

Iowa State University
Aerospace Engineering

AER E 322 Lab 8

Thin-Walled Section and Shear Center

Matthew Mehrkens, Peter Mikolitis, and Natsuki Oda

April 9, 2023

Aerospace Structures Laboratory Report
Lab 8 Thin-Walled Section and Shear Center

Section 4 Group 2

Matthew Mehrrens, Peter Mikolitis, and Natsuki Oda

AER E 322

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Contents

1	Pre-Lab	3
1.1	Introduction	3
1.2	Objectives	3
1.3	Hypothesis	4
2	Lab Work	5
2.1	Variables	5
2.1.1	Independent Variables	5
2.1.2	Dependent Variables	5
2.2	Work Assignments	6
2.3	Materials	6
2.4	Apparatus	7
2.5	Procedures	9
2.6	Data	9
3	Conclusion	12
3.1	Analysis	12
3.1.1	Problem 1	12
3.1.2	Problem 2	13
3.1.3	Problem 3	14
3.1.4	Problem 4	16
3.1.5	Problem 5	19
3.1.6	Problem 6	21
3.2	Sources of Error	21
3.3	Conclusion	22

Aerospace Structures Laboratory Report
Lab 8 Thin-Walled Section and Shear Center
Section 4 Group 2

Matthew Mehrtens, Peter Mikolitis, and Natsuki Oda

AER E 322

April 9, 2023

Spring 2023

References	22
A Graphs	24
A.1 Shear Center Graphs	24
B Code	28

Aerospace Structures Laboratory Report
Lab 8 Thin-Walled Section and Shear Center

Section 4 Group 2

Matthew Mehrtens, Peter Mikolitis, and Natsuki Oda

AER E 322

April 9, 2023

Spring 2023

Chapter 1

Pre-Lab

1.1. Introduction

To reduce the weight and cost of aircraft, aerospace engineers use thin-walled structures throughout aerostructures, particularly in the wings. In addition to being more weight and cost effective than traditional structures, they are generally equally strong. Unfortunately, one of the side effects of thin-walled structures is their tendency to twist under unsymmetrical loadings.

To evaluate the strength of thin-walled structures, we have to determine where the shear center is, *i.e.*, the point where applied loads bend and do not twist the structure. In this lab, we learned how to calculate the shear center and deflection of a thin-walled structure.

1.2. Objectives

Using the five provided specimens, we will apply twisting loads to thin-walled beams and make a number of measurements for each specimen. We will use the material learned in lecture and in the lecture notes to calculate the shear center of the thin-walled beams.

Aerospace Structures Laboratory Report
Lab 8 Thin-Walled Section and Shear Center

Section 4 Group 2

Matthew Mehrtens, Peter Mikolitis, and Natsuki Oda

AER E 322

April 9, 2023

Spring 2023

1.3. Hypothesis

Although our individual results from prelab varied significantly from each other, we expect the actual shear center values to at least correlate to the answers we calculated in prelab, even if they are off by a constant or a proportionate. We predict the cross sectional and the angle of opening will have a significant impact on the materials shear center and resistance or tendency to twist.

Aerospace Structures Laboratory Report
Lab 8 Thin-Walled Section and Shear Center

Section 4 Group 2

Matthew Mehrstens, Peter Mikolitis, and Natsuki Oda

AER E 322

April 9, 2023

Spring 2023

Chapter 2

Lab Work

2.1. Variables

2.1.1. Independent Variables

- Weight of mass; the applied load varied amongst the specimens
- Material of the specimen; the material of the element affects the local stiffness or strength and its tendency to twist
- Cross-section of the specimen; this variable controls the value of I which is used to derive the location of the shear center

2.1.2. Dependent Variables

- Deflection due to load; these deflections take the form of bending as observed in cantilevered beams or torsional deflections in the form of twist
- Shear center; the point about which applied loads cause bending but not torsion. The shear center is dependent on the cross-sectional layout.

Aerospace Structures Laboratory Report
Lab 8 Thin-Walled Section and Shear Center

Section 4 Group 2

Matthew Mehrrens, Peter Mikolitis, and Natsuki Oda

AER E 322

April 9, 2023

Spring 2023

2.2. Work Assignments

Refer to Table 2.1 for the distribution of work during this lab.

Table 2.1: Work assignments for AER E 322 Lab 8.

Task	Matthew	Peter	Natsuki
<i>Lab Work</i>			
Date Recording	X	X	X
Exp. Setup	X	X	X
Exp. Work	X	X	X
Exp. Clean-Up	X	X	X
<i>Report</i>			
Introduction	X		X
Objectives			X
Hypothesis	X		X
Variables		X	
Materials		X	
Apparatus		X	
Procedures		X	
Data			
Analysis	X	X	X
Conclusion	X	X	
References	X		
Appendix	X		
Revisions	X	X	
Editing	X		

2.3. Materials

- Five beam specimens as described in Table 2.2
- 100 g and 200 g mass
- Ruler
- Clamp; to secure one end of the beams

Aerospace Structures Laboratory Report
Lab 8 Thin-Walled Section and Shear Center

Section 4 Group 2

Matthew Mehrtens, Peter Mikolitis, and Natsuki Oda

AER E 322

April 9, 2023

Spring 2023

- Level

Table 2.2: Dimensions and specifications for the five different specimens.

Specimen ID	Cross section type	Height, h (inch)	Width, b (inch)	Thickness, t (inch)	Outer diameter, OD (inch)	Opening angle, $2\theta_0$ (deg)
I	Plastic C-channel	2.43	1.456	0.08	N/A	N/A
II	Metal C-channel	0.84	0.56	0.055	N/A	N/A
III	PVC circular open	N/A	N/A	0.071	1.66	3.1
IV	PVC circular open	N/A	N/A	0.071	1.66	36.3
V	PVC circular open	N/A	N/A	0.071	1.66	103.7

2.4. Apparatus

The five beams should be cantilevered with the clamp. On the free end, there is a cross beam on which the weight will be hung. Figure 2.1 shows the apparatus for the C-channel beam and Figure 2.2 shows the apparatus for the circular PVC beam.



Figure 2.1: One of the cantilevered C-channel beams from lab eight [1].

Aerospace Structures Laboratory Report
Lab 8 Thin-Walled Section and Shear Center
Section 4 Group 2

Matthew Mehrtens, Peter Mikolitis, and Natsuki Oda
April 9, 2023

AER E 322

Spring 2023

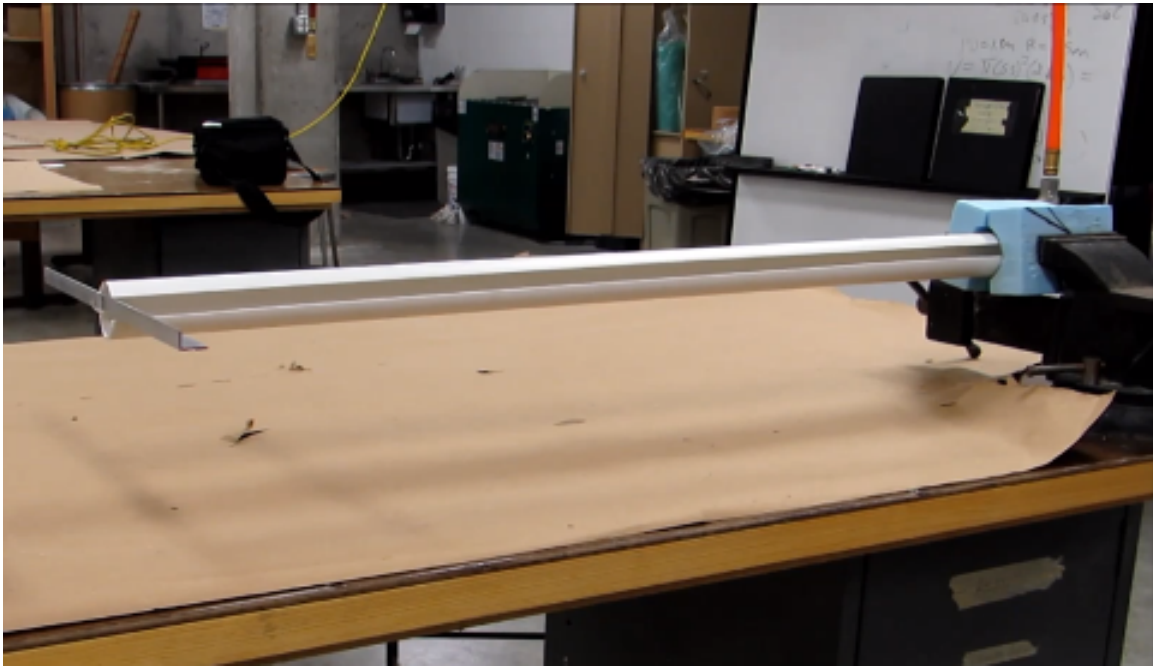


Figure 2.2: One of the cantilevered circular PVC beams from lab eight [1].

