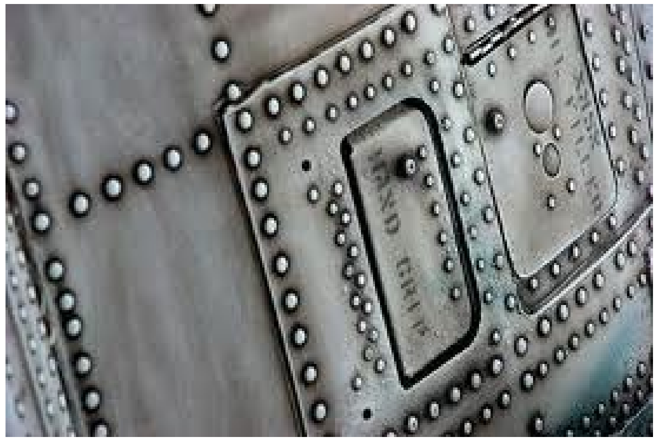
Week 4 Lecture:

Riveted Joint:
Design, Fabrication & Testing

Last update: February 5, 2023

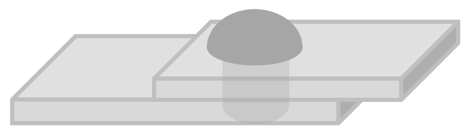
Riveted Joint Types

- One of most common connectors in aircraft structures

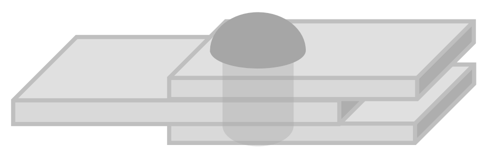


Source:
http://toolmonger.com/2009/10/01/reader-tip-strengthening-rivets/

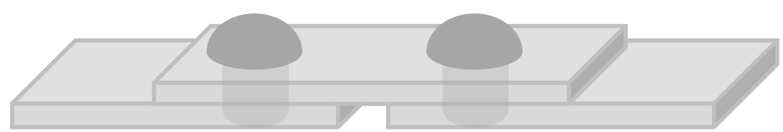
Single lap joint



Double lap joint



Butt joint



Failure Modes

Types of Failures

- shear failure of rivets
- bearing failure of joints
- tension failure of joints
- tearout failure of joints

Major aviation incident due to riveted joint failure: Aloha Airlines flight 243 (1988)

Source:http://mechanicsupport.blogspot.com/2011/10/aircraft-rivet-hole-fatigue-strength.html

Example of cracking at rivet joint

Source: http://deicinginnovations.com/?p=3015

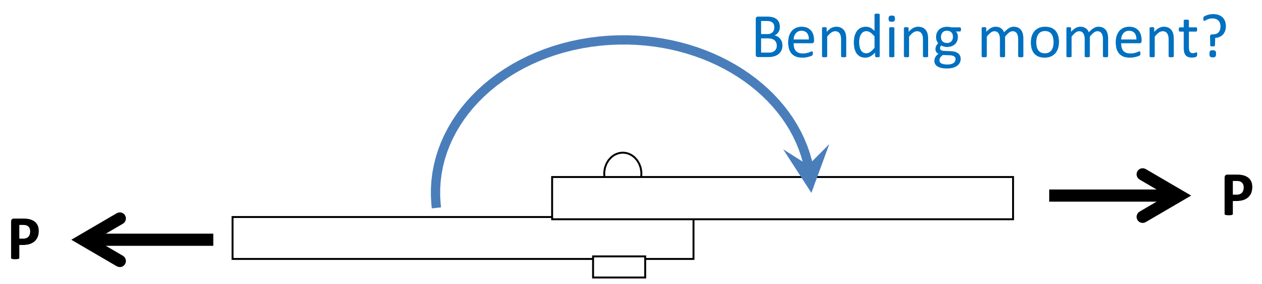
Fatigue and corrosion caused multi-site damage at a **lap joint** -> 18-foot long fuselage skin blew away !

