IOWA STATE UNIVERSITY

Aer E 322: Aerospace Structures Laboratory

Week 10 Lecture:

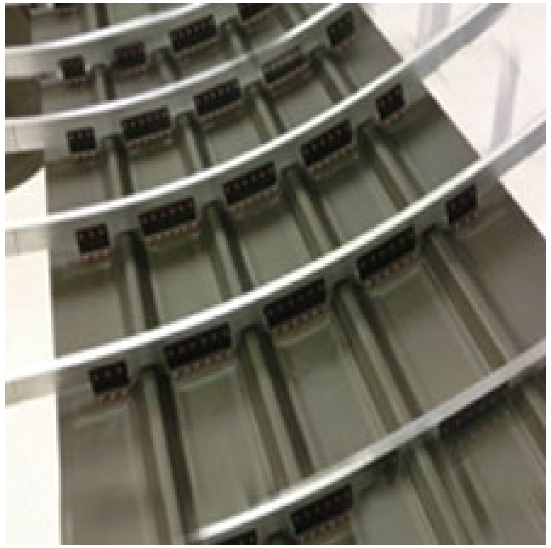
Thin-Walled Structures and Shear Center

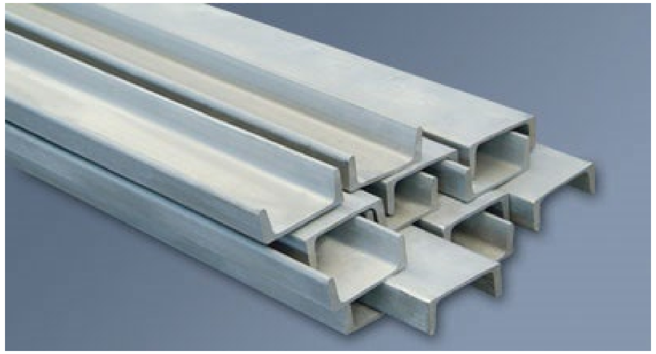
March 28, 2022

Thin-Walled Structures

- Thin-walled structural sections, consisting of closed tubes, open channels and their numerous variations, are subjected to transverse bending as well as compressive axial loads (column buckling, remember?)
- Thin-walled structures are essential to aeronautic and aerospace vehicles

Source: www.lookchem.com





Source: www.wxxnjs.com

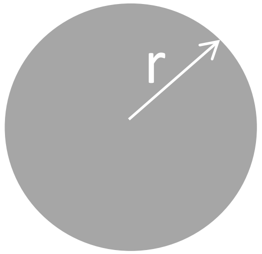
Thin-Walled Structures (cont'd)

Thin-walled sections are very cost-effective: light weight, space saver and equally strong.

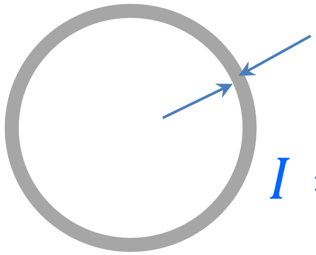
Consider, e.g. column buckling: $P_{cr} = \frac{\pi^2 EI}{L^2}$

solid

$I = \frac{\pi}{4} r^3 \cdot \mathrm{r}$



hollow, closed



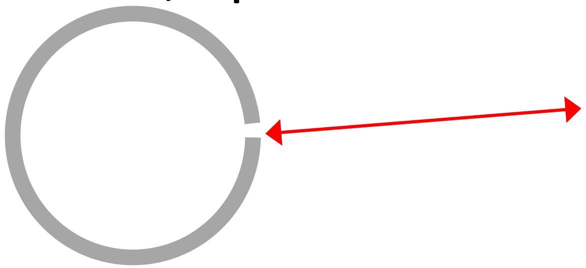
t, t<<r

$I \approx \frac{\pi}{4} r^3 \cdot 4\mathrm{t}$

Some drop of P_{cr} (due to drop on I) but big saving of material (and weight) !

hollow, opened

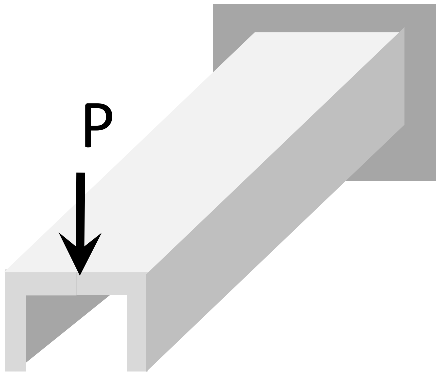
BUT ...



