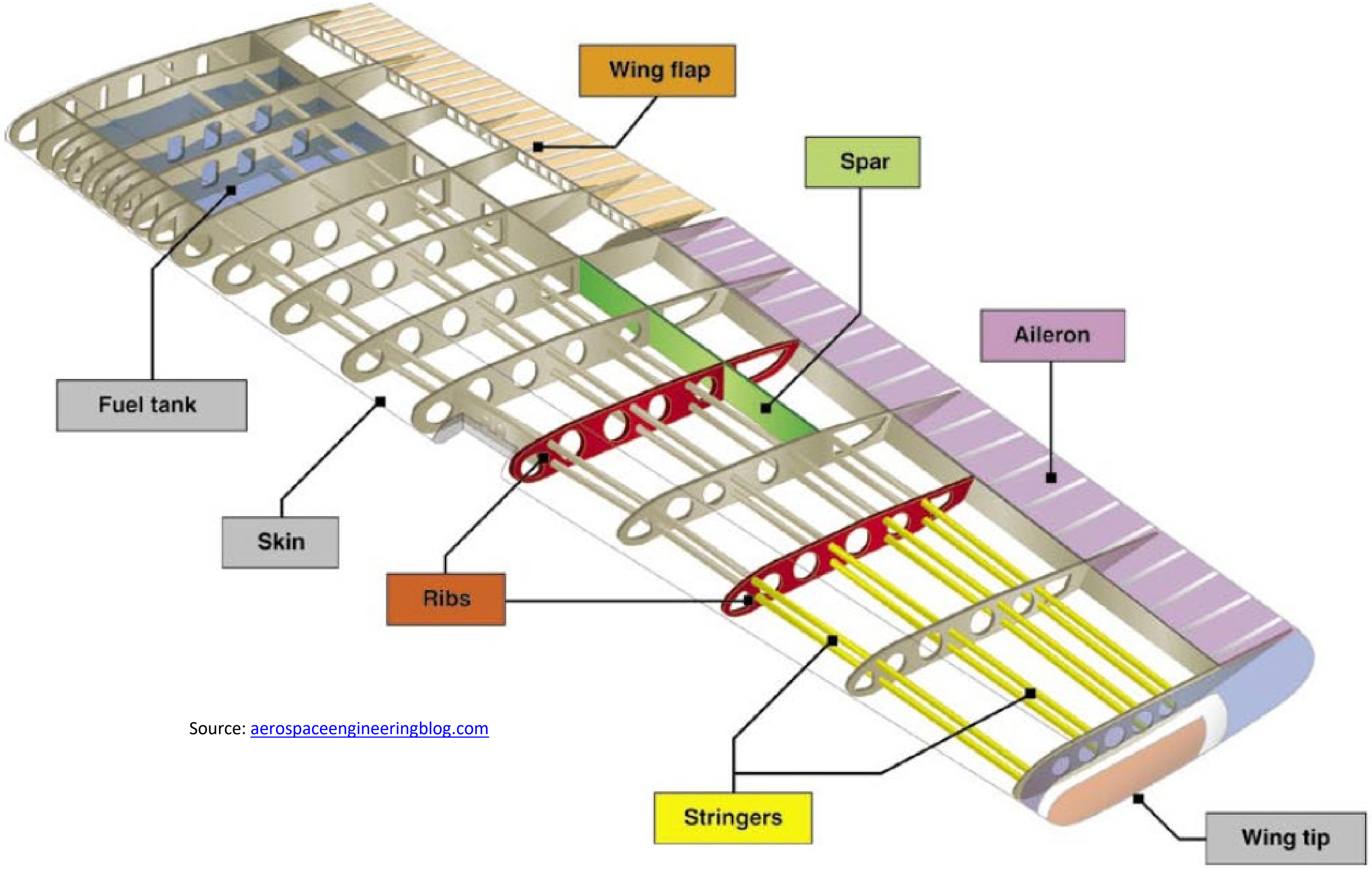
Week 3 Lecture:

