Lab 2: Stress Concentration

Objective

To observe and evaluate how stresses become concentrated around discontinuities such as a circular hole.

Introduction

<u>Stress concentration</u>

As you learned in Week 3 lecture, structural discontinuities such as holes or changes in width, diameter, etc. cause localized increases in stress known as "stress concentrations". A discontinuity can interrupt the stress path, if it exists in a structural or machine element. The stress at the discontinuity can then be significantly higher than the nominal stress in the section, and causes a stress concentration at the discontinuity. The ratio of the maximum stress due to stress concentration to the nominal stress is known as "stress concentration factor". Several stress concentration cases are shown in week 3 lecture notes and are repeated in Fig. 1 below.

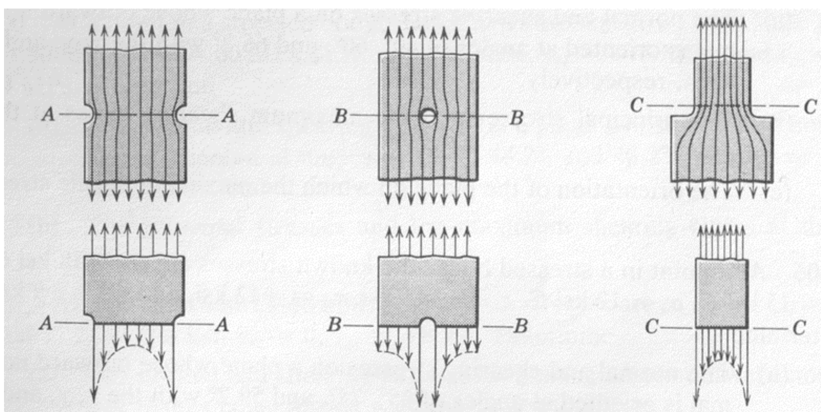

Figure 1 Stress concentration cases [3]

In this lab, we will investigate the stress concentration due to a central hole in a compressed specimen. For the special case where the width of the plate is considered very large with respect to the hole size, the stress concentration factor *K* can be determined theoretically. For one such "infinite" plate having a hole of radius "*a*" under extension, *t*he stresses at position (r, θ) are given by (Fig. 2; see also Week 3 lecture notes):

$$\sigma_r = \frac{\sigma_0}{2}\left[\left(1-\frac{a^2}{r^2}\right)-\left(1-\frac{4a^2}{r^2}+\frac{3a^4}{r^4}\right)\cos 2\theta\right] \tag{1}$$

$$\sigma_\theta = \frac{\sigma_0}{2}\left[\left(1+\frac{a^2}{r^2}\right)+\left(1+\frac{3a^4}{r^4}\right)\cos 2\theta\right] \tag{2}$$

$$\tau_{r\theta} = \frac{\sigma_0}{2}\left(1+\frac{2a^2}{r^2}-\frac{3a^4}{r^4}\right)\sin 2\theta \tag{3}$$

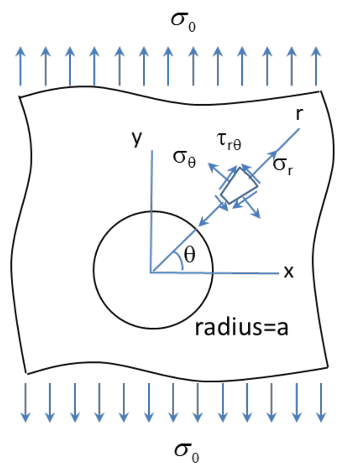

Figure 2 Stress setup around a circular hole in an infinite plate under tension

At the boundary of the hole (where r=a), these equations simplify to

$$\sigma_r = 0, \quad \sigma_\theta = \sigma_0\left(1 + 2\cos 2\theta\right), \quad \tau_{r\theta} = 0 \tag{4}$$

As stated above, stress concentration factor *K* by definition is

$$K = \frac{\sigma_{max} \text{ or } \tau_{max}}{\sigma_0} \tag{5}$$

When the plate width is not many times larger than the hole, the stresses in the vicinity of a single hole are somewhat larger than the stresses predicted by the infinite plate theory. You will need to find data about *K* versus hole diameter *d* and specimen width *w* from a suitable textbook or other definitive reference source.

