IOWA STATE UNIVERSITY

Aer E 322: Aerospace Structures Laboratory

Week 5 and 6 Lecture:

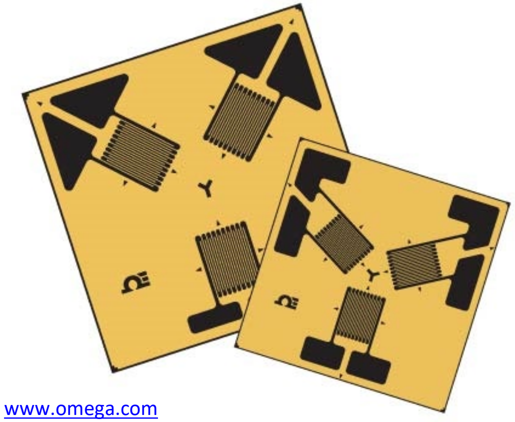
Strain Gages and Applications

February 13 and 20, 2023

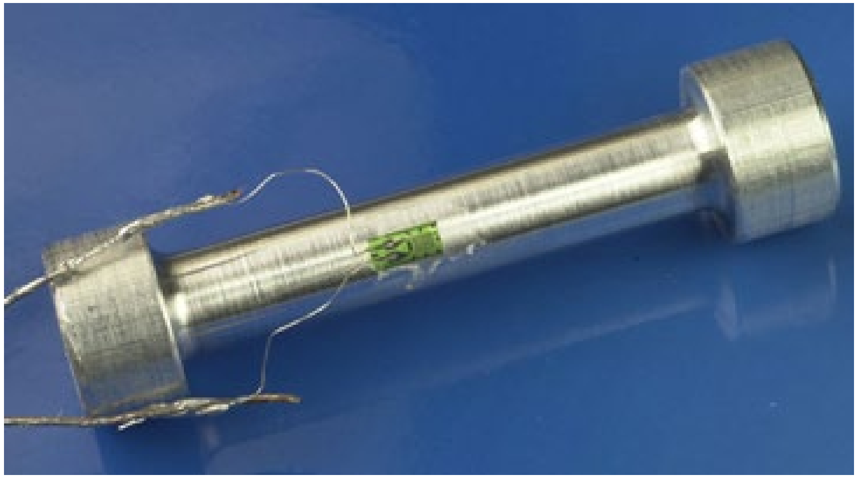
Strain Gages

- A strain gage is a small, thin metallic foil wrap-around pattern attached to the surface of the part of interest for measuring mechanical strain (hence mechanical load) by changing its electrical resistance when stressed (stretched or compressed) within the elastic limit
- Strain gages are commonly used in group and arranged in various orientations to measure deformations and thereby validate models and predictions of how external loads are distributed in the aircraft structures and the actual aerodynamic loads that occur during flight

rosette



Source: www.omega.com

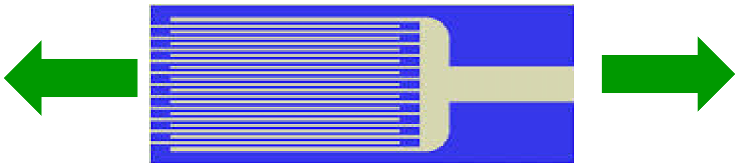


Source: http://www.doitpoms.ac.uk

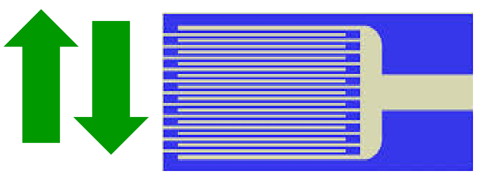
Strain Gages (cont'd)

$$R = \rho \frac{L}{A}$$

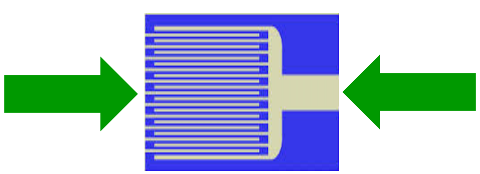
tension: resistance increases



resistance insensitive to lateral force



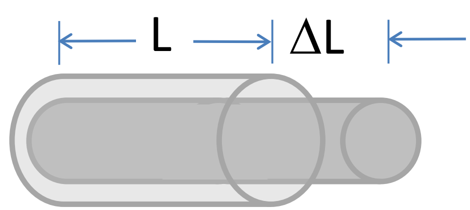
compression: resistance decreases



Gage factor (GF): ratio of change between electrical resistance R and mechanical strain ε

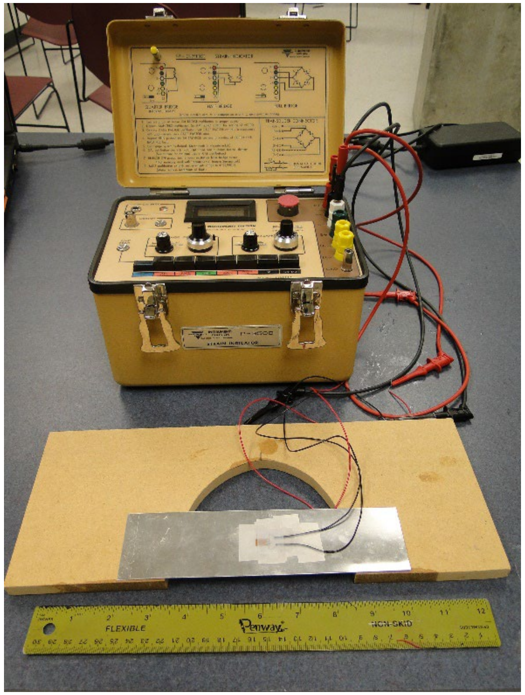
$$GF = \frac{\Delta R / R}{\Delta L / L} = \frac{\Delta R}{R}\frac{1}{\varepsilon}$$

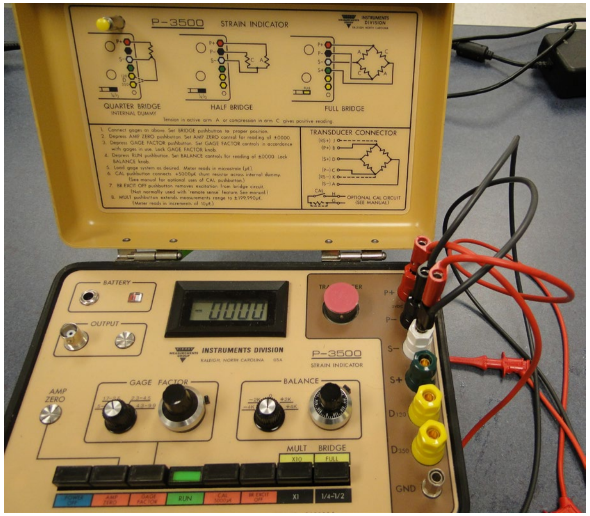
GF ~2 for metallic gage



Strain Measurements

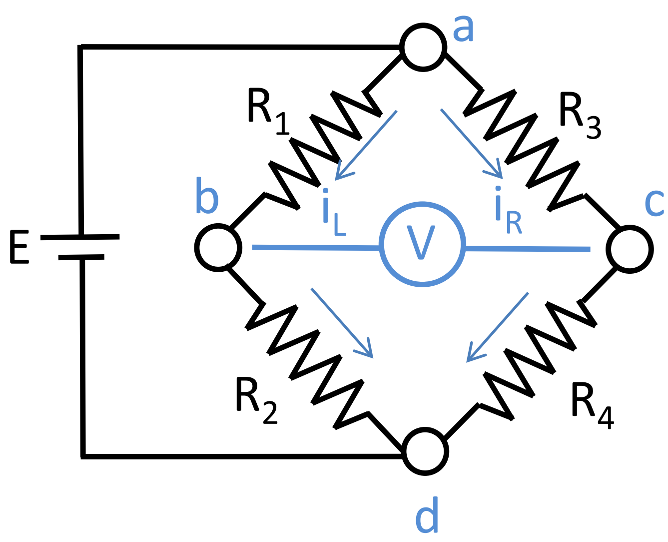
The strain gages are installed on the surface of samples and connected to the strain reader via electric wires to obtain the strain readings:





Wheatstone Bridge Circuit

The strain measurements done by the strain reader are based on the operations of Wheatstone bridge circuit:



By Ohm's law

$$E = i_L(R_1 + R_2) = i_R(R_3 + R_4)$$

$$E_b - E_a = i_L R_1 = \frac{R_1}{R_1 + R_2} E$$

Similarly

$$E_c - E_a = \frac{R_3}{R_3 + R_4} E$$

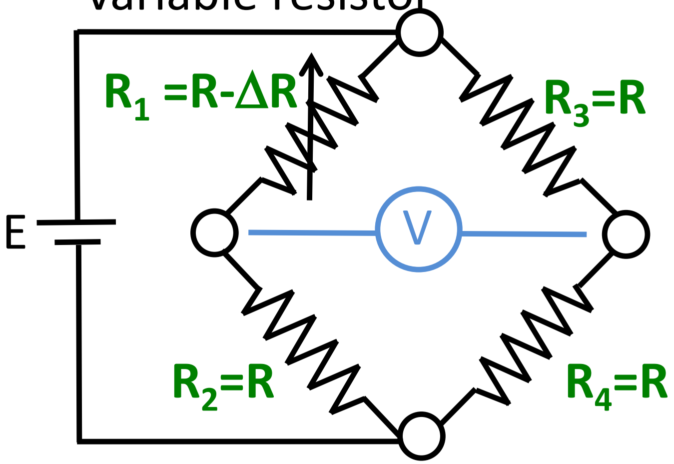
Then

$$V = E_c - E_b = \left(\frac{R_3}{R_3 + R_4} - \frac{R_1}{R_1 + R_2} \right) E$$

The above is the so-called Wheatstone bridge equation

Wheatstone Bridge Circuit: Quarter Bridge

Strain gage acts like variable resistor



From WB equation

$$\frac{V}{E} = \frac{R}{R+R} - \frac{R-\Delta R}{R-\Delta R+R}$$

$$= \frac{1}{2} - \frac{R-\Delta R}{2R-\Delta R}$$

$$= \frac{\Delta R}{2(2R-\Delta R)}$$

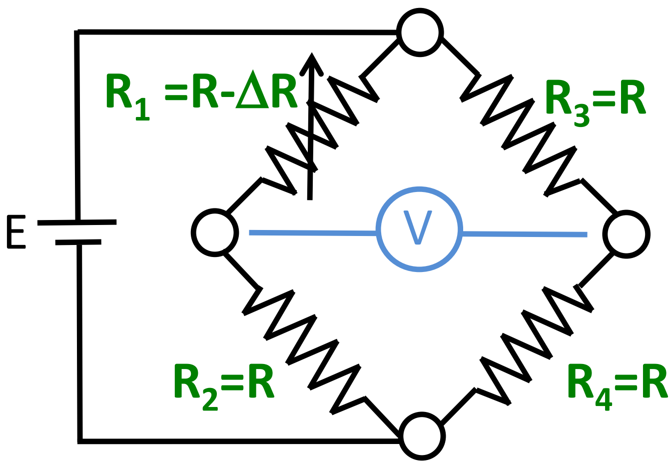
$$= \frac{\Delta R}{2}\left(\frac{1}{2R-\Delta R}\right)$$

$$= \frac{\Delta R}{2}\left[(2R)^{-1} + (2R)^{-2}(\Delta R) + \ldots\right]$$

Binomial theorem

$$(x+\Delta x)^n = x^n + nx^{n-1}\Delta x + \ldots$$

Wheatstone Bridge Circuit: Quarter Bridge (cont'd)



$$\frac{V}{E} = \frac{\Delta R}{2}\left[(2R)^{-1} + (2R)^{-2}(\Delta R) + \ldots\right]$$

$$\approx \frac{1}{4}\frac{\Delta R}{R}$$

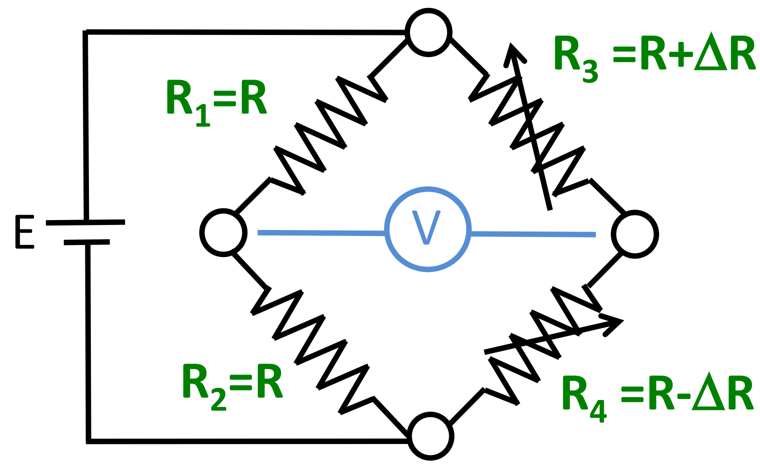
$$= \frac{1}{4} GF\,\varepsilon$$

Keep only leading term

and recall $GF = \dfrac{\Delta R}{R}\dfrac{1}{\varepsilon}$

Wheatstone Bridge Circuit: Half Bridge I

Gage adjacent; opposite change



$$\frac{V}{E} = \frac{R+\Delta R}{(R+\Delta R)+(R-\Delta R)} - \frac{R}{R+R}$$

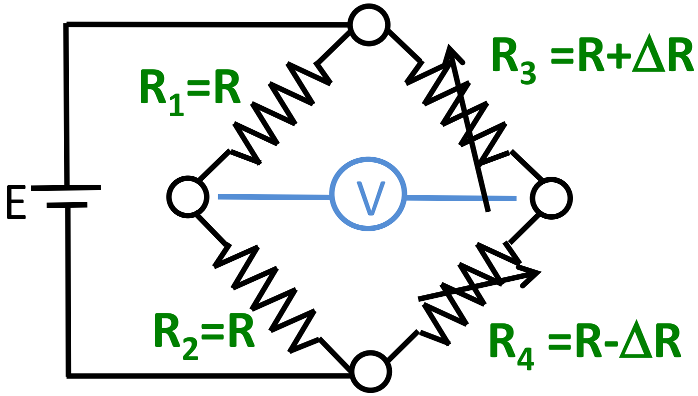
$$= \frac{R+\Delta R}{2R} - \frac{1}{2}$$