Aerospace Structures Laboratory Report
Lab 8 Thin-Walled Section and Shear Center

Section 4 Group 2

Matthew Mehrtens, Peter Mikolitis, and Natsuki Oda

AER E 322

April 9, 2023

Spring 2023

2.5. Procedures

Set up the five beams as described and shown in Section 2.4. Ensure the beams and the table are level. For specimen one, perform a preliminary measurement: measure the length from the edge of the clamp to the cross bar and measure the height from the table to a reference point at the same length as the cross bar, *e.g.*, the bottom of the beam.

For each of the five beams, perform the following procedure at least four times—placing the weight at least twice on each side of the cross beam:

1. Level the cross bar as best you can. For the circular specimens, this can be done by rotating the beam. Measure the height at both ends of the cross bar to calculate the initial angle of twist. For specimens one and two, if the initial angle of twist is greater than 1° , account for this baseline by subtracting the initial angle of twist from subsequent angle of twist calculations.
2. Place a weight on the cross beam at an arbitrary location (at least twice on both sides). Use the 100 g weight for specimens one, three, and five, and use the 200 g weight for specimens two and four.
3. Record the location of the weight along the cross bar and the height of both sides of the cross beam. Since the shear center is measured from the center of the circular beams and from the center of the vertical web of the C-beams, make sure to note where your cross beam measurements are referenced from. Later in the analysis you can adjust your cross beam measurements to be relative to the shear center.

For specimen one, put the weight at the shear center, *i.e.*, the point where an applied weight causes no twist, and measure the height at the reference point again. This should give you beam deflection.

2.6. Data

Note: Our group misinterpreted the lab instructions for this lab. Instead of measuring the height of the cross beam at both ends, we measured the height of the actual beam at both ends. The numbers we calculated in lab are, of course, meaningless. After looking through the lab instruction more closely, it is obvious we were supposed to measure the height of the cross beam to calculate the angle of twist.

Aerospace Structures Laboratory Report
Lab 8 Thin-Walled Section and Shear Center

Section 4 Group 2

Matthew Mehrtens, Peter Mikolitis, and Natsuki Oda

AER E 322

April 9, 2023

Spring 2023

To still be able to do the analysis properly, we asked a fellow group in our AER E 322 section, Section 4 Group 3, if we could use their data. They agreed; all data used in this report is courtesy of lab group 3 in AER E 322 section 4.

To calculate angle of twist, we used Equation 2.1 from the lab eight lecture notes [4]:

$$\theta = \sin^{-1} \left(\frac{h_R - h_L}{L} \right) \quad (2.1)$$

where h_R is the height from the table to the right end of the cross bar, h_L is the height from the table to the left end of the cross bar, and L is the length of the cross bar. Figure 2.3 shows this angle visually.

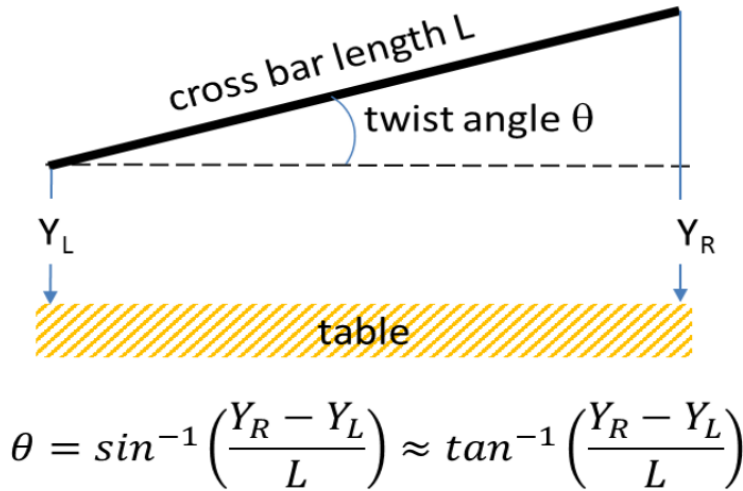


Figure 2.3: Visual representation of the angle of twist of the cross bar. [4]

The data we used in the analysis portion of this lab is shown in Table 2.3.

Aerospace Structures Laboratory Report
Lab 8 Thin-Walled Section and Shear Center

Section 4 Group 2

Matthew Mehrtens, Peter Mikolitis, and Natsuki Oda

AER E 322

April 9, 2023

Spring 2023

Table 2.3: Data collected from lab eight, courtesy of lab group 3 from AER E 322 section 4. h is the height of the cross bar at the left and right ends, L is the length of the cross bar, θ is the angle of twist, calculated using Equation 2.1, x is the distance to the weight on the cross bar, and m is the mass of the weight.

Specimen	h_L (cm)	h_R (cm)	L (cm)	θ (°)	x_L (cm)	x_R (cm)	m (g)
1	10.4	13.1	44.0	3.52	11.0	N/A	100
1	8.8	14.6	44.0	7.57	14.0	N/A	100
1	17.7	4.8	44.0	-17.0	N/A	9.2	100
1	18.7	3.4	44.0	-20.3	N/A	12.8	100
2	9.9	11.4	30.0	2.87	3.1	N/A	200
2	8.1	13.0	30.0	9.40	9.0	N/A	200
2	11.6	9.5	30.0	-4.01	N/A	3.3	200
2	12.6	8.3	30.0	-8.24	N/A	7.5	200
3	7.7	12.5	30.0	9.21	4.9	N/A	100
3	6.9	13.4	30.0	12.5	7.4	N/A	100
3	12.1	8.2	30.0	-7.47	N/A	9.8	100
3	11.5	9.1	30.0	-4.59	N/A	5.6	100
4	9.8	13.4	30.0	6.89	5.0	N/A	200
4	8.9	14.7	30.0	11.1	7.6	N/A	200
4	15.3	6.1	30.0	-17.9	N/A	7.5	200
4	14.2	7.7	30.0	-12.5	N/A	4.5	200
5	10.0	13.8	30.5	7.16	4.5	N/A	100
5	7.6	16.4	30.5	16.8	10.8	N/A	100
5	15.0	7.2	30.5	-14.8	N/A	5.6	100
5	17.2	5.8	30.5	-21.9	N/A	10.6	100

Chapter 3

Conclusion

3.1. Analysis

3.1.1. Problem 1

The measured shear centers were calculated with MATLAB by using the `polyfit` fit. `polyfit` takes the x and y values as an input—in our case the x and θ values—as well as an integer specifying what order line of best fit is desired and returns the corresponding polynomial coefficients. In this lab, the shear center data is generally linear, so we use a first order `polyfit` function. After being called, `polyfit` returns two values C_1 and C_2 which form the following linear equation:

$$y = C_1x + C_2$$

To find the x -intercept, we let $y = 0$ and solve for x . This is shown below:

$$\begin{aligned} 0 &= C_1x + C_2 \\ x &= -\frac{C_2}{C_1} \end{aligned}$$

This calculated distance is the measured distance from the reference center to the shear center of a material. The scatter plots, lines of best fit, and the point of shear center are shown in the graphs in Appendix [A.1](#).

