Iowa State University

Aerospace Engineering

AER E 322 Lab 5
Beam Deflection and Analysis

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Aerospace Structures Laboratory Summary
Lab 5 Beam Deflection and Analysis
Section 4 Group 2
Matthew Mehrtens, Peter Mikolitis, and Natsuki Oda
AER E 322 March 16, 2023 **Spring 2023**

1. Introduction

2. Objectives

3. Hypothesis

4. Work Assignments

Refer to Table 1 for the distribution of work during this lab.

Table 1: Work assignments for AER E 322 Lab <lab number>.

Task	**Matthew**	**Peter**	**Natsuki**
Lab Work			
Date Recording			
Exp. Setup			
Exp. Work			
Exp. Clean-Up			
Post Lab			
Report			
Introduction			
Objectives			
Hypothesis			
Materials			
Apparatus			
Procedures			
Data			
Analysis			
Conclusion			
Editing			

5. Materials

6. Apparatus

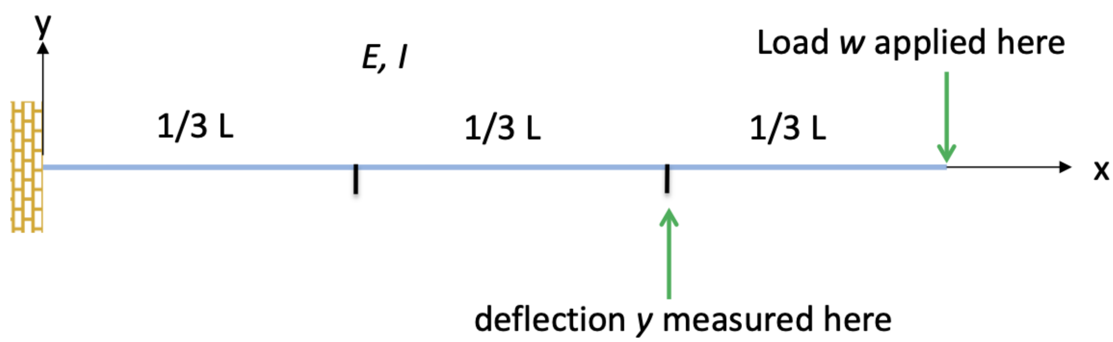

Figure 1: The beam configuration for experiments 1–3.

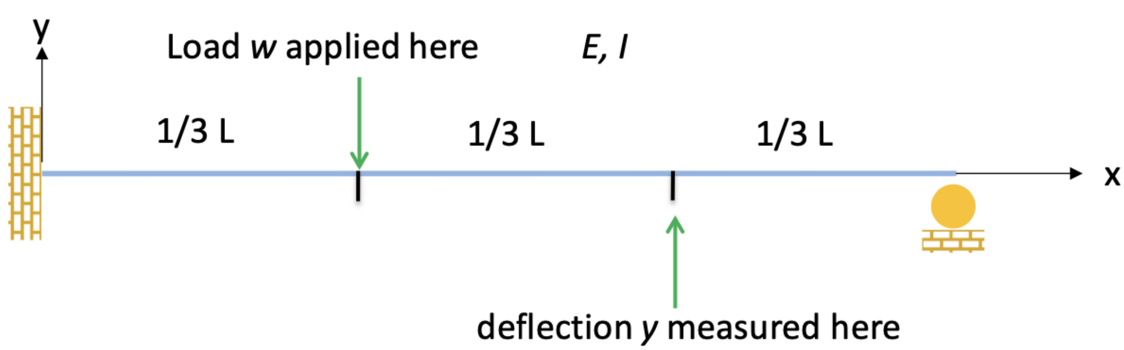

Figure 2: The beam configuration for experiment 4.