$$= \frac{1}{2}\frac{\Delta R}{R}$$

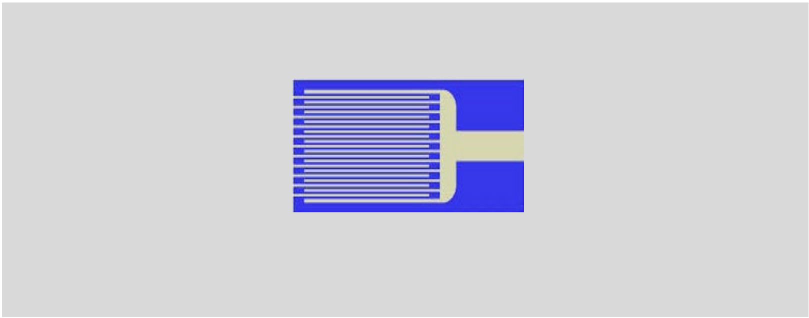
$$= \frac{1}{2} GF\varepsilon$$

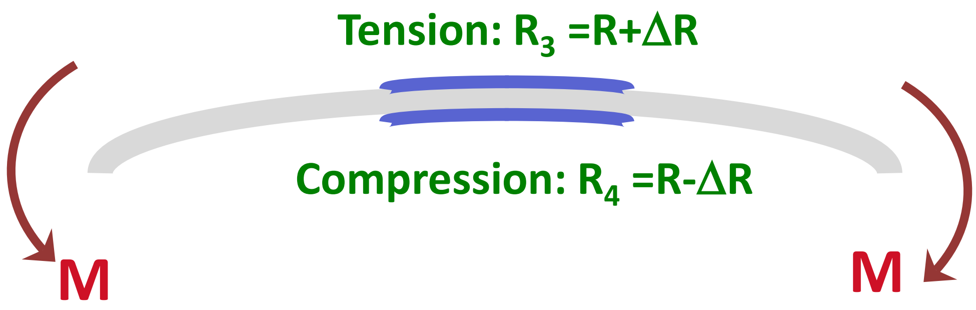
Bending Test:
Half Bridge I Application

Gage adjacent; opposite change



gages attached on both sides

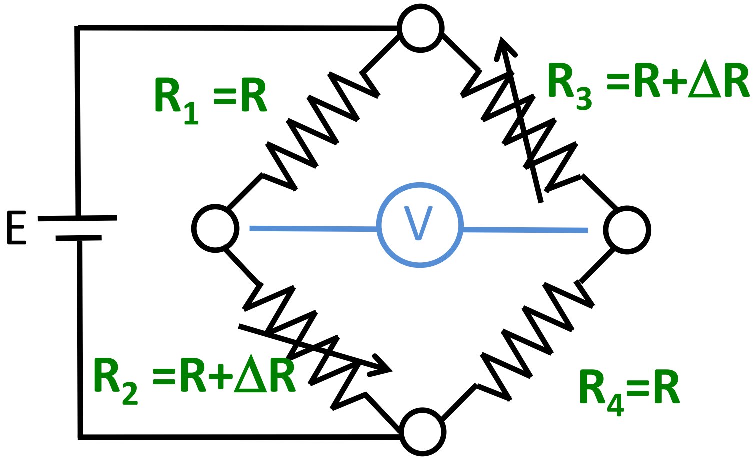




$$\frac{V}{E} = \frac{1}{2} GF\varepsilon$$

Wheatstone Bridge Circuit: Half Bridge II

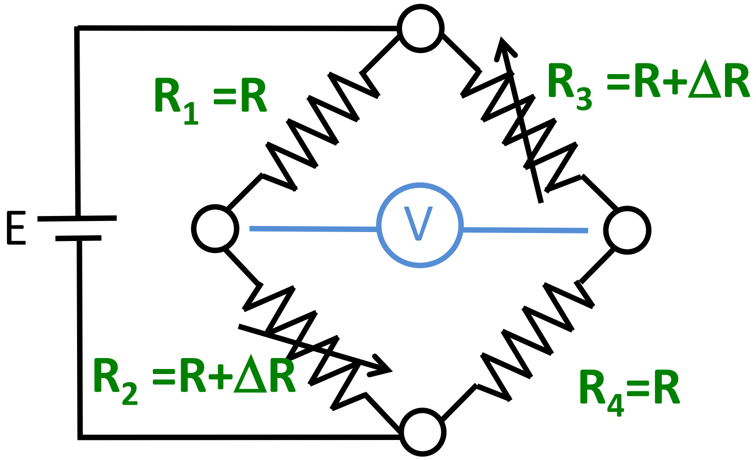
Gage opposite; like change



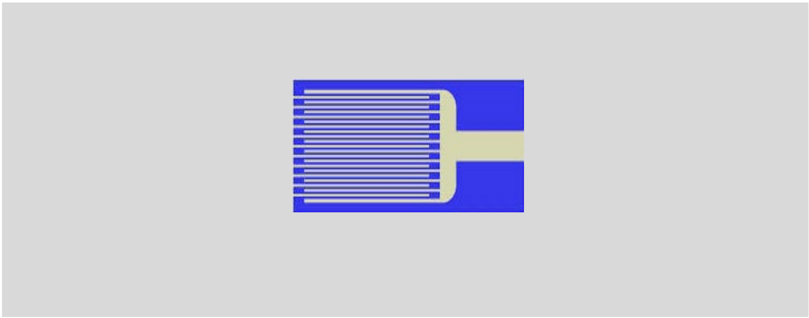
$$\frac{V}{E} = \frac{R+\Delta R}{R+\Delta R+R} - \frac{R}{R+R+\Delta R}$$
$$\approx \frac{1}{2} GF\,\varepsilon$$

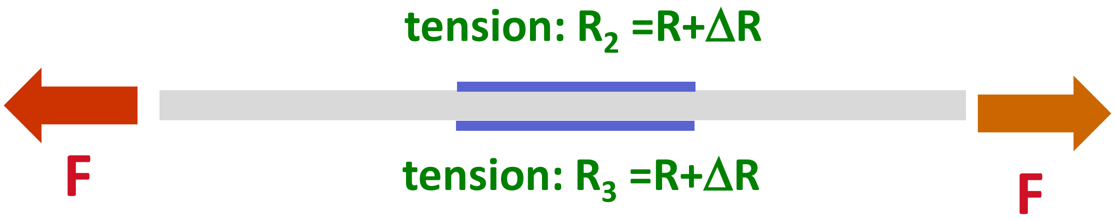
Tensile Test:
Half Bridge II Application

Gage opposite; like change



gages attached on both sides

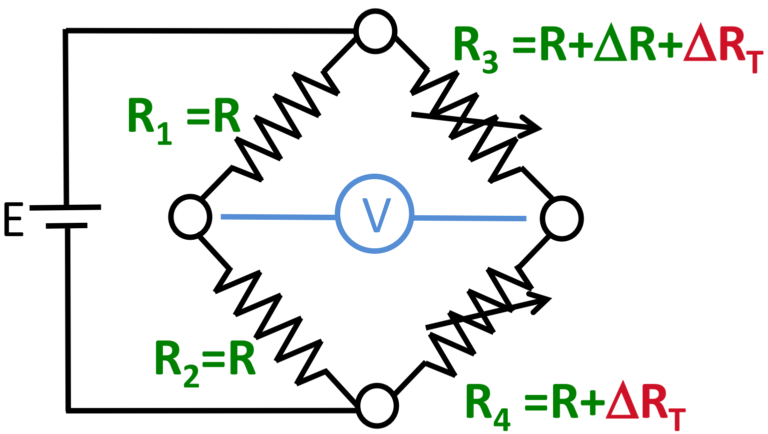




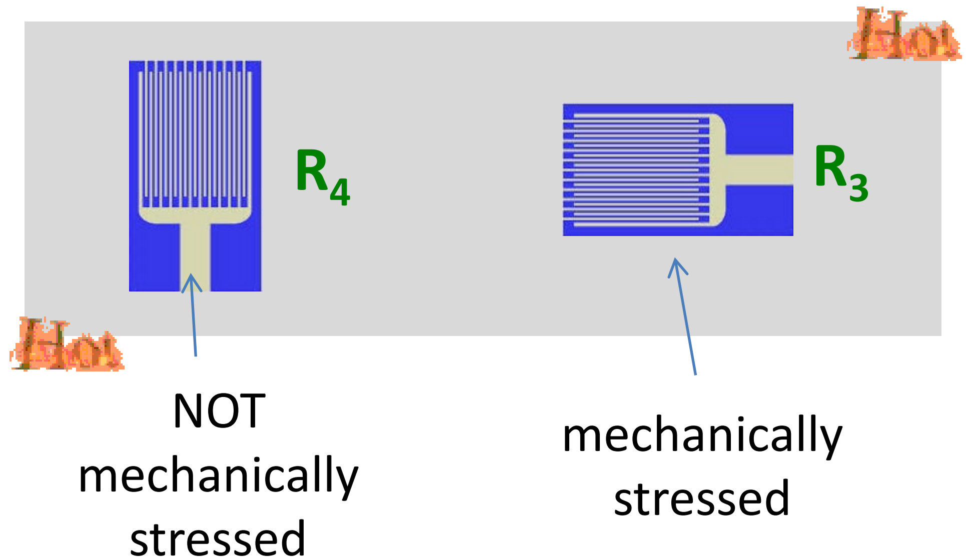
$$\frac{V}{E} = \frac{1}{2} GF\varepsilon$$

Temperature Compensation: Quarter/Half Bridge Application

Gage adjacent; like temp. change



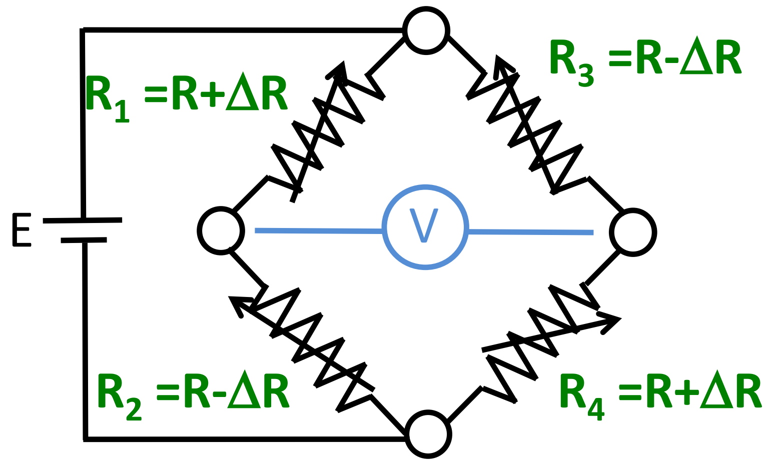
gages attached on same side of, e.g. a pressurized boiler wall



$$\frac{V}{E} = \frac{1}{4} GF \varepsilon$$

Temperature effort on strain is “cancelled” at the bridge

Wheatstone Bridge Circuit: Full Bridge



$$\frac{V}{E} = -GF\varepsilon$$

double the sensitivity;
true linearity

Wheatstone Bridge: Summary

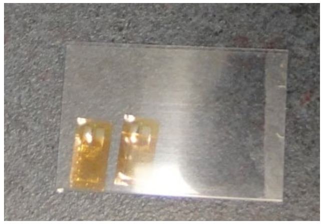
$$\frac{V}{E} = \beta GF\varepsilon \begin{cases} \text{quarter bridge: 1 gage, } \beta=1/4 \\ \text{half bridge: 2 gages, } \beta=1/2 \\ \text{full bridge: 4 gages, } \beta=1 \end{cases}$$