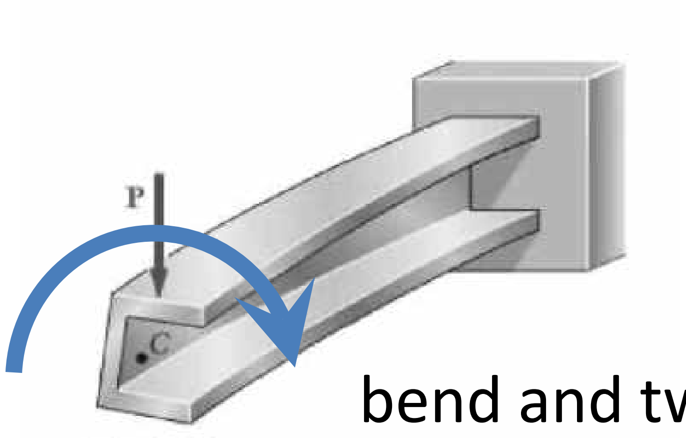
No free lunch: not as strong to torsion and bending – **once opened** !!!

Unsymmetric Loading - Shear Center

- When subjected to unsymmetric loading, thin-walled section not only bends but also twists



bend

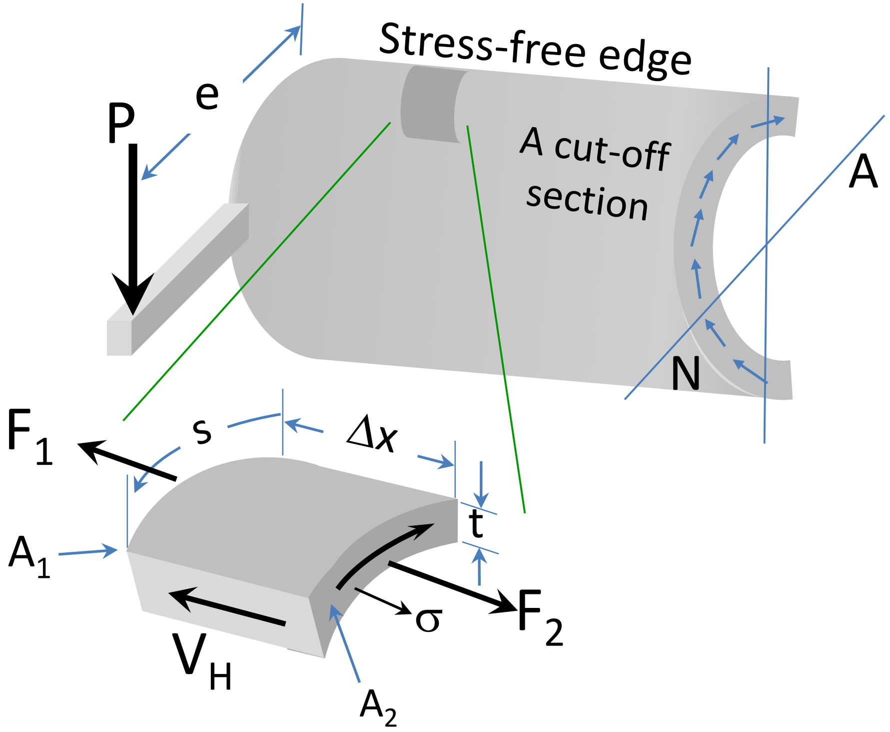


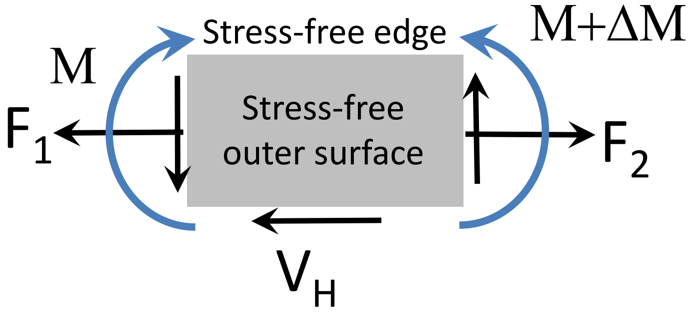
bend and twist

- Q: Where to apply the loading on unsymmetric thin-walled section so that it only bends and does not also twist?

