Lab 5: Beam Deflection and Analysis

Objective

Test the deflection of various beam configurations under load, and apply the basic beam theory learned in mechanics of materials and lecture to calculate what the deflections should be.

Introduction

Beams are basic components to aircraft structures. The ability to model and predict how beams react to loads and interact with each other is essential to the design of aircraft. From EM 324 mechanics of materials or the like, you have learned the basic Euler–Bernoulli beam theory [1]. In Week 7 lecture, we review beam deflection analysis using the superposition and discontinuity function methods [2]. In Prelab for this lab, you will apply these theories to predict the deflections of a cantilever beam setup. In the lab, you will physically measure the deflections from actual beam samples at various setups under different loads. Then in After Lab work, you confirm all your predictions and further derive solutions for statically indeterminate beams.

References

[1] R.C. Hibbeler, *Mechanics of Materials*, 10th Edition, Pearson, 2017, Ch. 12

[2] Week 7 lecture notes

Work to be done

To refresh your knowledge on beam deflection, you may want to review previous EM 324 materials and Week 7 lecture notes here in this course. For In Lab work, please watch the corresponding demo video which illustrates most of the testing procedures described below. Please also follow lab operation and safety protocol given in syllabus.

1. Prelab

For the prelab work, your task is to solve for the deflections of a cantilever beam setup, which corresponds to the test configurations (1)- (3) below. Please see separate prelab assignment from Canvas.

2. In lab

<u>Materials and configurations to be tested</u>

Five different test configurations are to be tested using long beam of square and rectangular cross section. The first three configurations correspond to the same cantilever beam setup (Fig. 1) as in Prelab, and the last two correspond to statically indeterminate configurations as shown in Figs. 2 and 3. All test configurations have the same beam length and material properties.

- Square cross-section dimensions: 12.8 x 12.8 mm
- Rectangular cross-section dimensions: 12.8 x 6.4 mm
- Beam length *L*:90cm.
- Young's modulus *E*: 10^7 psi or 68.9 GPa (for Aluminum 6061-T6511 alloy).

Equipment to be used

- Digital displacement gauge
- Wood base board and roller stand
- Vise
- Weights
- Level gauge and metal stand
- Ruler
- Wrench

Please note that, in the following references, "x grams load" or simply "x grams" should be interpreted as the "gravitational weight/force of x grams mass". Here the quantity of x purely measures the mass itself.

Test configurations (1)-(3)

For beam setups of three rectangular cross sections subjected to loads at the free end (Fig. 1), measure the deflections *y* in mm at 2/3 beam length *L* referenced from the fixed wall. The beam cross sections and loads are as follows:

(A) 12.8 x 6.4 mm (12.8 mm is the base and 6.4 mm is the height in the y direction) rectangular cross-section with 100 grams weight load.

(B) 6.4 x 12.8 mm rectangular cross-section with 200 grams weight load.

(C) 12.8 x 12.8 mm square cross-section with 500 grams weight load.

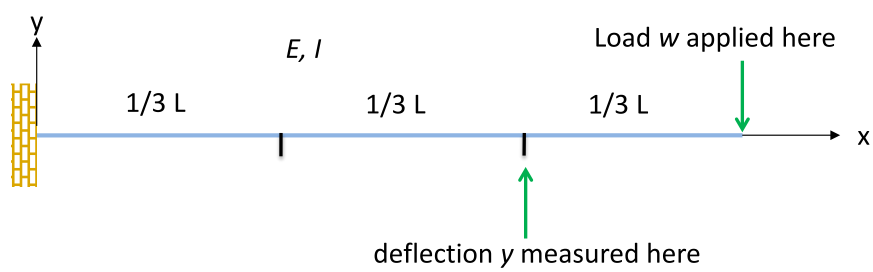


Figure 1. Cantilever beam setup for test configurations (1)-(3)

Test configuration (4)

A roller support is added to the cantilever beam setup at the free end to become a statically indeterminate beam structure (Fig. 2). The load w is applied at 1/3 L and the deflection y is measured at 2/3 L, both referenced from the fixed wall. The load and beam cross section are as follows.

(D) 12.8 x 6.4 mm (12.8 mm is the base and 6.4 mm is the height in the y direction) rectangular cross-section with 1000 grams weight load.