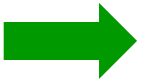
Wheatstone Bridge:
Lab 4a – Gage Installation

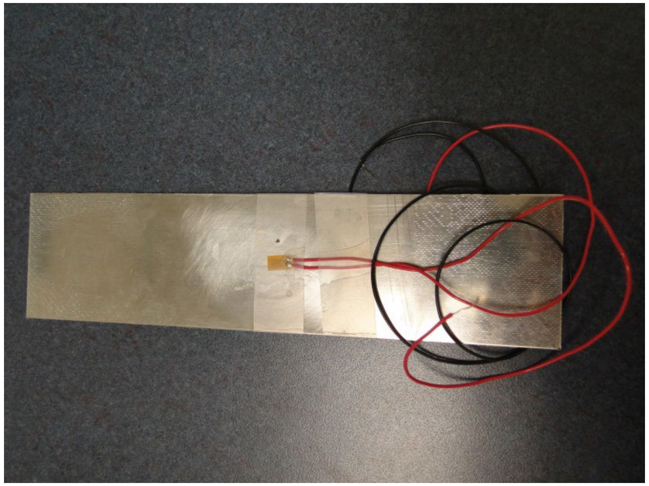
Follow gage application manuals and TA's instructions to attach strain gages to test specimen, using the materials and equipment shown

Wheatstone Bridge:
Lab 4a – Gage Installation (cont'd)

- Prepare surface of the 2024-O aluminum strip specimen
- glue two Omega SGD-5/350-LY13 strain gages on the strip, one on each side
- Solder connecting wires to gages

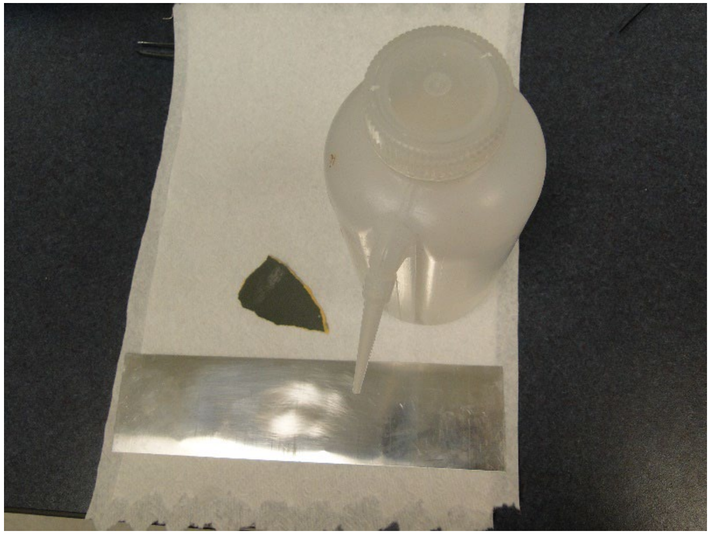


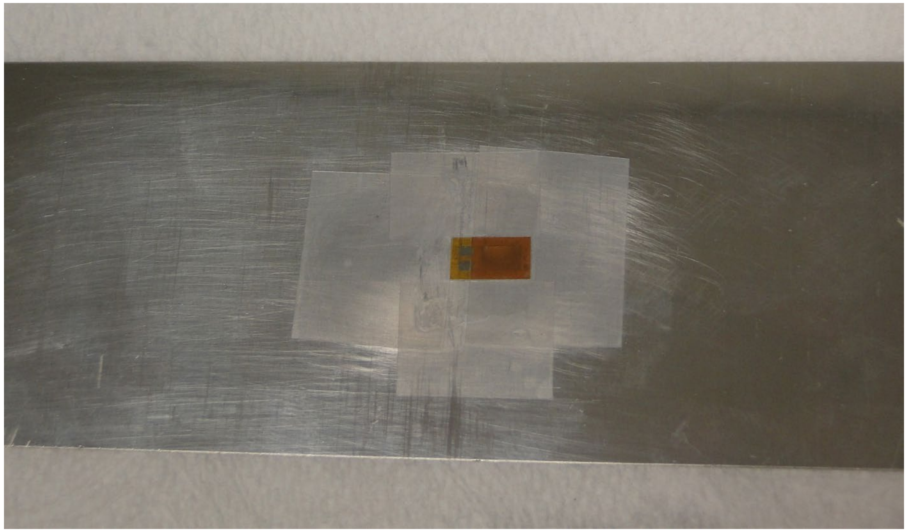




Lab 4a – Gage Installation (cont'd)

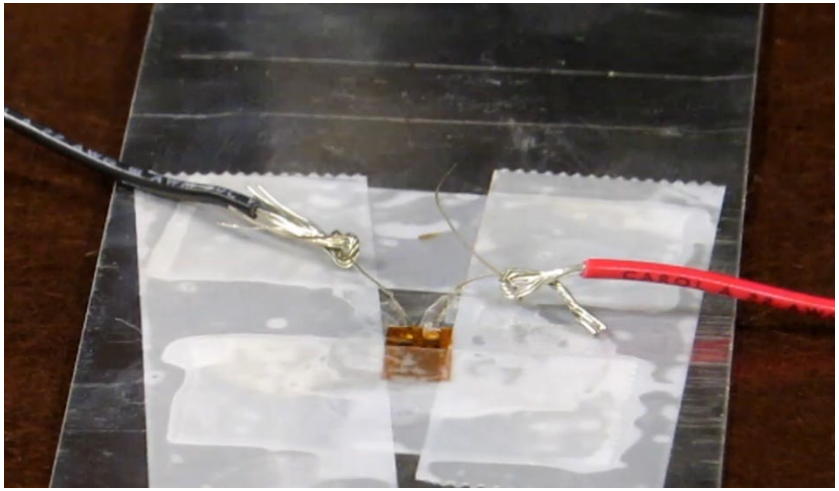
- Prepare surface of the aluminum sheet specimen through degreasing, sanding and further cleaning (using acetone as shown)
- Insulate the gages (except the terminals) with tapes

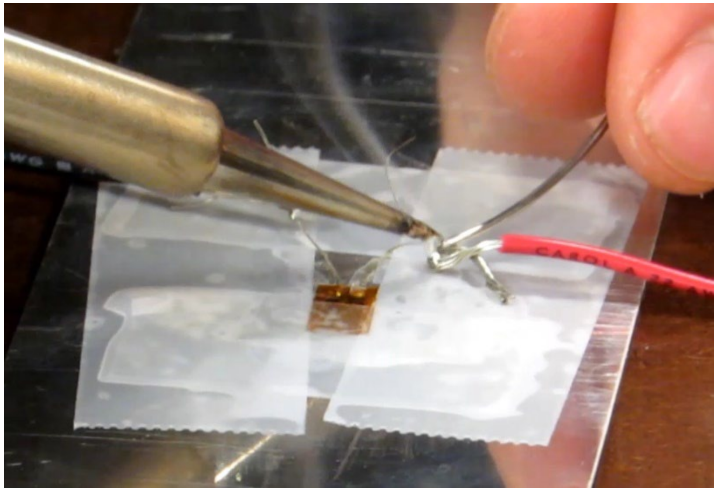




Lab 4a – Gage Installation (cont'd)