Load Analyses of Riveted Joints

- The actual load/stress distributions and deformation of riveted joints are complicated and remain active research topics of, e.g. finite element modeling (http://www.ijmerr.com/uploadfile/2015/0409/20150409045712593.pdf , https://smartech.gatech.edu/bitstream/handle/1853/10494/atre_amarendra_p_200605_phd.pdf)

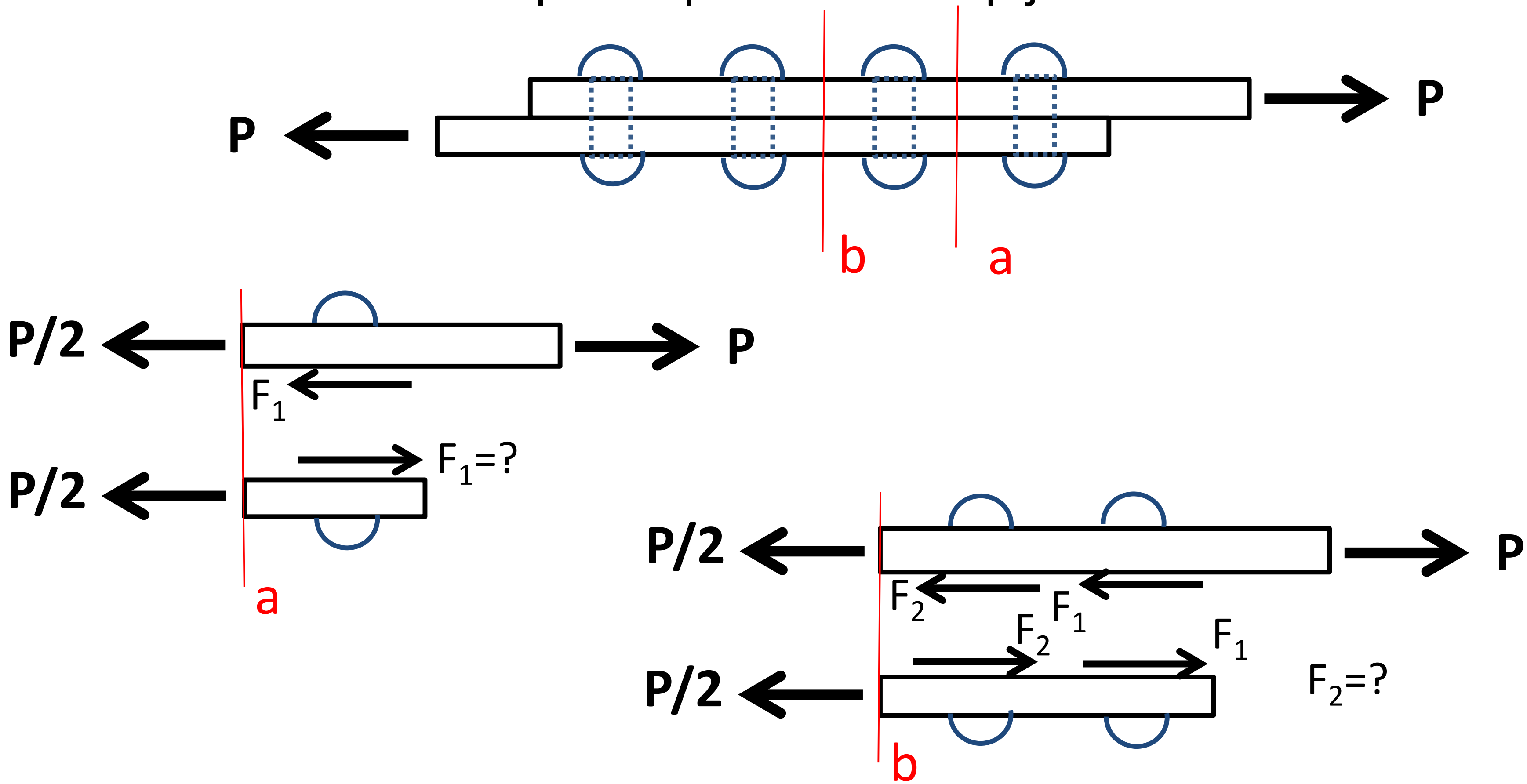
- To simplify the matters, general assumptions are ***needed***, e.g. stress is uniformly distributed over load area, all rivets take equal loads, rivets and holes are perfectly fitted, i.e. no slip, etc.

But



Load Analyses of Riveted Joints (cont'd)

Consider this quadruple-riveted lap joint



What happens to the "equal load" assumption?

