Aerospace Structures Pre-Laboratory
Lab 8 Thin-Walled Section and Shear Center

Section 4 Group 2
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AER E 322 March 30, 2023 **Spring 2023**

Question 1

(30 points) For specimens I and II (Table 1 below), derive an expression for the area moment of inertia, I, about its neutral axis in terms of h, b and t.

Hint: To derive the expression for I for specimens I and II, you can do one of two ways: (1) calculate I for the horizontal flanges and vertical web individually and then sum up or (2) use "subtraction" method by calculating I for a larger "outer" rectangular area and then subtracting it from I of the smaller "inner" area. You will also need parallel axis theorem. Also, as indicated in Figure 1, the shear center offset e for the C-channel beams is measured from the center of the vertical web, while e for the circular open-channel pipes is measured from the center of circular cross section. Likewise, height h is measured between mid-planes of top and bottom flanges, and r is mean radius, i.e., $r = \frac{OD-t}{2}$.

Table 1: Dimensions for the two types of cross sections of specimens.

Specimen ID	Cross section type	Height, h (inch)	Width, b (inch)	Thickness, t (inch)	Outer diameter, OD (inch)	Opening angle, $2\theta_0$ (deg)
I	Plastic C-channel	2.43	1.456	0.08	N/A	N/A
II	Metal C-channel	0.84	0.56	0.055	N/A	N/A
III	PVC circular open	N/A	N/A	0.071	1.66	3.1
IV	PVC circular open	N/A	N/A	0.071	1.66	36.3
V	PVC circular open	N/A	N/A	0.071	1.66	103.7

Solution:

Let the x-y axis be centered at the center of the vertical flange. The neutral axis of a symmetrical C-channel is on the horizontal x-axis. The moment of inertia, I, about the neutral axis can be calculated by finding the moment of inertia for each component (the two horizontal rectangles and a vertical rectangle) and then adding them together. The moment of inertia for a rectangle is shown in Equation 1. Since the centroid of the two horizontal rectangles does not intersect the centroid of the beam, we must use the parallel axis theorem, noted in Equation 2.

$$I_{\text{rect}} = \frac{1}{12}bh^3 \tag{1}$$

$$I = I_{\text{shape}} + Ay^2 \tag{2}$$

where A is the area of the shape and y is the distance from the centroid of the shape to the neutral axis.

Using these formulas, we can calculate the moment of inertia, I, for the C-channel beam as shown below:

$$I = I_{\text{top-rect}} + I_{\text{vert-rect}} + I_{\text{bot-rect}}$$
$$I = \frac{1}{12}bt^3 + bt\left(\frac{h}{2}\right)^2 + \frac{1}{12}bt^3 + bt\left(-\frac{h}{2}\right)^2 + \frac{1}{12}t(h-t)^3$$

Simplifying the above equation, we find that the general formula for calculating the moment of inertia is Equation 3

$$I = \frac{bt^3}{6} + \frac{bth^2}{2} + \frac{t(h-t)^3}{12} \tag{3}$$

Question 2

(25 points) Calculate the theoretical shear center of each of the five specimens using the following expressions as derived on pages 9 and 11 in lecture notes:

$$\text{C-channel: } e = \frac{h^2b^2t}{4I} \tag{4}$$

$$\text{Circular open-channel: } e = \frac{2r[\cos\theta_0(2\pi - 2\theta_0) + 2\sin\theta_0]}{2\pi - 2\theta_0 + \sin(2\theta_0)} \tag{5}$$

See Figure 1 for all the corresponding parameters h, b, t, r, and θ_0. Note that the total opening angle is $2\theta_0$.

Solution:

Shear centers are calculated in the MATLAB script attached to this document.

Question 3

(5 points) Tabulate the results.

Solution:

The results of the previous two questions are tabulated in Table 2.