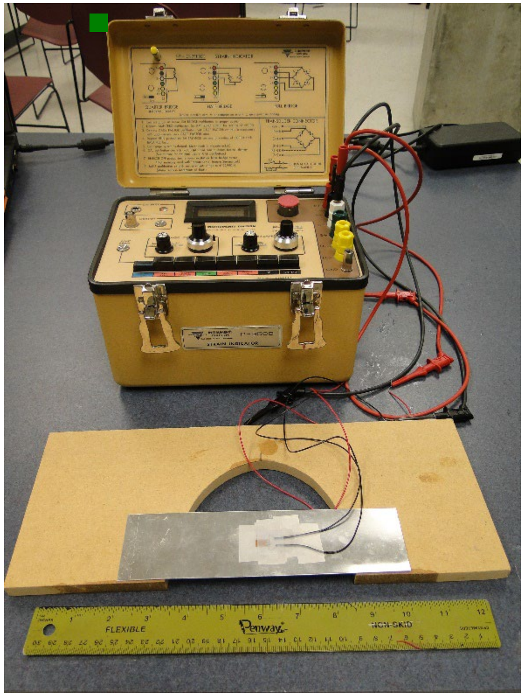
- (lower left) connecting the gage is now easy; just tie a knot to the gage leads
- (lower right) only solder the connecting joint



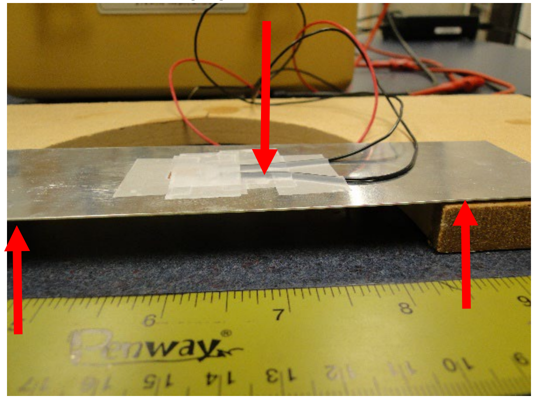


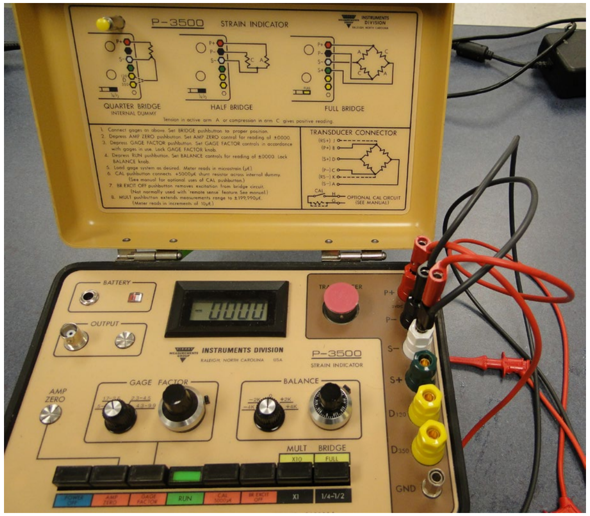
Lab 4b Gage Application - Bending Test

- A three-point bending is setup as shown
- The deflection in middle causes compression at gage on top side of AL specimen and tension at gage on the bottom side
- Half bridge I is wired (adjacent gages, opposite changes) to measure the strains on both sides and to compare with theory



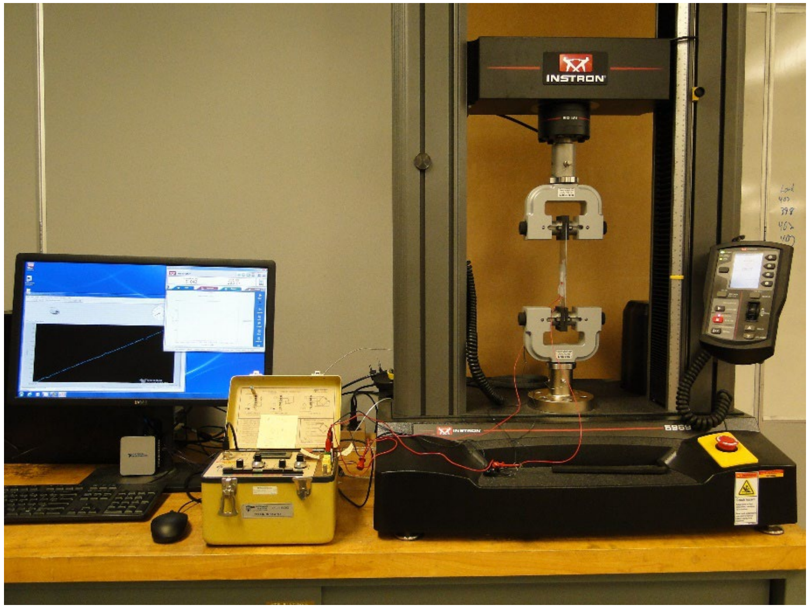
Load applied at middle

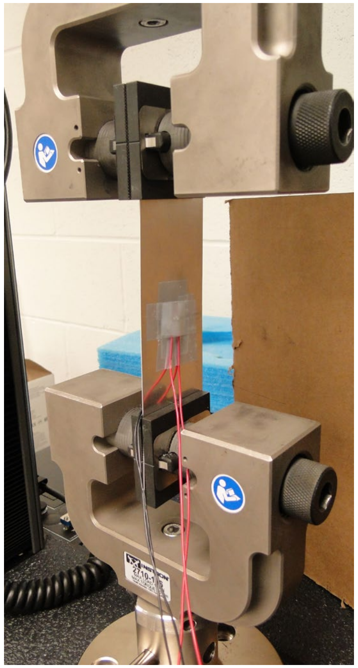


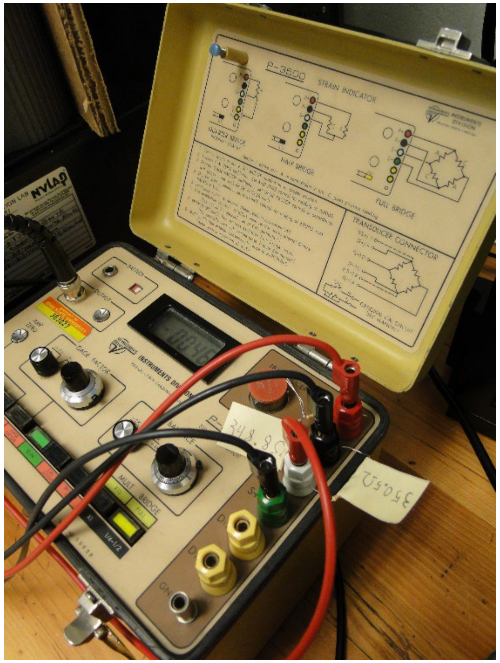


Lab 4b Gage Application - Tensile Test

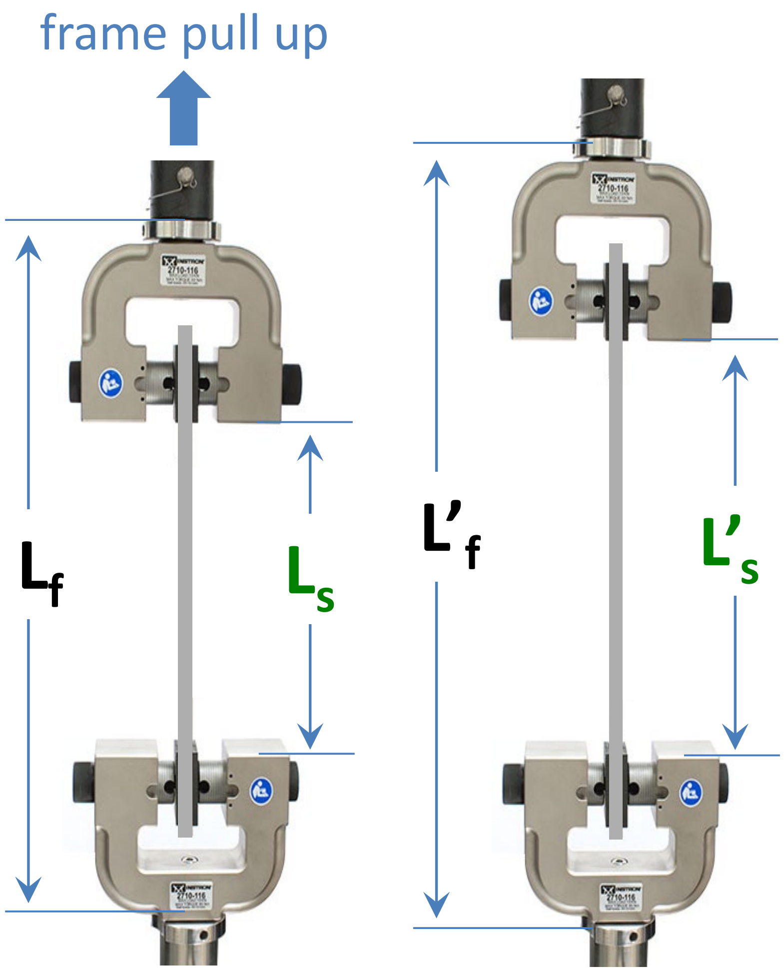
- Tensile test to perform by Instron load frame; load and elongation are to be measured
- The strains on both sides of AL specimen are also measured by the strain gage indicator and a data acquisition device and then routed to PC screen for display
- Half bridge II is setup using full bridge and two other reference resistors