Load Analyses of Riveted Joints (cont'd)

- The load/stress distributions in the riveted joints are considered in *average* sense
- When loads are approaching the inelastic yield limit, the load/stress distributions are "closer" to those assumptions
- Experimental data aid in to provide ad hoc rules

Rivet Joint – Design Criteria

- Several design criteria are in use (see Peery):
  fitting factor 1.15 for military and 1.2 for civil airplanes
  bearing factor 2 for landing gear in dynamic loading

- $$\text{Margin of safety} = \frac{\text{allowable stress}}{\text{stress from load}} - 1$$

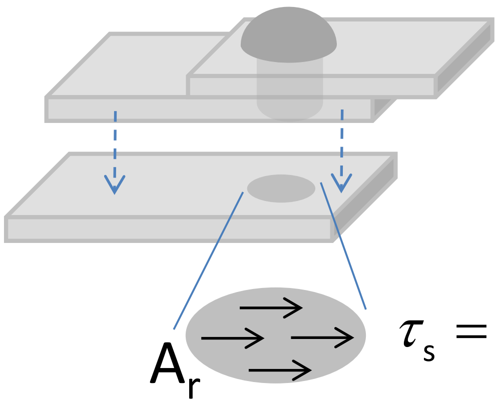
  Stress from load may include the fitting factors and safety factor 1.5

- For education purpose in this course, we use joint efficiency to predict the failure mode and location

$$\text{Joint efficiency} = \eta = \frac{\text{Ultimate load on a joint}}{\text{Ultimate load on the sheet}}$$

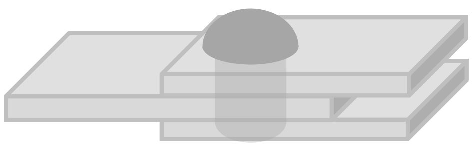
Failure Mode – Shear Failure of Rivets

single shear



$$\tau_s = \frac{P}{A_r}$$

double shear



$$\tau_s = \frac{P}{2A_r}$$

The cross-shearing stress, τ_s, in the rivets is uniformly distributed over all shear areas A_r. All rivets take equal loads.

Failure Mode – Shear Failure of Rivets (cont'd)

$$\text{Joint Efficiency} = \eta = \frac{A_r \tau_{su}}{A_{sheet} \sigma_{tu}} \times N$$

where

A_r = cross-sectional area of one rivet = $\pi r^2 = \pi d^2/4$

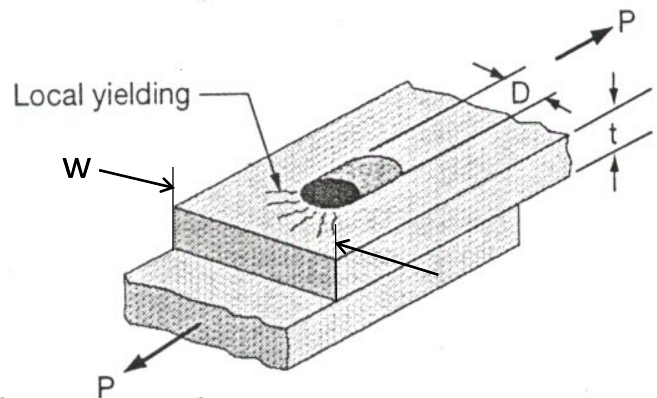
τ_{su}=ultimate shearing stress of rivets

N=number of rivets in the joint

A_{sheet} = cross-sectional area of each sheet = width(w) x thickness(t)

σ_{tu}=ultimate stress of plates/sheets in tension

Failure Mode – Bearing Failure of Joints



Assumption: Ultimate bearing stress, σ_{bu}, between the rivets and the plates is assumed to be uniformly distributed over the projection of the contact area. All rivets take equal loads.

$$\sigma_{bu} = \frac{P}{Dt}$$

D=diameter of rivet/hole, t=thickness of plate

Failure Mode – Bearing Failure of Joints (cont'd)

$$\text{joint efficiency} = \frac{\text{projected contact area} \times \sigma_{bu} \times N}{A_{sheet}\sigma_{tu}}$$

where

projected contact area =diameter of rivet/hole (D) x thickness (t)

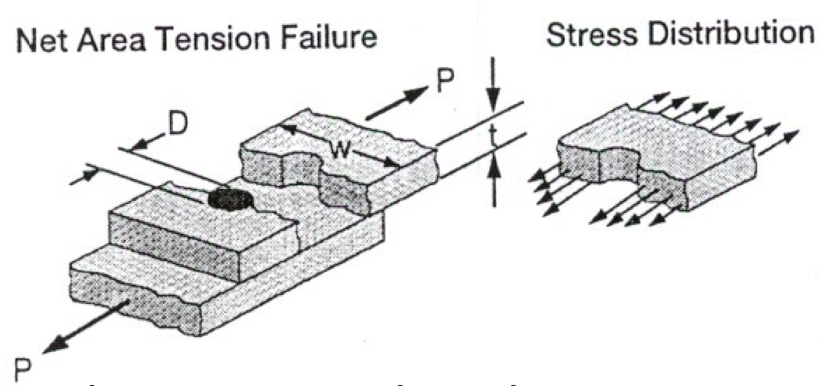
σ_{bu}=ultimate bearing stress between rivets and plates/sheets

