* $\boldsymbol{\Sigma}^i$ *from*

$$p(\boldsymbol{\Sigma} \mid \mathbf{B}^{i-1}, \mathbf{Q}^i, \mathbf{y}_{1:T}, \mathbf{S}_R(\cdot) > 0) \propto [\mathbf{S}_R(\cdot) > 0] p(\boldsymbol{\Sigma} \mid \mathbf{B}^{i-1}, \mathbf{y}_{1:T})$$

3. *Draw* $\mathbf{B}^i$ *from*

$$p(\mathbf{B} \mid \boldsymbol{\Sigma}^i, \mathbf{Q}^i, \mathbf{y}_{1:T}, \mathbf{S}_R(\cdot) > 0) \propto [\mathbf{S}_R(\cdot) > 0] p(\mathbf{B} \mid \boldsymbol{\Sigma}^i, \mathbf{y}_{1:T})$$

Applications

Small SVAR of the World Oil Market

- In the first application, we replicate Kilian and Murphy (2014). This paper adds oil inventories to the model Kilian and Murphy (2012) in order to identify speculative demand shocks. The tight restrictions used in this paper render the identified set small and the typical algorithm becomes infeasible.
- To get around this infeasibility, Kilian and Murphy (2014) consider an approach similar to the one in Chan Metthes, and Yu (2025) by exploiting permutations and sign alternation. As we will show below, our algorithm can handle this application in about half the time it takes when using Chan Metthes, and Yu (2025)'s accept-reject algorithm.

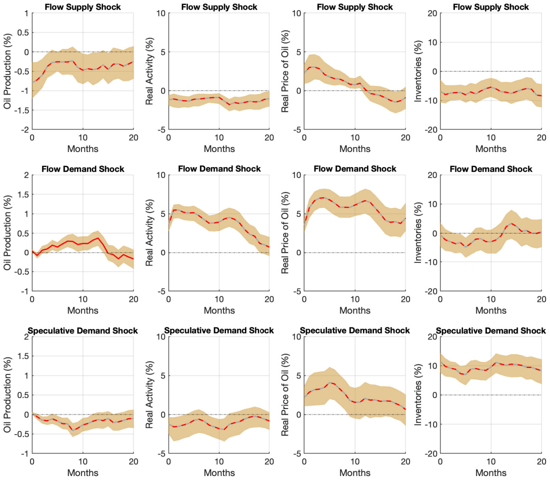
Small SVAR of the World Oil Market

Identifying Restrictions

	Sign Restrictions on Impact Impulse Responses		
Variable/Shock	Flow supply	Flow demand	Speculative demand
Oil production	-1	$+1$	$+1$
Real activity	-1	$+1$	-1
Real price of oil	$+1$	$+1$	$+1$
Inventories			$+1$
	Elasticity Bounds		
	Flow supply shock	Flow demand shock	Speculative demand shock
Price Elasticity of Oil Supply		0.025	0.025
	Sign Restrictions on Impulse Responses at Horizons 0 through 12		
	Flow supply shock	Flow demand shock	Speculative demand shock
Real activity	-1		
Real price of oil	$+1$		

Small SVAR of the World Oil Market

Impulse Responses



Computation Time: Gibbs vs Accept-Reject

Specification	Benchmark Model	Benchmark Model + Additional Restriction
Gibbs Sampler	0.03	
Accept-Reject	0.33	

Time (Hours) Per 1,000 Effective Draws

Small SVAR of the World Oil Market

Identifying Restrictions

	Sign Restrictions on Impact Impulse Responses		
Variable/Shock	Flow supply	Flow demand	Speculative demand
Oil production	-1	$+1$	$+1$
Real activity	-1	$+1$	-1
Real price of oil	$+1$	$+1$	$+1$
Inventories			$+1$
	Elasticity Bounds		
	Flow supply shock	Flow demand shock	Speculative demand shock
Price Elasticity of Oil Supply	$(-0.09, -0.07)$	0.025	0.025
	Sign Restrictions on Impulse Responses at Horizons 0 through 12		
	Flow supply shock	Flow demand shock	Speculative demand shock
Real activity	-1		
Real price of oil	$+1$		

Specification	Benchmark Model	Benchmark Model + Additional Restriction
Gibbs Sampler	0.03	0.10
Accept-Reject	0.33	7.92