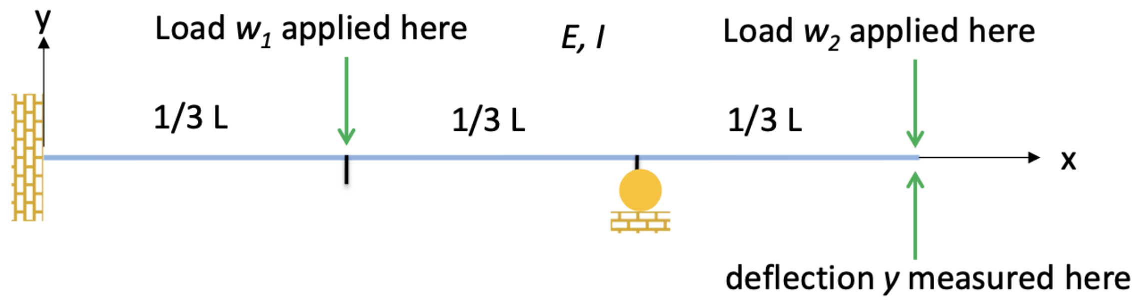

Figure 3: The beam configuration for experiment 5.

7. Procedures

8. Data

9. Analysis

To derive the following deflection expressions, we are given the following equations of superposition for a cantilever beam:

$$0 \leq x \leq L_1 : y = -\frac{wx^2}{6EI}(3L_1 - x) \tag{1}$$

$$L_1 \leq x \leq L : y = -\frac{wL_1^2}{6EI}(3x - L_1) \tag{2}$$

$$L_1 = L : y = -\frac{wx^2}{6EI}(3L - x) \tag{3}$$

where L is the length of the beam, E is the elastic modulus of the beam material, I is the beam moment of inertia, w is the applied load, L_1 is the distance from the fixed end of the beam to the applied load, and x is the point about which deflection is measured. Note that Equation 2 is invalid for $L_1 = L$.

The configuration for test four, shown in Figure 2, consists of a beam fixed at one end with a roller support on the other end. A load is being applied at $\frac{1}{3}$ the length, measured from the fixed end. To calculate the deflection at $\frac{2}{3}$ the length, *i.e.*, $x = \frac{2}{3}L$, we are given the following equation:

$$\nu = -\frac{w}{EI}\frac{23}{4374}L^3 \tag{4}$$

where w is the applied load, E is the elastic modulus, I is the moment of inertia, L is the length of the beam, and ν is the deflection of the beam at $x = \frac{2}{3}L$.

To derive this expression, we first note that the beam configuration is indeterminate. The roller support is redundant and can be replaced by a vertical applied load of magnitude, R_y. The beam is now effectively a cantilevered beam with two applied loads. Deflection at a point x can be calculated by summing the deflection at x from each of the applied loads.

The deflection due to the applied load at $x = \frac{1}{3}L$, ν_w, can be derived from Equation 2.

$$\begin{aligned}\nu_w(x) &= -\frac{wL_1^2}{6EI}(3x - L_1) \\ &= -\frac{w\frac{1}{9}L^2}{6EI}(3x - \frac{1}{3}L) \\ &= -\frac{wL^2}{162EI}(9x - L)\end{aligned}$$

This expression is true for all $x \in [\frac{1}{3}L, L]$.

The deflection due to the redundant reaction force at $x = L$, ν_{R_y}, can be derived from Equation 3.

$$\begin{aligned}\nu_{R_y}(x) &= -\frac{wx^2}{6EI}(3L - x) \\ &= -\frac{R_y x^2}{6EI}(3L - x)\end{aligned}$$

This expression is true for all $x \in [0, L]$.

To calculate the deflection at $x = \frac{2}{3}L$, we first need to determine the value of R_y. We know from Figure 2, at $x = L$ there is a roller support, and therefore, $\nu(x) = 0$ at $x = L$. We apply this boundary condition below to find R_y.

$$\begin{aligned}
\nu(L) &= \nu_w(L) + \nu_{R_y}(L) = 0 \\
0 &= -\frac{wL^2}{162EI}(9L - L) - \frac{R_yL^2}{6EI}(3L - L) \\
0 &= -\frac{4wL^3}{81EI} - \frac{R_yL^3}{3EI} \\
R_y &= -\frac{4w}{27}
\end{aligned}$$

Substituting in the derived expression for R_y into the earlier equation for ν_{R_y}, we find that

$$\nu_{R_y}(x) = \frac{2wx^2}{81EI}(3L - x)$$

Now that we have expressions for both ν_w and ν_{R_y} in terms of w, we can find the derive an equation for the deflection of the beam at $x = \frac{2}{3}L$ in terms of w, E, I, and L.

