# Iowa State University

## Aerospace Engineering

# AER E 322 Lab 5
# Beam Deflection and Analysis

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**Lab 5 Beam Deflection and Analysis**
Section 4 Group 2
Matthew Mehrtens, Peter Mikolitis, and Natsuki Oda


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# 4. Work Assignments

Refer to Table 1 for the distribution of work during this lab.

Table 1: Work assignments for AER E 322 Lab <lab number>.

| **Task** | **Matthew** | **Peter** | **Natsuki** |
|---|---|---|---|
| *Lab Work* | | | |
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# 5. Materials

# 6. Apparatus

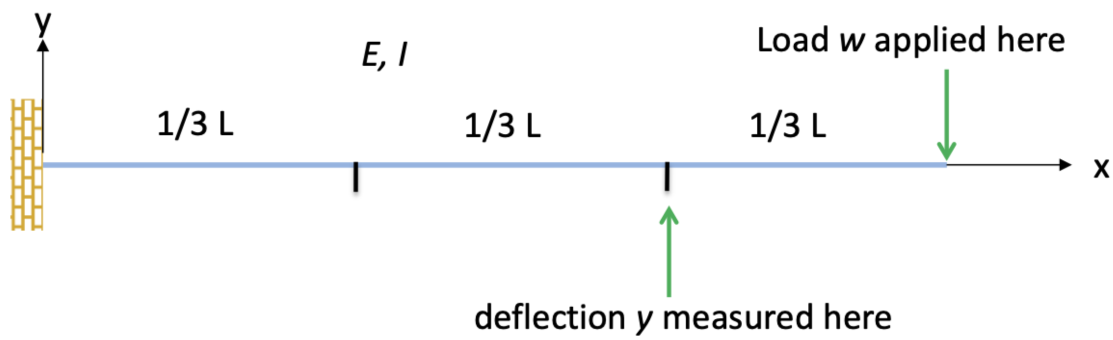


Figure 1: The beam configuration for experiments 1–3.

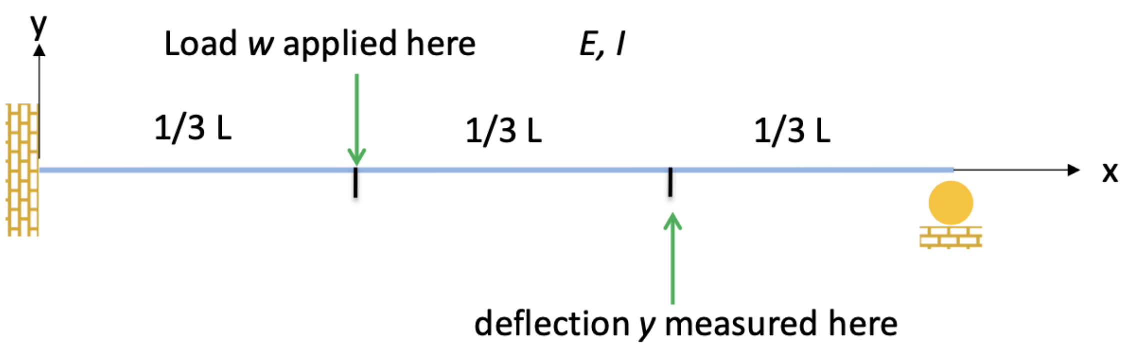


Figure 2: The beam configuration for experiment 4.

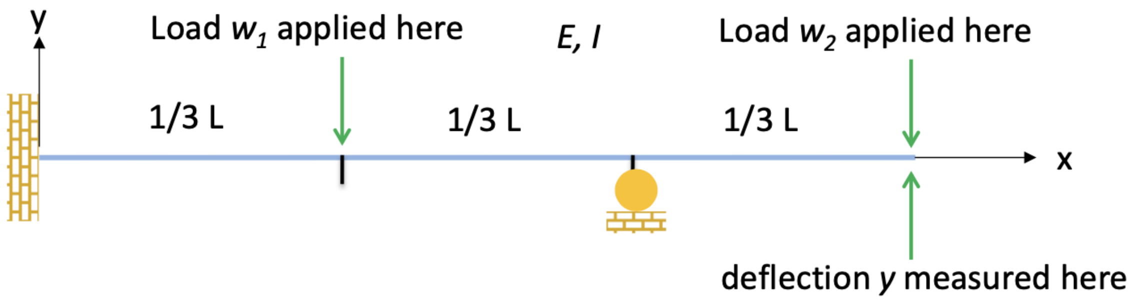


Figure 3: The beam configuration for experiment 5.

# 7. Procedures

# 8. Data

# 9. Analysis

To derive the following deflection expressions, we are given the following equations of superposition for a cantilever beam:

$$0 \leq x \leq L_1 : y = -\frac{wx^2}{6EI}(3L_1 - x) \tag{1}$$

$$L_1 \leq x \leq L : y = -\frac{wL_1^2}{6EI}(3x - L_1) \tag{2}$$

$$L_1 = L : y = -\frac{wx^2}{6EI}(3L - x) \tag{3}$$

where $L$ is the length of the beam, $E$ is the elastic modulus of the beam material, $I$ is the beam moment of inertia, $w$ is the applied load, $L_1$ is the distance from the fixed end of the beam to the applied load, and $x$ is the point about which deflection is measured. Note that Equation 2 is invalid for $L_1 = L$.