N=number of rivets in the joint

A_{sheet} = cross-sectional area of each sheet = width(w) x thickness(t)

σ_{tu}=ultimate stress of plates/sheets in tension

Failure Mode – Tension Failure of Joints



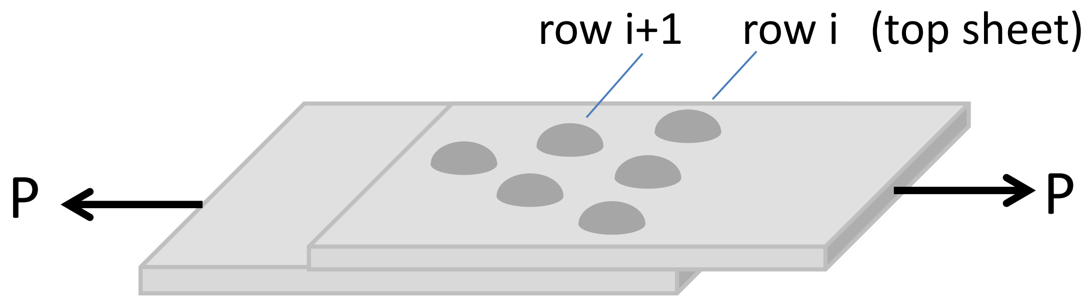
Assumption: tensile stress in the plate is assumed to be uniformly distributed over the net area of the plate at each row of rivets

$$\sigma_t = \frac{P}{A_{net}} = \frac{P}{(w-n_i d)t}$$

where n_i is number of rivets in row *i*

The allowable stress may be reduced by 10% to account for peak stresses at the hole edge

Failure Mode – Tension Failure of Joints (cont'd)



Given a sheet/plate, joint efficiency for each row:

$$\eta_t = \frac{\text{load on net area at row}}{\text{load fraction at row}} = \frac{A_{net}\sigma_{tu}}{\frac{N-n}{N} A_{sheet}\sigma_{tu}} = \frac{A_{net}}{\frac{N-n}{N} A_{sheet}}$$