Stress Concentration – Photoelasticity

Last update: January 31, 2022

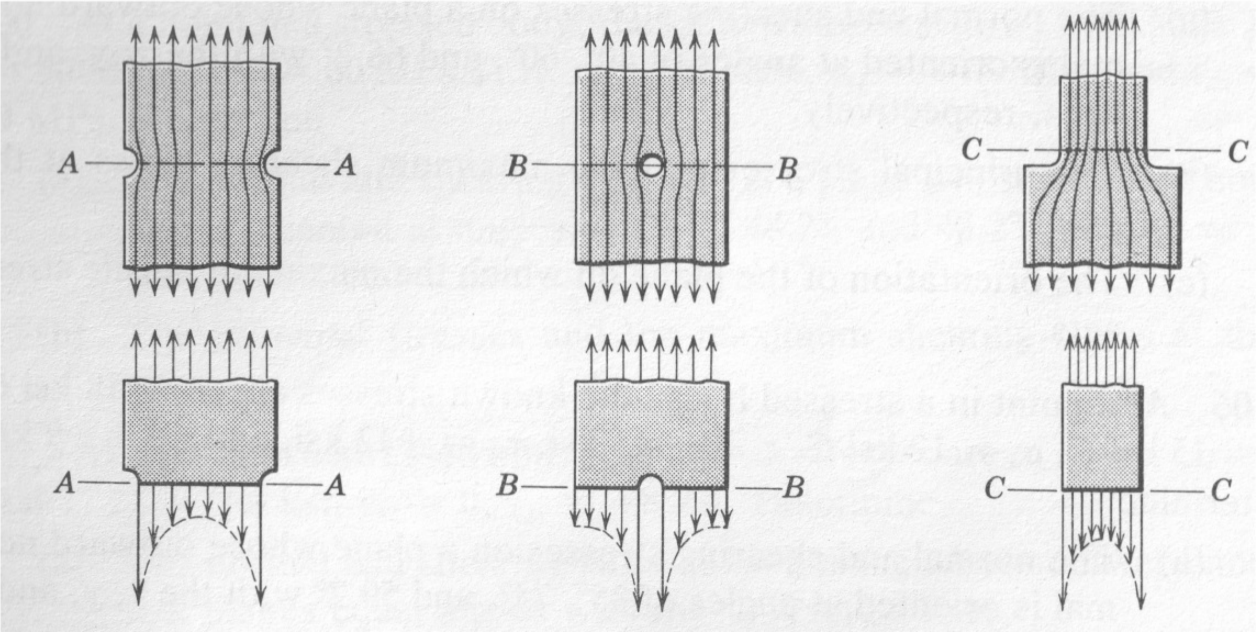


Source: aerospaceengineeringblog.com

Why so many holes/cutoffs?

Stress Concentration

- structural discontinuities such as holes or changes in width, diameter, etc. cause localized increases in stress, called "stress concentrations" to occur



Source: A. Higdon et al. *Mechanics of Materials* 4th ed, 1985.

- stress concentrations is not of big concern for static loading applied to ductile materials (such as aluminum), thanks to stress redistribution coming from inelastic yield
- *However, catastrophic fracture can occur if subjected to impact or repeated loading*

Stress Concentration Around a Circular Hole

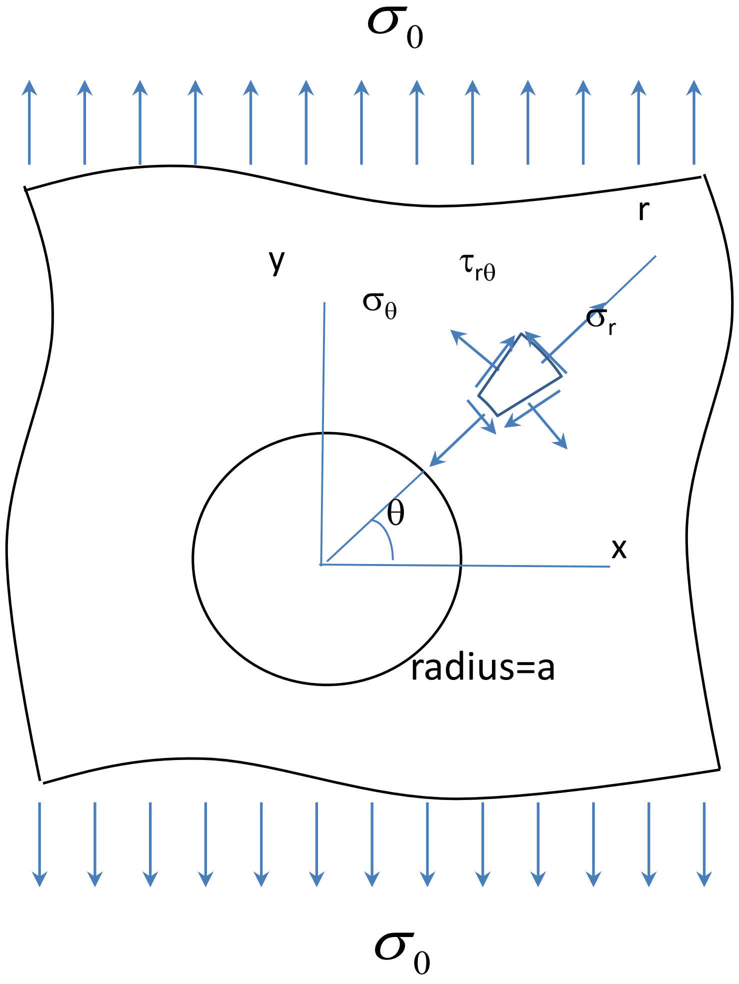
in a thin, infinite plate under uniaxial tensile loads