Aerospace Structures Laboratory Report
Lab 8 Thin-Walled Section and Shear Center

Section 4 Group 2

Matthew Mehrtens, Peter Mikolitis, and Natsuki Oda

AER E 322

April 9, 2023

Spring 2023

The theoretical shear centers were calculated using MATLAB and referring to the formulas from prelab [2], namely Equations 3.1 and 3.2.

$$\text{C-channel: } e = \frac{h^2 b^2 t}{4I} \quad (3.1)$$

$$\text{Circular open-channel: } e = \frac{2r[\cos \theta_0(2\pi - 2\theta_0) + 2 \sin \theta_0]}{2\pi - 2\theta_0 + \sin(2\theta_0)} \quad (3.2)$$

The measured shear centers and the theoretical shear center are shown in Table 3.1.

Table 3.1: Dimensions and specifications for the five different specimens.

Specimen	e_{meas} (cm)	e_{theor} (cm)	Error (%)
1	-0.800	-1.48	45.8
2	-0.322	-0.590	45.4
3	0.798	-4.03	119.8
4	-3.77	-3.87	2.79
5	-3.52	-3.21	9.54

Our theoretical calculations matched well for specimens four and five, but poorly for specimens one through three, with the worst match being for specimen three. If the outlier is removed from the specimen three data set, as shown in Figure A.6 in Appendix A.1, the error drops to a marginally improved 103.1%. Possible explanations for the source of this error are explained in Section 3.2.

3.1.2. Problem 2

First, we calculate the beam deflection theoretically using the method of superposition. Specifically, we refer to the max deflection formula, Equation 3.3, for a cantilevered beam shown on slide 14 of the week seven lecture notes [5].

$$\nu_{\text{max}} = \frac{-PL^3}{3EI} \quad (3.3)$$

where ν_{max} is the maximum deflection of the beam, P is the applied load, L is the length of the beam, E is the Young's Modulus, and I is the moment of inertia. Substituting the

Aerospace Structures Laboratory Report
Lab 8 Thin-Walled Section and Shear Center

Section 4 Group 2

Matthew Mehrtens, Peter Mikolitis, and Natsuki Oda

AER E 322

April 9, 2023

Spring 2023

proper values for this specimen, we find that

$$\begin{aligned}\nu_{\max} &= \frac{-(0.100 \text{ kg})(9.81 \text{ m s}^{-2})(1.10 \text{ m})^3}{(3 \times 10^9 \text{ Pa})(1.792 \times 10^{-7} \text{ m}^4)} \\ &= -2.43 \text{ mm}\end{aligned}$$

In lab, we measured the deflection to be $15.0 \text{ cm} - 15.2 \text{ cm} = -0.2 \text{ cm} = -2 \text{ mm}$. This very closely matches the theoretical calculation, the limiting factor being the ruler we used to measure the height.

3.1.3. Problem 3

We will use Figure 3.1 from the lab eight instructions [3] as a visual guide to our shear flow derivations.

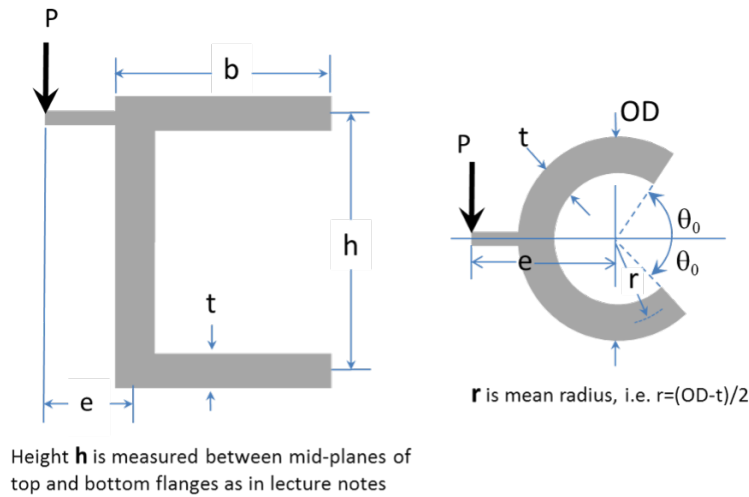


Figure 3.1: Schematic diagrams of the thin-walled cross section of the specimen: (left) C-channel and (right) circular open channel.

To calculate the shear stress distribution, we use Equation 3.4 from lecture notes [4]

$$\tau = -\frac{P}{It} \int_0^s y t ds \quad (3.4)$$

Aerospace Structures Laboratory Report
Lab 8 Thin-Walled Section and Shear Center

Section 4 Group 2

Matthew Mehrrens, Peter Mikolitis, and Natsuki Oda

AER E 322

April 9, 2023

Spring 2023

where P is an external load, I is the moment of inertia, s is the path along the cross section of the beam, and y is the distance from the axis of symmetry. We calculate I as shown in Equation 3.5

$$I = \frac{2bt^3 + t(h-t)^3 + 6bth^2}{12} \quad (3.5)$$

Below is our derivation for the shear stress distribution in each of the three sections of the C-channel beam. Note that since t is small and constant, we can simplify many of the integrals.

$$\begin{aligned} \tau_{12}(s_1) &= -\frac{P}{I} \int_0^{s_1} y ds_1, \quad \text{where } 0 \leq s_1 \leq b \\ &= -\frac{P}{I} \int_0^{s_1} \left(-\frac{h}{2}\right) ds_1 \\ &= \frac{Phs_1}{2I} \end{aligned}$$

$$\begin{aligned} \tau_{24}(s_2) &= \tau_2 - \frac{P}{I} \int_0^{s_2} y ds_2, \quad \text{where } 0 \leq s_2 \leq h \\ &= \tau_2 - \frac{P}{I} \int_0^{s_2} \left(s_2 - \frac{h}{2}\right) ds_2 \\ &= \tau_2 - \frac{P}{I} \left(\frac{s_2^2}{2} - \frac{hs_2}{2}\right) \\ &= \tau_2 - \frac{P}{2I}(s_2^2 - hs_2) \end{aligned}$$

$$\begin{aligned} \tau_{45}(s_4) &= \tau_4 - \frac{P}{I} \int_0^{s_4} y ds_4, \quad \text{where } 0 \leq s_4 \leq b \\ &= \tau_4 - \frac{P}{I} \int_0^{s_4} \left(\frac{h}{2}\right) ds_4 \\ &= \tau_4 - \frac{Phs_4}{2I} \end{aligned}$$

Programming these equations into MATLAB, for specimen one, we find the shear stress distributions shown below and the values of shear stress notated in Table 3.2. Both the

Aerospace Structures Laboratory Report
Lab 8 Thin-Walled Section and Shear Center

Section 4 Group 2

Matthew Mehrtens, Peter Mikolitis, and Natsuki Oda

AER E 322

April 9, 2023

Spring 2023

distributions and shear stresses are shown in Figure 3.2.

$$\begin{aligned}\tau_{12} &= s_1(168.9 \text{ kPa m}^{-1}) \\ \tau_{24} &= 6.248 \text{ kPa} + (2737 \text{ kPa m}^{-2})(-s_2^2 + s_2(0.06172 \text{ m})) \\ \tau_{45} &= 6.248 \text{ kPa} + s_4(-168.9 \text{ kPa m}^{-1})\end{aligned}$$

Table 3.2: Shear stresses in specimen one.

Point	τ (kPa)
1	0
2	6.248
3	8.854
4	6.248
5	0

For specimen two, we find the shear stress distributions shown below and the values of shear stress notated in Table 3.3. Both the distributions and shear stresses are shown in Figure 3.3.

$$\begin{aligned}\tau_{12} &= s_1(3839 \text{ kPa m}^{-1}) \\ \tau_{24} &= 54.61 \text{ kPa} + (1.799 \times 10^5 \text{ kPa m}^{-2})(-s_2^2 + s_2(0.02134 \text{ m})) \\ \tau_{45} &= 54.61 \text{ kPa} + s_4(-3839 \text{ kPa m}^{-1})\end{aligned}$$

Table 3.3: Shear stresses in specimen two.

Point	τ (kPa)
1	0
2	54.61
3	75.08
4	54.61
5	0

3.1.4. Problem 4

Since the C-channel beam is symmetrical about the x -axis, the shear center must lie on the x -axis. If it was above or below the x -axis, force applied at that location would cause torsion.