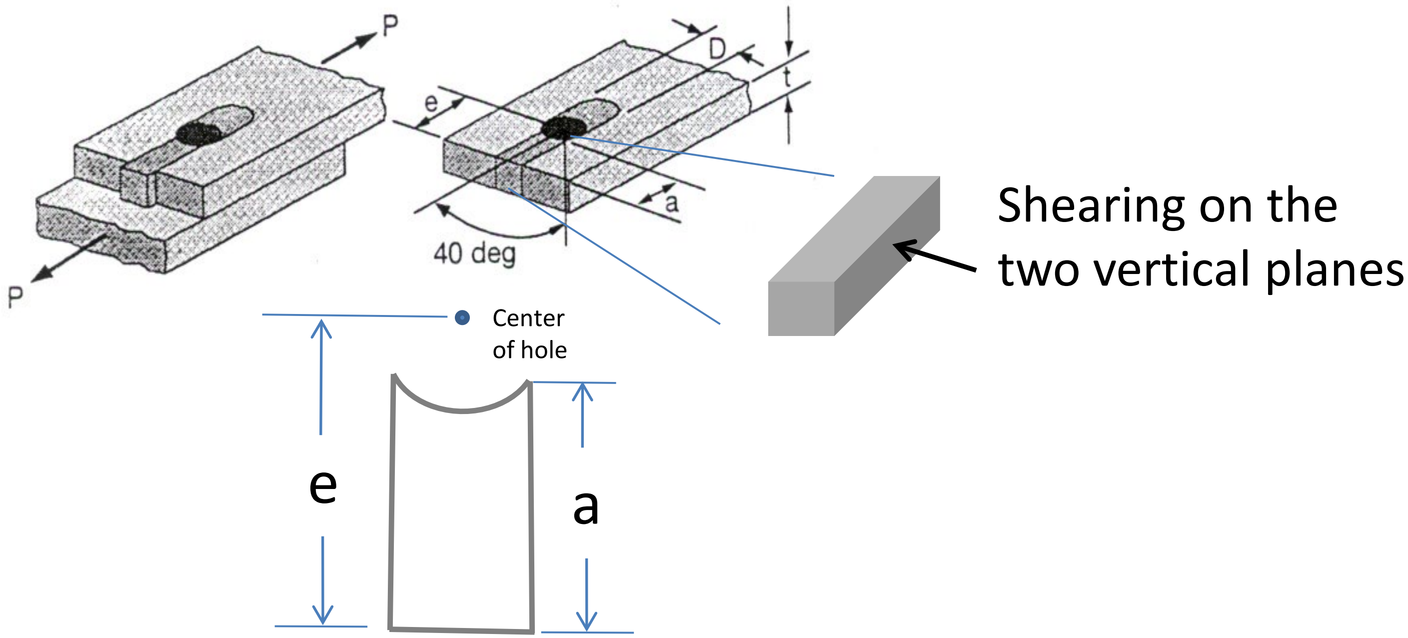
where

n = total number of rivets in all *previous* rows*,

N = total number of rivets in all rows

* "previous" as seen from the loaded end of the sheet. Each successive row has less stress on it because some of the force is held by the previous rows

Failure Mode – Tearout Failure of Joints



$$\tau_s = \frac{P}{N_e(2at)}, \; a = e - \frac{d}{2}\cos(40^0), \; \tau_s^{allow} = 0.85\tau_{sup}$$

Failure Mode – Tearout Failure of Joints (cont'd)

If for simplicity, let a=e and $\tau_s^{allow} = \tau_{sup}$,

$$\text{joint efficiency} = \eta = \frac{2\,t\,e\,\tau_{sup}}{A_{sheet}\sigma_{tu}} \times N_e$$