$$\begin{aligned}
\nu\left(\frac{2}{3}L\right) &= \nu_w\left(\frac{2}{3}L\right) + \nu_{R_y}\left(\frac{2}{3}L\right) \\
&= -\frac{wL^2}{162EI}\left(9\left(\frac{2}{3}L\right) - L\right) + \frac{2w\left(\frac{2}{3}L\right)^2}{81EI}\left(3L - \left(\frac{2}{3}L\right)\right) \\
&= -\frac{5wL^3}{162EI} + \frac{56wL^3}{2187EI} \\
&= \frac{wL^3}{EI}\left(-\frac{5}{162} + \frac{56}{2187}\right) \\
&= -\frac{23wL^3}{4374EI}
\end{aligned}$$

As expected, this equation matches the given expression in Equation 4. Calculating the theoretical deflection is then trivial. We know $w = (1000\,\mathrm{g})(\frac{1\,\mathrm{kg}}{1000\,\mathrm{g}})(9.81\,\frac{\mathrm{m}}{\mathrm{s}^2}) = 9.81\,\mathrm{N}$, $L = 0.90\,\mathrm{m}$, $E = 68.9 \times 10^9\,\mathrm{Pa}$, and $I = \frac{1}{12}(12.8\,\mathrm{mm})(6.4\,\mathrm{mm})^3(\frac{1\,\mathrm{m}}{1000\,\mathrm{mm}})^4 = 2.796 \times 10^{-10}\,\mathrm{m}^4$. Plugging these values into Equation 4, we find that

$$\begin{aligned}
\nu\left(\frac{2}{3}L\right) &= -\frac{23(9.81\,\mathrm{N})(0.90\,\mathrm{m})^3}{4374(68.9 \times 10^9\,\mathrm{Pa})(2.796 \times 10^{-10}\,\mathrm{m}^4)} \\
&= -1.952\,\mathrm{mm} \text{ or } 1.952\,\mathrm{mm} \downarrow
\end{aligned}$$

The configuration for test five, shown in Figure 3, consists of a beam fixed at one end with a roller support at $\frac{2}{3}$ the length. A load is being applied at $\frac{1}{3}$ the length and at the end, measured from the fixed end. To calculate the deflection at the free end of the beam, *i.e.*, $x = L$, we are given the following equation:

$$\nu = -\frac{L^3}{EI}\left(\frac{5w_2}{162} - \frac{w_1}{216}\right) \tag{5}$$

where w_1 is the applied load at $x = \frac{1}{3}L$, w_2 is the applied load at $x = L$, E is the elastic modulus, I is the moment of inertia, L is the length of the beam, and ν is the deflection of the beam at $x = L$.

To derive this expression, we first note that the beam configuration is indeterminate. The roller support is redundant and can be replaced by a vertical applied load of magnitude, R_y. The beam is now effectively a cantilevered beam with three applied loads. Deflection at a point x can be calculated by summing the deflection at x from each of the applied loads.

The deflection due to the applied load at $x = \frac{1}{3}$, ν_{w_1}, can be derived from Equation 2.

$$\begin{aligned}\nu_{w_1}(x) &= -\frac{wL_1^2}{6EI}(3x - L_1)\\ &= -\frac{w_1\frac{1}{9}L^2}{6EI}(3x - \frac{1}{3}L)\\ &= \frac{w_1L^2}{162EI}(9x - L)\end{aligned}$$

This expression is true for all $x \in [\frac{1}{3}L, L]$.

The deflection due to the redundant reaction force at $x = \frac{2}{3}L$, ν_{R_y}, can be derived from Equation 2.

$$\begin{aligned}\nu_{R_y}(x) &= -\frac{wL_1^2}{6EI}(3x - L_1)\\ &= -\frac{R_y\frac{4}{9}L^2}{6EI}(3x - \frac{2}{3}L)\\ &= -\frac{2R_yL^2}{81EI}(9x - 2L)\end{aligned}$$

This expression is true for all $x \in [\frac{2}{3}L, L]$.