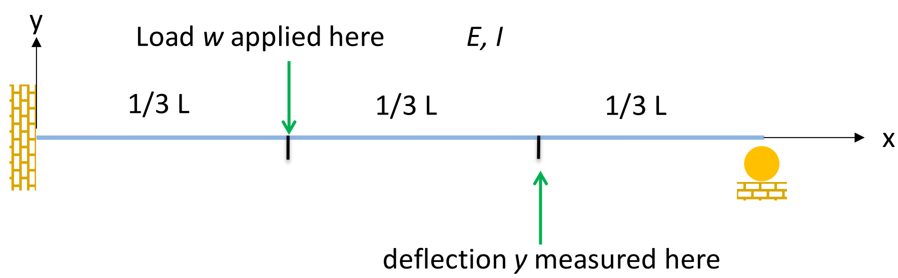


Figure 2. Test configuration (4)

Test configuration (5)

The roller support is now set at 2/3 L from the fixed wall. Two loads are applied: w_1 at 1/3 L from the fixed wall and w_2 at the free end. The deflection y is measured at the free end (Fig. 3). The load and beam cross section are as follows.

(E) 12.8 x 6.4 mm (12.8 mm is the base and 6.4 mm is the height in the y direction) rectangular cross-section with w_1=2500 grams weight and w_2 =200 grams weight.

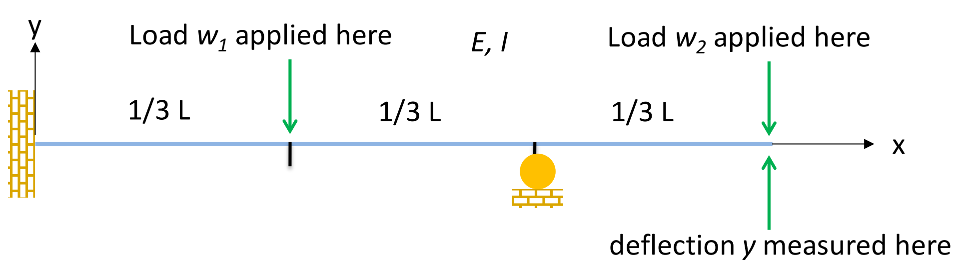


Figure 3. Test configuration (5)

Testing Procedures

- **i.** Double check the calculations you made in Prelab with your teammates'. Consolidate the team results and ensure the team has the correct predictions for these configurations, i.e. the first three test configurations (1)-(3).
- **ii.** The benchtop should have been leveled with gravity along the beam direction. Double check this using the level gauge: the bubble in the level gauge should stay in the middle. If the benchtop level is off, the height of bench can be adjusted by turning the screw-in stand at the end of each of the four bench legs with the wrench, as illustrated in the lab demo video.
- **iii.** The first three test configurations have the common cantilever beam setup as shown in Fig. 4. Begin the test by clamping down the test configuration (1) on the vertical wood board and tightening the wood board in the vise. Be sure the beam length sticking out of the edge of the wood board is 90 cm as used in the prelab calculations.
- **iv.** Once the beam length is correctly set, level the beam by placing the level gauge on top surface of the beam surface (Fig.5). Check the level at other locations on the beam.
- **v.** We will use the "needle" i.e. the PASCO displacement sensor to measure the beam deflections. Place the needle at the same specified locations as in your prelab predictions. Be sure the displacement sensor is pointing upward from below the beam, secured in the clamp and flush to the underside of the beam with as little spring load as possible (Fig. 6).
- **vi.** Repeat the measurements at least three times. Each time zero out the needle reading before re-applying the weight and re-taking the measurement.
- **vii.** Repeat the same process for test configurations (2) and (3). For test configuration (3), be sure to change the beam specimen to the square type. *Or you may choose to stay with the test configuration (1) and proceed with the last two test configurations (4) and (5) (Figs. 2 and 3) described above, since they have the same cross section. You can then return to test configurations (2) and (3) afterwards.*
- **viii.** For test configurations (4) and (5), you will need the wooden stand as the roller support. When setting up, place the stand as close to the vertical wood board as possible. Level the beam by using the level gauge as before. Make sure the top round edge of the wood stand is in firm (but not tight) contact with the bottom surface of the beam. Swing the stand underneath the beam in the axial direction toward the free end. To obtain a good leveling, make sure you get the same firmness of contact and the bubble of the level gauge stays in the middle throughout the swinging (Fig. 5).
- **ix.** Again, repeat measurements at least three times for each test configuration. The overview of test configuration (5) is shown in Fig. 7. Extra care should be taken to set the needle pointing downward at the same location where the hook of the weight is hung (Fig. 8).
- **x.** For all five test configurations, take a few photos to be used later in lab report.