where

N_e=Number of rivets in the row closest to the edge of either plate

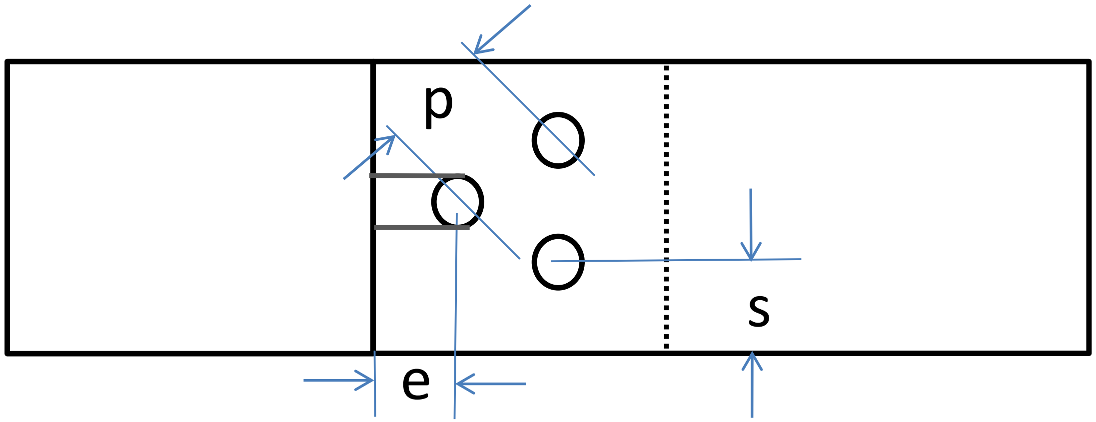
t = thickness of the plate

e=distance of rivet center from the edge

τ_{sup}=ultimate shear stress of the plate

σ_{tu}=ultimate stress of plates/sheets in tension

Design Rules of Thumb



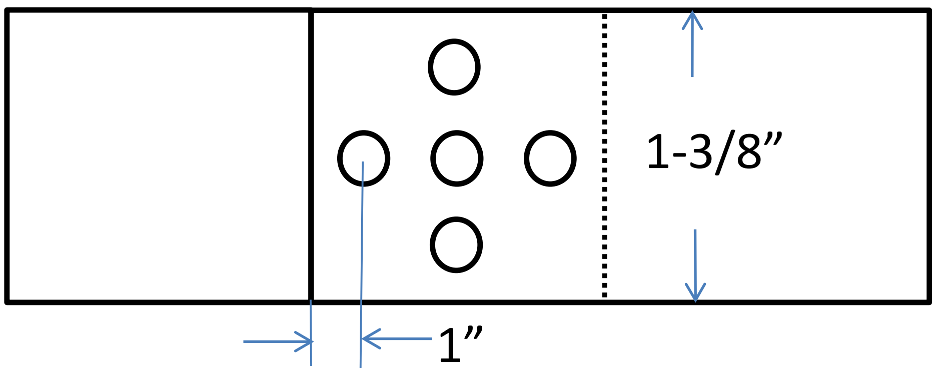
d: rivet/hole diameter

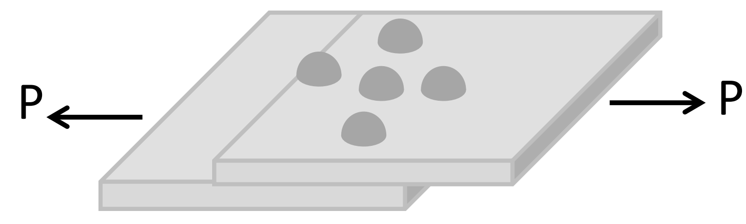
- Distance to the edge: e >=2d
- Inter-rivet spacing: p >=4d
- Distance to the side: s >=2d
- Rivet length: 1.5d

A Workout Example: Part 1

A lap joint made of two Al 2024-T3 sheets and 5 Al 2117-T3 rivets as shown. Find: all the joint efficiencies

Rivet diameter: 5/32”, sheet thickness: 0.025”





Material strength

Al 2024-T3 sheet: ultimate tensile=70ksi, ultimate bearing=124ksi, shearing=41ksi

Al 2117-T3 rivet: ultimate shearing=30ksi

Source:
A. Skorupa and M. Skorupa, *Riveted Lap Joints in Aircraft Fuselage: Design, Analysis and Properties*, Springer, 2012.

A Workout Example: Part 1 (cont'd)

(i) Shear failure

$$\eta_s = \frac{A_r \tau_{su}}{A_{sheet}\sigma_{tu}} = \frac{(\pi/4)(5/32)^2(30000)}{1\,{}^3\!/_8 * .025 * 70000} \times 5 = 1.20$$

(ii) Bearing Failure

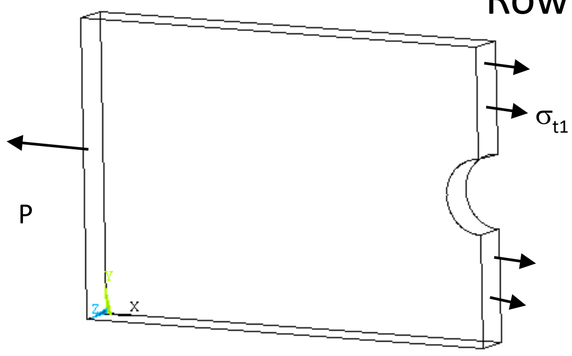
$$\eta_b = \frac{\text{Contact Area} * \sigma_{bu}}{A_{sheet}\sigma_{tu}} \times N = \frac{(5/32)*0.025*124000}{1\,{}^3\!/_8 * 0.025 * 70000} \times 5 = 1.01$$

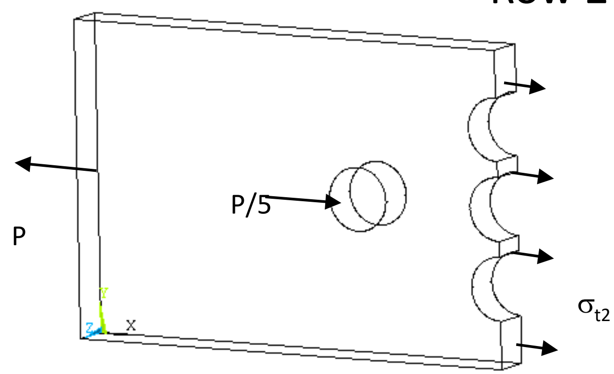
(iii) Tearout Failure

$$\eta_{to} = \frac{2\ t\ e\ \tau_{sup}}{\sigma_{tu} A_{sheet}} \times N_e = \frac{2*0.025*1*41000}{70000 * 1\,{}^3\!/_8 * .0025} x1 = 0.85$$

A Workout Example: Part 1 (cont'd)