→ Shear center

Stress Analysis in Thin-Walled Open Section





top view

For the small patch taken from the cut-off section of the loaded cantilever beam, normal stress σ acting on cross section area A_1 is given by

$$\sigma = \frac{My}{I}$$

Then the resultant force F_1 is

s measured from right edge of the A_1 area (more about this later)

$$F_1 = \int_{A_1} \sigma dA = \int_{A_1} \frac{My}{I} dA = \int_0^s \frac{My}{I} t ds$$

Similarly the resultant force F_2 is

$$F_2 = \int_0^s \frac{(M + \Delta M)y}{I} t ds$$

Stress Analysis in Thin-Walled Open Section (cont'd)

In equilibrium $V_H = F_2 - F_1 = \dfrac{\Delta M}{I}\int_0^s y(tds)$

By definition, the average shearing stress is $\tau = \lim_{\Delta x \to 0} \dfrac{V_H}{t\Delta x} = \dfrac{dM}{dx}\dfrac{1}{It}\int_0^s y(tds)$

or $\tau = \dfrac{VQ}{It}$

where $V = \dfrac{dM}{dx}$ is the shear force at the beam section,

and $Q = \int_0^s y(tds)$ is the first moment w.r.t. neutral axis of the cross section area

Shear Flow

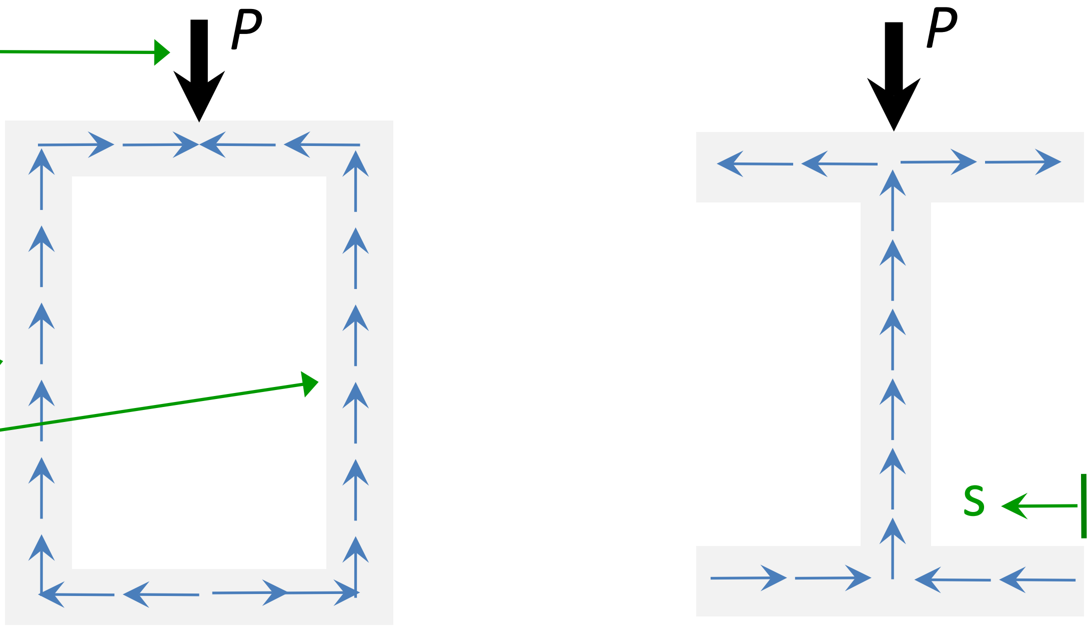
- A useful concept for analyzing thin-walled section, shear flow ***q***, is defined as *the internal shearing force per unit length of the thin section*

$$q = \tau t = \frac{VQ}{I} \qquad Q = \int_0^s y(t\,ds) \approx \underbrace{t\int_0^s y\,ds}$$

