Aerospace Structures Pre-Laboratory
Lab 5 Beam Deflection and Analysis
Section 4 Group 2
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Apply the basic beam theory, which you learned in E M 324 or the like, to directly solve for the deflections for the cantilever beam setup as shown in Fig. 1. Please note that, in the following references, "x grams load" or simply "x grams" should be interpreted as the "gravitational weight/force of x grams mass". Here the quantity of x purely measures the mass itself.

Question 1

(30 pts) Derive a general formula by using either method of superposition or discontinuity function method and show the following expression for deflections at $\frac{1}{3}$ beam length measured from the free end, i.e.,

$$\begin{aligned} x &= \frac{2}{3}L \\ \nu &= -\frac{w}{EI}\frac{14}{81}L^3 \end{aligned} \tag{1}$$

From the *Cantilevered Beam Slopes and Deflections* table on slide 14 of the lecture slides, we are given

$$\nu = -\frac{wx^2}{6EI}(3L - x) \tag{2}$$

As described, we let $x = \frac{2}{3}L$ and simplify Equation 2:

$$\begin{aligned} \nu &= -\frac{w(\frac{2}{3}L)^2}{6EI}\left(3L - \frac{2}{3}L\right) \\ \nu &= -\frac{w\frac{4}{9}L^2}{6EI}\left(\frac{7}{3}L\right) \\ \nu &= -\frac{w\frac{28}{27}L^3}{6EI} \\ \nu &= -\frac{w}{EI}\frac{28}{162}L^3 \\ \nu &= -\frac{w}{EI}\frac{14}{81}L^3 \end{aligned}$$

Question 2

(20 pts) Calculate the deflections in mm *at* $\frac{1}{3}$ *beam length measured from the free end for three beams of rectangular cross section subjected to loads at the free end. Let the beam length be* $90\,\mathrm{cm}$ *and use* $E = 68.9\,\mathrm{GPa}$ *(for Aluminum 6061-T6511 alloy). The beam cross sections and loads are as follows:*

(A) $12.8\,\mathrm{mm} \times 6.4\,\mathrm{mm}$ **(**$12.8\,\mathrm{mm}$ **is the base and** $6.4\,\mathrm{mm}$ **is the height) rectangular cross-section with** $100\,\mathrm{g}$ **load.**

We are given $L = 0.90\,\mathrm{m}$, $E = 68.9 \times 10^9\,\mathrm{Pa}$, $b = 0.0128\,\mathrm{m}$, and $h = 0.0064\,\mathrm{m}$. We can calculate w as shown below

$$\begin{aligned} w &= (100\,\mathrm{g})\left(\frac{1\,\mathrm{kg}}{1000\,\mathrm{g}}\right)\left(9.81\,\frac{\mathrm{m}}{\mathrm{s}^2}\right) \\ w &= 0.9810\,\mathrm{N} \end{aligned}$$

and we can calculate I as follows

$$\begin{aligned} I &= \frac{1}{12}(0.0128\,\mathrm{m})(0.0064\,\mathrm{m})^3 \\ I &= 2.796 \times 10^{-10}\,\mathrm{m}^4 \end{aligned}$$

Substituting these values into Equation 1, we find that

$$\begin{aligned} \nu_A &= -\frac{0.9810\,\mathrm{N}}{(68.9 \times 10^9\,\mathrm{Pa})(2.796 \times 10^{-10}\,\mathrm{m}^4)}\frac{14}{81}(0.90\,\mathrm{m})^3 \\ \nu_A &= -6.416\,\mathrm{mm} \\ &\text{or} \\ \nu_A &= 6.416\,\mathrm{mm}\downarrow \end{aligned}$$

(B) $6.4\,\mathrm{mm} \times 12.8\,\mathrm{mm}$ **rectangular cross-section with** $200\,\mathrm{g}$ **load.**

We are given $L = 0.90\,\mathrm{m}$, $E = 68.9 \times 10^9\,\mathrm{Pa}$, $b = 0.0064\,\mathrm{m}$, and $h = 0.0128\,\mathrm{m}$. We can calculate w as shown below

$$\begin{aligned} w &= (200\,\mathrm{g})\left(\frac{1\,\mathrm{kg}}{1000\,\mathrm{g}}\right)\left(9.81\,\frac{\mathrm{m}}{\mathrm{s}^2}\right) \\ w &= 1.962\,\mathrm{N} \end{aligned}$$

and we can calculate I as follows

$$I = \frac{1}{12}(0.0064\,\text{m})(0.0128\,\text{m})^3$$
$$I = 1.119 \times 10^{-9}\,\text{m}^4$$

Substituting these values into Equation 1, we find that

$$\nu_A = -\frac{1.962\,\text{N}}{(68.9 \times 10^9\,\text{Pa})(1.119 \times 10^{-9}\,\text{m}^4)}\frac{14}{81}(0.90\,\text{m})^3$$
$$\nu_A = -3.208\,\text{mm}$$
or
$$\nu_A = 3.208\,\text{mm}\downarrow$$

(C) $12.8\,\text{mm} \times 12.8\,\text{mm}$ **square cross-section with** $500\,\text{g}$ **load.**

We are given $L = 0.90\,\text{m}$, $E = 68.9 \times 10^9\,\text{Pa}$, $b = 0.0128\,\text{m}$, and $h = 0.0128\,\text{m}$. We can calculate w as shown below

$$w = (500\,\text{g})\left(\frac{1\,\text{kg}}{1000\,\text{g}}\right)\left(9.81\,\frac{\text{m}}{\text{s}^2}\right)$$
$$w = 4.905\,\text{N}$$

and we can calculate I as follows

$$I = \frac{1}{12}(0.0128\,\text{m})(0.0128\,\text{m})^3$$
$$I = 2.237 \times 10^{-9}\,\text{m}^4$$

Substituting these values into Equation 1, we find that

$$\nu_A = -\frac{4.905\,\text{N}}{(68.9 \times 10^9\,\text{Pa})(2.237 \times 10^{-9}\,\text{m}^4)}\frac{14}{81}(0.90\,\text{m})^3$$
$$\nu_A = -4.010\,\text{mm}$$
or
$$\nu_A = 4.010\,\text{mm}\downarrow$$