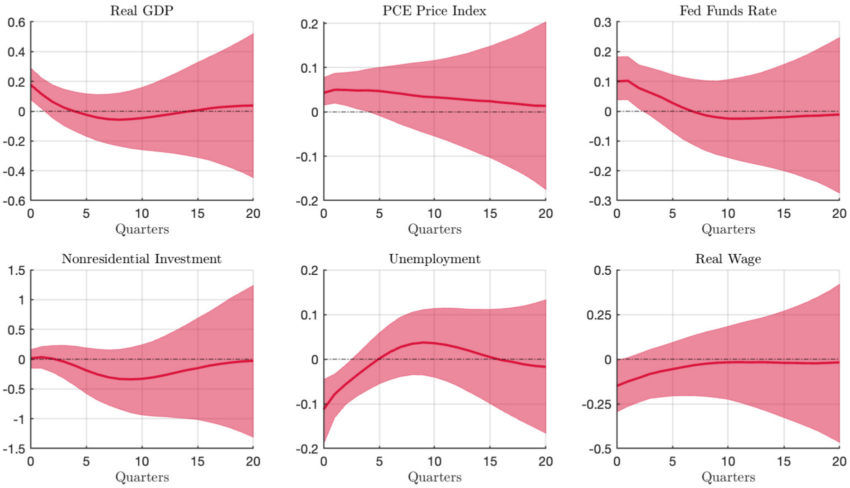
Time (Hours) Per 1,000 Effective Draws

A Large SVAR of the U.S. Economy

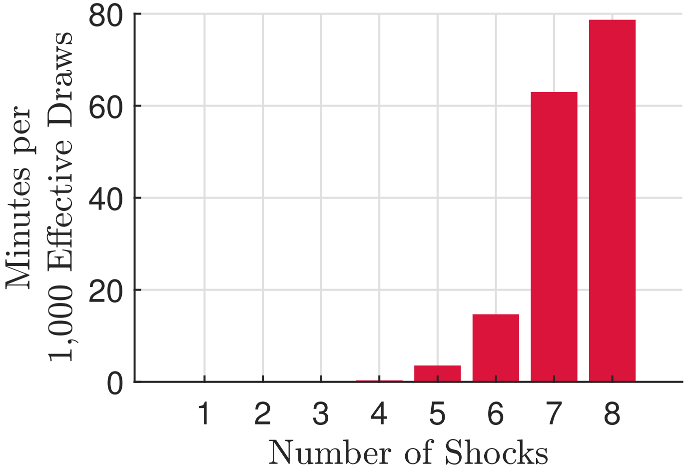
- We re-visit the structural analysis in Chan, Metthes, Yu (2025) who use Crump, Eusepi, Giannone, Qian, Sbordone (2025)'s large SVAR model of the U.S. economy to identify 8 structural shocks
- The model includes 35 variables typically monitored at the Federal Reserve System. The SVAR is specified at quarterly frequency (1973:Q2–2019:Q4)
- We assume a Minnesota prior for the reduced-form parameters and we set the hyper-parameters following Giannone, Lenza, Primiceri (2015). We follow the conventional approach and impose a Haar distribution over the set of orthogonal matrices
- For identification purposes, Chan, Metthes, Yu (2025) use sign restrictions on the contemporaneous impulse responses as well as by ranking restrictions. In total, there 105 sign restrictions are imposed