for thin wall sections, wall thickness *t* can be taken as constant

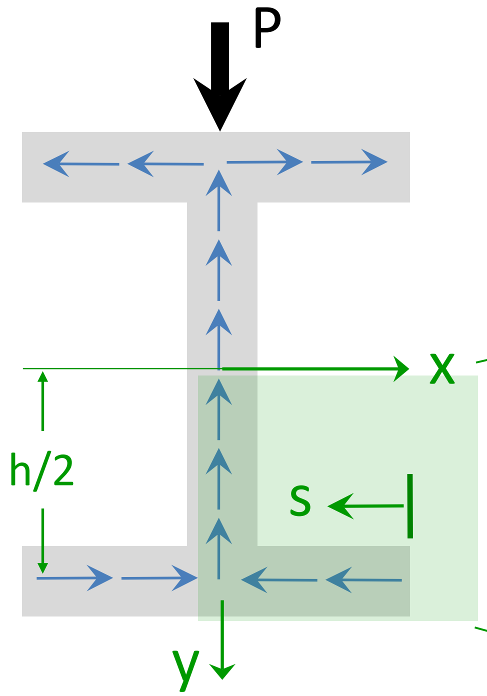
External load *P* applied at the free end (see pages 4 and 5)

Shear flow on the cut-off cross section (as shown on page 5) opposing the external load *P*

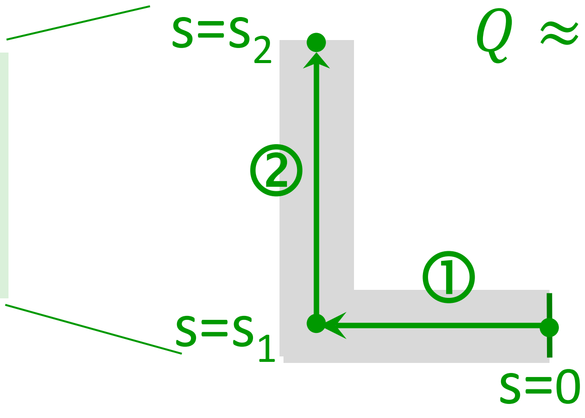


Evaluating Q integral

The Q integral is known as a *line integral* mathematically. The integration follows an arbitrary path defined by the variable ***s***. The integrand ***y*** is a function of ***s*** varying with the location of ***s***



$$Q = \int_0^s y(t\,ds) \approx t\int_0^s y\,ds$$



$$Q \approx \overbrace{t\int_{s=0}^{s=s_1}\left(\frac{h}{2}\right)ds}^{①} + \overbrace{t\int_{s=s_1}^{s=s_2}\left(\frac{h}{2}-s-s_1\right)ds}^{②}$$

$$\parallel \qquad s' = s - s_1$$

$$t\int_{s'=0}^{s'=s_2-s_1=\frac{h}{2}}\left(\frac{h}{2}-s'\right)ds'$$

Determining Shear Center

For simple geometry, the shear center can be found by summing up moments from individual parts as shown