The configuration for test four, shown in 2, consists of a beam fixed at one end with a roller support on the other end. A load is being applied at $\frac{1}{3}$ the length, measured from the fixed end. To calculate the deflection at $\frac{2}{3}$ the length, *i.e.*, $x = \frac{2}{3}L$, we are given the following equation:

$$\nu = -\frac{w}{EI}\frac{23}{4374}L^3 \tag{4}$$

where $w$ is the applied load, $E$ is the elastic modulus, $I$ is the moment of inertia, $L$ is the length of the beam, and $\nu$ is the deflection of the beam at $x = \frac{2}{3}L$.

To derive this expression, we first note that the beam configuration is indeterminate. The roller support is redundant and can be replaced by a vertical applied load of magnitude, $R_y$. The beam is now effectively a cantilevered beam with two applied loads. Deflection at a point $x$ can be calculated by summing the deflection at $x$ from each of the applied loads.

The deflection due to the applied load at $x = \frac{1}{3}L$, $\nu_w$, can be derived from Equation 2.

$$\begin{aligned}\nu_w(x) &= -\frac{wL_1^2}{6EI}(3x - L_1)\\ &= -\frac{w\frac{1}{9}L^2}{6EI}(3x - \frac{1}{3}L)\\ &= -\frac{wL^2}{162EI}(9x - L)\end{aligned}$$

This expression is true for all $x \in [L_1, L]$.

The deflection to the redundant reaction force at $x = L$, $\nu_{R_y}$, can be derived from Equation 3.

$$\begin{aligned}\nu_{R_y}(x) &= -\frac{wx^2}{6EI}(3L - x)\\ &= -\frac{R_y x^2}{6EI}(3L - x)\end{aligned}$$

This expression is true for all $x \in [0, L]$.

To calculate the deflection at $x = \frac{2}{3}L$, we first need to determine the value of $R_y$. We know from Figure 2, at $x = L$, there is a roller support, and therefore, $\nu(x) = 0$. We apply this boundary condition below to find $R_y$.

$$\begin{aligned}
\nu(L) &= \nu_w(L) + \nu_{R_y}(L) = 0 \\
0 &= -\frac{wL^2}{162EI}(9L - L) - \frac{R_y L^2}{6EI}(3L - L) \\
0 &= -\frac{4wL^3}{81EI} - \frac{R_y L^3}{3EI} \\
R_y &= -\frac{4w}{27}
\end{aligned}$$

Substituting in the derived expression for $R_y$ into the earlier equation for $\nu_{R_y}$, we find that

$$\nu_{R_y} = \frac{2wx^2}{81EI}(3L - x)$$

Now that we have expressions for both $\nu_w$ and $\nu_{R_y}$ in terms of $w$, we can find the derive an equation for the deflection of the beam at $x = \frac{2}{3}L$ in terms of $w$, $E$, $I$, and $L$.

$$\begin{aligned}
\nu\left(\frac{2}{3}L\right) &= \nu_w\left(\frac{2}{3}L\right) + \nu_{R_y}\left(\frac{2}{3}L\right) \\
&= -\frac{wL^2}{162EI}\left(9\left(\frac{2}{3}L\right) - L\right) + \frac{2w\left(\frac{2}{3}L\right)^2}{81EI}\left(3L - \left(\frac{2}{3}L\right)\right) \\
&= -\frac{5wL^3}{162EI} + \frac{56wL^3}{2187EI} \\
&= \frac{wL^3}{EI}\left(-\frac{5}{162} + \frac{56}{2187}\right) \\
&= -\frac{23wL^3}{4374EI}
\end{aligned}$$

As expected, this equation matches the given expression in Equation 4. Calculating the theoretical deflection is then trivial. We know $w = (1000\,\mathrm{g})(\frac{1\,\mathrm{kg}}{1000\,\mathrm{g}})(9.81\,\frac{\mathrm{m}}{\mathrm{s}^2}) = 9.81\,\mathrm{N}$, $L = 0.90\,\mathrm{m}$, $E = 68.9 \times 10^9\,\mathrm{Pa}$, and $I = \frac{1}{12}(12.8\,\mathrm{mm})(6.4\,\mathrm{mm})^3(\frac{1\,\mathrm{m}}{1000\,\mathrm{mm}})^4 = 2.796 \times 10^{-10}\,\mathrm{m}^4$. Plugging these values into Equation 4, we find that

$$\begin{aligned}
\nu\left(\frac{2}{3}L\right) &= -\frac{23(9.81\,\mathrm{N})(0.90\,\mathrm{m})^3}{4374(68.9 \times 10^9\,\mathrm{Pa})(2.796 \times 10^{-10}\,\mathrm{m}^4)} \\
&= -1.952\,\mathrm{mm} \text{ or } 1.952\,\mathrm{mm} \downarrow
\end{aligned}$$

# 10. Conclusion