The effects of stress concentration are less significant for static loading of ductile materials because yielding will occur in the region of high stress and therefore it redistributes the peak stresses. However, repeated loading or impact loading may result in fracture. Static loads may cause immediate fracture in the case where a stress concentration exists in a brittle material.

Photoelasticity

In this lab, we will also explore the field of photoelasticity and apply the photoelastic effect to visualize stress patterns. The photoelastic effect comes from *birefringence* (difference in the speed of light for different optical polarizations) induced by stresses. It applies to transparent materials that exhibit stress-induced birefringence. Under light, when the sample is not loaded, it is uniformly lit. When the sample is loaded, a pattern of fringes appears and changes with the loading, that represents the various stress states in the sample.

Specifically, under plane stress conditions, each fringe (constant color) is a line that represents a constant value of the difference in principal stresses σ_{p1}-σ_{p2}. If you count the number of fringes that cross a point as you apply a load, that number of fringes is proportional to the stress σ_{p1}-σ_{p2}. At the boundary of the hole, σ_r=0, so it is one of the principal stresses, and therefore the number of fringes is proportional to σ_θ. In the bulk of the bar, away from the hole, there is no lateral stress, so σ_x=0 is one of the principal stresses, and the number of fringes is proportional to σ_y.

References

[1] Week 3 lecture notes
[2] D. Peery, *Aircraft Structures*, 1950, pp302-303
[3] A. Higdon et al., *Mechanics of Materials* 4e, 1985, pp62-64
[4] J. W. Dally and W. F. Riley, *Experimental Stress Analysis* 3e 1991

Work to be done

1. Prelab

Study the demo video, Week 3 lecture notes and lab manual. Then work and submit the prelab to Canvas no later than the night before your lab day.

2. In lab

As always, safety comes first. You MUST wear safety goggles in this lab.

We will use a light-field circular polariscope for testing stress concentration. The polariscope consists of a frame that is designed to hold several knobs for applying compressive load to the sample and top and bottom face sheets that are polarizers bonded to quarter-wave plates (Fig. 3). Be careful with the polarizers as they are easily scratched. As shown in Fig. 3, the polariscope is raised above the bench by using supporting blocks on both ends. This would allow the scope to be illuminated with sufficient light from below, so that the fringe patterns can be visualized clearly.

In this lab, we will test three samples, namely holed, notched and arched samples, as pictured in Fig. 4. The first holed sample is a long bar with a circular hole in the middle, which matches with the canonical problem described above (Fig. 2). You should begin the test by removing the top face sheet and inserting the test sample into the scope. Then align the long size of the bar with that of scope and level the bar to the plane of knobs. Next, move all four lateral knobs around to hold the sample evenly in place. The trick is to have the knobs touch the sample lightly with as little load as possible; that is, you don't see any fringe appears under light. If the sample is set up satisfactorily in the scope, put the face sheet back on. Then start applying compressive load by turning the end knob slowly, and the fringe patterns will appear at the point of loading and around the hole. Do not tighten the end knob further after the fringe patterns have changed 2-3 times; or you may damage the scope. Remember to take some good photos of the fringe patterns. Do this at various stages as the axial force increases.

Continue the experiment by repeating the same process to the notched and arched samples. Note that two loading cases will be applied to the arched sample: axial (along the long side of the scope) and lateral. The setup of arched sample and its fringe pattern under lateral load are illustrated in Fig. 5.

Figure 3 Polariscope for stress concentration

3. After Lab

Below are questions related to the data and observations you have from the experiment. Answer them in the Analysis section of the report summary.