Aerospace Structures Laboratory Report
Lab 8 Thin-Walled Section and Shear Center
Section 4 Group 2

Matthew Mehrtens, Peter Mikolitis, and Natsuki Oda

AER E 322

April 9, 2023

Spring 2023

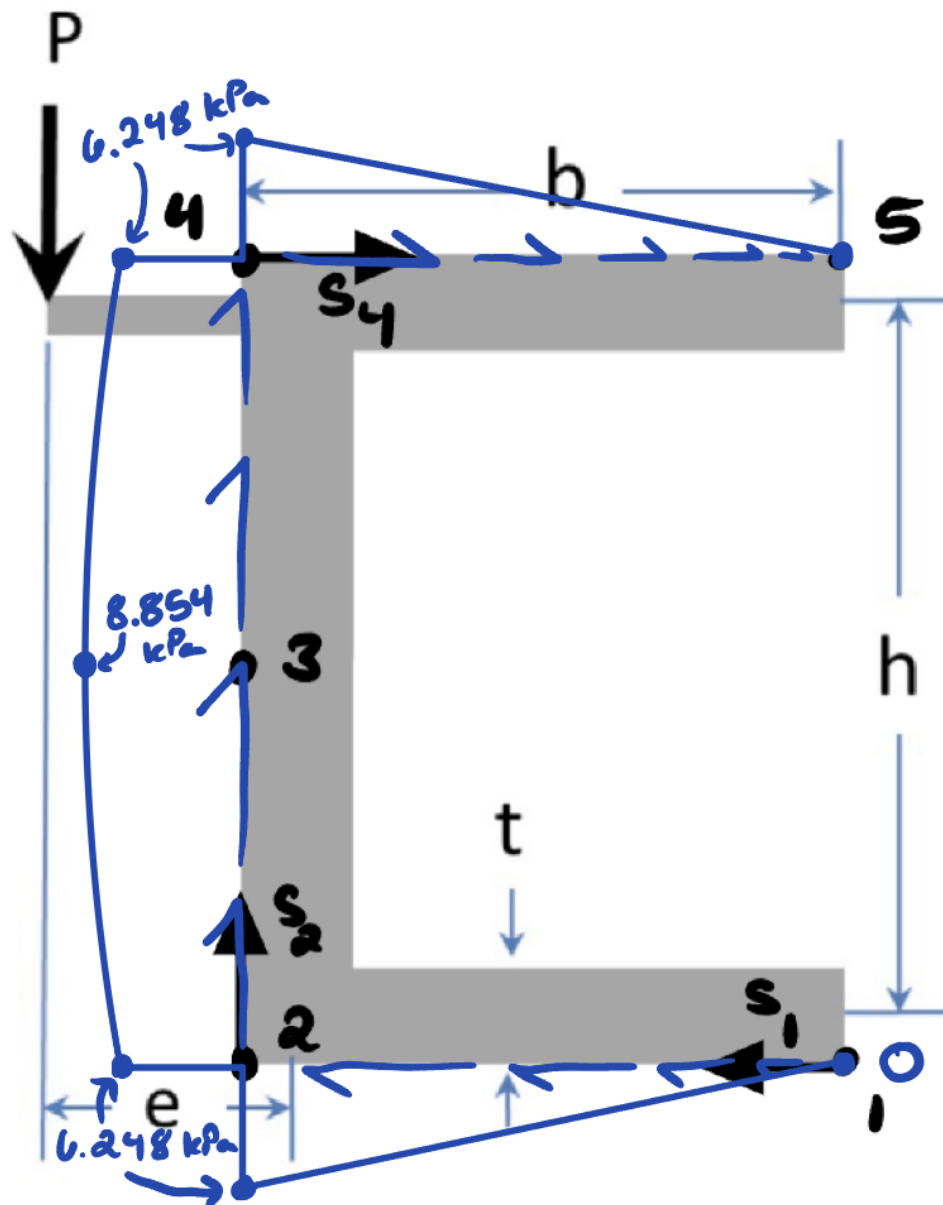
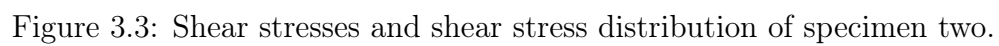


Figure 3.2: Shear stresses and shear stress distribution of specimen one.

April 9, 2023

Spring 2023



Aerospace Structures Laboratory Report
Lab 8 Thin-Walled Section and Shear Center

Section 4 Group 2

Matthew Mehrtens, Peter Mikolitis, and Natsuki Oda

AER E 322

April 9, 2023

Spring 2023

In both specimen, the x -axis is located at $\frac{h}{2}$. The x -coordinate for the shear centers is found using Equation 3.1 and is tabulated in Table 3.1.

Assuming the x axis aligns with the axis of symmetry for the C-channel beam and the y axis aligns with the axis of symmetry for the vertical web of the web, the exact (x, y) coordinate positions for specimen one and two are as follows.

Specimen 1: $(x, y) = (-1.48 \text{ cm}, 0)$

Specimen 2: $(x, y) = (-0.590 \text{ cm}, 0)$

These coordinates are shown visually with a red dot in Figures 3.4 and 3.5. Each square of the grid is equivalent to a square with dimensions $t \times t$, where t is the thickness of the beam.

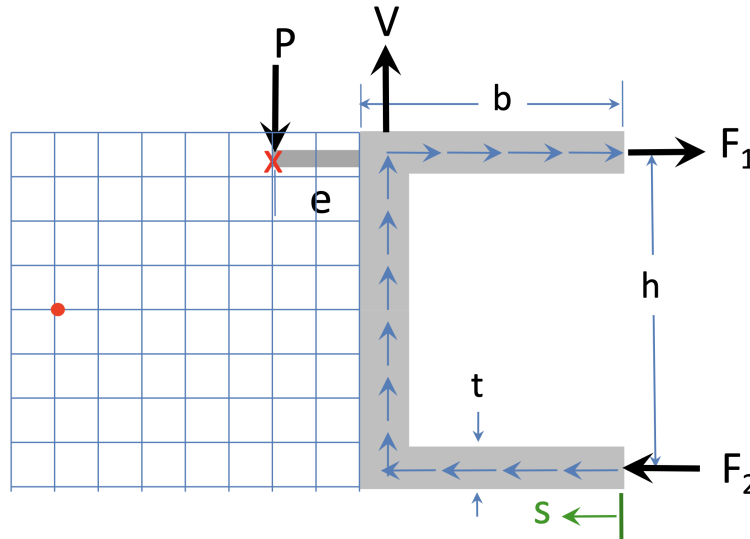


Figure 3.4: Shear center grid for specimen one. The shear center is marked with the red dot. Each grid square is a $t \times t$ square, where t is the thickness of the beam.

3.1.5. Problem 5

If the height of the C-channel beam were to increase a hundredfold or approach infinity, the shape of the C-channel beam would begin to resemble a rectangle. Specifically, the horizontal flanges would have less of an effect on the structure. As the vertical flange increases in area,

Aerospace Structures Laboratory Report
Lab 8 Thin-Walled Section and Shear Center

Section 4 Group 2

Matthew Mehrtens, Peter Mikolitis, and Natsuki Oda

AER E 322

April 9, 2023

Spring 2023

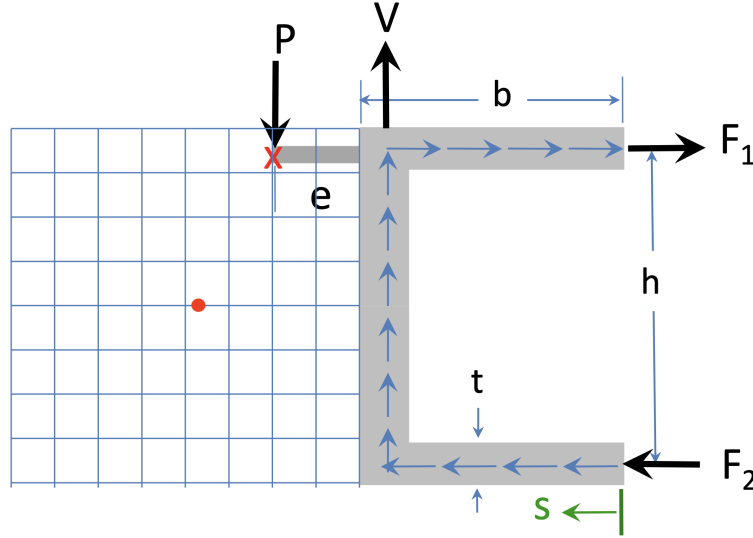


Figure 3.5: Shear center grid for specimen two. The shear center is marked with the red dot. Each grid square is a $t \times t$ square, where t is the thickness of the beam.