Lab 4b Tensile Test – Ideal

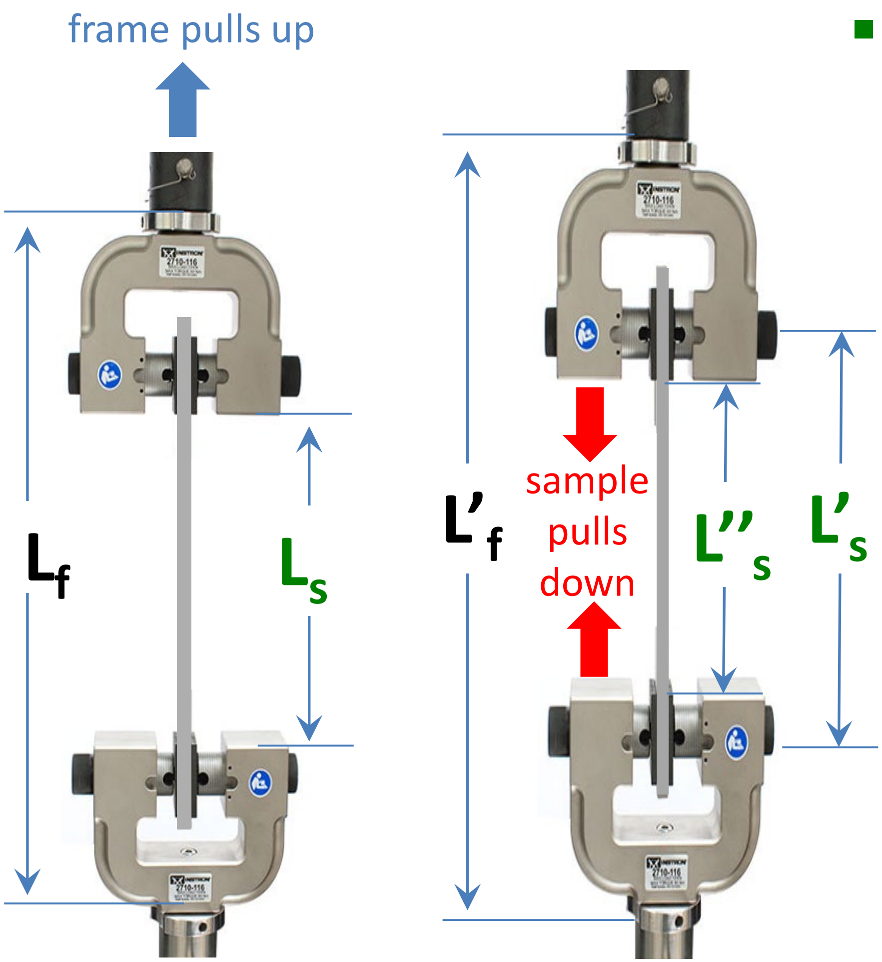


- For the tensile test performed at the load frame, the frame pulls the sample up through the grips. The *ideal* situation is that the elongation measured by the load frame, ΔL_f, is attributed *entirely* to the elongation of the sample, ΔL_s, i.e.

$$L_f' - L_f = \Delta L_f = \Delta L_s = L_s' - L_s$$

measured = desired

Lab 4b Tensile Test – Reality



- In reality, however, the sample also pulls the grips down, and when this pulling strength (stiffness) can not be ignored, the sample elongation is measured larger than it actually is. This is especially true for the relatively "soft" flat-end grips used in the lab.

$$L'_s = L''_s + L_g$$

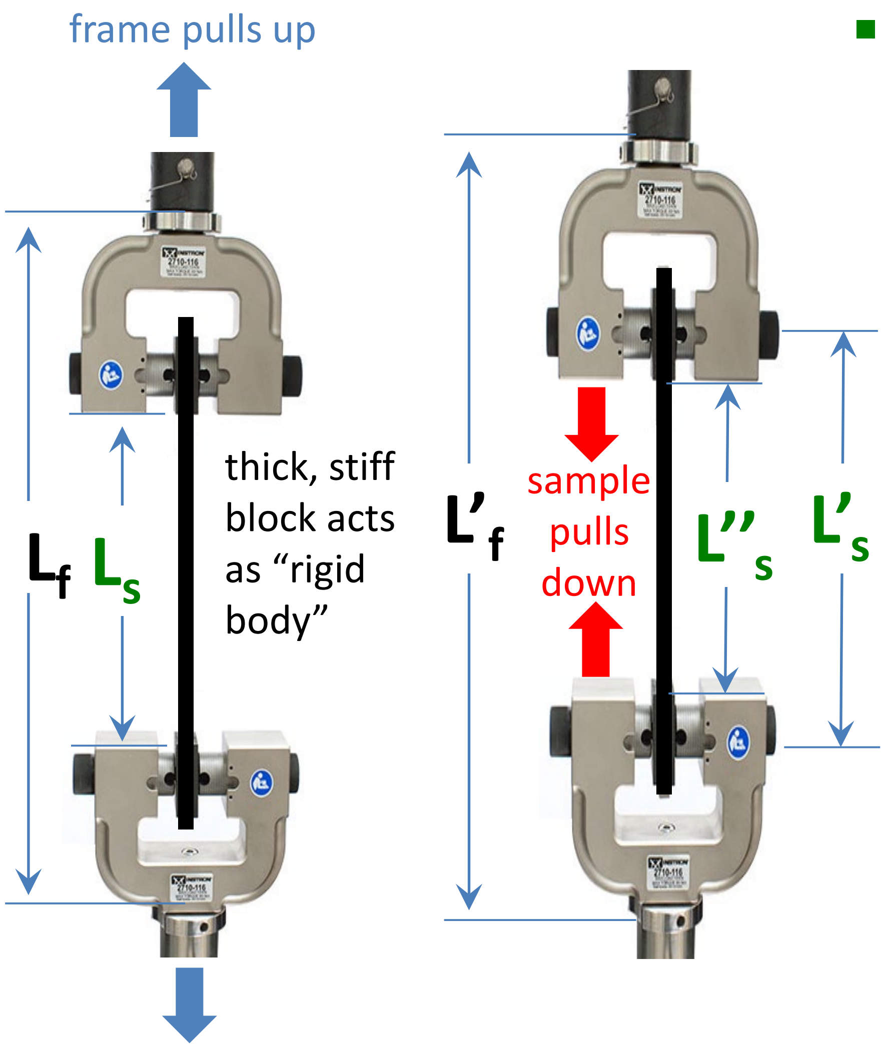
stretch of grip (mainly)

$$L'_s - L_s = L''_s - L_s + L_g$$

$$L_f - L'_f = \Delta L_f = \Delta L_s + L_g$$

measured > desired

Lab 4b Tensile Test – Calibration



- To calibrate this unwanted “system” elongation L_g, one simple trick is to test separately a “rigid body” (approximated by a thick and much stiffer block) under the same condition. The elongation measured would mostly be L_g.

