Iowa State University

Aerospace Engineering

AER E 322 Lab 6
Composite Laminate Design

Matthew Mehrtens, Peter Mikolitis, and Natsuki Oda

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1. Introduction

In this lab, we aimed to design a composite layup pattern with the maximum resistance to bending by utilizing the lecture notes and the online composite calculator. Firstly, we studied the lecture notes carefully to gain a better understanding of composite materials and their properties. Next, we experimented with the online composite calculator to design a layup pattern that would maximize the resistance to bending. Through a series of five or more trials, we arrived at what we believe is the ultimate value of κ_{xy}, the twisting curvature.

2. Objectives

The objectives of this lab experiment were to:

1. Understand the concepts and theory presented in the week seven and eight lectures regarding composite materials.
2. Analyze how the twisting curvature, κ_{xy}, changes based on alterations to the layup pattern.
3. Utilize an online composite calculator to experimentally determine the maximum value of κ_{xy}. The calculator was used to simulate different layup patterns and determine the maximum twisting curvature values that could be achieved under given material properties.
4. Find the maximum value of κ_{xy} given the default online calculator configuration. By using the online composite calculator and experimentally determining the twisting curvature values for various layup patterns, we aimed to identify the maximum twisting curvature value achievable for a given set of material properties.

3. Hypothesis

We predict that a layup design with 2 to 3 layers in between 30° and 60° will result in a maximum absolute twisting curvature. We suspect layup angles at 0° and 90° will have at best no effect and at worst a negative effect on the twisting curvature.

4. Work Assignments

Refer to Table 1 for the distribution of work during this lab.

Table 1: Work assignments for AER E 322 Lab 6.

Task	**Matthew**	**Peter**	**Natsuki**
Lab Work			
Date Recording	X		
Exp. Setup	X	X	X
Exp. Work	X	X	X
Report			
Introduction	X		X
Objectives	X	X	
Hypothesis			X
Materials		X	
Apparatus		X	
Procedures			X
Data	X		
Analysis	X	X	X
Conclusion	X		
Editing	X		

5. Materials

- Week seven and eight lecture notes
- Online composite calculator

6. Apparatus

The simulated beam in this lab was a cantilevered, metal 6 inch $\times$ 1 inch beam. A simulated torsional bending moment was applied to the free end of the beam.

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7. Procedures

First, we reviewed the week seven and eight lecture notes individually and as a group to learn the theory of laminate composites. After loading the calculator, we set the following parameters:

$$t_{\text{layer}} = 150\,\mu\text{m}$$
$$\sigma_x = 0$$
$$M_x = 1\,\text{N m}$$

After these values were set, we experimented with different layup configurations to observe how each configuration effected the twisting curvature, κ_{xy}.

8. Data

The value of κ_{xy} from our five designs in shown in Table 2.

Table 2: Values of κ_{xy} with respect to each of the different layup configurations.

Design	Layup Pattern	M_x [N m]	$\lvert\kappa_{xy}\rvert$ [m^{-1}]
1	0/90	1	1.37×10^{-17}
2	0/30/60/90	1	0.0761
3	0/45/90	1	0.0864
4	±45	1	28.3
5	±27	1	46.6

9. Analysis

Laminate Analysis

We are given in the lecture notes that

$$\mathbf{B} = \frac{1}{2}\sum_{i=1}^{K}\overline{\mathbf{S}}_i(z_{i+1}^2 - z_i^2) \tag{1}$$

where $\mathbf{B}$ is the coupling stiffness matrix, K is the number of laminate layers, $\overline{\mathbf{S}}_i$ is the stiffness matrix for the ith layer, and z is the distance from the midplane to the surface of a layer. Positive z is above the midplane and negative z is below the midplane. The top layer of the composite ($i = 1$) is bounded above by z_1 and below by z_2, the second layer from the top ($i = 2$) is bounded above by z_2 and below by z_3, and the bottom layer of the composite ($i = K$) is bounded above by z_K and below by z_{K+1}. In general, the ith layer is bounded above by z_i and below by z_{i+1}. We must show that for a symmetrical laminate layup, $\mathbf{B}$ is zero. In order for $\mathbf{B}$ to be zero, the summation from Equation 1

$$\sum_{i=1}^{K} \overline{\mathbf{S}}_i (z_{i+1}^2 - z_i^2)$$

must be zero. We begin the proof by expanding the summation as shown below.