the shear center will move closer to the vertical axis of symmetry of the vertical flange. Since e is measured from that vertical axis of symmetry, e should approach 0. We can prove this by substituting I from Equation 3.5 into Equation 3.1 and taking the limit as h approaches infinity. This proof is shown below:

$$\begin{aligned}
 e &= \frac{h^2 b^2 t}{4I} \\
 &= \frac{12h^2 b^2 t}{2bt^3 + t(h-t)^3 + 6bth^2} \\
 &= \frac{12h^2 b^2 t}{2bt^3 + th^3 - 3t^2h^2 + 3t^3h - t^4 + 6bth^2} \\
 &= \frac{12h^2 b^2}{2bt^2 + h^3 - 3th^2 + 3t^2h - t^3 + 6bh^2}
 \end{aligned}$$

Aerospace Structures Laboratory Report
Lab 8 Thin-Walled Section and Shear Center

Section 4 Group 2

Matthew Mehrtens, Peter Mikolitis, and Natsuki Oda

AER E 322

April 9, 2023

Spring 2023

Since there is an h^3 term in the denominator and only an h^2 term in the numerator, the denominator will grow faster than the numerator and therefore,

$$\lim_{h \rightarrow \infty} (e) = \lim_{h \rightarrow \infty} \left(\frac{12h^2b^2}{2bt^2 + h^3 - 3th^2 + 3t^2h - t^3 + 6bh^2} \right) = 0$$

3.1.6. Problem 6

During the increase of both opening angle and radius, shape of the section going not to look like C . Increasing open angle, the open portion gets itself bigger. If the open angle reaches 180° ($2\theta_0 = 180^\circ$), shape of cross section will look like the circular pipe split in half along to path of that. However, the area value of this section will be bigger, depending on the value of increment. The equation below expresses the area of section with the circular given:

$$A = \pi \left(r + \frac{t}{2} \right)^2 \frac{360 - 2\theta_0}{360} - \pi \left(r - \frac{t}{2} \right)^2 \frac{360 - 2\theta_0}{360}$$

The area will increase depending on how the value of the open-angle and radius increase. That is because the radius increases quadratically. But after open angle reaches 90° , the value will start to decrease because shear center will approach radius.

Plotting Equation 3.2, the equation for the shear center offset of a circular open-channel beam, and letting θ_0 get arbitrarily close to π , we find that the shear center offset approaches r as $\theta_0 \rightarrow \pi$.

3.2. Sources of Error

Some of our shear center calculations had a significant amount of error. If the lab table or the beam was not initially level, all our measurements would be systematically incorrect. If the thickness of the beams or the dimensions were not exactly as described in the lab instructions, this would also result in higher error. Since we were measuring by hand, there is a possibly for random human error to propagate through our calculations due to the inconsistent way in which we measure things visually. Additionally, we did not “zero” out the cross bar after each test, so there may have been an initial angle of twist value after the first test was measured.

Aerospace Structures Laboratory Report
Lab 8 Thin-Walled Section and Shear Center

Section 4 Group 2

Matthew Mehrtens, Peter Mikolitis, and Natsuki Oda

April 9, 2023

AER E 322

Spring 2023

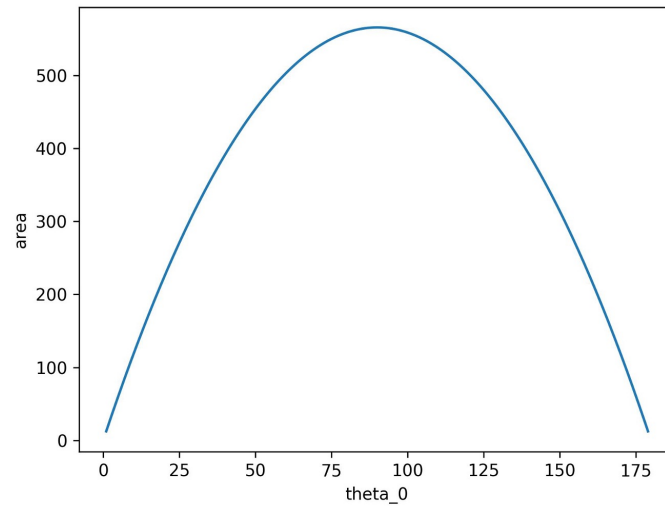


Figure 3.6: The changes when both radius and degree of open angle increases one by one.

3.3. Conclusion

Due to the numerous sources of error described in Section 3.2, our shear center calculations did not match well on specimens one through three. We were able to accurately predict the location of the shear center in specimens four and five, however.

Additionally, we accurately predicted the deflection of specimen one undergoing a loading at its shear center. We examined what effect changing the cross section would have on the shear center offset of a C-channel and open-channel beam.

References

- [1] Thomas Chiou. *Demo Video: shear center*. Iowa State University. 2023. URL: https://canvas.iastate.edu/courses/97229/pages/demo-video-shear-center?module_item_id=5127327.
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Aerospace Structures Laboratory Report
Lab 8 Thin-Walled Section and Shear Center

Section 4 Group 2

Matthew Mehrtens, Peter Mikolitis, and Natsuki Oda

AER E 322

April 9, 2023

Spring 2023

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Aerospace Structures Laboratory Report
Lab 8 Thin-Walled Section and Shear Center
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April 9, 2023

AER E 322

Spring 2023

Appendix A

Graphs

A.1. Shear Center Graphs

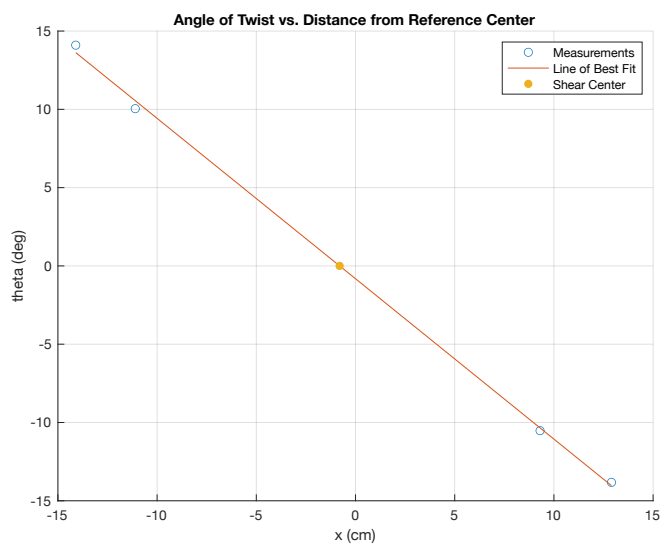


Figure A.1: The measured angles of twist for specimen one with the measured shear center.

Aerospace Structures Laboratory Report
Lab 8 Thin-Walled Section and Shear Center

Section 4 Group 2

Matthew Mehrtens, Peter Mikolitis, and Natsuki Oda

AER E 322

April 9, 2023

Spring 2023

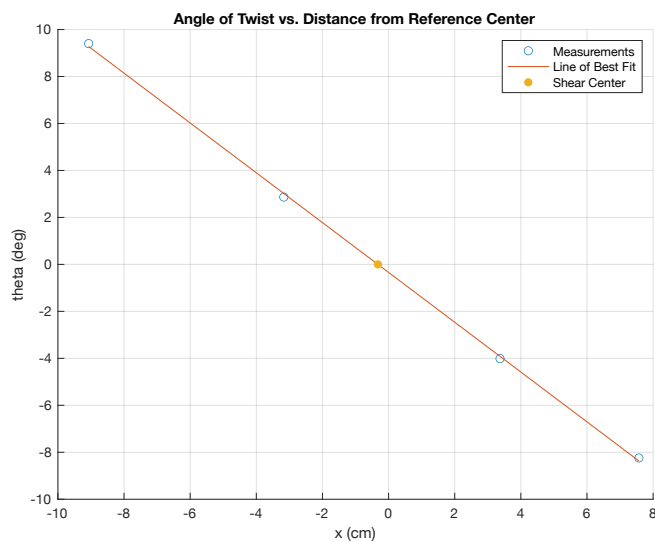


Figure A.2: The measured angles of twist for specimen two with the measured shear center.