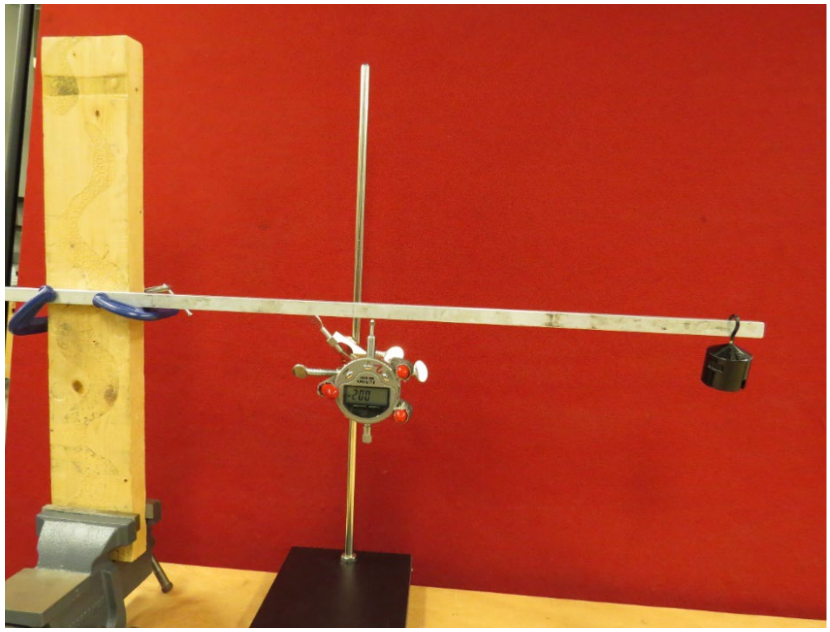


Figure 4. Basic cantilever setup for test configurations (1)-(3)

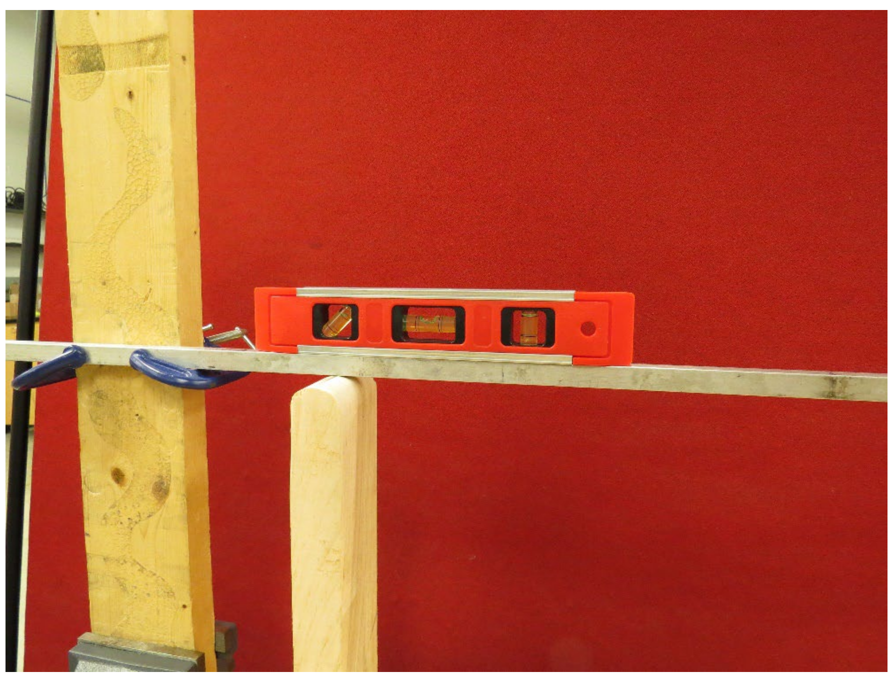


Figure 5. Leveling the beam using level gauge and roller stand

Figure 6. The PASCO displacement sensor setup (pointing upward) for test configurations (1)-(4).