By using superposition method with stress function in linear elasticity theory, we can derive the following solutions for stresses:

$$\sigma_r = \frac{\sigma_0}{2}\left[\left(1-\frac{a^2}{r^2}\right)-\left(1-\frac{4a^2}{r^2}+\frac{3a^4}{r^4}\right)\cos 2\theta\right]$$

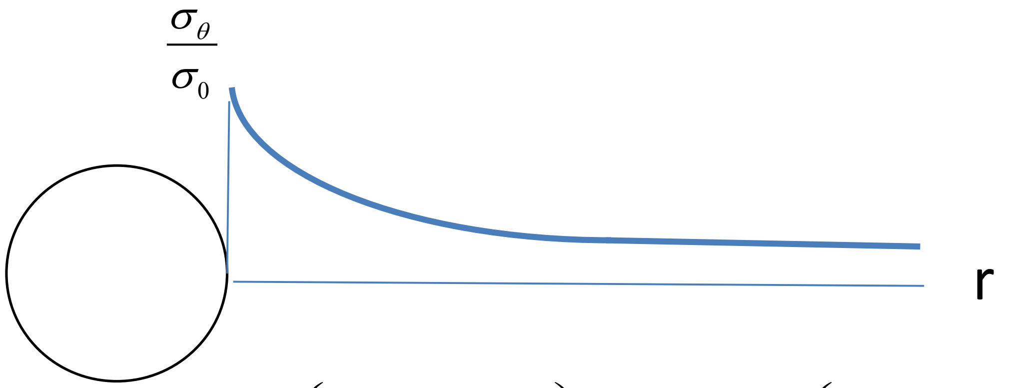
$$\sigma_\theta = \frac{\sigma_0}{2}\left[\left(1+\frac{a^2}{r^2}\right)+\left(1+\frac{3a^4}{r^4}\right)\cos 2\theta\right]$$

$$\tau_{r\theta} = \frac{\sigma_0}{2}\left(1+\frac{2a^2}{r^2}-\frac{3a^4}{r^4}\right)\sin 2\theta$$



Stress Concentration Around a Circular Hole

in a thin, infinite plate under uniaxial tensile loads



$$\theta = 0: \quad \sigma_r = \frac{\sigma_0}{2}\left(\frac{3a^2}{r^2} - \frac{3a^4}{r^4}\right), \ \sigma_\theta = \frac{\sigma_0}{2}\left(2 + \frac{a^2}{r^2} + \frac{3a^4}{r^4}\right), \ \tau_{r\theta} = 0$$

$$r = a: \quad \sigma_r = 0, \quad \sigma_\theta = \sigma_0\left(1 + 2\cos 2\theta\right), \quad \tau_{r\theta} = 0$$

Stress concentration factor= $\dfrac{\text{max. stress}}{\text{nominal stress, } \sigma_0}$ =?