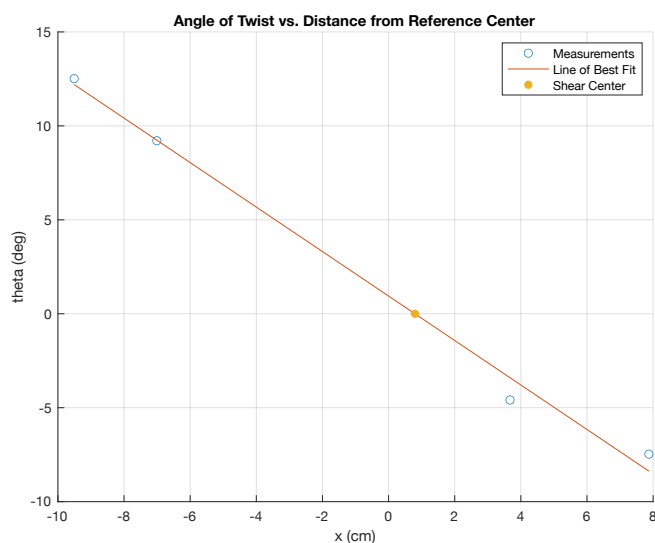


Figure A.3: The measured angles of twist for specimen three with the measured shear center.

Aerospace Structures Laboratory Report
Lab 8 Thin-Walled Section and Shear Center

Section 4 Group 2

Matthew Mehrtens, Peter Mikolitis, and Natsuki Oda

AER E 322

April 9, 2023

Spring 2023

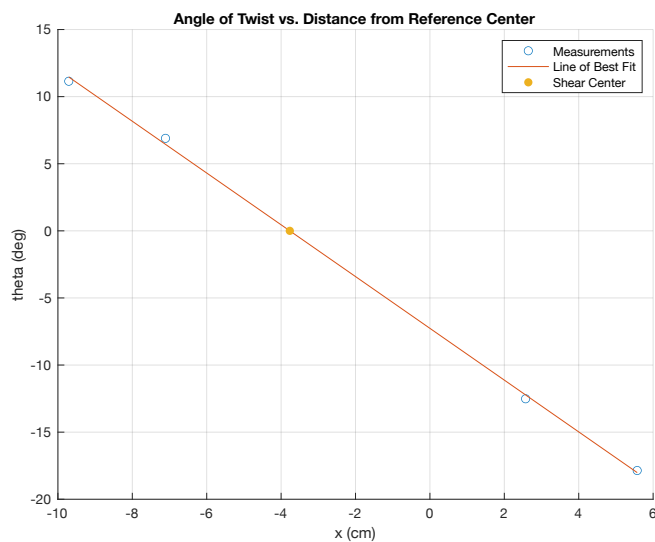


Figure A.4: The measured angles of twist for specimen four with the measured shear center.

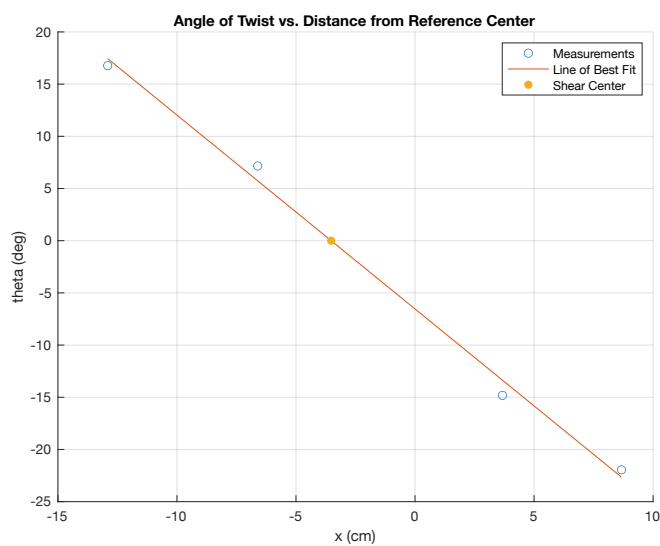


Figure A.5: The measured angles of twist for specimen five with the measured shear center.

Aerospace Structures Laboratory Report
Lab 8 Thin-Walled Section and Shear Center

Section 4 Group 2

Matthew Mehrtens, Peter Mikolitis, and Natsuki Oda

AER E 322

April 9, 2023

Spring 2023

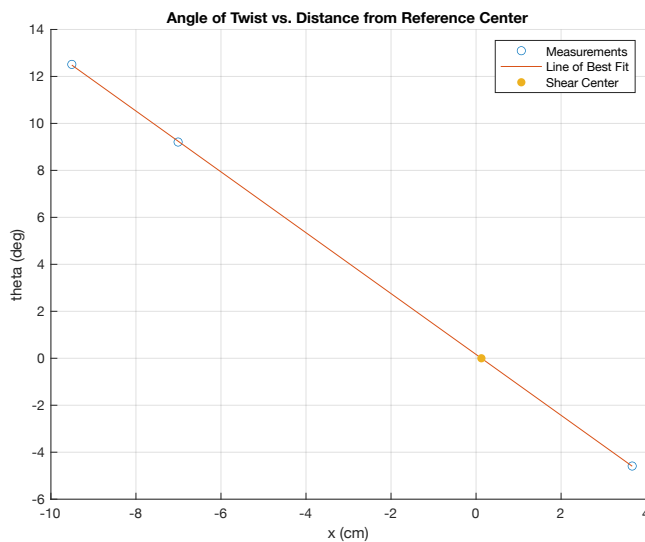


Figure A.6: The measured angles of twist for specimen three with the measured shear center and the final outlier data point removed.

Aerospace Structures Laboratory Report
Lab 8 Thin-Walled Section and Shear Center
Section 4 Group 2

