Aerospace Structures Pre-Laboratory
Lab 7 Column Buckling
Section 4 Group 2
Matthew Mehrtens
AER E 322 March 22, 2023 **Spring 2023**

Question 1

Review Week 9 lecture and corresponding reference book materials (Peery is available online from ISU library).

Question 2

(15 points) For the end condition of one free and one fixed (lecture note page 12), why is the effective length twice as long as the actual length? Could you come up with a simple explanation? Hint: think of "mirror"...

The effective length has to be measured from two points on the same vertical axis. The first point is at the free-end, distance L above the ground, and if you imagine the beam is mirrored through the ground, the other point would be at a distance L *beneath* the ground. Therefore, the effective length of the beam is $L + L = 2L$.

Question 3

(25 pts) Derive the formulas for critical load P and slenderness ratio $\frac{L}{\rho}$ of a circular rod and a rectangular bar subjected to axial loading, in terms of π, length L, modulus of elasticity E and specimen radius R (for circular rod) or cross-sectional dimensions B and/or H (for rectangular bar).

We know that

$$P_{cr} = \frac{\pi^2 EI}{L^2} \tag{1}$$
$$\frac{L}{\rho} = L\sqrt{\frac{A}{I}} \tag{2}$$

For the circular rod, we know that the moment of inertia is

$$I = \frac{\pi}{64}(2R)^4$$

Substituting I into Equations 1 and 2, we derive the following equations for P_{cr} and $\frac{L}{\rho}$:

$$P_{cr} = \frac{\pi^2 E \frac{\pi}{64}(2R)^4}{L^2} = \frac{\pi^3 E R^4}{4L^2} \tag{3}$$

$$\frac{L}{\rho} = L\sqrt{\frac{\pi R^2}{\frac{\pi}{64}(2R)^4}} = \frac{2L}{R} \tag{4}$$

For the beam, we know the moment of inertia is defined as

$$I = \frac{1}{12}BH^3$$

Substituting I into Equations 1 and 2, we derive the following equations for P_{cr} and $\frac{L}{\rho}$:

$$P_{cr} = \frac{\pi^2 E \frac{1}{12} B H^3}{L^2} = \frac{\pi^2 E B H^3}{12L^2} \tag{5}$$

$$\frac{L}{\rho} = L\sqrt{\frac{BH}{\frac{1}{12}BH^3}} = \frac{2L}{H}\sqrt{3} \tag{6}$$

Question 4

(30 pts) Use the formulas from question three to calculate the Ps and $\frac{L}{\rho}$s for metal specimens made of stainless 304 annealed cold finish steel (elastic modulus $E = 29\,000\,\mathrm{ksi}$ and yield strength $\sigma^Y = 35\,\mathrm{ksi}$) and 6061-T6 aluminum ($E = 10\,000\,\mathrm{ksi}$ and $\sigma^Y = 40\,\mathrm{ksi}$) with the sizes and end conditions given in Table 1. What equivalent lengths will you use for the pivot-pivot and pivot-fixed end conditions? For the $0.25\,\mathrm{in} \times 1\,\mathrm{in}$ aluminum specimen, which dimension do you choose to calculate the slenderness ratio? Hint: see the workout example on pages 14 to 15 in lecture notes. You may want to write yourself a little computer program for these calculations. Tabulate your calculations on the Ps and $\frac{L}{\rho}$s. Also list the effective lengths you used.

I wrote a MATLAB script to solve these problems. Table 2 shows the resultant P_cr and $\frac{L}{\rho}$ values. The code output and code are shown below.

For configuration 2, we use $1''$ to be the base and $0.25''$ to be the height since this results in the smallest moment of inertia, *i.e.*, the least resistance to bending. For configurations 1 to 4, $L_{eff} = L$. For configurations 5, based on the lecture notes, $L_{eff} = 0.7L$.

Aerospace Structures Pre-Laboratory
Lab 7 Column Buckling
Section 4 Group 2
Matthew Mehrtens
AER E 322 March 22, 2023 **Spring 2023**

Table 1: Five column buckling test sets.

Specimen ID	Material	Cross-Section [in]	Length [in]	End Condition
I	aluminum	$\frac{3}{8}$ dia.	30	both pivot (round)
II	aluminum	0.25×1	30	both pivot (round)
III	steel	$\frac{1}{4}$ dia.	30	both pivot (round)
IV	steel	$\frac{1}{4}$ dia.	24	both pivot (round)
V	steel	$\frac{1}{4}$ dia.	27.5 (30 original)	one pivot, one fixed

I believe there is a typo in the table given for this question. We are given the length of configuration 5 is $27.5''$, but it says the original length was $30''$. If the original length is $30''$, then the effective length should be $0.7 * 30'' = 21'' \neq 27.5''$. I will assume the effective length is $21''$ for this problem.

Table 2: Five column buckling test sets.

Configuration	P_{cr} [lbf]	$\frac{L}{\rho}$ []
1	106.5	320.0
2	142.8	415.7
3	60.98	480.0
4	95.28	384.0
5	124.4	336.0

```
========== Configuration 1 ==========
P_cr   =  106.452 [lbf]
L/rho  =      320 []
========== Configuration 2 ==========
P_cr   =  142.789 [lbf]
L/rho  =  415.692 []
========== Configuration 3 ==========
P_cr   =  60.9797 [lbf]
L/rho  =      480 []
========== Configuration 4 ==========
P_cr   =  95.2808 [lbf]
L/rho  =      384 []
========== Configuration 5 ==========
P_cr   =  124.448 [lbf]
```

```
L/rho  =      336 []
```

```matlab
% Lab 7 Prelab Calculations
% AER E 322 Spring 2023
% Matthew Mehrtens
clear,close all;

% Constants
config          = 5;        % []

E_steel         = 29000e3;  % [psi]
sigma_Y_steel   = 35e3;     % [psi]
E_al            = 10000e3;  % [psi]
sigma_Y_al      = 40e4;     % [psi]

B               = 1;        % [in]
H               = 0.25;     % [in]
D               = 1 / 4;    % [in]
R               = D / 2;    % [in]
L_eff           = 21;       % [in]

% Set constants
steel = true; % Set this to true for steel; false for aluminum
if steel
    E = E_steel;
    sigma_Y = sigma_Y_steel;
else
    E = E_al;
    sigma_Y = sigma_Y_al;
end

% Calculate Beam
calc_beam = false; % Set this to true for a beam; false for a rod
if calc_beam
    A = B * H; % [in^2]
    P_cr = pi^2 * E * B * H^3 / (12 * L_eff^2); % [lbf]
    slenderness_ratio = 2 * L_eff * sqrt(3 * A / (B * H^3)); % []
else
    A = pi * R^2; % [in^2]
    P_cr = pi^3 * E * R^4 / (4 * L_eff^2); % [lbf]
    slenderness_ratio = 2 * L_eff / R^2 * sqrt(A / pi); % []
end

fprintf("========== Configuration %d ==========\n" + ...
    "P_cr   = %8g [lbf]\n" + ...
    "L/rho  = %8g []\n", [config, P_cr, slenderness_ratio]);
```