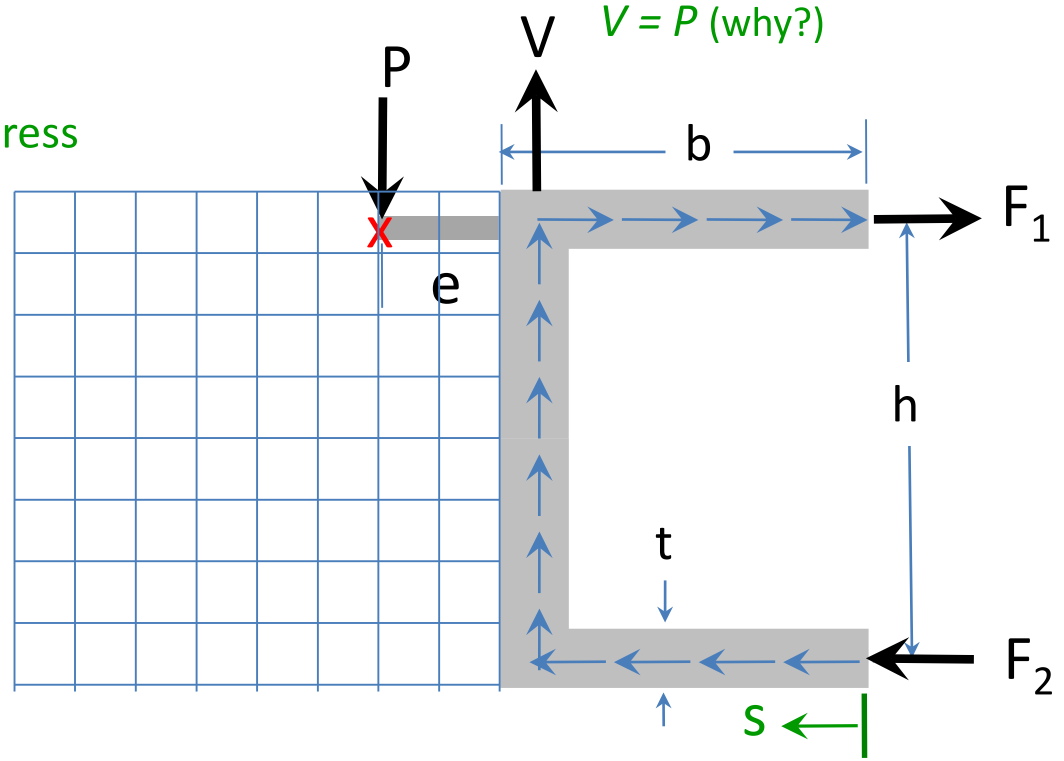
$$\tau = \frac{V}{It}\int_0^s y(tds)$$

$$\tau_{flange} = \frac{V}{It}\int_0^s \frac{h}{2}(tds) = \frac{Vhs}{2I}$$

what shape of stress distribution ?

$$\text{average } \tau_{flange} = \frac{1}{2}\left[\frac{Vhs}{2I}(s=b) - \frac{Vhs}{2I}(s=0)\right]$$

$$F_2 = (\text{avg } \tau_{flange})(area) = \frac{1}{2}\frac{Vhb}{2I}bt = \frac{Vhb^2t}{4I}$$



$$Ve = F_2h \quad \Rightarrow \quad e = \frac{h^2b^2t}{4I}$$

why?

Question: (1) what properties shear center depends on and (2) where exactly is the shear center here?

Determining Shear Center (cont'd)

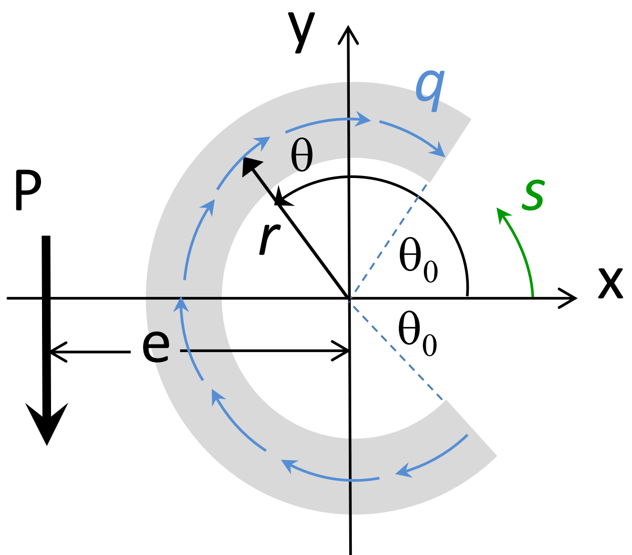
For more elaborate section, we need a little calculus to help out

$$q = \tau t = \frac{VQ}{I}$$

$$Q(\theta; r, t) = \overbrace{\int_0^s y(tds) = \int_{\theta_0}^{\theta} r\sin\theta trd\theta}^{\text{Converted to polar coordinates}} = tr^2 \int_{\theta_0}^{\theta} \sin\theta d\theta$$

Full length of the C section

$$I = I_x(r,t) = \int_0^c y^2(tds) = tr^3 \int_{\theta_0}^{2\pi-\theta_0} \sin^2\theta d\theta$$



r: mean radius

Let $\underline{q} = (\sin\theta\hat{i} - \cos\theta\hat{j})\frac{VQ}{I}$, then the resultant shear force is

$$\underline{V} = \int \underline{q}ds = \int_{\theta_0}^{2\pi-\theta_0} \underline{q}rd\theta$$

Determining Shear Center (cont'd)