Matthew Mehrrens, Peter Mikolitis, and Natsuki Oda

AER E 322

April 9, 2023

Spring 2023

Appendix B

Code

See the attached pages for our calculation script and output.

```
% AER E 322 Lab 8 Analysis Script
% Spring 2023 Section 4 Group 2
clear,clc,close all;
```

```
% Calculate Angle of Twist and x for Specimen 1
```

```
u = symunit;
L_1 = 44.0*u.cm; % [cm]
t_1 = 0.08*u.in; % [in]
t_1 = unitConvert(t_1,u.cm); % [cm]

x_11 = -(11.0*u.cm + t_1 / 2); % [cm]
x_12 = -(14.0*u.cm + t_1 / 2); % [cm]
x_13 = 9.2*u.cm + t_1 / 2; % [cm]
x_14 = 12.8*u.cm + t_1 / 2; % [cm]
x_1 = [x_11 x_12 x_13 x_14]; % [cm]

h_L_1 = [10.4 8.8 17.7 18.7].*u.cm; % [cm]
h_R_1 = [13.1 14.6 4.8 3.4].*u.cm; % [cm]
aot_1 = asind((h_R_1 - h_L_1) ./ L_1)*u.deg; % [deg]
err_1 = asind((9.0*u.cm - 14.0*u.cm) / L_1)*u.deg; % [deg]
aot_1 = aot_1 - err_1; % [deg]
```

```
% Calculate Angle of Twist and x for Specimen 2
```

```
L_2 = 30.0*u.cm; % [cm]
t_2 = 0.055*u.in; % [in]
t_2 = unitConvert(t_2,u.cm); % [cm]

x_21 = -(3.1*u.cm + t_2 / 2); % [cm]
x_22 = -(9.0*u.cm + t_2 / 2); % [cm]
x_23 = 3.3*u.cm + t_2 / 2; % [cm]
x_24 = 7.5*u.cm + t_2 / 2; % [cm]
x_2 = [x_21 x_22 x_23 x_24]; % [cm]

h_L_2 = [9.9 8.1 11.6 12.6].*u.cm; % [cm]
h_R_2 = [11.4 13.0 9.5 8.3].*u.cm; % [cm]
aot_2 = asind((h_R_2 - h_L_2) ./ L_2)*u.deg; % [deg]
```

```
% Calculate Angle of Twist and x for Specimen 3
```

```
L_3 = 30.0*u.cm; % [cm]
t_3 = 0.071*u.in; % [in]
t_3 = unitConvert(t_3,u.cm); % [cm]
OD_3 = 1.66*u.in; % [in]
OD_3 = unitConvert(OD_3,u.cm); % [cm]

x_31 = -(4.9*u.cm + OD_3 / 2); % [cm]
x_32 = -(7.4*u.cm + OD_3 / 2); % [cm]
x_33 = 9.8*u.cm + t_3 - OD_3 / 2; % [cm]
x_34 = 5.6*u.cm + t_3 - OD_3 / 2; % [cm]
x_3 = [x_31 x_32 x_33 x_34]; % [cm]
```

```
h_L_3 = [7.7 6.9 12.1 11.5].*u.cm; % [cm]
h_R_3 = [12.5 13.4 8.2 9.1].*u.cm; % [cm]
aot_3 = asind((h_R_3 - h_L_3) ./ L_3)*u.deg; % [deg]
```

```
% Calculate Angle of Twist and x for Specimen 4
```

```
L_4 = 30.0*u.cm; % [cm]
t_4 = 0.071*u.in; % [in]
t_4 = unitConvert(t_4,u.cm); % [cm]
```

```

OD_4 = 1.66*u.in; % [in]
OD_4 = unitConvert(OD_4,u.cm); % [cm]

x_41 = -(5.0*u.cm + OD_4 / 2); % [cm]
x_42 = -(7.6*u.cm + OD_4 / 2); % [cm]
x_43 = 7.5*u.cm + t_4 - OD_4 / 2; % [cm]
x_44 = 4.5*u.cm + t_4 - OD_4 / 2; % [cm]
x_4 = [x_41 x_42 x_43 x_44]; % [cm]

h_L_4 = [9.8 8.9 15.3 14.2]*u.cm; % [cm]
h_R_4 = [13.4 14.7 6.1 7.7]*u.cm; % [cm]
aot_4 = asind((h_R_4 - h_L_4) ./ L_4)*u.deg; % [deg]

% Calculate Angle of Twist and x for Specimen 5
L_5 = 30.5*u.cm; % [cm]
t_5 = 0.071*u.in; % [in]
t_5 = unitConvert(t_5,u.cm); % [cm]
OD_5 = 1.66*u.in; % [in]
OD_5 = unitConvert(OD_5,u.cm); % [cm]

x_51 = -(4.5*u.cm + OD_5 / 2); % [cm]
x_52 = -(10.8*u.cm + OD_5 / 2); % [cm]
x_53 = 5.6*u.cm + t_5 - OD_5 / 2; % [cm]
x_54 = 10.6*u.cm + t_5 - OD_5 / 2; % [cm]
x_5 = [x_51 x_52 x_53 x_54]; % [cm]

h_L_5 = [10.0 7.6 15.0 17.2]*u.cm; % [cm]
h_R_5 = [13.8 16.4 7.2 5.8]*u.cm; % [cm]
aot_5 = asind((h_R_5 - h_L_5) / L_5)*u.deg; % [deg]

fprintf("Measured Shear Centers:\n")

% Plot and Calculate Shear Center of Specimen 1
c_1 = polyfit(double(separateUnits(x_1)), double(separateUnits(aot_1)), 1);
lobf_x_1 = linspace(double(separateUnits(x_1(2))), ...
    double(separateUnits(x_1(end))), ...
    1000); % [cm]
lobf_y_1 = c_1(1) * lobf_x_1 + c_1(2); % [deg]
e_1 = (-c_1(2) / c_1(1))*u.cm; % [cm]

fprintf("e_1 = %g [cm]\n", separateUnits(e_1));

figure(1);
scatter(separateUnits(x_1), separateUnits(aot_1), ...
    "DisplayName", "Measurements");
hold on;
plot(lobf_x_1, lobf_y_1, "DisplayName", "Line of Best Fit");
scatter(separateUnits(e_1), 0, "filled", "DisplayName", "Shear Center");
hold off;
title("Angle of Twist vs. Distance from Reference Center");
xlabel("x (cm)");
ylabel("theta (deg)");
legend;
grid on;

% Plot and Calculate Shear Center of Specimen 2
c_2 = polyfit(double(separateUnits(x_2)), double(separateUnits(aot_2)), 1);
lobf_x_2 = linspace(double(separateUnits(x_2(2))), ...

```

```

    double(separateUnits(x_2(end))), ...
    1000); % [cm]
    lobf_y_2 = c_2(1) * lobf_x_2 + c_2(2); % [deg]
    e_2 = (-c_2(2) / c_2(1))*u.cm; % [cm]

    fprintf("e_2 = %g [cm]\n", separateUnits(e_2));

    figure(2);
    scatter(separateUnits(x_2), separateUnits(aot_2), ...
        "DisplayName", "Measurements");
    hold on;
    plot(lobf_x_2, lobf_y_2, "DisplayName", "Line of Best Fit");
    scatter(separateUnits(e_2), 0, "filled", "DisplayName", "Shear Center");
    hold off;
    title("Angle of Twist vs. Distance from Reference Center");
    xlabel("x (cm)");
    ylabel("theta (deg)");
    legend;
    grid on;

    % Plot and Calculate Shear Center of Specimen 3
    c_3 = polyfit(double(separateUnits(x_3)), double(separateUnits(aot_3)), 1);
    lobf_x_3 = linspace(double(separateUnits(x_3(2))), ...
        double(separateUnits(x_3(end - 1))), ...
        1000); % [cm]
    lobf_y_3 = c_3(1) * lobf_x_3 + c_3(2); % [deg]
    e_3 = (-c_3(2) / c_3(1))*u.cm; % [cm]

    fprintf("e_3 = %g [cm]\n", separateUnits(e_3));

    figure(3);
    scatter(separateUnits(x_3), separateUnits(aot_3), ...
        "DisplayName", "Measurements");
    hold on;
    plot(lobf_x_3, lobf_y_3, "DisplayName", "Line of Best Fit");
    scatter(separateUnits(e_3), 0, "filled", "DisplayName", "Shear Center");
    hold off;
    title("Angle of Twist vs. Distance from Reference Center");
    xlabel("x (cm)");
    ylabel("theta (deg)");
    legend;
    grid on;

    % Plot and Calculate Shear Center of Specimen 4
    c_4 = polyfit(double(separateUnits(x_4)), double(separateUnits(aot_4)), 1);
    lobf_x_4 = linspace(double(separateUnits(x_4(2))), ...
        double(separateUnits(x_4(end - 1))), ...
        1000); % [cm]
    lobf_y_4 = c_4(1) * lobf_x_4 + c_4(2); % [deg]
    e_4 = (-c_4(2) / c_4(1))*u.cm; % [cm]

    fprintf("e_4 = %g [cm]\n", separateUnits(e_4));

    figure(4);
    scatter(separateUnits(x_4), separateUnits(aot_4), ...
        "DisplayName", "Measurements");
    hold on;
    plot(lobf_x_4, lobf_y_4, "DisplayName", "Line of Best Fit");