1. (35%) Normalize the three stresses in Equations 1-3 by dividing both sides of equations by σ_o *and* compute the three normalized stresses along θ=0^o from $r=a$ to $r=5a$. Plot the results in three separate charts. Which stress reaches maximum and where does it occur? What value is the stress concentration factor then (recall the definition of Equation 5)? Comment how rapid the decay of the stress concentration away from the hole. This is typical of the redistribution of stress in the neighborhood of a discontinuity.

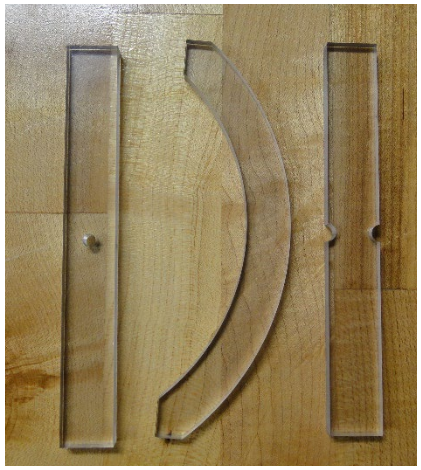

Figure 4 Three test samples (left to right): holed, arched and notched samples

Figure 5 Arched sample setup and fringe pattern at lateral load

2. (10%) From Equation 4, can you find a "singular point" where all stresses vanish? Hint: simple trigonometry.

3. (55%) Using Equations 1-3, calculate and plot the fringe patterns and see how well they match with what you observed in the lab. Be sure to indicate which direction the loading is in. These patterns are only approximate, as the equations do not take into account the finite width of the specimen. You may use any computing tool of your choice; the

suggestions below assume the use of MATLAB.

Hints/tips:

i. Define the edges by writing down *x* and *y* values. Make *x* a row vector and *y* a column vector:
```
x=-10:.1:10;
y=(-12:.1:12)';
```
ii. Define x positions on the grid by projecting down the row vector *x* and projecting across the column vector *y*:
```
xgrid=ones(size(y,1),1)*x;
ygrid=y*ones(1,size(x,2));
```
iii. Now you can calculate *r* and θ at each point using vectorized operations:
```
r=sqrt(xgrid.^2+ygrid.^2);
theta=atan2(ygrid,xgrid);
```
(Note the use of '.^' to indicate element-by-element power. You will similarly need to use '.*' and './' for element-by-element multiplication and division.)

iv. We can assign arbitrary values for hole radius *a* and nominal stress σ_o as all we are looking for is the pattern:
```
a=2;
Sigma_0=1;
```
v. Calculate `sigma_r, sigma_t,` and `tau_rt` from the equations.

vi. Mask out the hole itself by setting the stresses to zero (or NaN).
```
sigma_r(find(r < a))=0;
sigma_t(find(r < a))=0;
tau_rt(find(r < a))=0;
```
vii. Now you can use, for example, `imagesc(sigma_t)` to view σ_θ. Does what you see make sense? Which direction is the stress in?

viii. Finally you can evaluate difference of principal stresses (the diameter of Mohr's circle, or twice the maximum in-plane shearing stress)

$$\sigma_{p1}-\sigma_{p2}=2\sqrt{\left(\frac{\sigma_r-\sigma_\theta}{2}\right)^2+\tau_{r\theta}^2} \tag{6}$$

which will be proportional to the fringe count. Visualize this with `imagesc` as before. You can make the plot look closer to what you will see in lab by using a cyclic colormap:
```
colormap([jet;jet;jet])
```
(You can reset to the single-cycle colormap with `colormap(jet);`.)

Does your fringe pattern image match with one of your photos taken from the experiment?

4. Report

Work with your group members to write a report summary at least 5 pages (including all figures and code listing). Follow the report template "AerE 322 lab report template.pdf", summary instruction "AerE 322 lab summary instruction.pdf" and the check list "prelabs_labreports.pdf" that are provided to you via e-mail or online in class Canvas site. In addition to answers/analyses to all the After Lab work items, you should include descriptions of the experiment specimens, test equipment, and experiment procedures. Also include all necessary pictures, graphs, tables, etc. to better present your work. Make sure your hypotheses are clearly made and if your results support them and why.