$$L_s' = L_g + L_s''$$

$$\underbrace{L_s' - L_s}_{\parallel} = L_g + \underbrace{L_s'' - L_s}_{\sim 0}$$

$$\overbrace{L_f - L_f'} = \Delta L_f \approx L_g + \sim 0$$

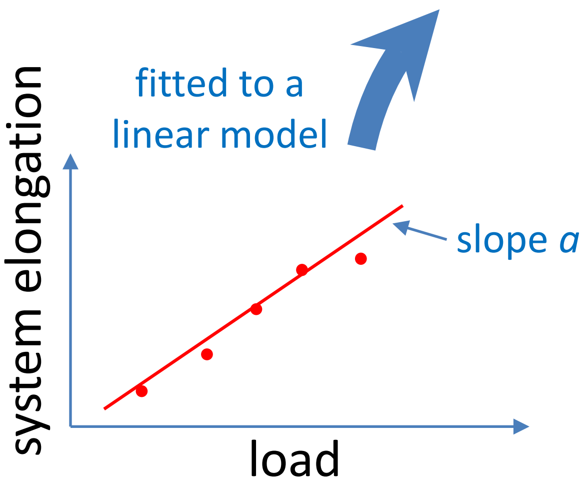
measured ~ system error

Lab 4b Tensile Test – Correction

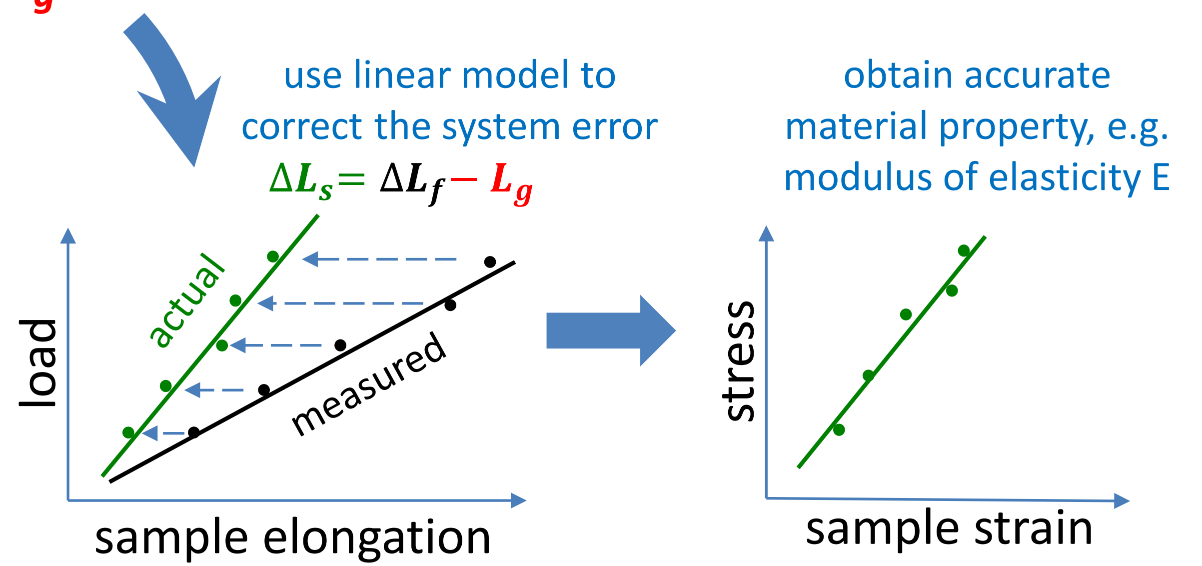
- To obtain accurate measurement in the tensile test of the sample, we need to correct the ”system” error by subtracting the calibrated L_g from the measured L_f in e.g. strain vs. stress plot

$$L_f' - L_f - L_g = \underbrace{\Delta L_f}_{\text{measured}} - L_g = \underbrace{\Delta L_s}_{\text{actual}} = L_s'' - L_s$$