And resultant moment about the section center (x,y)=(0,0) can be evaluated:

$$\underline{M} = M\hat{k} = \underline{r} \times \underline{V} = \int \underline{r} \times \underline{q} ds$$

$$= \int_{\theta_0}^{2\pi-\theta_0} (r\cos\theta\hat{i} + r\sin\theta\hat{j}) \times (\sin\theta\hat{i} - \cos\theta\hat{j}) \frac{VQ}{I} r d\theta$$

Finally

$$e = \overbrace{\left|\frac{\underline{M}}{\underline{V}}\right|}^{\text{Why?}} = \frac{2r\left[\cos\theta_0(2\pi - 2\theta_0) + 2\sin\theta_0\right]}{2\pi - 2\theta_0 + \sin 2\theta_0}$$

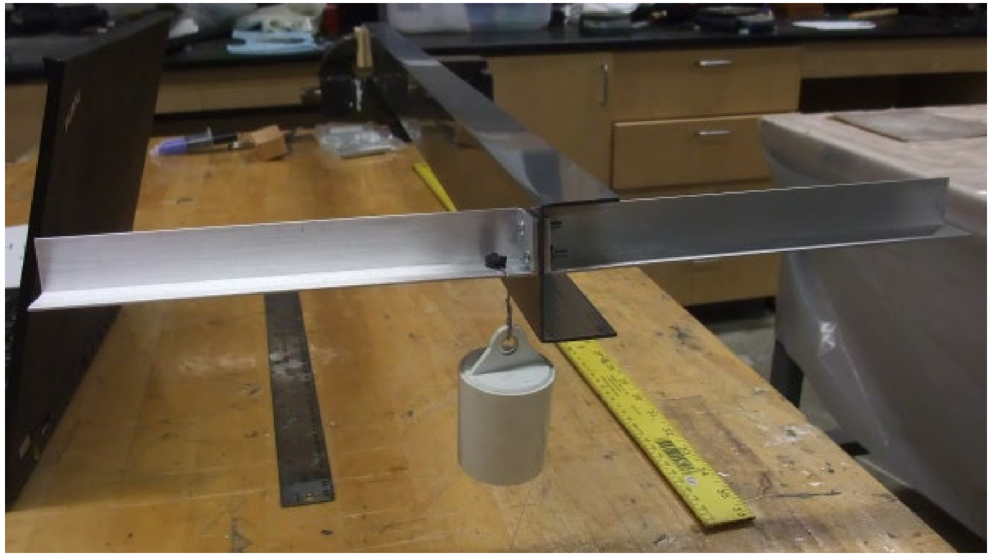
$|V| = P$

Can you work out the details?

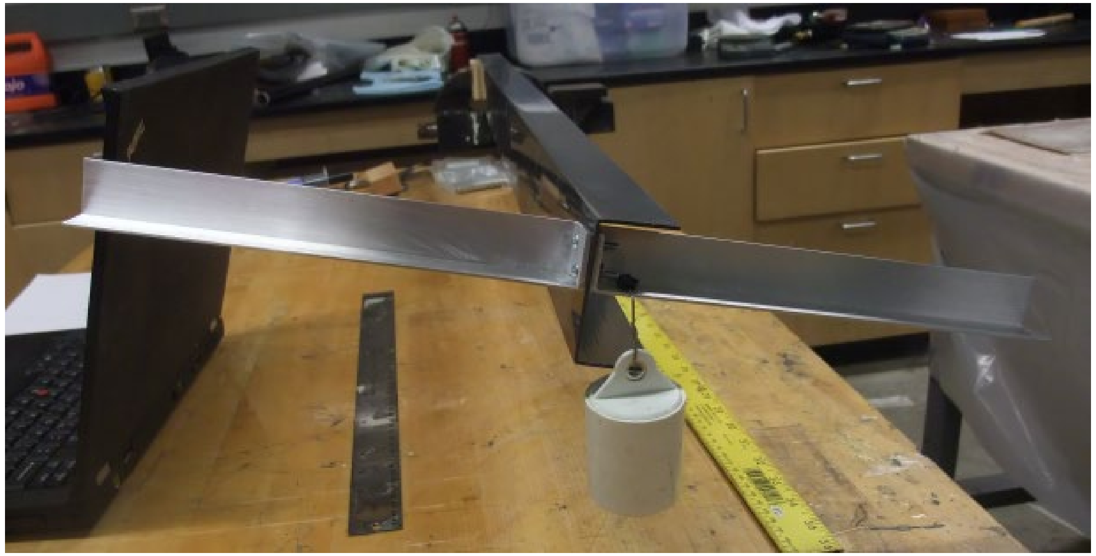
Tips: (1) you may need double angle formulas;
(2) be aware of the fact that odd function integrated over symmetric limits is zero;
(3) be sure use radian, not degree