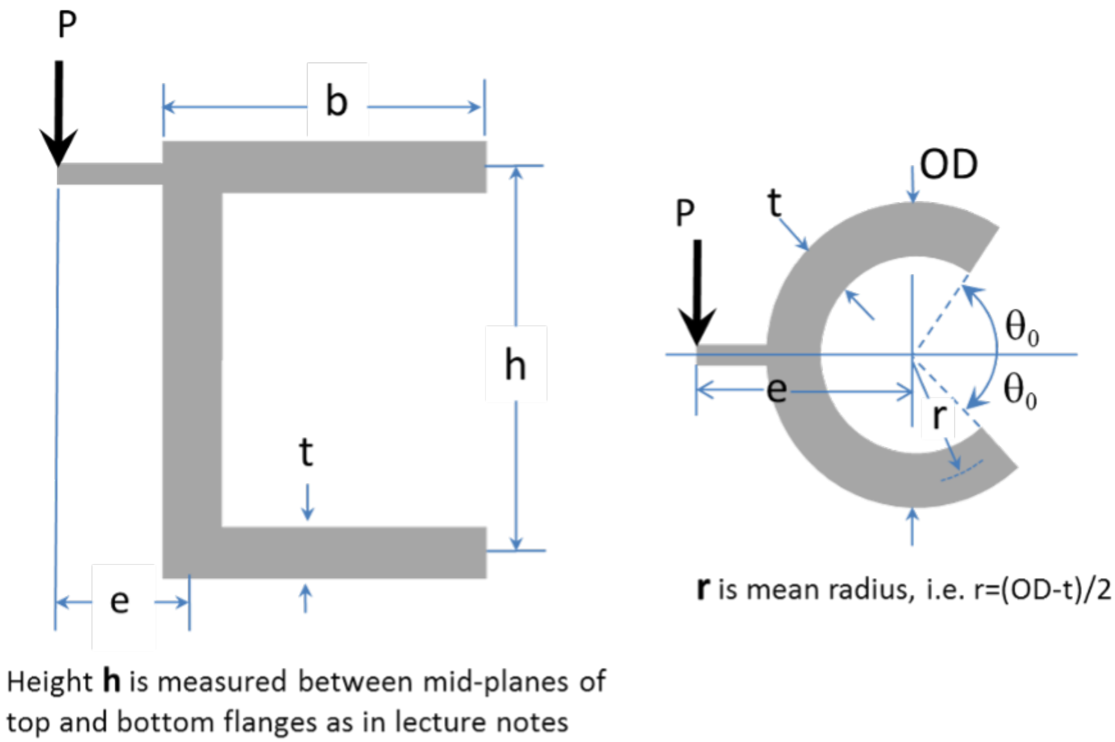

Figure 1: Schematic diagrams of the thin-walled cross section of specimen: (left) C-channel and (right) circular open channel.

Table 2: The moments of inertia and shear centers of specimens 1–5.

Specimen ID	Moment of Inertia, I (inch^4)	Shear Center, e (inch)
1	0.4305	0.5815
2	0.013 10	0.2323
3	N/A	1.588
4	N/A	1.507
5	N/A	0.9655

```
% AER E 322 Prelab 8 Spring 2023
% Matthew Mehrtens
% Section 4 Group 2
clear,clc;

% Formula for Moment of Inertia of Specimen 1 & 2
I = @(h, b, t) b * t^3 / 6 + b * t * h^2 / 2 ...
    + t * (h - t)^3 / 12; % [inch^4]

% Formula for r
r = @(OD, t) (OD - t) / 2; % [inch]

% Specimen 1 Parameters
h_1 = 2.43; % [inch]
b_1 = 1.456; % [inch]
t_1 = 0.08; % [inch]
I_1 = I(h_1, b_1, t_1); % [inch^4]

% Specimen 2 Parameters
h_2 = 0.84; % [inch]
b_2 = 0.56; % [inch]
t_2 = 0.055; % [inch]
I_2 = I(h_2, b_2, t_2); % [inch^4]

% Specimen 3 Parameters
t_3 = 0.071; % [inch]
OD_3 = 1.66; % [inch]
theta_0_3 = 3.1 / 2; % [deg]
r_3 = r(OD_3, t_3); % [inch]

% Specimen 4 Parameters
t_4 = 0.071; % [inch]
OD_4 = 1.66; % [inch]
theta_0_4 = 36.3 / 2; % [deg]
r_4 = r(OD_4, t_4); % [inch]

% Specimen 5 Parameters
t_5 = 0.071; % [inch]
OD_5 = 1.66; % [inch]
theta_0_5 = 103.7 / 2; % [deg]
r_5 = r(OD_5, t_5); % [inch]

% Shear center calculations
e_c = @(h, b, t, I) h^2 * b^2 * t / (4 * I); % [inch]
e_circ = @(r, theta_0) 2 * r ...
    * (cosd(theta_0) * (2 * pi - 2 * theta_0) + 2 * sind(theta_0)) ...
    / (2 * pi - 2 * theta_0 + sind(2 * theta_0)); % [inch]

e_1 = e_c(h_1, b_1, t_1, I_1); % [inch]
e_2 = e_c(h_2, b_2, t_2, I_2); % [inch]
e_3 = e_circ(r_3, theta_0_3); % [inch]
e_4 = e_circ(r_4, theta_0_4); % [inch]
e_5 = e_circ(r_5, theta_0_5); % [inch]

% Print output
fprintf("I_1 = %g [inch^4]\n" + ...
    "I_2 = %g [inch^4]\n" + ...
    "e_1 = %g [inch]\n" + ...
```

```
    "e_2 = %g [inch]\n" + ...
    "e_3 = %g [inch]\n" + ...
    "e_4 = %g [inch]\n" + ...
    "e_5 = %g [inch]\n", ...
    [I_1, I_2, e_1, e_2, e_3, e_4, e_5]);
```

```
I_1 = 0.430545 [inch^4]
I_2 = 0.0130989 [inch^4]
e_1 = 0.581496 [inch]
e_2 = 0.232275 [inch]
e_3 = 1.58844 [inch]
e_4 = 1.50667 [inch]
e_5 = 0.965536 [inch]
>>
```