$$\begin{aligned}\sum_{i=1}^{K} \overline{\mathbf{S}}_i (z_{i+1}^2 - z_i^2) &= \overline{\mathbf{S}}_1 (z_2^2 - z_1^2) + \overline{\mathbf{S}}_2 (z_3^2 - z_2^2) + \cdots + \overline{\mathbf{S}}_{K-1} (z_K^2 - z_{K-1}^2) + \overline{\mathbf{S}}_K (z_{K+1}^2 - z_K^2) \\ &= \overline{\mathbf{S}}_1 z_2^2 - \overline{\mathbf{S}}_1 z_1^2 + \overline{\mathbf{S}}_2 z_3^2 - \overline{\mathbf{S}}_2 z_2^2 + \cdots \\ &\quad + \overline{\mathbf{S}}_{K-1} z_K^2 - \overline{\mathbf{S}}_{K-1} z_{K-1}^2 + \overline{\mathbf{S}}_K z_{K+1}^2 - \overline{\mathbf{S}}_K z_K^2\end{aligned}$$

Since the composite is symmetrical, we assume $\overline{\mathbf{S}}_1 = \overline{\mathbf{S}}_K$, $\overline{\mathbf{S}}_2 = \overline{\mathbf{S}}_{K-1}, \cdots$. Additionally, due to the symmetry and according to our definition of z, we know that $z_1 = -z_{K+1}$, $z_2 = -z_K, \cdots$. It follows that $z_1^2 = z_{K+1}^2$, $z_2^2 = z_K^2, \cdots$. We then come up with the following equivalencies:

$$\begin{aligned}\overline{\mathbf{S}}_1 z_1^2 &= \overline{\mathbf{S}}_K z_{K+1}^2 \\ \overline{\mathbf{S}}_1 z_2^2 &= \overline{\mathbf{S}}_K z_K^2 \\ \overline{\mathbf{S}}_2 z_2^2 &= \overline{\mathbf{S}}_{K-1} z_K^2 \\ \overline{\mathbf{S}}_2 z_3^2 &= \overline{\mathbf{S}}_{K-1} z_{K-1}^2\end{aligned}$$

We now continue the expansion of the summation below substituting the $\overline{\mathbf{S}}_K$ and $\overline{\mathbf{S}}_{K-1}$ terms with their equivalent $\overline{\mathbf{S}}_1$ and $\overline{\mathbf{S}}_2$ terms:

$$\begin{aligned}\sum_{i=1}^{K} \overline{\mathbf{S}}_i (z_{i+1}^2 - z_i^2) &= \overline{\mathbf{S}}_1 z_2^2 - \overline{\mathbf{S}}_1 z_1^2 + \overline{\mathbf{S}}_2 z_3^2 - \overline{\mathbf{S}}_2 z_2^2 + \cdots + \overline{\mathbf{S}}_2 z_2^2 - \overline{\mathbf{S}}_2 z_3^2 + \overline{\mathbf{S}}_1 z_1^2 - \overline{\mathbf{S}}_1 z_2^2 \\ &= 0\end{aligned}$$

As shown above, if the composite is symmetric, all the terms in the summation will cancel out. Therefore, in a symmetric composite, the coupling stiffness matrix, $\mathbf{B}$, is zero.

For a composite material, we are given the following matrix equation, shown in Equation 2, that relates the external forces, N and M, to the internal stresses and strains, ε^0 and κ.

$$\begin{bmatrix} \mathbf{N} \\ \mathbf{M} \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{B} & \mathbf{D} \end{bmatrix} \begin{bmatrix} \boldsymbol{\varepsilon}^0 \\ \boldsymbol{\kappa} \end{bmatrix} \tag{2}$$

The **A-B-B-D** matrix is called the laminate stiffness matrix. It is a 6×6 matrix—each element itself is a 3×3 matrix. $\boldsymbol{\varepsilon}^0$ and $\boldsymbol{\kappa}$ can be further broken down as shown below:

$$\begin{bmatrix} \boldsymbol{\varepsilon}^0 \\ \boldsymbol{\kappa} \end{bmatrix} = \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix}$$

Furthermore, the **A-B-B-D** matrix can be inverted to get Equation 3:

$$\begin{bmatrix} \boldsymbol{\varepsilon}^0 \\ \boldsymbol{\kappa} \end{bmatrix} = \begin{bmatrix} \mathbf{a} & \mathbf{b} \\ \mathbf{c} & \mathbf{d} \end{bmatrix} \begin{bmatrix} \mathbf{N} \\ \mathbf{M} \end{bmatrix} \tag{3}$$

where $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$, and $\mathbf{d}$ are defined below:

$$\begin{aligned} \mathbf{a} &= \mathbf{A}^{-1} + \mathbf{A}^{-1}\mathbf{B}(\mathbf{D}^*)^{-1}\mathbf{B}\mathbf{A}^{-1} \\ \mathbf{b} &= -\mathbf{A}^{-1}\mathbf{B}(\mathbf{D}^*)^{-1} \\ \mathbf{c} &= \mathbf{B}^T \\ \mathbf{d} &= (\mathbf{D}^*)^{-1} \\ \mathbf{D}^* &= \mathbf{D} - \mathbf{B}\mathbf{A}^{-1}\mathbf{B} \end{aligned}$$