A Large SVAR of the U.S. Economy

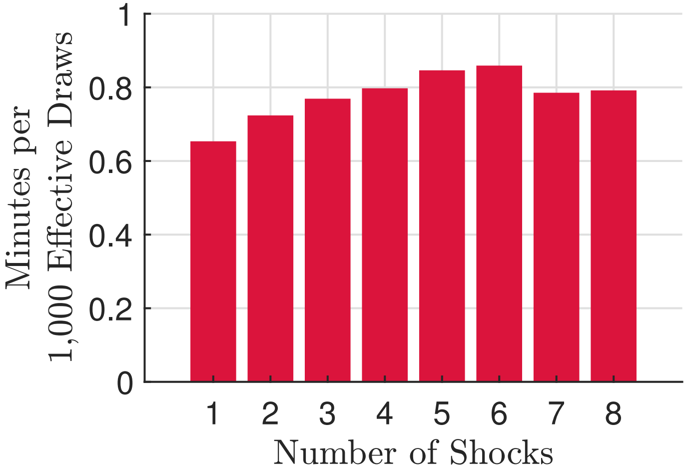
Impulse Responses to a Demand Shock



Accept-Reject

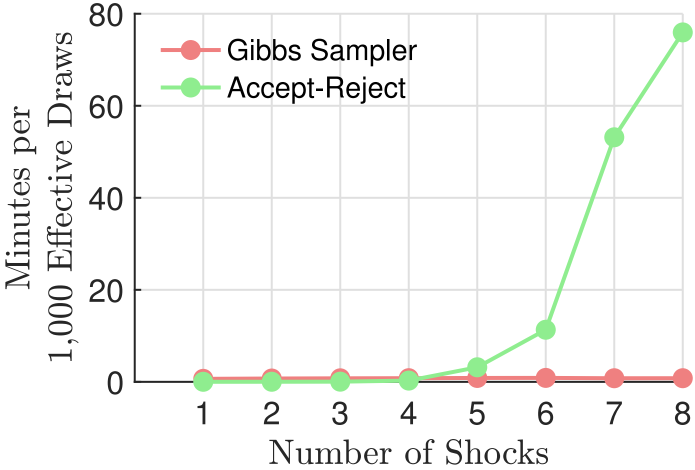


Gibbs

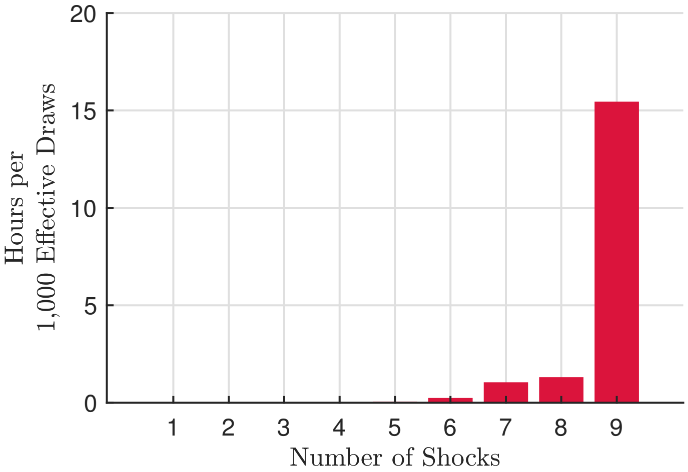


Minutes per
1,000 Effective Draws
0
0.2
0.4
0.6
0.8
1
1
2
3
4
5
6
7
8
Number of Shocks

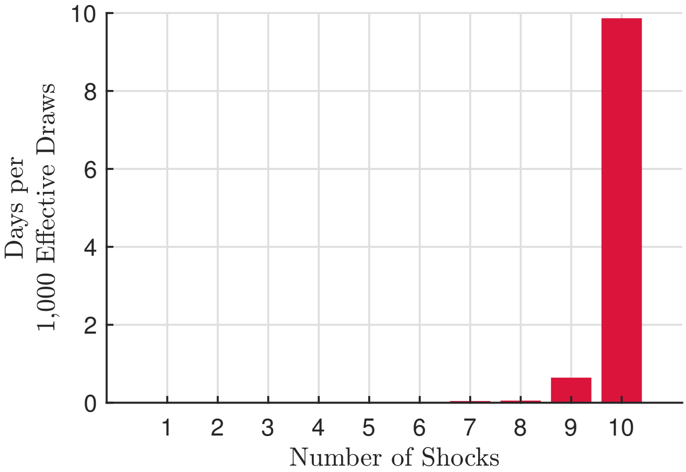
Accept-Reject vs Gibbs



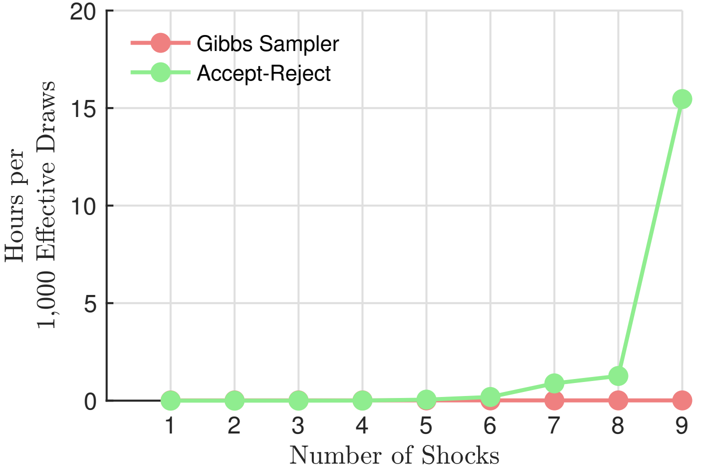
Accept-Reject



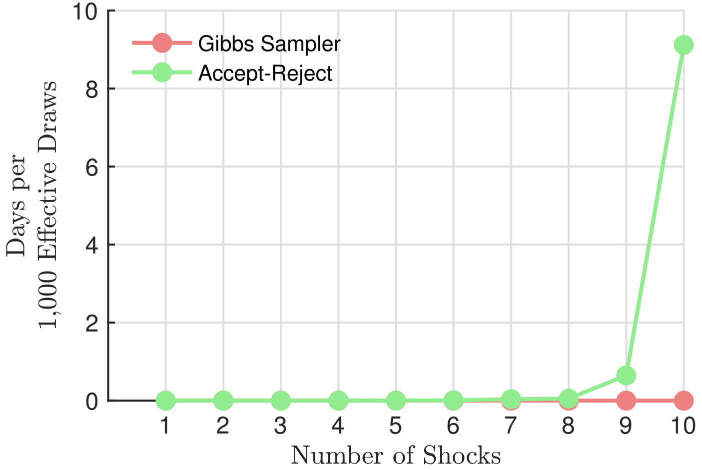
Accept-Reject



Accept-Reject vs Gibbs



Accept-Reject vs Gibbs



Conclusion

Conclusion

- We develop a new algorithm for inference based on sign-identified SVARs
  - The key insight is to break apart from the accept-reject tradition associated with sig-identified SVAR
  - We show that embedding an elliptical slice sampling within a Gibbs sampler approach can deliver dramatic gains in speed and turn previously infeasible applications into feasible ones
- We provide a tractable example to illustrate the power of the elliptical slice sampling applied to sign-identified SVARs
- We demonstrate the usefulness of our algorithm by applying it to a well-known small-SVAR model of the oil market featuring a tight identified set as well as to large SVAR model with more than 100 sign restrictions