(iv) Tension Failure of bottom sheet





Row 1

$$\eta_{t1} = \frac{A_{net1}}{A_{sheet}} = \frac{(1\,{}^{3}\!/_{8} - {}^{5}\!/_{32})*.025}{\frac{5\text{-}0}{5}\,1\,{}^{3}\!/_{8}*.025} = 0.89$$

Row 2

$$\eta_{t2} = \frac{A_{net2}}{(5\text{-}1)/5\;A_{sheet}} = \frac{(1\,{}^{3}\!/_{8} - 3\,{}^{5}\!/_{32})*.025}{{}^{4}\!/_{5}*1\,{}^{3}\!/_{8}*.025} = 0.82$$

A Workout Example: Part 1 (cont'd)

Row 3

$$\eta_{t3} = \frac{A_{net3}}{(5\text{-}4)/5\ A_{sheet}} = \frac{(1\,3/8 - 5/32)*.025}{1/5 * 1\,3/8 * .025} = 4.43$$

Thus, the joint is likely to fail at Row 2 in tension because it has the least efficiency

Warning! You will need to repeat the calculations for the other sheets if the sheets have different dimensions and/or have unsymmetric layout such as 1-3-2

A Workout Example: Part 2

Now, let P = 1640 lbs. Determine:

a. Average shearing stress in the rivets

b. The average bearing stress between the rivet and plate

c. The maximum average tensile stress and state where it occurs

Assume the shear force on each of the rivets (in single shear) is equal to F_s, then $5F_s = 1640$, therefore $F_s = 328$ lbs and

a.

$$\tau_s = F_s/A = 328/[\pi\,(5/32)^2/4] = 17{,}106\text{psi},$$

$$\text{vs. } \tau_{su} = 30{,}000\text{psi}$$

A Workout Example: Part 2 (cont'd)

b.

$$\sigma_{br} = \frac{F_{br}}{\text{contact area}} = \frac{328}{(5/32)*0.025} = 83{,}968\text{psi}$$

with $\sigma_{bu} = 124{,}000\text{psi}$

c.

At Row 1

$\sigma_{t1} * A_{t1} = 1640$

$\sigma_{t1}(1\,3/8 - 5/32) * 0.025 = 1640$ σ_{t1} = 53,826 psi in tension

At Row 2

$\sigma_{t2} * A_{t2}$ = P-P/5 = 1312 lbs

$\sigma_{t2}(1\,3/8 - 3\text{x}\,5/32) * 0.025 = 1312$ σ_{t2} = 57,909 psi in tension

A Workout Example: Part 2 (cont'd)

At Row 3

$\sigma_{t3} * A_{t3}$ = P-4P/5 =328 lbs

$$\sigma_{t3}\left(1\tfrac{3}{8} - \tfrac{5}{32}\right) * 0.025 = 328$$

σ_{t3} = 10,765psi in tension

A Workout Example: Part 3

Given the stress limits and the layout of this joint design, determine the maximum load that can cause failure in each of the four failure modes.

1 shear failure mode (in rivets)

The shear load F_s that the rivets in the joint hold is maximized when the shearing stress in the rivets reaches ultimate shearing stress τ_{su}=30,000psi. Since there are five rivets in the joint, under the equal load assumption F_s would be

$$F_s = 5\tau_{su}A = 5\times 30{,}000 \times [\pi(5/32)^2/4]=2876 \text{ lbs}$$

2 bearing failure mode (in joint sheet)

Similarly, given σ_{bu}=124,000psi and five rivets, the maximum load F_b this joint can hold in bearing failure mode is

$$F_b = 5\sigma_{bu}A = 5\times 124{,}000 \times [(5/32) \times 0.025]=2422 \text{ lbs}$$

A Workout Example: Part 3 (cont'd)

3 tension failure mode (in joint sheet)

As learned from page 14, in tension failure row 1 sheet carries the largest load applied to the joint and the succeeding rows carry less and less loads. However, how high the largest load can reach is determined by the stress level occurred in a critical row, which is not necessarily row 1. This can be seen in Part 2c on pages 23-24 where the tensile stress in row 2 sheet is higher than that in rows 1 and 3. If the applied load P continues increasing, row 2 stress will reach ultimate tensile stress σ_{tu} first and dictate how high P can be. We can now calculate the load in row 2 sheet, F_{t2}:

$$F_{t2} = \sigma_{tu} A_{t2} = 70{,}000 \times \left[\left(1\,{}^{3}/_{8} - 3\text{x}\,{}^{5}/_{32}\right) \times 0.025\right] = 1586 \text{ lbs}$$