For symmetric laminates, $\mathbf{a} = \mathbf{A}^{-1}$, $\mathbf{B} = \mathbf{b} = \mathbf{c} = 0$, and $\mathbf{d} = \mathbf{D}^{-1}$. By examining Equation 2, it is clear that $\mathbf{A}$ is not related to the twisting curvature, κ_{xy}. Only $\mathbf{B}$ and $\mathbf{D}$ are related to the twisting curvature. But since $\mathbf{B} = 0$, only $\mathbf{D}$ affects κ_{xy}, and because $\mathbf{d} = \mathbf{D}^{-1}$, $\mathbf{d}$ also effects κ_{xy}.

Therefore, of the variables $\mathbf{A}$, $\mathbf{B}$, $\mathbf{D}$, $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$, and $\mathbf{d}$, only $\mathbf{D}$ and $\mathbf{d}$ have any effect on the twisting curvature, κ_{xy} in a symmetrical composite.

Design Process In the design process, the goal was to find a layup design that is maximally resistant to torsional bending, *i.e.*, maximize $|\kappa_{xy}|$.

1. For our first design, we chose a very natural $[0/90]_\mathrm{s}$ design which resulted in $\kappa_{xy} = 1.37 \times 10^{-17}\,\mathrm{m}^{-1}$. Based on our hypothesis, we assumed correctly this would be a very bad design, but it provided a good starting point from which to make iterations upon our design. It was clear the lack of layers oriented between 0° and 90° was causing our design to have hardly any resistance to torsion.

2. For our second design, we chose $[0/30/60/90]_\mathrm{s}$ which resulted in $\kappa_{xy} = -0.0761\,\mathrm{m}^{-1}$. This is a significant improvement over the first design, but is still far from optimal. We chose this design to determine whether or not the addition of more layers between 0° and 90° would increase the resistance to torsion. It did increase κ_{xy}, but it was unclear whether or not the increase was due to the addition of more layers or the addition to layers in between 0° and 90°.

3. For our third design, we chose $[0/45/90]_\mathrm{s}$ which resulted in $\kappa_{xy} = -0.0864\,\mathrm{m}^{-1}$. Our reasoning for this was to see if reducing the number of composite layers increased the resistance to bending. The result was that, yes, reducing the number of composite layers did increase the resistance to torsional bending, but only nominally. Based on our hypothesis, we expected the value of $kappa_{xy}$ to become smaller relative to the last design, but our hypothesis was proven wrong. At this point in the design process, we are led to conclude that less composite layers is better for increasing the resistance to torsional bending.

4. For our fourth design, we chose $[45]_\mathrm{s}$ which resulted in $\kappa_{xy} = -28.3\,\mathrm{m}^{-1}$. We chose this design since we knew that $[0/90]_\mathrm{s}$ was very bad at reducing torsional bending effects and because we concluded that less composite layers were better at increasing torsional resistance. This partially confirmed our hypothesis that having layup angles in between 0° and 90° would increase resistance to torsional bending. We also noted that a layup design of $[\pm 45]_\mathrm{s}$ resulted in the same magnitude of κ_{xy} just with opposite signs.

5. For our fifth design, we chose $[27]_\mathrm{s}$ which resulted in $\kappa_{xy} = -46.6\,\mathrm{m}^{-1}$. We came to this design by first increasing the previous design angle by 1°. When we noted that this decreased the magnitude of κ_{xy}, we tried decreasing the layup angle which resulted in the magnitude of κ_{xy} increasing. We decreased the layup angle in steps of 1° degree until the magnitude of κ_{xy} started to decrease again. This led us to the optimal value of κ_{xy} being $\pm 46.6\,\mathrm{m}^{-1}$ based on a layup design of $[\pm 27]_\mathrm{s}$. This somewhat counter-intuitive

result contradicts the majority of our hypothesis. We suspect this particular result is also very dependent on the geometric properties of the beam in question.

10. Conclusion

Our hypothesis was partially correct. We accurately predicted that layup angles of 0° and 90° had a worse affect on κ_{xy}. We did not predict, however, that the largest κ_{xy} would occur with only two layers. We were correct that maximum angle would be between 0° and 90°, but it was slightly below our expected range of 30° to 60° at 27°.

In general we learned that the layup angle has a significant affect on the material's resistance to torsion. Additionally, we learned that adding more layers does not necessarily increase the resistance to torsion or any other desired material property.