```



```

scatter(separateUnits(e_4), 0, "filled", "DisplayName", "Shear Center");
hold off;
title("Angle of Twist vs. Distance from Reference Center");
xlabel("x (cm)");
ylabel("theta (deg)");
legend;
grid on;

% Plot and Calculate Shear Center of Specimen 5
c_5 = polyfit(double(separateUnits(x_5)), double(separateUnits(aot_5)), 1);
lobf_x_5 = linspace(double(separateUnits(x_5(2))), ...
    double(separateUnits(x_5(end))), ...
    1000); % [cm]
lobf_y_5 = c_5(1) * lobf_x_5 + c_5(2); % [deg]
e_5 = (-c_5(2) / c_5(1))*u.cm; % [cm]

fprintf("e_5 = %g [cm]\n", separateUnits(e_5));

figure(5);
scatter(separateUnits(x_5), separateUnits(aot_5), ...
    "DisplayName", "Measurements");
hold on;
plot(lobf_x_5, lobf_y_5, "DisplayName", "Line of Best Fit");
scatter(separateUnits(e_5), 0, "filled", "DisplayName", "Shear Center");
hold off;
title("Angle of Twist vs. Distance from Reference Center");
xlabel("x (cm)");
ylabel("theta (deg)");
legend;
grid on;

% Calculate moments of inertia and theoretical shear centers
fprintf("\nTheoretical Shear Centers:\n")
[I_1, I_2, e_1_th, e_2_th, e_3_th, e_4_th, e_5_th] = calc_shear_centers;
I_1 = unitConvert(I_1, u.m^4); % [m^4]
I_2 = unitConvert(I_2, u.m^4); % [m^4]
e_1_th = unitConvert(e_1_th, u.cm); % [cm]
e_2_th = unitConvert(e_2_th, u.cm); % [cm]
e_3_th = unitConvert(e_3_th, u.cm); % [cm]
e_4_th = unitConvert(e_4_th, u.cm); % [cm]
e_5_th = unitConvert(e_5_th, u.cm); % [cm]

% Calculate relative errors
err = @(meas, theor) abs((theor - meas) / theor) * 100; % [%]
e_1_err = err(e_1, e_1_th);
e_2_err = err(e_2, e_2_th);
e_3_err = err(e_3, e_3_th);
e_4_err = err(e_4, e_4_th);
e_5_err = err(e_5, e_5_th);

fprintf("\nRelative Errors:\n" + ...
    "e_1_err = %g%\n" + ...
    "e_2_err = %g%\n" + ...
    "e_3_err = %g%\n" + ...
    "e_4_err = %g%\n" + ...
    "e_5_err = %g%\n", ...
    [e_1_err, e_2_err, e_3_err, e_4_err, e_5_err]);

```

% General Equations of Shear Flow

```
tau_12 = @(P, h, I, s_1) P * h * s_1 / (2 * I); % [Pa]
tau_24 = @(P, h, I, b, s_2) tau_12(P, h, I, b) ...
    - P / (2 * I) * (s_2^2 - h * s_2); % [Pa]
tau_45 = @(P, h, I, b, s_4) tau_24(P, h, I, b, h) ...
    - P * h * s_4 / (2 * I); % [Pa]
```

% Calculate shear flow in specimen 1

```
fprintf("\nShear Flow Calculations for Specimen 1:\n")
P_1 = unitConvert(100*u.g, u.kg) * (9.81*u.m/u.s^2); % [N]
h_1 = unitConvert(2.43*u.in, u.m); % [m]
b_1 = unitConvert(1.456*u.in, u.m); % [m]
```

```
syms s;
```

```
tau_12_1 = unitConvert(tau_12(P_1, h_1, I_1, s), u.kPa); % [kPa]
tau_24_1 = unitConvert(tau_24(P_1, h_1, I_1, b_1, s), u.kPa); % [kPa]
tau_45_1 = unitConvert(tau_45(P_1, h_1, I_1, b_1, s), u.kPa); % [kPa]
tau_1_1 = unitConvert(subs(tau_12_1, s, 0), u.kPa); % [kPa]
tau_2_1 = unitConvert(subs(tau_12_1, s, b_1), u.kPa); % [kPa]
tau_3_1 = unitConvert(subs(tau_24_1, s, h_1 / 2), u.kPa); % [kPa]
tau_4_1 = unitConvert(subs(tau_24_1, s, h_1), u.kPa); % [kPa]
tau_5_1 = unitConvert(subs(tau_45_1, s, b_1), u.kPa); % [kPa]
```

```
fprintf("tau_12_1 = "); disp(vpa(tau_12_1, 4));
fprintf("tau_24_1 = "); disp(vpa(tau_24_1, 4));
fprintf("tau_45_1 = "); disp(vpa(tau_45_1, 4));
fprintf("tau_1_1 = "); disp(vpa(tau_1_1, 4));
fprintf("tau_2_1 = "); disp(vpa(tau_2_1, 4));
fprintf("tau_3_1 = "); disp(vpa(tau_3_1, 4));
fprintf("tau_4_1 = "); disp(vpa(tau_4_1, 4));
fprintf("tau_5_1 = "); disp(vpa(tau_5_1, 4));
```

% Calculate shear flow in specimen 2

```
fprintf("Shear Flow Calculations for Specimen 2:\n")
P_2 = unitConvert(200*u.g, u.kg) * (9.81*u.m/u.s^2); % [N]
h_2 = unitConvert(0.84*u.in, u.m); % [m]
b_2 = unitConvert(0.56*u.in, u.m); % [m]
```

```
syms s;
```

```
tau_12_2 = unitConvert(tau_12(P_2, h_2, I_2, s), u.kPa); % [kPa]
tau_24_2 = unitConvert(tau_24(P_2, h_2, I_2, b_2, s), u.kPa); % [kPa]
tau_45_2 = unitConvert(tau_45(P_2, h_2, I_2, b_2, s), u.kPa); % [kPa]
tau_1_2 = unitConvert(subs(tau_12_2, s, 0), u.kPa); % [kPa]
tau_2_2 = unitConvert(subs(tau_12_2, s, b_2), u.kPa); % [kPa]
tau_3_2 = unitConvert(subs(tau_24_2, s, h_2 / 2), u.kPa); % [kPa]
tau_4_2 = unitConvert(subs(tau_24_2, s, h_2), u.kPa); % [kPa]
tau_5_2 = unitConvert(subs(tau_45_2, s, b_2), u.kPa); % [kPa]
```

```
fprintf("tau_12_2 = "); disp(vpa(tau_12_2, 4));
fprintf("tau_24_2 = "); disp(vpa(tau_24_2, 4));
fprintf("tau_45_2 = "); disp(vpa(tau_45_2, 4));
fprintf("tau_1_2 = "); disp(vpa(tau_1_2, 4));
fprintf("tau_2_2 = "); disp(vpa(tau_2_2, 4));
fprintf("tau_3_2 = "); disp(vpa(tau_3_2, 4));
fprintf("tau_4_2 = "); disp(vpa(tau_4_2, 4));
fprintf("tau_5_2 = "); disp(vpa(tau_5_2, 4));
```

Measured Shear Centers:

e_1 = -0.799851 [cm]
e_2 = -0.322254 [cm]
e_3 = 0.797676 [cm]
e_4 = -3.76638 [cm]
e_5 = -3.52081 [cm]

Theoretical Shear Centers:

I_1 = 0.430545 [inch^4] = 17.9206 [cm^4]
I_2 = 0.0130989 [inch^4] = 0.545217 [cm^4]
e_1 = -0.581496 [inch] = -1.477 [cm]
e_2 = -0.232275 [inch] = -0.589979 [cm]
e_3 = -1.58843 [inch] = -4.03461 [cm]
e_4 = -1.52533 [inch] = -3.87433 [cm]
e_5 = -1.26541 [inch] = -3.21415 [cm]

Relative Errors:

e_1_err = 45.8463%
e_2_err = 45.3787%
e_3_err = 119.771%
e_4_err = 2.78628%
e_5_err = 9.54079%

Shear Flow Calculations for Specimen 1:

tau_12_1 = 168.9*s*([kPa]/[m])

tau_24_1 = 6.248*[kPa] + 2737.0*(- 1.0*s^2 + s*0.06172*[m])*([kPa]/[m]^2)

tau_45_1 = 6.248*[kPa] + (-168.9*s)*([kPa]/[m])

tau_1_1 = 0.0

tau_2_1 = 6.248*[kPa]

tau_3_1 = 8.854*[kPa]

tau_4_1 = 6.248*[kPa]

tau_5_1 = 0.0

Shear Flow Calculations for Specimen 2:

tau_12_2 = 3839.0*s*([kPa]/[m])

tau_24_2 = 54.61*[kPa] + 1.799e+5*(- 1.0*s^2 + s*0.02134*[m])*([kPa]/[m]^2)

tau_45_2 = 54.61*[kPa] + (-3839.0*s)*([kPa]/[m])

tau_1_2 = 0.0

tau_2_2 = 54.61*[kPa]

tau_3_2 = 75.08*[kPa]

tau_4_2 = 54.61*[kPa]

tau_5_2 = 0.0

>>