About the Experiment

- In Lab 8, you will experimentally determine the shear center of similar channel sections



Weight right at shear center; no twist

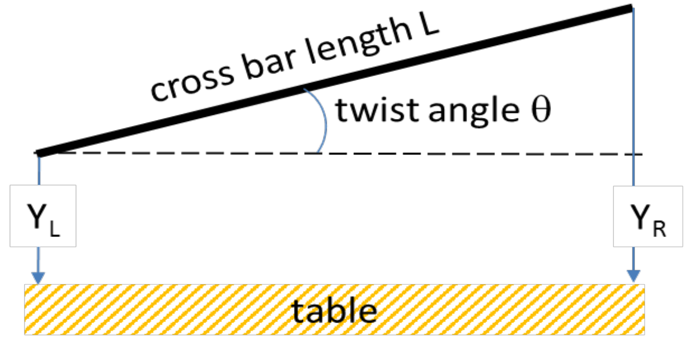


Weight not at shear center; cross bar twists

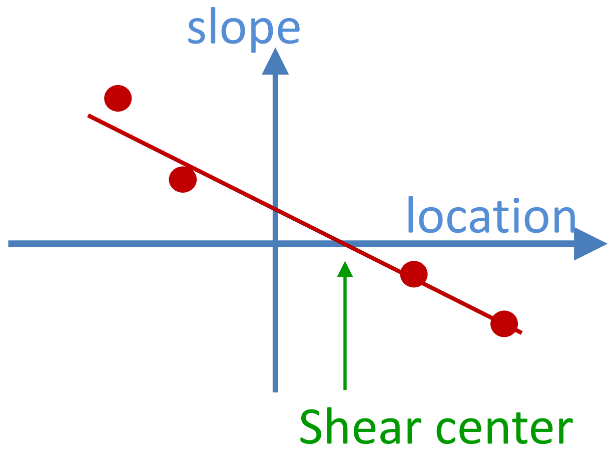
About the Experiment (cont'd)

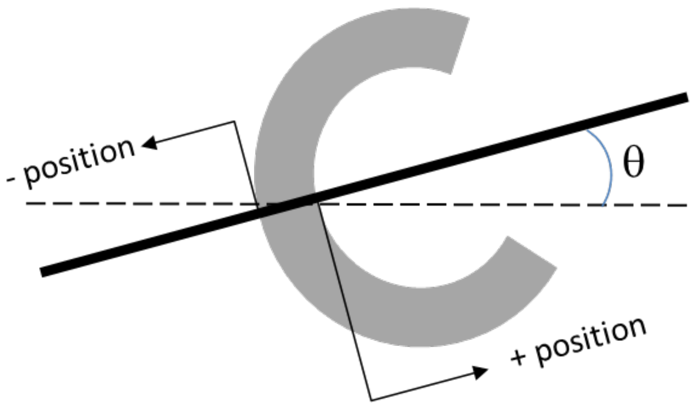
- By measuring the height difference of the two ends of cross bar, you can determine slope (or twist angle) of the cross bar at various locations of the weight (left figure below). You then fit a line through these data points: the zero crossing (i.e. zero slope) location is the shear center (middle figure below)
- To get around of the blockage of the section wall, you will need to use different references for the weight location (right figure below)

See lab assignment for details



$$\theta = sin^{-1}\left(\frac{Y_R - Y_L}{L}\right) \approx tan^{-1}\left(\frac{Y_R - Y_L}{L}\right)$$





Main References

1. Peery Ch. 6
2. A. Higdon et al., *Mechanics of Materials*, 4th ed. 1985, Chs. 5&6
3. F. Beer et al., *Mechanics of Materials*, 5th ed. 2009, Ch. 6