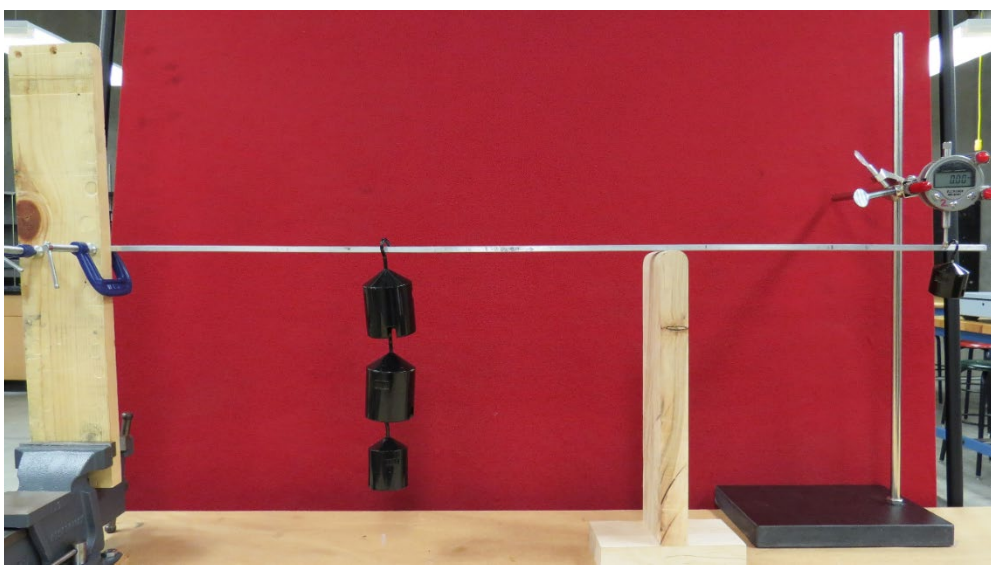


Figure 7. Beam setup for test configuration (5)

Figure 8. The PASCO displacement sensor setup (pointing downward at the same location as the weight is hung) for test configuration (5).

3. After Lab

In After Lab analyses, you will first derive formulas for calculating deflections of the more elaborate test configurations (4) and (5). You will then compare these predictions with experimental measurements obtained in the lab.

<u>Derivation of deflection formula for test configuration (4) (30 points)</u>

By using method of superposition (and the formulas given in Fig. 9) or discontinuity function method, derive the following expression for deflections at 2/3 beam length measured from the left fixed end for the setup shown in Fig. 2:

$$x = \frac{2}{3}L, \;\; y = -\frac{w}{EI}\frac{23}{4374}L^3$$

Hints: Note that Fig. 2 setup is statically indeterminate, so you will need a compatibility condition for the extra unknown. Where is the best location to set up this compatibility condition? Regarding arithmetic, you may want to express most numbers as their prime factorizations.

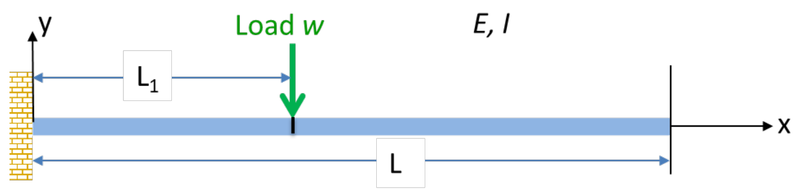
Calculate deflection in mm for 12.8 x 6.4 mm (12.8 mm is the base and 6.4 mm is the height) rectangular cross-section with 1000 grams load. Beam length L=90 cm and E = 68.9 GPa.

Derivation of deflection formula for test configuration (5) (30 points)

Likewise, by using method of superposition (and the formulas given in Fig. 9) or discontinuity function method, derive the following expression for deflection measured at the right free end for the setup shown in Fig. 3:

$$x = L, \;\; y = -\frac{L^3}{EI}\left(\frac{5w_2}{162} - \frac{w_1}{216}\right)$$

Calculate deflection in mm for again 12.8 x 6.4 mm (12.8 mm is the base and 6.4 mm is the height) rectangular cross-section with w_1=2500 grams and w_2=200 grams. L=90 cm and E = 68.9 GPa.



$$0 \le x \le L_1: \; y = -\frac{wx^2}{6EI}(3L_1 - x)$$

$$L_1 \le x \le L: \; y = -\frac{wL_1^2}{6EI}(3x - L_1) \quad \text{Invalid if } L_1 = L$$

$$L_1 = L: \quad y = -\frac{wx^2}{6EI}(3L - x)$$

Figure 9. Deflection formulas for a concentrated load w applied at any point of a cantilever beam

Comparison between model predictions and experimental results (8% each for the five configurations)

Compare your deflection calculations for the five test configurations with the experimental values you obtained in lab. Tabulate the comparisons and comment the accuracy between the calculations and experiments. Discuss the sources and levels of error in the experiments.

4. Report Summary

Work with your group members to write a short summary about 5-10 pages. Include all calculations, In Lab experiment results and the After Lab analysis in terms of graphs, tables, derivations, statements, code listing, etc.