The deflection due to the applied load at $x = L$, ν_{w_2}, can be derived from Equation 3.

$$\begin{aligned}\nu_{w_2}(x) &= -\frac{wx^2}{6EI}(3L - x)\\ &= -\frac{w_2{}^2}{x}6EI(3L - x)\end{aligned}$$

This expression is for all $x \in [0, L]$.

To calculate the deflection at $x = L$, we first need to determine the value of R_y. We know from Figure 3), at $x = \frac{2}{3}L$ there is a roller support, and therefore, $\nu(x) = 0$ at $x = \frac{2}{3}L$. We apply this boundary condition below to find R_y.

$$\begin{aligned}\nu(\frac{2}{3}L &= \nu_{w_1}(\frac{2}{3}L) + \nu_{R_y}(\frac{2}{3}L) + \nu_{w_2}(\frac{2}{3}L) = 0\\ 0 &= -\frac{w_1L^2}{162EI}(9(\frac{2}{3}L) - L) - \frac{2R_yL^2}{81EI}(9(\frac{2}{3}L) - 2L) - \frac{w_2\frac{4}{9}L^2}{6EI}(3L - (\frac{2}{3}L)\\ 0 &= -\frac{5w_1}{162} - \frac{8R_y}{81} - \frac{14w_2}{81}\\ R_y &= -\frac{5}{16}(w_1 + \frac{28w_2}{5})\end{aligned}$$

Now that we have expressions for the deflection due to all the forces, we can derive an equation for the deflection of the beam at $x = L$ using the method of superposition.

$$\begin{aligned}\nu(L) &= \nu_{w_1}(L) + \nu_{R_y}(L) + \nu_{w_2}(L)\\ &= -\frac{w_1L^2}{162EI}(9L - L) - \frac{2\left(-\frac{5}{16}\left[w_1 + \frac{28w_2}{5}\right]\right)L^2}{81EI}(9L - 2L) - \frac{w_2L^2}{6EI}(3L - L)\\ &= -\frac{4w_1L^3}{81EI} + \frac{35\left(w_1 + \frac{28w_2}{5}\right)L^3}{648EI} - \frac{w_2L^3}{3EI}\\ &= -\frac{L^3}{EI}\left[\frac{4w_1}{81} - \frac{35w_1}{648} - \frac{49w_2}{162} + \frac{w_2}{3}\right]\\ &= -\frac{L^3}{EI}\left(\frac{5w_2}{162} - \frac{w_1}{216}\right)\end{aligned}$$

As expected, this equation matches the given expression in Equation 5. Calculating the theoretical deflection is then trivial. We know $w_1 = (2500\,\mathrm{g})(\frac{1\,\mathrm{kg}}{1000\,\mathrm{g}})(9.81\,\frac{\mathrm{m}}{\mathrm{s}^2}) = 24.53\,\mathrm{N}$,

$w_2 = (200\,\mathrm{g})(\frac{1\,\mathrm{kg}}{1000\,\mathrm{g}})(9.81\,\frac{\mathrm{m}}{\mathrm{s}^2}) = 19.62\,\mathrm{N}$, $L = 0.90\,\mathrm{m}$, $E = 68.9 \times 10^9\,\mathrm{Pa}$, and $I = \frac{1}{12}(12.8\,\mathrm{mm}) \cdot (6.4\,\mathrm{mm})^3(\frac{1\,\mathrm{m}}{1000\,\mathrm{mm}})^4 = 2.796 \times 10^{-10}\,\mathrm{m}^4$. Plugging these values into Equation 5, we find that

$$\begin{aligned}\nu(L) &= -\frac{0.90\,\mathrm{m}}{(68.9 \times 10^9\,\mathrm{Pa})(2.796 \times 10^{-10}\,\mathrm{m}^4)}\left(\frac{5(19.62\,\mathrm{N})}{162} - \frac{24.53\,\mathrm{N}}{216}\right)\\ &= 2.005\,\mathrm{mm} \text{ or } 2.005\,\mathrm{mm}\uparrow\end{aligned}$$

10. Conclusion