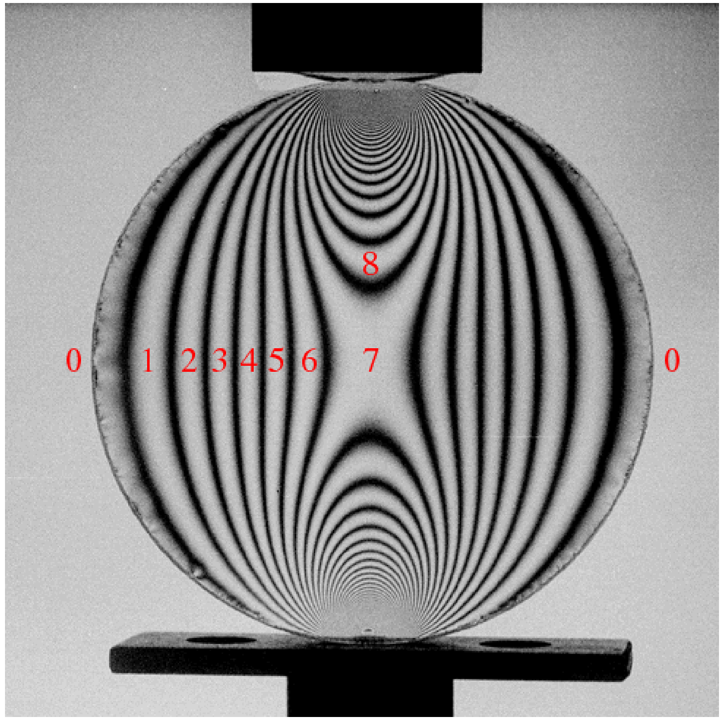
Can you find a “singular point” where all stresses=0?

Photoelasticity

- A “full field” technique by viewing transparent birefringent model of structural member under polarized light. The resulting optical patterns (or fringes) are then analyzed to determine the stress distribution in the model
- The difference between principal stresses is proportional to the fringe numbers as given by the stress-optic law

$$\sigma_{p1} - \sigma_{p2} = 2\tau_{max} = \frac{c}{t} N$$

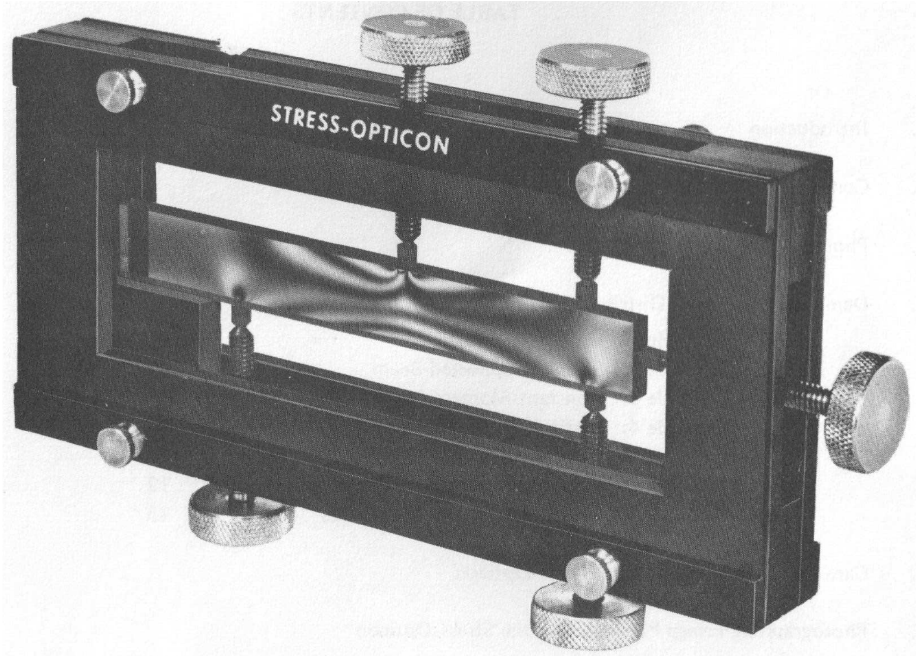
in which c is the stress optic coefficient, t is specimen thickness and N is fringe order number



Source: Univ. Illinois at Urbana-Champaign

Photoelasticity (cont'd)

- In the stress concentration experiment, a light-field circular polariscope is used to examine specimen under lights
- Another way of measurement is to count the number of fringes that cross a specific point of interest on the specimen as the load is applied



Lab 2 – Stress Concentration

Examine stress distributions of holed, notched and arched samples