system elongation $L_g = a \times \textbf{load} + b$

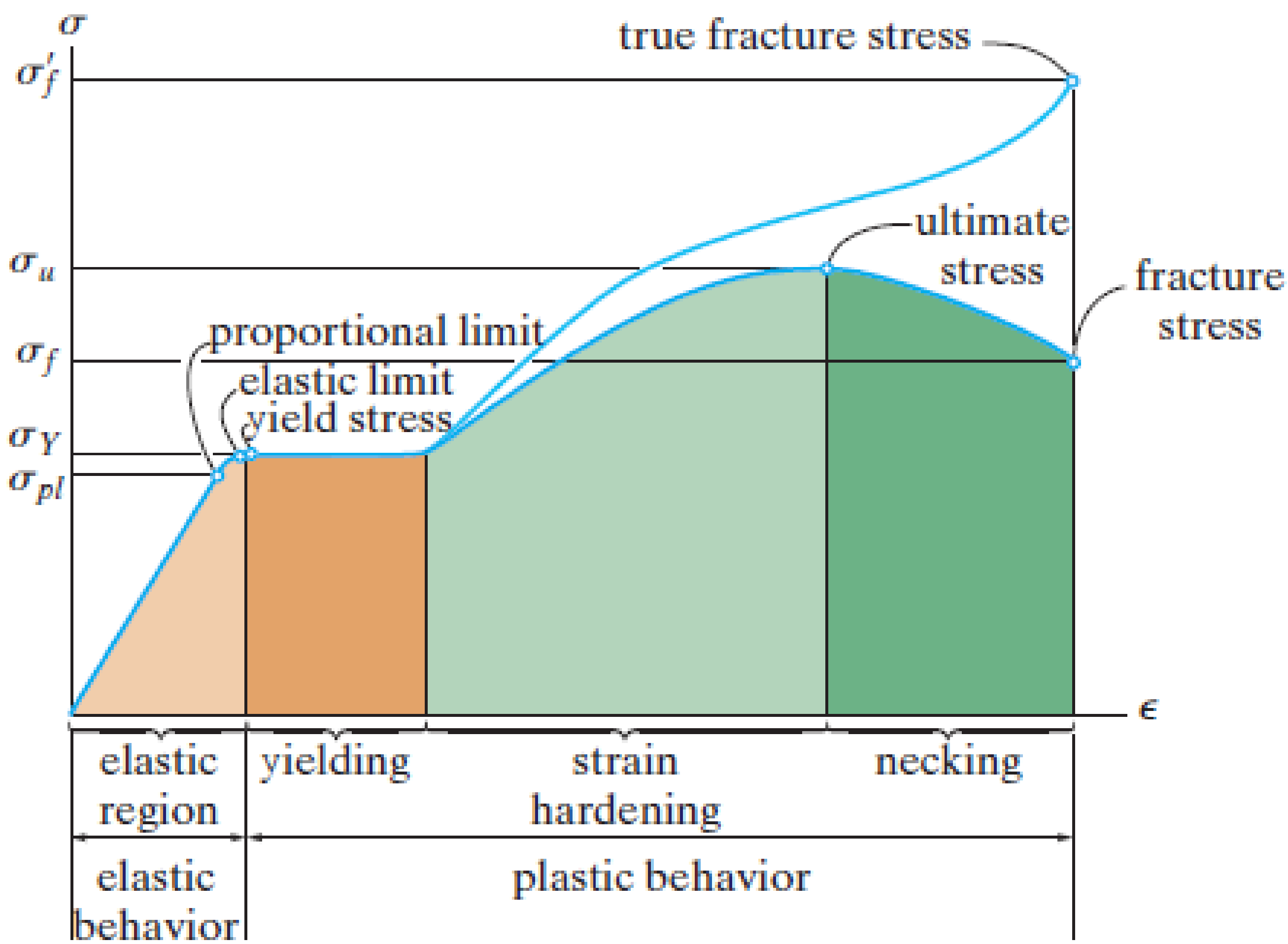


calibration test



sample test

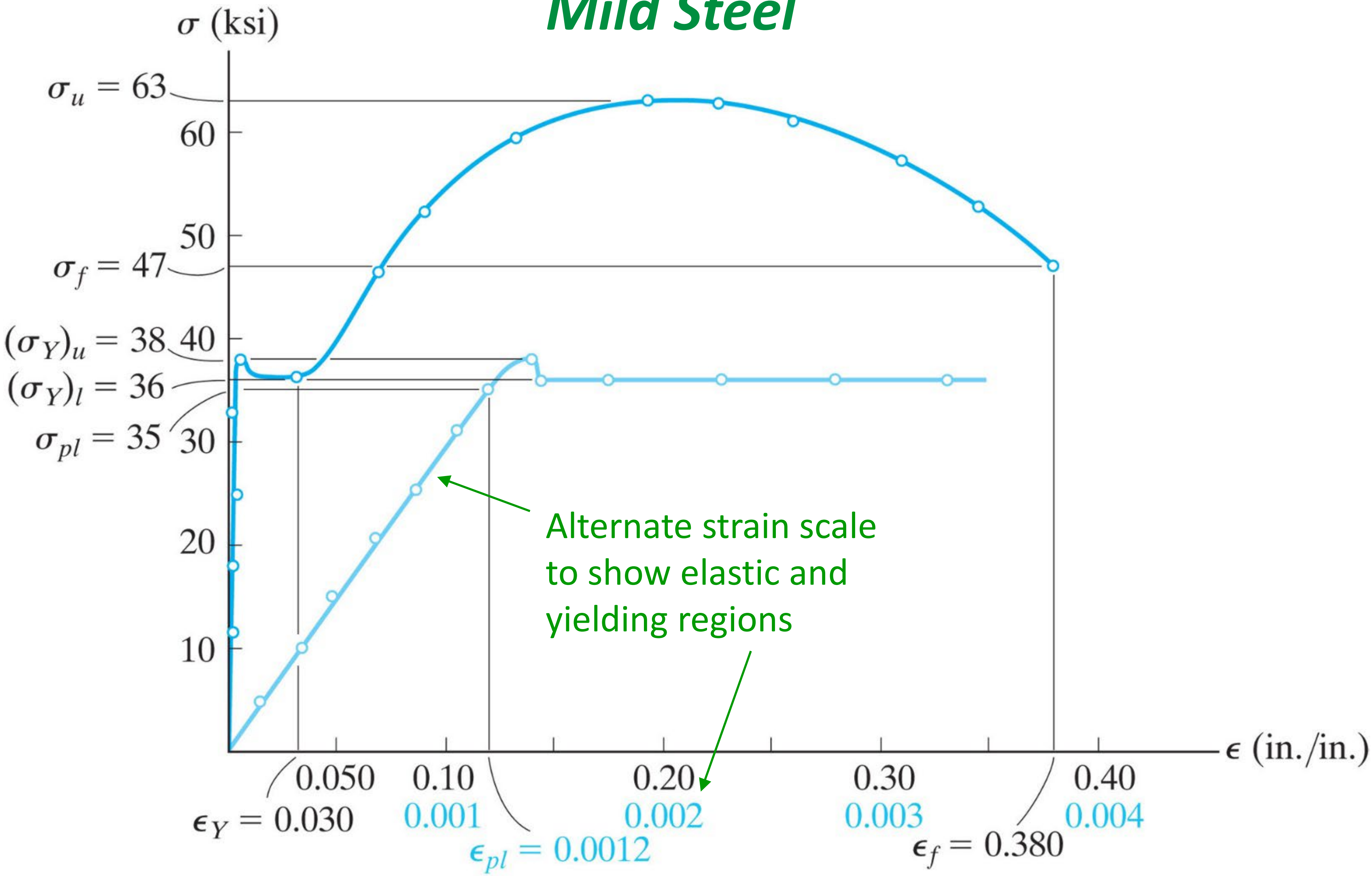
Review of Stress–Strain Diagram *Idealized*



Conventional and true stress-strain diagrams
for ductile material (steel) (not to scale)

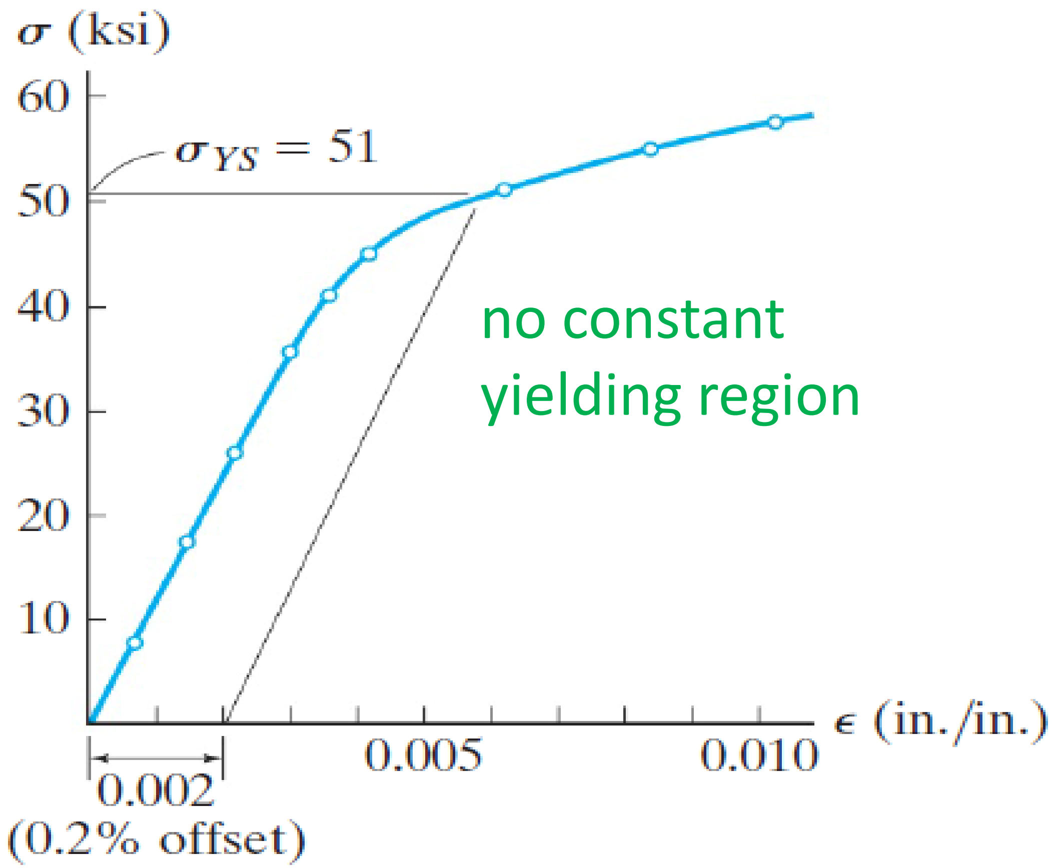
Review of Stress–Strain Diagram ***Actual***

Mild Steel



Review of Stress–Strain Diagram *Actual* *(cont'd)*

Most metals such as the Aluminum alloy do not have well-defined yield point. The **0.2% offset method** is often used to set yield strength at the stress level when strain = 0.002

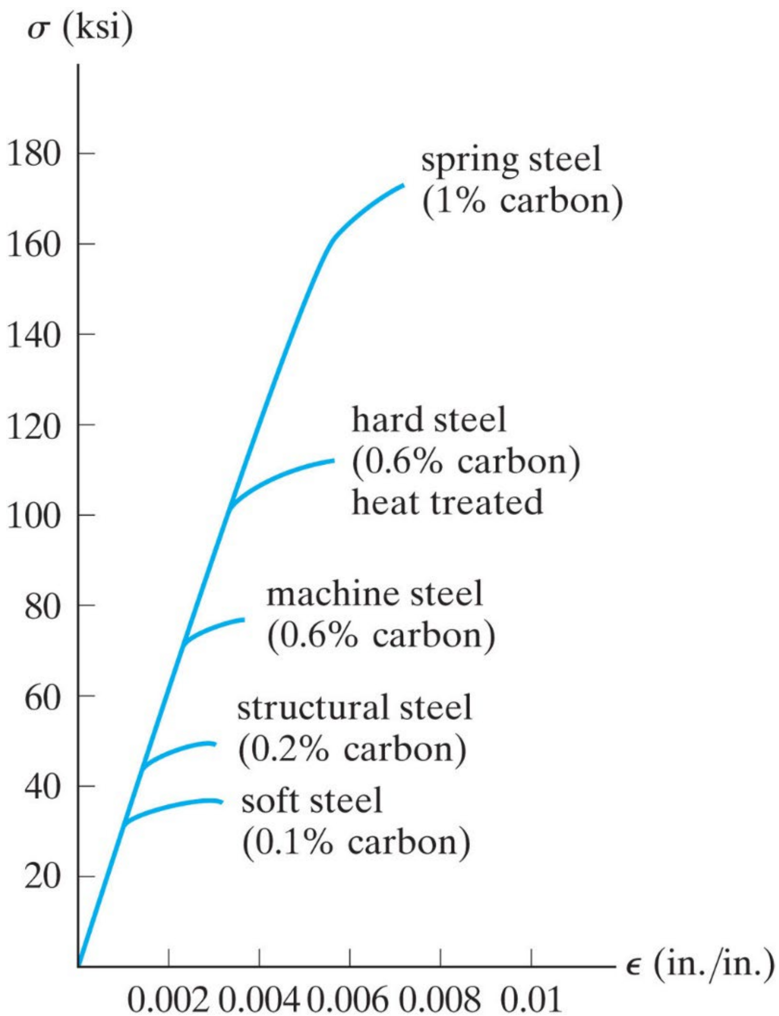


no constant yielding region

Yield strength for an aluminum alloy

Linear Elastic Behavior and Hooke's Law

Proportional Limits of Various Steels



$$\boxed{\sigma = E\varepsilon}$$

- This Constant "E", called "**modulus of elasticity**" and/or "**Young's modulus**", is the slope of the linear (proportional) stress strain curve
- For steels, E ≈ 200 GPa [~29x10^3 ksi] at ambient temperature. Other metals, not predominately made of iron atoms, have differing values of E