Stress in row 2 sheet, $\sigma_{t2} = \sigma_{tu}$=70,000 psi

A Workout Example: Part 3 (cont'd)

By equal load assumption, row 1 rivet carries *P/5* load and row 2 sheet carries the rest load of *4P/5* = F_{t2} (see the free body diagram for row 2 on page 20). Then

$P = \frac{5}{4}F_{t2}$=1982 lbs = load carried by row 1 sheet = F_{t1}

Stress in row 1 sheet, $\sigma_{t1} = F_{t1}/\ A_{t1}$ =1982 / $\left[\left(1\ {}^{3}/_{8} - {}^{5}/_{32}\right) \times 0.025\right]$ =65064 psi

For row 3, we have similarly, $\sigma_{t3} x A_{t3}$ = *P - 4P/5* = $F_{t3} = 396 lbs$

Stress in row 3, σ_{t3} = F_{t3}/A_{t3} = 396 / $\left[\left(1\ {}^{3}/_{8} - {}^{5}/_{32}\right) \times 0.025\right]$ =13013psi

A Workout Example: Part 3 (cont'd)

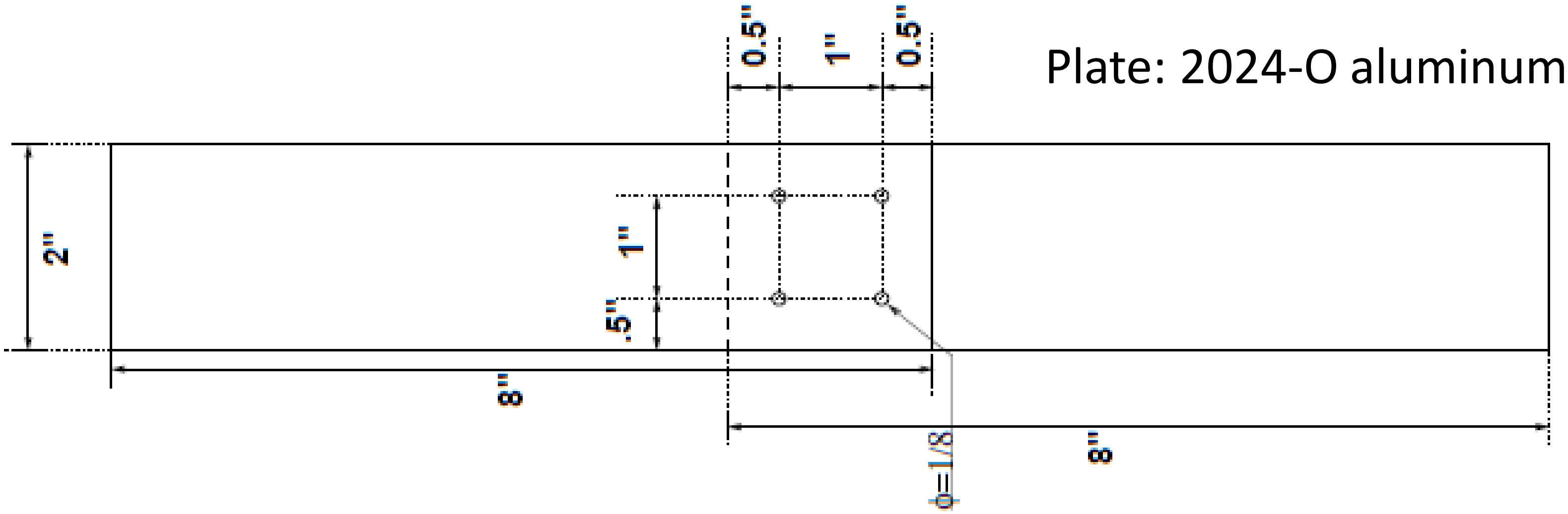
4 tearout failure mode (in joint sheet)

The load in tearout failure, F_{to}, is maximized when the stress reaches ultimate shearing stress τ_{sup}:

$$F_{to} = 2 \times t \times e \times \tau_{sup} \times N_e = 2 \times 0.025 \times 1 \times 41000 \times \ 1$$
=2050lbs

Re-design Rivet Joints

- In Lab 3, you are to re-design the lap joint below and apply tensile test to the new sample and see where and how it will fail



Lab 3 prelab is due 11:59pm to Canvas the night before the lab;

Lab 3 report (in summary format) is due 11:59pm to Canvas the night of the specific due date